Homomorphic Machine Learning with SEAL

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Homomorphic Machine Learning

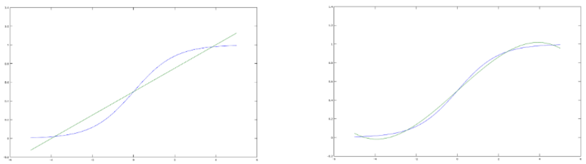
To run learning phase of machine learning in encrypted state, what is the big problem? **message size problem**

Data: X, y
Result: W
$\alpha :$ learning rate, $N :$ number of samples, $K :$ number of variables;
initialization W (as origin point);
initialization r (as origin point);
for i *in* $[0, N)$ **do**
  compute inner product $\langle \vec{X}_i, \vec{W} \rangle = V_i$;
  compute approximate sigmoid f to V_i;
end
for i *in range* $[0, K)$ **do**
  compute derivative $\Delta_i = \sum_{j \in B_k} (Y_j - f(V_j)) \cdot X_{j,i}$;
  add residue to derivative $\Delta_i = \Delta_i + r_i$;
  extract sign of derivative $\mathsf{sign} = 1$ if $\Delta_i > 0$ otherwise -1;
  update weight vector element $W_i = W_i + \alpha \cdot \mathsf{sign}$;
  update residue vector element $r_i = \Delta_i + \alpha \cdot \mathsf{sign}$;
end

Instead of using gradient decent, “1-Bit Stochastic Gradient Descent” used only sign information in learning and this solves the message size problem [1].

Polynomial Approximate of Sigmoid Function

- **Mini-max polynomial approximate**
  - Range [-5, 5]
  - Degree 1 and 3 for each

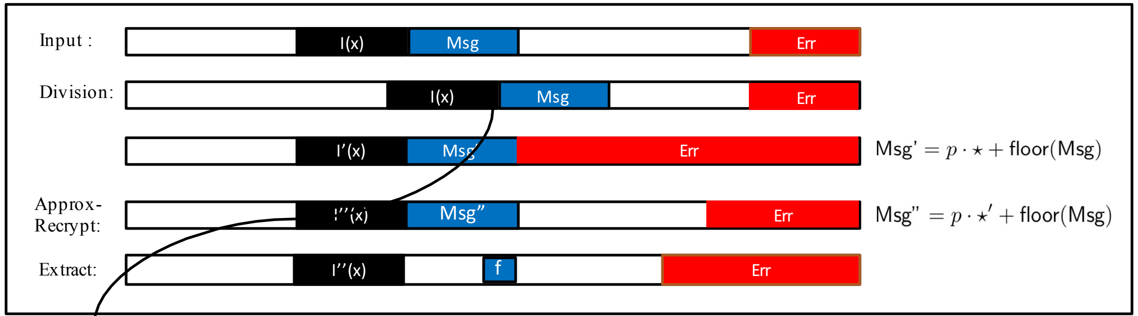


(a) $f(x) = 0.5 + 0.125x$ (b) $f(x) = 0.5 + 0.197x - 0.004x^3$

Figure 1: Mini-max approximate for sigmoid

Homomorphic Flooring using SEAL

FV style encryption of message m: $c_1(x) + c_2(x) \cdot sk(x) = q \cdot I(x) + \Delta \cdot \mathsf{msg} + \mathsf{err}$



$$(c_1(x), c_2(x)) \Rightarrow \frac{q}{p^r} \cdot m(x) + e(x) + q \cdot I(x)$$

$$\left(\frac{c_1(x)}{p^{r-1}}, \frac{c_2(x)}{p^{r-1}}\right) \Rightarrow \frac{q}{p^r} \cdot \frac{m(x)}{p^{r-1}} + \frac{e(x)}{p^{r-1}} + \frac{q}{p^{r-1}} \cdot I(x)$$

$$= \frac{q}{p^r} \cdot \left(p \cdot I(x) + \left\lfloor \frac{m(x)}{p^{r-1}} \right\rfloor\right) + [m(x)]_{p^{r-1}} + \frac{e(x)}{p^{r-1}}$$

- Only need to keep lowest digit part, this makes our recrypt faster with approx-recrypt.
- Full process is little-bit expensive than bootstrapping, but we can perfume **bootstrapping** and **flooring** at once.
- Sign Extraction can be expressed by floor function, and we can adapt SIMD technique.

$$-p^2 < x < p^2 \Rightarrow 2 \cdot \left\lfloor \frac{x}{p^2} \right\rfloor + 1 = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

Encryption Parameter

- We used SEAL with RNS-FV version and bootstrapping implementation [2].
- Each coefficient modulus chain is 60-bit prime integer

$\texttt{coeff\_mod\_count} = 16$
$\texttt{poly\_modulus} = X^{32768} + 1$
$\texttt{plain\_modulus} = 127^3$

Setting and Result

- Timing for each iteration is same and bootstrapping is included in homomorphic floor, so we can perform any number of iteration (total running time will increase linear to iteration number).
- Number of samples: 1000 ~ 2000
- Number of features: 18 (include 1 dummy)
- Timing result: 300 seconds per iteration, $\texttt{num\_thread} = 8$

[1] Seide, Frank, et al. "1-bit stochastic gradient descent and its application to data-parallel distributed training of speech dnns." *Fifteenth Annual Conference of the International Speech Communication Association*. 2014.
[2] Bajard, Jean-Claude, et al. "A full RNS variant of FV like somewhat homomorphic encryption schemes." *Selected Areas in Cryptography-SAC*. 2016.