Chap4: Models for stationary time series (4.1-4.3)

4.4 Show that when θ is replaced by $1/\theta$, the autocorrelation function for an MA(1) process does not change.

4.2 Sketch the autocorrelation functions for the following MA(2) models with parameters as specified:

(a) $\theta_1 = 0.5$ and $\theta_2 = 0.4$.

(b) $\theta_1 = 1.2$ and $\theta_2 = -0.7$.

(c) $\theta_1 = -1$ and $\theta_2 = -0.6$.

4.24 Let $\{e_t\}$ be a zero-mean, unit-variance white noise process. Consider a process that begins at time $t = 0$ and is defined recursively as follows. Let $Y_0 = c_1 e_0$ and $Y_1 = c_2 Y_0 + e_1$. Then let $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$ for $t > 1$ as in an AR(2) process.

(a) Show that the process mean is zero.

(b) For particular values of ϕ_1 and ϕ_2 within the stationarity region for an AR(2) model, show how to choose c_1 and c_2 so that both $Var(Y_0) = Var(Y_1)$ and the lag 1 autocorrelation between Y_1 and Y_0 match that of a stationary AR(2) process with parameters ϕ_1 and ϕ_2.

(c) Once the process $\{Y_t\}$ is generated, show how to transform it to a new process that has any desired mean and variance. (This exercise suggests a convenient method for simulating stationary AR(2) processes.)

4.25 Consider an "AR(1)" process satisfying $Y_t = \phi Y_{t-1} + e_t$, where ϕ can be ***any*** number and $\{e_t\}$ is a white noise process such that e_t is independent of the past $\{Y_{t-1}, Y_{t-2}, \ldots\}$. Let Y_0 be a random variable with mean μ_0 and variance σ_0^2.

(a) Show that for $t > 0$ we can write

$$Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \phi^3 e_{t-3} + \cdots + \phi^{t-1} e_1 + \phi^t Y_0.$$

(b) Show that for $t > 0$ we have $E(Y_t) = \phi^t \mu_0$.

(c) Show that for $t > 0$

$$Var(Y_t) = \begin{cases} \dfrac{1-\phi^{2t}}{1-\phi^2}\sigma_e^2 + \phi^{2t}\sigma_0^2 & \text{for } \phi \neq 1 \\ t\sigma_e^2 + \sigma_0^2 & \text{for } \phi = 1 \end{cases}$$

(d) Suppose now that $\mu_0 = 0$. Argue that, if $\{Y_t\}$ is stationary, we must have $\phi \neq 1$.

(e) Continuing to suppose that $\mu_0 = 0$, show that, if $\{Y_t\}$ is stationary, then $Var(Y_t) = \sigma_e^2/(1-\phi^2)$ and so we must have $|\phi| < 1$.

Deadline for submission: 23:00, 29 Sep