Task 1

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 9x_2 + 4x_3 = 9 \\ 8x_1 + x_2 + 2x_3 = 15 \end{cases}$$

a). Gaussian Elimination

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 9 & 4 & 9 \\ 8 & 1 & 2 & 15 \end{array}\right] \xrightarrow[r_3 - 8r_1]{r_2 - 2r_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 5 & -2 & 7 \\ 0 & -15 & -22 & 7 \end{array}\right]$$

$$\xrightarrow{r_2/5} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & -15 & -22 & 7 \end{array}\right] \xrightarrow[r_3 + 15r_2]{r_1 - 2r_2} \left[\begin{array}{ccc|c} 1 & 0 & 3.8 & -1.8 \\ 0 & 1 & -0.4 & 1.4 \\ 0 & 0 & -[illegible] & [illegible] \end{array}\right]$$

$$\xrightarrow{r_3/-28} \left[\begin{array}{ccc|c} 1 & 0 & 3.8 & -1.8 \\ 0 & 1 & -0.4 & 1.4 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{r_1 - 3.8r_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -0.4 & 1.4 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{r_2 + 0.4r_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\therefore \begin{cases} x_1 = 2 \\ x_2 = 1 \\ x_3 = -1 \end{cases}$$

b). LU Decomposition

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 4 \\ 8 & 1 & 2 \end{bmatrix}$$

$$r_2 = r_2 - 2r_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ 8 & 1 & 2 \end{bmatrix} \quad r_3 = r_3 - 8r_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ 0 & -15 & -22 \end{bmatrix}$$

$$r_3 = r_3 + 3r_2 \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ 0 & 0 & -28 \end{bmatrix}$$

$$L: \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 8 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 15 \end{bmatrix} \quad \begin{aligned} &y_1 = 1 \ldots ① \\ &2y_1 + y_2 = 9 \ldots ② \\ &8y_1 + (-3)y_2 + y_3 = 15 \ldots ③ \end{aligned}$$

$$U: \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -2 \\ 0 & 0 & -28 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 28 \end{bmatrix} \quad \begin{aligned} &x_1 + 2x_2 + 3x_3 = 1 \ldots ③^+ \\ &5x_2 - 2x_3 = 7 \ldots ②^+ \\ &-28x_3 = 28 \ldots ①^+ \end{aligned}$$

① $y_1 = 1$

② $2(1) + y_2 = 9$

$2 + y_2 = 9$

$y_2 = 7$

③ $8(1) + (-3)(7) + y_3 = 15$

$8 - 21 + y_3 = 15$

$y_3 = 15 + 21 - 8$

$y_3 = 28$

①$^+$ $-28x_3 = 28$

$x_3 = -1$

②$^+$ $5x_2 - 2(-1) = 7$

$5x_2 + 2 = 7$

$5x_2 = 5$

$x_2 = 1$

③$^+$ $x_1 + 2(1) + 3(-1) = 1$

$x_1 + 2 - 3 = 1$

$x_1 - 1 = 1$

$x_1 = 2$

$\therefore \{(2, 1, -1)\}$

c). NO.

The coefficient of the system is not diagonally dominant. In this case, the diagonal elements are 1, 9, 2. However, the sum of the absolute value of the other elements in each row is greater than the absolute value of the diagonal element in the 2nd and 3rd row

d). Multiply each element by 2/9 in 2nd row and each element by 4 in the 3rd row

$$\begin{bmatrix} 1 & 2 & 3 \\ 4/9 & 5/9 & -8/9 \\ 32/9 & 4/9 & 8/9 \end{bmatrix}$$

∴ However the result will be different with the LU Decomposition results, due to the changes in the coefficients.