Time Series Analysis

Homework of week 7

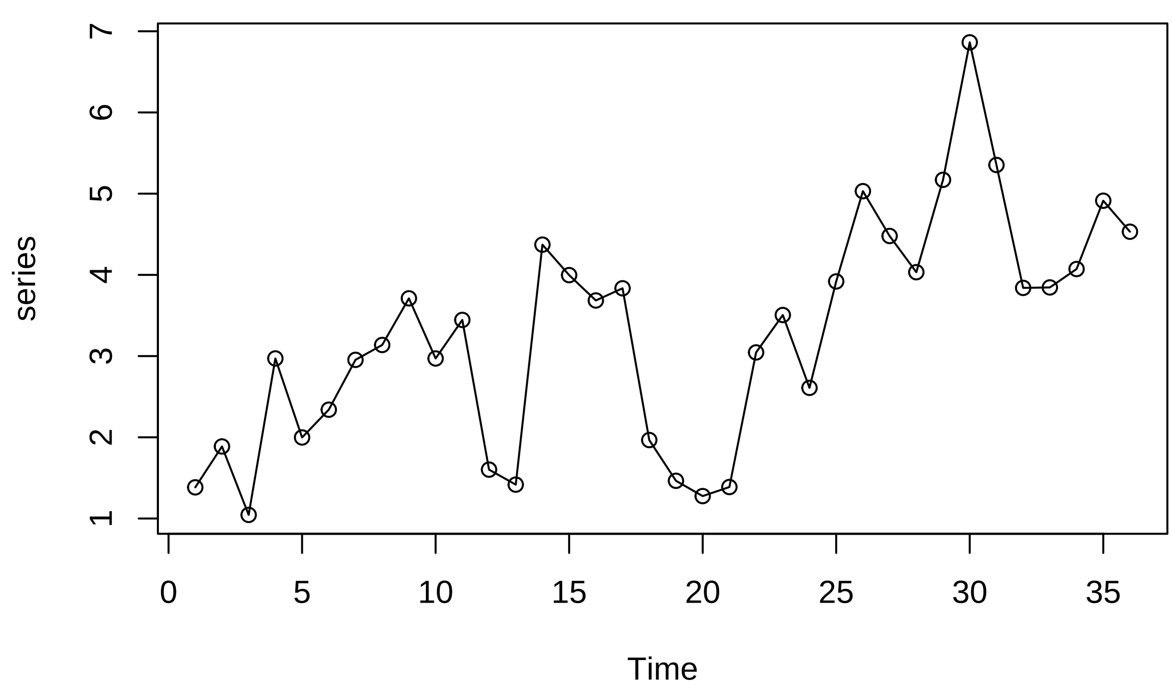
Hanbin Liu 11912410

6.32

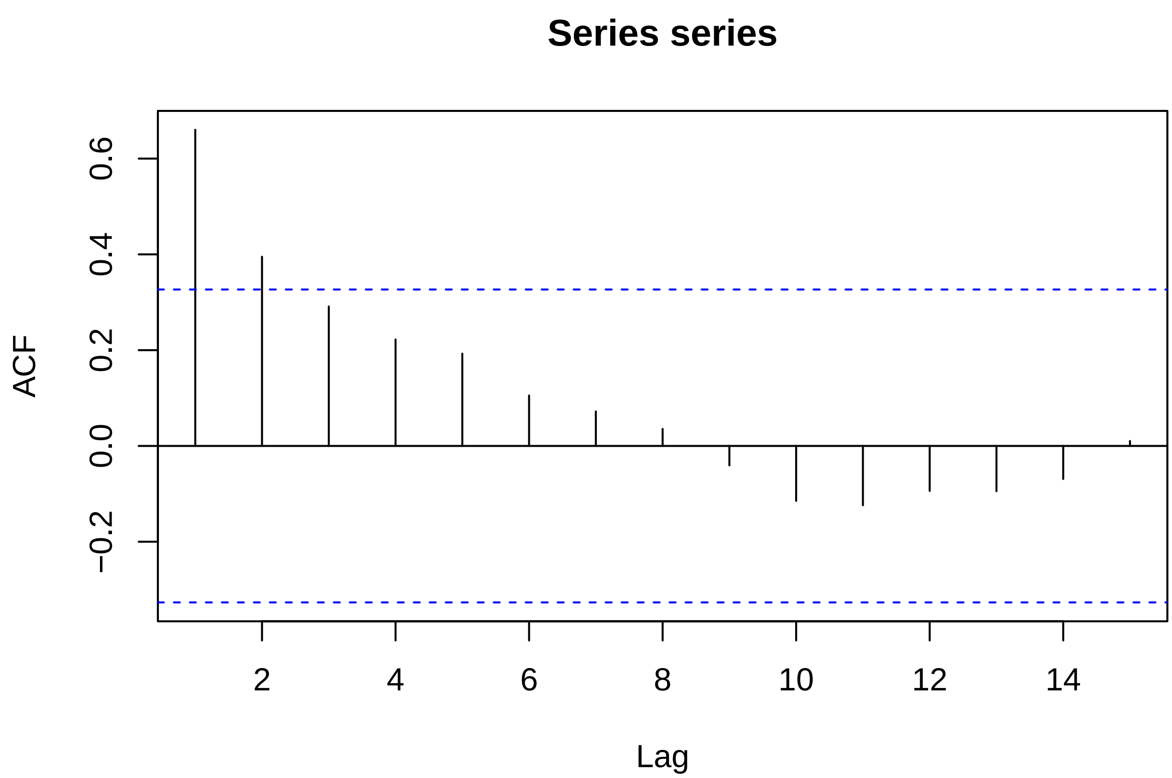
```
set.seed(274135)
series <- arima.sim(n = 36, list(ar = 0.95))
```

(a)

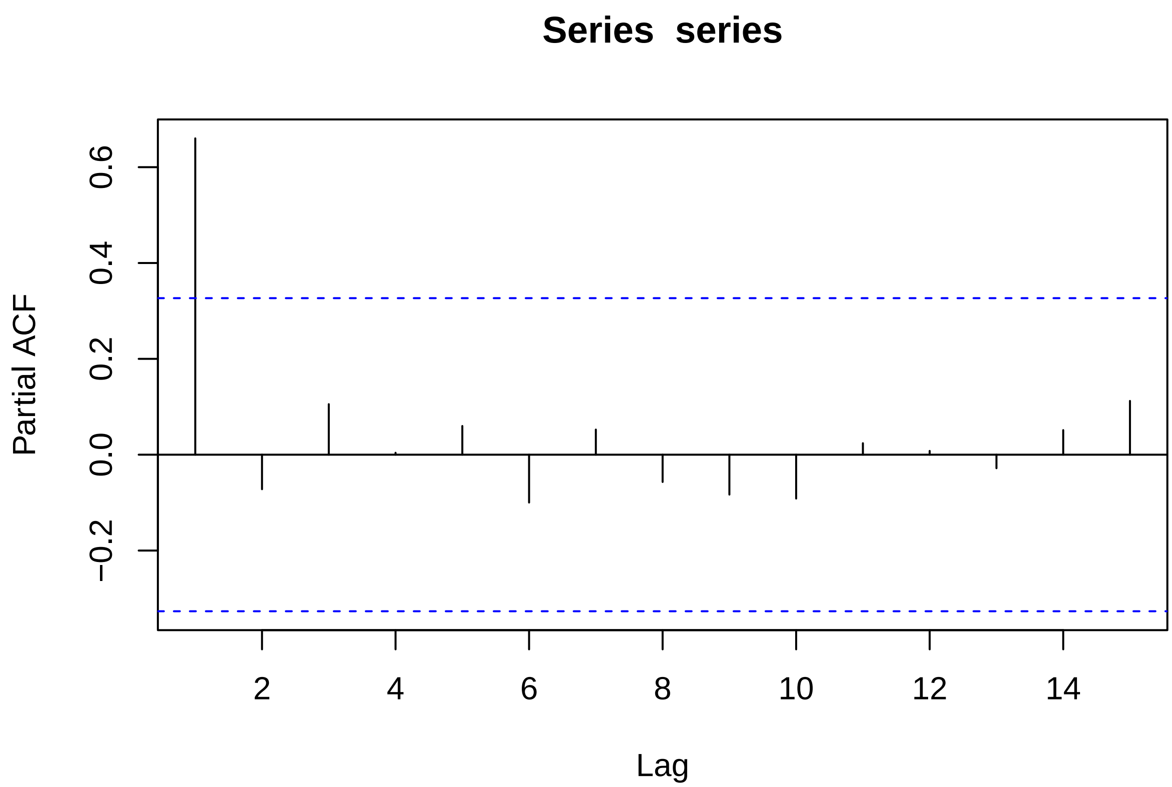
```
plot(series, type = 'o')
```



```
acf(series)
```




```
pacf(series)
```



The ACF and PACF graphs would lead us to at least entertain an AR(1) model. However, the “upward trend” in the time series plot suggests nonstationarity of some kind

(b)

```
library(uroot)
ADF.test(series, selectlags=list(Pmax=0), itsd=c(1,0,0))
```

```
## Warning in interpolpval(code = code, stat = adfreg[, 3], N = N): p-value is
## greater than printed p-value
```

```
##   --------- ------ - ------ ----
##   Augmented Dickey & Fuller test
##   --------- ------ - ------ ----
##
##   Null hypothesis: Unit root.
##   Alternative hypothesis: Stationarity.
##
## ----
##   ADF statistic:
##
##          Estimate Std. Error t value Pr(>|t|)
## adf.reg   -0.323      0.124  -2.597      0.1
##
```

```
##   Lag orders: 0
##   Number of available observations: 35
```

The Dickey-Fuller test results suggest that we consider a nonstationary model for these data.

(c)

```r
ar(diff(series))
```

```
##
## Call:
## ar(x = diff(series))
##
## Coefficients:
##       1        2
## -0.1573  -0.3090
##
## Order selected 2  sigma^2 estimated as  1.115
```

```r
ADF.test(series, selectlags=list(mode=c(1,2)), itsd=c(1,0,0))
```

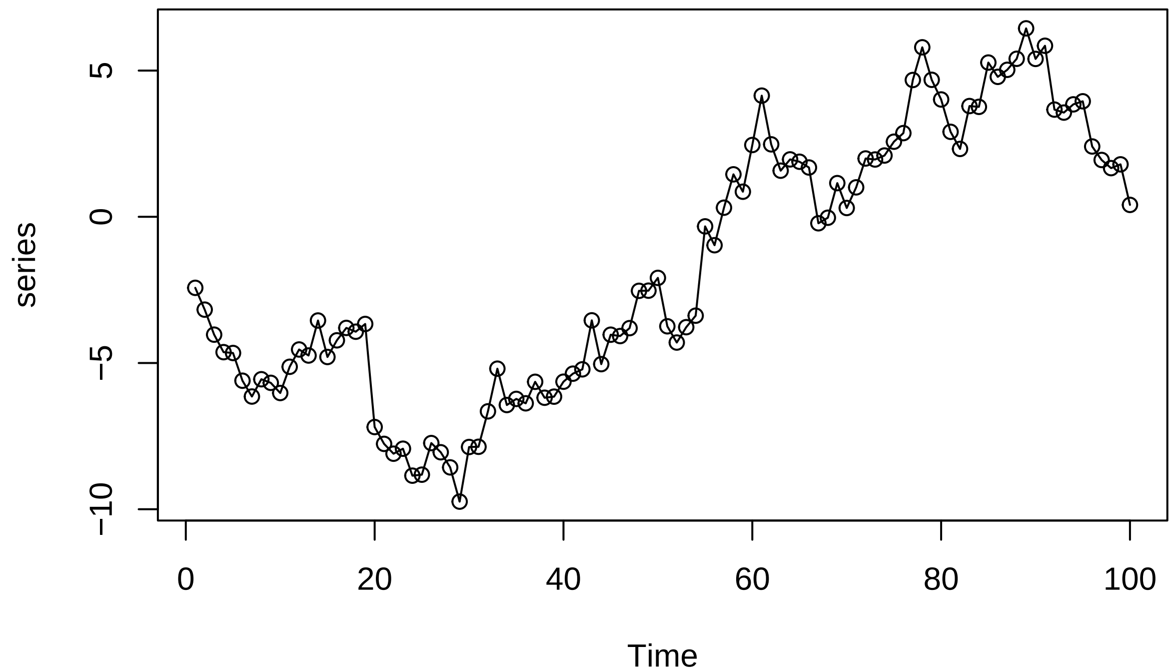
```
## Warning in interpolpval(code = code, stat = adfreg[, 3], N = N): p-value is
## greater than printed p-value
```

```
##   --------- ------ - ------ ----
##   Augmented Dickey & Fuller test
##   --------- ------ - ------ ----
##
##   Null hypothesis: Unit root.
##   Alternative hypothesis: Stationarity.
##
## ----
##   ADF statistic:
##
##          Estimate Std. Error t value Pr(>|t|)
## adf.reg   -0.335      0.155  -2.156      0.1
##
##   Lag orders: 1 2
##   Number of available observations: 33
```

The augmented Dickey-Fuller test, also suggests a unit root for this series.

(d)

```r
set.seed(274153)
series=arima.sim(n=100,list(ar=0.95))
plot(series,type='o')
```




```
ADF.test(series, selectlags=list(Pmax=0), itsd=c(1,0,0))
```

```
## Warning in interpolpval(code = code, stat = adfreg[, 3], N = N): p-value is
## greater than printed p-value
```

```
##   --------- ------ - ------ ----
##   Augmented Dickey & Fuller test
##   --------- ------ - ------ ----
##
##   Null hypothesis: Unit root.
##   Alternative hypothesis: Stationarity.
##
## ----
##   ADF statistic:
##
##         Estimate Std. Error t value Pr(>|t|)
## adf.reg   -0.024      0.023  -1.073      0.1
##
##   Lag orders: 0
##   Number of available observations: 99
```

```
# The Dickey-Fuller test results suggest that we consider a nonstationary model for these data.
ar(diff(diff(series)))
```

```
##
```

```
## Call:
## ar(x = diff(diff(series)))
##
## Coefficients:
##       1        2        3        4
## -0.8531  -0.6263  -0.4173  -0.2979
##
## Order selected 4  sigma^2 estimated as  1.235
```

```
ADF.test(series, selectlags=list(mode=c(1,2,3,4)), itsd=c(1,0,0))
```

```
## Warning in interpolpval(code = code, stat = adfreg[, 3], N = N): p-value is
## greater than printed p-value
```

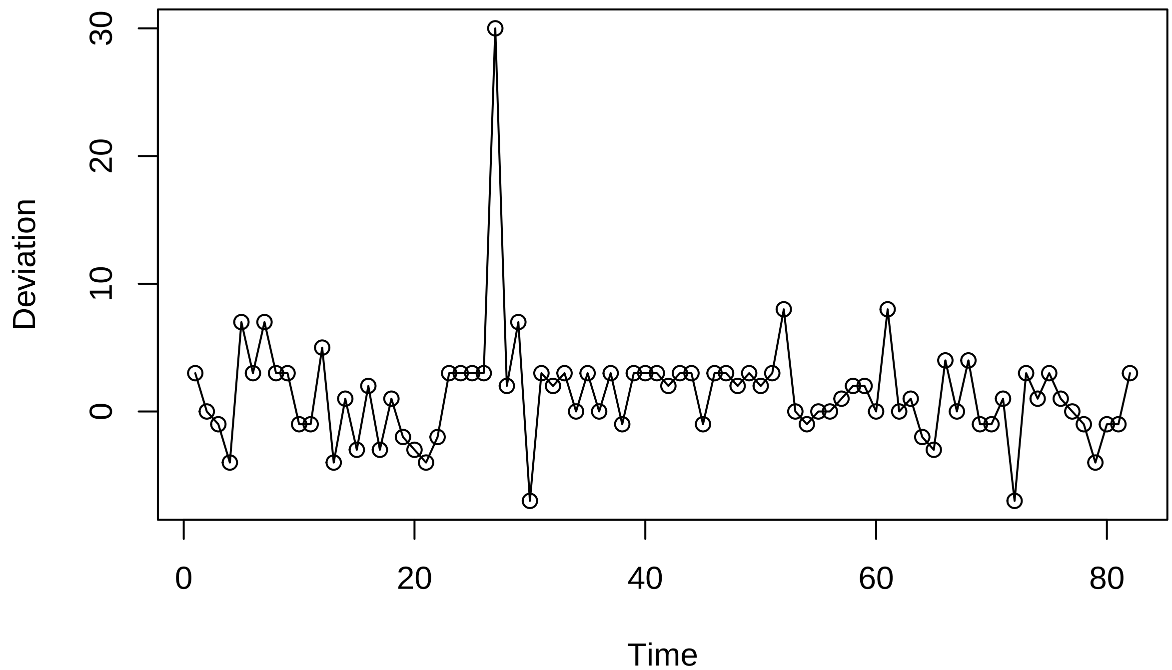
```
##   --------- ------ - ------ ----
##   Augmented Dickey & Fuller test
##   --------- ------ - ------ ----
##
##   Null hypothesis: Unit root.
##   Alternative hypothesis: Stationarity.
##
## ----
##   ADF statistic:
##
##         Estimate Std. Error t value Pr(>|t|)
## adf.reg   -0.025      0.024   -1.05      0.1
##
##   Lag orders: 1 2 3 4
##   Number of available observations: 95
```

```
# The augmented Dickey-Fuller test, also suggests a unit root for this series.
```

6.33

(a)

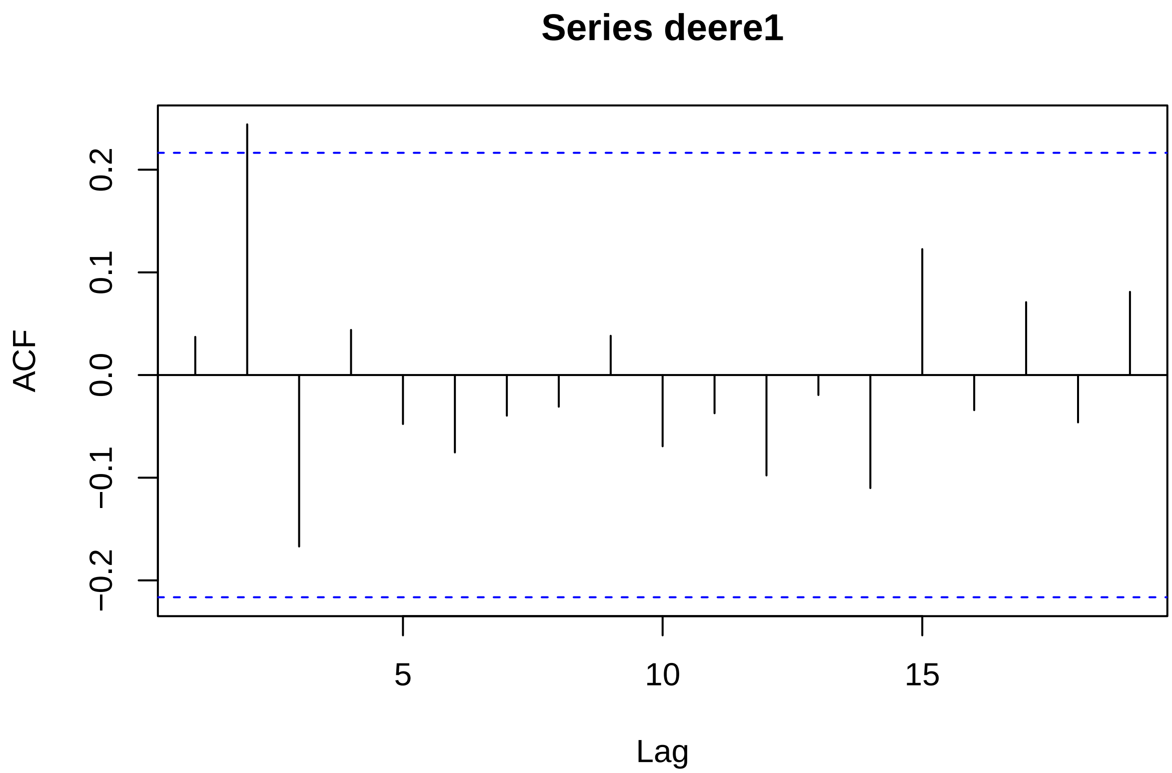
```
data(deere1)
plot(deere1, type = 'o', ylab = 'Deviation')
```



Except for one point of 30 at t = 27 the process seems relatively stable and stationary.

(b)

```
acf(deere1)
```



The graph indicates a statistically significant autocorrelation at lag 2.

(c)

```
t <- which(deere1 == max(deere1)); t; deere1[t]
```

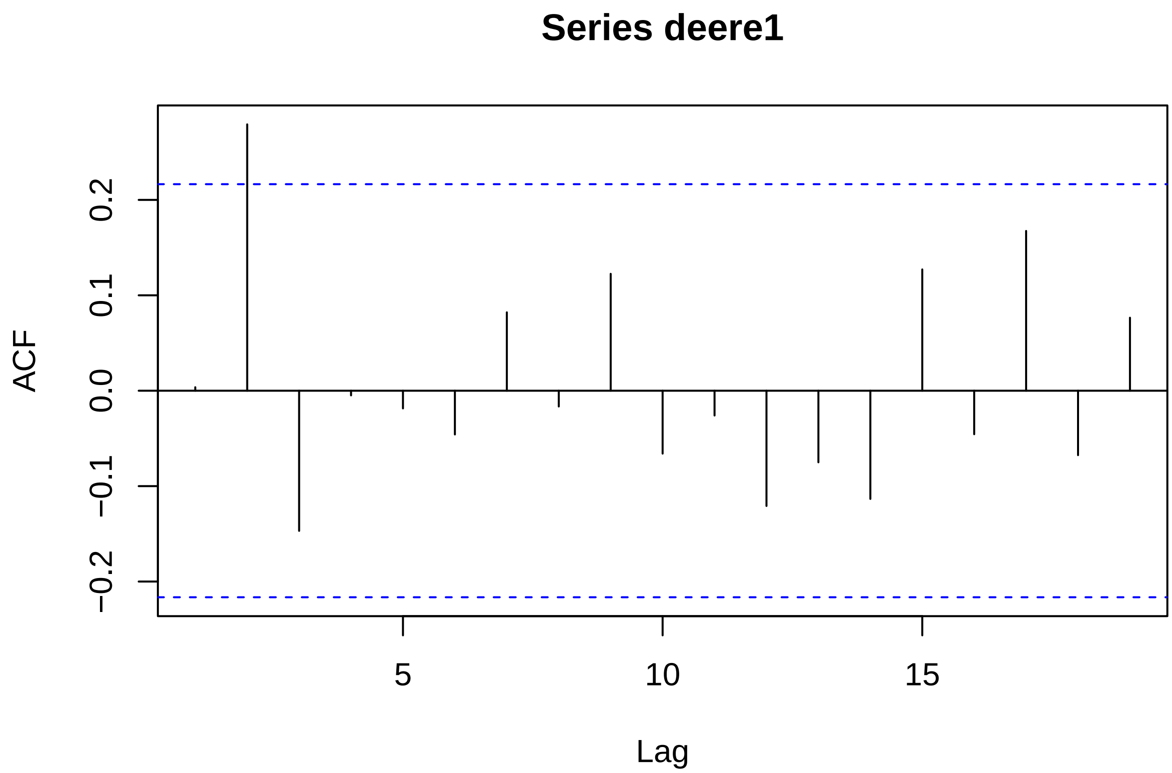
```
## [1] 27
```

```
## [1] 30
```

```
deere1[t] <- max(deere1[-t]); deere1[t]
```

```
## [1] 8
```

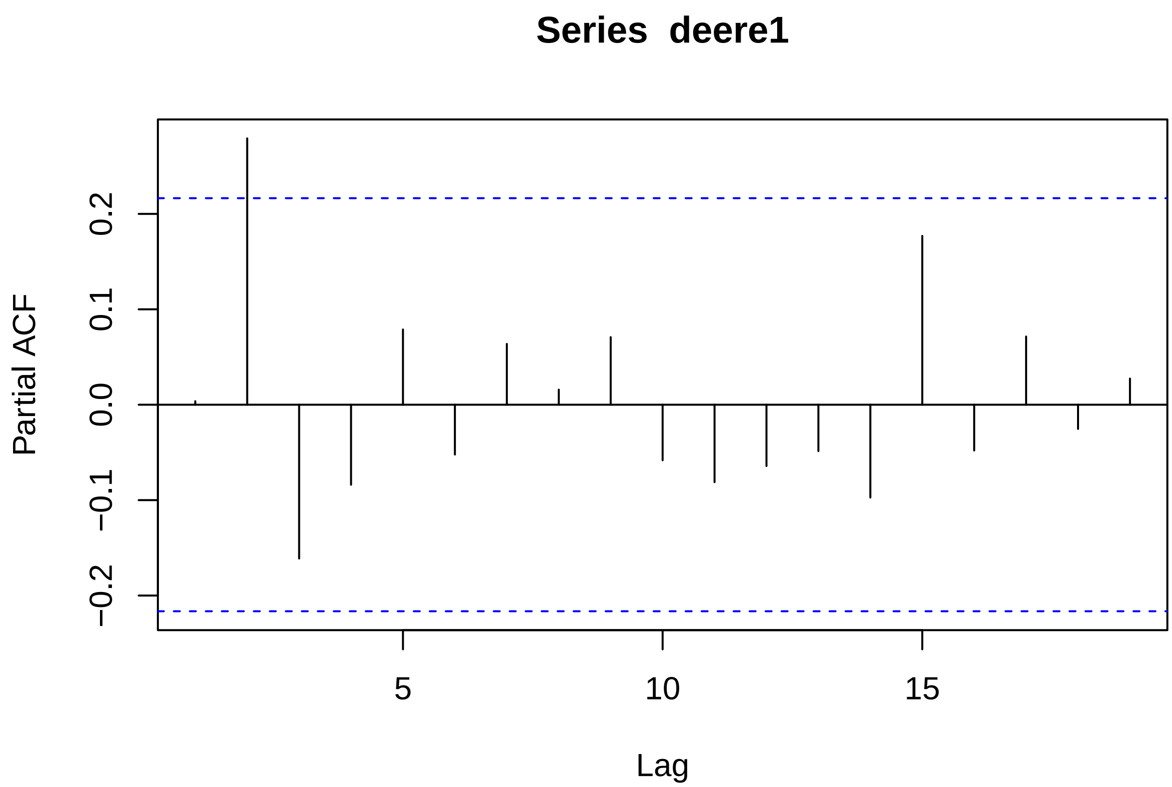
```
acf(deere1)
```



We replaced the unusual value of 30 at time 27 with the next largest value of 8. This had only a small effect on the sample autocorrelation function.

(d)

```
pacf(deere1)
```



This pacf suggests an AR(2) model for the series.

7.4

(a)

We have $e_1 = Y_1 = 0, e_2 = Y_2 + \theta Y_1 = -1, e_3 = Y_3 + \theta Y_2 + \theta^2 Y_1 = \frac{1}{2} - \theta$. Then

$$\sum_t e_t^2 = 1 + (\theta - \frac{1}{2})^2 = \theta^2 - \theta + \frac{5}{4},$$

which implies that $\hat{\theta} = \frac{1}{2}$.

(b)

The estimate of the noise variance is given by

$$\hat{\sigma}_e^2 = \frac{1}{n-1} \sum_t e_t^2 = \frac{1}{3-1} \cdot 1 = \frac{1}{2}.$$

7.7

MA(1)model: $Y_t = e_t - \theta e_{t-1}$. Then $\rho_1 = \frac{-\theta}{1+\theta^2}$. Replace ρ_1 with r_1, we have

$$r_1 = \frac{-\theta}{1+\theta^2},$$

which has two roots $\theta_1 = \frac{-1+\sqrt{1-4r_1^2}}{2r_1}, \theta_2 = \frac{-1-\sqrt{1-4r_1^2}}{2r_1}$. Note that $\theta_1\theta_2 = 1$. If $\theta_i > 0$, then $\theta_2 > 1 > \theta_1$. If $\theta_i < 0$, then $\theta_2 < -1 < \theta_1$. Thus, $\hat{\theta} = \theta_1 = \frac{-1+\sqrt{1-4r_1^2}}{2r_1}$ since $|\theta_1| < 1$.

Additional problem

The conditional maximum likelihood function is given by

$$\begin{aligned} L(\phi_0, \phi_1, \phi_2, \sigma_e^2 | Y_1, Y_2) &= \prod_{t=3}^{T} f(Y_t | Y_{t-1}, ..., Y_2, Y_1) \\ &= \prod_{t=3}^{T} (2\pi\sigma_e^2)^{-\frac{1}{2}} \exp\Big\{ -\frac{1}{2\sigma_e^2} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})^2 \Big\} \\ &= (2\pi\sigma_e^2)^{-\frac{T-2}{2}} \exp\Big\{ -\frac{1}{2\sigma_e^2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})^2 \Big\}. \end{aligned}$$

The log-likelihood is

$$l(\phi_0, \phi_1, \phi_2, \sigma_e^2 | Y_1, Y_2) = -\frac{T-2}{2} \log(2\pi\sigma_e^2) - \frac{1}{2\sigma_e^2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})^2.$$

We have

$$\begin{cases} 0 = \frac{\partial l}{\partial \sigma_e^2} = -\frac{T-2}{2} \cdot \frac{1}{\sigma_e^2} + \frac{1}{2\sigma_e^4} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})^2 \\ 0 = \frac{\partial l}{\partial \phi_0} = -\frac{1}{\sigma_e^2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})(-1) \\ 0 = \frac{\partial l}{\partial \phi_1} = -\frac{1}{\sigma_e^2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})(-Y_{t-1}) \\ 0 = \frac{\partial l}{\partial \phi_2} = -\frac{1}{\sigma_e^2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})(-Y_{t-2}) \end{cases}$$

Thus, the equation of the MLE $\phi_0, \phi_1, \phi_2, \sigma_e^2$ should satisfy is

$$\begin{cases} \sigma_e^2 = \frac{1}{T-2} \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2})^2 \\ 0 = \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2}) \\ 0 = \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2}) Y_{t-1} \\ 0 = \sum_{t=3}^{T} (Y_t - \phi_0 - \phi_1 Y_{t-1} - \phi_2 Y_{t-2}) Y_{t-2} \end{cases}$$