STAT 153 - Introduction to Time Series
Practice Midterm

Fall 2022, UC Berkeley

1. The `UKgas` dataset in R gives Quarterly Observations on the UK gas consumption from the first quarter of 1960 to the last quarter of 1986 (there are 108 observations in total). A plot of the data is given in Figure 1. Consider the two periodograms given in Figure 2. One of these is the correct periodogram for the logarithm of the UK gas data while the other is the periodogram for some other dataset. Identify the correct periodogram for the logarithm of the UK gas data giving reasons for your answer.

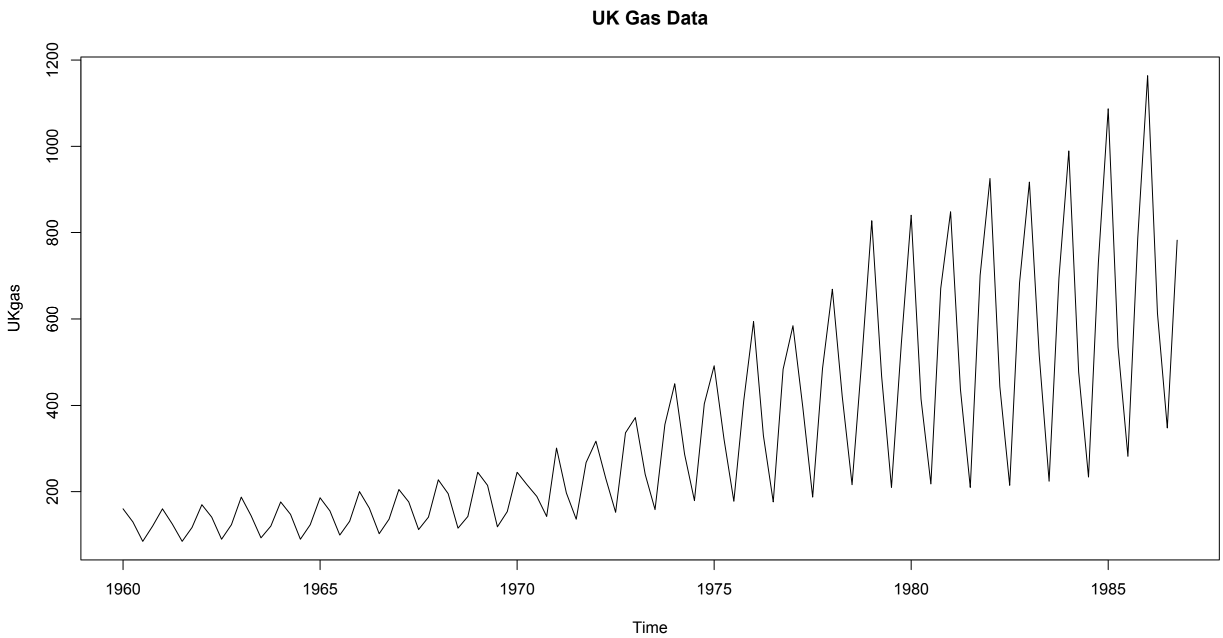

Figure 1: UKgas Data

2. The data plotted in Figure 3 gives (seasonally adjusted) monthly observations on the retail sales (in millions of dollars) of Furniture Stores. For each of the following models, indicate whether they are adequate for this dataset giving reasons:

- **Model One**: $Y_t = \beta_0 + \beta_1 t + Z_t$ with $Z_t \overset{\text{i.i.d}}{\sim} N(0, \sigma^2)$. This model has three parameters β_0, β_1 and σ.

- **Model Two**: $Y_t = \beta_0 + \beta_1 t + \beta_2 (t - \omega)_+ + Z_t$ with $Z_t \overset{\text{i.i.d}}{\sim} N(0, \sigma^2)$. This model has five parameters $\beta_0, \beta_1, \beta_2, \omega, \sigma$. Here x_+ denotes $\max(x, 0)$.

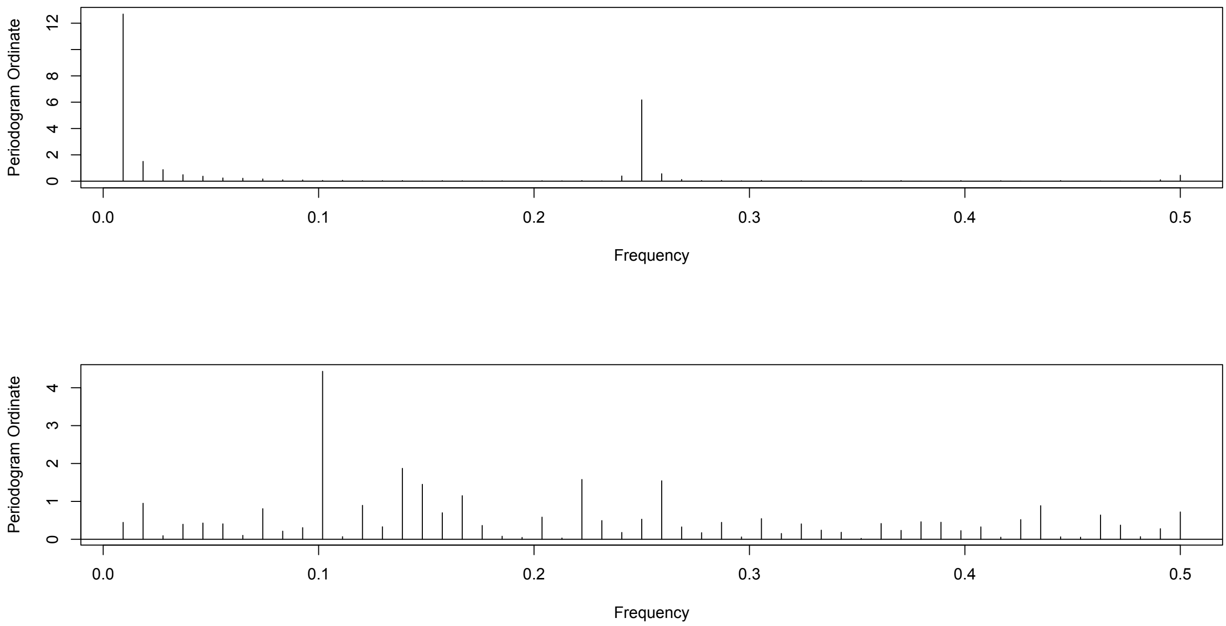

Figure 2: Two Periodograms

- **Model Three**: $Y_t = \beta_0 + \beta_1 t + \beta_2 (t - \omega_1)_+ + \beta_3 (t - \omega_2)_+ + Z_t$ with $Z_t \overset{\text{i.i.d}}{\sim} N(0, \sigma^2)$. This model has seven parameters $\beta_0, \beta_1, \beta_2, \beta_3, \omega_1, \omega_2, \sigma$.

3. For a time series dataset $y_1, \dots, y_n$, I would like to fit the model:

$$Y_t = \beta_0 + [\beta_1 \cos(2\pi f t) + \beta_2 \sin(2\pi f t)] \exp(-\omega t) + Z_t \qquad \text{with } Z_t \overset{\text{i.i.d}}{\sim} N(0, \sigma^2).$$

This fits a sinusoid to the data with an exponential decay. The model has six unknown parameters $\beta_0, \beta_1, \beta_2, f, \omega, \sigma$. Suppose my main interest is in the parameters f and ω. Describe a procedure for estimating f and ω along with proper uncertainty quantification.

4. For a time series dataset $y_1, \dots, y_n$, I would like to fit the model:

$$Y_t = \beta_0 + \beta_1 t + Z_t \qquad \text{with } Z_t \overset{\text{i.i.d}}{\sim} C(0, \sigma).$$

where $C(0, \sigma)$ is the Cauchy density with scale σ (its density is $x \mapsto \frac{\sigma}{\pi(\sigma^2 + x^2)}$). This model has the three unknown parmaeters β_0, β_1, σ. Describe a numerical procedure for estimating β_1 and quantifying uncertainty.

5. I have an observed time series dataset $y_1, \dots, y_n$ with $n = 100$. For this dataset, I am considering the two models:

- **Model One**: $y_1, \dots, y_n \overset{\text{i.i.d}}{\sim} N(\mu, \sigma^2)$. This model has two unknown parameters μ and σ^2.

- **Model Two**: $y_1, \dots, y_n$ are independent with

$$y_1, \dots, y_{n/2} \overset{\text{i.i.d}}{\sim} N(\mu_1, \sigma^2) \quad \text{and} \quad y_{(n/2)+1}, \dots, y_n \overset{\text{i.i.d}}{\sim} N(\mu_2, \sigma^2).$$

This model has three unknown parameters μ_1, μ_2 and σ^2.

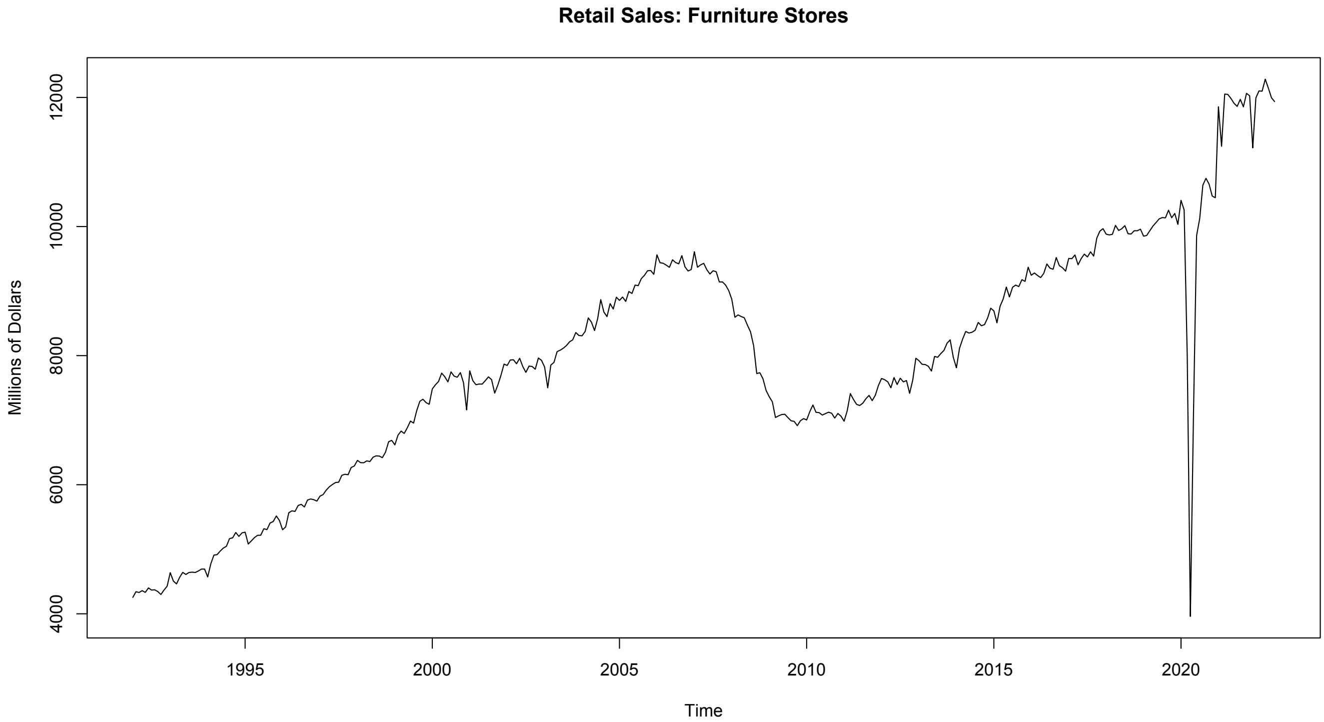

Figure 3: FRED Data

a) Describe a Bayesian model selection procedure for deciding between the above two models.

b) Write down the AIC of each of the two models explicitly in terms of $y_1, \ldots, y_n$.