Assignment 7

Chapter 6 Model specification

6.32 Simulate a stationary time series of length $n = 36$ according to an AR(1) model with $\phi = 0.95$. This model is stationary, but just barely so. With such a series and a short history, it will be difficult if not impossible to distinguish between stationary and nonstationary with a unit root.

(a) Plot the series and calculate the sample ACF and PACF and describe what you see.

(b) Perform the (augmented) Dickey-Fuller test on the series with $k = 0$ in Equation (6.4.1) on page 128. (With $k = 0$ this is the Dickey-Fuller test and is not augmented.) Comment on the results.

(c) Perform the augmented Dickey-Fuller test on the series with k chosen by the software—that is, the "best" value for k. Comment on the results.

(d) Repeat parts (a), (b), and (c) but with a new simulation with $n = 100$.

Hints: the Augmented Dickey-Fuller test can be coded as

```
ADF.test(series,selectlags=list(mode=c(1,2,3)),itsd=c(1,0,0))
```

6.33 The data file named deere1 contains 82 consecutive values for the amount of deviation (in 0.000025 inch units) from a specified target value that an industrial machining process at Deere & Co. produced under certain specified operating conditions.

(a) Display the time series plot of this series and comment on any unusual points.

(b) Calculate the sample ACF for this series and comment on the results.

(c) Now replace the unusual value by a much more typical value and recalculate the sample ACF. Comment on the change from what you saw in part (b).

(d) Calculate the sample PACF based on the revised series that you used in part (c). What model would you specify for the revised series? (Later we will investigate other ways to handle outliers in time series modeling.)

Chapter 7 Parameter estimation

7.4 Consider an MA(1) process for which it is *known* that the process mean is zero. Based on a series of length $n = 3$, we observe $Y_1 = 0$, $Y_2 = -1$, and $Y_3 = ½$.

(a) Show that the conditional least-squares estimate of θ is ½.

(b) Find an estimate of the noise variance. (Hint: Iterative methods are not needed in this simple case.)

7.7 Verify

$$\hat{\theta} = \frac{-1 + \sqrt{1 - 4r_1^2}}{2r_1} \qquad (7.1.4)$$

for MA(1) model $Y_t = e_t - \theta e_{t-1}$ in page 150, where

$$e_t \sim (0, \sigma_e^2).$$

Additional problem. For AR(2) model

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma^2).$$

To get the conditional maximum likelihood function given (Y_1, Y_2), denoted by $L(\phi_0, \phi_1, \phi_2, \sigma_e^2 \mid Y_1, Y_2)$, and derive the equation of the MLE $\phi_0, \phi_1, \phi_2, \sigma_e^2$ should satisfy.

Deadline： before classes on 02 Nov, 2021