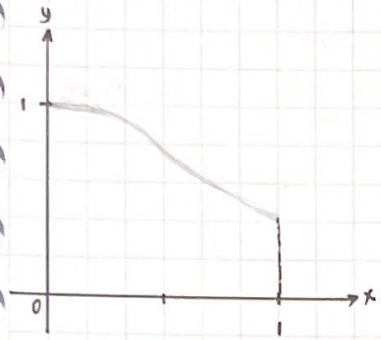
Task 9

2602067874 - Hansel Adlitia Harrono
2602067719 - Ignatius Kennard

$f(x) = e^{-x^2}$, $0 \le x \le 1$



a). composite trapezoidal 2 and 4 panels = 0.7470

$n = 2 \rightarrow h = \frac{1-0}{2} = \frac{1}{2}$

$$I = \frac{1/2}{2}\left[1e^{-(0)^2} + 2e^{-(1/2)^2} + 1e^{-(1)^2}\right]$$

$$= \frac{1}{4}\left[e^0 + 2e^{-1/4} + e^{-1}\right]$$

$$= \frac{1}{4}\left[1 + 2(0.7788) + 0.3679\right]$$

$$= \frac{1}{4}\left[2.9255\right]$$

$$= 0.7314$$

$n = 4 \rightarrow h = \frac{1-0}{4} = \frac{1}{4}$

$$I = \frac{1/4}{2}\left[1e^{-(0)^2} + 2e^{-(1/4)^2} + 2e^{-(2/4)^2} + 2e^{-(3/4)^2} + 1e^{-(1)^2}\right]$$

$$= \frac{1}{8}\left[e^0 + 2e^{-1/16} + 2e^{-4/16} + 2e^{-9/16} + e^{-1}\right]$$

$$= \frac{1}{8}\left[1 + 2(0.9394) + 2(0.7788) + 2(0.5698) + 0.3679\right]$$

$$= \frac{1}{8}\left[5.9439\right]$$

$$= 0.7430$$

b). composite Simpson's 1/3, 4 panels.

$n = 4 \rightarrow h = \frac{1-0}{4} = \frac{1}{4}$

$$I = \frac{1/4}{3}\left[1e^{-0^2} + 4e^{-(1/4)^2} + 2e^{-(2/4)^2} + 4e^{-(3/4)^2} + 1e^{-(1)^2}\right]$$

$$= \frac{1/4}{3}\left[8.9623\right]$$

$$= 0.7469$$

c). composite Simpson's 3/8, 3 panels

$n = 3 \rightarrow \frac{1-0}{3} = \frac{1}{3}$

$$I = \frac{3(1/3)}{8}\left[1e^{-(0)^2} + 3e^{-(1/3)^2} + 3e^{-(2/3)^2} + 1e^{-(1)^2}\right]$$

$$= \frac{1}{8}\left[e^0 + 3e^{-1/9} + 3e^{-4/9} + e^{-1}\right]$$

$$= \frac{1}{8}\left[1 + 3(0.8948) + 3(0.6412) + (0.3679)\right]$$

$$= \frac{1}{8}\left(\text{A}5.9759\right)$$

d). Romberg Integration

$n = 2 \rightarrow 0.7314$

$n = 4 \rightarrow 0.7430$

$$\frac{4(0.7430) - 0.7314}{4-1}$$

Romberg's Integration = 0.7469