

Lab 9

Problem 1a.)

$$f := x \rightarrow e^{-x^2}$$

$$f := x \mapsto e^{-x^2} \quad (1)$$

$$\text{taylor}(f(x), x=0, 7)$$

$$1 - x^2 + \frac{1}{2} x^4 - \frac{1}{6} x^6 + O(x^8) \quad (2)$$

Problem 1b.)

$$g := x \rightarrow \tan(x)$$

$$g := x \mapsto \tan(x) \quad (3)$$

$$\text{taylor}(g(x), x=0, 7)$$

$$x + \frac{1}{3} x^3 + \frac{2}{15} x^5 + O(x^7) \quad (4)$$

Problem 2a.)

$$f := x \rightarrow \frac{1}{x}$$

$$f := x \mapsto \frac{1}{x} \quad (5)$$

$$\text{taylor}(f(x), x=1, 7)$$

$$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 - (x-1)^5 + (x-1)^6 + O((x-1)^7) \quad (6)$$

Problem 2b.)

$$g := x \rightarrow \ln(x)$$

$$g := x \mapsto \ln(x) \quad (7)$$

$$\text{taylor}(g(x), x=1, 7)$$

$$(x-1) - \frac{1}{2} (x-1)^2 + \frac{1}{3} (x-1)^3 - \frac{1}{4} (x-1)^4 + \frac{1}{5} (x-1)^5 - \frac{1}{6} (x-1)^6 + O((x-1)^7) \quad (8)$$

Problem 2c.)

$$h := x \rightarrow \tan(x)$$

$$h := x \mapsto \tan(x) \quad (9)$$

$$\text{taylor}(h(x), x = 1, 7)$$

$$\begin{aligned} & \tan(1) + (1 + \tan(1)^2) (x - 1) + (1 + \tan(1)^2) \tan(1) (x - 1)^2 + \left(\frac{1}{3} + \frac{4 \tan(1)^2}{3} \right. \\ & \quad \left. + \tan(1)^4 \right) (x - 1)^3 + \left(\frac{5 \tan(1)^3}{3} + \frac{2 \tan(1)}{3} + \tan(1)^5 \right) (x - 1)^4 + \left(\frac{2}{15} \right. \\ & \quad \left. + \frac{17 \tan(1)^2}{15} + 2 \tan(1)^4 + \tan(1)^6 \right) (x - 1)^5 + \left(\frac{77 \tan(1)^3}{45} + \frac{17 \tan(1)}{45} \right. \\ & \quad \left. + \frac{7 \tan(1)^5}{3} + \tan(1)^7 \right) (x - 1)^6 + O((x - 1)^7) \end{aligned} \quad (10)$$

Problem 3

$$f := x \rightarrow \text{sqrt}(x)$$

$$f := x \mapsto \sqrt{x} \quad (11)$$

$$\text{taylor}(f(x), x = 1, 3)$$

$$1 + \frac{1}{2} (x - 1) - \frac{1}{8} (x - 1)^2 + O((x - 1)^3) \quad (12)$$

$$P[2] := x \mapsto 1 + \frac{1}{2} (x - 1) - \frac{1}{8} (x - 1)^2$$

$$P_2 := x \mapsto \frac{1}{2} + \frac{x}{2} - \frac{(x - 1)^2}{8} \quad (13)$$

$$\text{abs}(P[2](1.5) - f(1.5))$$

$$0.005994871 \quad (14)$$

$$\text{taylor}(f(x), x = 1, 4)$$

$$1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3 + O((x-1)^4) \quad (15)$$

$$P[3] := x \mapsto 1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3$$

$$P_3 := x \mapsto \frac{1}{2} + \frac{x}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16} \quad (16)$$

$$\text{abs}(P[3](1.5) - f(1.5))$$

$$0.001817629 \quad (17)$$

$$\text{taylor}(f(x), x=1, 5)$$

$$1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3 - \frac{5}{128} (x-1)^4 + O((x-1)^5) \quad (18)$$

$$P[4] := x \mapsto 1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3 - \frac{5}{128} (x-1)^4$$

$$P_4 := x \mapsto \frac{1}{2} + \frac{x}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16} - \frac{5 \cdot (x-1)^4}{128} \quad (19)$$

$$\text{abs}(P[4](1.5) - f(1.5))$$

$$0.000623777 \quad (20)$$

$$\text{taylor}(f(x), x=1, 6)$$

$$1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3 - \frac{5}{128} (x-1)^4 + \frac{7}{256} (x-1)^5 + O((x-1)^6) \quad (21)$$

$$P[5] := x \mapsto 1 + \frac{1}{2} (x-1) - \frac{1}{8} (x-1)^2 + \frac{1}{16} (x-1)^3 - \frac{5}{128} (x-1)^4 + \frac{7}{256} (x-1)^5$$

$$P_5 := x \mapsto \frac{1}{2} + \frac{x}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16} - \frac{5 \cdot (x-1)^4}{128} + \frac{7 \cdot (x-1)^5}{256} \quad (22)$$

$$\text{abs}(P[5](1.5) - f(1.5))$$

$$0.000230715 \quad (23)$$

Problem 4:

$$f := x \mapsto \cos(x) + \sin(x)$$

$$f := x \mapsto \cos(x) + \sin(x) \quad (24)$$

$$\text{taylor}(f(x), x=0, 2)$$

$$1 + x + O(x^2) \quad (25)$$

$$P[1] := x \mapsto 1 + x$$

$$P_1 := x \mapsto x + 1 \quad (26)$$

$$\text{taylor}(f(x), x=0, 3)$$

$$1 + x - \frac{1}{2} x^2 + O(x^3) \quad (27)$$

$$P[2] := x \mapsto 1 + x - \frac{1}{2} x^2$$

$$P_2 := x \mapsto 1 + x - \frac{1}{2} \cdot x^2 \quad (28)$$

$$\text{taylor}(f(x), x=0, 4)$$

$$1 + x - \frac{1}{2} x^2 - \frac{1}{6} x^3 + O(x^4) \quad (29)$$

$$P[3] := x \mapsto 1 + x - \frac{1}{2} x^2 - \frac{1}{6} x^3$$

$$P_3 := x \mapsto 1 + x - \frac{1}{2} \cdot x^2 - \frac{1}{6} \cdot x^3 \quad (30)$$

$$\text{taylor}(f(x), x=0, 5)$$

$$1 + x - \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4 + O(x^5) \quad (31)$$

$$P[4] := x \mapsto 1 + x - \frac{1}{2} x^2 - \frac{1}{6} x^3 + \frac{1}{24} x^4$$

$$P_4 := x \mapsto 1 + x - \frac{1}{2} \cdot x^2 - \frac{1}{6} \cdot x^3 + \frac{1}{24} \cdot x^4 \quad (32)$$

for n from 1 to 4 do

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plot( {g(x), P[n](x)}, x=-Pi..Pi, y=-4..4)
end do

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