Assignment 10 Chapter 9 Forecasting

EXERCISES

9.1 For an AR(1) model with $Y_t = 12.2$, $\phi = -0.5$, and $\mu = 10.8$,

(a) Find $\hat{Y}_t(1)$.

(b) Calculate $\hat{Y}_t(2)$ in two different ways.

(c) Calculate $\hat{Y}_t(10)$.

9.2 Suppose that annual sales (in millions of dollars) of the Acme Corporation follow the AR(2) model $Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + e_t$ with $\sigma_e^2 = 2$.

(a) If sales for 2005, 2006, and 2007 were \$9 million, \$11 million, and \$10 million, respectively, forecast sales for 2008 and 2009.

9.3 Suppose that the monthly simple yield of a bond follows the following first-order autoregressive model

$$R_t = 0.2R_{t-1} + a_t,$$

where a_t's are white noises with $\sigma_a = \sqrt{var(a_t)}$. It is known that $R_{100} = 0.01$.

Please

(i) calculate the forward 1-step and forward 2-step forecasts with T = 100 as the forecasting origin;

(ii) calculate standard deviation of the corresponding prediction error;

(iii) calculate the autocorrelation coefficient of this return series with the interval 1 and 2.

9.13 Simulate an ARMA(1,1) process with $\phi = 0.7$, $\theta = -0.5$, and $\mu = 100$. Simulate 50 values but set aside the last 10 values to compare forecasts with actual values.

(a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of ϕ, θ, and μ.

(b) Using the estimated model, forecast the next ten values of the series. Plot the series together with the ten forecasts. Place a horizontal line at the estimate of the process mean.

(c) Compare the ten forecasts with the actual values that you set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

(e) Repeat parts (a) through (d) with a new simulated series using the same values of the parameters and same sample size.

9.14 Simulate an IMA(1,1) process with $\theta = 0.8$ and $\theta_0 = 0$. Simulate 35 values, but set aside the last five values to compare forecasts with actual values.

(a) Using the first 30 values of the series, find the value for the maximum likelihood estimate of θ.

(b) Using the estimated model, forecast the next five values of the series. Plot the series together with the five forecasts. What is special about the forecasts?

(c) Compare the five forecasts with the actual values that you set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

(e) Repeat parts (a) through (d) with a new simulated series using the same values of the parameters and same sample size.

Addition problem: Consider the following time series model:

$$Y_t = 0.3Y_{t-1} - 0.02Y_{t-2} + \varepsilon_t + 0.6\varepsilon_{t-1} - 0.07\varepsilon_{t-2}^2,$$

where ε_t is $i.i.d$ with the standard normal distribution.

Questions:

1) Reduce the above model and write down its reduced form.

2) Assume $Y_T = 0.14, \quad \varepsilon_T = 0.03$, what is the two-step-ahead forecast of Y_T and the corresponding 95% prediction interval for this forecast. You need to keep 2 non-zero digits after the decimal point for the calculation.

(Hints:

$$\hat{Y}_T(h) \pm 1.96(1 + \psi_1^2 + ... + \psi_{h-1}^2)^{1/2})).$$

Deadline: before class next Tuesday (30, Nov)