CAE LAB 4

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Date: 2023.11.16

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Subject: Impact hammer test

1. Prelab

1.1. Investigation about impact hammer test

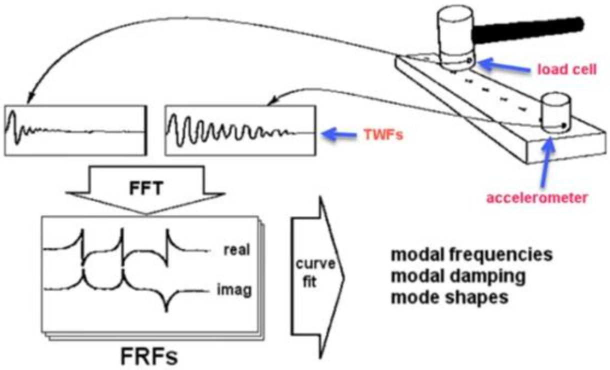

Figure 1. Impact hammer test

Modal testing is a common method of characterizing the vibrations of a structure by imparting a known force and measuring the response of the structure. By measuring both the input to the structure and the response, the frequency response of the structure can be calculated. Calculating the frequency response over multiple locations, either simultaneously or individually, will yield data that can be used to estimate the dynamic response of the structure. The scale of a modal test can vary greatly. Test structures can be as small as silicon wafers used in electronics, and as large as multistory industrial sifters used at rock quarries. The size and geometry of the test structure will play a role is choosing how to excite it. The two most common methods are impact testing using a modal hammer and shaker testing. After collection, the data can be processed using ME'scope, a popular modal analysis software from Vibrant Technologies. The result of the measurements and processing would be an animated model of the operating deflection shapes (ODS) that clearly illustrates the movement of the structure. Most commonly, these models are analyzed to identify modal frequencies. At these frequencies the structure vibrates with minimal input energy. Exciting the structure at these frequencies can easily cause damage to the system. Characterizing the response of the structure mean that the design can be changed to reduce the response, or the operating conditions can be adjusted to avoid failures.

1.2. FFT for following signal and the sketch of spectrum of frequency domain

1) Impulse signal

$$X(\omega) = \int x(t) \cdot e^{-j\omega t} dt = \int \delta(t) \cdot e^{-j\omega t} dt$$

$$\delta(t) = 1 \ for\ t = 0 \ \ and \ \ 0\ for\ t \neq 0$$

$$\int \delta(t) \cdot e^{-j\omega t} dt = \int 1 \cdot e^{-j\omega t} = 1$$

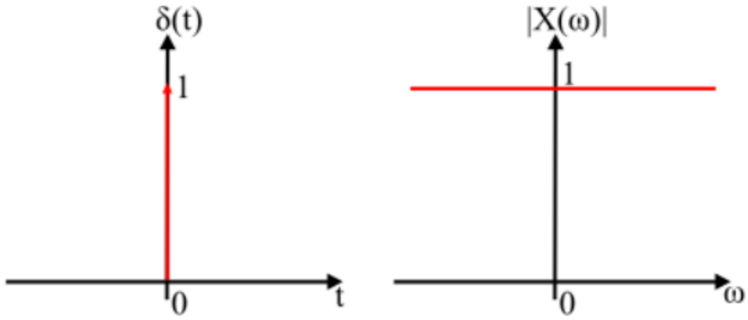

Figure 2. Time domain and FFT result of impulse signal

2) Unit step function

$$X(\omega) = \int x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = u(t) = 1\ for\ t \geq 0\ and\ 0\ for\ t < 0$$

In order to find the Fourier transform of the unit step function, express the unit step function in terms of signum function as

$$u(t) = \frac{1}{2} + \frac{1}{2} sgn(t) = \frac{1}{2}[1 + sgn(t)]$$

$$X(\omega) = \int \frac{1}{2}[1 + sgn(t)] \cdot e^{-j\omega t} dt$$

$$X(\omega) = \frac{1}{2}[F\{1\} + F\{sgn(t)\}]$$

Following that, the FFT result of unit step function can be calculated as follows:

$$X(\omega) = infinite\ when\ \omega = 0\ and\ 0\ when\ \omega\ is\ posivie\ \inf \&\ negative\ inf$$

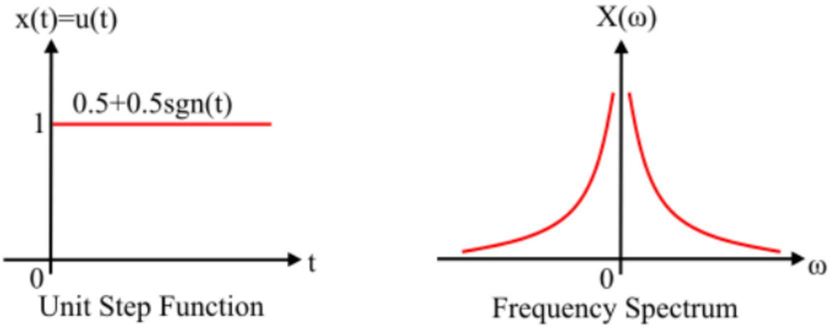

Figure 3. Time domain and FFT result of unit step signal

3) sinusoidal signal

$$X(\omega) = \int x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \sin(\omega_0 t) = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

$$X(\omega) = F\{\sin(\omega_0 t)\} = F\{\frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})\}$$

$$X(\omega) = \frac{\pi}{j}\{\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\}$$

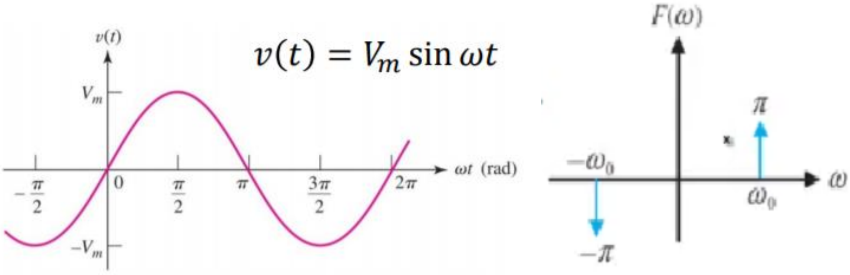

Figure 4. Time domain and FFT result of sinusoidal signal

2. Results

2.1. Experimental values

Firstly, damping ratio can be calculated with the time-domain based signal.

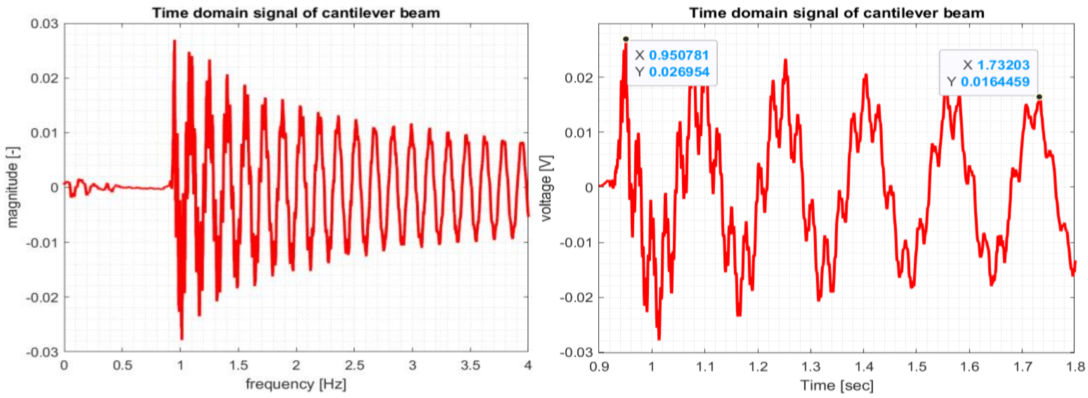

Figure 5. Time domain signal

In the underdamped signal, characteristic equation gets conjugate complex poles. And with setting $n = 5$, σ and ζ value can be calculated as follows:

$$\delta = \frac{1}{n}\ln\left(\frac{x_k}{x_{k+n}}\right), \qquad \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

$$\delta = \frac{1}{5}\ln\left(\frac{0.026954}{0.0164459}\right) = 0.0988$$

Following that, damping ratio value can be calculated as follows:

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.0157$$

1) Steel with cantilever beam condition

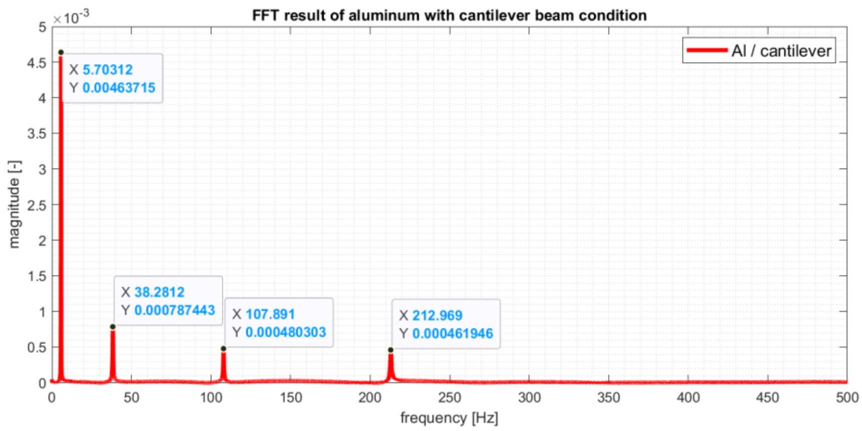

Figure 6. FFT result of steel with cantilever beam condition

As can be seen in the **Figure 6**, four resonance frequencies can be observed in the frequency domain, $5.7\ [Hz], 38.28[Hz], 107.89[Hz], 212.97[Hz]$.

2) Brass with cantilever beam condition

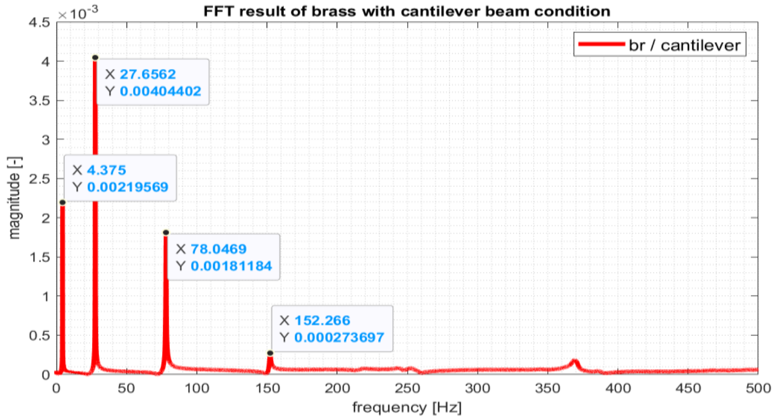

Figure 7. FFT result of brass with cantilever beam condition

As can be seen in the **Figure 7**, four resonance frequencies can be observed in the frequency domain, $4.38\ [Hz], 27.66[Hz], 78.05[Hz], 152.27[Hz]$.

3) Aluminum with cantilever beam condition

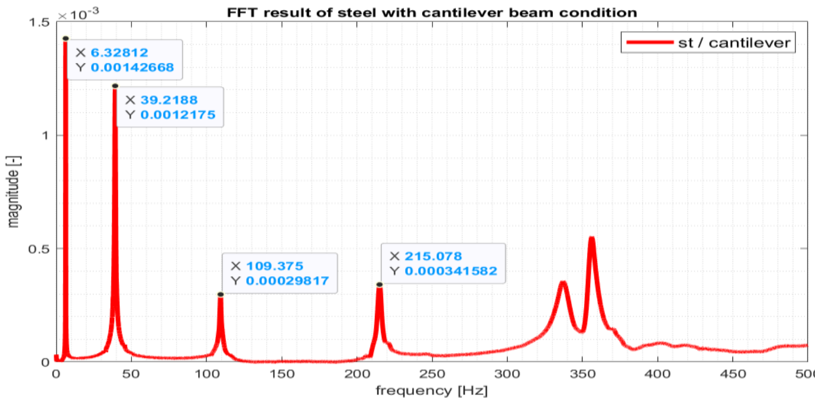

Figure 8. FFT result of aluminum with cantilever beam condition

As can be seen in the **Figure 8**, four resonance frequencies can be observed in the frequency domain, $6.33\ [Hz], 39.22[Hz], 109.37[Hz], 215.09[Hz]$.

4) Steel with both fixed boundary condition

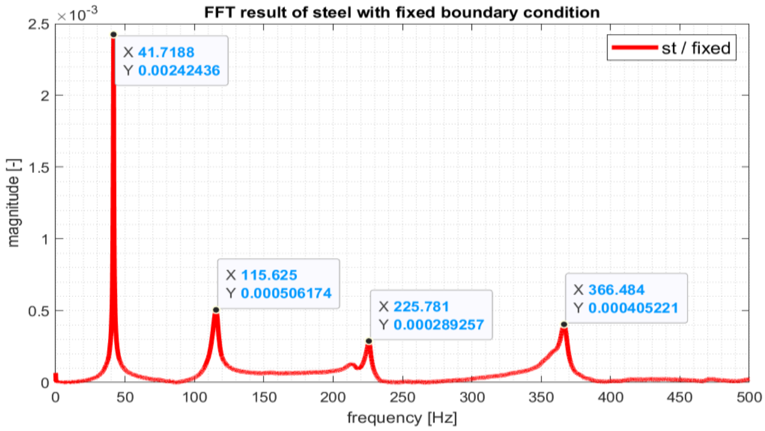

Figure 9. FFT result of steel with fixed boundary condition

As can be seen in the **Figure 9**, four resonance frequencies can be observed in the frequency domain, $41.72\ [Hz], 115.62[Hz], 225.78[Hz], 366.48[Hz]$.

2.2. Theoretical values

To get the proper theoretical values, the specifications for the specimen and some properties of them are set with values below:

$$cantilver : 19mm * 3.2mm * 625mm$$

$$fixed : 19mm * 3.2mm * 600mm$$

Table 2.1. Main properties of materials used in experiment

	Steel	Brass	Aluminum
Elastic modulus $[GPa]$	200	105	70
Density $[kg/m^3]$	7850	8740	2700

$$I = \frac{b \cdot h^3}{12} = 5.1883 \cdot 10^{-11}\ [m^4]$$

$$A = w \cdot h = 6.08 \cdot 10^{-5}\ [m^2]$$

Under the cantilever beam condition, multiplication of beta and length are defined as follows, and the beta values can be calculated with known length, $l = 0.625\ [m]$.

Table 2.2. Beta values for cantilever boundary condition

	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$
$\boldsymbol{\beta \cdot l}$	1.875104	4.694091	7.854757	10.995541
$\boldsymbol{\beta}$	3.1252	7.8235	13.0913	18.3259

And the following resonance frequency values can be calculated with following equation:

$$\omega = \sqrt{\frac{\beta^4 \cdot E \cdot I}{\rho \cdot A}}, \ \ resonance\ freq = \frac{\omega}{2 \cdot \pi}$$

Table 2.3. Theoretical resonance frequency values of each material: cantilever condition

	Steel	**Brass**	**Aluminum**
$\boldsymbol{1st\ freq}$	$7.2478\ [Hz]$	$4.9769\ [Hz]$	$7.3113\ [Hz]$
$\boldsymbol{2nd\ freq}$	$45.4213\ [Hz]$	$31.1902\ [Hz]$	$45.8192\ [Hz]$
$\boldsymbol{3rd\ freq}$	$127.1811\ [Hz]$	$87.3336\ [Hz]$	$128.2949\ [Hz]$
$\boldsymbol{4th\ freq}$	$249.2242\ [Hz]$	$171.1390\ [Hz]$	$251.4069\ [Hz]$

Under both side fixed condition, multiplication of beta and length are defined as follows, and the beta values can be calculated with known length, $l = 0.60\ [m]$.

Table 2.4. Beta values for both fixed boundary condition

	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$
$\boldsymbol{\beta \cdot l}$	4.730041	7.853205	10.995608	14.137165
$\boldsymbol{\beta}$	7.8867	13.0887	18.3260	23.5619

And the following resonance frequency values can be calculated with following equation:

$$\omega = \sqrt{\frac{\beta^4 \cdot E \cdot I}{\rho \cdot A}}, \ \ resonance\ freq = \frac{\omega}{2 \cdot \pi}$$

Table 2.3. Theoretical resonance frequency values of steel: both side fixed condition

	$\boldsymbol{1st\ freq\ [Hz]}$	$\boldsymbol{2nd\ freq\ [Hz]}$	$\boldsymbol{3rd\ freq\ [Hz]}$	$\boldsymbol{4th\ freq\ [Hz]}$
Steel	45.9249	127.1308	249.2273	411.9853

2.3. Simulation values

1) Steel with cantilever beam condition

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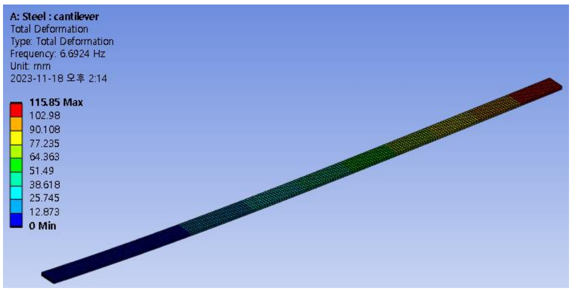
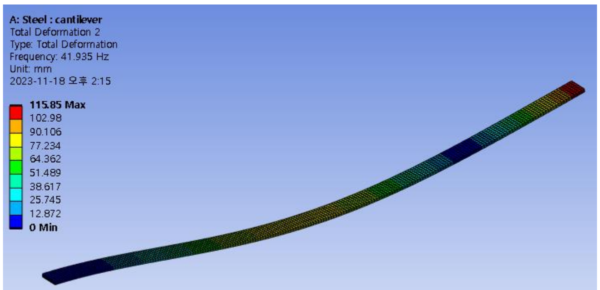
<tr><td>
</td><td>
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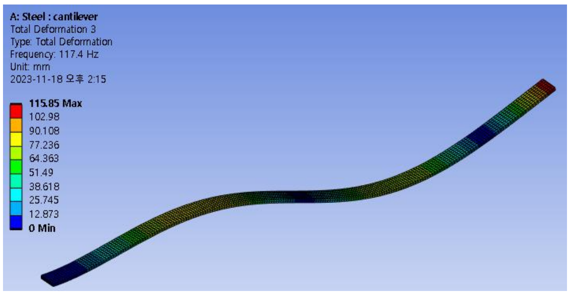
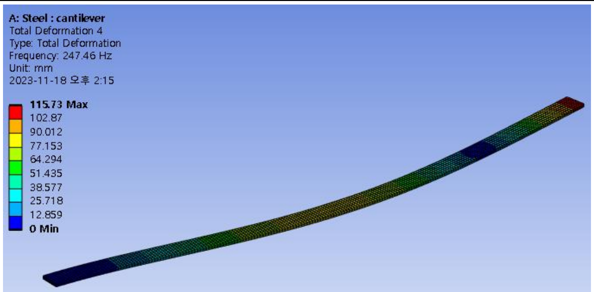
<tr><td>$6.6924\ [Hz]$</td><td>$41.935\ [Hz]$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$117.4\ [Hz]$</td><td>$247.46\ [Hz]$</td></tr>
</table>

Figure 10. Ansys simulation result of steel: cantilever beam

2) Brass with cantilever beam condition

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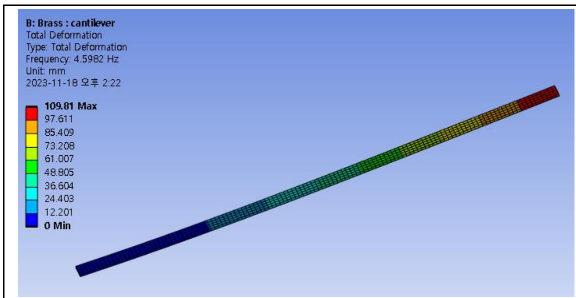
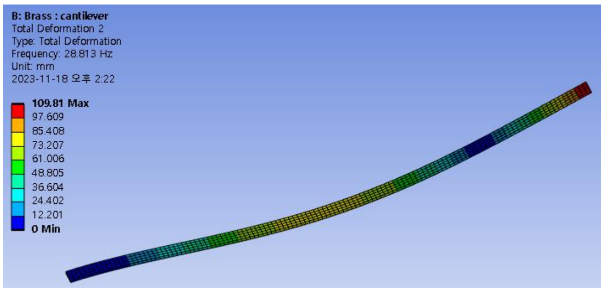
<tr><td>
</td><td>
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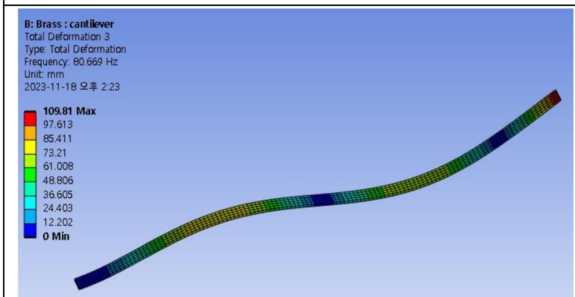
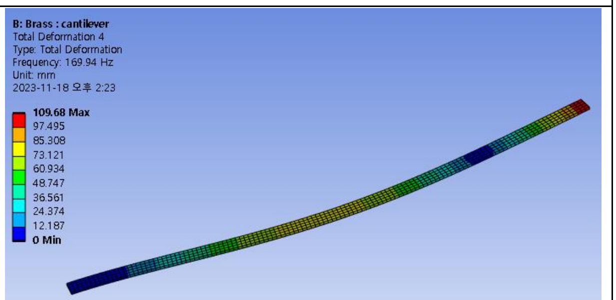
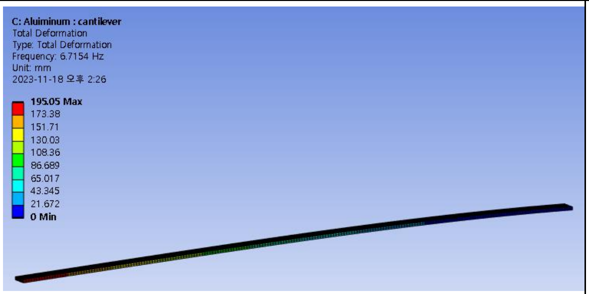
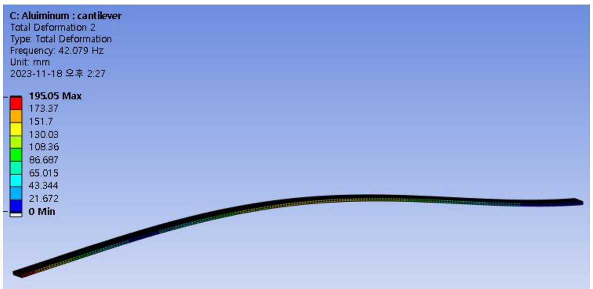
<tr><td>$4.5982\ [Hz]$</td><td>$28.813\ [Hz]$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$80.669\ [Hz]$</td><td>$169.94\ [Hz]$</td></tr>
</table>

Figure 11. Ansys simulation result of brass: cantilever beam

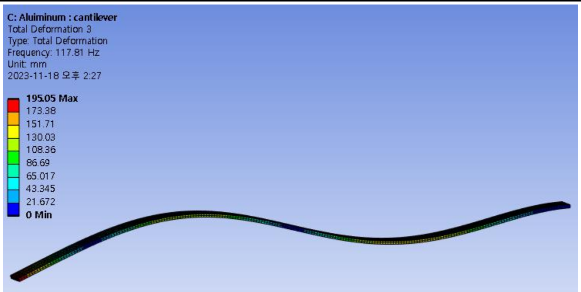
3) Aluminum with cantilever beam condition



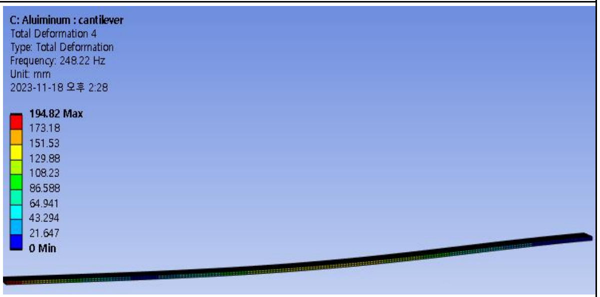
$6.7154\ [Hz]$



$42.079\ [Hz]$



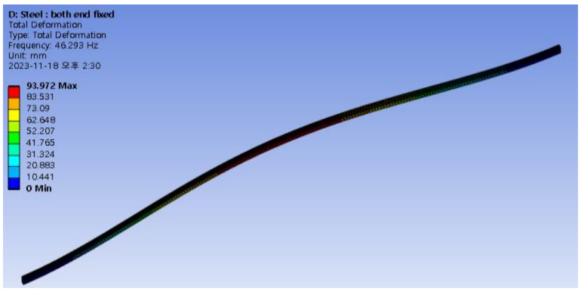
$117.81\ [Hz]$



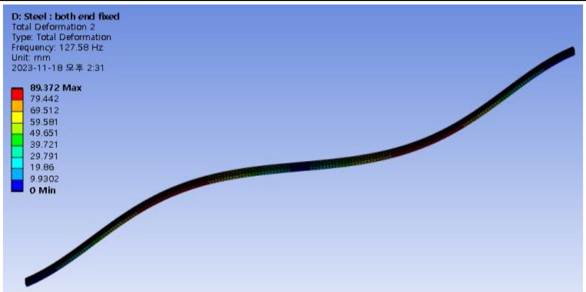
$248.22\ [Hz]$

Figure 12. Ansys simulation result of aluminum: cantilever beam

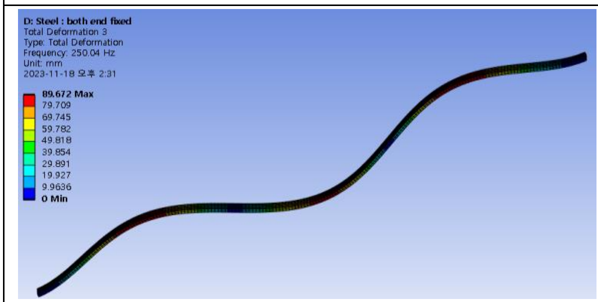
4) Steel with both fixed condition



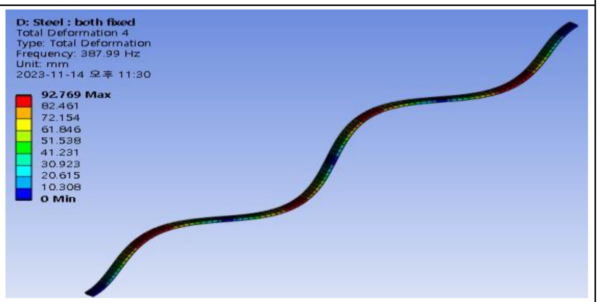
$46.293\ [Hz]$



$127.58\ [Hz]$



$250.04\ [Hz]$



$413.32\ [Hz]$

Figure 13. Ansys simulation result of aluminum: cantilever beam

3. Discussion and analysis

3.1. Comparison of experimental, theoretical, simulation values

Table 3.1. Steel beam: cantilever beam

	$1st\ f_n\ [Hz]$	$2nd\ f_n\ [Hz]$	$3rd\ f_n\ [Hz]$	$4th\ f_n\ [Hz]$
Experiment	5.7	38.28	107.89	212.97
Theory	7.2478	45.4213	127.1811	249.2242
Simulation	6.6924	41.935	117.4	247.46

Table 3.2. Brass beam: cantilever beam

	$1st\ f_n\ [Hz]$	$2nd\ f_n\ [Hz]$	$3rd\ f_n\ [Hz]$	$4th\ f_n\ [Hz]$
Experiment	4.38	27.66	78.05	152.27
Theory	4.9769	31.1902	87.3336	171.1390
Simulation	4.5982	28.813	80.669	169.94

Table 3.3. Aluminum beam: cantilever beam

	$1st\ f_n\ [Hz]$	$2nd\ f_n\ [Hz]$	$3rd\ f_n\ [Hz]$	$4th\ f_n\ [Hz]$
Experiment	6.33	39.22	109.37	215.09
Theory	7.3113	45.8192	128.2949	250.4069
Simulation	6.7154	42.079	117.81	248.22

Table 3.4. Steel beam: both fixed beam

	$1st\ f_n\ [Hz]$	$2nd\ f_n\ [Hz]$	$3rd\ f_n\ [Hz]$	$4th\ f_n\ [Hz]$
Experiment	41.72	115.62	225.78	366.48
Theory	45.9249	127.1308	249.2273	411.9853
Simulation	46.293	127.58	250.04	413.32

$$\omega = \sqrt{\frac{\beta^4 \cdot E \cdot I}{\rho \cdot A}}$$

First, as can be seen in all specimens, the experimental values were calculated to be lower than the theoretical values. It can be said that the resonance frequency value is proportional to the elastic modulus of the material according to the equation above. When we went through the process of estimating the elastic modulus of the specimen using the previous strain gauge and deflection values, we were able to confirm that the actual elastic modulus was estimated to be lower than the theoretical elastic modulus. Accordingly, it is possible to confirm phenomena in which the value of the resonance frequency obtained experimentally is smaller than the theoretical value. Additionally, if there is a difference between the length of the specimen actually used during the experiment and the length used to calculate the theoretical value, it affects the β value and ultimately has a significant impact on the resonance frequency value.

3.2. Influence of material of specimen and boundary conditions

3.2.1. Influence of material

$$\omega = \sqrt{\frac{\beta^4 \cdot E \cdot I}{\rho \cdot A}}$$

Equation above shows that under same boundary condition, area and moment of inertia, the following relationship is established.

$$\omega \propto \sqrt{\frac{E}{\rho}}$$

So, it is necessary to compare the values of elastic modulus and density ratio for each material. Materials with similar values will have similar resonant frequencies under the same conditions mentioned above. The units of elastic modulus and density used in the ratios below are ignored.

$$\sqrt{\frac{\mathrm{E}}{\rho}}_{\mathrm{steel}} = \sqrt{\frac{200}{7850}} = 0.1596$$

$$\sqrt{\frac{\mathrm{E}}{\rho}}_{\mathrm{brass}} = \sqrt{\frac{105}{8740}} = 0.1096$$

$$\sqrt{\frac{\mathrm{E}}{\rho}}_{\mathrm{aluminum}} = \sqrt{\frac{70}{2700}} = 0.1610$$

When comparing the above results, the ratio of elastic modulus and density of steel and aluminum is similar, so they have similar natural frequencies under same conditions mentioned above. However, in the case of brass, it can be confirmed that it has a lower natural frequency than steel and aluminum since its ratio mentioned above is lower steel and aluminum. Ultimately, under the same cross-sectional area, moment of inertia, and boundary conditions, if the material changes, the size of the resonance frequency can be compared according to the ratio of the corresponding elastic modulus and density.

3.2.2. Material of boundary condition

When comparing the natural frequency of the experiment when a steel beam was fixed with a cantilever and that when both sides foxed, the natural frequency of support at both ends was much higher. This is because the stiffness of the beam increases when both sides are supported compared to when one side is fixed. Under the same material, experiment result shows that the first resonance frequency with both fixed condition is 8 times higher than that with one side fixed although the whole length for both condition is slightly different, $25\ [mm]$.

3.3. Discuss the differences between the FFT results with hand pressing the end of the beam and the impact hammer experiment.

First, using an impact hammer to provide excitation within the shortest possible time is to apply a signal as close to the impulse signal in the time domain as possible. In the case of an impulse signal, when FFT is taken, it has components corresponding to all frequency bands in all frequency regions. As can be seen in the Prelab part, the impulse signal in the time domain has a magnitude equal to 1 in the frequency domain. In other words, using this technique has the advantage of finding a resonance frequency corresponding to a high frequency. Conversely, the act of pressing and then releasing with the hand has an initial condition in the time domain, then has a signal form that becomes 0, and has a spectrum in the frequency domain as shown in **Figure 13**. Compared to the frequency spectrum of the impulse delta signal, it can be seen that the size of the components of the spectrum of the signal becomes smaller as it moves toward the high frequency region. In other words, the signal has the disadvantage that it is difficult to find a high-frequency resonance frequency.

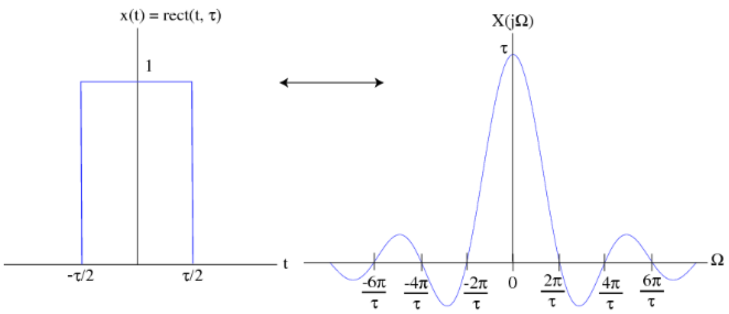


Figure 14. Frequency domain spectrum of time limited signal

3.4. Effect of the location of the acceleration sensor on the experimental results

If an acceleration sensor is attached to a part with a large amplitude, the difference between the signals can be clearly measured, so the natural frequency can be clearly measured. However, there is a part with an amplitude of 0, that is, a part where the vibration signal is not measured. This part is called a node, and if an acceleration sensor is attached to the node, the natural frequency in the corresponding mode cannot be found, so the acceleration sensor must be attached avoiding this part. Since the node appears in a different position for each mode, a correct experiment requires measuring the acceleration sensor several times while changing the attachment position, finding a point that does not match the node point for all modes, and attaching it to that point.