Task 10

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1.) Use Gauss-Legendre 4d.p, 3 points for $\int_{0.5}^{3.5} x\sqrt{(16-x^2)^3}\,dx$

$a = 0.5 \quad b = 3.5 \quad f(x) = x\sqrt{(16-x^2)^3}$

$$\frac{3.5-0.5}{2}\int_{-1}^{1} f\left(\frac{3.5-0.5}{2}x + \frac{3.5+0.5}{2}\right)dx$$

$$= \frac{3}{2}\int_{-1}^{1} f\left(\frac{3}{2}x+2\right)dx$$

$$= \frac{3}{2}\int_{-1}^{1}\left(\frac{3}{2}x+2\right)\cdot\sqrt{\left(16-\left(\frac{3}{2}x+2\right)^2\right)^3}\,dx$$

$$\to f(0) = \frac{3}{2}\left(\frac{3}{2}(0)+2\right)\cdot\sqrt{\left(16-\left(\frac{3}{2}(0)+2\right)^2\right)^3}\,dx$$

$$= 124.7077$$

$$I = 0.888889\cdot f(0) + 0.555556\cdot f(0.774597) + 0.555556\cdot f(-0.774597)$$

$$= 0.888889(124.7077) + 0.555556(69.7474) + 0.55556(75.2183)$$

$$= 191.3879$$

2.) Gauss-Chebyshev 4d.p, 2 points $\int_{-1}^{1}(1-x)^{1.5}\cdot\cos x\,dx$

$$\hookrightarrow \int_{-1}^{1}(1-x^2)^{3/2}\cos x\,dx$$

$$= \int_{-1}^{1}\left(\sqrt{1-x^2}\right)^3\cos x\,dx$$

$$= \int_{-1}^{1}\frac{\left(\sqrt{1-x^2}\right)^4}{\sqrt{1-x^2}}\cos x\,dx$$

$$\to f(x) = \left(\sqrt{1-x^2}\right)^4\cos x$$

$$I = \frac{\pi}{n}\sum_{k=1}^{n} f\left(\cos\left(\frac{2k-1}{2n}\pi\right)\right)$$

$$= \frac{\pi}{1}\sum_{k=1}^{1} f\left(\cos\left(\frac{2(1)-1}{2(1)}\pi\right)\right)$$

$$= \pi + \left(\cos\left(\frac{1}{2}\pi\right)\right)$$

$$= \pi + 4(0)$$

$$= \pi\left(\sqrt{1-0^2}\right)^4\cos 0$$

$$= \pi(1) = \pi \text{ or } \boxed{3.141}$$

4 d.p

3.) Gauss-Laguerre, 4 d.p, 3 points $\int_{0}^{\infty} e^{-x}\cdot x^2\,dx$

$f(x) = x^2$

$$I = 0.711093\,f(0.415775) + 0.278517\,f(2.294280) + 0.0103892\,f(6.289945)$$

$$= 0.711093(0.1729) + 0.278517(5.2637) + 0.0103892(39.5634)$$

$$= 2$$