CAE LAB 3

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Subject: Resonant frequency of multi-story building

1. Prelab

1.1. Find the dynamic equation for the system below, derive the displacement response of the system to external force, and explain the resonance phenomenon.

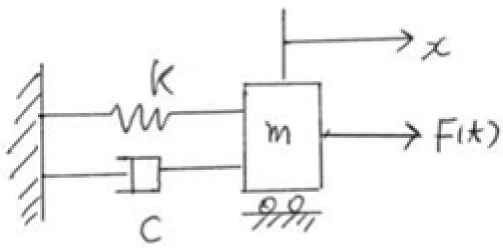


Figure 1. mck system

The differential equation of $F_0 \sin(\omega t)$ external forced mck system is as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 \sin(\omega t)$$

For convenience, the equation for the second standard system is expressed as follows.

$$\ddot{x(t)} + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 = \frac{F_0 \sin(\omega t)}{m}$$

Since the system is a non-homogeneous system, it has a special solution.

$$x_p = X_1 \sin(\omega t) + X_2 \cos(\omega t) = X_0 \sin(\omega t - \theta)$$

$$X_0 = \sqrt{X_1^2 + X_2^2}, \qquad \theta = \arctan\left(\frac{X_2}{X_1}\right)$$

When the answer is derived by substituting the special solution into the existing differential equation, the result is as follows.

$$X_0 = \frac{F_0}{\sqrt{(w_n^2 - \omega^2) + (2\zeta\omega_n\omega)^2}}, \qquad \theta = \arctan\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$

$$x_p = \frac{F_0}{\sqrt{(w_n^2 - \omega^2) + (2\zeta\omega_n\omega)^2}} \cdot \sin(\omega t - \arctan(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}))$$

Next, the form of the general solution is determined from the following homogeneous characteristic equation. Since the system is an underdamped system, that is, the characteristic equation assumes two complex conjugate solutions, it is calculated as follows.

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

$$\lambda = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Accordingly, the general solution is determined as follows.

$$x_h = e^{-\zeta\omega_n t}(C_1\cos(\omega_d t) + C_1\sin(\omega_d t)) = C \cdot e^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$$

The final solution form, that is, the displacement equation, is as follows.

$$x_{tot} = x_h + x_p = C \cdot e^{-\zeta\omega_n t}\sin(\omega_d t + \psi) + X_0\sin(\omega t - \theta)$$

$$C, \psi : determined\ by\ initial\ condition$$

$$\zeta, \omega\_n, \omega\_d, X, \omega, \theta : properties\ of\ system$$

The amplitude of the particular solution X normalized with the static deflection $\frac{F}{k}$ caused by the external force is defined as follows:

$$M = \frac{X}{(\frac{F_0}{k})} = \frac{1}{\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

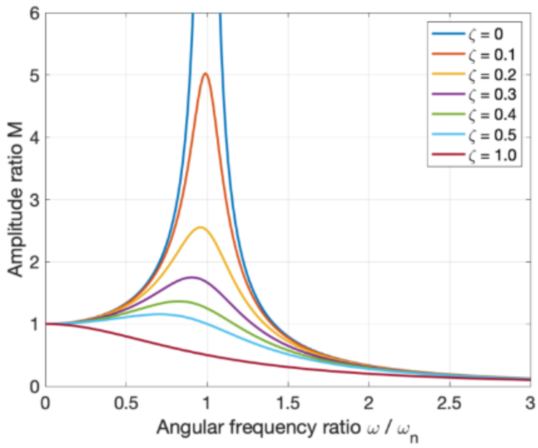


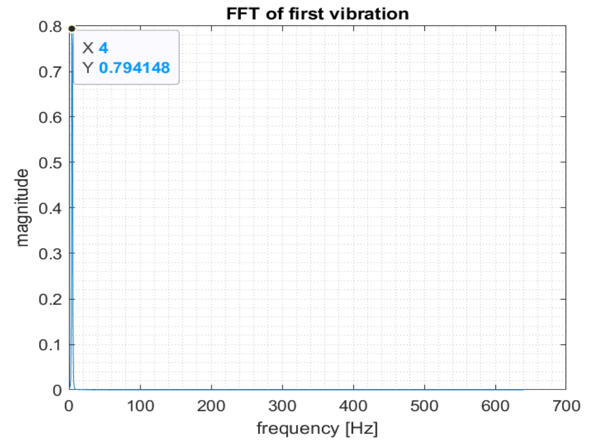
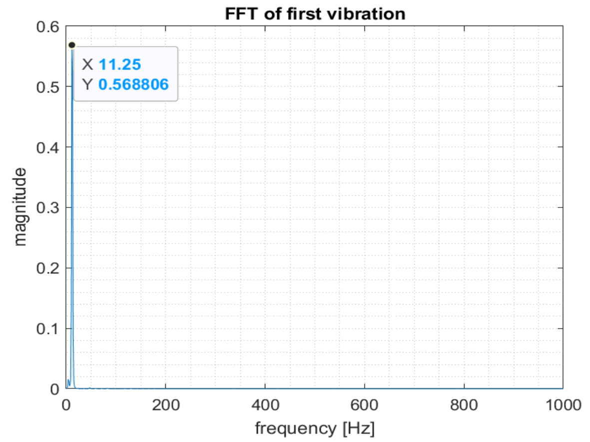
Figure 2. Resonance frequency phenomenon

As can be seen in the **Figure 2**, resonance occurs when $\omega_n = \omega$ and ζ goes to zero. It means that to avoid resonance of the system, either damping ratio value has to be increased or alter the driving frequency ω.

2. Results

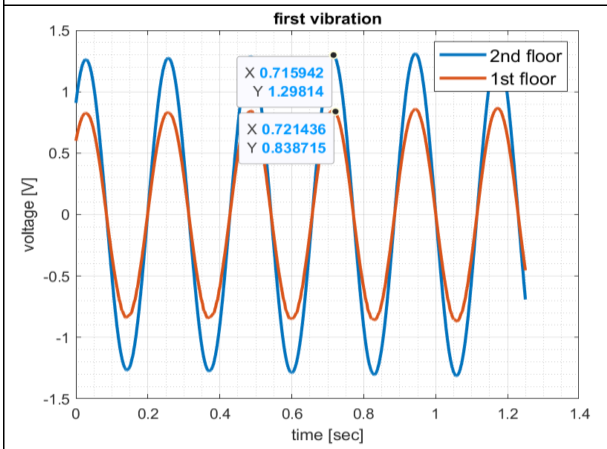
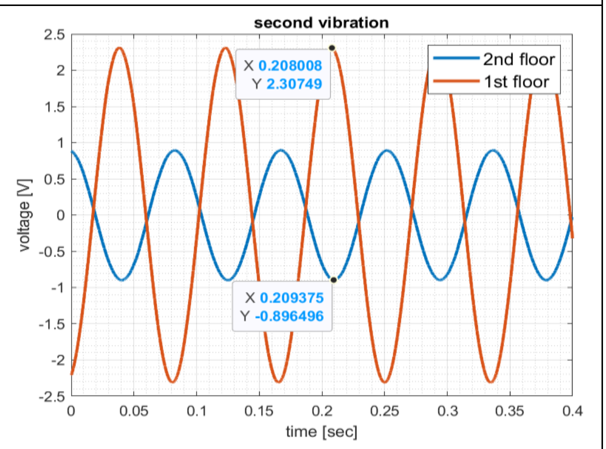
2.1. Finding the two resonance frequency and the mode amplitude.

Table 2.1. Resonance frequency of two modes

Mode 1	Mode 2
	

As can be seen in the **Table 2.1**, the resonance frequencies confirmed in mode 1 and mode 2 are $4\ [Hz]$ and $11.25\ [Hz]$, respectively. Those can be found in the frequency domain. And the amplitude can be found in the time domain.

Table 2.2. Mode shape

Mode 1	Mode 2
	

Mode shape ratio of mode 1 and the mode 2 can be calculated as follows. That of mode 2 shows a negative value since the wave shows that the phase is reversed as can be seen in the **Table 2.2.**

$$mode\ shape\ ratio\ of\ mode\ 1 = \frac{1.29814[V]}{0.838715[V]} = 1.5478$$

$$mode\ shape\ ratio\ of\ mode\ 2 = \frac{-0.8965[V]}{2.30749[V]} = -0.3885$$

2.2. Model it as a two-degree-of-freedom mass-spring and find the resonance frequencies and mode shapes for the two-side bends.

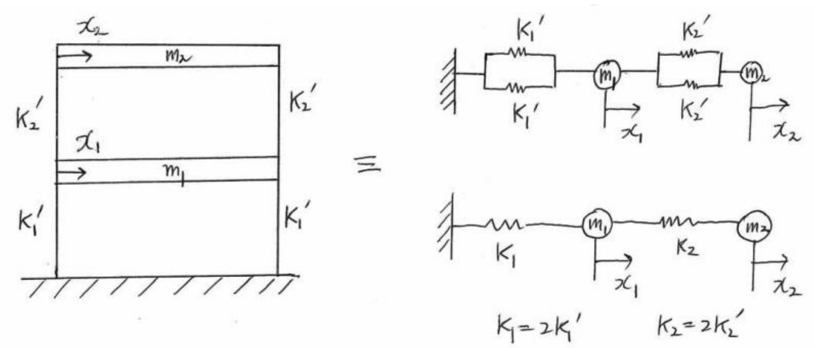


Figure 3. mass-spring system modeling of building

Equation of motion can be written as follows:

$$m_1\ddot{x_1} + (k_1 + k_2)x_1 - k_2x_2 = 0$$

$$m_2\ddot{x_2} - k_2x_1 + k_2x_2 = 0$$

And the equation can be written with matrix format for harmonic motion as follows:

$$\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

To get the proper displacement, inverse matrix must not exist, that means the determinant of first matrix should be zero. And the following result of that equation is as follows:

$$\det\left(\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix}\right) = 0$$

$$\omega^2 = \frac{(m_2k_1 + m_2k_2 + m_1k_2) \pm \sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_1m_2k_1k_2}}{2 \cdot m_1 \cdot m_2}$$

And by considering the boundary conditions, the spring constant can be calculated as follows:

$$k = \frac{24EI}{l^3}$$

$$E = 70[GPa], \qquad I = \frac{1}{12} \cdot (0.05[m]) \cdot (0.0015[m])^3 = 1.4063 \cdot 10^{-11}[m^4], \qquad l = 0.22[m]$$

$$k = 2.2187 \cdot 10^3 \left[\frac{N}{m}\right], \qquad k = k_1 = k_2$$

And considering the mass of two floors and the equation above, resonance frequency of each mode can be calculated as follows:

$$m_1 = 1.0\,[kg], \qquad m_2 = 1.37[kg], \qquad k = 2.2187 \cdot 10^3 \left[\frac{N}{m}\right]$$

$$\omega^2 = \frac{(m_2k_1 + m_2k_2 + m_1k_2) \pm \sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_1m_2k_1k_2}}{2 \cdot m_1 \cdot m_2}$$

$$\omega^2 = \frac{(2m_2 + m_1) \cdot k \pm \sqrt{(2m_2 + m_1)^2 \cdot k^2 - 4m_1m_2k^2}}{2 \cdot m_1 \cdot m_2}$$

Calculation result of two resonance frequencies with the equation above is as follows:

$$freq_1 = \frac{\omega_1}{2\pi} = 4.1092\,[Hz]$$

$$freq_2 = \frac{\omega_2}{2\pi} = 11.685\,[Hz]$$

And the mode shape ratio of this model can be calculated with below matrix equation. Cause each mode has different ω value, it has two different mode shape ratios.

$$r_1 = \frac{-\omega_1^2 m_1 + 2k}{k} = -0.4295, \qquad \omega_1 = 73.4190\,[\frac{rad}{s}]$$

$$r_2 = \frac{-\omega_2^2 m_1 + 2k}{k} = 1.6996, \qquad w_2 = 25.8187\,[\frac{rad}{s}]$$

2.3. Find the five resonance frequencies and mode shapes with ANSYS analysis.

The material of vertical part and the horizontal part is set with aluminum 6061-T6, structural steel, respectively.

	Mode	Frequency [Hz]
1	1.	4.5743
2	2.	12.704
3	3.	36.236
4	4.	118.38
5	5.	187.16

Figure 4. five resonance frequency values in ANSYS

As can be seen in the **Figure 5,** mode shape ratio with first resonance frequency is positive, and that with second resonance frequency is negative. The results are as follows:

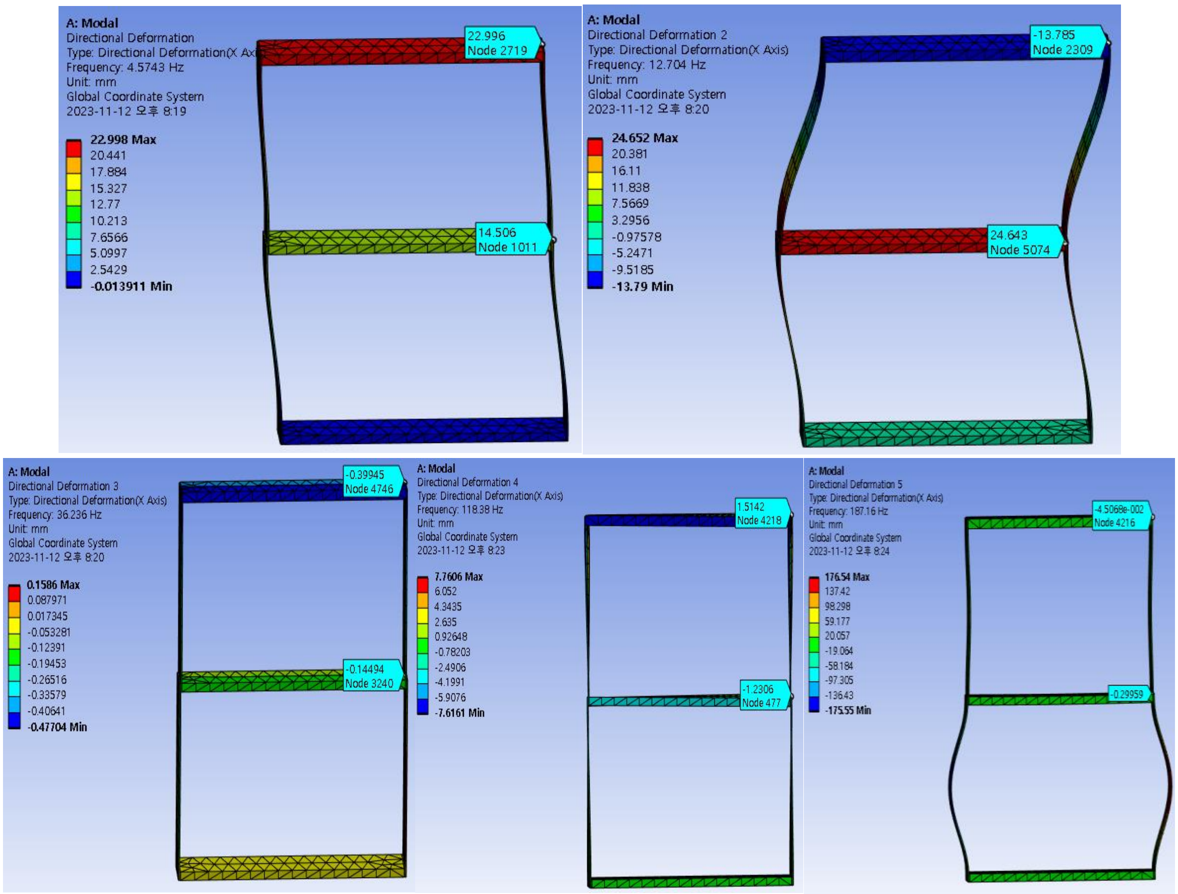


Figure 5. result of each simulation results: resonance frequency

And the mode shape ratio for each case is specified in the **Table 2.3**:

Table 2.3. Mode shape ratio of each result

	1st	2nd	3rd	4th	5th
Mode shape ratio	$\frac{22.996[mm]}{14.506[mm]} = 1.58$	$-\frac{13.785[mm]}{24.643[mm]} = -0.56$	$\frac{0.39945[mm]}{0.14494[mm]} = 2.75$	$-\frac{1.5142[mm]}{-1.2306[mm]} = -1.23$	$\frac{0.045[mm]}{0.29959[mm]} = 0.15$

3. Discussions

3.1. Comparison of experimental, theoretical, simulation values

Table 3.1. Comparison of values

	1^{st} f_n	2^{nd} f_n	1^{st} mode shape	2^{nd} mode shape
Experiment	4.0 $[Hz]$	11.25 $[Hz]$	1.5478	-0.3885
Theory	4.1092 $[Hz]$	11.695 $[Hz]$	1.6996	-0.4295
Simulation	4.5743 $[Hz]$	12.704 $[Hz]$	1.5848	-0.56

As can be seen in **Table 3.1**, it can be verified that the theoretical values for all parameters are within the error range of $10.0[\%]$ of the experimental values. It is necessary to analyze the cause of the first and second resonance frequencies being smaller than the experimental and simulation values. When this building is modeled with mass-spring system, the spring constant value k is determined as follows:

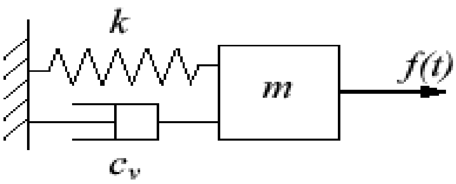
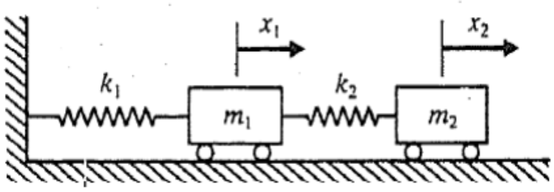
$$k = \frac{24EI}{L^3}$$

And it can be said that the resonance frequency is proportional to the spring constant value. And as can be referred in the previous labs, the actual elastic modulus of a material is generally lower than the theoretical value. Accordingly, actual resonance frequency can be observed with lower value than theoretical value. Furthermore, mode shape ratio becomes negative value when the resonance frequency goes up from first one to second one.

3.2. Vibration characteristics of multi-degree-of-freedom structures compared to single-degree-of-freedom structures.

Single DOF system and a multi DOF system can be tabularized as follows:

Table 3.2. single DOF system and multi DOF system

Single DOF system	**Multi DOF system**
	

A multi-degree-of-freedom system is a system in which several types of equivalent inertial masses are set because the entire system is centered on one type of equivalent inertial mass. Depending on the number of equivalent inertial masses, 2-degree-of-freedom, 3-degree-of-freedom systems, etc. exist. In a multi-degree-of-freedom system, the concept of a mode is installed to compare different variants by looking at the natural frequency corresponding to the degree of freedom. And the system with two mass factors and spring factors is modeled as follows:

$$2\mathrm{m}\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0$$

$$\mathrm{m}_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0$$

If each displacement is expressed as $\mathrm{x(t)} = \mathrm{X}\cos(\omega t + \phi)$ Therefore, there are two natural frequencies ω, and if the natural frequencies are expressed under the conditions $k\_1 = k\_2 = k, E\_1 = E\_2 = E$, it is organized as follows.

$$\omega_1 = 0.5412\sqrt{\frac{k}{m}}$$

$$\omega_2 = 1.3066\sqrt{\frac{k}{m}}$$

3.3. A review of methods to prevent resonance when a large cooling fan is installed on the roof of a building, causing resonance in the building.

The reason why cooling fans cause resonance in buildings is because the natural frequencies of the cooling fans are the same or similar. In general, to reduce resonance, the cause of vibration is removed, or the natural frequency is changed. However, in this situation where the cause cannot be removed, it is solved by changing the natural frequency. Since it is difficult to change the natural frequency of the building, a buffer or vibration pad must be installed on the contact surface between the cooling fan and the building to absorb the vibration of the cooling fan. do.