Appendices

A. Some suggestions to make traveling from one point to another simpler.

B. How to decide whether a point is inside a convex quadrilateral

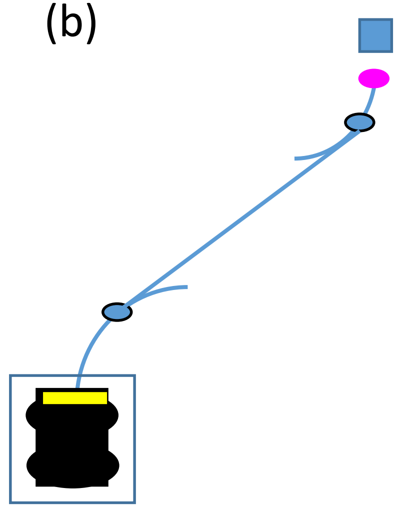
C. How to decide whether two line segments cross each othe

Appendix A

Some suggestions to make traveling from one point to another simpler by making 90 degree turns as much as possible. This will result in a path which is not the shortest, but it simplifies the calculation a lot.

I. For the scenario where robot and image facing opposite directions

(b)

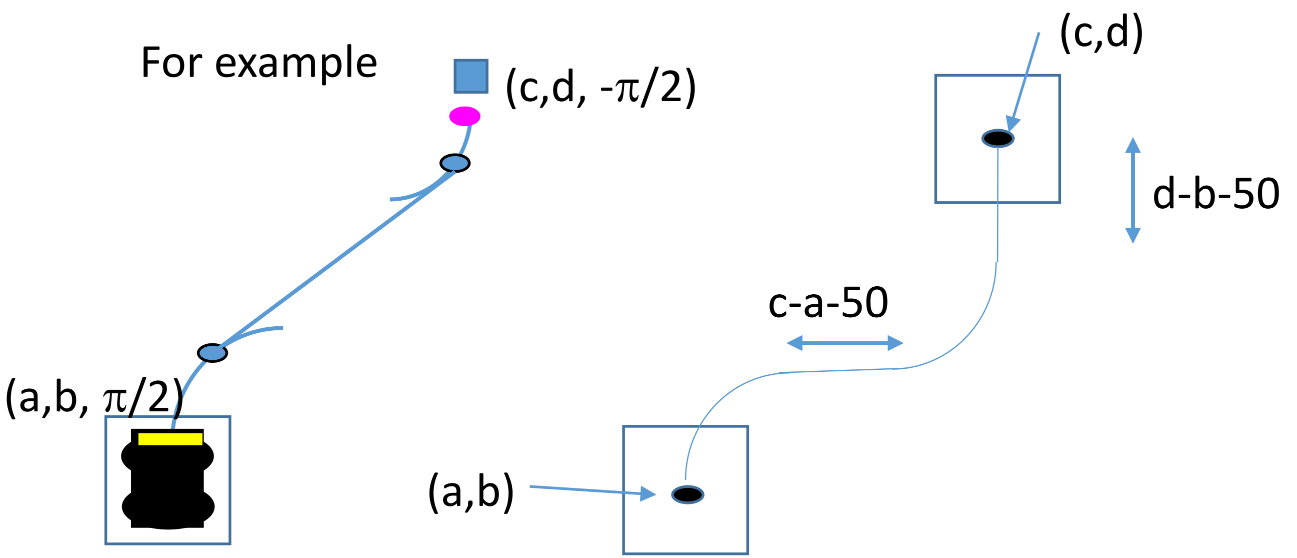


In the briefing lecture, we said

- Need to turn right, go straight, turn left, changing directions 2 times
- Find two points at the two changes of directions and the point to stop

But you may try a simpler way as suggested here first

A very simple path by always turning 90 degrees

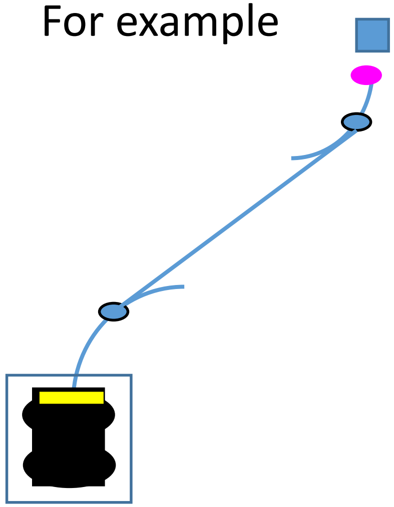


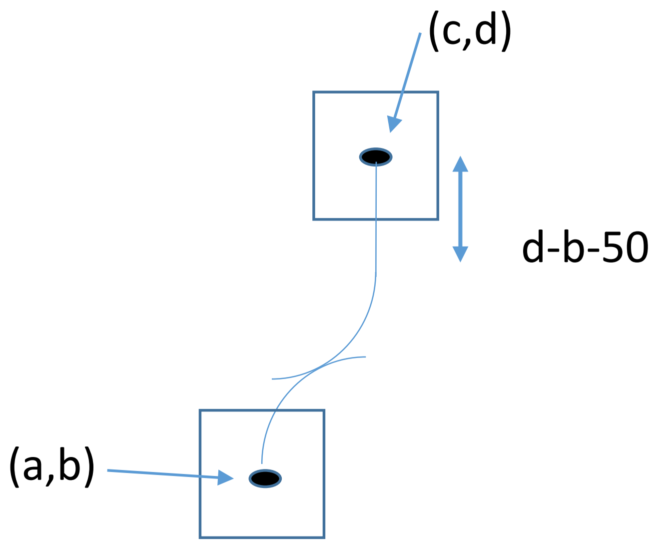
Robot is at (a, b, $\pi/2$) and need to get to (c, d,$\pi/2$)

1) Consider the example c>a and d>b
2) If c-a-50>0, d-b-50>0, start from (a,b), turn right 90 degrees till facing east, go straight for c-a-50, turn left 90 degrees, go straight for d-b-50
3) If c-a-50=0, the right turn is followed immediately by left turn
4) If d-b-50 = 0, stop after the left turn

A very simple path by always turning 90 degrees

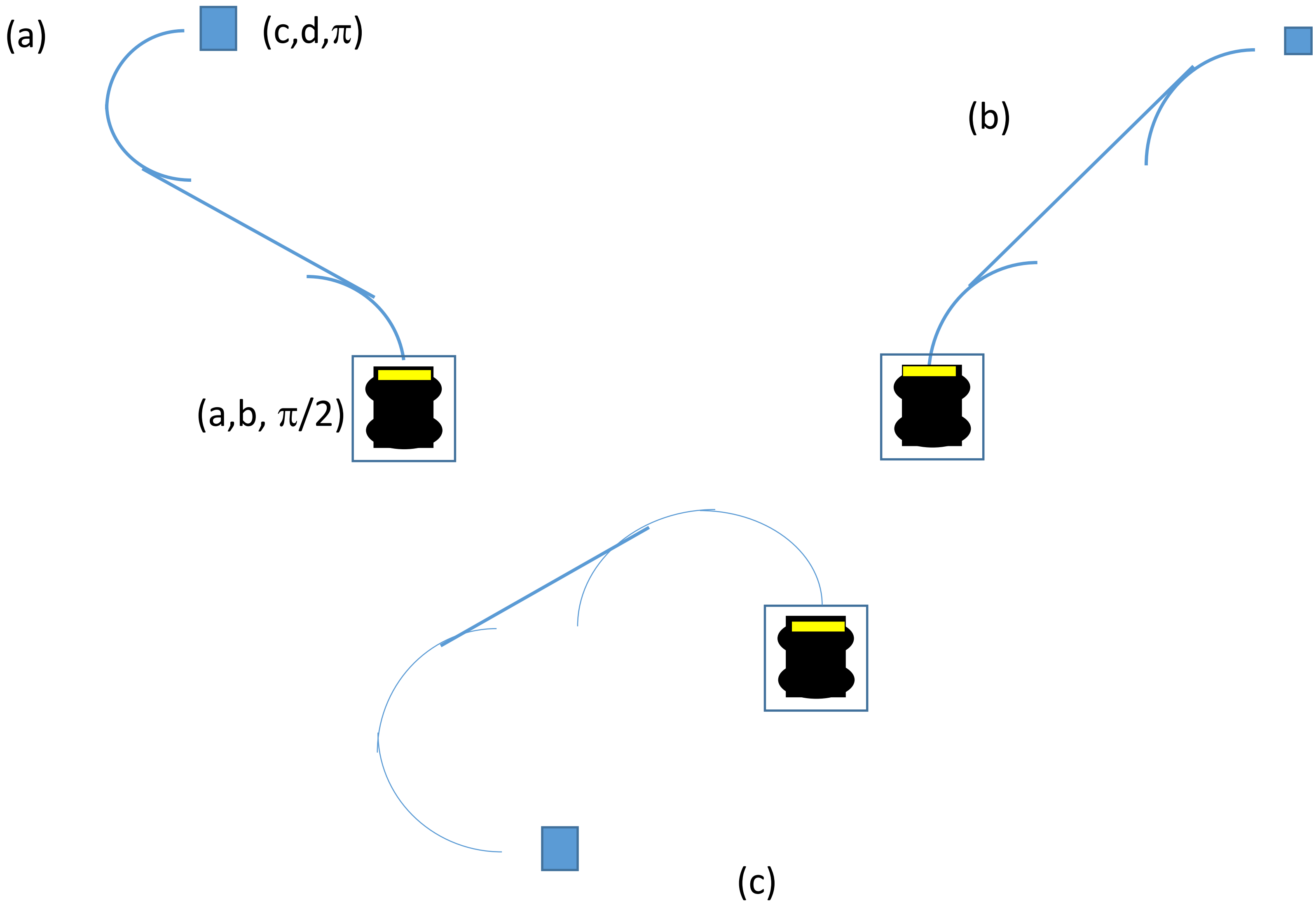
For example





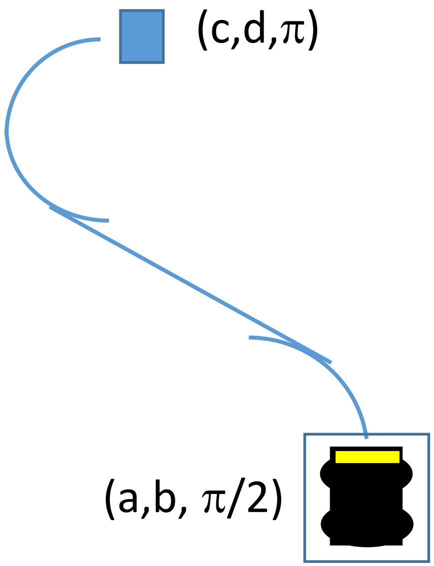
5) If $c-a<50$ and $c>a$, turn right till x-coordinate is $(c+a)/2$, turn left till x-coordinate is c, go straight till y-coordinate is d
6) If $d-b<50$, the simplest is to reverse till y-coordinate is b' such that $d-b'=50$

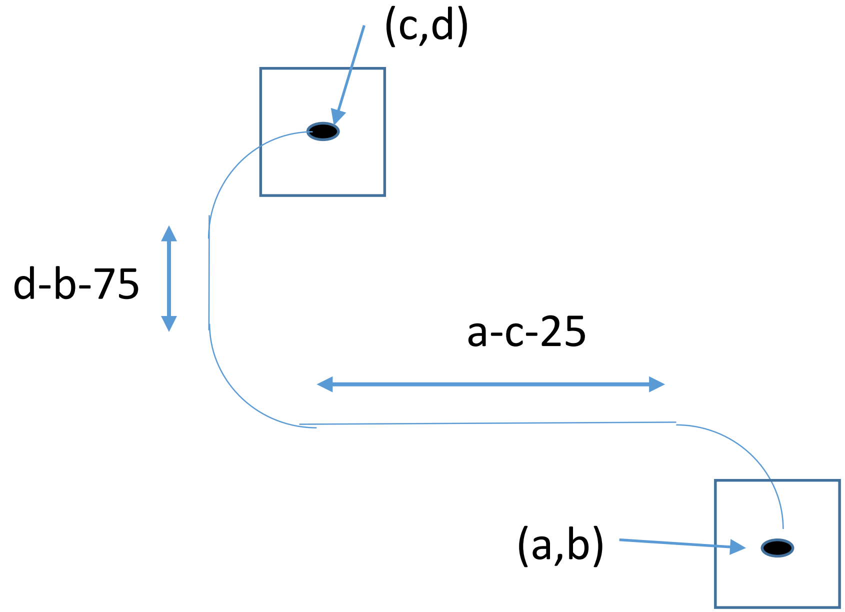
II. Robot and image facing has a difference of $\pi/2$ or $-\pi/2$



A very simple path by always turning 90 degrees

E.g.



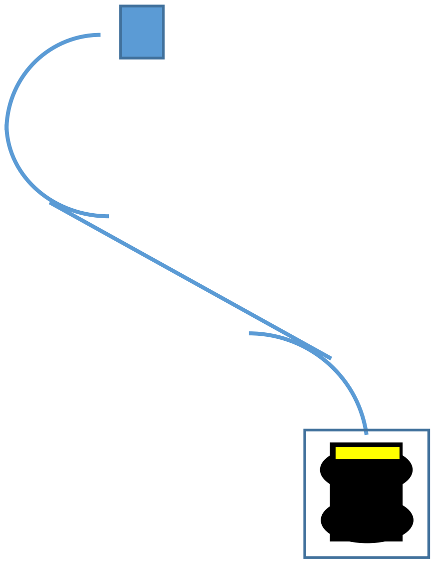


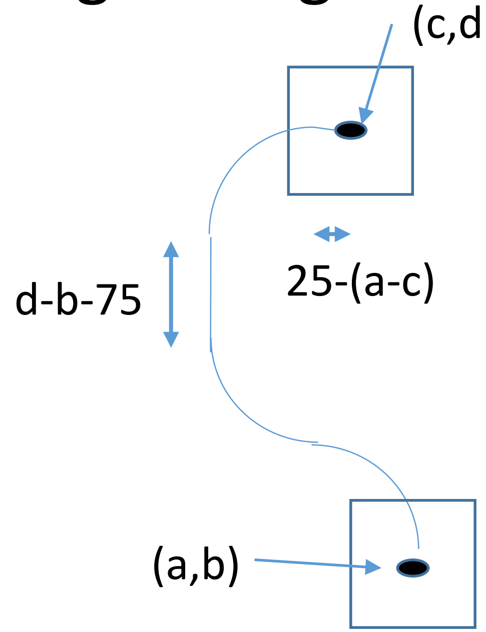
Robot is at (a, b, $\pi/2$) and need to get to (c, d, 0)

1) Consider $c<a$ and $d>b$, a-c-25>0, d-b-75>0
2) Start from (a,b), turn left 90 degrees till facing west, go straight for a-c-25, turn right 90 degrees, go straight for d-b-75, turn right 90 degrees
3) If a-c-25=0, the left turn is followed immediately by right turn
4) If d-b-75 = 0, the 2nd right turn is followed immediately by the 1st right turn

A very simple path by always turning 90 degrees

E.g.





Robot is at (a, b, $\pi/2$) and need to get to (c, d, π)

5) If a-c<25, the left turn is followed immediately by right turn, then go straight for d-b-75, turn right 90 degrees, go straight for 25-(a-c)
6) If d-b<75, the simplest is to reverse till y-coordinate is b' such that d-b'=75

Appendix B

How to decide whether a point is inside a convex quadrilateral.
If you need to decide whether the Robot will go to a location which is inside a virtual obstacle's space, you may use this.

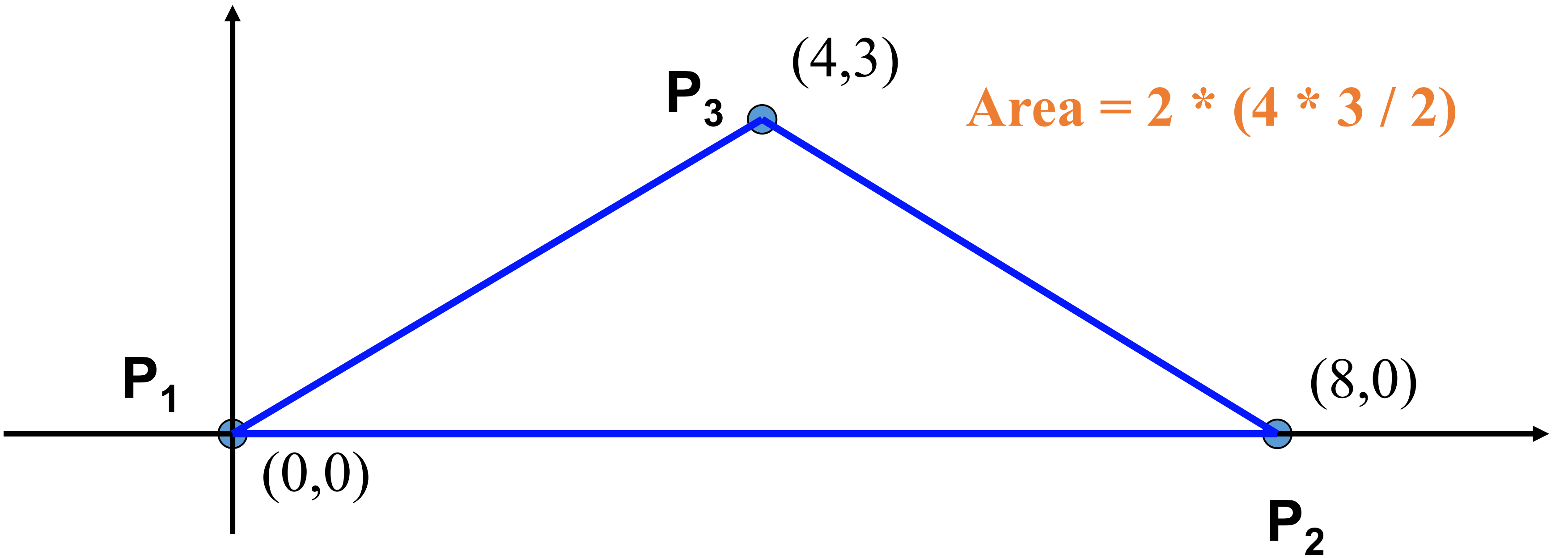
- **A useful fact from analytical geometry:**

  $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$ and $P_3 = (x_3, y_3)$ are three arbitrary points in the plane, then the area of the triangle $\Delta P_1P_2P_3$ is equal to one half of the magnitude of the determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = x_1y_2 + x_3y_1 + x_2y_3 - x_3y_2 - x_2y_1 - x_1y_3$$

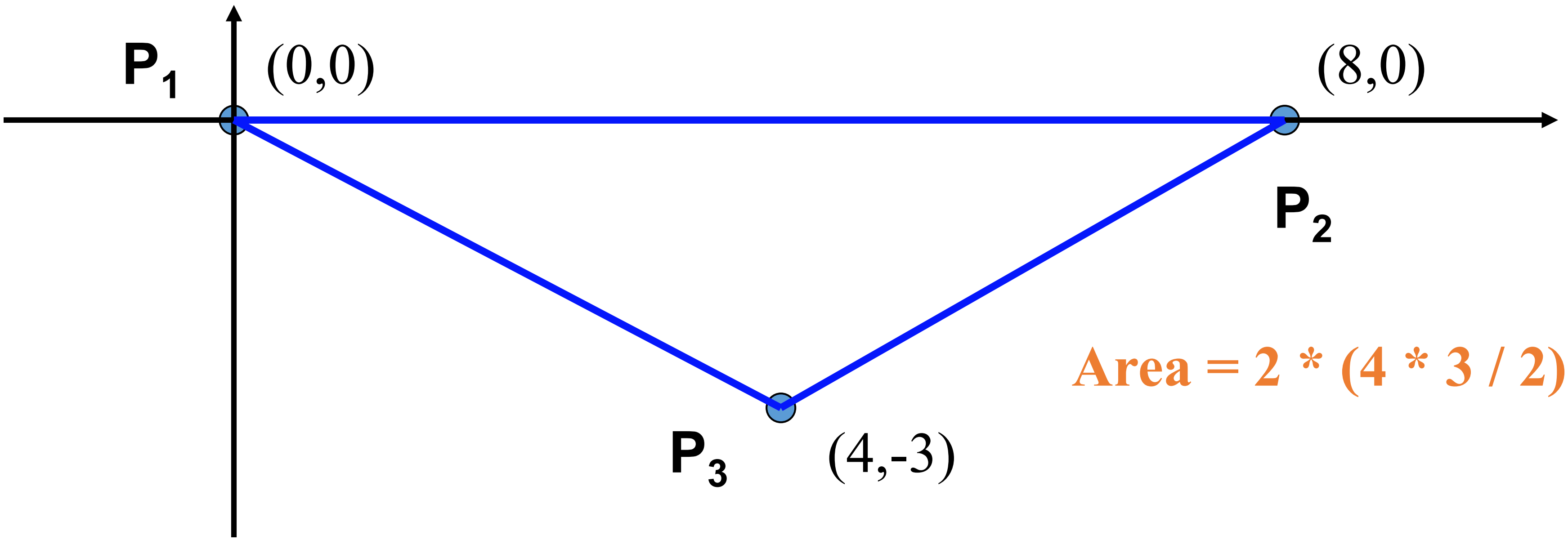
The sign of the determinant is positive if and only if the point P_3 is to the left of the line from P_1 to P_2 .

For example:



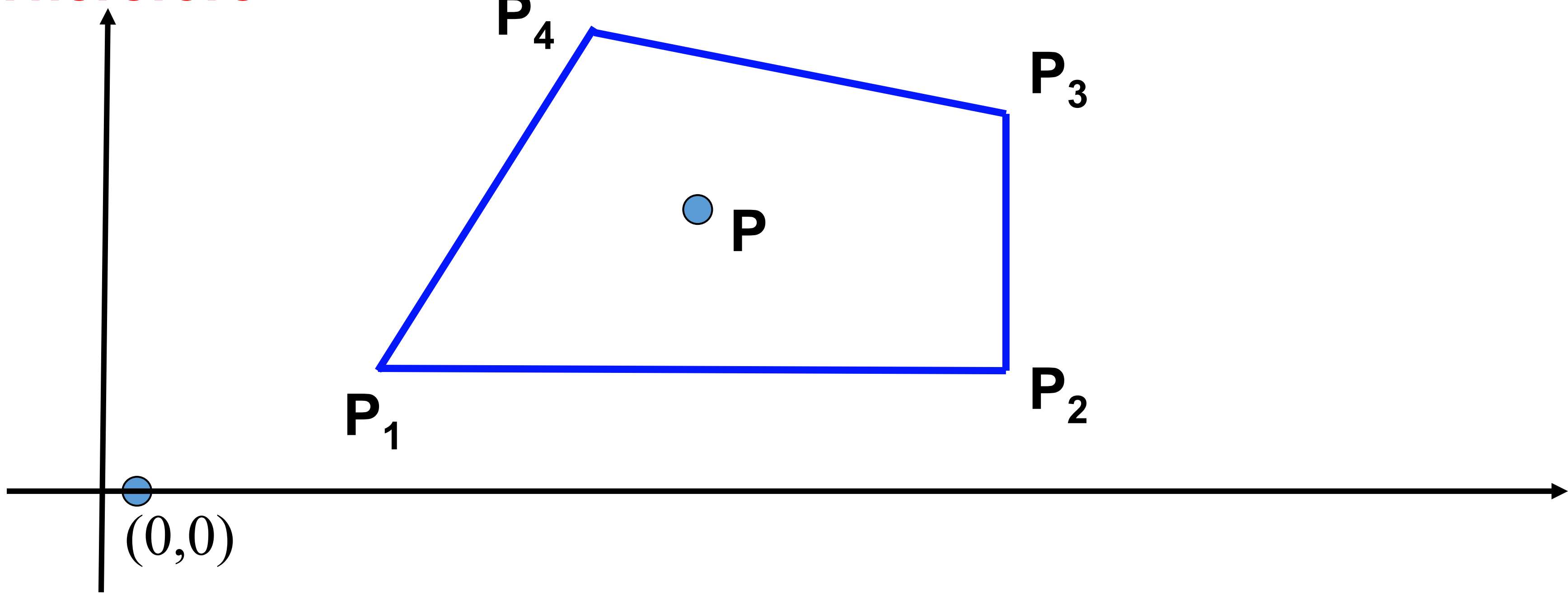
$$\begin{vmatrix} 0 & 0 & 1 \\ 8 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} = 0 + 0 + 24 - 0 - 0 - 0 = 24$$

For example:



$$\begin{vmatrix} 0 & 0 & 1 \\ 8 & 0 & 1 \\ 4 & -3 & 1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 - 0 = -24$$

Therefore



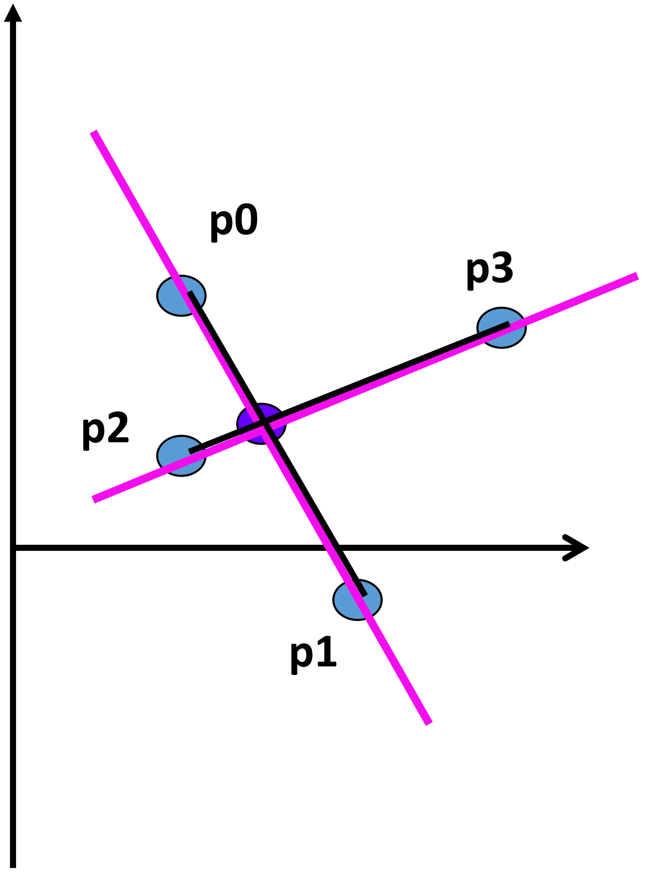
If the point P is to the left of the line from P_1 to P_2 , to the left of the line from P_1 to P_2 , to the left of the line from P_1 to P_2 , to the left of the line from P_1 to P_2 , then P is inside the convex quadrilateral.

Appendix C

How to decide whether two line segments cross each other.
If you want to decide whether a robot’s travelling straight will get into a virtual obstacle’s space, you may check whether the straight line segment of the robot’s path cross any boundary of a virtual obstacle’s space.

Determine whether 2 line segments intersect (method 1)

- **We are given line segments (p0, p1) and (p2, p3) on the plane and we have to determine whether they intersect or not.**
- **We find the intersection point of the 2 lines that contain the 2 line segments respectively, and then check that the intersection is on the segments.**



Recap:

- **The straight line through two points (x_0, y_0), (x_1, y_1) in the coordinate plane can be defined by the equation**

  $$ax + by = c,$$

  where $a = y_1 - y_0$, $b = x_0 - x_1$, $c = x_0y_1 - y_0x_1$

- **So we find the 2 lines**

  $$a_1x + b_1y = c_1, L_1 \text{ through p0, p1}$$

  $$a_2x + b_2y = c_2 \; L_2 \text{ through p2, p3}$$

  and compute the intersection (x, y).

- **For example, we substitute**

  $$y = \frac{c_1 - a_1 x}{b_1}$$

  into

  $$x = \frac{c_2 - b_2 y}{a_2}$$

- Finally we check that:
  $\min(x0, x1) \le x \le \max(x0, x1)$ and
  $\min(x2, x3) \le x \le \max(x2, x3)$

- Cases that need special treatment:

  (1) The system of 2 equations has no solution

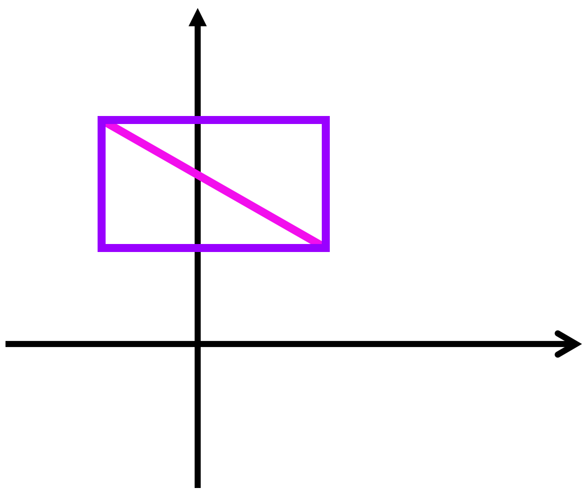
  (2) The system of 2 equations has infinite solutions

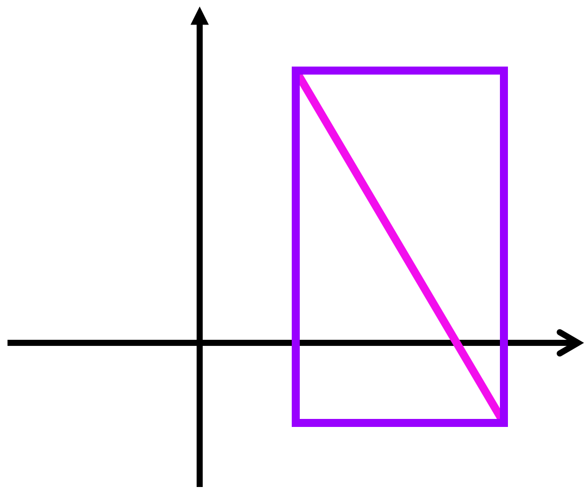
- This method needs divisions in several places
- Division generates truncation errors that can produce the wrong final results
- However we do not need to find the intersection point in order to detect its existence

Determine whether 2 line segments intersect (method 2)

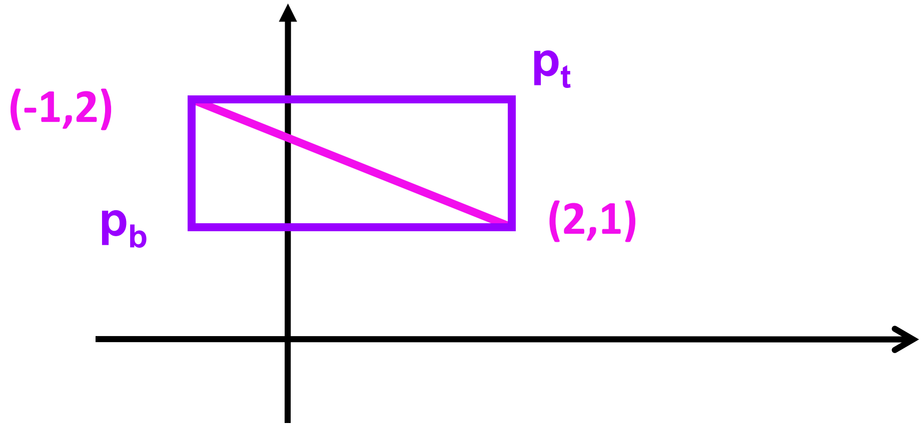
- A two-stage process is used to determine whether 2 line segments intersect.
- The 1st stage is quick rejection: 2 line segments cannot intersect if their bounding boxes do not intersect.
- The 2nd stage decides whether each segment "straddles" the line containing the other.

Definition: The bounding box of a geometric figure is the smallest rectangle that contains the figure and whose segments are parallel to the x-axis and y-axis.





The bounding box of a line segment is represented by the rectangle (p_b, p_t) with the lower left point $p_b = (x_b, y_b)$ and upper right point $p_t = (x_t, y_t)$ where $x_b = \min(x_1, x_2)$,
$y_b = \min(y_1, y_2)$, $x_t = \max(x_1, x_2)$ and $y_t = \max(y_1, y_2)$.



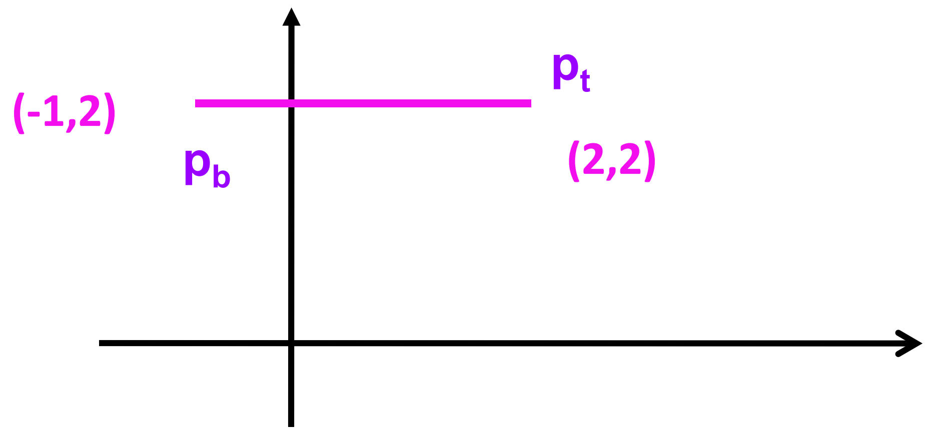
$p_b = (-1, 1)$

$p_t = (2, 2)$

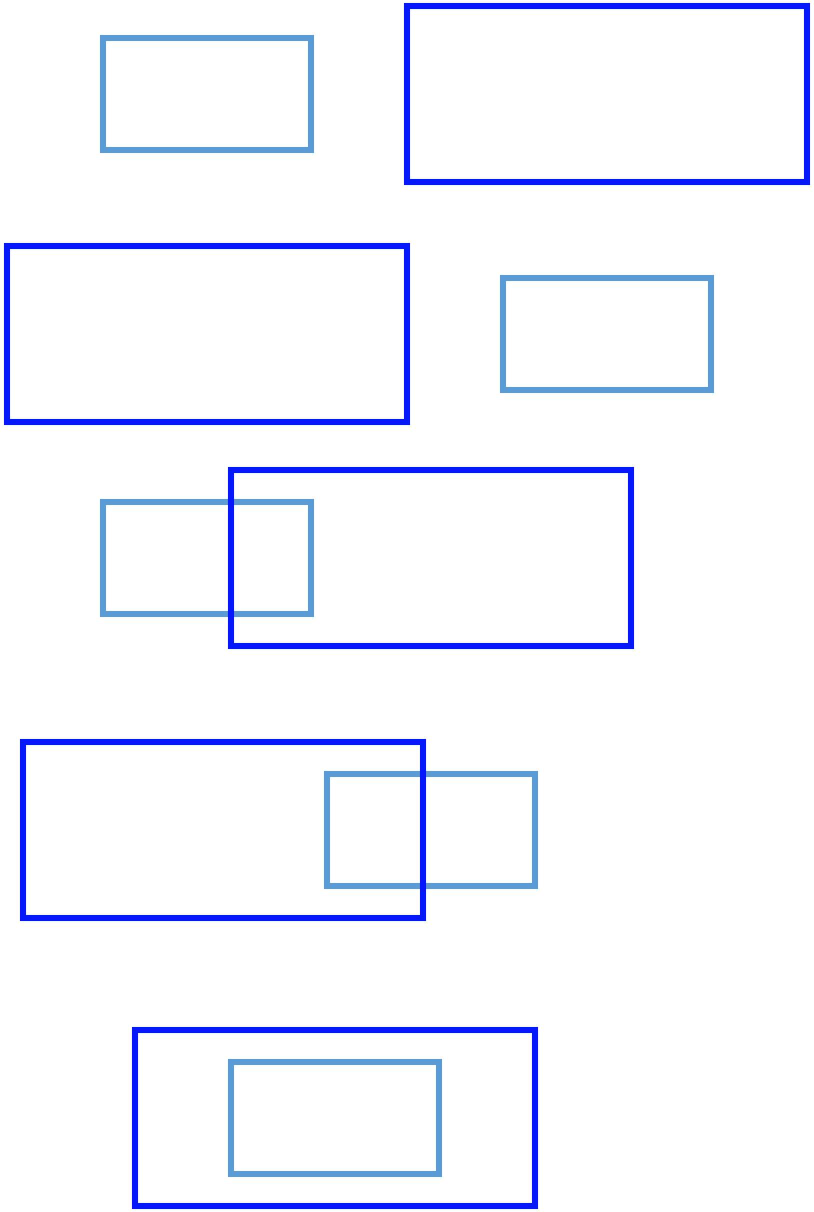
A special case:

$p_b = (-1, 2)$

$p_t = (2, 2)$



$\boxed{x_1 < x_2 \text{ and } x_3 < x_4}$



$x_1 < x_2 < x_3 < x_4$

$x_3 < x_4 < x_1 < x_2$

$x_1 < x_3 < x_2 < x_4$

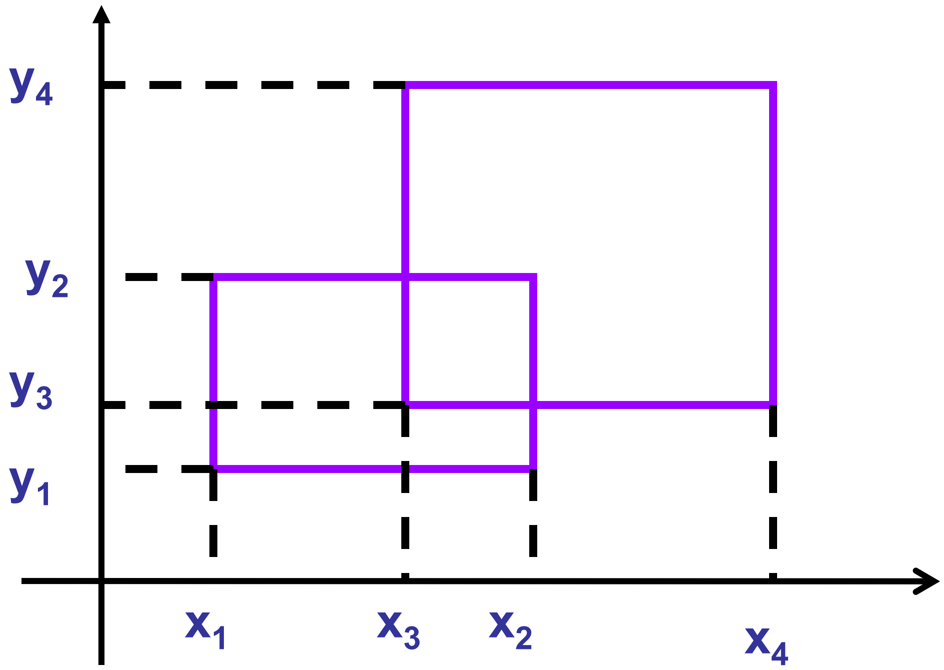
$x_3 < x_1 < x_4 < x_2$

$x_3 < x_1 < x_2 < x_4$

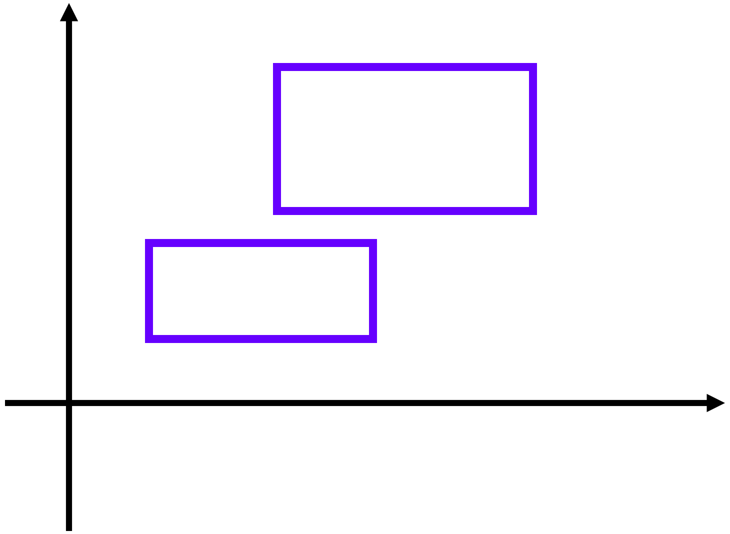
$x_1 < x_3 < x_4 < x_2$

Two rectangles, represented by their lower left and upper right points (p_b, p_t) and (p_b', p_t') respectively, intersect if and only if

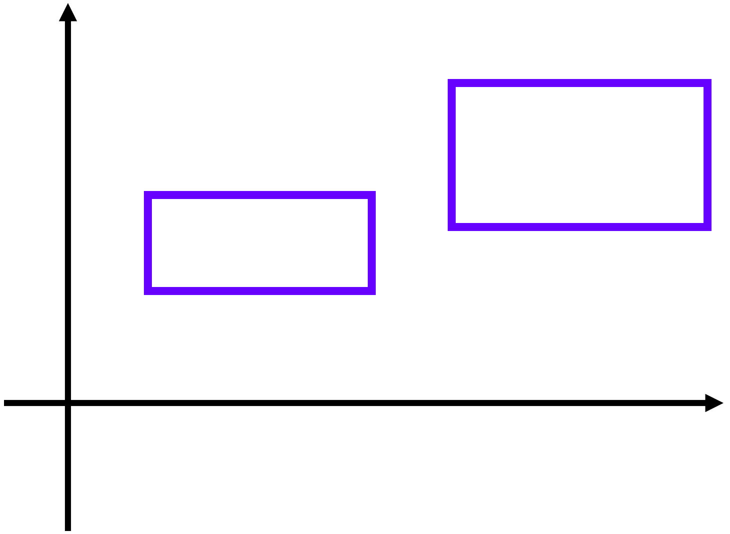
$(x_2 \geq x_3) \wedge (x_4 \geq x_1) \wedge (y_2 \geq y_3) \wedge (y_4 \geq y_1)$ is true.



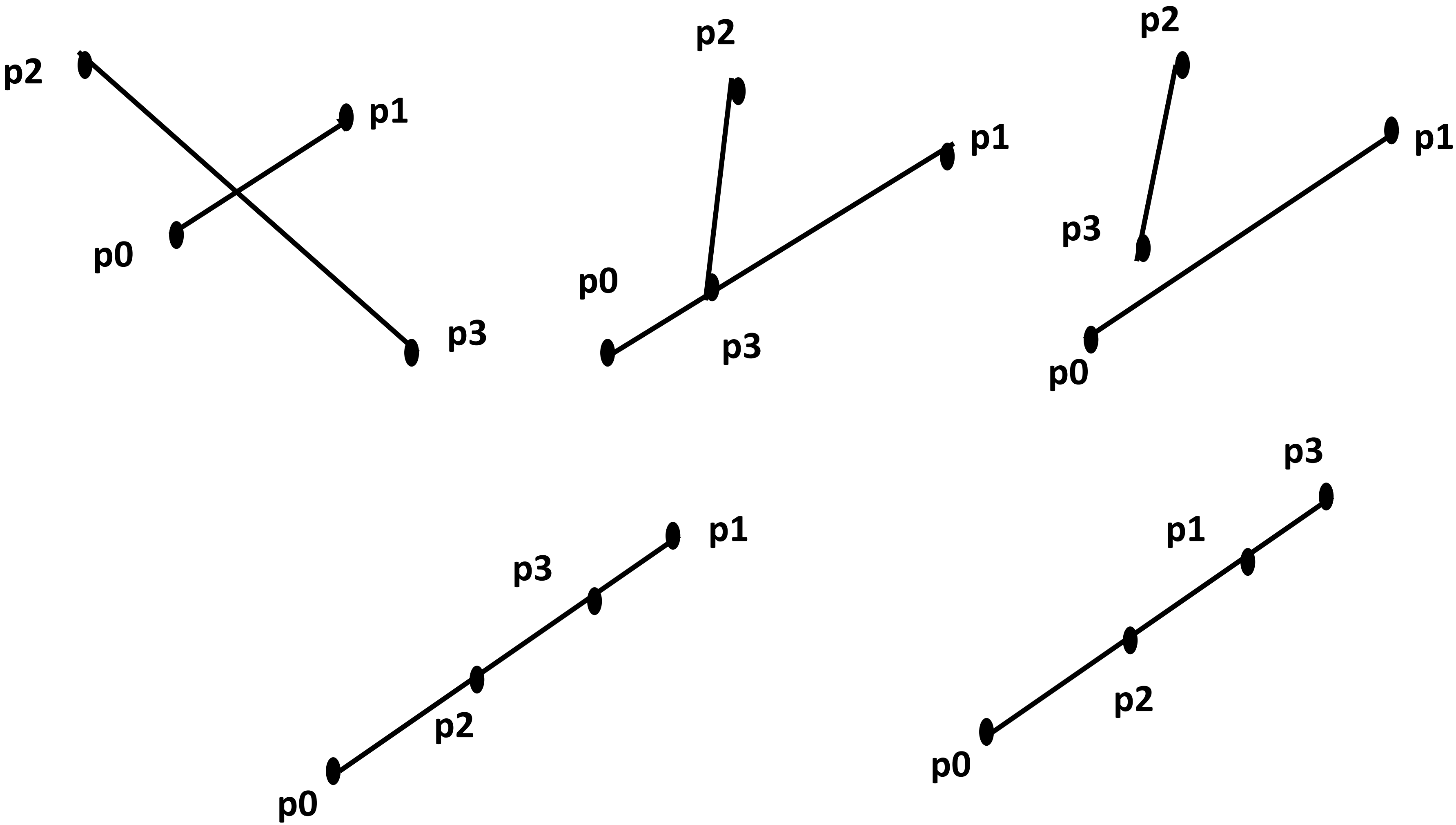
$(x_2 \geq x_3) \wedge (x_4 \geq x_1)$ only:



$(y_2 \geq y_3) \wedge (y_4 \geq y_1)$ only:

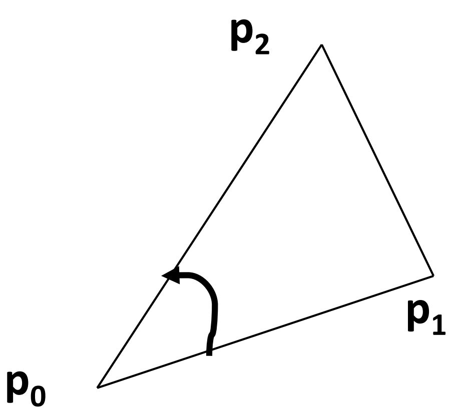


After the 2 line segments pass the rejection test (so two bounding boxes do intersect): check whether each line segment “straddles” the line containing the other



Determining whether p_2 is on the left of line segment $\overline{p_0 p_1}$

For examples



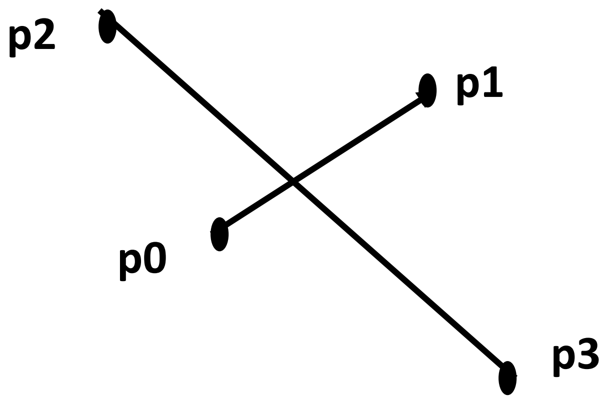
$p_0 = (2, 3)$, $p_1 = (5, 4)$, $p_2 = (4, 7)$,

$(p_2 - p_0) \times (p_1 - p_0)$

$$= \begin{vmatrix} 4-2 & 5-2 \\ 7-3 & 4-3 \end{vmatrix}$$

$= 2 - 12 = -10$

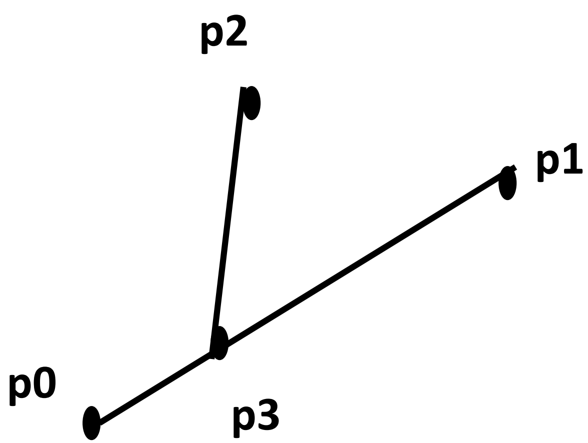
p_2 is on the left of line segment $\overline{p_0 p_1}$.



$(p2 - p0) \times (p1 - p0) < 0$

$(p3 - p0) \times (p1 - p0) > 0$

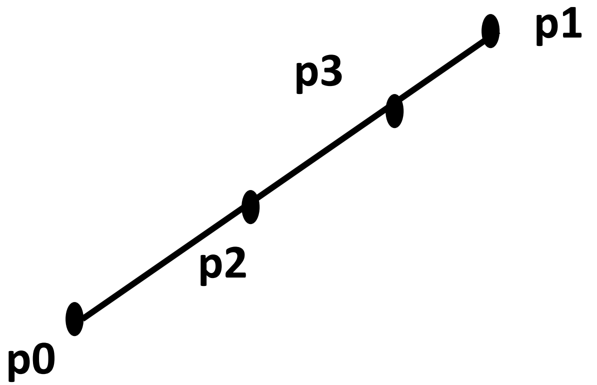
P2 is on the left of p0p1, p3 is on the right of p0p1



$(p2 - p0) \times (p1 - p0) < 0$

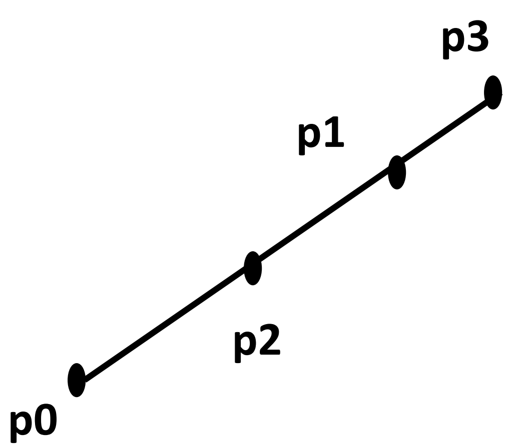
$(p3 - p0) \times (p1 - p0) = 0$

P2 is on the left of p0p1, p3 is collinear to p0p1



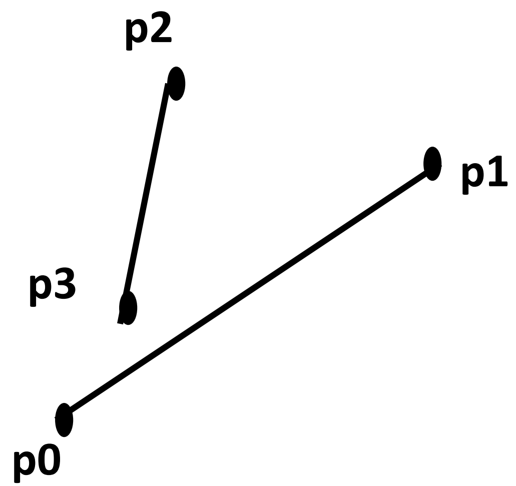
$$(p2 - p0) \times (p1 - p0) = 0$$

$$(p3 - p0) \times (p1 - p0) = 0$$



$$(p2 - p0) \times (p1 - p0) = 0$$

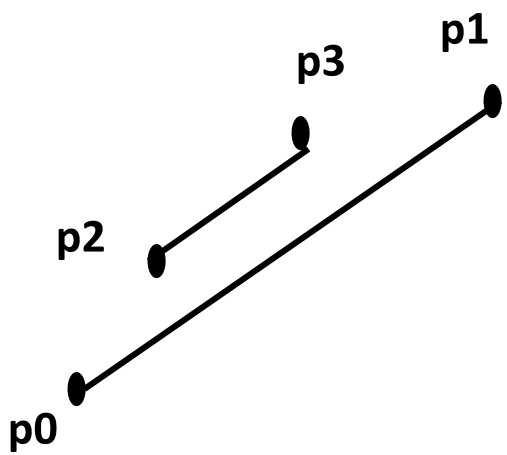
$$(p3 - p0) \times (p1 - p0) = 0$$



$$(p2 - p0) \times (p1 - p0) < 0$$

$$(p3 - p0) \times (p1 - p0) < 0$$

P2 and p3 are both on the left of p0p1. BUT p1 is on the left of p2p3 and p0 is on the right of p2p3



Conclusion:

Two line segments intersect if and only if

(i) they pass the rejection test and

(ii) $(p2 - p0) \times (p1 - p0)$ and $(p3 - p0) \times (p1 - p0)$ do not have the same signs (should test from both segments)