

## Exercise

### A Random Binary String

A random binary string contains  $n$  random bits. For  $1 \leq i \leq n$ , the probability that the  $i$ -th random bit is 1 is  $p_i$ ,  $0 \leq p_i \leq 1$ . That is, the probability for the  $i$ -th random bit to be 0 is  $1 - p_i$ . Given  $n$ , another integer  $k$ ,  $2 \leq k \leq n \leq 200$ , and  $p_1, p_2, p_3, \dots, p_n$ , your program should output the probability that the random binary string **does not** contain  $k$  (or more) consecutive 1's. For example, let  $n=4$ ,  $p_1=0.9$ ,  $p_2=0.8$ ,  $p_3=0.7$ ,  $p_4=0.6$ , and  $k=2$ , we have the following table.

Random binary string, $S$	Probability, $S$ occurs	$S$ Contains 2 (or more) consecutive 1's
0000	$0.1 \times 0.2 \times 0.3 \times 0.4 = 0.0024$	
0001	$0.1 \times 0.2 \times 0.3 \times 0.6 = 0.0036$	
0010	$0.1 \times 0.2 \times 0.7 \times 0.4 = 0.0056$	
0011	$0.1 \times 0.2 \times 0.7 \times 0.6 = 0.0084$	Yes
0100	$0.1 \times 0.8 \times 0.3 \times 0.4 = 0.0096$	
0101	$0.1 \times 0.8 \times 0.3 \times 0.6 = 0.0144$	
0110	$0.1 \times 0.8 \times 0.7 \times 0.4 = 0.0224$	Yes
0111	$0.1 \times 0.8 \times 0.7 \times 0.6 = 0.0336$	Yes
1000	$0.9 \times 0.2 \times 0.3 \times 0.4 = 0.0216$	
1001	$0.9 \times 0.2 \times 0.3 \times 0.6 = 0.0324$	
1010	$0.9 \times 0.2 \times 0.7 \times 0.4 = 0.0504$	
1011	$0.9 \times 0.2 \times 0.7 \times 0.6 = 0.0756$	Yes
1100	$0.9 \times 0.8 \times 0.3 \times 0.4 = 0.0864$	Yes
1101	$0.9 \times 0.8 \times 0.3 \times 0.6 = 0.1296$	Yes
1110	$0.9 \times 0.8 \times 0.7 \times 0.4 = 0.2016$	Yes
1111	$0.9 \times 0.8 \times 0.7 \times 0.6 = 0.3024$	Yes

The probability that the random binary string **does not** contain 2 or more consecutive 1's is  $0.0024 + 0.0036 + 0.0056 + 0.0096 + 0.0144 + 0.0216 + 0.0324 + 0.0504 = 0.14$ . Note that since  $n$  and  $k$  can be as large as 200, your program must have an efficient algorithm in order to finish the computation in time.

#### Input Format

The input file contains several (at most 10) test cases. Each test case is given in a separated line with integers  $n$ ,  $k$ , and probabilities  $p_1, p_2, p_3, \dots, p_n$ . You can assume that  $2 \leq k \leq n \leq 200$  and no invalid probability value will be given as input. All integers and probabilities are separated by one or more space. The last test case will

be followed by a single line containing the integer 0.

### **Output Format**

For each test case, your program should output a single line containing a floating point number to indicate the probability that the random binary string **does not** contain  $k$  (or more) consecutive 1's. The floating point number must be rounded to 5 digits after decimal point.

### **Sample Input**

```
4 2 0.9 0.8 0.7 0.6
10 6 0 1 0.7 0.8 0.9 0.8 0.7 0.6 0.5 0.432
0
```

### **Sample Output**

```
0.14000
0.66841
```