EM Algorithm for Gaussian Mixture Model

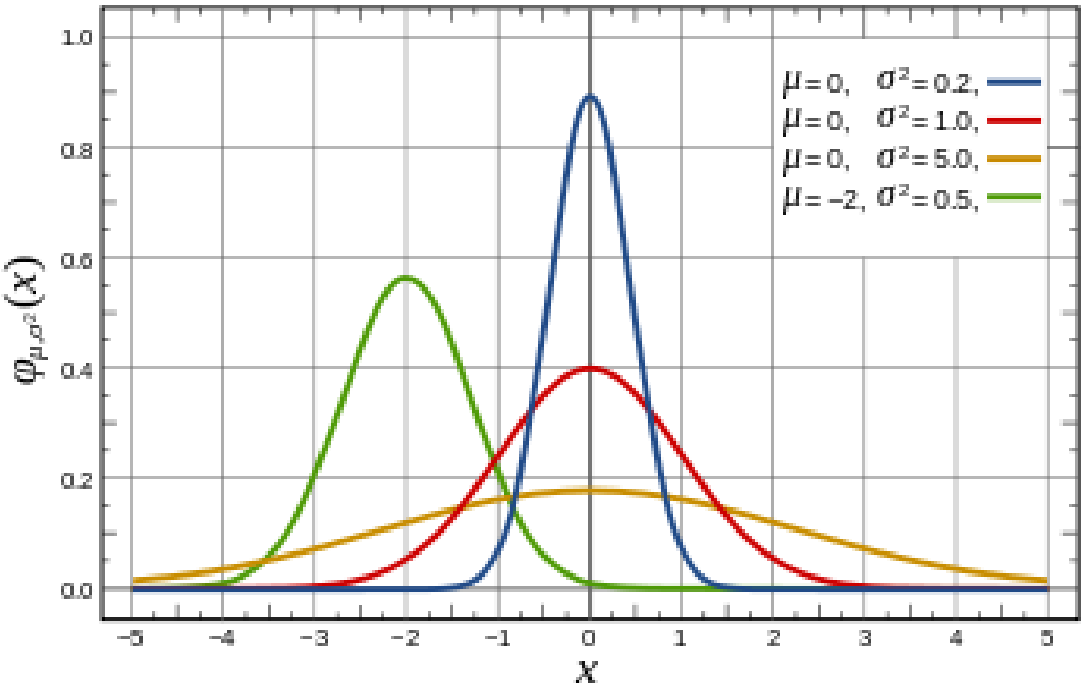
April 2013

Introduction of the Guassian Mixture Model

Recap : The Guassian distribution

The Guassian distribution:

$$\mathcal{N}(x|\mu,\Sigma) = \frac{1}{(2\pi)^{D/2}}\frac{1}{|\Sigma|^{1/2}}exp\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\} \quad (1)$$



Introduction of the Guassian Mixture Model

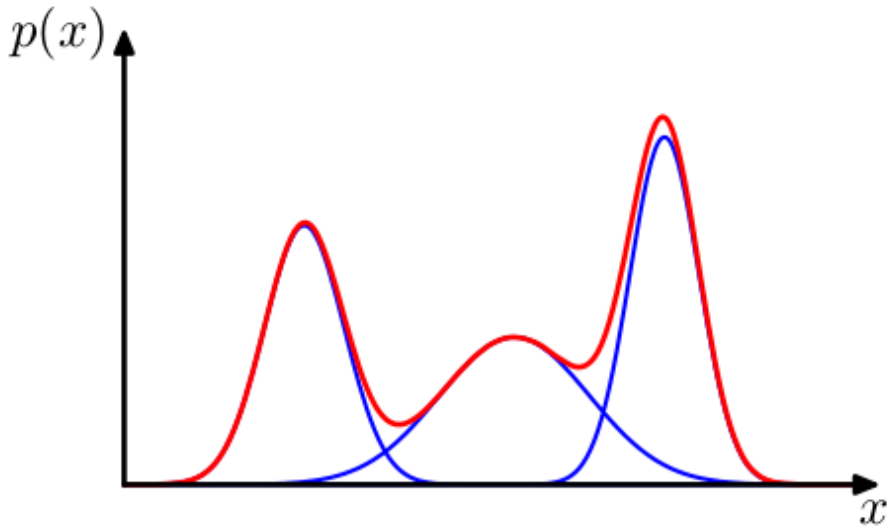
The Guassian Mixture distribution

The Guassian Mixture distribution is a linear superposition of Guassians:

$$p(x) = \Sigma_{k=1}^{K} \pi_k \mathcal{N}(x|\mu_k, \Sigma_k) \tag{2}$$

Subject to:

$$\Sigma_{k=1}^{K} \pi_k = 1 \tag{3}$$



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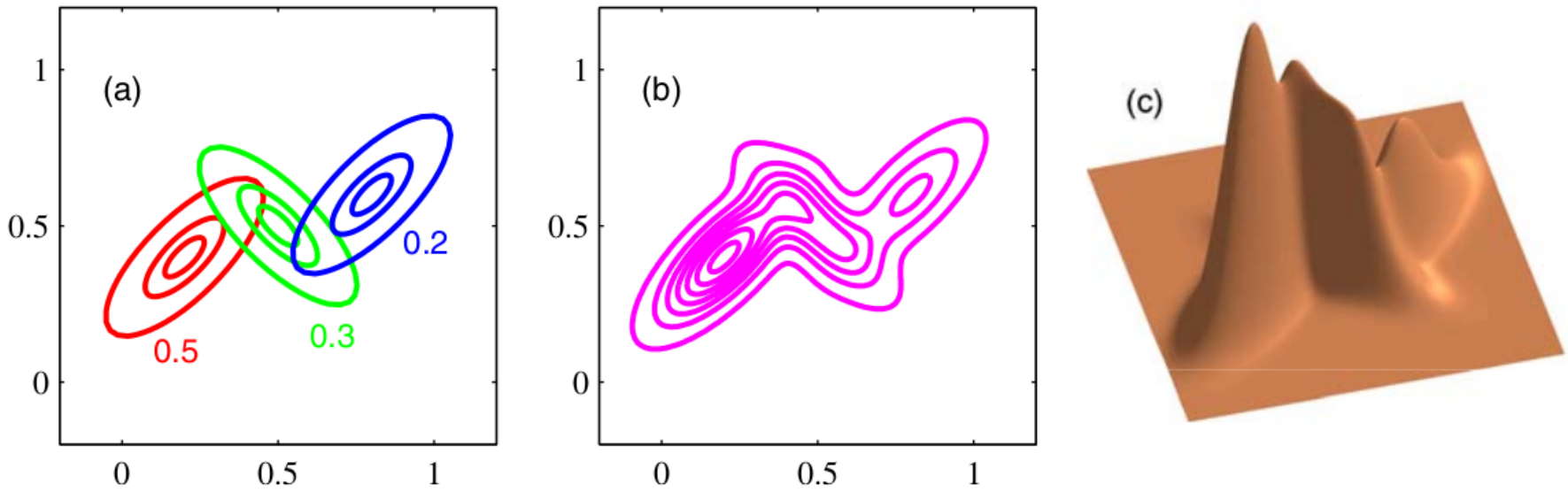
The Guassian Mixture distribution

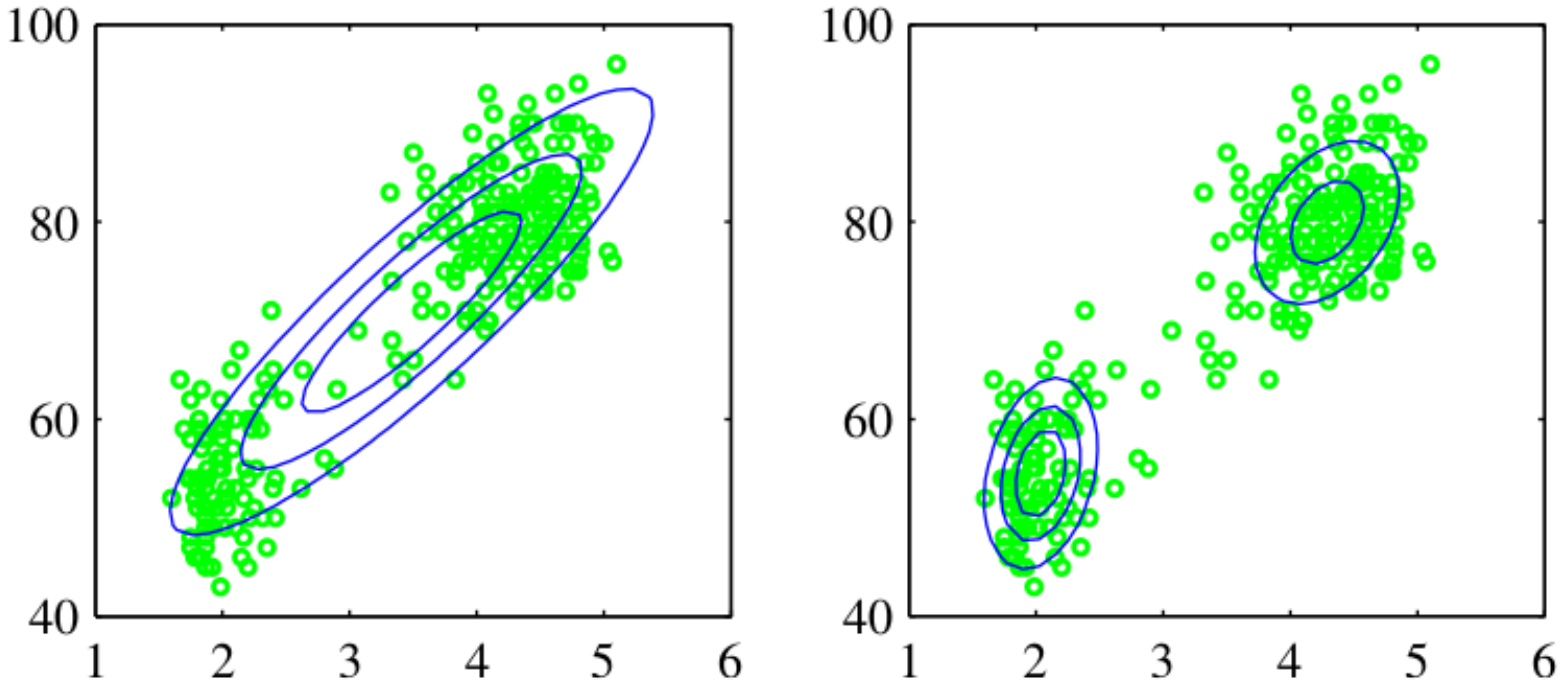


A 2-dimension example of GMM

Introduction of the Guassian Mixture Model

Now, for a Guassian Mixture Model, given the parameters:
k, the number of Guassian components
$\pi_1 ... \pi_k$, the mixture weights of the components
$\mu_1 ... \mu_k$, the mean of each component
$\Sigma_1 ... \Sigma_k$, the variance of each component
We can generate samples $s_1, s_2 ... s_n$ from the distribution.

Why do we need Guassian Mixture



In this example, we see that Guassian Mixture describes the data better a single Guassian.

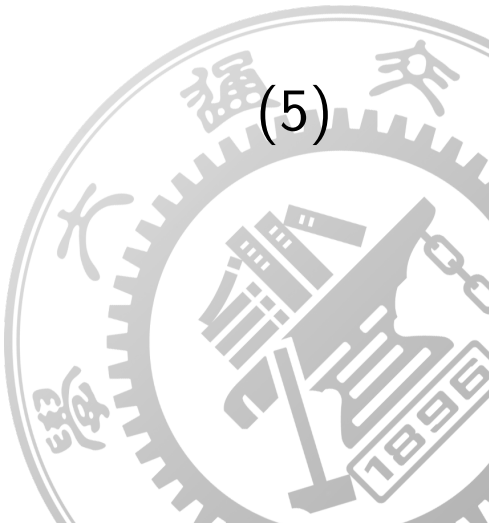
Another representation of the Guassian Mixture

Given a Guassian Mixture model, we introduce K-dimensional binary random variable z which only one element z_k is euqal to 1 and the others are all 0.

$$z = (0, 0, .., 1, 0, ..0) \tag{4}$$

So there are K possible states for z.And we let

$$p(z_k = 1) = \pi_k \tag{5}$$



Even more structural elements

Verse, Quote and Quotation

This is a text in verse style.

This is a quote.

While this is a quotation. Note how it has a larger indentation in the first line.

Maths

Including mathematical formulae into Beamer presentations is easy

Beamer's biggest strength for scientific presentations is its ability to use the full power of LaTeX's mathematical displays.

$$\begin{aligned} D_{\mathrm{KL}}(P_0, P_\infty) &= \sum_{\gamma\delta} P_0^{\gamma\delta} \log P_0^{\gamma\delta} - \sum_{\gamma\delta} P_0^{\gamma\delta} \log P_\infty^{\gamma\delta} \\ &= -H(P_0) - \langle \log P_\infty \rangle_0 \end{aligned} \tag{6}$$

Structuring Texts

Lists

1. Of course Beamer can do enumerated lists
2. It also knows how to do columns. This is helpful if you want to put figures next to text.

- bulleted lists are not numbered
- Beamer can do a lot more. For overlays, figures with captions, etc., have a look at [aaPicutre]. But don't get carried away! Simple is nearly always better.

Installation Instructions

These instructions assume you are using a packaged LaTeX distribution, like MikTex or TeXLive. If you have a custom installation, chances are you are proficient enough to interpret these instructions accordingly.

1. install beamer. If you are using a LaTeX distribution, it's most probably already installed. Otherwise, see [aaPicutre]
2. find the beamer package directory. It's typically in [texroot]/tex/latex/beamer/. Change there.
3. copy the file `beamercolorthemecambridgeuk.sty` to ./themes/color/.
4. copy the file `beamerthemeCambridgeUK.sty` to ./themes/theme/.
5. run `sudo texhash`, or the equivalent on your system[1]

[1] Under MikTex on Windows, open Start → MikTex → Settings and run

Bibliography

- Some pictures are from Wiki or PRML
- Tantau, Till
  The Beamer class
  http://latex-beamer.sourceforge.net/
- University of Cambridge
  Identity Guidelines – first edition, May 2008
  http://www.admin.cam.ac.uk/offices/...
  communications/services/identityguidelines/