Linear_Model_with_R

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Chapter 1

1.2 Initial Data Analysis

Exercises

Q1

q1a

```
library(ggplot2)
```

```
data(teengamb, package = "faraway")
```

```
head(teengamb)
```

```
  sex status income verbal gamble
1   1     51   2.00      8    0.0
2   1     28   2.50      8    0.0
3   1     37   2.00      6    0.0
4   1     28   7.00      4    7.3
5   1     65   2.00      8   19.6
6   1     61   3.47      6    0.1
```

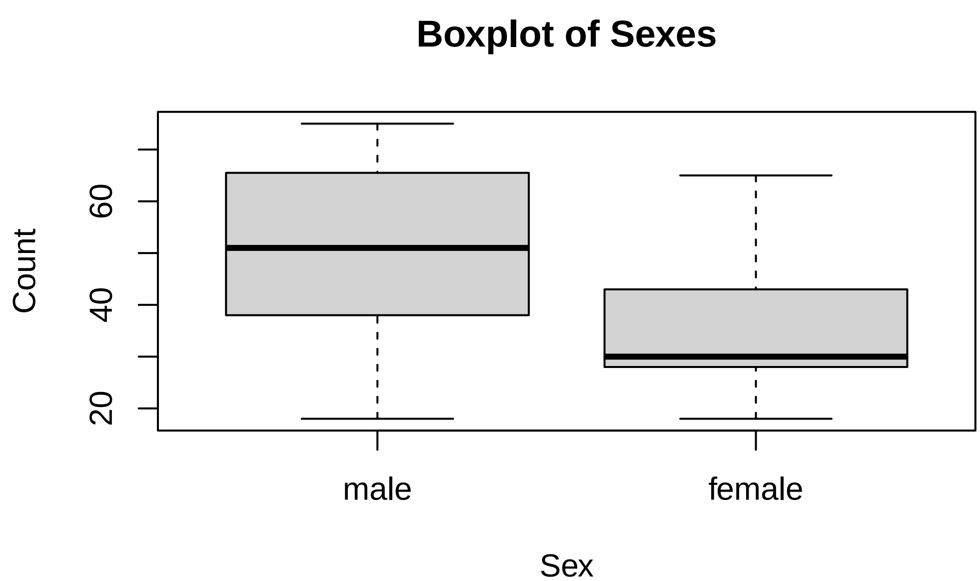
```
teengamb$sex = factor(teengamb$sex)
levels(teengamb$sex) = c("male", "female")
summary(teengamb$sex)
```

```
  male female
    28     19
```

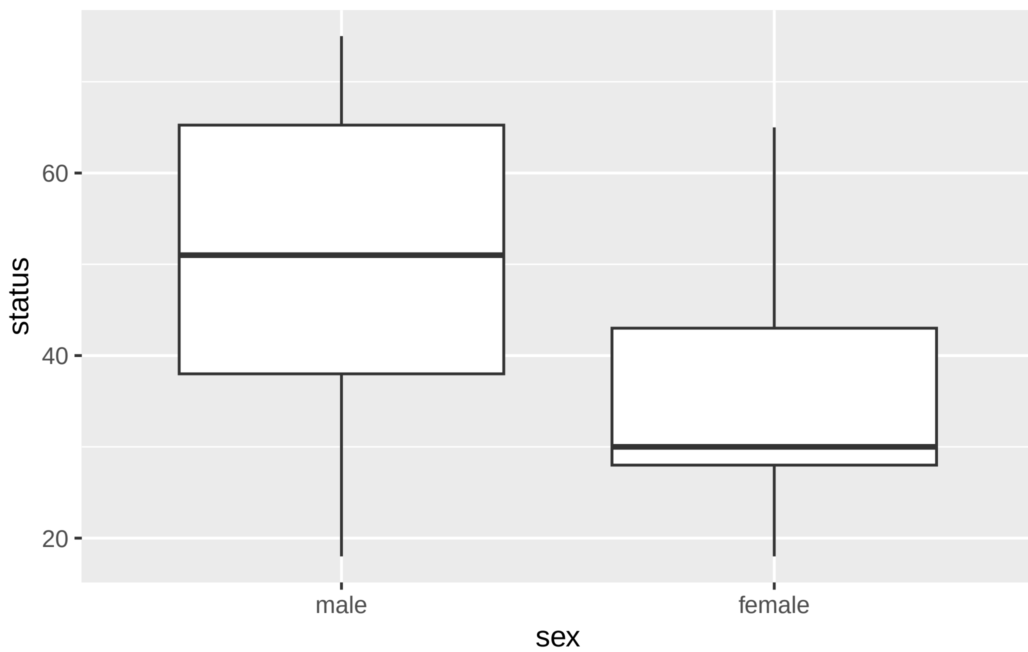
There are 28 males and 19 females.

q1b

```
# Base graphics
boxplot(status ~ sex, data = teengamb, main = "Boxplot of Sexes", xlab = "Sex", ylab = "Coun
```



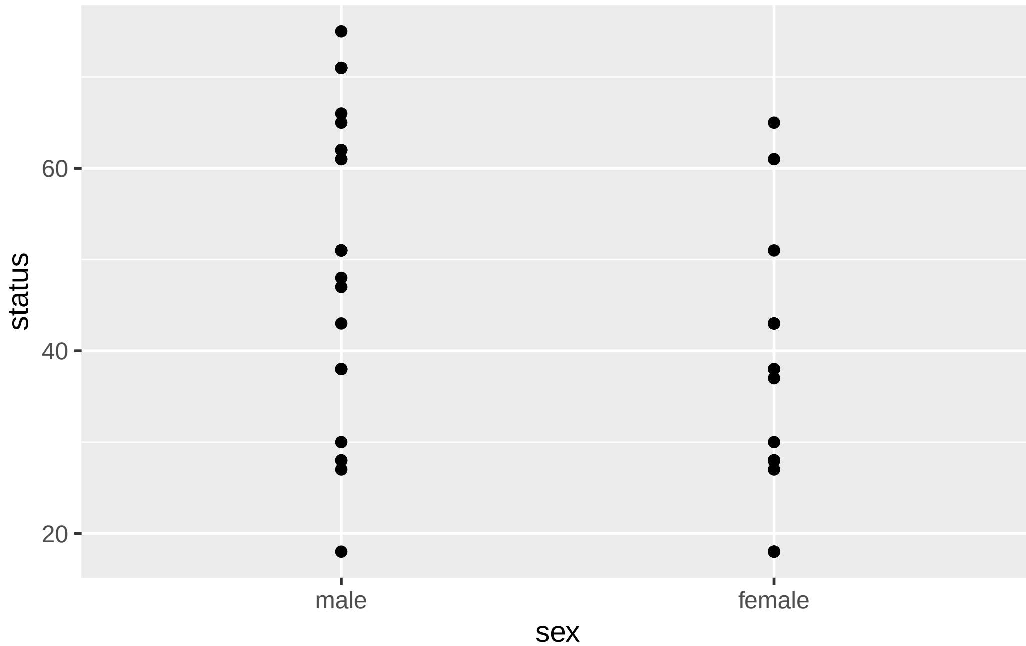

```
# ggplot2 package
ggplot(data = teengamb, aes(x = sex, y = status)) + geom_boxplot()
```



Both methods differ only in appearance.

q1c

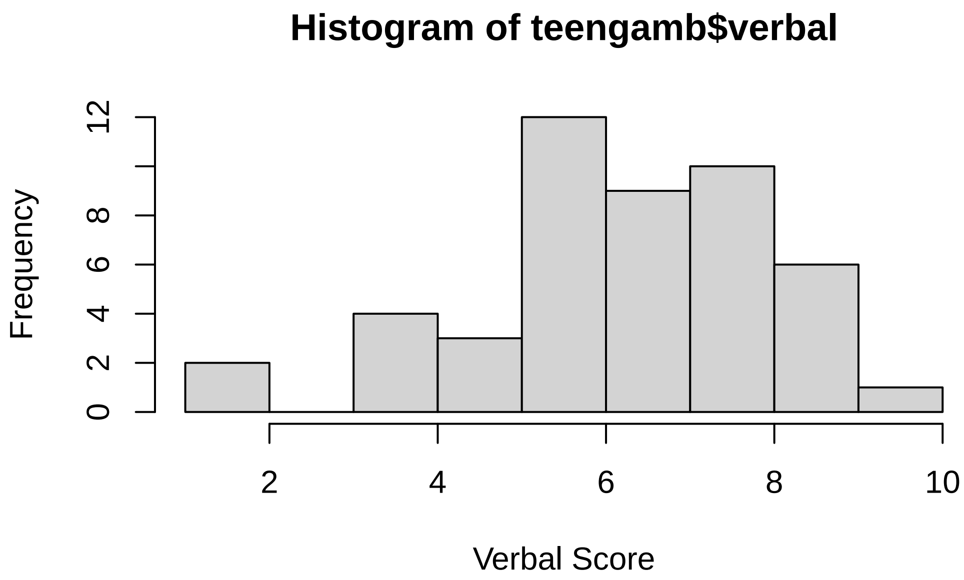
```
ggplot(data = teengamb, aes(x = sex, y = status)) + geom_point()
```



No. We see fewer points due to overplotting.

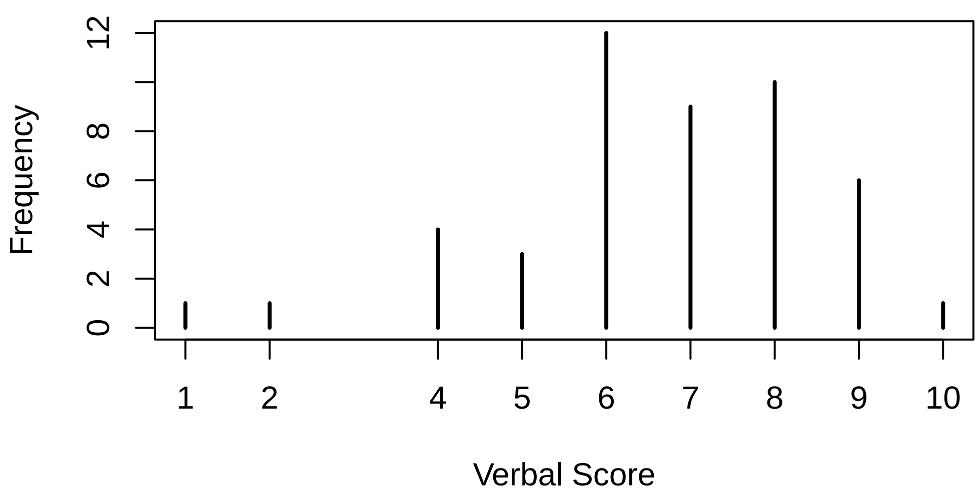
q1d

```
hist(x = teengamb$verbal, xlab = "Verbal Score")
```



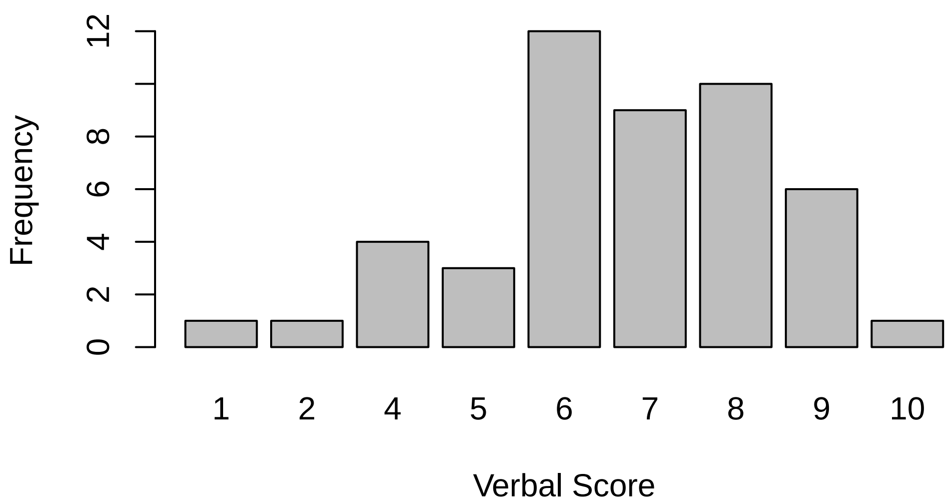
The x-axis is difficult to interpret because the ticks marks align with the boundaries of the blocks, rather than being centred on them.

```
plot(table(teengamb$verbal), xlab = "Verbal Score", ylab = "Frequency")
```



Cleaner but the thin lines are non-standard way of visualising histogram.

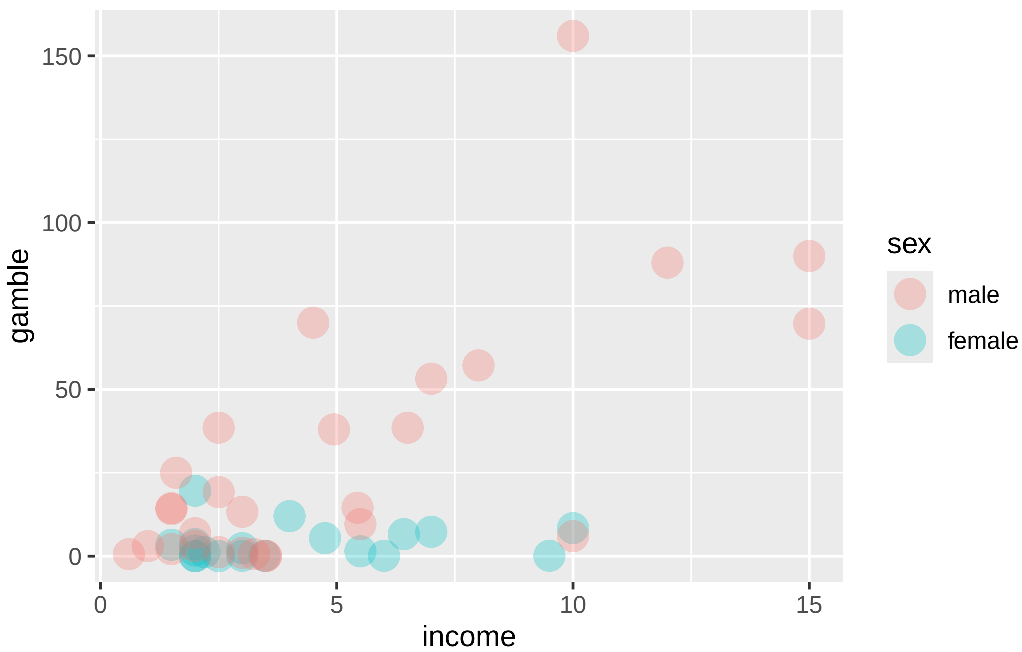
```
barplot(table(teengamb$verbal), xlab = "Verbal Score", ylab = "Frequency")
```



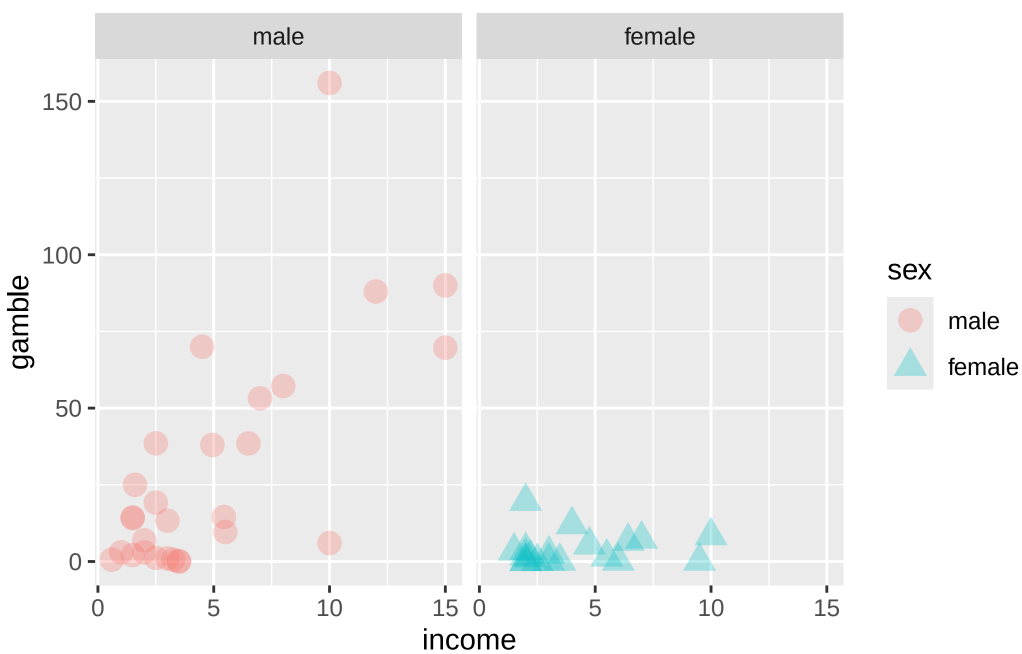
The plot does not leave a gap for the verbal score of 3, even though this value is missing from the dataset.

q1e

```
ggplot(data = teengamb, aes(x = income, y = gamble, colour = sex)) + geom_point(size = 5, al
```




```
ggplot(data = teengamb, aes(x = income, y = gamble, shape = sex, colour = sex)) + geom_point
```



The faceted plot, because it is easier to distinguish the two sexes.

q1f

```
summary(teengamb)
```

```
     sex         status          income           verbal          gamble     
 male  :28   Min.   :18.00   Min.   : 0.600   Min.   : 1.00   Min.   :  0.0  
 female:19   1st Qu.:28.00   1st Qu.: 2.000   1st Qu.: 6.00   1st Qu.:  1.1  
             Median :43.00   Median : 3.250   Median : 7.00   Median :  6.0  
             Mean   :45.23   Mean   : 4.642   Mean   : 6.66   Mean   : 19.3  
             3rd Qu.:61.50   3rd Qu.: 6.210   3rd Qu.: 8.00   3rd Qu.: 19.4  
             Max.   :75.00   Max.   :15.000   Max.   :10.00   Max.   :156.0  
```

The `gamble` variable is the most skewed, because its minimum and lower quartile are much closer to the median than its upper quartile and maximum are.

Q2

q1a

```
data(uswages, package = "faraway")
head(uswages)
```

```
        wage educ exper race smsa ne mw so we pt
6085  771.60   18    18    0    1  1  0  0  0  0
23701 617.28   15    20    0    1  0  0  0  1  0
16208 957.83   16     9    0    1  0  0  1  0  0
2720  617.28   12    24    0    1  1  0  0  0  0
9723  902.18   14    12    0    1  0  1  0  0  0
22239 299.15   12    33    0    1  0  0  0  1  0
```

```
usw = uswages[, c("wage", "ne", "mw", "we", "so")]
head(usw)
```

```
        wage ne mw we so
6085  771.60  1  0  0  0
23701 617.28  0  0  1  0
16208 957.83  0  0  0  1
2720  617.28  1  0  0  0
9723  902.18  0  1  0  0
22239 299.15  0  0  1  0
```

q1b

```r
sum(usw$ne * usw$wage) / sum(usw$ne)
```

```
[1] 631.6591
```

q1c

```r
tapply(usw$wage, usw$ne, mean)["1"]
```

```
       1 
631.6591 
```

```r
all.equal(
    sum(usw$ne * usw$wage) / sum(usw$ne),
    unname(tapply(usw$wage, usw$ne, mean)["1"]))
```

```
[1] TRUE
```

Yes it does.

q1d

```r
head(rowSums(usw[, -1]))
```

```
 6085 23701 16208  2720  9723 22239 
    1     1     1     1     1     1 
```

Each row should have exactly one `1` because these columns are indicators for mutually exclusive geographic regions (one region per individual).

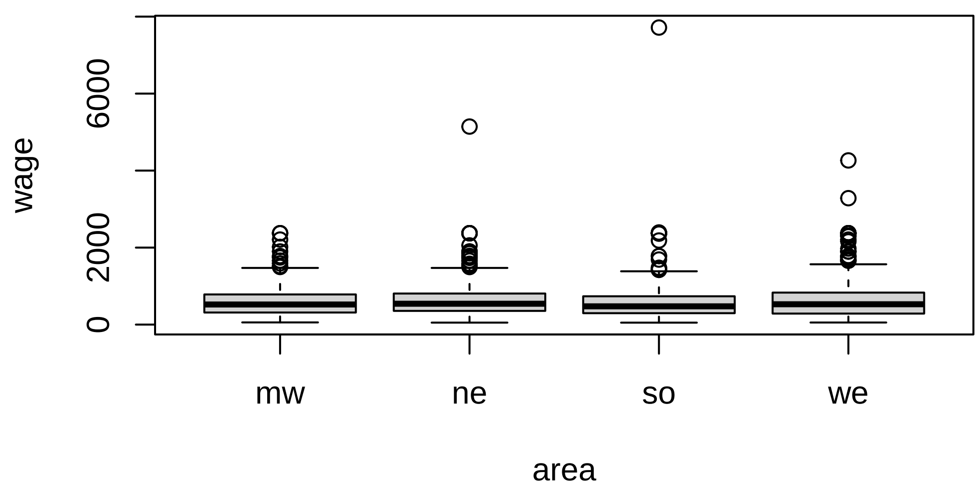
q1e

```
usw$area = c("ne", "mw", "we", "so")[apply(usw[, -1], 1, which.max)]
head(usw)
```

```
        wage ne mw we so area
6085  771.60  1  0  0  0   ne
23701 617.28  0  0  1  0   we
16208 957.83  0  0  0  1   so
2720  617.28  1  0  0  0   ne
9723  902.18  0  1  0  0   mw
22239 299.15  0  0  1  0   we
```

q1f

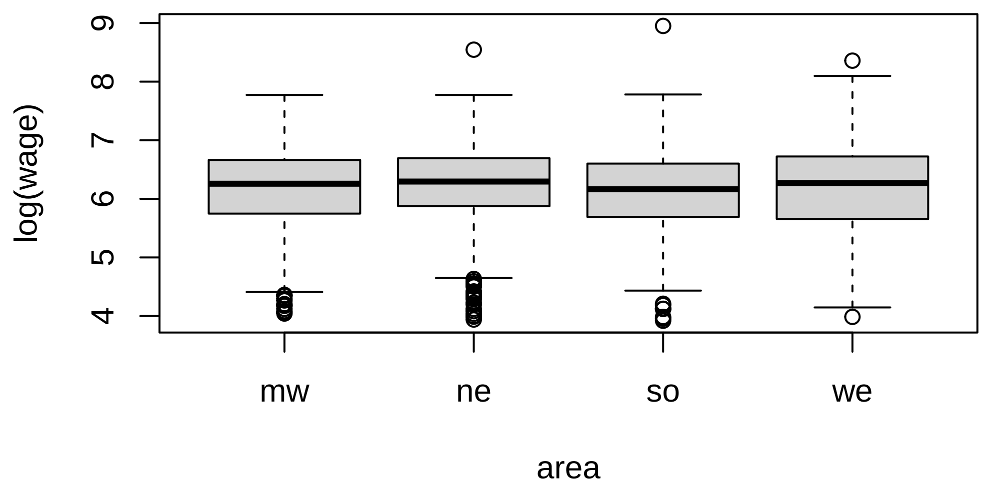
```
boxplot(formula = wage ~ area, data = usw)
```



The distributions are highly skewed.

q1g

```r
boxplot(log(wage) ~ area, data = usw)
```



This is better because it is now easier to distinguish differences in the distributions.

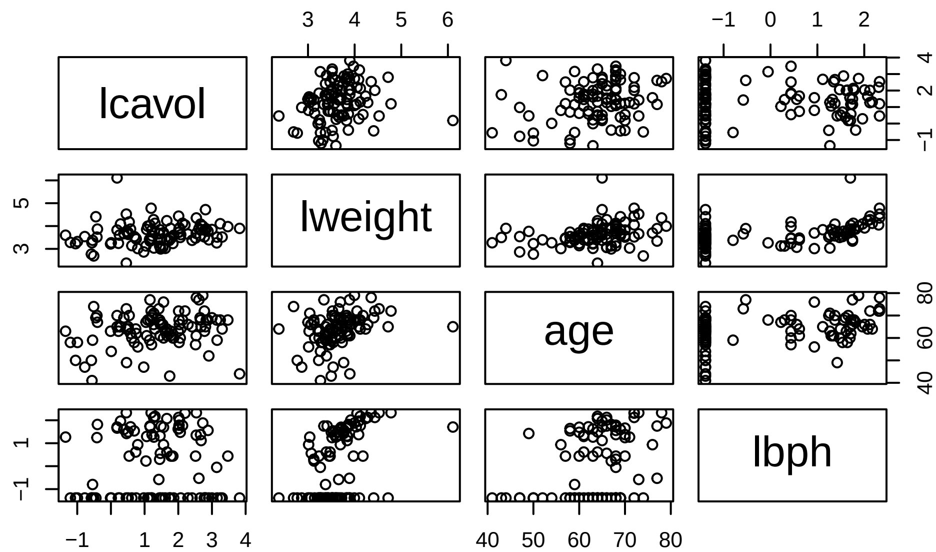
Q3

q3a

```r
data(prostate, package = "faraway")
head(prostate)
```

```
      lcavol lweight age      lbph svi      lcp gleason pgg45     lpsa
1 -0.5798185  2.7695  50 -1.386294   0 -1.38629       6     0 -0.43078
2 -0.9942523  3.3196  58 -1.386294   0 -1.38629       6     0 -0.16252
3 -0.5108256  2.6912  74 -1.386294   0 -1.38629       7    20 -0.16252
4 -1.2039728  3.2828  58 -1.386294   0 -1.38629       6     0 -0.16252
5  0.7514161  3.4324  62 -1.386294   0 -1.38629       6     0  0.37156
6 -1.0498221  3.2288  50 -1.386294   0 -1.38629       6     0  0.76547
```

```r
pairs(prostate[,1:4])
```



`lbph` has many identical values.

q3b

```
cor(prostate[,1:4])
```

```
            lcavol   lweight       age       lbph
lcavol  1.00000000 0.1941284 0.2249999 0.02734971
lweight 0.19412839 1.0000000 0.3075247 0.43493174
age     0.22499988 0.3075247 1.0000000 0.35018592
lbph    0.02734971 0.4349317 0.3501859 1.00000000
```

There are four assumptions to check before performing a Pearson correlation test. - The two variables (the variables of interest) need to be using a continuous scale. - The two variables of interest should have a linear relationship, which you can check with a scatterplot. - There should be no spurious outliers. - The variables should be normally or near-to-normally distributed.

Given the non-normal distribution of lbph, its Pearson correlation coefficients are influenced by it, which can misrepresent the overall strength of the relationships to other variables.

q3c

```r
nrow(prostate[prostate$lbph == min(prostate$lbph),]) / nrow(prostate)
```

```
[1] 0.443299
```

44%.

```r
exp(min(prostate$lbph))
```

```
[1] 0.2500001
```

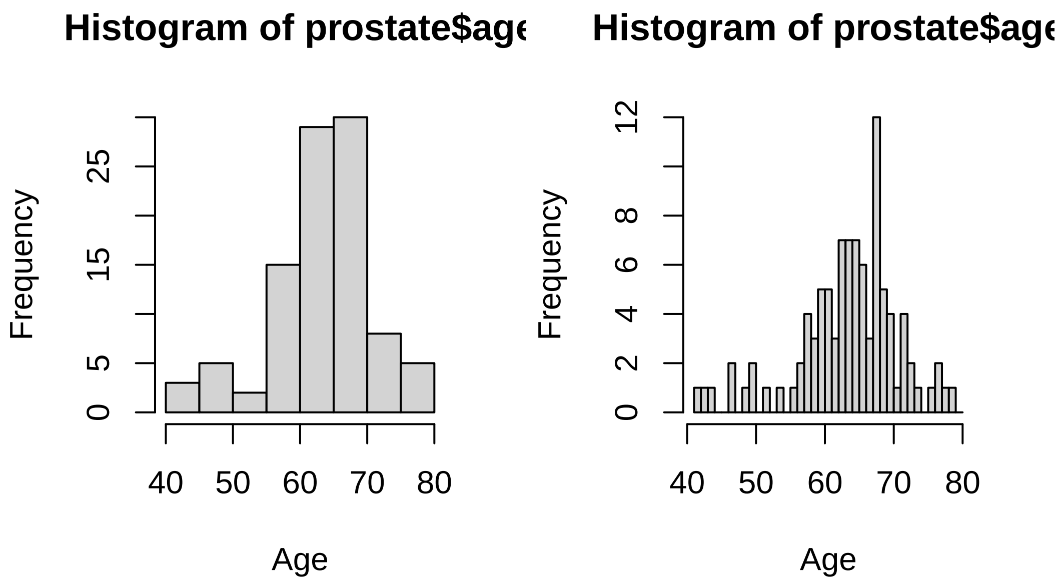
It seems likely that the transform $log(x + 0.25)$ was used to avoid the problem of $log(0)$. There were many cases with $bph = 0$.

q3d

```r
age_range <- range(prostate$age)
one_year_breaks <- seq(
    from = age_range[1],
    to = age_range[2] + 1,
    by = 1
)
par(mfrow = c(1, 2))

hist(x = prostate$age, xlab = "Age")

hist(x = prostate$age, xlab = "Age", breaks = one_year_breaks)
```



- **Default Plot**: This plot provides a smoother, more general overview of the age distribution. It's easy to see the central tendency and the overall shape. This plot is good for quickly understanding the general pattern.
- **One-Year Bin Plot**: This plot is much more jagged, noisy, and granular. The overall shape is still visible but is obscured by the high degree of variation between individual years. It looks less like a smooth distribution and more like a collection of sharp spikes.

A larger binwidth is chosen to emphasise the overall structure of the age distribution. This approach is appropriate as we are not concerned with the fine-grained detail in the data.

q3e

```
xtabs(~ gleason + svi, data = prostate)
```

```
       svi
gleason  0  1
      6 35  0
      7 37 19
      8  1  0
      9  3  2
```

The most common combination is a Gleason score of 7 with no seminal vesicle invasion, which occurred in 37 of the 97 patients in the study.

Q4

q4a

```r
data(sat, package = "faraway")
head(sat)
```

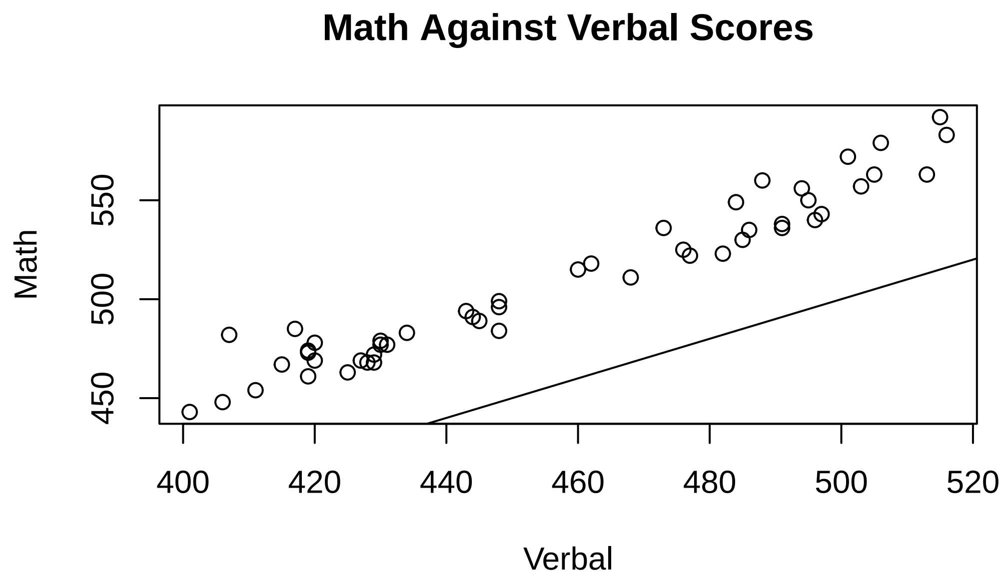
```
           expend ratio salary takers verbal math total
Alabama     4.405  17.2 31.144      8    491  538  1029
Alaska      8.963  17.6 47.951     47    445  489   934
Arizona     4.778  19.3 32.175     27    448  496   944
Arkansas    4.459  17.1 28.934      6    482  523  1005
California  4.992  24.0 41.078     45    417  485   902
Colorado    5.443  18.4 34.571     29    462  518   980
```

```r
table(sat$verbal + sat$math == sat$total)
```

```
TRUE
  50
```

q4b

```r
plot(x = sat$verbal, y = sat$math, main = "Math Against Verbal Scores", xlab = "Verbal", ylab
abline(0, 1)  # y = x is equal to intercept = 0 and slope = 1
```



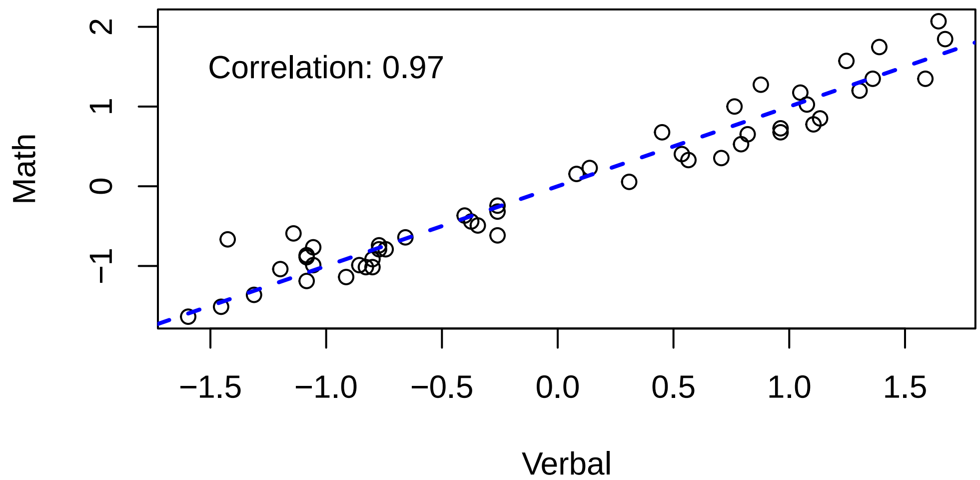
The distributions differ; students generally score higher on their maths tests than on their verbal tests.

There is a strong, positive, linear relationship between verbal and math scores. As students' verbal scores increase, their math scores also tend to increase.

```
# hist(x = sat$verbal)
# plot(density(x = sat$verbal))
```

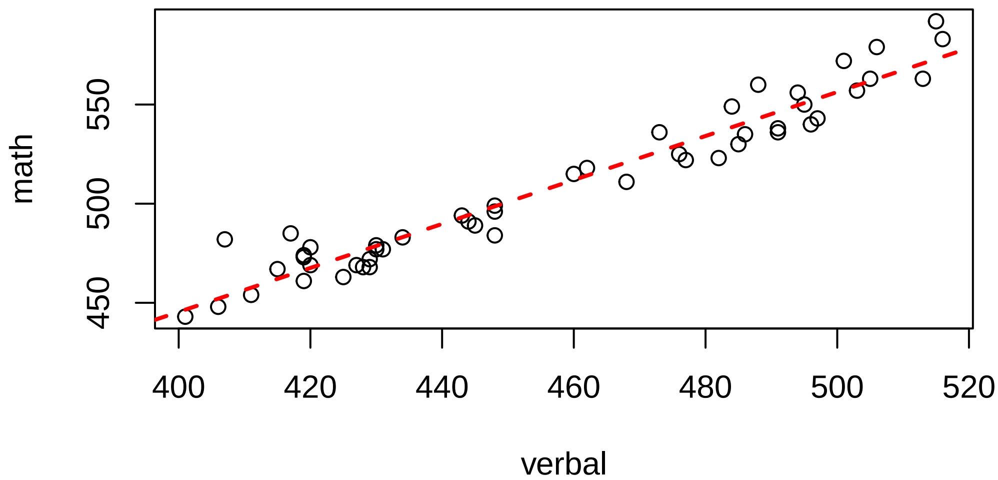
q4c

```
plot(scale(sat$math) ~ scale(sat$verbal), xlab = "Verbal", ylab = "Math")
abline(a = 0, b = 1, lty = 2, lwd = 2, col = "blue")
text(x = -1.0, y = 1.5, label = paste0("Correlation: ", round(cor(sat$verbal, sat$math), 3)))
```



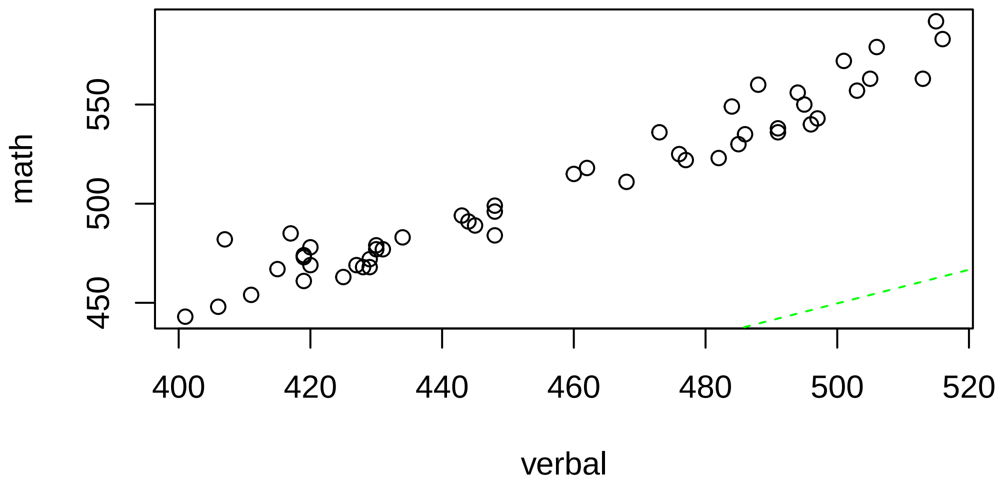
q4d

```
q4d_lm_coef <- coef(lm(sat$math ~ sat$verbal))
plot(math ~ verbal, data = sat)
abline(lm(sat$math ~ sat$verbal), col = "red", lty = 2, lwd = 2)
text(x = -0.5, y = 1.25, label = paste0("Intercept: ", round(q4d_lm_coef[1], 18)), cex = 1)
text(x = -0.5, y = 1.0, label = paste0("Slope: ", round(q4d_lm_coef[2], 3)), cex = 1)
```



q4e

```
q4e_lm_coef <- coef(lm(sat$verbal ~ sat$math))
plot(math ~ verbal, data = sat)
abline(lm(sat$verbal ~ sat$math), col = "green", lty = 2)
text(x = -0.5, y = 1.25, label = paste0("Intercept: ", round(q4e_lm_coef[1], 18)), cex = 1)
text(x = -0.5, y = 1.0, label = paste0("Slope: ", round(q4e_lm_coef[2], 3)), cex = 1)
```



q4f

Remake the plot for clearer visual.

- The model gives us: verbal = intercept + slope $*$ math
- Rearrange it to the equation below so we can plot it on our graph
- verbal − intercept = slope $*$ math
- math = (verbal / slope) − (intercept / slope)
- math = (-intercept / slope) + (1/slope) $*$ verbal
- So, the new intercept is $-\frac{\text{intercept}}{\text{slope}}$ and the new slope is $\frac{1}{\text{slope}}$

```
mean_verbal <- mean(sat$verbal)
mean_math <- mean(sat$math)


plot(math ~ verbal, data = sat,
    main = "Intersection of Two Regression Lines",
    xlab = "Verbal Score",
    ylab = "Math Score",
    pch = 19, col = "gray")


lm_math_on_verbal <- lm(math ~ verbal, data = sat)
```

```r
abline(lm_math_on_verbal, col = "red", lwd = 2)


lm_verbal_on_math <- lm(verbal ~ math, data = sat)


coeffs <- coef(lm_verbal_on_math)
intercept_for_plot <- -coeffs[1] / coeffs[2]
slope_for_plot <- 1 / coeffs[2]

# Now we can add this rearranged line to our plot
abline(a = intercept_for_plot, b = slope_for_plot, col = "blue", lwd = 2)


# This point should be exactly where the two lines cross
points(x = mean_verbal, y = mean_math,
    col = "purple",
    pch = 19,
    cex = 2.5)

text(x = -1.0, y = 1.25, label = paste0("Intercept: ", round(q4d_lm_coef[1], 18)), cex = 1)
text(x = -1.0, y = 1.0, label = paste0("Slope: ", round(q4d_lm_coef[2], 3)), cex = 1)

text(x = 0.8, y = -0.75, label = paste0("Intercept [Exchanged]: ", round(q4e_lm_coef[1], 18))
text(x = 0.8, y = -0.5, label = paste0("Slope [Exchanged]: ", round(q4e_lm_coef[2], 3)), cex
```

q4g(i)

Based on the equation math $= 0.97 \cdot$ verbal $+ 3.41 \times 10^{-16}$

```
print(q4d_lm_coef[2] * (mean(sat$verbal) + 20) + q4d_lm_coef[1])
```

```
sat$verbal
  530.9593
```

q4g(ii)

Based on the equation verbal $= 0.97 \cdot$ math $- 3.08 \times 10^{-16}$

```
print(q4e_lm_coef[2] * (mean(sat$math) + 20) + q4e_lm_coef[1])
```

```
sat$math
474.1179
```

q4g(iii)

```r
print(q4d_lm_coef[2] * (mean(sat$verbal)) + q4d_lm_coef[1])
```

```
sat$verbal
    508.78
```

q4g(iv)

Without any information about the verbal score, we can only use the mean verbal score.

```r
print(q4d_lm_coef[2] * (mean(sat$verbal)) + q4d_lm_coef[1])
```

```
sat$verbal
    508.78
```

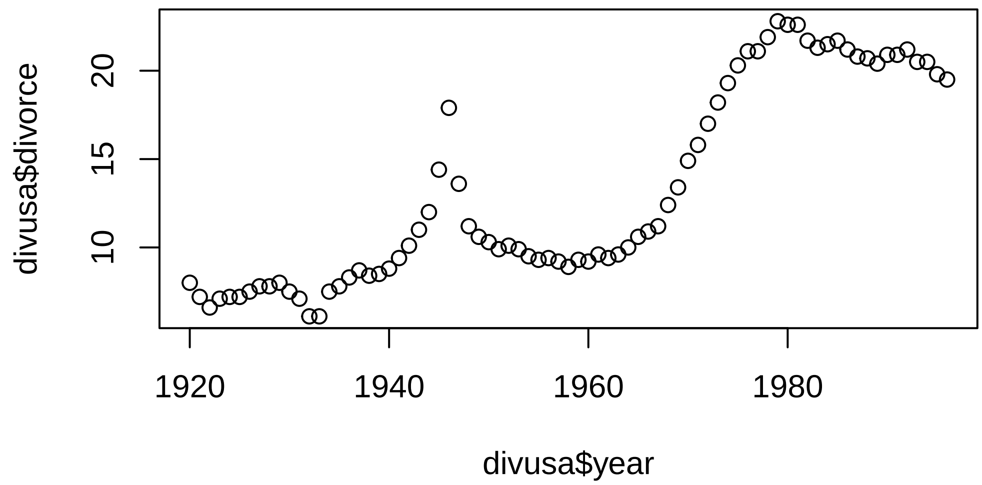
Q5

q5a

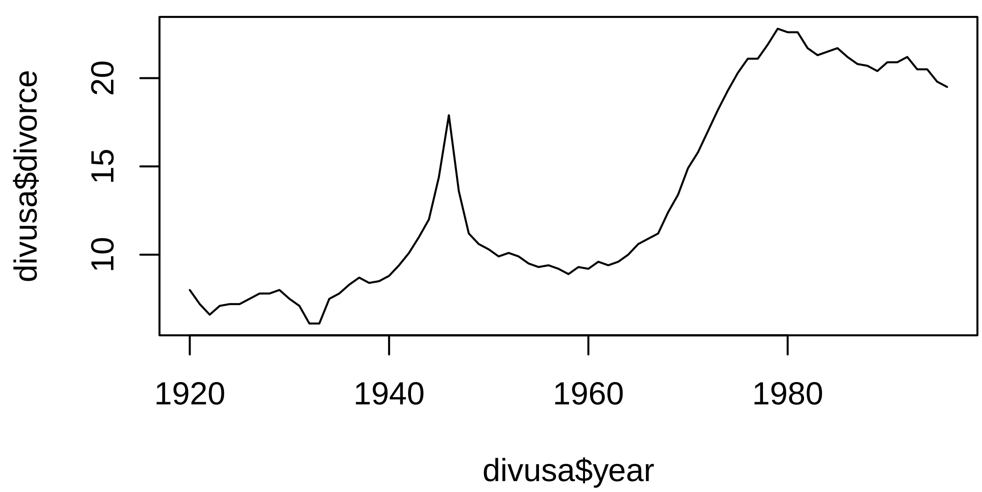
```r
data(divusa, package = "faraway")
head(divusa)
```

```
  year divorce unemployed femlab marriage birth military
1 1920     8.0        5.2  22.70     92.0 117.9   3.2247
2 1921     7.2       11.7  22.79     83.0 119.8   3.5614
3 1922     6.6        6.7  22.88     79.7 111.2   2.4553
4 1923     7.1        2.4  22.97     85.2 110.5   2.2065
5 1924     7.2        5.0  23.06     80.3 110.9   2.2889
6 1925     7.2        3.2  23.15     79.2 106.6   2.1735
```

```r
plot(x = divusa$year, y  = divusa$divorce)
```



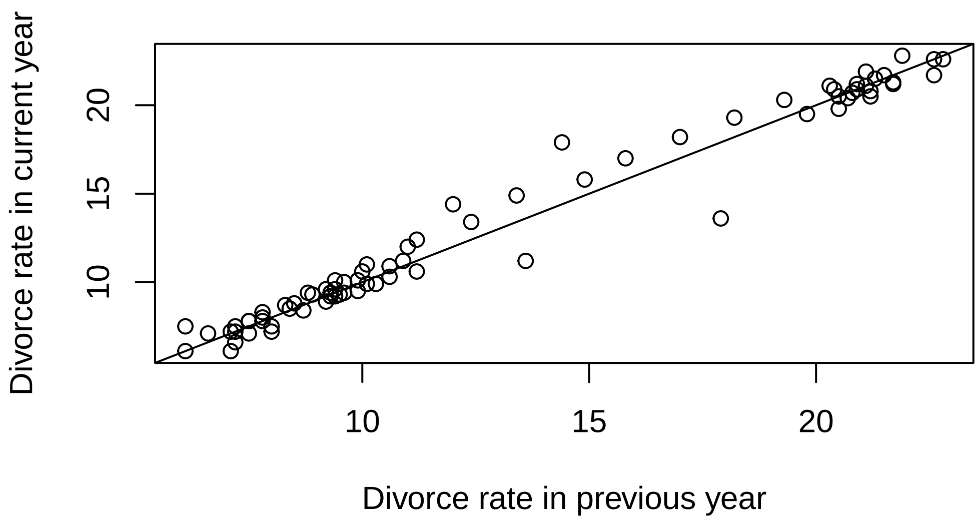

```
plot(x = divusa$year, y = divusa$divorce, type = "l")
```



Line plot is preferable for time ordered data.

q5b

```r
plot(
    y = divusa$divorce[-1],
    x = divusa$divorce[-nrow(divusa)],
    xlab = "Divorce rate in previous year",
    ylab = "Divorce rate in current year"
)
abline(0, 1)
```



Prediction is possible; divorce rate in successive years is strongly correlated.

q5c

```r
q5c_coef <- coef(lm(divorce ~ year, data= divusa))
q5c_coef
```

```
 (Intercept)         year
-422.9752984    0.2228009
```

Based on the equation divorce $= 0.22 \cdot \text{year} - 422.98$.

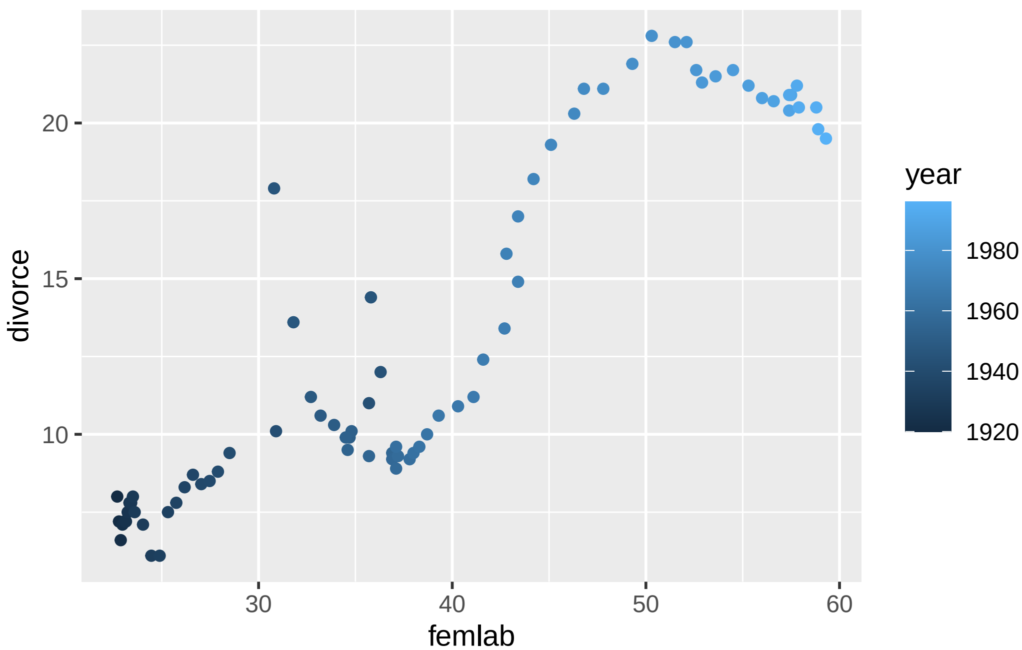
```r
(100 - q5c_coef[1]) / q5c_coef[2]
```

```
(Intercept)
   2347.277
```

After rearraging the equation, the year when divorce rate hits 100% is 2347.277. It is not a realistic prediction, as this extrapolation over time is sure to stop at some point.

q5d

```r
ggplot(data = divusa, aes(x = femlab, y = divorce, colour = year)) + geom_point()
```



- Historically, divorce rates were lower during periods when female participation in the labor force was also lower.
- Also, both variables start low in the distant past as indicated by the darker coloured points and progress to higher values over time as indicated by the lighther coloured points.
- But the change is not linear, especially during the Great Depression which caused a shift in the divorce rate.