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HomeWork 4:

Problem 1:

$$expand\left(\left(3 \cdot x^7 - 5 \cdot x^2 - x + 11\right) \cdot \left(x^3 - 2 \cdot x^2 + 1\right) \cdot \left(17 \cdot x^{10} - 9 \cdot x^8 - 11 \cdot x^5 + 6\right)\right);$$

$$51\,x^{20} - 102\,x^{19} - 27\,x^{18} + 105\,x^{17} - 145\,x^{15} + 219\,x^{14} + 266\,x^{13} - 573\,x^{12} - 134\,x^{11} \qquad \textbf{(1)}$$
$$+ 503\,x^{10} - 126\,x^9 - 242\,x^8 + 315\,x^7 + 11\,x^6 - 151\,x^5 + 54\,x^4 + 78\,x^3 - 162\,x^2 - 6\,x + 66$$

Problem 2:

$$factor\left(2 \cdot x^4 + 5 \cdot x^3 - x^2 + 15 \cdot x - 21\right)$$
$$(x - 1)\,(2\,x + 7)\,\left(x^2 + 3\right) \qquad \textbf{(2)}$$

Problem 3:

$$ifactor(8711211817102)$$
$$(2)\,(23)\,(189374169937) \qquad \textbf{(3)}$$

Problem 4A:

$$f := x \mapsto x^3 - 5 \cdot x^2 - 7 \cdot x$$
$$f := x \mapsto x^3 - 5 \cdot x^2 - 7 \cdot x \qquad \textbf{(4)}$$

$\xrightarrow{\text{differentiate}}$

$$x \mapsto 3 \cdot x^2 - 10 \cdot x - 7 \qquad \textbf{(5)}$$

$\xrightarrow{\text{differentiate}}$

$$x \mapsto 6 \cdot x - 10 \qquad \textbf{(6)}$$

Problem 4B:

$$f := x \mapsto \frac{-2 \cdot x}{sqrt\left(x^2 + 1\right)}$$

$$f := x \mapsto -\frac{2 \cdot x}{\sqrt{x^2 + 1}} \qquad \textbf{(7)}$$

$$x \mapsto -\frac{2}{\sqrt{x^2+1}} + \frac{2 \cdot x^2}{\left(\sqrt{x^2+1}\right)^2 \cdot \sqrt{x^2+1}} \tag{8}$$

differentiate →

$$x \mapsto \frac{6 \cdot x}{\left(\sqrt{x^2+1}\right)^2 \cdot \sqrt{x^2+1}} - \frac{4 \cdot x^3}{\left(\sqrt{x^2+1}\right)^3 \cdot (x^2+1)} - \frac{2 \cdot x^3}{\left(\sqrt{x^2+1}\right)^2 \cdot (x^2+1)^{3/2}} \tag{9}$$

Problem 4C:

$$f := x \mapsto$$
$$\frac{x}{2} + \ln\left(\frac{x}{x+3}\right)$$

$$\frac{x}{2} + \ln\left(\frac{x}{x+3}\right) \tag{10}$$

differentiate w.r.t. x →

$$\frac{1}{2} + \frac{\left(\dfrac{1}{x+3} - \dfrac{x}{(x+3)^2}\right)(x+3)}{x} \tag{11}$$

differentiate w.r.t. x →

$$\frac{\left(-\dfrac{2}{(x+3)^2} + \dfrac{2x}{(x+3)^3}\right)(x+3)}{x} - \frac{\left(\dfrac{1}{x+3} - \dfrac{x}{(x+3)^2}\right)(x+3)}{x^2} \tag{12}$$

$$+ \frac{\dfrac{1}{x+3} - \dfrac{x}{(x+3)^2}}{x}$$

Problem 4D:

$$f := x \mapsto \frac{3 \cdot x}{2}\left(1 + 4 \cdot e^{-\frac{x}{3}}\right)$$

$$f := x \mapsto \frac{3 \cdot x \cdot \left(1 + 4 \cdot e^{-\frac{x}{3}}\right)}{2} \tag{13}$$

differentiate →

$$x \mapsto \frac{3}{2} + 6 \cdot e^{-\frac{x}{3}} - 2 \cdot x \cdot e^{-\frac{x}{3}} \tag{14}$$
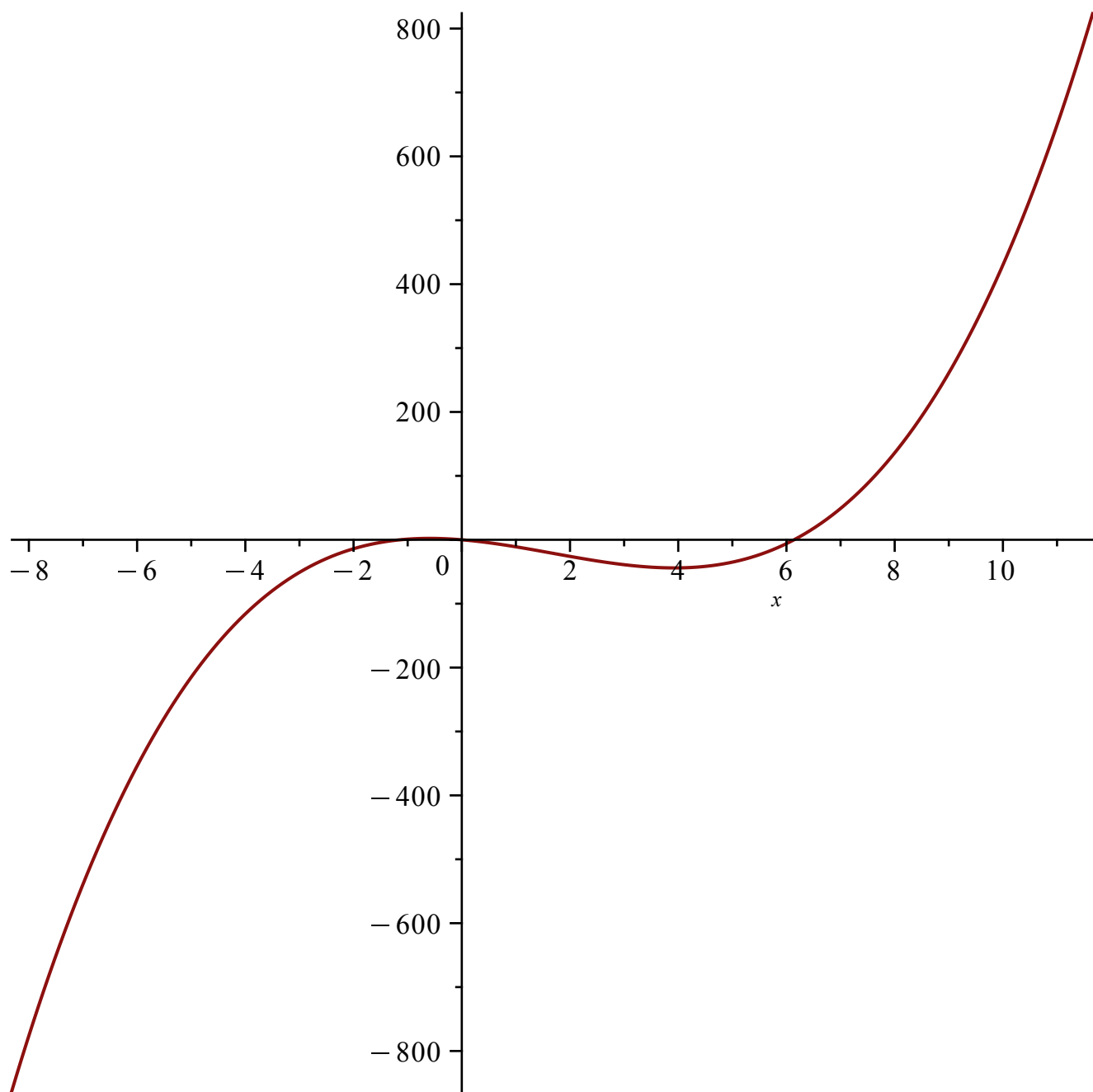
differentiate

$$x \mapsto -4 \cdot e^{-\frac{x}{3}} + \frac{2 \cdot x \cdot e^{-\frac{x}{3}}}{3} \tag{15}$$

Problem 5A:

$$f := x \mapsto x^3 - 5 \cdot x^2 - 7 \cdot x$$

$$f := x \mapsto x^3 - 5 \cdot x^2 - 7 \cdot x \tag{16}$$

$$plot\left(x^3 - 5 \cdot x^2 - 7 \cdot x\right)$$

$\mathrm{D}(f)(x) = 3x^2 - 10x - 7$

$solve(\mathrm{D}(f)(x) > 0, x)$

$$\left(-\infty, \frac{5}{3} - \frac{\sqrt{46}}{3}\right), \left(\frac{5}{3} + \frac{\sqrt{46}}{3}, \infty\right) \tag{17}$$

$evalf(\%)$

$$(-\infty, -0.594109994), (3.927443328, \infty) \tag{18}$$

$solve(\mathrm{D}(f)(x) < 0, x)$

$$\left(\frac{5}{3} - \frac{\sqrt{46}}{3}, \frac{5}{3} + \frac{\sqrt{46}}{3}\right) \tag{19}$$

evalf (%)

$$(-0.594109994, 3.927443328)$$ **(20)**

fsolve $(\mathrm{D}(f)(x) = 0, x)$;

$$-0.5941099944, 3.927443328$$ **(21)**

Maximum: -0.5941099944, Minimum: 3.927443328

Problem 5B:

$$f := x \mapsto -\frac{2 \cdot x}{\mathrm{sqrt}\left(x^2 + 1\right)}$$

$$f := x \mapsto -\frac{2 \cdot x}{\sqrt{x^2 + 1}}$$ **(22)**

$$plot\left(\frac{-2 \cdot x}{\mathrm{sqrt}\left(x^2 + 1\right)}\right)$$
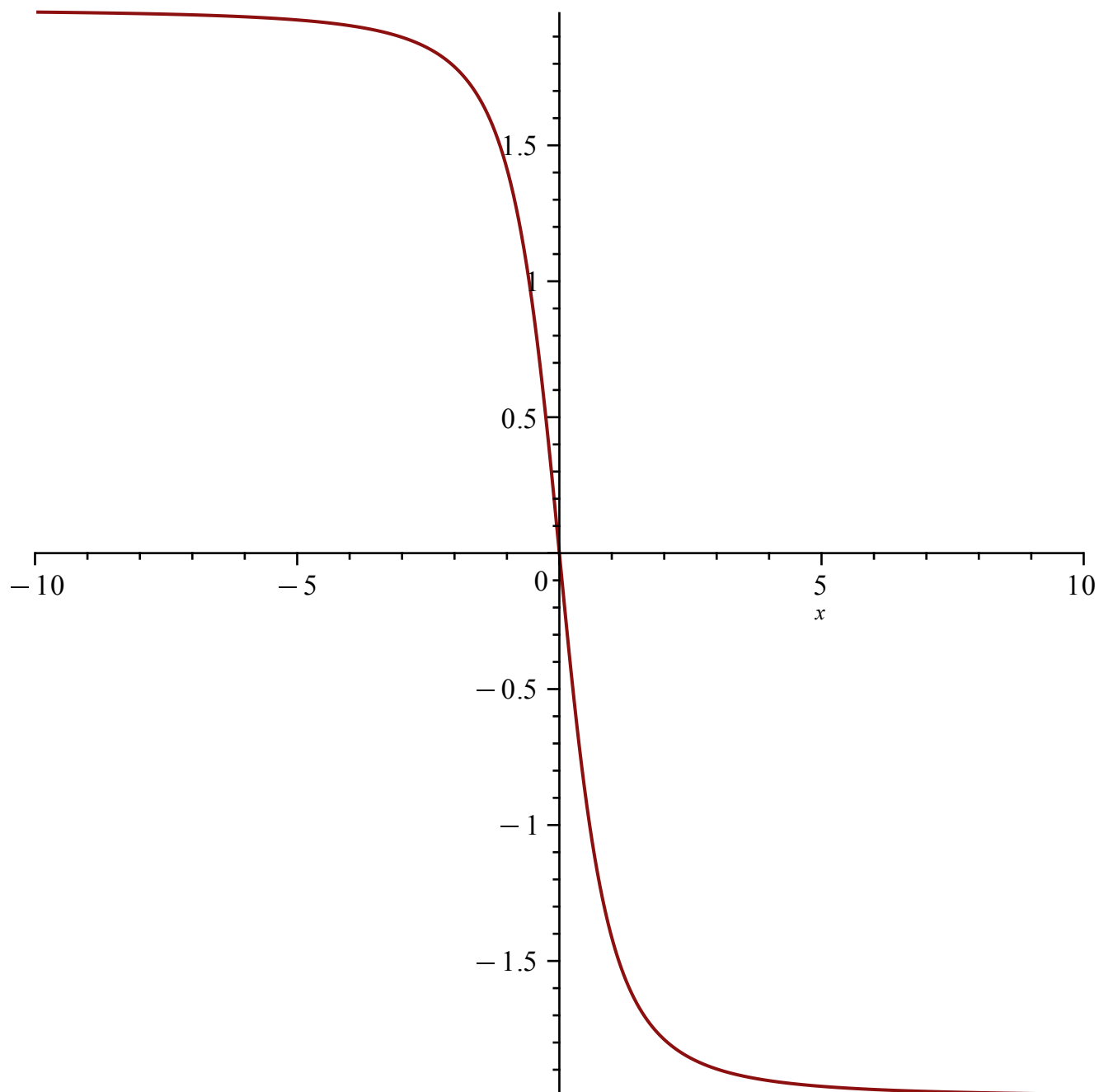
$D(f)(x); simplify(\%);$

$$-\frac{2}{\sqrt{x^2+1}} + \frac{2x^2}{\left(x^2+1\right)^{3/2}}$$

$$-\frac{2}{\left(x^2+1\right)^{3/2}} \tag{23}$$

$fsolve(D(f)(x) = 0, x);$

$$fsolve\left(-\frac{2}{\sqrt{x^2+1}} + \frac{2x^2}{\left(x^2+1\right)^{3/2}} = 0, x\right) \tag{24}$$

NO maximum NO minimum

$fsolve\left(\left(\mathrm{D}@@2\right)\left(f\right)\left(x\right)=0,x\right);$

$$0. \tag{25}$$

inflection point at x=0.

$Limit\left(f\left(x\right),x=\text{infinity}\right)=limit\left(f\left(x\right),x=\text{infinity}\right);$

$$\lim_{x\to\infty}\left(-\frac{2\,x}{\sqrt{x^2+1}}\right)=-2 \tag{26}$$

$Limit\left(f\left(x\right),x=-\text{infinity}\right)=limit\left(f\left(x\right),x=-\text{infinity}\right);$

$$\lim_{x\to-\infty}\left(-\frac{2\,x}{\sqrt{x^2+1}}\right)=2 \tag{27}$$
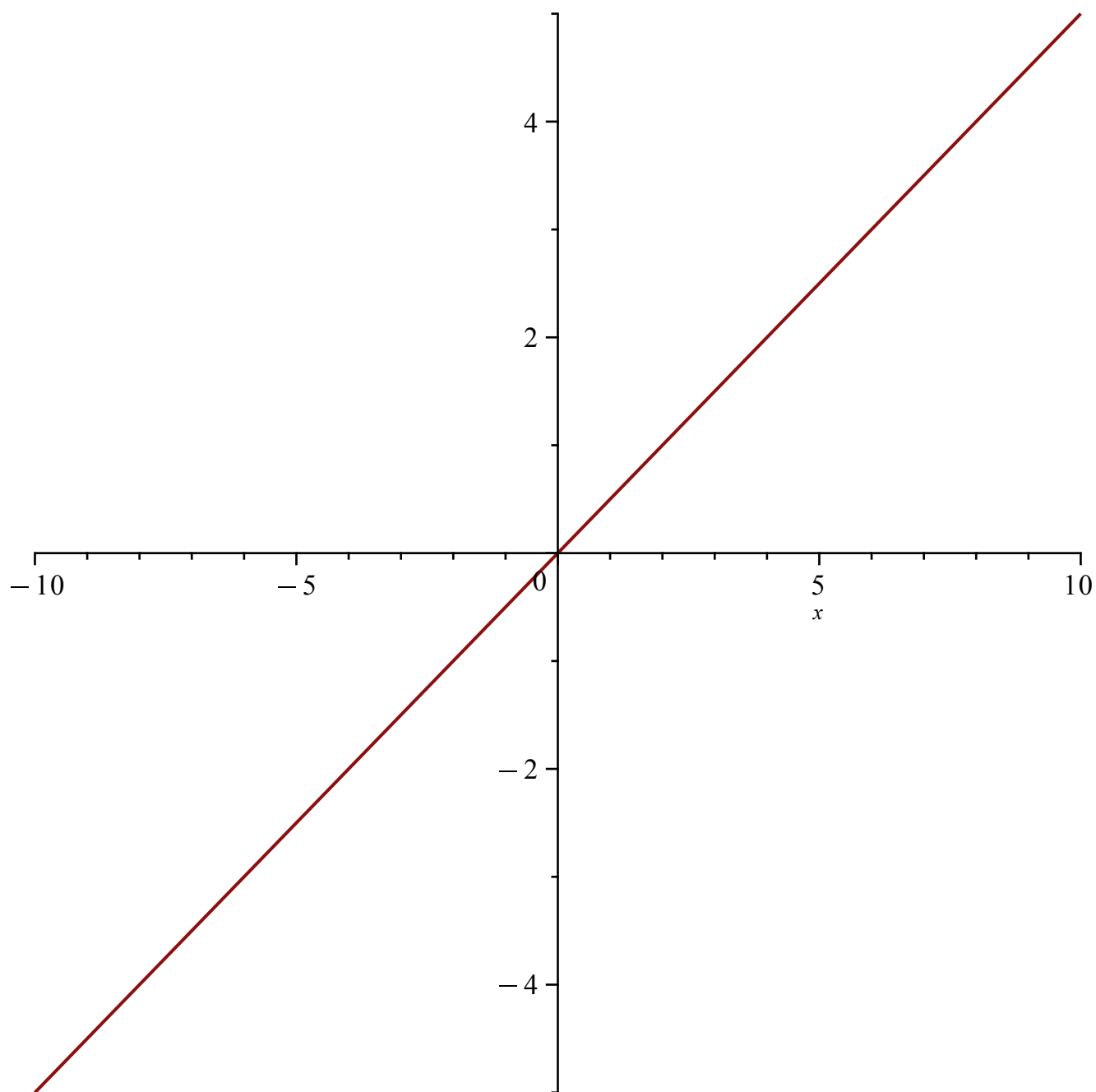
y=2 and y=-2 are horizontal asymptotes

Problem 5C:

$$g:=x\mapsto\frac{x}{2}+\ln\left(\frac{x}{x+3}\right)$$

$$g:=x\mapsto\frac{x}{2}+\frac{\ln\cdot x}{3+x} \tag{28}$$

$plot\left(\left\{g\left(x\right),\frac{x}{2}\right\},x\right);$

Warning, expecting only range variable x in expression 1/2*x+ln*x/(x+3) to be plotted but found name ln

$solve\left(\dfrac{x}{x+3} > 0\right);$

$$(-\infty, -3), (0, \infty)$$

(29)

$D(g)(x);$

$$\frac{1}{2} + \frac{\ln}{x+3} - \frac{\ln x}{(x+3)^2}$$

(30)

$simplify(\%);$

$$\frac{x^2 + 6\ln + 6x + 9}{2(x+3)^2}$$

(31)

NO maximum NO minimum

$(D@@2)(g)(x); simplify(\%);$

$$-\frac{2\ln}{(x+3)^2} + \frac{2\ln x}{(x+3)^3}$$
$$-\frac{6\ln}{(x+3)^3}$$

(32)
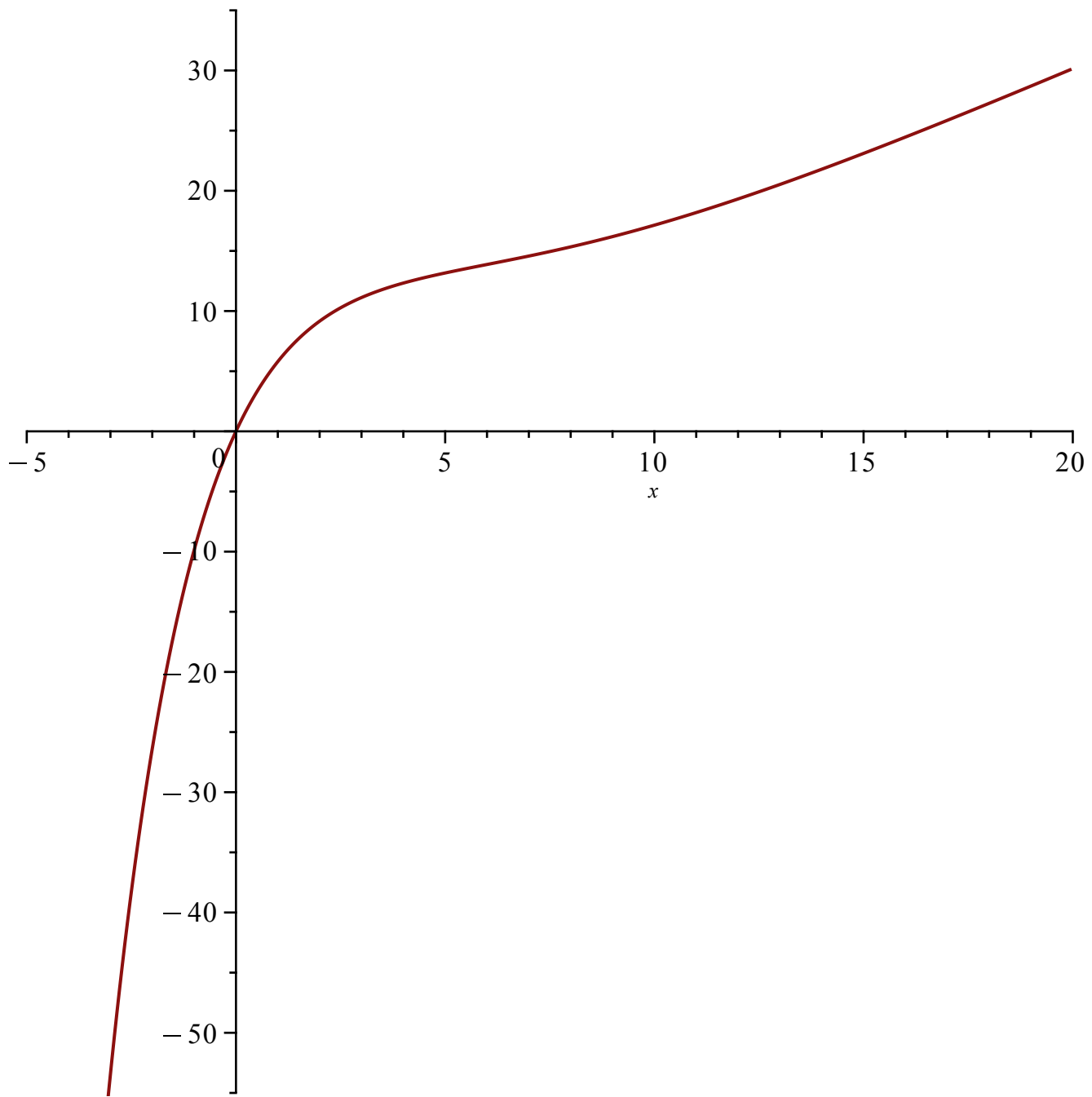
no inflection point

Problem 5D:

$h := x \mapsto \frac{3\cdot x}{2}\cdot\left(1 + 4\cdot\exp\left(-\frac{x}{3}\right)\right);$

$$h := x \mapsto \frac{3\cdot x\cdot\left(1 + 4\cdot e^{-\frac{x}{3}}\right)}{2}$$

(33)

$plot(h(x), x = -5..20);$

D(h)(x);

$$\frac{3}{2} + 6\,e^{-\frac{x}{3}} - 2\,x\,e^{-\frac{x}{3}}$$

(34)

$solve(\mathrm{D}(h)(x) > 0, x)$;

$$x$$

(35)

NO maximum NO minimum

$(D@@2)(h)(x); \text{simplify}(\%);$

$$-4\,e^{-\frac{x}{3}} + \frac{2\,x\,e^{-\frac{x}{3}}}{3}$$

$$\frac{2\,e^{-\frac{x}{3}}\,(-6+x)}{3} \tag{36}$$

$\text{fsolve}((D@@2)(h)(x) = 0, x);$

$$6. \tag{37}$$

$\text{solve}((D@@2)(h)(x) > 0, x);$

$$(6, \infty) \tag{38}$$

$\text{solve}((D@@2)(h)(x) < 0, x);$

$$(-\infty, 6) \tag{39}$$

concave down for x <6, concave up for x>6, inflection point at x=6.

Problem 6:

for *n* **from** 1 **to** 200 **do if** *isprime*$\left(n^2 + n + 1\right)$ **then** *print*$\left(n,\ n^2 + n + 1\right);$
    end if; **end do**;

$$1, 3$$
$$2, 7$$
$$3, 13$$
$$5, 31$$
$$6, 43$$
$$8, 73$$
$$12, 157$$
$$14, 211$$
$$15, 241$$
$$17, 307$$
$$20, 421$$
$$21, 463$$
$$24, 601$$
$$27, 757$$

33, 1123

38, 1483

41, 1723

50, 2551

54, 2971

57, 3307

59, 3541

62, 3907

66, 4423

69, 4831

71, 5113

75, 5701

77, 6007

78, 6163

80, 6481

89, 8011

90, 8191

99, 9901

101, 10303

105, 11131

110, 12211

111, 12433

117, 13807

119, 14281

131, 17293

138, 19183

141, 20023

143, 20593

147, 21757

150, 22651

153, 23563

155, 24181

161, 26083

162, 26407

(40)

Problem 7:

$N := 0 :$

for n **from** 1 **to** 200 **do if** $isprime(n^2 + n + 1)$ **then** $N := N + 1; print("There are ", N, "primes.");$
    end if; **end do**;