MA50260 Statistical Modelling

Lecture 19: GLMMs - Inference and Model Selection

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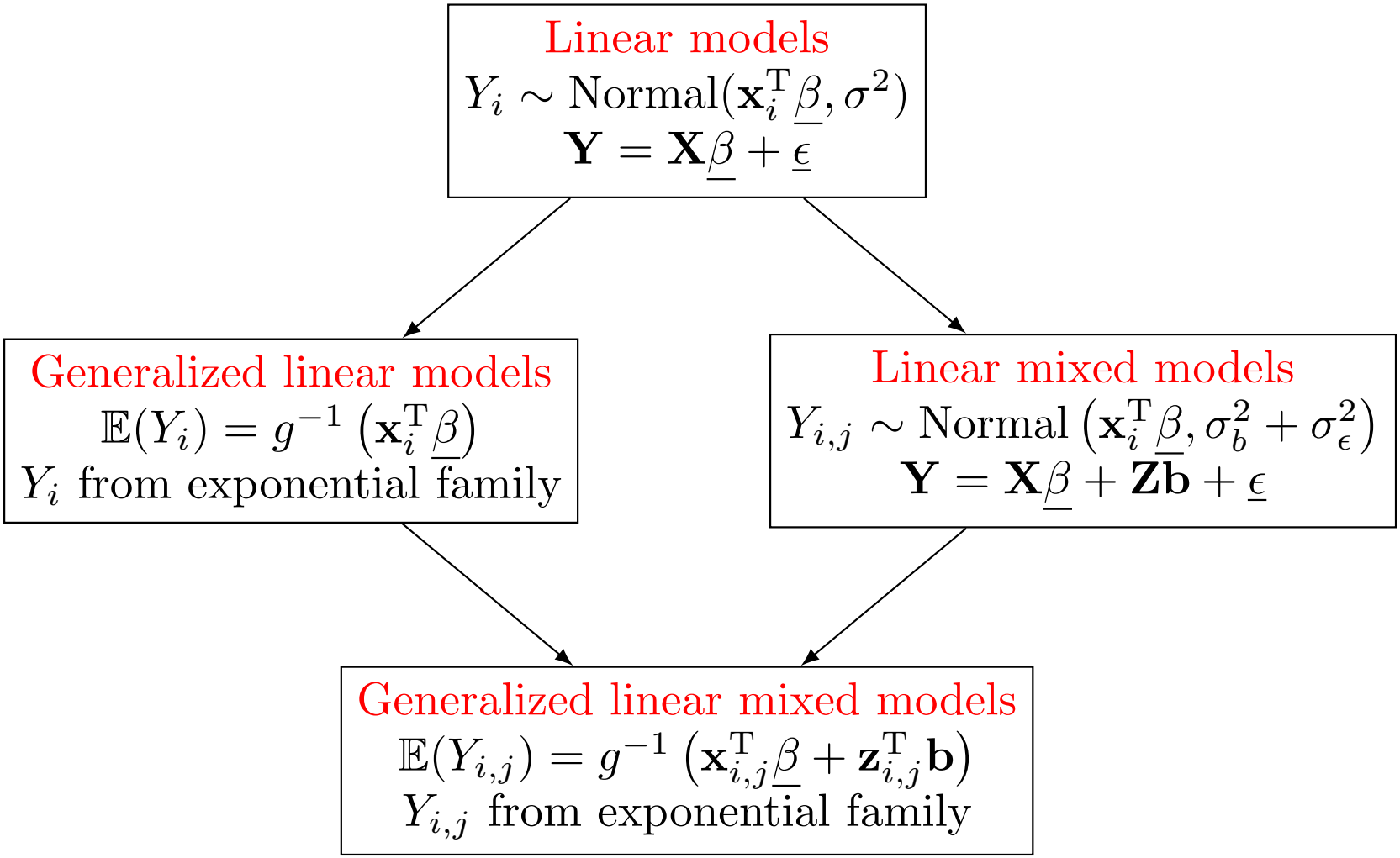
April 23, 2024

Generalised Linear Mixed Models



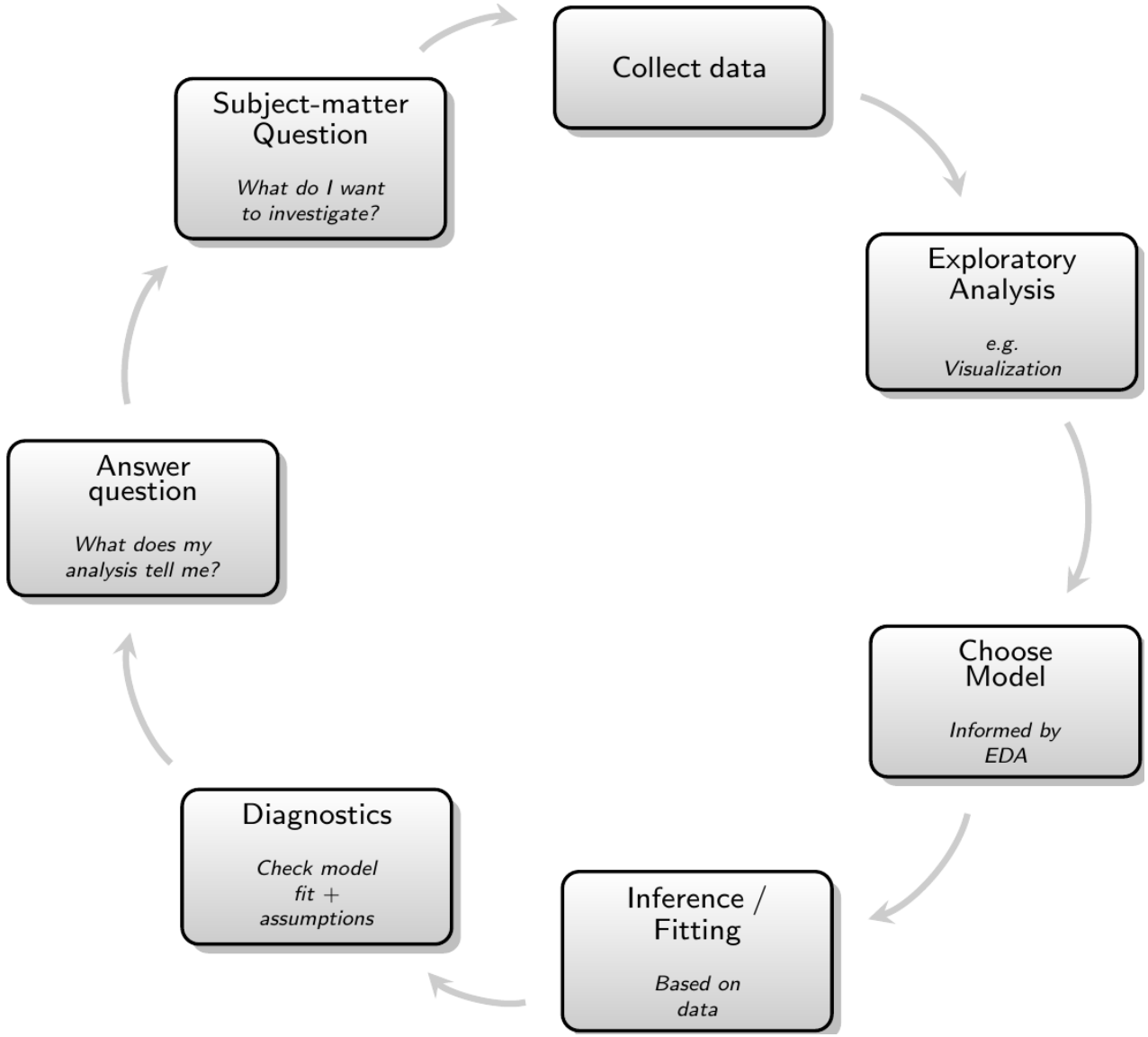
GLMMs - Estimation

In the last lecture, we considered two approaches for estimating GLMMs:

- Penalized quasi likelihood
- Gauss-Hermite quadrature

How do these methods compare?

Philosophy of Statistical Modelling



Sampling distribution of $\hat{\underline{\beta}}$

For a GLM, we have

$$\hat{\underline{\beta}}(\mathbf{Y}) \sim \mathrm{MVN}_{\boldsymbol{p}}\left(\underline{\beta}\,,\, \phi(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X})^{-1}\right).$$

Further, for a (normal) linear mixed model,

$$\hat{\underline{\beta}}(\mathbf{Y}) \sim \mathrm{MVN}_{\boldsymbol{p}}(\underline{\beta}, (\mathbf{X}^{\mathrm{T}}(\mathbf{Z}\mathcal{D}\mathbf{Z}^{\mathrm{T}} + \sigma_{\epsilon}^{2}\mathbf{I}_{n})^{-1}\mathbf{X})^{-1}).$$

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The sampling distribution for the GLMM is

$$\hat{\underline{\beta}}(\mathbf{Y}) \sim \mathrm{MVN}_{\boldsymbol{p}}(\underline{\beta}, (\mathbf{X}^{\mathrm{T}}(\mathbf{Z}\mathcal{D}\mathbf{Z}^{\mathrm{T}} + \mathbf{W}^{-1}\phi)^{-1}\mathbf{X})^{-1}).$$

Can you show that this result agrees with the results above?

Example - Seed Germination

We estimate the model defined in Lecture 17

```
load("seeds.rda")
estim <- glmer(
    cbind(Germinated,NotGerminated) ~ (1 | Plate),
    nAGQ = 25, data = seeds, family = binomial
)
```

Let's consider the estimated fixed effect

```
round( summary(estim)$coefficients, 3 )
```

```
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -0.377      0.263   -1.43    0.153
```

How can we give a 95% confidence interval for β_1?

Exercise - Ohio Wheeze Data

Let's consider the results for the Ohio wheeze data with 536 children

```
round( summary( estimGH )$coefficients[,1:2], 3 )
```

```
##             Estimate Std. Error
## (Intercept)   -3.102      0.219
## age           -0.176      0.068
## smoke          0.399      0.273
```

Which of the fixed effects is significant?

Model Comparison

Similar to GLMs, we can compare models using the AIC.

How is the AIC defined?

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Similar to GLMs, we can compare models using the AIC.

How is the AIC defined?

Let's consider the seeds data set.

If we assume $p_1 = \ldots, p_{10}$, the AIC is

```
## [1] 64.30628
```

For the GLMM, the AIC is

```
## [1] 25.1
```

Which model would we choose?

The GLMM because it has the lower AIC.