

Final Project

Mechanical Vibrations

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Defining the data given in the project essay

```
clc,clear,close all

m = [ 2.294  1.941  1.943  2.732  0.774  0.774] * 1e3;

k = [2155.059  662.275  1455  778.436  75.332  75.332];

kt = 1e5;

L = [10.12  20.21  21.97  25  45.3  45.3];
```

Calculating the moment of inertia of masses 5 & 6 to be used as the moment of inertia of the bar connecting them

(Further detail is explained in the project report)

```
I = m(5) * L(5) ^ 2 + m(6) * L(6) ^ 2;
```

Defining the mass and stiffness coefficient matrices to obtain the natural frequencies and modeshapes

```
M = [m(1) 0 0 0 0 0
      0 m(2) 0 0 0 0
      0 0 m(3) 0 0 0
      0 0 0 m(4) 0 0
      0 0 0 0 m(5) 0
      0 0 0 0 0 m(6) 0]
```

```

0 0 0 0 0 0 I];

K = [k(1)+k(2) -k(2) 0 0 0 0 0
      -k(2) k(2)+k(3) -k(3) 0 0 0 0
      0 -k(3) k(3)+k(4) -k(4) 0 0 0
      0 0 -k(4) k(4)+k(5)+k(6) -k(5) -k(6) k(5)*L(5)-k(6)*L(6)
      0 0 0 -k(5) k(5) 0 -k(5)*L(5)
      0 0 0 -k(6) 0 k(6) k(6)*L(6)
      0 0 0 k(5)*L(5)-k(6)*L(6) -k(5)*L(5) k(6)*L(6) k(5)*L(5)^2+k(6)*L(6)^2+kt];

C = 0.005 * K;

```

Defining the forces

```

w = 1.75;

tspan = 0:0.1:50;

tf = tspan;

g = sin(w*tf);

F = [306.25
      612.5
      918.75
      1225
      0
      0
      0];

```

Obtaining natural frequencies and mode shapes

```
[V , D] = eig(K , M);
```

```
ModeShapes = V
```

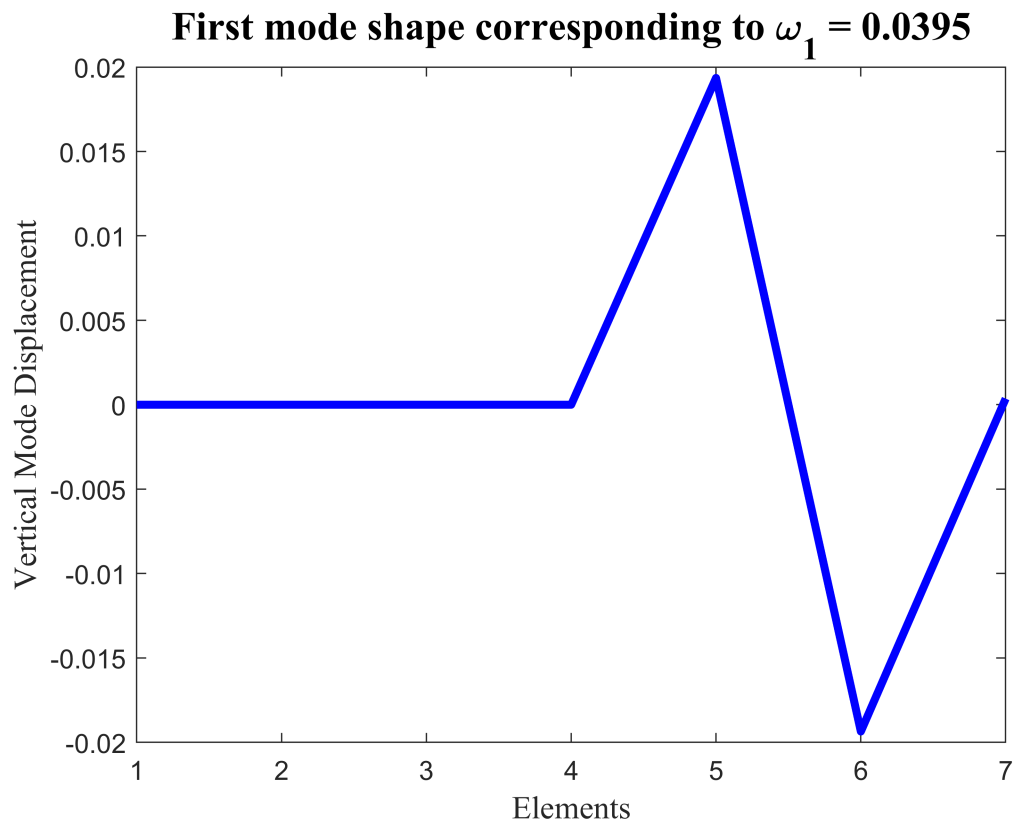
```
ModeShapes = 7×7
-0.0000 -0.0015 0.0021 0.0000 0.0055 0.0191 -0.0059
-0.0000 -0.0063 0.0081 0.0000 0.0134 -0.0005 0.0152
-0.0000 -0.0081 0.0092 0.0000 0.0077 -0.0086 -0.0152
-0.0000 -0.0108 0.0082 0.0000 -0.0129 0.0027 0.0027
0.0194 -0.0175 -0.0182 -0.0165 0.0030 -0.0002 -0.0001
-0.0194 -0.0175 -0.0182 0.0165 0.0030 -0.0002 -0.0001
0.0004 -0.0000 -0.0000 0.0004 0.0000 -0.0000 -0.0000
```

```
omega_n = sqrt(D)
```

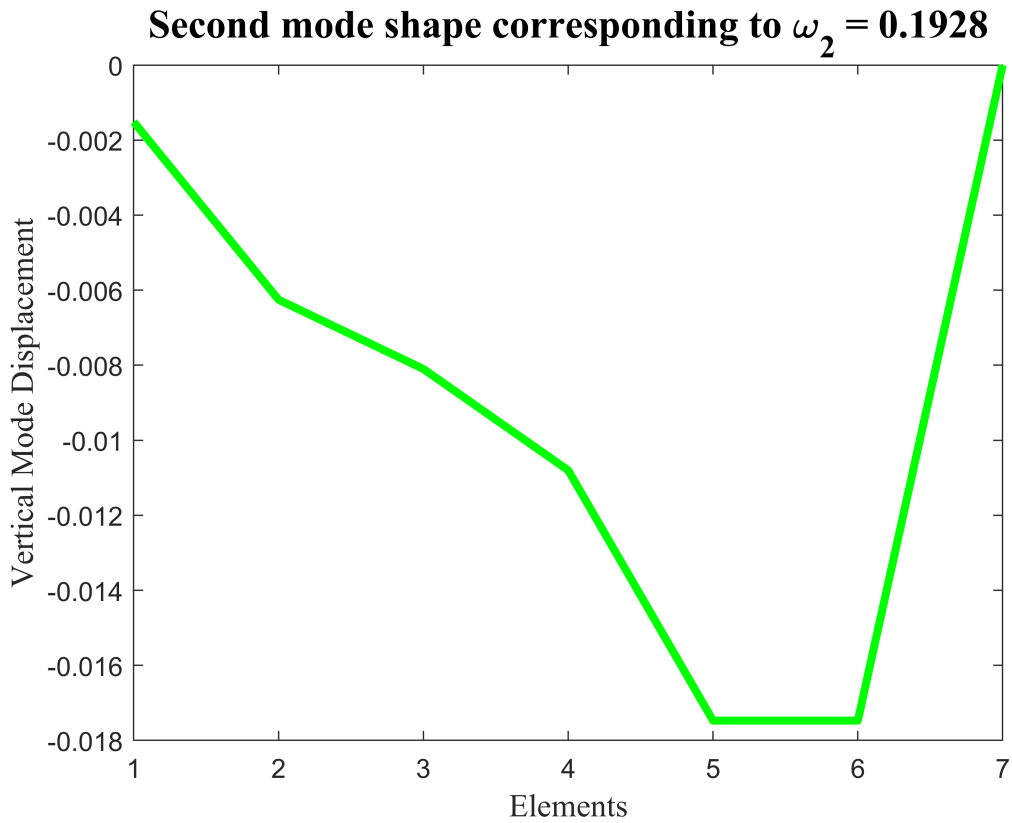
```
omega_n = 7×7
0.1203 0 0 0 0 0 0
0 0.1928 0 0 0 0 0
0 0 0.3756 0 0 0 0
0 0 0 0.4601 0 0 0
0 0 0 0 0.7220 0 0
0 0 0 0 0 1.1115 0
0 0 0 0 0 0 1.4038
```

Plotting the mode shapes

```
plot(ModeShapes(:,1) , 'color' , "b" , 'LineWidth' , 3)
title("First mode shape corresponding to \omega_1 = 0.0395", 'fontName', 'Times New Roman', 'FontSize', 12)
ylabel("Vertical Mode Displacement", 'fontName', 'Times New Roman' , 'FontSize', 12)
xlabel("Elements", 'fontname', 'Times New Roman' , 'FontSize', 12)
```



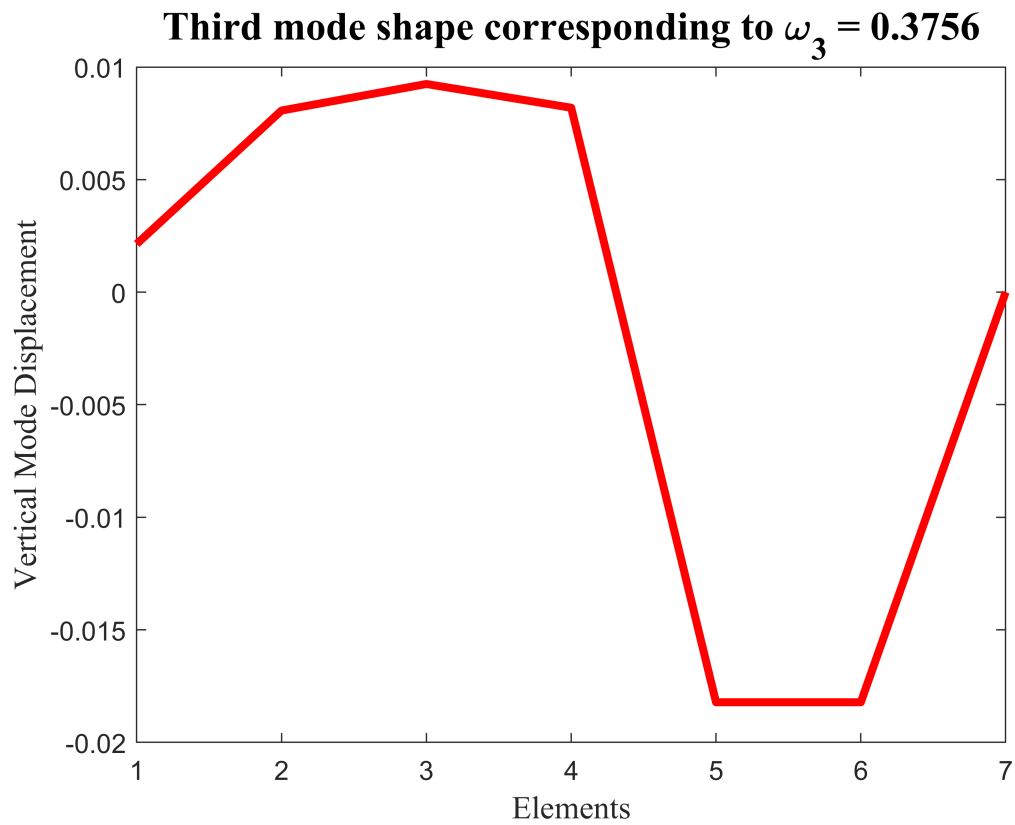
```
plot(ModeShapes(:,2) , "color" , "g" , 'LineWidth' , 3)
title("Second mode shape corresponding to \omega_2 = 0.1928",'fontName','Times New Roman','FontSize',12)
ylabel("Vertical Mode Displacement",'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Elements",'fontname','Times New Roman' , 'FontSize', 12)
```



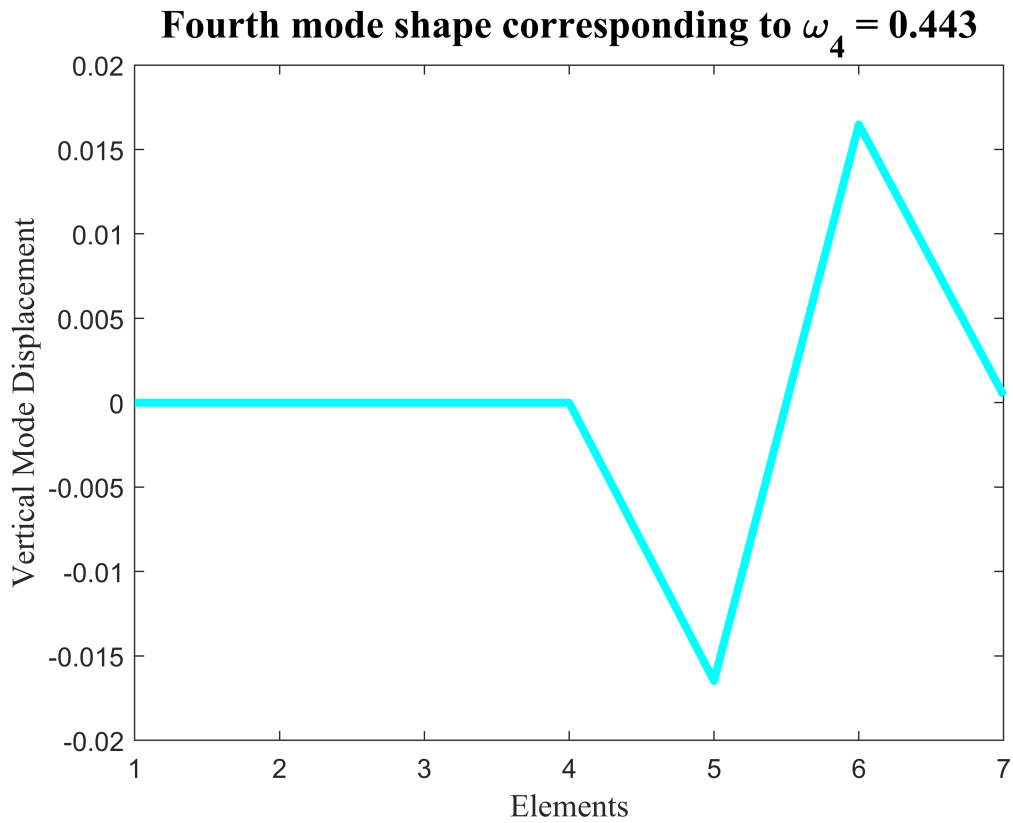
```

plot(ModeShapes(:,3) , "color" , "r" , 'LineWidth' , 3)
title("Third mode shape corresponding to \omega_3 = 0.3756",'fontName','Times New Roman','FontSize',12)
ylabel("Vertical Mode Displacement",'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Elements",'fontname','Times New Roman' , 'FontSize', 12)

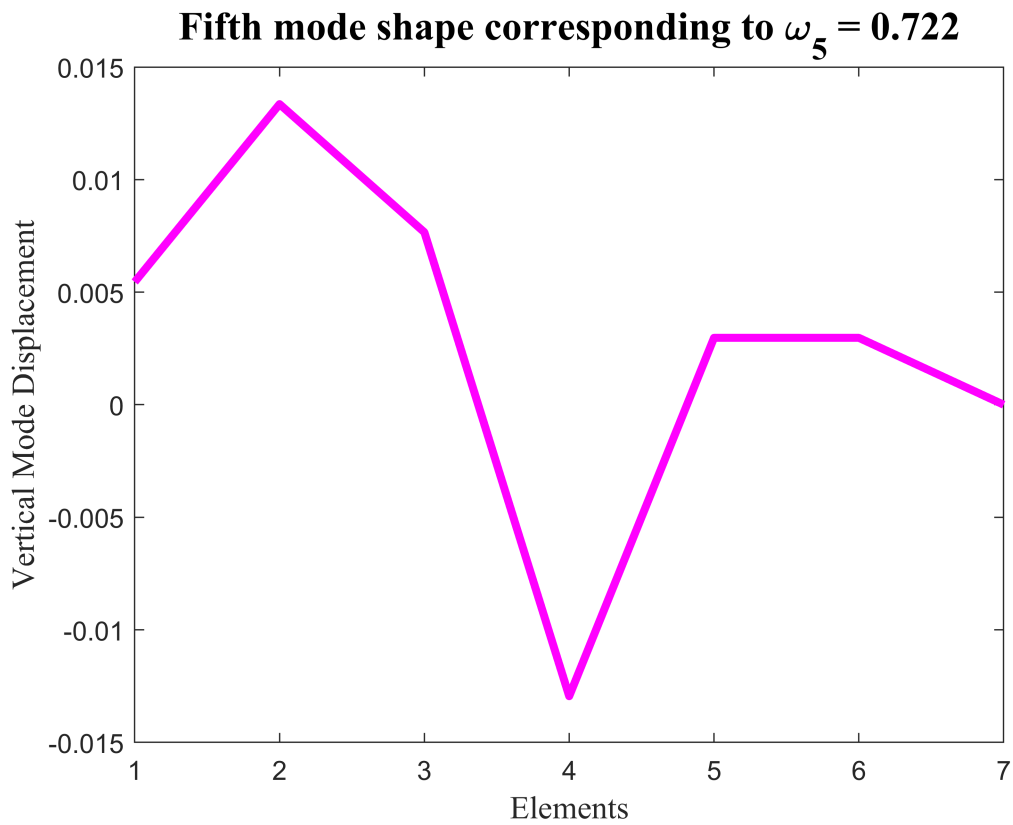
```



```
plot(ModeShapes(:,4) , "color" , "c" , 'LineWidth' , 3)
title("Fourth mode shape corresponding to \omega_4 = 0.443", 'fontName', 'Times New Roman', 'FontSize', 12)
ylabel("Vertical Mode Displacement", 'fontName', 'Times New Roman' , 'FontSize', 12)
xlabel("Elements", 'fontname', 'Times New Roman' , 'FontSize', 12)
```



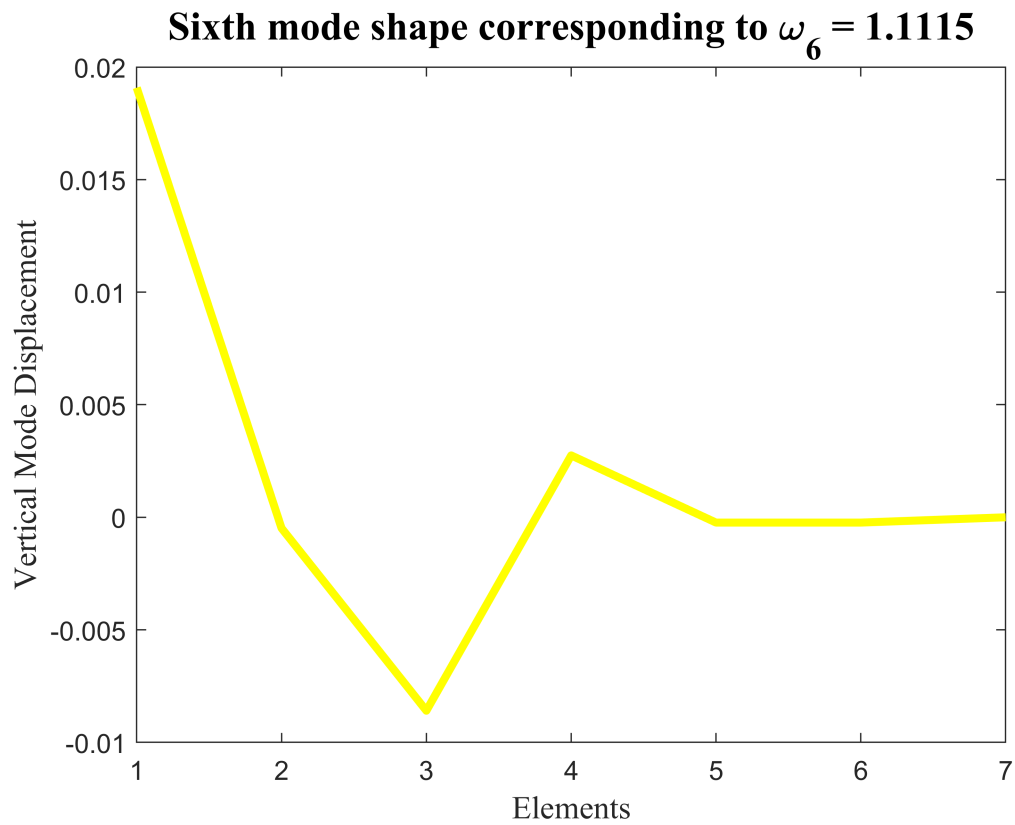
```
plot(ModeShapes(:,5) , "color" , "m" , 'LineWidth' , 3)
title("Fifth mode shape corresponding to \omega_5 = 0.722",'fontName','Times New Roman','FontSize',12)
ylabel("Vertical Mode Displacement",'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Elements",'fontname','Times New Roman' , 'FontSize', 12)
```



```

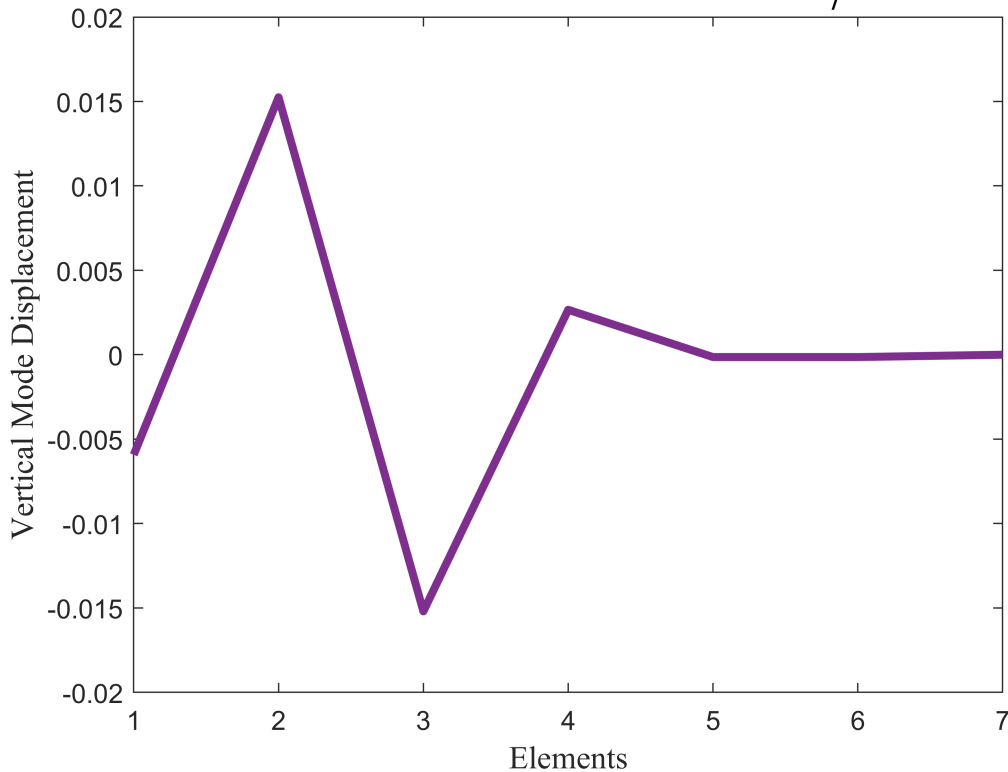
plot(ModeShapes(:,6) , "color" , "y" , 'LineWidth' , 3)
title("Sixth mode shape corresponding to \omega_6 = 1.1115",'fontName','Times New Roman','FontSize', 12)
ylabel("Vertical Mode Displacement",'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Elements",'fontname','Times New Roman' , 'FontSize', 12)

```

```
plot(ModeShapes(:,7) , "color" , [0.494, 0.1840, 0.556] , 'LineWidth' , 3)
title("Seventh mode shape corresponding to \omega_7 = 1.4038", 'fontName', 'Times New Roman', 'FontSize', 12)
ylabel("Vertical Mode Displacement", 'fontName', 'Times New Roman' , 'FontSize', 12)
xlabel("Elements", 'fontname', 'Times New Roman' , 'FontSize', 12)
```

Seventh mode shape corresponding to $\omega_7 = 1.4038$



Defining the initial conditions and defining the forced response of the system using ode45

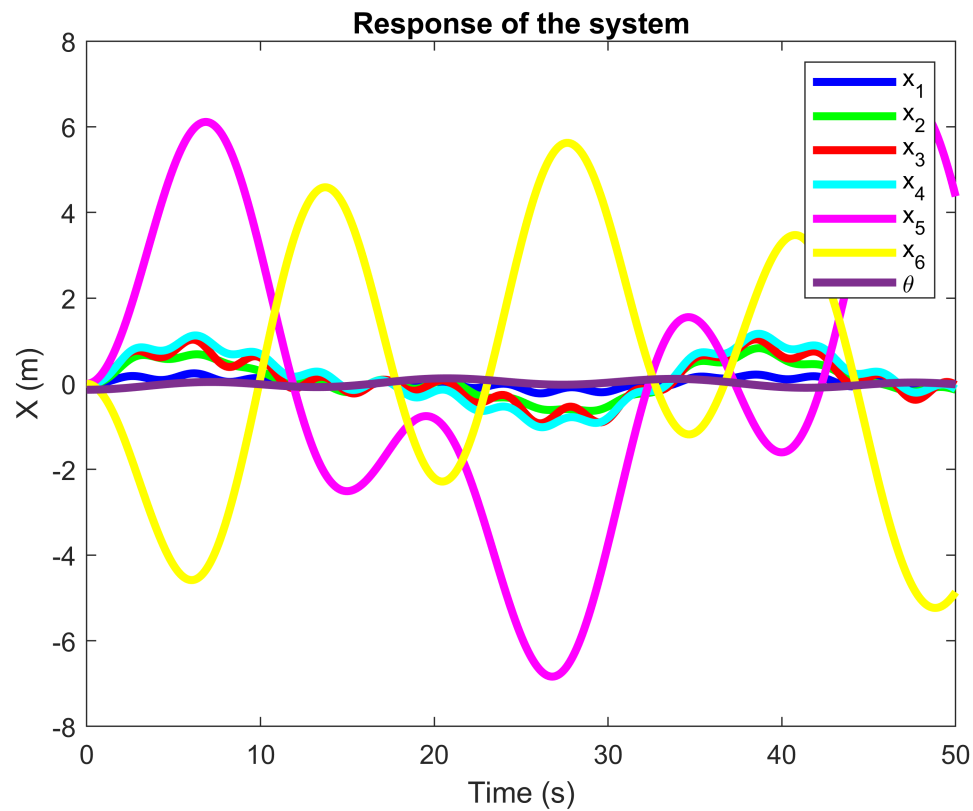
```
x0 = [ 0 0 0 0 0 0 8*pi/180 0 0 0 0 0 0];

opts = odeset('RelTol',1e-2,'AbsTol',1e-4);

[t,x] = ode45(@(t,x) odefcn(t , x , M , K , C , tf , g , F) , tspan , x0 , opts);
```

Plotting the force response of the system all together

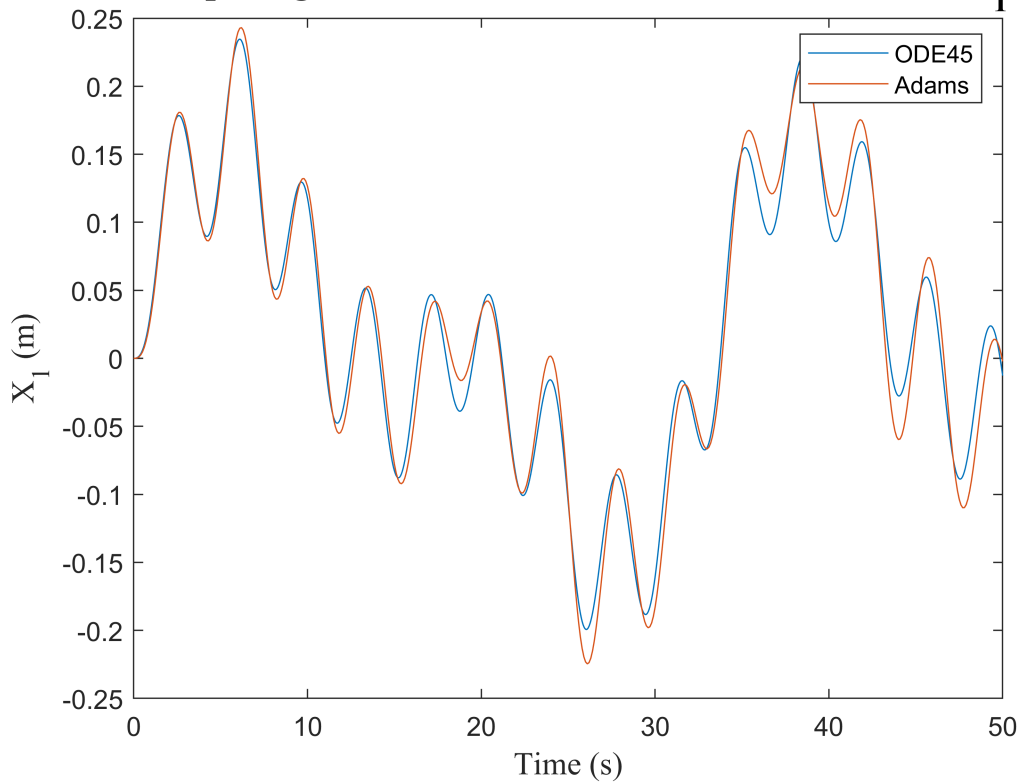
```
plot(t,x(:,1) , 'color' , "b" , 'LineWidth' , 3)
hold on
plot(t,x(:,2) , "color" , "g" , 'LineWidth' , 3)
plot(t,x(:,3) , "color" , "r" , 'LineWidth' , 3)
plot(t,x(:,4) , "color" , "c" , 'LineWidth' , 3)
plot(t,x(:,5) , "color" , "m" , 'LineWidth' , 3)
plot(t,x(:,6) , "color" , "y" , 'LineWidth' , 3)
plot(t,-x(:,7) , "color" , [0.494, 0.1840, 0.556] , 'LineWidth' , 3)
title("Response of the system")
ylabel("X (m)")
xlabel("Time (s)")
legend(["x_1","x_2","x_3","x_4","x_5","x_6","\theta"])
hold off
```



Comparing results from ODE45 and Adams

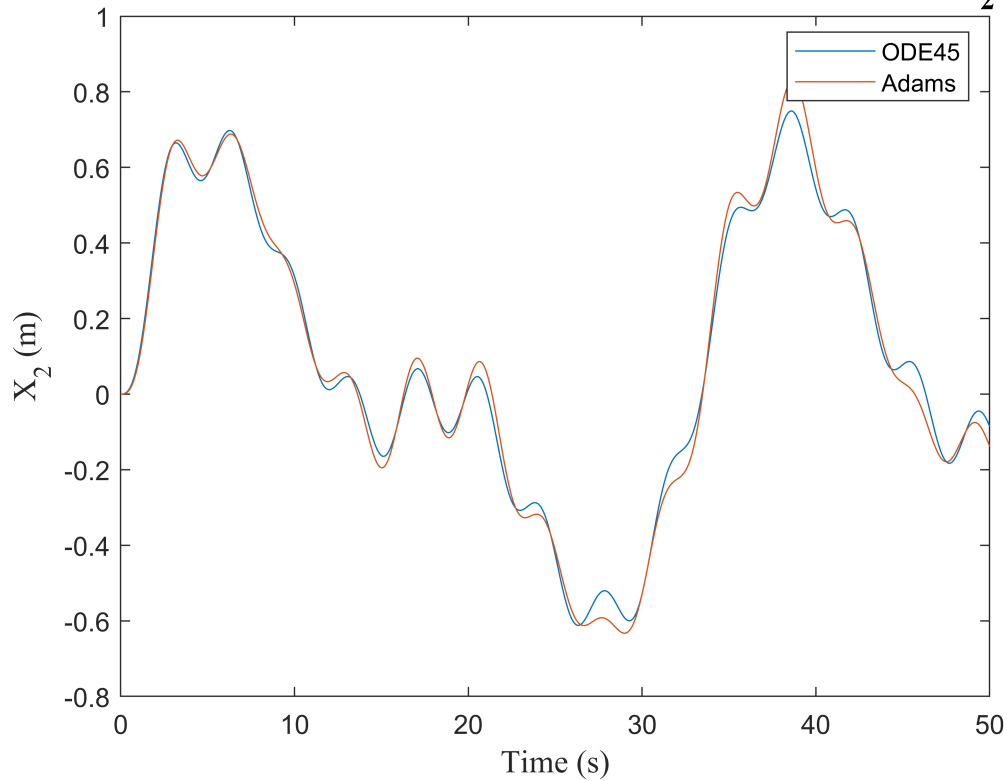
```
x1 = load('x_m1_adams.mat');
t1 = load('t_m1_adams.mat');
plot(t1.t_m1_adams , x1.x_m1_adams)
hold on
plot(t,x(:,1))
hold off
legend('ODE45','Adams')
ylabel("X_1 (m)", 'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName','Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_1', 'fontName','Times New Roman', 'FontSize', 12)
```

Comparing Adams Results with ODE45 Results for M_1



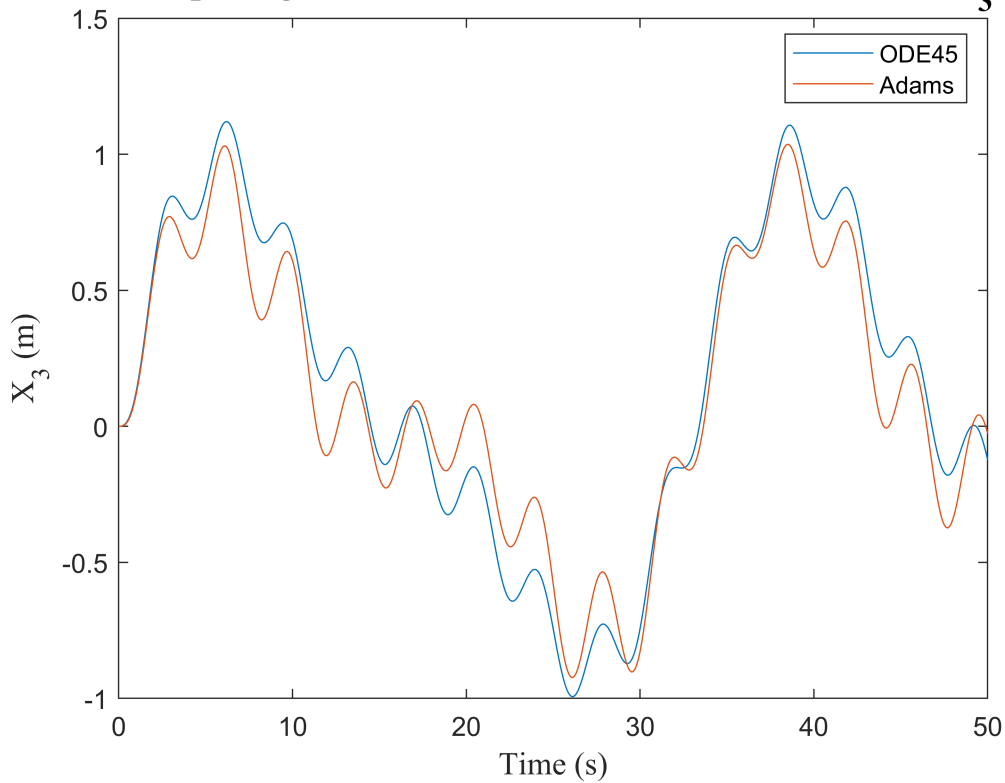
```
x2 = load('x_m2_adams.mat');
t2 = load('t_m2_adams.mat');
plot(t2.t_m2_adams , x2.x_m2_adams)
hold on
plot(t,x(:,2))
hold off
legend('ODE45','Adams')
ylabel("X_2 (m)", 'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName','Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_2', 'fontName','Times New Roman', 'FontSize', 12)
```

Comparing Adams Results with ODE45 Results for M_2



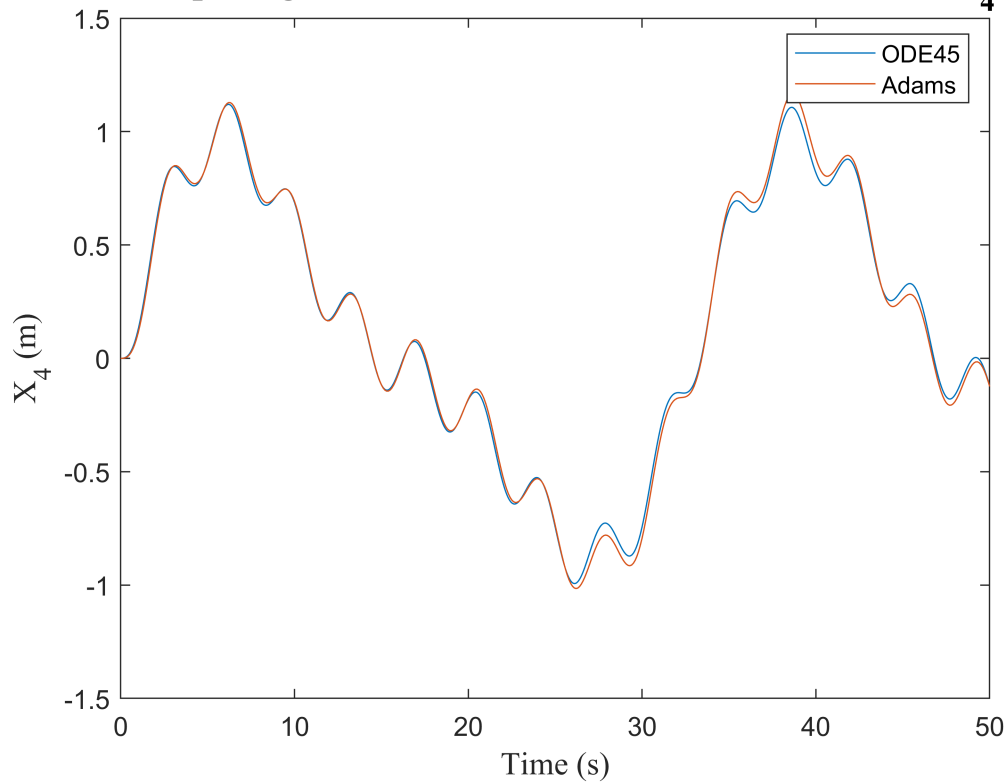
```
x3 = load('x_m3_adams.mat');
t3 = load('t_m3_adams.mat');
plot(t3.t_m3_adams , x3.x_m3_adams)
hold on
plot(t,x(:,3))
hold off
legend('ODE45','Adams')
ylabel("X_3 (m)", 'fontName', 'Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName', 'Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_3', 'fontName', 'Times New Roman', 'FontSize', 12)
```

Comparing Adams Results with ODE45 Results for M_3

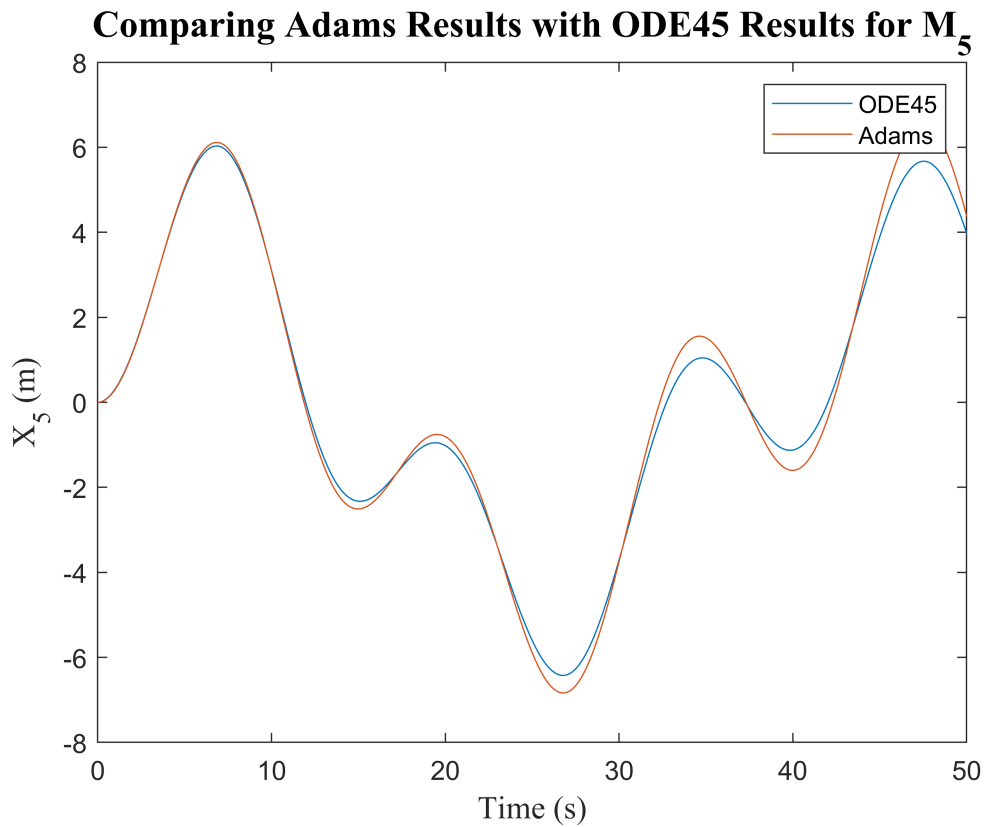


```
x4 = load('x_m4_adams.mat');
t4 = load('t_m4_adams.mat');
plot(t4.t_m4_adams , x4.x_m4_adams)
hold on
plot(t,x(:,4))
hold off
legend('ODE45','Adams')
ylabel("X_4 (m)", 'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName','Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_4', 'fontName','Times New Roman', 'FontSize', 12)
```

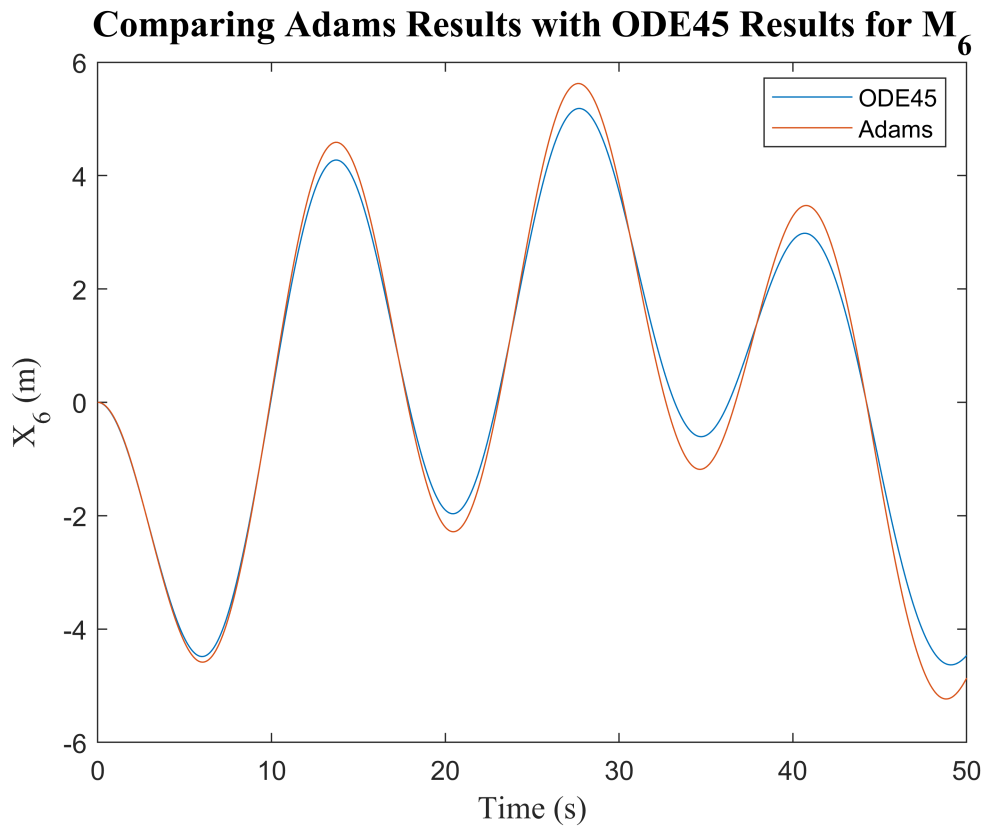
Comparing Adams Results with ODE45 Results for M_4



```
x5 = load('x_m5_adams.mat');
t5 = load('t_m5_adams.mat');
plot(t5.t_m5_adams , x5.x_m5_adams)
hold on
plot(t,x(:,5))
hold off
legend('ODE45','Adams')
ylabel("X_5 (m)", 'fontName', 'Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName', 'Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_5', 'fontName', 'Times New Roman', 'FontSize', 12)
```

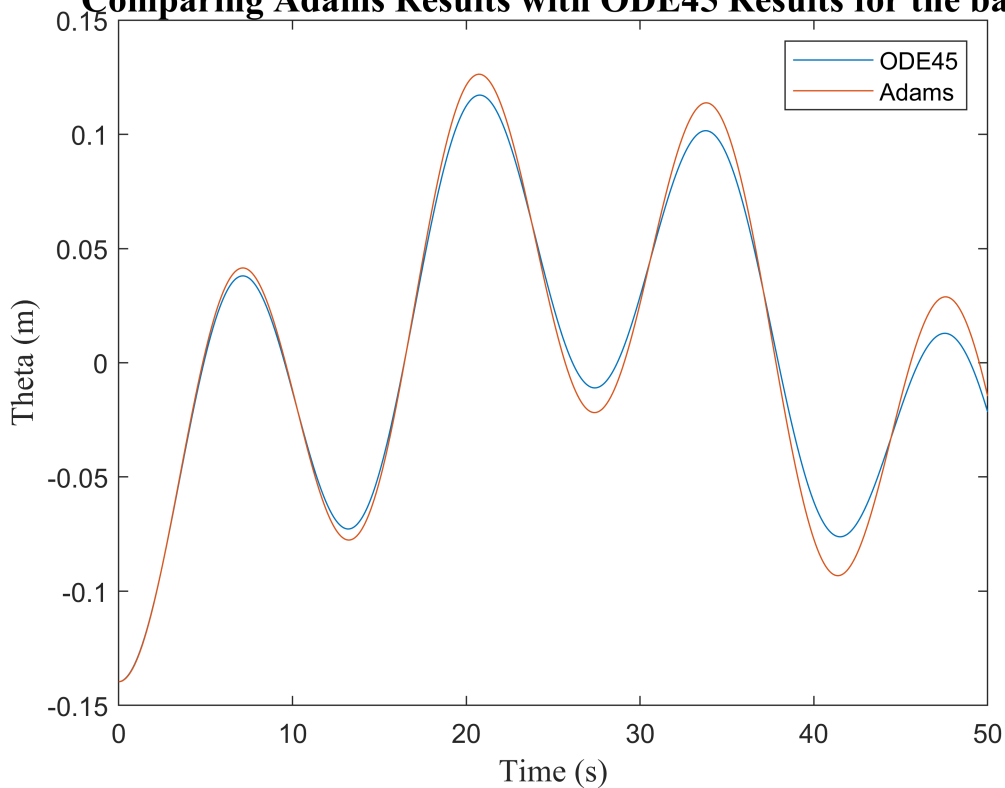


```
x6 = load('x_m6_adams.mat');
t6 = load('t_m6_adams.mat');
plot(t6.t_m6_adams , x6.x_m6_adams)
hold on
plot(t,x(:,6))
hold off
legend('ODE45','Adams')
ylabel("X_6 (m)", 'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName','Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for M_6', 'fontName','Times New Roman', 'FontSize', 12)
```

```
teta7 = load('theta_barr_adams.mat');
t7 = load('t_barr_adams.mat');
plot(t7.t_barr_adams , teta7.theta_barr_adams)
hold on
plot(t,-x(:,7))
hold off
legend('ODE45','Adams')
ylabel("Theta (m)", 'fontName','Times New Roman' , 'FontSize', 12)
xlabel("Time (s)", 'fontName','Times New Roman' , 'FontSize', 12)
title('Comparing Adams Results with ODE45 Results for the bar', 'fontName','Times New Roman', 'FontSize', 12)
```

Comparing Adams Results with ODE45 Results for the bar



Plotting the force response of the system one by one

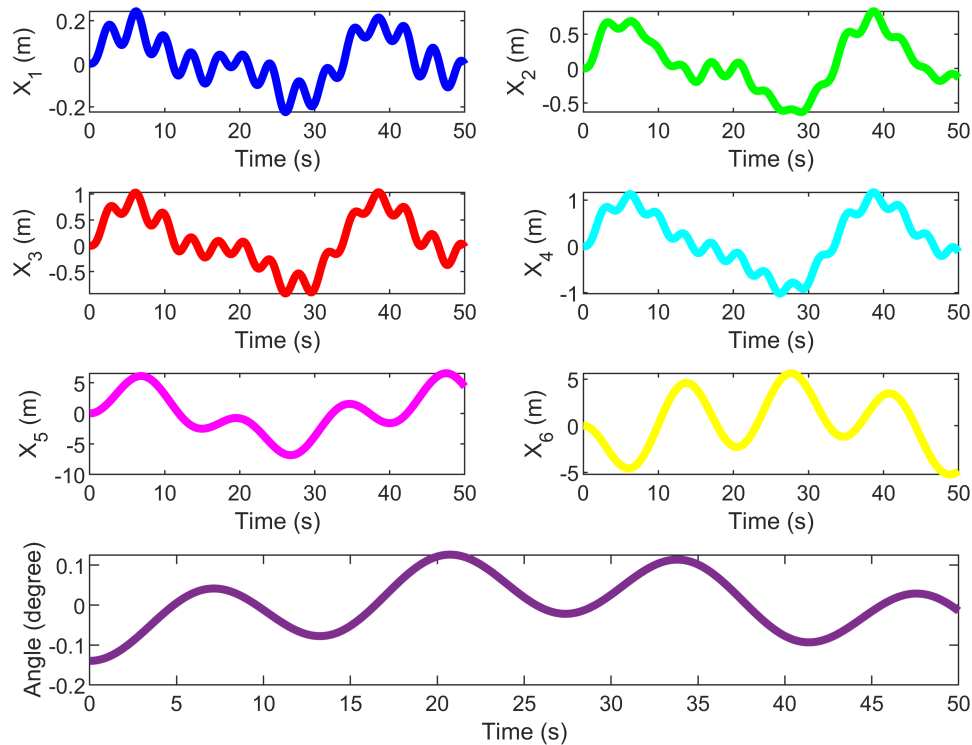
```
figure
subplot(4,2,1)
plot(t,x(:,1) , 'color' , "b" , 'LineWidth' , 3)
ylabel("X_1 (m)")
xlabel("Time (s)")
subplot(4,2,2)
plot(t,x(:,2) , "color" , "g" , 'LineWidth' , 3)
ylabel("X_2 (m)")
xlabel("Time (s)")
subplot(4,2,3)
plot(t,x(:,3) , "color" , "r" , 'LineWidth' , 3)
ylabel("X_3 (m)")
xlabel("Time (s)")
subplot(4,2,4)
plot(t,x(:,4) , "color" , "c" , 'LineWidth' , 3)
ylabel("X_4 (m)")
xlabel("Time (s)")
subplot(4,2,5)
plot(t,x(:,5) , "color" , "m" , 'LineWidth' , 3)
ylabel("X_5 (m)")
xlabel("Time (s)")
subplot(4,2,6)
plot(t,x(:,6) , "color" , "y" , 'LineWidth' , 3)
ylabel("X_6 (m)")
```

```

xlabel("Time (s)")
subplot(4,2,[7,8])
plot(t,-x(:,7) , "color" , [0.494, 0.1840, 0.556] , 'LineWidth' , 3)
ylabel("Angle (degree)")
xlabel("Time (s)")
sgtitle("The Plot of each Degree of Freedom Individually")

```

The Plot of each Degree of Freedom Individually



Defining the ode45 function

```

function dxdt = odefcn(t , x , M , K , C , tf , g , F)

g = interp1(tf,g,t); % To interpolate all the values in between the points to get a smooth graph
% I don't exactly know how that works, it was on mathworks and I used it
% the reference site is given in the appendix part

F = F*g;

dxdt = zeros(14 , 1);

xx = [x(1) ; x(2) ; x(3) ; x(4) ; x(5) ; x(6) ; x(7)] ;

xdot = [x(8) ; x(9) ; x(10) ; x(11) ; x(12) ; x(13) ; x(14)];

xddot = (F - K * xx - C * xdot);

```

```
dxdt(1) = x(8);  
dxdt(2) = x(9);  
dxdt(3) = x(10);  
dxdt(4) = x(11);  
dxdt(5) = x(12);  
dxdt(6) = x(13);  
dxdt(7) = x(14);  
  
dxdt(8) = (xddot(1))/M(1,1);  
dxdt(9) = (xddot(2))/M(2,2);  
dxdt(10) = (xddot(3))/M(3,3);  
dxdt(11) = (xddot(4))/M(4,4);  
dxdt(12) = (xddot(5))/M(5,5);  
dxdt(13) = (xddot(6))/M(6,6);  
dxdt(14) = (xddot(7))/M(7,7);  
  
end
```