CAE LAB 1

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Subject : Estimation of elastic modulus using strain guage

1. Introduction

The elasticity coefficient is determined by measuring deflection and strain caused by the bending of a beam. Strain gauges are affixed to metal specimens, and a bridge measurement circuit is established to measure voltage signals using LabVIEW. From these values, strain is calculated, and subsequently, the elasticity coefficient is derived. Additionally, a comparison is made with the coefficient obtained from measuring the deflection of the beam using a dial gauge. The objective of this experiment is to comprehend the relationship between deflection, strain, and stress resulting from bending moments applied to the beam and to gain practical experience in attaching strain gauges, configuring bridge circuits, and acquiring voltage signals through LabVIEW, thereby laying the foundation for fundamental principles in mechanical measurements.

(1) Strain guage

- Guage factor of strain gage

Gauge factor is defined as the ratio of fractional change in electrical resistance to the fractional change in length (strain). The Gauge factor for metallic strain guages is typically around 2.0. Definition of gauge factor (K) can be written as follows :

$$K = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon}$$

L : initial length of installed guage

R : initial resistance of installed guage

ΔL : change in length

ΔR : change in resistance

- Analysis of voltage output formula of a bridge circuit that measures voltage by resistance change according to change in the length of the strain gauge.

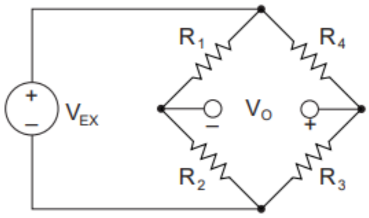

Figure 1.1. Weatstone Bridge circuit

The output voltage in the above circuit is calculated as follows :

$$V_0 = [\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}] \times V_{EX}$$

Any change in resistance in any arm of the bridge will result in a nonzero output voltage. Therefore, replacement of any resistance to active strain guage will unbalance the bridge and produce a nonzero output voltage, due to the any changes in the strain guage resistance. All equations for this procedure can be written as follows :

$$\Delta R = R_G \times GF \times \varepsilon$$

$$e_{out} = E \times \frac{\Delta R \times G}{4R}$$

- Change in output voltage when a $100k\ [Ohm]$ resistor is connected in parallel to an arm with a gauge in a bridge circuit and amplifier circuit.

$$e_{out} = E \times \frac{\Delta R}{4 \times R} \times G_{amp}$$

$$e_{out} = 5.0[V] \times \frac{\Delta R}{4 \times 120.0[\Omega]} \times 1500 - 1$$

$$\Delta R = 120.0[\Omega] - \frac{1}{\frac{1}{120.0[\Omega]} + \frac{1}{100.0[k\Omega]}} = 120.0[\Omega] - 119.8562[\Omega] = 0.1438[\Omega]$$

With calculated ΔR and the above equation, e_{out} can be calculated as follows :

$$e_{out} = 5[V] \times \frac{0.1438[\Omega]}{4 \times 120[\Omega]} \times 1500 \approx 2.25[\mathrm{V}]$$

(2) stress and deflection

- From the material yield point of view, the weight that can be hung at the center of a aliminum with a simply supported beam length of $400\ [mm]$

The yield strength value and given conditions of aluminium are as follows.

$$\sigma_Y = 110\ [Mpa]$$

$$l = 400[mm] = 0.4[m]$$

$$b = 20[mm] = 0.02[m]$$

$$h = 3.0[mm] = 0.003[m]$$

In order to calculate the maximum weight that can be carried, you must set a safety factor and use the formula below to calculate the maximum load accordingly.

$$SF = \frac{\sigma_Y}{\sigma_{max}}\ , \qquad \sigma_{max} = \frac{(M_{max} \times c\ )}{I}$$

Therefore, the formula for the maximum moment of the simply supported beam is as follows.

$$M_{max} = \frac{W}{2} \times \frac{L}{2} = \frac{mg}{2} \times \frac{0.4}{2}\ [Nm]$$

After that, when the secondary moment of inertia value is obtained and the safety factor is set to 2, the process of calculating the maximum mass is as follows.

$$I = \frac{(bh^3)}{12} = \frac{1}{12} \times 0.02[m] \times (0.003[m])^3 = 4.5 \times 10^{-11}\ [m^4]$$

$$\sigma_{max} = \frac{\sigma_Y}{SF} = \frac{110[Mpa]}{2.0} = 55\ [Mpa]$$

$$\sigma_{max} = \frac{(mg \times 0.1) \times 0.0015[m]}{I} = 32700000 \times m = 55\ [MPa]$$

$$m_{max} = \frac{55[Mpa]}{32700000} = 1.68[kg]$$

- The measuring range of the dial gauge that can measure the deflection of the central part of the above simply supported beam is 7.0mm. How much weight can be hung in the center?

The elastic modulus of aluminium is $E = 70.0[Gpa]$.

The mass value must be obtained when the measurement range is maximum, that is, when $\delta_{max} = 7.0[mm]$.

$$\delta_{max} = \frac{PL^3}{48EI} \leq 0.007[m]$$

$$P = m \times 9.81[m/s^2]$$

$$\delta_{max} = \frac{PL^3}{48EI} = \frac{m \times 9.81[m/s^2] \times 0.4^3[m^3]}{48 \times 70[GPa] \times 4.5 \times 10^{-11}[m^4]} \leq 0.007[m]$$

$$m \leq 1.68[kg]$$

Therefore, max mass can be hung in the center part is $1.68[kg]$.

2. Experiment setup and procedure

2.1. Experiment setup

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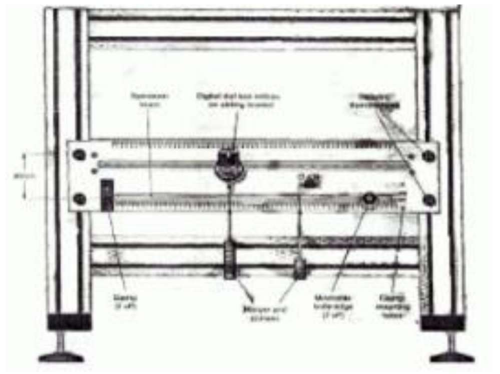
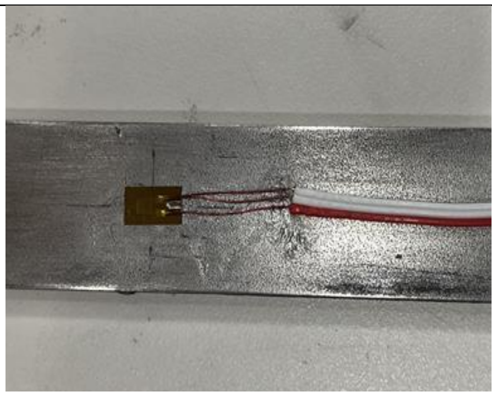
<td></td>
<td></td>
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<td>Dial guage</td>
<td>Strain guage</td>
<td>DN-AM100</td>
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Figure 2.1. Experiment devices

In this experiment, a simple support form was utilized for the dial gauge. Second one in **Figure 2.1.** is a strain guage designed to measure the changes in resistance resulting from a change in length. And the third one in **Figure 2.1** illustrates an amplifier use to amplify and filter the output voltage from a bridge circuit equipped with a strain gauge. In this experiment, a low-pass filter with an amplification factor of 1500 and a cutoff frequency of $10\ [Hz]$ was utilized. The experiment employed a half-bridge circuit for the bridge circuit configuration, comprising four resistors arranged in a diamond shape. However, instead of physically implementing the circuit, a bridge circuit was assembled by connecting the strain gauge to a custom-made PCB board. The comprehensive schematic illustrating the connections between the strain amplifier and this bridge circuit is depicted in **Figure 2.2.**

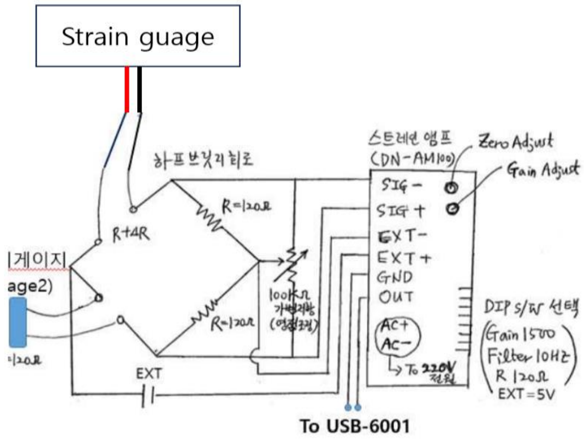


Figure 2.2. Circuit diagram for strain measurement

2.1. Experiment procedure

1) First, the strain gauge is attached using adhesive so that it is located in the center of the specimen being tested. Here, since the load is placed on the center of the specimen, it is important to install it properly in the center. In addition, it is better to attach it well as shown in **Figure 2.1** so that the lines connected to the strian gauge do not contact each other during the attachment process.

2) Connect to gage 1 of the half bridge circuit PCB board of the strain gauge, and connect the 120 ohm resistor directly to gage 2 part to play the role of temperature compensation. Afterwards, a strain amplifier is used to remove noise and amplify voltage. Connect four wires (SIG+, SIG-, EXT+, EXT-) between the bridge circuit and the strain amplifier.

3) Lastly, DAQ is used to read the voltage value. To use differential mode, connect the final output voltage to one side of the analog input of the DAQ and ground to one side. The detailed circuit diagram is specified in **Figure 2.2.**

4) Attach the test specimen to the dial gauge, increase the load, and measure the final voltage value. By back-estimating the strain using the output voltage, it is possible to calculate the amount of deflection.

3. Experiment result

The main parameters of the cross-section measured in advance for the test specimen are as follows.

$$\text{Specimen cross section}: \ 20.03\ [mm] \times 2.79\ [mm]$$

$$\text{Secondary moment of inertia}: \ 3.63\ \times 10^{-11}\ [m^4]$$

The settings of additional experimental equipment are as follows.

$$\text{Strain guage}: \ GF = 2.10\ , R = 120.0\ [\Omega]$$

$$\text{Amp input voltage}: \ E = 5.0\ [V],\ gain = 1500$$

The values obtained through the experiment are as follows.

Deflection values are based on values obtained from dial gauge equipment.

Table 3.1. Measured value of e_{out} and deflection

weight $[kg]$	e_{out} $[V]$	deflection $[mm]$
0.2	0.36	1.1
0.4	0.815	2.14
0.6	1.3	3.19
0.8	1.7	4.27
1.0	2.5	5.31

Based on the above values, the bending moment and strain acting on the specimen must be calculated inversely.

The specimen was fixed with a simple support beam and calculated as $L = 0.4[m]$.

First, the moment is defined as follows.

$$M = \frac{PL}{4} = \frac{mgL}{4}\ [N \times m]$$

Therefore, calculating the moment value according to each load is as follows.

Table 3.2. Bending moment value according to load

weight $[kg]$	Bending moment $[Nm]$
0.2	0.1962
0.4	0.3924
0.6	0.5886
0.8	0.7848
1.0	0.981

Next, the process of calculating strain according to the voltage value coming from the experimental equipment is as follows.

$$\text{deflection : } \delta = \frac{PL^3}{48EI} = \frac{mgL^3}{48EI} \; [m]$$

$$\text{resistance variation : } \quad \Delta R = \frac{4R \times e_{out}}{E \times G} \; [\Omega]$$

$$\text{strain : } \varepsilon = \frac{\Delta R}{R \times GF} \; [-]$$

$$\text{stress : } \sigma = E\varepsilon = \frac{Mc}{I} \; [Pa]$$

Table 3.3. Moment, resistance change, and strain value according to load

weight $[kg]$	Bending moment $[N \times m]$	$\Delta R \; [\Omega]$	$\varepsilon \; [-]$
0.2	0.1962	0.02304	0.091×10^{-3}
0.4	0.3924	0.05216	0.207×10^{-3}
0.6	0.5886	0.0832	0.33×10^{-3}
0.8	0.7848	0.1088	0.43×10^{-3}
1.0	0.981	0.16	0.63×10^{-3}

The elastic modulus of a specimen can be calculated in the following two ways, and the elastic modulus value calculated according to load is as follows.

$$E_1 = \frac{Mc}{\varepsilon \times I} \;\; [Pa], \;\; E_2 = \frac{PL^3}{48\delta I} \;\; [Pa]$$

The elastic modulus value according to load can be calculated using the equation as follows.

Table 3.4. Calculation of elastic modulus according to load

하중 $[kg]$	$E_1 \; [GPa]$	$E_2 \; [GPa]$
0.2	82.59	65.04
0.4	72.96	67.44
0.6	68.61	67.87
0.8	69.95	67.6
1.0	59.45	67.95

Afterwards, the result of linear regression analysis for calculating elastic modulus of E1, E2 are as follows :

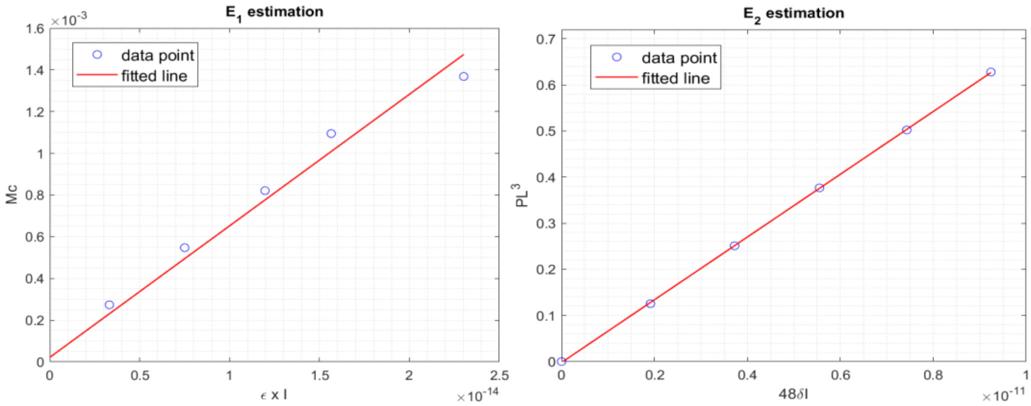


Figure 3.1. Elastic modulus estimation using polyfit

$$E1_{est} = 63.09\ [GPa], \qquad E2_{est} = 68.06\ [GPa]$$

4. Discussion and analysis

Comparison of experimental value and theoretical value of important parameters is as follows :

Table 4.1. Comparison table of important parameters

	Theoretical deflection	Experimental deflection	Theoretical Stress	Experimental stress
$\mathbf{0.2}\ [\boldsymbol{kg}]$	$1.0\ [mm]$	$1.1\ [mm]$	$7.51\ [MPa]$	$7.55\ [MPa]$
$\mathbf{0.4}\ [\boldsymbol{kg}]$	$2.06\ [mm]$	$2.14\ [mm]$	$15.02\ [MPa]$	$15.10\ [MPa]$
$\mathbf{0.6}\ [\boldsymbol{kg}]$	$3.09\ [mm]$	$3.19\ [mm]$	$25.23\ [MPa]$	$22.65\ [MPa]$
$\mathbf{0.8}\ [\boldsymbol{kg}]$	$4.12\ [mm]$	$4.27\ [mm]$	$30.03\ [MPa]$	$30.20\ [MPa]$
$\mathbf{1.0}\ [\boldsymbol{kg}]$	$5.15\ [mm]$	$5.31\ [mm]$	$37.54\ [MPa]$	$37.75\ [MPa]$

(1) Discuss the location of the maximum load magnitude determined for each group experiment.

From the material yield point of view, the weight that can be hung at the center of a alliminum with a simply supported beam length of $400\ [mm]$.

The yield strength value and given conditions of alliminum are as follows.

$$\sigma_Y = 110\ [Mpa]$$

$$l = 400[mm] = 0.4[m]$$

$$b = 20[mm] = 0.02[m]$$

$$h = 3.0[mm] = 0.003[m]$$

In order to calculate the maximum weight that can be carried, you must set a safety factor and use the formula below to calculate the maximum load accordingly.

$$SF = \frac{\sigma_Y}{\sigma_{max}}, \qquad \sigma_{max} = \frac{(M_{max} \times c\ )}{I}$$

Therefore, the formula for the maximum moment of the simply supported beam is as follows.

$$M_{max} = \frac{W}{2} \times \frac{L}{2} = \frac{mg}{2} \times \frac{0.4}{2}$$

After that, when the secondary moment of inertia value is obtained and the safety factor is set to 2, the process of calculating the maximum mass is as follows.

$$I = \frac{(bh^3)}{12} = \frac{1}{12} \times 0.02[m] \times (0.003[m])^3 = 4.5 \times 10^{-11}\ [m^4]$$

$$\sigma_{max} = \frac{\sigma_Y}{SF} = \frac{110[Mpa]}{2.0} = 55\ [Mpa]$$

$$\sigma_{max} = \frac{(mg \times 0.1) \times 0.0015[m]}{I} = 32700000 \times m = 55\ [MPa]$$

$$m_{max} = \frac{55[Mpa]}{32700000} = 1.68[kg]$$

Maximum mass used in the experiment for checking strain was $1.0\ [kg]$. So, it is reasonable to use maximum load in this experiment when considering safety factor as 2 in the material yield view.

(2) Discuss the noise in the experimental data and the adequacy of the applied filter.

Noise that must be considered in this experiment includes noise generated from the power supply and noise generated while passing through each circuit, such as signal lights. In the experiment, by using a low pass filter with a cutoff frequency of $10\ [Hz]$, the intended output signal is passed and high frequency signals such as noise are filtered. From that perspective, the feasibility of the designed low pass filter can be evaluated.

(3) Compare and discuss the elastic modulus values obtained from deflection measurements and the elastic modulus values obtained from strain measurements, and compare them with the standard values of material properties.

First, this will compare the values obtained using each method with the theoretical values to determine the error, and then compare and analyze the cause. The percentile relative error components for each method are:

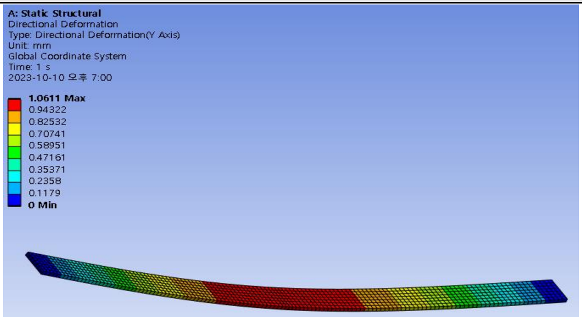
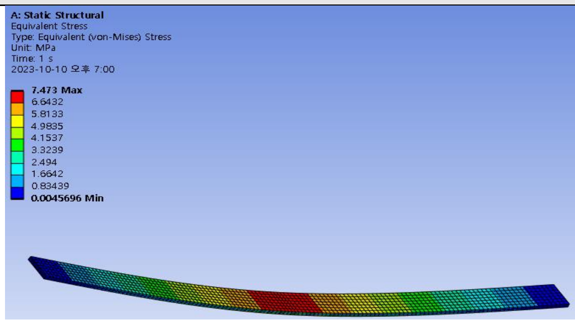
$$pre\ of\ E_1 = \frac{70[GPa] - 63.09[GPa]}{63.09[GPa]} = 10.95\ [\%]$$

$$pre\ of\ E_2 = \frac{70[GPa] - 68.06[GPa]}{68.06[GPa]} = 2.85\ [\%]$$

First, if we analyze the error factors of the second elastic modulus obtained through dial gauge, it is as follows. First, simple support was assumed, but as the DOF for vertical movement is not completely constrained, a larger deflection than the theoretical deflection may occur. Accordingly, the value of the experimentally obtained elastic modulus can become smaller. Additionally, it assumes an Euler-Bernoulli beam, and the theory assumes that a rod or beam has a small deformation angle and deformation, and that a straight and uniform beam is present. Accordingly, the shape of the beam often has a slight curvature or is subject to an uneven load, and errors may occur. Second, it is necessary to analyze the causes of error in the elastic modulus obtained through the voltage value from the strain gauge. The internal bridge circuit in the strain gauge measures the change in resistance, and accordingly measures the output voltage multiplied by the gain. In the case of gain, theoretically it is 1500 times, but in the actual environment, there may be an error with the theoretical value, resulting in an error. In addition, as the signal changes in resistance and passes through the amp for amplification and DAQ for reading, a subtle noise signal can be introduced through those procedures. Although this can be resolved by applying a low pass filter, it is impossible to completely eliminate it. Accordingly, if the bandwidth of the signal is known to some extent, it would be a good idea to reduce the impact of noise by applying a band pass filter that passes only the corresponding bandwidth.

(4) Create an ANSYS simulation model to analyze stress and deflection, and compare simulation values with theoretical and experimental values at three load values.

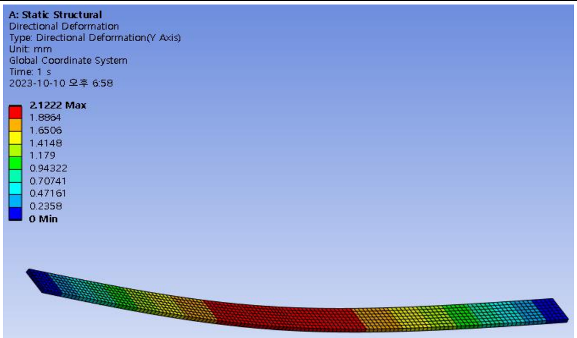
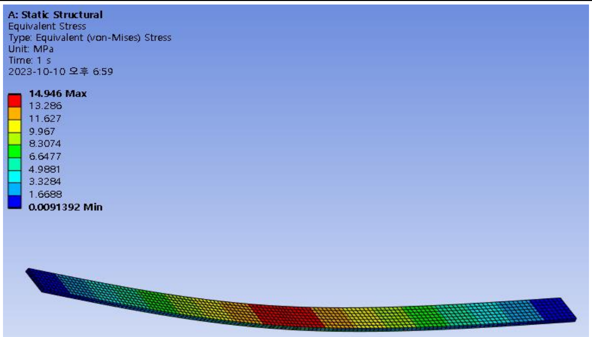
The results of ANSYS simulation for $0.2\ [kg], 0.4\ [kg], and\ 0.6\ [kg]$ are as follows. All simulation process was conducted with young's modulus $68.06\ [GPa]$.

Deflection in $0.2\ [kg]$	Stress in $0.2\ [kg]$
	

Theoretical value in this case can be calculated as follows :

$$P = 0.2 \times 9.81 = 1.962\ [N]$$

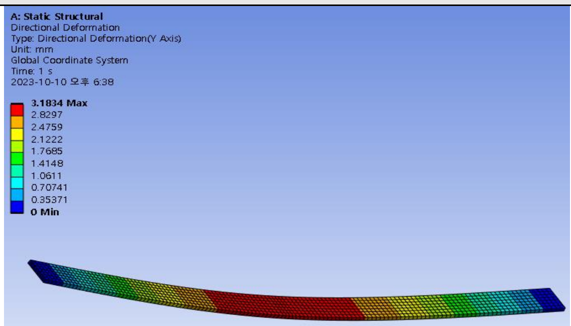
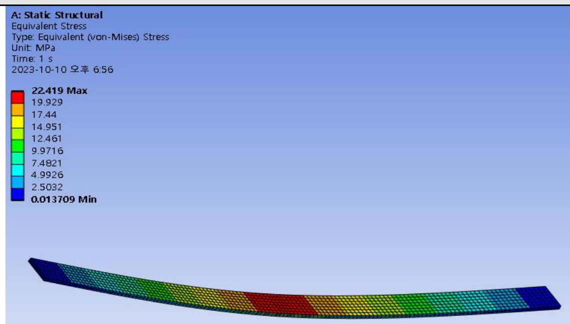
$$\delta = \frac{PL^3}{48EI} = 1.0\ [mm], \qquad \sigma = \frac{Mc}{I} = 7.55[MPa]$$

Deflection in $0.4\ [kg]$	Stress in $0.4\ [kg]$
	

Theoretical value in this case can be calculated as follows :

$$P = 0.4 \times 9.81 = 3.9240\ [N]$$

$$\delta = \frac{PL^3}{48EI} = 2.1\ [mm], \qquad \sigma = \frac{Mc}{I} = 15.1[MPa]$$

Deflection in $0.6\ [kg]$	Stress in $0.6\ [kg]$
	

Theoretical value in this case can be calculated as follows :

$$P = 0.6 \times 9.81 = 5.88\ [N]$$

$$\delta = \frac{PL^3}{48EI} = 3.1\ [mm], \qquad \sigma = \frac{Mc}{I} = 22.7[MPa]$$