

Task 11

Question

$y' = y + e^x$, $y(0) = 2$, $h = 0.1$, 4 d.p., find $y(0.2)$

1.) RK-1

2.) RK-2

3.) RK-4

4.) Conclusion of 1, 2, 3 if actual value $y(0.2) = 2.6871$

1.) RK-1

| n | x | y |
|---|-------------------|---|
| 0 | 0 | 2 |
| 1 | $0 + 0.1 = 0.1$ | $2 + 0.1 F(0.1, 2) = 2 + 0.1(2 + e^0) = 2.3$ |
| 2 | $0.1 + 0.1 = 0.2$ | $2.3 + 0.1 F(0.2, 2.3) = 2.3 + 0.1(2.3 + e^{0.1}) = 2.6405$ |

RK-1 $\rightarrow y(0.2) = 2.6405 //$

2.) RK-2

1st iteration

$$K_0 = 0.1 F(0, 2) = 0.1 [2 + e^0] = 0.3$$

$$K_1 = 0.1 F\left(0 + \frac{1}{2}(0.1), 2 + \frac{1}{2}(0.3)\right) = 0.1 F(0.05, 2.15) \\ = 0.1 (2.15 + e^{0.05}) = 0.3201271096 \approx 0.3201$$

$$x = 0 + 0.1 = 0.1$$

$$y(0.1) = 2 + 0.3201 = 2.3201$$

2nd iteration

$$K_0 = 0.1 F(0.1, 2.3201) = 0.1 [2.3201 + e^{0.1}] = 0.3425270918 \approx 0.3425$$

$$K_1 = 0.1 F\left(0.1 + \frac{1}{2}(0.1), 2.3201 + \frac{1}{2}(0.3425)\right) = 0.1 F(0.15, 2.4914) \\ = 0.1 (2.4914 + e^{0.15}) = 0.3653239293 \approx 0.3653$$

$$x = 0.1 + 0.1 = 0.2$$

$$y(0.2) = 2.3201 + 0.3653 = 2.6854$$

RK-2 $\rightarrow y(0.2) = 2.6854 //$

$$x = 0 \quad y = 2$$

3.) RK-4

1st iteration:

$$K_0 = 0.1 \cdot [2 + e^0] = 0.3$$

$$K_1 = 0.1 \cdot F\left[0 + \frac{1}{2}(0.1), 2 + \frac{1}{2}(0.3)\right] = 0.1 \cdot F(0.05, 2.15) \\ = 0.1 \cdot [2.15 + e^{0.05}] = 0.3201$$

$$K_2 = 0.1 \cdot F\left[0 + \frac{1}{2}(0.1), 2 + \frac{1}{2}(0.3201)\right] = 0.1 \cdot F(0.05, 2.1601) \\ = 0.1 \cdot [2.1601 + e^{0.05}] = 0.3211$$

$$K_3 = 0.1 \cdot F[0 + 0.1, 2 + (0.3211)] = 0.1 \cdot F(0.1, 2.3211) \\ = 0.1(2.3211 + e^{0.1}) = 0.3427 \quad (0.325770716)$$

$$x = 0 + 0.1 = 0.1$$

$$y(0.1) = 2 + \frac{1}{6}(0.3 + 2(0.3201) + 2(0.3211) + 0.3427) \\ = 2.3209$$

2nd iteration:

$$K_0 = 0.1 \cdot F(x, y) = 0.1 [2.3209 + e^{0.1}] = 0.3426$$

$$K_1 = 0.1 \cdot F\left(0.1 + \frac{1}{2}(0.1), 2.3209 + \frac{1}{2}(0.3426)\right) = 0.1 \cdot F(0.15, 2.4922) \\ = 0.1(2.4922 + e^{0.15}) = 0.3654$$

$$K_2 = 0.1 \cdot F\left(0.1 + \frac{1}{2}(0.1), 2.3209 + \frac{1}{2}(0.3654)\right) = 0.1 \cdot F(0.15, 2.5036) \\ = 0.1(2.5036 + e^{0.15}) = 0.3665$$

$$K_3 = 0.1 \cdot F(0.1 + 0.1, 2.3209 + (0.3665)) = 0.1 \cdot F(0.2, 2.6874) \\ = 0.1(2.6874 + e^{0.2}) = 0.3909$$

$$x = 0.1 + 0.1 = 0.2$$

$$y(0.2) = 2.3209 + \frac{1}{6}(0.3426 + 2(0.3654) + 2(0.3665) + 0.3909)$$

$$= 2.687116667 \approx 2.6871$$

(since the values above are rounded
up, this is not = to the exact
value)

$$\boxed{\text{RK-4} \rightarrow \text{Approx } y(0.2) = 2.6871}$$

4.) In conclusion, we could say that the higher the amount of RK method used, the closer the value is to the exact value. RK-1 (3.6405), RK-2 (2.6854), RK-4 (2.6871)