Task 5.

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Given $f(x) = \sqrt[3]{x}$. use 5 d.p.

a). Find Taylor series for $f(x)$ at $x = 8$ (4 Non-zero Terms).

b). What is the maximum error occured?

c). Approximate $\sqrt[3]{8.1}$ using part (a).

d). what is the actual error? compare (b) and (c).

a). $f(x) = \sqrt[3]{x}$

$f(x) : f(8) + \frac{f'(8)}{1!}(x-8) + \frac{f''(8)}{2!}(x-8)^2 + \frac{f'''(8)}{3!}(x-8)^3 + \dots$

▶ $x = 8$

$\therefore x^{\frac{1}{3}}$

$: 2 + \frac{0.08333}{1!}(x-8)^1 + \frac{-0.00694}{2!}(x-8)^2 + \frac{0.00144}{3!}(x-8)^3 + \frac{-0.00048}{4!}(x-8)^4$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \rightarrow 0.08333$

$f''(x) = -\frac{2}{9}x^{-\frac{5}{3}} \rightarrow -0.00694$

$f'''(x) = \frac{10}{27}x^{-\frac{8}{3}} \rightarrow 0.00144$

$f^{(4)}(x) = -\frac{80}{81}x^{-\frac{11}{3}} \rightarrow -0.00048$

$f(8) : 2$

$\Rightarrow f(x) : 2 + 0.08333(x-8)^1 - 0.00347(x-8)^2 + 0.00024(x-8)^3 + \dots$

~~$- 0.00002(x-8)^4$~~

b). $|E_n(x)| \leq \frac{M}{(n+1)!} \cdot (x-a)^{n+1}$ $\rightarrow$ $|E_n(x)| \leq \frac{0.00048}{(3+1)!} \times (8.1-8)^{3+1} = 0.000000002$ or 2×10^{-9}

$M : |f^{(n+1)}(x)| = |f^{(4)}(x)|$

$: \left|-\frac{80}{81}x^{-\frac{11}{3}}\right|$

$: \left|-\frac{80}{81}(8)^{-\frac{11}{3}}\right|$

$: 0.00048$

c). $f(8.1) = 2 + 0.08333(x-8)^1 - 0.00347(x-8)^2 + 0.00024(x-8)^3 + \dots$

$= 2 + 0.08333(8.1-8) - 0.00347(8.1-8)^2 + 0.00024(8.1-8)^3 + \dots$

$= 2.00830$

d). $\sqrt[3]{8.1} = 2.008299$

≈ 2.00830

Actual ~~Error~~ : $2.00830 - 2.00830 : 0$

$\therefore$ Result of Taylor's series in inaccurate since 0 is < 0.000000002 or 2×10^{-9}