CAE LAB 2

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Subject : Combined stress

1. Prelab

1.1. Generalized Hook's Law

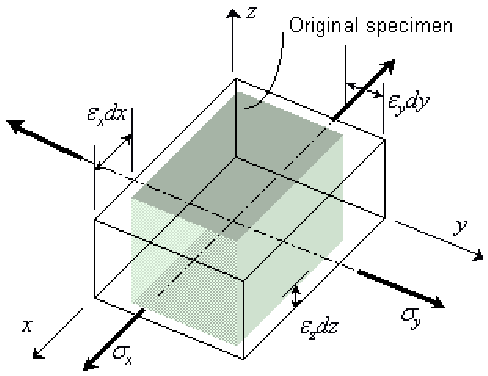

Figure 1. Generalized Hooke's law

The generalized Hooke's Law reveals that strain can exist without stress. For example, if the member is experiencing a load in the y-direction (which in turn causes a stress in the y-direction), the Hooke's Law shows that strain in the x-direction does not equal to zero. This is because as material is being pulled outward by the y-plane, the material in the x-plane moves inward to fill in the space once occupied, just like an elastic band becomes thinner as you try to pull it apart. In this situation, the x-plane does not have any external force acting on them but they experience a change in length. Therefore, it is valid to say that strain exist without stress in the x-plane.

$$\varepsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$$

$$\varepsilon_y = -v\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\sigma_x}{E} - v\frac{\sigma_y}{E} + v\frac{\sigma_z}{E}$$

1.2. Strain gauge gadget

$$\varepsilon_x' = \varepsilon_x \cos^2\theta + \varepsilon_y\, sin^2\theta + \gamma_{xy}\, sin\,\theta cos\theta$$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y\, sin^2\theta_a + \gamma_{xy}\, sin\,\theta_a cos\theta_a\ \ (\theta_a = 45\ [deg])$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y\, sin^2\theta_b + \gamma_{xy}\, sin\,\theta_b cos\theta_b\ \ (\theta_a = 0\ [deg])$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y\, sin^2\theta_c + \gamma_{xy}\, sin\,\theta_c cos\theta_c\ (\theta_a = -45\ [deg])$$

According to the equations above, $\varepsilon_a, \varepsilon_b, \varepsilon_c$ can be calculated with following equations :

$$\varepsilon_a = \frac{\varepsilon_x}{2} + \frac{\varepsilon_y}{2} + \frac{\gamma_{xy}}{2}$$

$$\varepsilon_b = \varepsilon_x$$

$$\varepsilon_c = \frac{\varepsilon_x}{2} + \frac{\varepsilon_y}{2} - \frac{\gamma_{xy}}{2}$$

With the equations above, $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ can be expressed with following equations :

$$\varepsilon_x = \varepsilon_b$$

$$\varepsilon_y = \varepsilon_a + \varepsilon_c - \varepsilon_b$$

$$\gamma_{xy} = \varepsilon_a - \varepsilon_c$$

Finally, $\sigma_x, \sigma_y, \tau_{xy}$ can be calculated as follows :

$$\sigma_x = \frac{E \cdot (\varepsilon_x + \nu\varepsilon_y)}{(1-\nu^2)}$$

$$\sigma_y = \frac{E \cdot (\varepsilon_y + \nu\varepsilon_x)}{(1-\nu^2)}$$

$$\tau_{xy} = G \cdot \gamma_{xy} = \frac{E}{2(1+\nu)} \cdot \gamma_{xy}$$

1.3. Maximum shear stress, Principal stress, Von-Mises stress

Maximum shear stress equation :

$$\tau_{max} = \pm\sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + (\tau_{xy})^2}$$

Principal stress equation :

$$\sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Von-Mises stress :

$$\sigma_{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

2. Experimental Results

2.1. Amp gain verification

Table 2.1. Calibration voltage of strain gauge amplifier

	Gage_a ($+45\ deg$)	Gage_b ($0\ deg$)	Gage_a ($-45\ deg$)
$e_{out\_CAL}\ [V]$	2.20	2.23	2.30

2.2. Elastic modulus estimation

Table 2.2. Displacement & e_{out} value

	Displacement $[mm]$	e_{out} for gage b $[V]$
1 $[kg]$	2.67	0.34
2 $[kg]$	5.31	0.69
3 $[kg]$	8.40	1.08

2.3. Stress measurement

Table 2.4. Strain rosette voltage value for a load of $3.0\ [kg]$

	e_{out} for gage a	e_{out} for gage b	e_{out} for gage c
1 time	0.82	0.95	-0.25
2 time	0.84	0.93	-0.32
3 time	0.81	0.93	-0.19
Average	0.82	0.94	-0.25

3. Experimental results and analysis

3.1. Estimation of elastic modulus using two different methods

Considering the values obtained in **Table 2.2** and the boundary condition that the L beam is a cantilever, it is possible to estimate the elastic modulus of the material used in the experiment. The steps below explain the processes.

First, the formula related to the deflection of the cantilever beam is as follows, and the process of calculating the key parameter values to utilize the corresponding value is as follows.

$$\delta = \frac{PL^3}{3EI}, \qquad E = \frac{PL^3}{3\delta I}$$

$$L = 335\ [mm], \qquad b = 40\ [mm], \qquad h = 6\ [mm]$$

$$I = \frac{bh^3}{12} = 7.20 \cdot 10^{-10}\ [m^4]$$

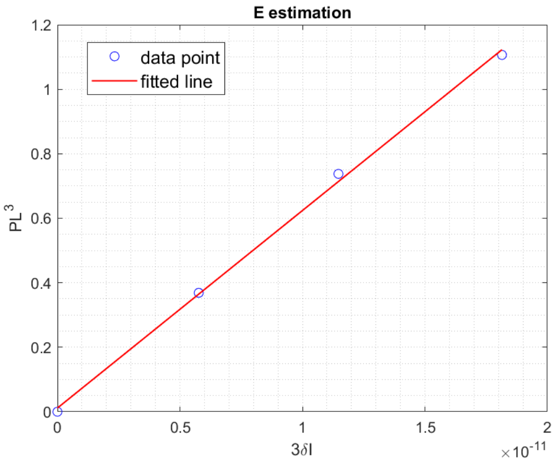

Figure 2. Estimation of elastic modulus with deflection value

$$E_{def} = 61.43\ [GPa]$$

In addition, the elastic modulus can be estimated as above using the strain gage value measured using gage b, which is heading $0\ [deg]$, and the process is as follows.

$$\text{resistance variation :} \quad \Delta R = \frac{4R \times e_{out}}{E \times G}\ [\Omega]$$

$$\text{strain :}\ \varepsilon = \frac{\Delta R}{R \times GF}\ [-]$$

Table 3.1. Main parameters for estimating elastic modulus

weight $[kg]$	Bending moment $[N \times m]$	$\Delta R\ [\Omega]$	$\varepsilon\ [-]$
1 $[kg]$	1.4715	0.02	$8.5133e-05$
2 $[kg]$	2.9430	0.04	$1.7276e-04$
3 $[kg]$	4.4145	0.069	$2.7047e-04$

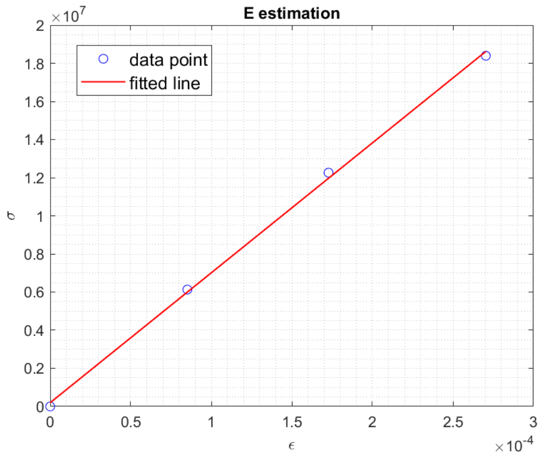

Figure 3. Estimation of elastic modulus with deflection value

$$E_{strain\_gage} = 68.14\ [GPa]$$

3.2. Bending stress, shear stress comparison (theoretical & experimental value)

$$\Delta R = \frac{E \cdot G}{4 \cdot R \cdot e_{out}}$$

$$epsilon = \frac{\Delta R}{R \cdot GF}$$

Table 3.2. ΔR, epsilon values for each gages

	Gage a	Gage b	Gage c
Average $e_{out}\ [V]$	0.82	0.94	-0.25
$\Delta R\ [\Omega]$	0.0525	0.0602	-0.016
Epsilon ε	$2.05 \cdot 10^{-4}$	$2.35 \cdot 10^{-4}$	$-6.26 \cdot 10^{-5}$

$$\varepsilon_x = \varepsilon_b = 2.35 \cdot 10^{-4}$$

$$\varepsilon_y = \varepsilon_a + \varepsilon_c - \varepsilon_b = 2.05 \cdot 10^{-4} - 6.26 \cdot 10^{-5} - 2.35 \cdot 10^{-4} = -9.26 \cdot 10^{-5}$$

$$\gamma_{xy} = \varepsilon_a - \varepsilon_c = 2.67 \cdot 10^{-4}$$

With the generalized Hooke's law, $\sigma_x, \sigma_y, \tau_{xy}$ can be calculated as follows, and the poisson's ratio is 0.3, and the elastic modulus for this material is set with $E_{strain\_gage} = 68.14\ [GPa]$.

$$\sigma_x = \frac{E \cdot (\varepsilon_x + \nu\varepsilon_y)}{(1-\nu^2)} = 15.54\ [MPa]$$

$$\sigma_y = \frac{E \cdot (\varepsilon_y + \nu\varepsilon_x)}{(1-\nu^2)} = -1.69\ [MPa]$$

$$\tau_{xy} = G \cdot \gamma_{xy} = \frac{E}{2(1+\nu)} \cdot \gamma_{xy} = 7.02\ [MPa]$$

3.3. Principal stress, max shear stress, von-mises stress value (theoretical & experimental & simulation value)

3.3.1. Theoretical value

Theoretical value of elastic modulus is set with $70\,[GPa]$. Firstly, τ_{max} value can be calculated with the following process, where torque arm is $100\ [mm]$:

$$c = \frac{1}{3}\left(1 - 0.63 \cdot \frac{h}{b}\right) \quad where \quad \frac{b}{h} > 5$$

$$\tau_{xy} = \frac{T}{cbh^2}, \quad T = 3\ [kg] \cdot 9.81 \left[\frac{m}{s^2}\right] \cdot 0.1[m] = 6.77\ [MPa]$$

To claculate theoretical bending stress value and the shear stress have to be calculated. Moment arm value has to be considered with $150\ [mm]$ since the bending arm in the aspect of strain gadget is $150\ [mm]$, not $335\ [mm]$.

$$M = 3\ [kg] \cdot 9.81 \left[\frac{m}{s^2}\right] \cdot 0.15\ [m] = 4.41\ [Nm]$$

$$\sigma_{max} = \frac{Mc}{I} = 18.4\ [MPa]$$

$$\sigma_{principal} = \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{xy}^2} = 20.61\ [MPa]$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{xy}^2} = 11.4\ [MPa]$$

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3 \cdot {\tau_{xy}}^2} = 21.82\ [MPa]$$

3.3.2. Experimental value

With the calculated values above, principal stress, maximum shear stress, von-mises stress can be calculated with following equations :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}, \qquad \sigma_1 = 18.04\ [MPa]$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}, \qquad \tau_{max} = 11.10\ [MPa]$$

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3 \cdot {\tau_{xy}}^2}, \qquad \sigma_{VM} = 20.44\ [MPa]$$

3.3.3. Simulation value

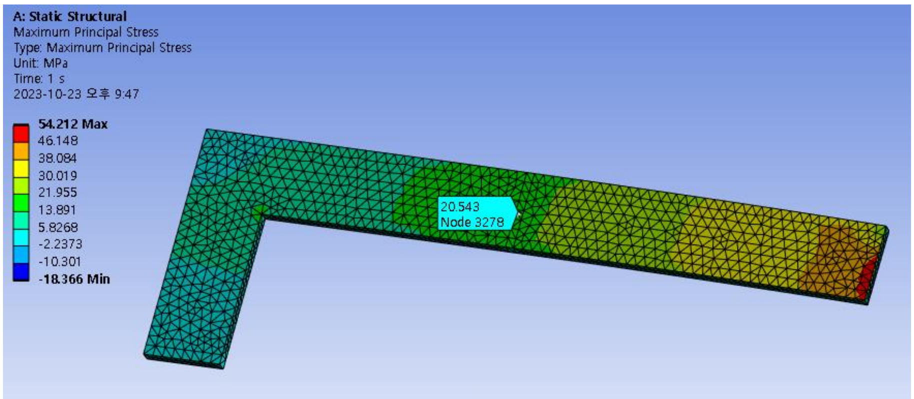

Figure 4. Maximum principal stress

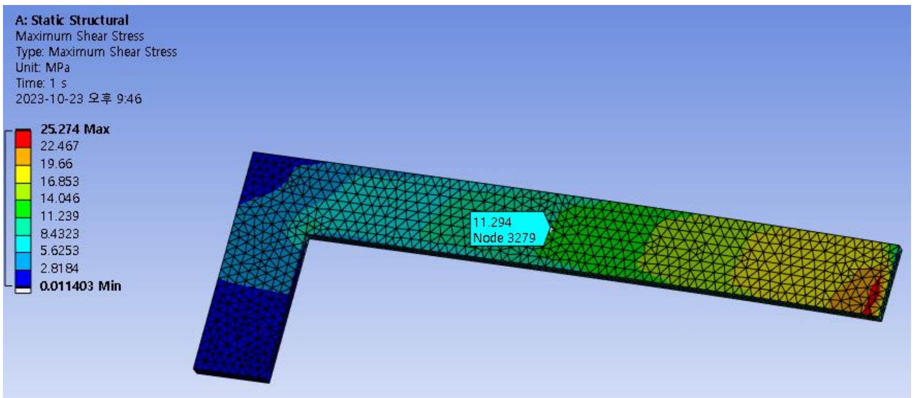

Figure 5. Maximum shear stress

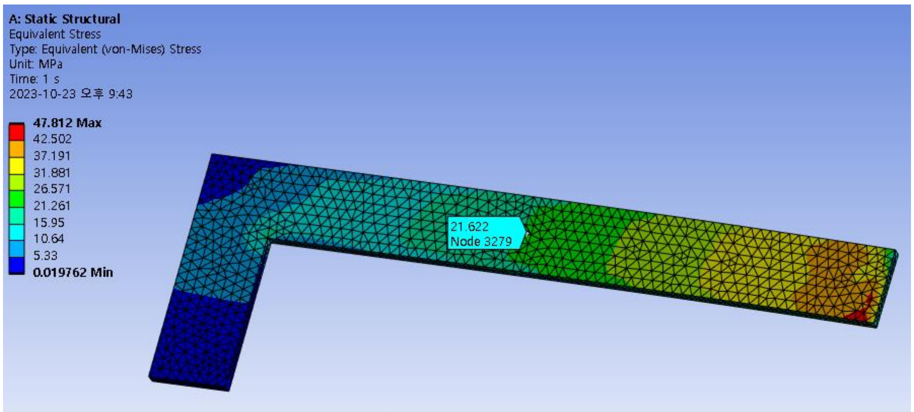

Figure 6. Von-Mises stress

Table 3.1. Comparison of theoretical, experimental, simulation values

	Principal stress $[MPa]$	Maximum shear stress $[MPa]$	Von-Mises stress $[MPa]$
Theory	20.61	11.4	21.82
Experiment	18.04	11.10	20.44
Simulation	20.543	11.29	21.62

The most noticeable difference is the experimental value of principal stress. As can be seen, the strain is calculated as the change in resistance of the gage, and the change in resistance is proportional to the elastic modulus. As can be seen in **Figure 3**, the experimentally obtained elastic modulus regressed to a value smaller than the theoretical value. As a result, the principal stress value came out to be smaller than the theoretical value.

3.4. Considering the safety factor with 3, what is the maximum mass that can be loaded on the L beam in this experiment ?

Suppose that the maximum load considering the safety factor 3 from the yield point of the material is m $[kg]$.

$$\sigma_{max} = \frac{Mc}{I} = \frac{(m \cdot 9.81) \cdot 0.335 \cdot 0.003}{7.2 \cdot 10^{-10}}$$

$$\tau_{xy} = \frac{(A \cdot 9.81) \cdot 0.1}{4.3464 \cdot 10^{-7}}$$

$$\sigma_{principal} = \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{xy}^2} \leq \frac{280\ [MPa]}{3.0} = 93.33\ [MPa]$$

where $280\,[MPa]$ is tensile yield strength of material (AL 6061-T6) used in this experiment. Accordingly, max load that can be used to prevent material yield considering safety factor as 3 is

$$m = 6.64\ [kg]$$

3.4. In addition to the above considerations, discuss what you observed and what you learned through the experiment.

In the case of lab1, the strain effect due to temperature was eliminated by setting the resistance for the dummy gage on the other side. Here, temperature compensation is performed in the same way, and it can be seen that it is very important. This is because it is so sensitive that the voltage value can change significantly just by blowing air on the dymmy gage at zero load. Accordingly, it can be seen that in order to obtain an accurate voltage output value and a corresponding strain value, an appropriate temperature compensation term setting is necessary. Additionally, the principal stress value can be obtained by combining the values of strain gauges installed at different angles. In other words, it was found that the maximum value of the stress acting on the cross section and the von-mises equivalent stress could be obtained, and thus it was possible to design considering the actual safety factor of the material.