1. Demonstrate what happens when we insert the keys 5, 28, 19,15, 20, 33,12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$. you must show all the steps to arrive at the answers. Simply stating the answers will result in 0 points awarded. (20 points)

2. Consider a hash table of size $m = 1000$ and a corresponding hash function $h(k) = \lfloor m(kA \bmod 1) \rfloor$ for $A = (\sqrt{5} - 1)/2$. Compute the locations to which the keys 61, 62, 63, 64, and 65 are mapped. You must show all the steps to arrive at the answers. Simply stating the answers will result in 0 points awarded. (20 points)

3. Consider inserting the keys 10, 22, 31,4,15, 28, 17, 88,59 into a hash table of length $m = 11$ using open addressing with the auxiliary hash function $h'(k) = k$. Illustrate the result of inserting these keys using the following three commonly used techniques to compute the probe sequences.
   a. linear probing
   b. quadratic probing with $c_1 = 1$ and $c_2 = 3$
   c. double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \bmod (m - 1))$

   You must show all the steps to arrive at the answers. Simply stating the answers will result in 0 points awarded. (20 points)

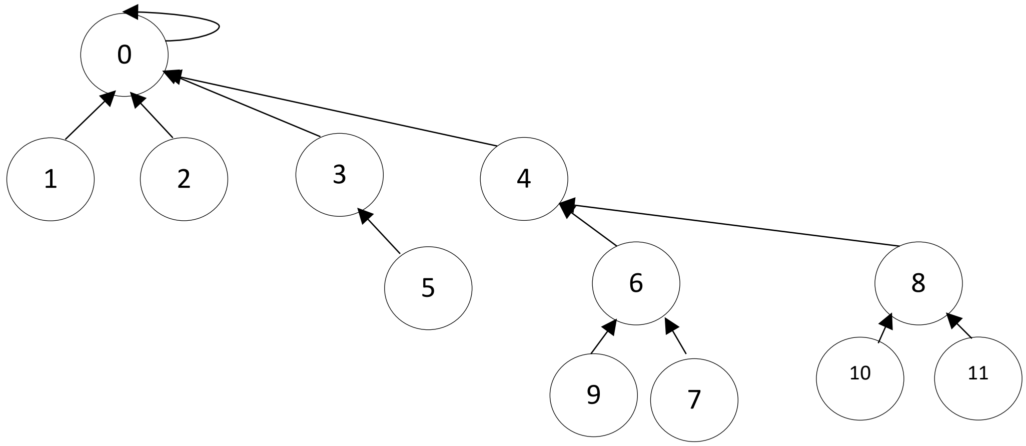
4. Suppose that CONNECTED-COMPONENTS is run on the undirected graph $G = (V, E)$, where $V = \{a, b, c, d, e, f, g, h, i, j, k\}$ and the edges of E are processed in the order (d, i), (f, k), (g, i), (b, g), (a, h), (i, j), (d, k), (b, j), (d, f), (g, j), (a, e). List the vertices in each connected component after each iteration of lines 3–5.

```
CONNECTED-COMPONENTS(G)
1  for each vertex v ∈ G.V
2      MAKE-SET(v)
3  for each edge (u, v) ∈ G.E
4      if FIND-SET(u) ≠ FIND-SET(v)
5          UNION(u, v)
```

   You must show all the steps to arrive at the answers. Simply stating the answers will result in 0 points awarded. (20 points)

5. Assume that the first listed element is the representative for each set in a disjoint set. You must show all the steps to arrive at the answers. Simply stating the answers will result in 0 points awarded. (20 points)
   a. Show the data structure of the disjoint set S: {{0, 1, 2}, {3, 5}} as it is being built using both the list and tree representation.
   b. Consider the following commands applied to the disjoint set S: {{0, 1, 2}, {3, 5}}.
      MAKE-SET (4)
      UNION (1,5)
      UNION (4,5)
      Show the resulting data structure after applying the commands on the following representations:
      i. the list representation with weighted union heuristic applied
      ii. the tree representation with union-by-rank applied
   c. Given the following tree representation of a disjoint set (see below), use path-compression when executing FIND-SET(9) command using the following procedure. Show the results after each step.

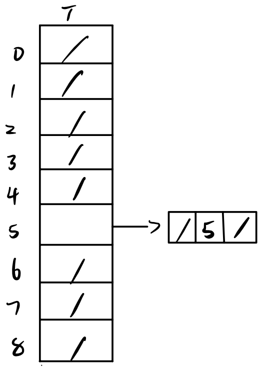
```
FIND-SET(x)
if x ≠ x.p
    x.p = FIND-SET(x.p)
return x.p
```


0
1
2
3
4
5
6
8
9
7
10
11

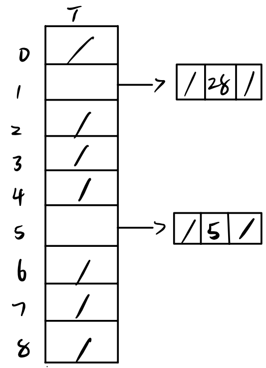
1. keys: 5, 28, 19, 15, 20, 33, 12, 17, 10

$h(k) = k \bmod 9$

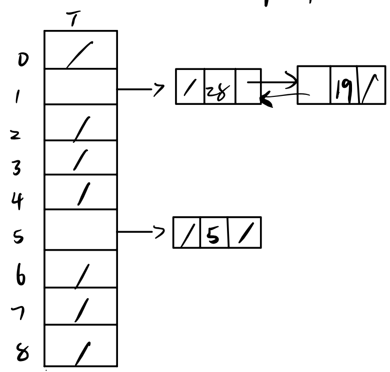
① $h(5) = 5 \bmod 9 = 5$



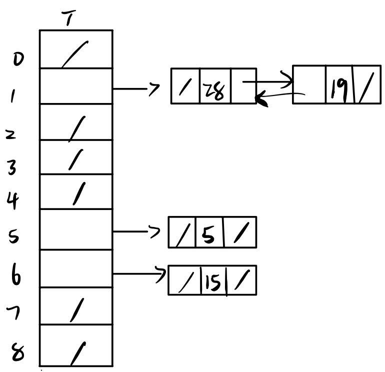
② $h(28) = 28 \bmod 9 = 1$



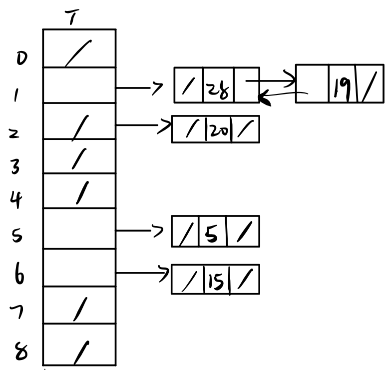
③ $h(19) = 19 \bmod 9 = 1$



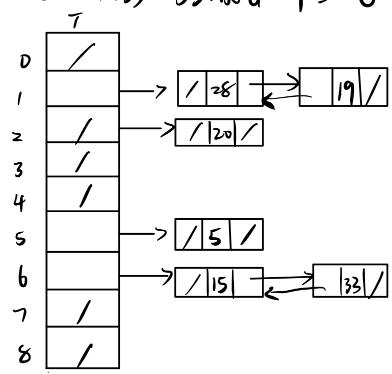
④ $h(15) = 15 \bmod 9 = 6$



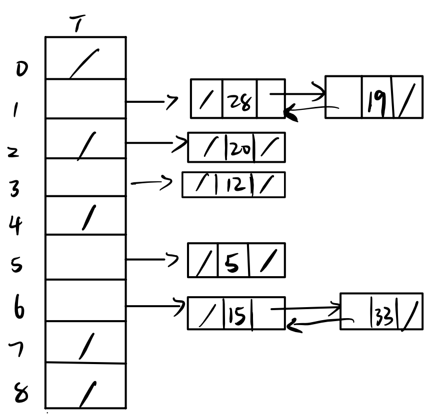
⑤ $h(20) = 20 \bmod 9 = 2$



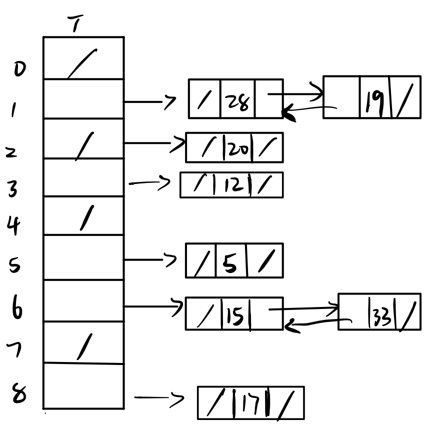
⑥ $h(33) = 33 \bmod 9 = 6$



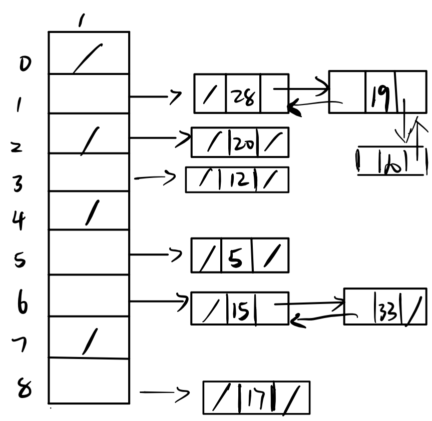
⑦ $h(12) = 12 \bmod 9 = 3$



⑧ $h(17) = 17 \bmod 9 = 8$



⑨ $h(10) = 10 \bmod 9 = 1$



2、 $m = 1000$, $h(k) = \lfloor m(kA \bmod 1) \rfloor$, $A = (\sqrt{5}-1)/2 \approx 0.31$

$h(61) = \lfloor 100 \times (61 \times 0.62 \bmod 1) \rfloor$

$= \lfloor 100 \times 0.7 \rfloor$

$= 700$

$h(62) = \lfloor 100 \times (62 \times 0.62 \bmod 1 \rfloor$

$= \lfloor 100 \times 0.318 \rfloor$

$= 318$

$h(63) = \lfloor 100 \times (63 \times 0.62 \bmod 1) \rfloor$

$= \lfloor 100 \times 0.936 \rfloor$

$= 936$

$h(64) = \lfloor 100 \times (64 \times 0.62 \bmod 1) \rfloor$

$= \lfloor 100 \times 0.554 \rfloor$

$= 554$

$h(65) = \lfloor 100 \times (65 \times 0.62 \bmod 1) \rfloor$

$= \lfloor 100 \times 0.172 \rfloor$

$= 172$

3. a. linear probing

$h(k,i) = (h'(k) + i) \bmod m$

$= (k+i) \bmod 11 \quad , i = 0,1,2 \ldots 10$

$k=10, \quad h'(k) = 10$

0	1	2	3	4	5	6	7	8	9	10
										10

$k=22, \quad h'(k) = 22, \quad h(k) = 22 \bmod 11 = 0$

0	1	2	3	4	5	6	7	8	9	10
22										10

$k=31, \quad h'(k) = 31, \quad h(k) = 31 \bmod 11 = 9$

0	1	2	3	4	5	6	7	8	9	10
22									31	10

$k=4, \quad h'(k) = 4, \quad h(k) = 4 \bmod 11 = 4$

0	1	2	3	4	5	6	7	8	9	10
22				4					31	10

$k=15, \quad h'(k) = 15, \quad h(k) = 15 \bmod 11 = 4$, Collisions!

$h(k) = (15+1) \bmod 11 = 5$

0	1	2	3	4	5	6	7	8	9	10
22				4	15				31	10

$k=28, \quad h'(k) = 28, \quad h(k) = 28 \bmod 11 = 6$

0	1	2	3	4	5	6	7	8	9	10
22				4	15	28			31	10

$k=17, \quad h'(k) = 17, \quad h(k) = 17 \bmod 11 = 6$, Collisions!

$h(k) = (17+1) \bmod 11 = 7$

0	1	2	3	4	5	6	7	8	9	10
22				4	15	28	17		31	10

$k=88, \quad h'(k) = 88, \quad h(k) = 88 \bmod 11 = 0$, Collisions!

$h(k) = (88+1) \bmod 11 = 1$

0	1	2	3	4	5	6	7	8	9	10
22	88			4	15	28	17		31	10

k = 59 , h'(k) = 59 , h(k) = 59 mod 11 = 4 , Collisions !

h(k) = (59 + 4) mod 11 = 8

0	1	2	3	4	5	6	7	8	9	10
22	88			4	15	28	17	59	31	10

b. quadratic probing with $c_1 = 1$ and $c_2 = 3$

k = 10 , h'(k) = 10 , h(k) = 10 mod 11 = 10   19 - 11

k = 22 , h'(k) = 22 , h(k) = 22 mod 11 = 0

k = 31 , h'(k) = 31 , h(k) = 31 mod 11 = 9

k = 4 , h'(k) = 4 , h(k) = 4 mod 11 = 4

k = 15 , h'(k) = 15 , h(k) = 15 mod 11 = 4 , Collisions !

h(k) = (15 + 1 + 3) mod 11 = 8

0	1	2	3	4	5	6	7	8	9	10
22				4				15	31	10

k = 28 , h'(k) = 28 , h(k) = 28 mod 11 = 6

0	1	2	3	4	5	6	7	8	9	10
22				4		28		15	31	10

k = 17 , h'(k) = 17 , h(k) = 17 mod 11 = 6 , Collisions !

h(k) = (17 + 1 + 3) mod 11 = 10 , Collisions !

h(k) = (17 + 2 + 12) mod 11 = 9 , Collisions !

h(k) = (17 + 3 + 27) mod 11 = 3

k = 88 , h'(k) = 88 , h(k) = 88 mod 11 = 0 , Collisions !

h(k) = (88 + 1 + 3) mod 11 = 4 , Collisions !

h(k) = (88 + 2 + 12) mod 11 = 3 , Collisions !

h(k) = (88 + 3 + 27) mod 11 = 8 , Collisions !

h(k) = (88 + 4 + 3×16) mod 11 = 8 , Collisions

h(k) = (88 + 5 + 3×25) mod 11 = 3 , Collisions

h(k) = (88 + 8 + 3×64) mod 11 = 2

0	1	2	3	4	5	6	7	8	9	10
22		88	17	4		28		15	31	10

k = 59 , h'(k) = 59 , h(k) = 59 mod 11 = 4 , Collisions !

h(k) = (59 + 1 + 3) mod 11 = 8 , Collisions

h(k) = (59 + 2 + 12) mod 11 = 7

0	1	2	3	4	5	6	7	8	9	10
22		88	17	4		28	59	15	31	10

c. double hasing with $h_1(k) = k$, $h_2(k) = 1 + (k \bmod (m-1))$

$k = 10$, $h_1(k) = 10$, $h(k) = 10 \bmod 11 = 10$

$k = 22$, $h_1(k) = 22$, $h(k) = 22 \bmod 11 = 0$

$k = 31$, $h_1(k) = 31$, $h(k) = 31 \bmod 11 = 9$

$k = 4$, $h_1(k) = 4$, $h(k) = 4 \bmod 11 = 4$

$k = 15$, $h_1(k) = 15$, $h(k) = 15 \bmod 11 = 4$. Collisions!

$h(k) = [15 + 1 + (15 \bmod (11-1))] \bmod 11 = 21 \bmod 11 = 9$. Collisions!

$h(k) = [15 + 2 + 2(15 \bmod 10)] \bmod 11 = 5$

0	1	2	3	4	5	6	7	8	9	10
22				4	15				31	10

$k = 28$, $h_1(k) = 28$, $h(k) = 28 \bmod 11 = 7$, Collisions!

$h(k) = [28 + 1 + (28 \bmod 10)] \bmod 11 = 6$

0	1	2	3	4	5	6	7	8	9	10
22				4	15	28			31	10

$k = 17$, $h_1(k) = 17$, $h(k) = 17 \bmod 11 = 6$, Collisions!

$h(k) = (17 + 1 + 17 \bmod 10) \bmod 11 = 3$

0	1	2	3	4	5	6	7	8	9	10
22			17	4	15	28			31	10

$k = 88$, $h_1(k) = 88$, $h(k) = 88 \bmod 11 = 0$. Collisions!

$h(k) = (88 + 1 + 88 \bmod 10) \bmod 11 = 9$. Collisions!

$h(k) = (88 + 2 + 2 \times 8) \bmod 11 = 7$

$k = 59$, $h_1(k) = 59$, $h(k) = 59 \bmod 11 = 4$, Collisions!

$h(k) = (59 + 1 + 59 \bmod 10) \bmod 11 = 3$. Collisions

$h(k) = (59 + 2 + 9 \times 2) \bmod 11 = 2$

0	1	2	3	4	5	6	7	8	9	10
22		59	17	4	15	28	88		31	10

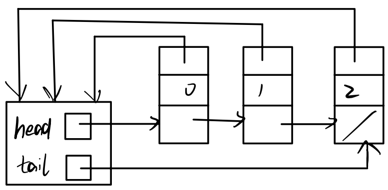
4.

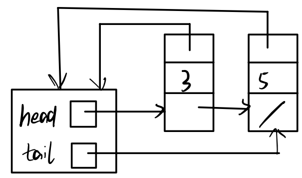
(✓/× — if true / false)

Edge processed	Collection of disjoint sets
initial sets	{a} {b} {c} {d} {e} {f} {g} {h} {i} {j} {k}
(d, i) ✓	{a} {b} {c} {d, i} {e} {f} {g} {h} {j} {k}
(f, k) ✓	{a} {b} {c} {d, i} {e} {f, k} {g} {h} {j}
(g, i) ✓	{a} {b} {c} {e} {f, k} {g, d, i} {h} {j}
(b, g) ✓	{a} {b, g, d, i} {c} {e} {f, k} {h} {j}
(a, h) ✓	{a, h} {b, g, d, i} {c} {e} {f, k} {j}
(i, j) ✓	{a, h} {b, g, d, i, j} {c} {e} {f, k}
(d, k) ✓	{a, h} {b, g, d, i, j, f, k} {c} {e}
(b, j) ×	{a, h} {b, g, d, i, j, f, k} {c} {e}
(d, f) ×	{a, h} {b, g, d, i, j, f, k} {c} {e}
(g, j) ×	{a, h} {b, g, d, i, j, f, k} {c} {e}
(a, e) ✓	{a, h, e} {b, g, d, i, j, f, k} {c}

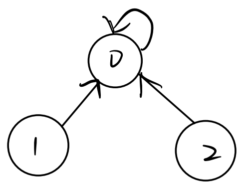
5.

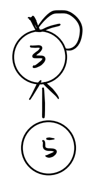
a. list: $S_1 = \{0, 1, 2\}$ $S_2 = \{3, 5\}$





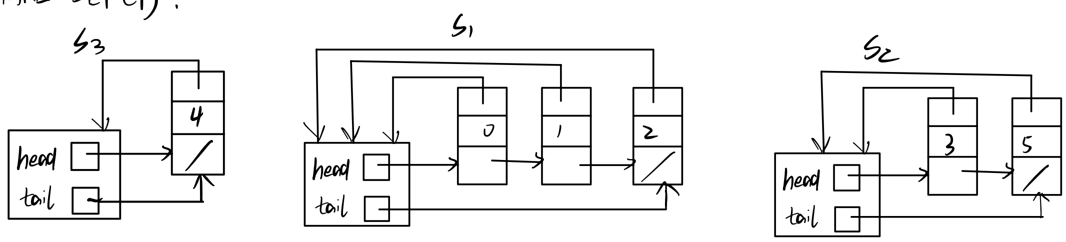
tree: $S_1 = \{0, 1, 2\}$ $S_2 = \{3, 5\}$



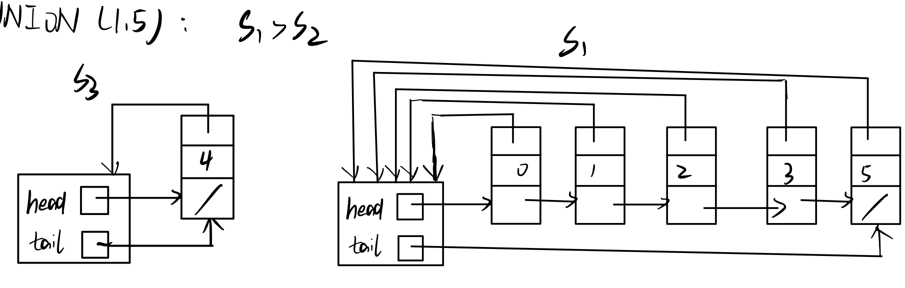


b. i: weighted union heuristic

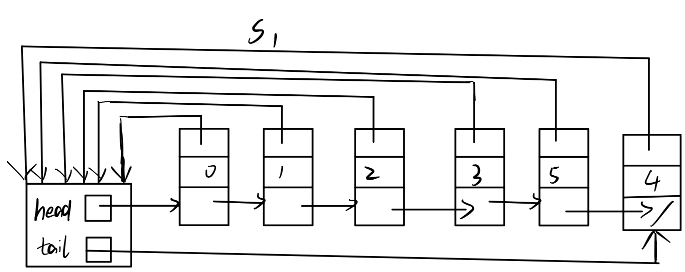
MAKE-SET(4):



UNION(1,5): $S_1 > S_2$

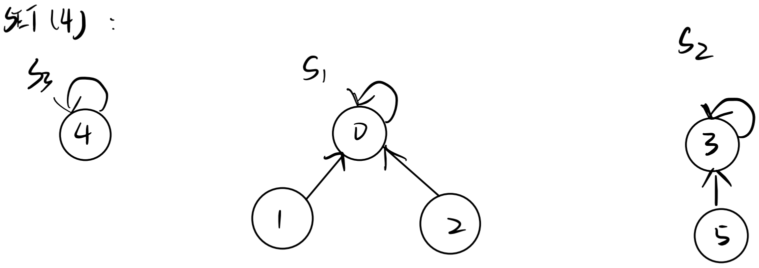


UNION(4,5): $S_1 > S_3$

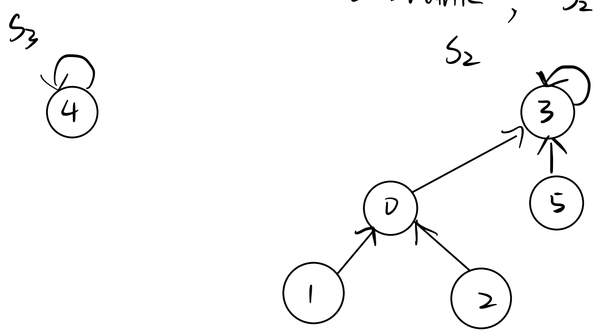


ii: union - by - rank

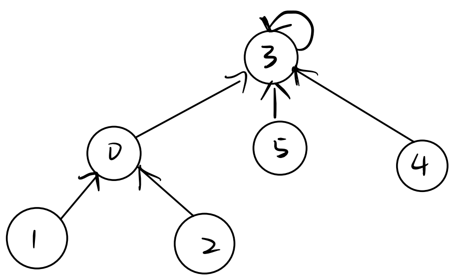
MAKE-SET (4) :



UNION (1,5) : $S_1.rank = S_2.rank$, $S_2.rank+1$



UNION (4,5) : $S_2.rank > S_3.rank$

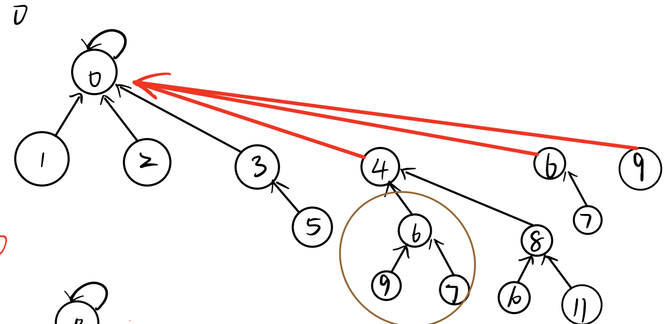


c. The final path is 9 -> 6 -> 4 -> 0

① $x = 9$, $9 \neq 9.p$ is true

$9.p =$ FIND-SET$(9.p)$

$=$ FIND-SET(6)



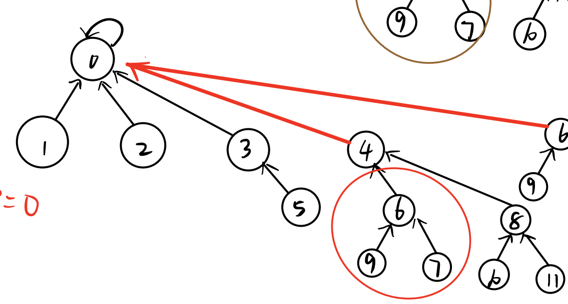
$9.p = 0$

② $x = 6$, $6 \neq 6.p$ is true

$6.p =$ FIND-SET$(6.p)$

$=$ FIND-SET(4)

$6.p = 0$

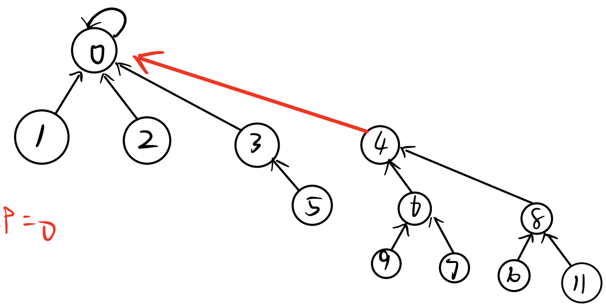


③ $X = 4,\ 4 \neq 4.P$ is true

$4.P = \text{FIND-SET}(4.P)$

$= \text{FIND-SET}(0)$

$4.P = 0$



④ $X = 0,\ 0 \neq 0.P$ is false

Final: