Time Series Analysis

Homework of week 15

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1.

(a)

$$r_t = \epsilon_t = \sqrt{h_t} v_t, \quad h_t = 0.003 + 0.81 h_{t-1} + 0.07 \epsilon_{t-1}$$

which implies that $\alpha_0 = 0.003, \alpha_1 = 0.07, \beta_1 = 0.81$. Then

$$\alpha_0 > 0,\ \alpha_1 > 0,\ \beta_1 > 0,\ \alpha_1 + \beta_1 = 0.07 + 0.81 = 0.88 < 1$$

Hence, the above GARCH$(1,1)$ model is stationary.

(b)

we have

$$h_{T+1} = 0.003 + 0.81 h_T + 0.07 \epsilon_T^2$$

so that

$$h_T(1) = E(h_{T+1} | F_T) = 0.003 + 0.81 h_T + 0.007 r_T^2$$

Then

$$h_{1190}(1) = 0.003 + 0.81 \times 0.0034 + 0.07 \times (-0.0012)^2 = 0.0058$$

and

$$h_{t+1} = 0.003 + 0.07 h_t v_t^2 + 0.81 h_t = 0.003 = 0.88 h_t + 0.07 h_t (v_t^2 - 1)$$

$$h_{1190}(2) = E(h_{1190+2}) | F_t) = 0.003 + 0.88 h_T(1) = 0.0081$$

□

2.

The yield rate is given by

$$(0.5, 0.3, 0.2) \begin{pmatrix} 0.1 \\ 0.2 \\ 0.15 \end{pmatrix} = 0.14$$

The variance of the yield rate is

$$(0.5, 0.3, 0.2) \begin{pmatrix} 0.1 & 0.04 & 0.03 \\ 0.04 & 0.2 & -0.04 \\ 0.03 & -0.04 & 0.6 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.3 \\ 0.2 \end{pmatrix} = 0.0802$$

Since $\Phi^{-1}(x) = -1.65$ for $\alpha = 0.05$, $R \sim N(0.14, 0.0802)$, it then follows that $0.14 = \sqrt{0.0802} \times (-1.65) = -0.327$ and $VaR = -100 \times (-0.3273) = 32.73$. □