

Answer each of the following problems. If a grammar is requested, you may express it as a formal context-free grammar, or in an equivalent BNF form.

(BNF is just a shorthand notation for context-free grammars.)

(~ is used for the NOT function)

Consider the following BNF grammar:

$$\begin{aligned} \langle F \rangle &\rightarrow 0 \mid 1 \mid \langle V \rangle \mid \sim \langle F \rangle \mid (\langle F \rangle \wedge \langle F \rangle) \mid (\langle F \rangle \vee \langle F \rangle) \\ \langle V \rangle &\rightarrow P \mid Q \mid R \mid U \mid W \mid X \end{aligned}$$

Part 1:

- 1) In addition to production rules, a context free grammar explicitly defines sets of terminal symbols, non-terminal symbols, and a start symbol. What are the terminal and non-terminal symbols of this language? (10 points)

Terminal symbols are literal symbols that may appear in the outputs of the production rules of a formal grammar and which cannot be changed using the rules of the grammar. Applying the rules recursively to a source string of symbols will usually terminate in a final output string consisting only of terminal symbols.

Consider a grammar defined by two rules. Using pictoric marks interacting with each other:

1. The symbol \sqcap can become $\sqcap \sqcap$
2. The symbol \sqcap can become \sqcap

Nonterminal symbols are those symbols that can be replaced. They may also be called simply *syntactic variables*. A formal grammar includes a *start symbol*, a designated member of the set of nonterminals from which all the strings in the language may be derived by successive applications of the production rules. In fact, the language defined by a grammar is precisely the set of *terminal* strings that can be so derived.

Context-free grammars are those grammars in which the left-hand side of each production rule consists of only a single nonterminal symbol. This restriction is non-trivial; not all languages can be generated by context-free grammars. Those that can are called context-free languages. These are exactly the languages that can be recognized by a non-deterministic push down automaton. Context-free languages are the theoretical basis for the syntax of most programming languages.

Here \sqcap is a terminal symbol because no rule exists which would change it into something else. On the other hand, \sqcap has two rules that can change it, thus it is nonterminal. A formal language defined or *generated* by a particular grammar is the set of strings that can be produced by the grammar *and that consist only of terminal symbols*.

So in this language, 0, 1, ~, ^, v, P, Q, R, U, W, X are terminal

F, V are not terminal

- 2) Describe this grammar in plain English. (10 Points)

If we talk about describing the grammar in plain english, so in the first equation we are referring to 0,1 as the terminal symbols and we are adding an OR condition (|) in b/w: The non terminal symbol ϵ evaluated with a or condition with the not of non terminal symbol $\langle F \rangle$, Similarly, the condition is evaluated

with ϵ and with the non terminal AND Condition b/w two $\langle F \rangle$. -

Likewise in the second equation, we are only taking into consideration the terminal symbols, these are basically evaluated using an OR Condition in between.

- 3) Is this grammar ambiguous? That is, is there a string in this language that has two different (leftmost) derivations? (10 points)

Grammar is unambiguous. This is because the parentheses in $\langle F \rangle \wedge \langle F \rangle$ and $\langle F \rangle \vee \langle F \rangle$ already specify the precedence.

However, if one considers logical operations, perhaps there is an ambiguity. For example, $\sim(P \vee Q)$ and $(\sim P \wedge \sim Q)$ are equal in logical operations, but they have different derivations.

$$\langle F \rangle \Rightarrow \sim \langle F \rangle$$

$$\Rightarrow \sim(\langle F \rangle \vee \langle F \rangle)$$

$$\Rightarrow \sim(\langle V \rangle \vee \langle F \rangle)$$

$$\Rightarrow \sim(\langle P \rangle \vee \langle F \rangle)$$

$$\Rightarrow \sim(\langle P \rangle \vee \langle V \rangle)$$

$$\Rightarrow \sim(\langle P \rangle \vee \langle Q \rangle)$$

$$\langle F \rangle \Rightarrow (\langle F \rangle \wedge \langle F \rangle)$$

$$\Rightarrow (\sim \langle F \rangle \wedge \sim \langle F \rangle)$$

$$\Rightarrow (\sim \langle V \rangle \wedge \sim \langle F \rangle)$$

$$\Rightarrow (\sim \langle P \rangle \wedge \sim \langle F \rangle)$$

$$\Rightarrow (\sim \langle P \rangle \wedge \sim \langle V \rangle)$$

$$\Rightarrow (\sim \langle P \rangle \wedge \sim \langle Q \rangle)$$

The above two strings, which are equal in logical operations, possess different derivatives. If this case is considered, the grammar is ambiguous.

Part 2:

- 4) Draw a derivation tree for $(\sim(P \vee Q) \wedge R)$ (15 points)

Definition of ambiguous: A grammar is ambiguous if and only if it generates a sentential form that has two or more distinct parse trees.

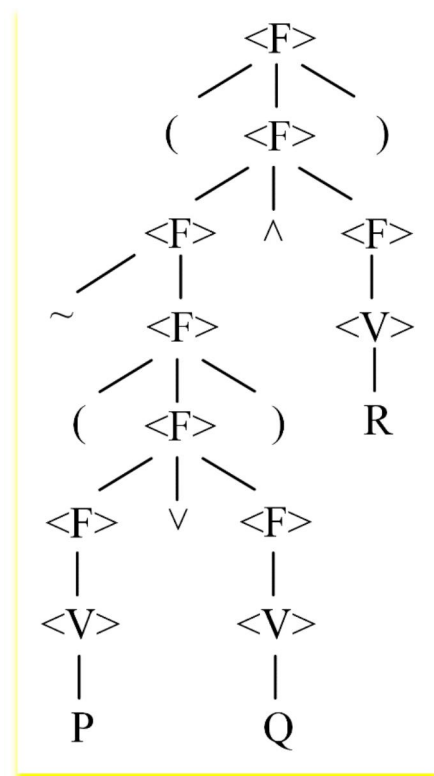
$\langle F \rangle \rightarrow (\langle F \rangle \wedge \langle F \rangle) \rightarrow (\sim \langle F \rangle \wedge \langle V \rangle) \rightarrow (\sim(\langle F \rangle \vee \langle F \rangle) \wedge \langle V \rangle)$

$\rightarrow (\sim(\langle V \rangle \vee \langle V \rangle) \wedge \langle V \rangle) \rightarrow (\sim(P \vee Q) \wedge R)$

HINT: What are the derivations of the language? What are the possible outcomes based on the terminals and the non-terminal values?

Part 3:

- 5) Show a parse tree for the sentence " $(\sim(P \vee Q) \wedge R)$ ". (10 points)



- 6) Write a BNF (or context-free) grammar for the language composed of sentences of the following form: (15 points)

$a^{2k} \quad x^{2j-1} \quad b^{3k}$

I.e., sentences consisting of a sequence of a's, followed by a sequence of x's of odd length, in turn followed by a sequence of b's of length exactly $3/2$ as long as the sequence of a's, where $k > 0$ and $j > 0$.

(E.g., the language includes aaaaxxxxxxxxxxbbbb and aaxbbb but not aaxbbb or bbxaa or abx or axxxbb.)

$S \rightarrow AXB$

$A \rightarrow aaA|aa$

$X \rightarrow x|xXx$

$B \rightarrow bbbB|bbb$

Part 4:

- 7) Can you modify your grammar so that the resulting language is the proper subset of the original language in which $j=k$ (thus resulting in sentences containing one fewer x's than a's)? Justify your answer. (10 points)

S --> MB

B --> bbbB|bbb

M --> aax|aaMxx

Part 5:

The syntax of the Klingon language is quite simple, yet only Klingons can speak it without making mistakes. The alphabet of the language is {o,r,z,&}, where & stands for a space. The Klingon language is such that every phrase has to be of odd number of syllables.

Grammar:

<WORD> = <SYLLABLE>

<ID> -> o | r | z | &

<PLOSIVE> -> <STOP>

<STOP> -> &

<SYLLABLE> -> (<SYLLABLE>)

| <SYLLABLE> + <PLOSIVE>

| <SYLLABLE> + <SYLLABLE> + <SYLLABLE> + <PLOSIVE>

| <ID>

The rule for <WORD> is constructed in such a way that words can only have an odd number of <SYLLABLE>'s. The rules for <SYLLABLE>, <PLOSIVE>, and <STOP> can be combined to make the following <SYLLABLE>'s **(none other are possible)**:

ro or ror roz oro

zo oz zoz zor ozo

- 8) Identify which of the following speakers is the secret agent in Klingon disguise? (10 points)

"Vijr" is the secret agent. Reason:

Consider Vijr's message "rozozrooz". None of the syllables have "rz" as substring of a syllable or a syllable itself. So this message must be formed using "rozoz" + "rooz".

- 9) Explain why you believe they are the secret agent (hint: they will be the person whose words aren't odd syllables) (10 points)

Romulus: ro & ororozozo & roz & zorrozo

Vulca: orzoroozor & ozo

Viijr: zoz & oz & oroozoz & rozozrooz

Only way to reach "rooz" is using 2 syllables, "ro" + "oz" (because there are no 1 letter or 4 letter syllables)

Only way to reach "rozoz" is using 2 syllables, "ro" + "zoz" OR "roz" + "oz" (because there are no 1 letter or 5 letter syllables)

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