

Decision Making

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This week

Today

Reasoning about probabilities

Markov Decision Processes

Thursday

Applications for MDPs

Coursework



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A problem

- The probability that somebody has a disease Q is 1% (prevalence).
- If a person has Q then the probability that they test positive is 90% (sensitivity)
- If a person does not have Q, then nevertheless they have a 9% chance of testing positive (a false positive)
- A person tests positive. What is the chance of them having the disease?



Medical Advice

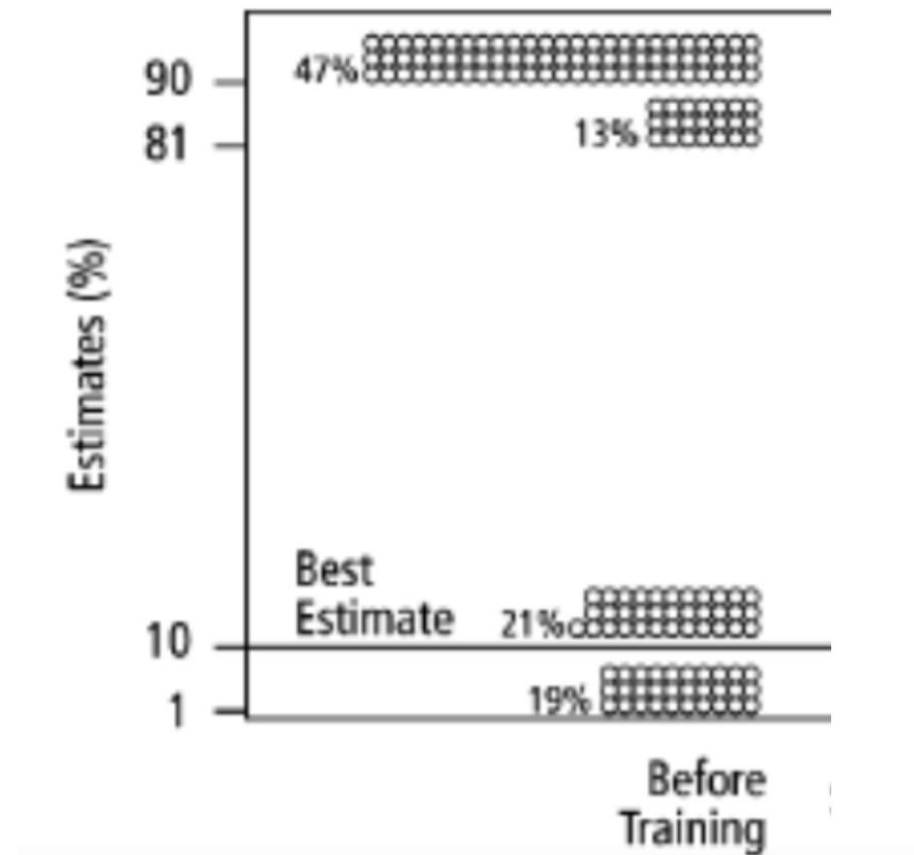
- The probability that somebody has a disease Q is 1% (prevalence).
- If a person has Q then the probability that they test positive is 90% (sensitivity)
- If a person does not have Q, then nevertheless they have a 9% chance of testing positive (a false positive)
- A person tests positive.

What advice should a Medical Advice System give?

1. The probability that they have disease Q is about 81%.
2. Out of 10 people with a positive test, about 9% have disease Q.
3. Out of 10 people with a positive test, about 1 has Q.
4. The probability that they have Q is about 1%.



Gigerenzer et al. (2008)



Natural Frequencies

Assume you conduct screening for Q in a certain region. You know the following information about the people in this region:

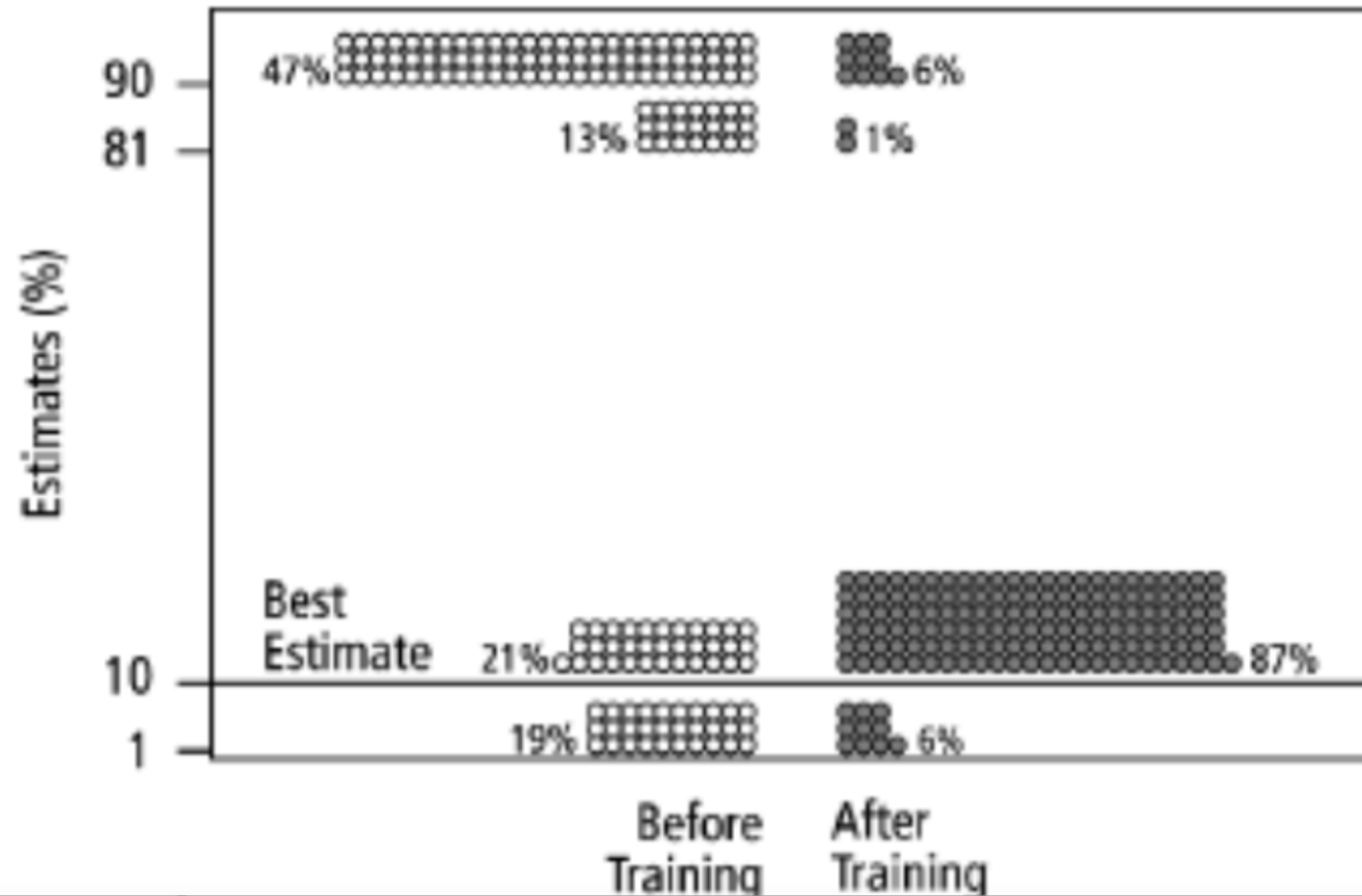
Ten out of every 1,000 people have Q

Of these 10 people, 9 test positive

Of the 990 people without cancer, about 89 nevertheless test positive



After training



Bayes' Law

$$p(\theta | D) = \frac{p(D | \theta) p(\theta)}{p(D)}.$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}.$$



Applied to our problem

Q = has disease

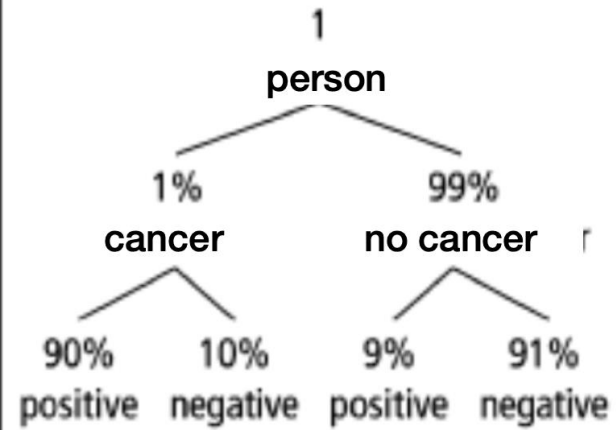
T = has positive test

$$P(Q|T) = \frac{P(T|Q)P(Q)}{P(T)}$$


$$P(Q|T) = 0.0918$$
$$= \sim 9\%$$



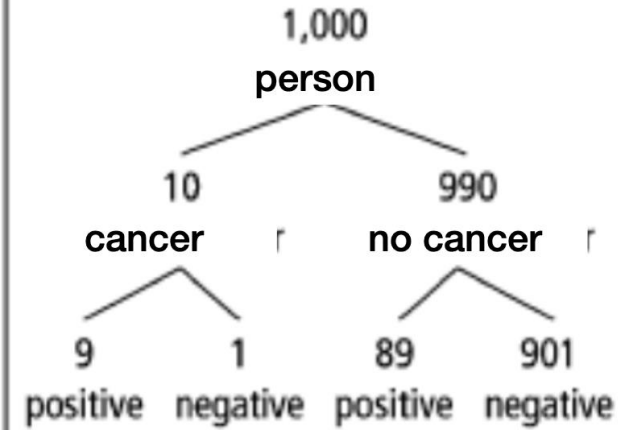
Conditional Probabilities




$$p(\text{cancer} | \text{test positive}) = \frac{.01 \times .9}{.01 \times .9 + .99 \times .09}$$



Natural Frequencies



$$p(\text{cancer} | \text{test positive}) = \frac{9}{9 + 89}$$





Discussion

Humans appear to be poor at probabilistic reasoning.

Intelligent Interactive Systems tend to very good.

Is this fair?

How should we design IIS to provide advice & support to humans?