Chapter 12-Group assignment (3 person per group) 13

To submit this assignment on Dec 23 before class.

12.7 Simulate a GARCH(1,1) process with $\alpha = 0.1$ and $\beta = 0.8$ and of length 500. Plot the time series and inspect its sample ACF, PACF, and EACF. Are the data consistent with the assumption of white noise?

(a) Square the data and identify a GARCH model for the raw data based on the sample ACF, PACF, and EACF of the squared data.

(b) Identify a GARCH model for the raw data based on the sample ACF, PACF and EACF of the absolute data. Discuss and reconcile any discrepancy between the tentative model identified with the squared data and that with the absolute data.

(c) Perform the McLeod-Li test on your simulated series. What do you conclude?

(d) Repeat the exercise but now using only the first 200 simulated data. Discuss your findings.

2. For GARCH(1,1) model

$$\begin{cases} X_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \varpi + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2 \end{cases}$$

where $\varepsilon_t \sim t(d)$ and $t(d)$ is the t-distribution with degree of d. Let $\theta = (\omega, \alpha, \beta)'$.

i) Compute the log-likelihood function of L(θ).

ii) $Set\ \omega = 0.02, \alpha = 0.25, \beta = 0.35, \sigma_1^2 = 0.05, d = 4,$ simulate the above GARCH(1,1) process with sample size=1000 and display the time series plot.

iii) Based on the above samples simulated, write a R programme to find the MLEs of θ and their standard errors (SE) for the above GARCH(1,1) model.

If someone cannot find a teammate and hopes to finish this assignment alone, 2iii) can be an option question(选做题).