Time Series Analysis

Homework of week 10

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9.1

(a)

$$\hat{Y}_t(1) = \mu + \phi(Y_t - \mu) = 10.8 - 0.5(12.2 - 10.8) = 10.1$$

(b)

Method 1:

$$\hat{Y}_t(2) = \mu + \phi(\hat{Y}_t(1) - \mu) = 10.8 - 0.5(10.1 - 10.8) = 11.15$$

Method 2:

$$\hat{Y}_t(2) = \mu + \phi^2(Y_t - \mu) = 10.8 + 0.5^2(12.2 - 10.8) = 11.15$$

(c)

Since

$$Y_{t+1} - \mu = \phi(Y_t - \mu),$$

it follows that

$$Y_{t+10} - \mu = \phi^{10}(Y_t - \mu).$$

Thus,

$$\hat{Y}_t(10) = \mu + \phi^{10}(Y_t - \mu) = 10.8 + (-0.5)^1 0(12.2 - 10.8) = 10.80137$$

9.2

(a)

$$\hat{Y}_{2007}(1) = 5 + 1.1Y_{2007} - 0.5Y_{2006} = 5 + 1.1 * 10 - 0.5 * 11 = 10.5.$$

$$\hat{Y}_{2007}(2) = 5 + 1.1\hat{Y}_{2007}(1) - 0.5Y_{2007} = 5 = 1.1 * 10.5 - 0.5 * 10 = 11.55.$$

(b)

Suppose

$$Y_t - \mu = e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + ...,$$

then

$$e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + ... = 1.1(e_{t-1} + \psi_1 e_{t-2} + \psi_2 e_{t-3} + ...) - 0.5(e_{t-2} + \psi_1 e_{t-3} + \psi_2 e_{t-4} + ...) + e_t$$

which implies that

$$\psi_1 = 1.1.$$

9.3

(a)

$$\hat{R}_{100}(1) = 0.2 * R_{100} = 0.2 * 0.01 = 0.002$$

and

$$\hat{R}_{100}(1) = 0.2 * \hat{R}_{100}(1) = 0.2 * 0.002 = 0.0004.$$

(b)

$$e_{100}(1) = R_{101} - \hat{R}_{100}(1) = 0.2 * 0.01 + a_t - 0.002 = a_t$$

and

$$e_{100}(2) = R_{102} - \hat{R}_{100}(2) = 0.2(0.2R_{100} + a_{101}) + a_{102} - 0.004 = 0.2a_{101} + a_{102}.$$

Thus,

$$sd(e_{100}(1)) = \sigma_a$$

and

$$sd(e_{100}(2)) = \sqrt{var(0.2a_{101} + a_{102})} = \sqrt{1.04}\sigma_a$$

(c)

By

$$\rho_k = \phi^k,$$

we have

$$\rho_1 = 0.2$$

and

$$\rho_2 = 0.04.$$

9.13

```
set.seed(172456)
series <- arima.sim(n=50,list(ar=0.7,ma=0.5))+100
actual <- window(series,start=41)
series <- window(series,end=40)
```

(a)

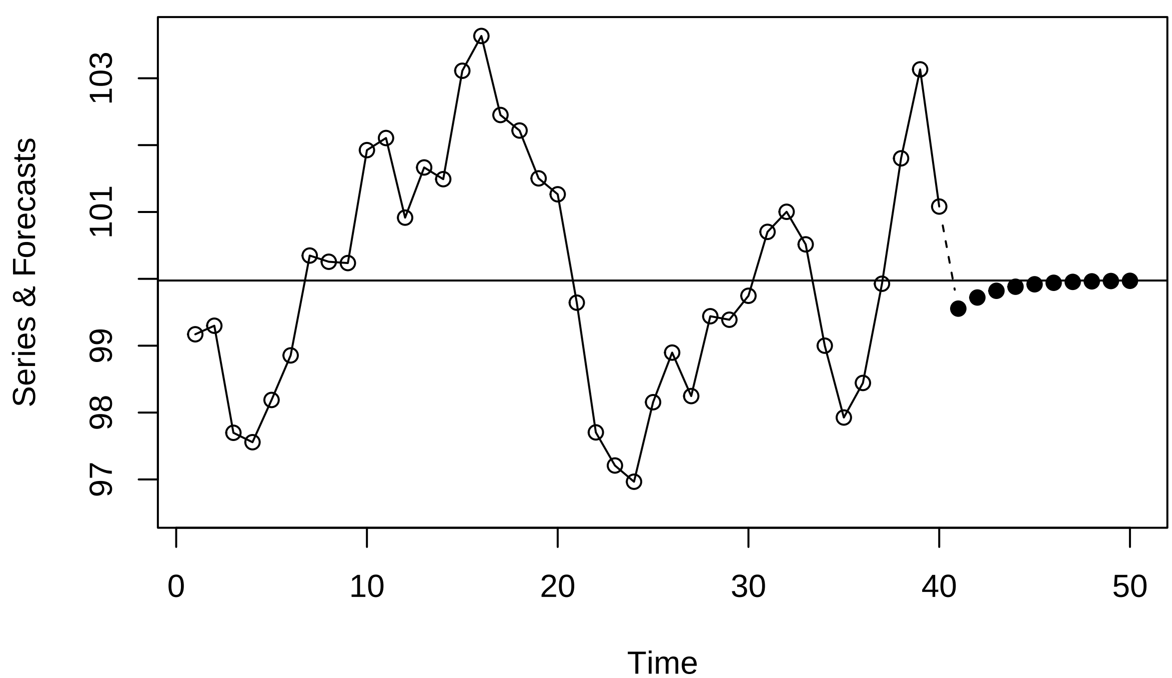
```
model <- arima(series,order=c(1,0,1));model
```

```
##
## Call:
## arima(x = series, order = c(1, 0, 1))
##
## Coefficients:
##          ar1     ma1  intercept
##       0.6047  0.6908    99.9749
## s.e.  0.1585  0.2521     0.5845
##
## sigma^2 estimated as 0.8161:  log likelihood = -53.6,  aic = 113.19
```

Taking the standard errors into account, the maximum likehood estimates are reasonably close to the true values in this simulation.

(b)

```
result <- plot(model, n.ahead=10,ylab='Series & Forecasts',col=NULL,pch=19)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



The forecasts approach the series mean fairly quickly.

(c)

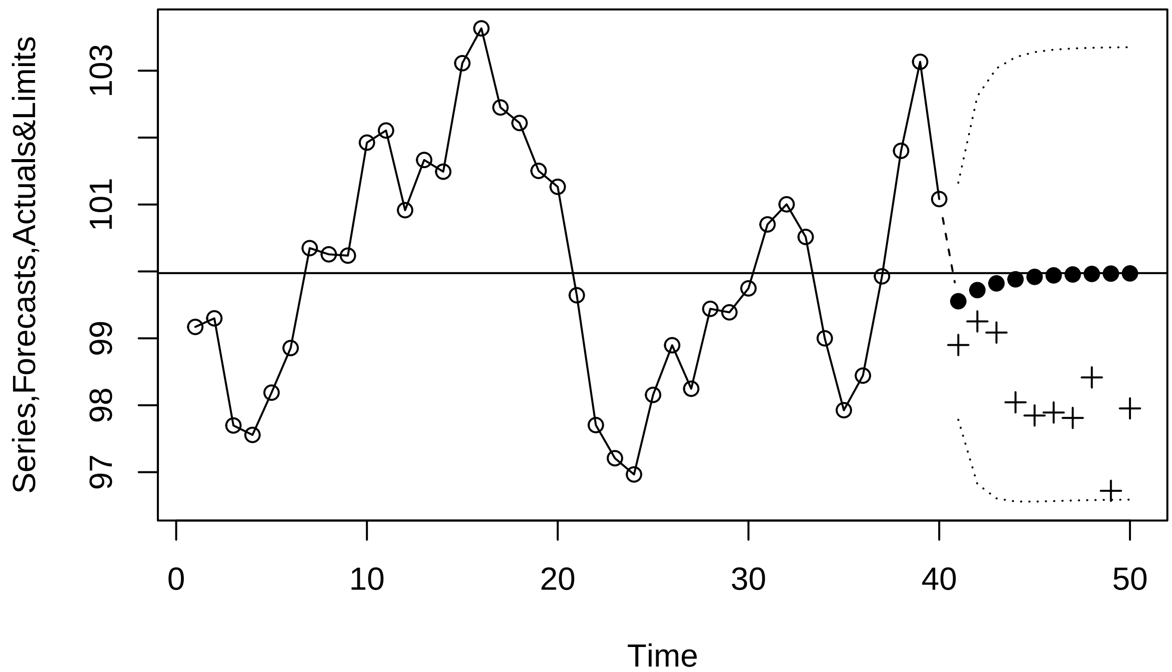
```
forecast <- result$pred
cbind(actual,forecast)
```

```
## Time Series:
## Start = 41
## End = 50
## Frequency = 1
##      actual forecast
## 41 98.90034 99.55445
## 42 99.25304 99.72067
## 43 99.08626 99.82118
## 44 98.04358 99.88196
## 45 97.84692 99.91871
## 46 97.89159 99.94093
## 47 97.81065 99.95437
## 48 98.41574 99.96250
## 49 96.72142 99.96741
## 50 97.95263 99.97038
```

See part(d) for a graphical comparision.

(d)

```
plot(model,n.ahead=10,ylab='Series,Forecasts,Actuals&Limits',pch=19)
points(x=(41:50),y=actual,pch=3)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



This series is quite erratic but the actual series values are contained within the forecast limits. The forecasts decay to the estimated process mean rather quickly and the prediction limits are quite wide.

(e)

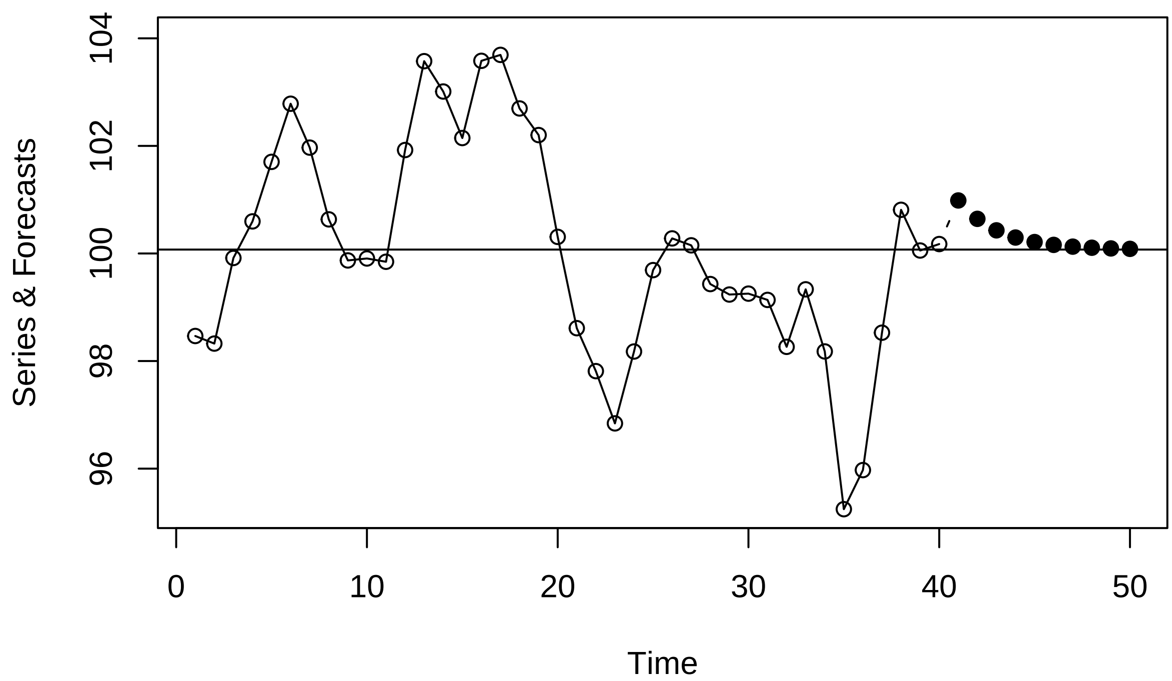
```
set.seed(996)
series <- arima.sim(n=50,list(ar=0.7,ma=0.5))+100
actual <- window(series,start=41)
series <- window(series,end=40)
```

```
# (a)
model <- arima(series,order=c(1,0,1));model
```

```
##
## Call:
## arima(x = series, order = c(1, 0, 1))
##
## Coefficients:
##          ar1     ma1  intercept
##       0.6256  0.7549   100.0725
## s.e.  0.1300  0.1490     0.6781
##
## sigma^2 estimated as 0.9243:  log likelihood = -56.24,  aic = 118.48
```

Taking the standard errors into account, the maximum likehood estimates are reasonably close to the true values in this simulation.

```
# (b)
result <- plot(model, n.ahead=10,ylab='Series & Forecasts',col=NULL,pch=19)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



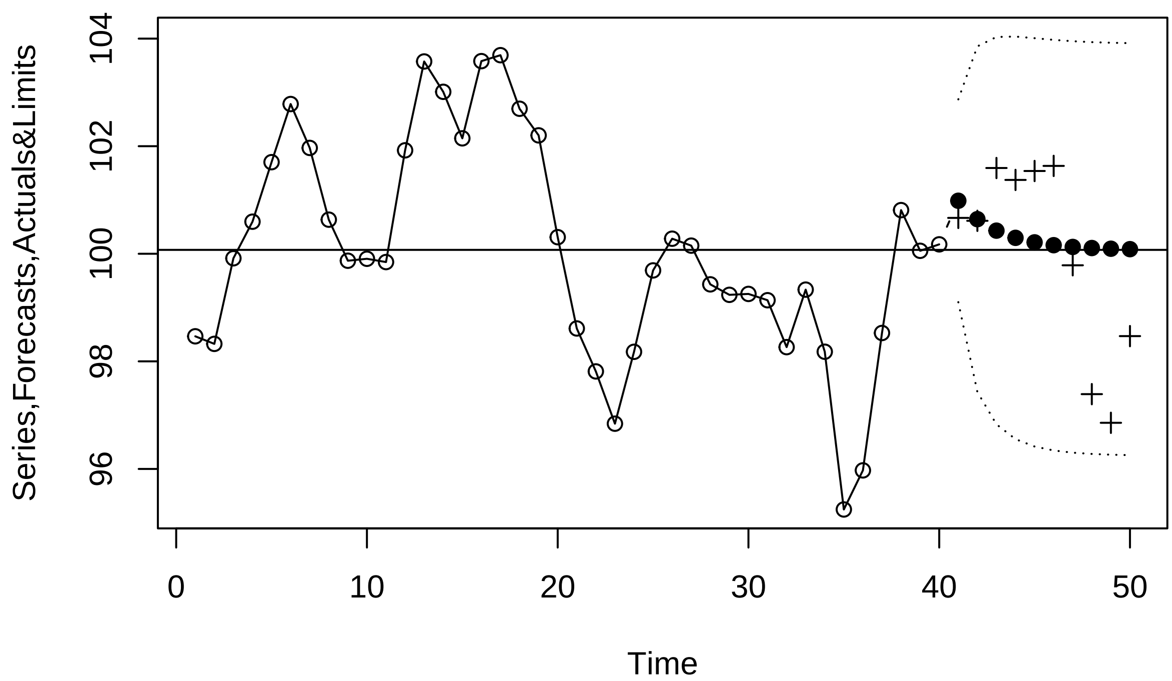
The forecfasts approach the series mean fairly quickly.

```
# (c)
forecast <- result$pred
cbind(actual,forecast)
```

```
## Time Series:
## Start = 41
## End = 50
## Frequency = 1
##       actual forecast
## 41 100.66700 100.9857
## 42 100.61282 100.6438
## 43 101.59437 100.4299
## 44 101.37280 100.2961
## 45 101.53910 100.2124
## 46 101.63390 100.1600
## 47  99.78625 100.1272
## 48  97.39050 100.1067
```

```
## 49   96.85739 100.0939
## 50   98.46938 100.0859
```

```
# (d)
plot(model,n.ahead=10,ylab='Series,Forecasts,Actuals&Limits',pch=19)
points(x=(41:50),y=actual,pch=3)
abline(h=coef(model)[names(coef(model))=='intercept'])
```



This series is quite erratic but the actual series values are contained within the forecast limits. The forecasts decay to the estimated process mean rather quickly and the prediction limits are quite wide.

9.14

```
set.seed(127456)
series <- arima.sim(n=35,list(order=c(0,1,1),ma=-0.8))[-1]
# delete initial term as it is always = 0
actual <- window(series,start=31)
series <- window(series,end=30)
```

(a)

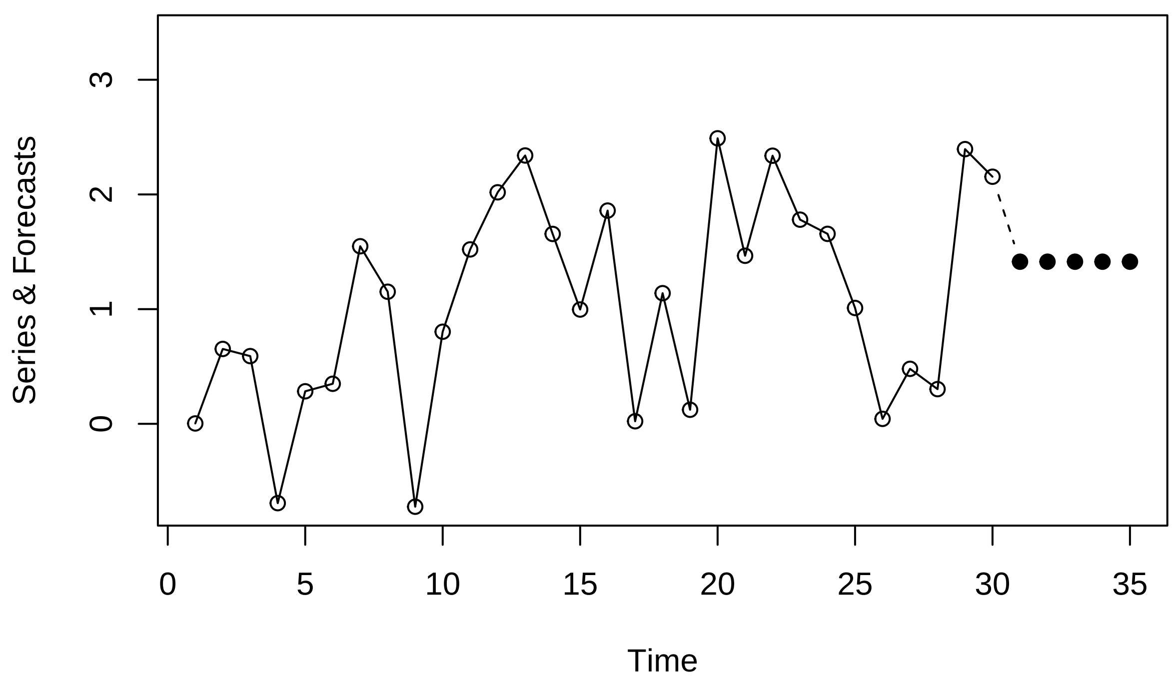
```r
model <- arima(series,order=c(0,1,1));model
```

```
##
## Call:
## arima(x = series, order = c(0, 1, 1))
##
## Coefficients:
##           ma1
##       -0.7696
## s.e.   0.1832
##
## sigma^2 estimated as 0.8449:  log likelihood = -39.15,  aic = 80.31
```

Taking the standard errors into account, the maximum likehood estimate is quite close to the true value in this simulation.

(b)

```r
result <- plot(model,n.ahead=5,ylab='Series & Forecasts',col=NULL,pch=19)
```



For this model the forecasts are constant for all lead times.

(c)

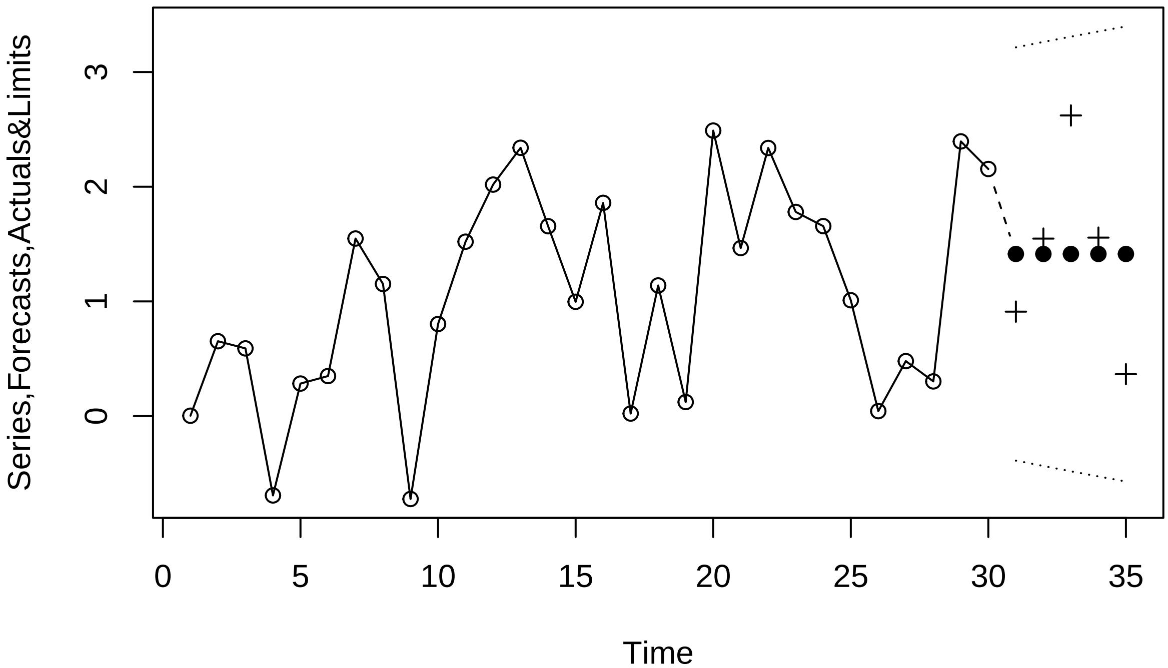
```
forecast <- result$pred
cbind(actual,forecast)
```

```
## Time Series:
## Start = 31
## End = 35
## Frequency = 1
##       actual forecast
## 31 0.9108642 1.413627
## 32 1.5476147 1.413627
## 33 2.6211930 1.413627
## 34 1.5560880 1.413627
## 35 0.3657792 1.413627
```

For this model the forecasts are the same at all lead times. See part (d) for a graphical comparison

(d)

```
plot(model,n.ahead=5,ylab='Series,Forecasts,Actuals&Limits',pch=19)
points(x=(31:35),y=actual,pch=3)
```



The forecast limits contain all of the actual values but they are quite wide.

(e)

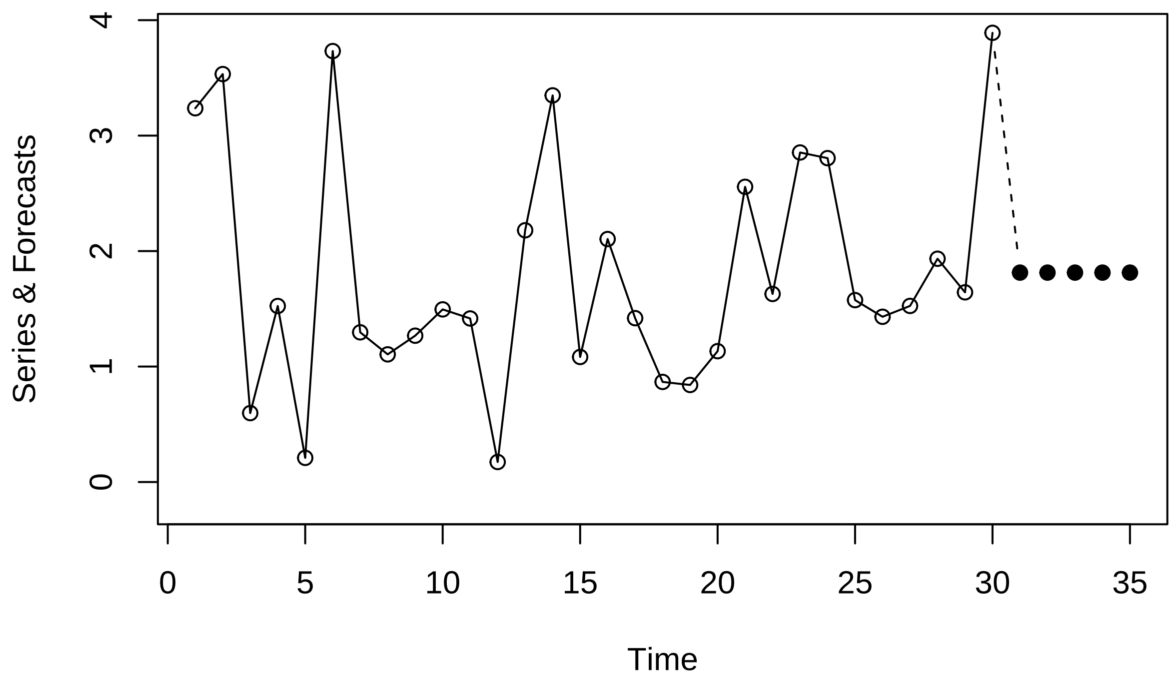
```
set.seed(996)
series <- arima.sim(n=35,list(order=c(0,1,1),ma=-0.8))[-1]
# delete initial term as it is always = 0
actual <- window(series,start=31)
series <- window(series,end=30)
```

```
# (a)
model <- arima(series,order=c(0,1,1));model
```

```
##
## Call:
## arima(x = series, order = c(0, 1, 1))
##
## Coefficients:
##           ma1
##       -1.0000
## s.e.   0.1445
##
## sigma^2 estimated as 1.024:  log likelihood = -43.19,  aic = 88.38
```

Taking the standard errors into account, the maximum likehood estimate is quite close to the true value in this simulation.

```
# (b)
result <- plot(model,n.ahead=5,ylab='Series & Forecasts',col=NULL,pch=19)
```



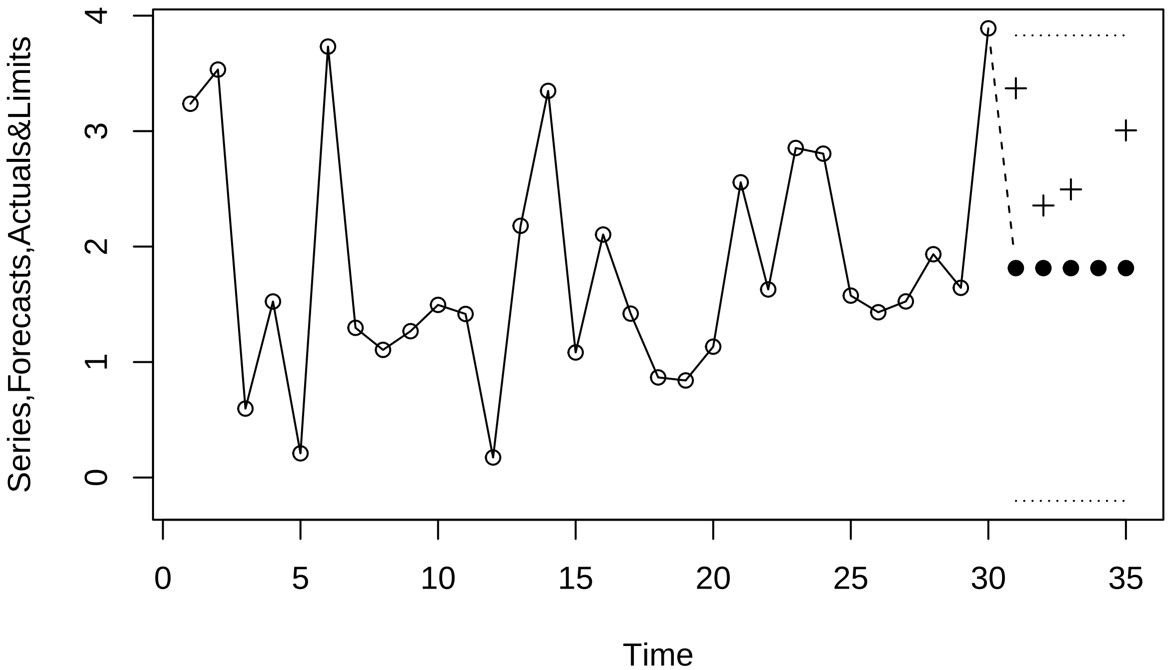
For this model the forecasts are constant for all lead times.

```
# (c)
forecast <- result$pred
cbind(actual,forecast)
```

```
## Time Series:
## Start = 31
## End = 35
## Frequency = 1
##      actual forecast
## 31 3.370915 1.813581
## 32 2.355926 1.813581
## 33 2.495300 1.813581
## 34 4.473598 1.813581
## 35 3.006651 1.813581
```

For this model the foecasts are the same at all lead times.

```
# (d)
plot(model,n.ahead=5,ylab='Series,Forecasts,Actuals&Limits',pch=19)
points(x=(31:35),y=actual,pch=3)
```



The forecast limits contain all of the actual values but they are quite wide.

Additional Problem

(1)

$$
\begin{aligned}
(1-0.3L+0.02L^2)Y_t &= (1+0.6L-0.07L^2)\epsilon_t \\
(1-0.1L)(1-0.2L)Y_t &= (1-0.1L)(1+0.7L)\epsilon_t \\
(1-0.2L)Y_t &= (1+0.7L)\epsilon_t
\end{aligned}
$$

The reduced form is

$$Y_t = 0.2Y_{t-1} + \epsilon_t + 0.7\epsilon_{t-1}$$

(2)

$$
\begin{aligned}
\hat{Y}_T(1) &= 0.2Y_T + 0.7\epsilon_T \\
\hat{Y}_T(2) &= 0.2\hat{Y}_T(1) = 0.04Y_T + 0.14\epsilon_T
\end{aligned}
$$

$$
\begin{aligned}
e_T(2) &= Y_{T+2} - \hat{Y}_T(2) \\
&= 0.2Y_{T+1} + \epsilon_{T+2} + 0.7\epsilon_{T+1} - (0.04Y_T + 0.14\epsilon_T) \\
&= 0.2(Y_{T+1} - 0.2Y_T) + \epsilon_{T+2} + 0.7\epsilon_{T+1} - 0.14\epsilon_T \\
&= 0.2(\epsilon_{T+1} + 0.7\epsilon_T) + \epsilon_{T+2} + 0.7\epsilon_{T+1} - 0.14\epsilon_T \\
&= \epsilon_{T+2} + 0.9\epsilon_{T+1}
\end{aligned}
$$

$Y_T = 0.14, \epsilon_T = 0.03$, then $\hat{Y}_T(2) = 0.04Y_T + 0.14\epsilon_T = 0.0098$. $\psi_1 = 0.9$, thus the 95% prediction interval is given by

$$\hat{Y}_T(2) \pm 1.96(1 + \psi_1^2)^{1/2} = [-2.62711031323, 2.64671031323] \approx [-2.63, 2.65]$$