

# Statistical Calculation and Software

## Assignment 3

Hanbin Liu 11912410

### 3.1

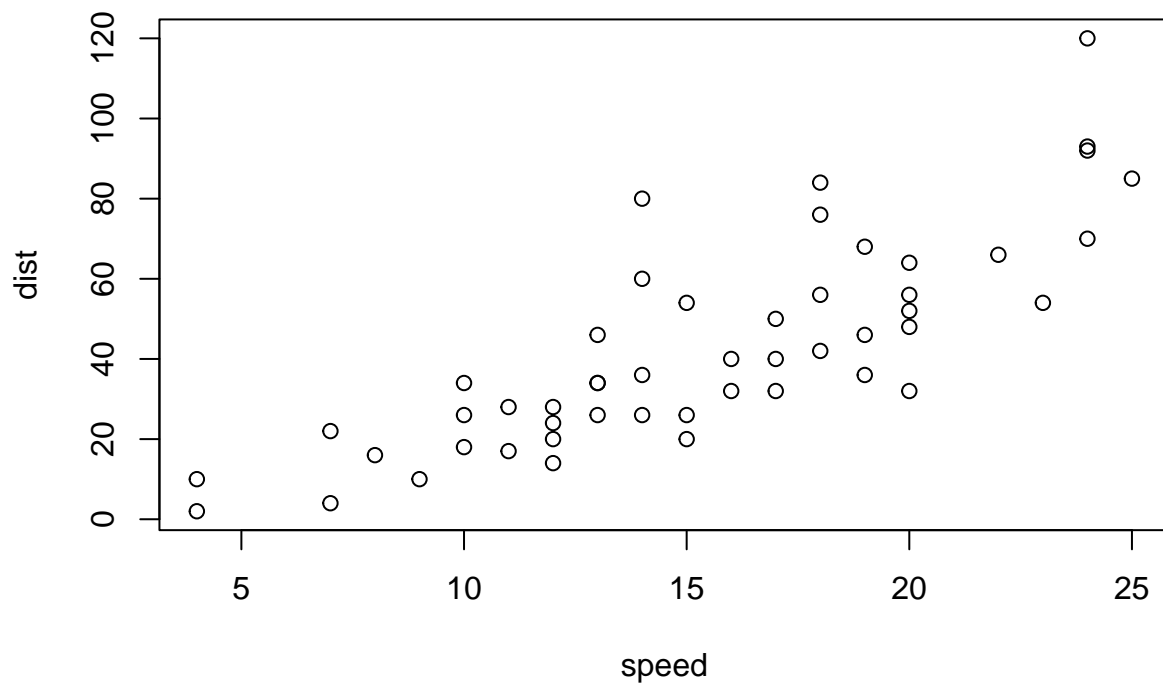
```
data(cars)
any(is.na(cars))
```

```
## [1] FALSE
```

(a)

```
attach(cars)
```

```
plot(speed, dist)
```



```
nw_box <- ksmooth(speed, dist, kernel = 'box', bandwidth = 1)
nw_box
```

```
## $x
## [1] 4.000000 4.212121 4.424242 4.636364 4.848485 5.060606 5.272727
## [8] 5.484848 5.696970 5.909091 6.121212 6.333333 6.545455 6.757576
## [15] 6.969697 7.181818 7.393939 7.606061 7.818182 8.030303 8.242424
## [22] 8.454545 8.666667 8.878788 9.090909 9.303030 9.515152 9.727273
## [29] 9.939394 10.151515 10.363636 10.575758 10.787879 11.000000 11.212121
## [36] 11.424242 11.636364 11.848485 12.060606 12.272727 12.484848 12.696970
## [43] 12.909091 13.121212 13.333333 13.545455 13.757576 13.969697 14.181818
## [50] 14.393939 14.606061 14.818182 15.030303 15.242424 15.454545 15.666667
## [57] 15.878788 16.090909 16.303030 16.515152 16.727273 16.939394 17.151515
## [64] 17.363636 17.575758 17.787879 18.000000 18.212121 18.424242 18.636364
## [71] 18.848485 19.060606 19.272727 19.484848 19.696970 19.909091 20.121212
## [78] 20.333333 20.545455 20.757576 20.969697 21.181818 21.393939 21.606061
## [85] 21.818182 22.030303 22.242424 22.454545 22.666667 22.878788 23.090909
## [92] 23.303030 23.515152 23.727273 23.939394 24.151515 24.363636 24.575758
## [99] 24.787879 25.000000
##
## $y
## [1] 6.00000 6.00000 6.00000 NA NA NA NA NA
## [9] NA NA NA NA 13.00000 13.00000 13.00000 13.00000
## [17] 13.00000 16.00000 16.00000 16.00000 16.00000 16.00000 10.00000 10.00000
## [25] 10.00000 10.00000 26.00000 26.00000 26.00000 26.00000 26.00000 22.50000
```

```
## [33] 22.50000 22.50000 22.50000 22.50000 21.50000 21.50000 21.50000 21.50000
## [41] 21.50000 35.00000 35.00000 35.00000 35.00000 50.50000 50.50000 50.50000
## [49] 50.50000 50.50000 33.33333 33.33333 33.33333 33.33333 33.33333 36.00000
## [57] 36.00000 36.00000 36.00000 40.66667 40.66667 40.66667 40.66667 40.66667
## [65] 64.50000 64.50000 64.50000 64.50000 64.50000 50.00000 50.00000 50.00000
## [73] 50.00000 50.00000 50.40000 50.40000 50.40000 50.40000 NA NA
## [81] NA NA NA 66.00000 66.00000 66.00000 66.00000 66.00000
## [89] 54.00000 54.00000 54.00000 54.00000 93.75000 93.75000 93.75000 93.75000
## [97] 93.75000 85.00000 85.00000 85.00000
```

```
nw_gaussian <-
  ksmooth(speed, dist, kernel = 'normal', bandwidth = 1)
nw_gaussian
```

```
## $x
## [1] 4.000000 4.212121 4.424242 4.636364 4.848485 5.060606 5.272727
## [8] 5.484848 5.696970 5.909091 6.121212 6.333333 6.545455 6.757576
## [15] 6.969697 7.181818 7.393939 7.606061 7.818182 8.030303 8.242424
## [22] 8.454545 8.666667 8.878788 9.090909 9.303030 9.515152 9.727273
## [29] 9.939394 10.151515 10.363636 10.575758 10.787879 11.000000 11.212121
## [36] 11.424242 11.636364 11.848485 12.060606 12.272727 12.484848 12.696970
## [43] 12.909091 13.121212 13.333333 13.545455 13.757576 13.969697 14.181818
## [50] 14.393939 14.606061 14.818182 15.030303 15.242424 15.454545 15.666667
## [57] 15.878788 16.090909 16.303030 16.515152 16.727273 16.939394 17.151515
## [64] 17.363636 17.575758 17.787879 18.000000 18.212121 18.424242 18.636364
## [71] 18.848485 19.060606 19.272727 19.484848 19.696970 19.909091 20.121212
## [78] 20.333333 20.545455 20.757576 20.969697 21.181818 21.393939 21.606061
## [85] 21.818182 22.030303 22.242424 22.454545 22.666667 22.878788 23.090909
## [92] 23.303030 23.515152 23.727273 23.939394 24.151515 24.363636 24.575758
## [99] 24.787879 25.000000
##
## $y
## [1] 6.00000 6.00000 6.00000 6.00000 6.00000 6.00000 6.00000 NA
## [9] 13.00000 13.00000 13.00000 13.00000 13.00144 13.00673 13.03127 13.14104
## [17] 13.56304 14.55558 15.47362 15.69961 15.18509 13.49127 11.45293 10.82344
## [25] 12.16487 16.67077 22.32167 25.03552 25.74906 25.78105 25.29895 24.11914
## [33] 23.03072 22.57818 22.32735 21.96800 21.67726 21.65002 22.03337 23.66535
## [41] 27.87816 32.44321 34.55172 35.77912 38.52125 44.01605 48.39480 49.75723
## [49] 49.22462 46.06958 39.87539 35.33184 33.84805 33.67226 34.21093 35.19083
## [57] 35.84004 36.27550 37.21687 38.92205 40.24251 41.05715 42.90506 48.53854
## [65] 57.30442 62.43897 63.77348 63.18850 60.11286 54.78641 51.37114 50.32450
## [73] 50.14902 50.23953 50.34994 50.38814 50.39741 50.39944 50.40417 50.49097
## [81] 52.17829 61.91842 65.75199 65.98211 65.91667 65.61949 64.40428 60.98363
## [89] 57.01260 56.24933 60.78352 73.39266 86.47830 91.93045 93.31359 93.49584
## [97] 92.99142 91.10208 87.88560 85.83190
```

(b)

```
tt1 <-
  loess(
    dist ~ speed,
    data = cars,
```

```

    span = 0.15,
    degree = 2,
    family = 'gaussian'
  )
tt2 <-
  loess(
    dist ~ speed,
    data = cars,
    span = 0.5,
    degree = 2,
    family = 'gaussian'
  )

```

(c)

Nadaraya-Watson Kernel Regression model:

```

##
nw_fitted_box <-
  ksmooth(
    speed,
    dist,
    kernel = 'box',
    bandwidth = 1,
    x.points = speed
  )$y
nw_fitted_box

```

```

## [1] 6.00000 6.00000 13.00000 13.00000 16.00000 10.00000 26.00000 26.00000
## [9] 26.00000 22.50000 22.50000 21.50000 21.50000 21.50000 21.50000 35.00000
## [17] 35.00000 35.00000 35.00000 50.50000 50.50000 50.50000 50.50000 33.33333
## [25] 33.33333 33.33333 36.00000 36.00000 40.66667 40.66667 40.66667 64.50000
## [33] 64.50000 64.50000 64.50000 50.00000 50.00000 50.00000 50.40000 50.40000
## [41] 50.40000 50.40000 50.40000 66.00000 54.00000 93.75000 93.75000 93.75000
## [49] 93.75000 85.00000

```

```

##
nw_fitted_gaussian <-
  ksmooth(
    speed,
    dist,
    kernel = 'normal',
    bandwidth = 1,
    x.points = speed
  )$y
nw_fitted_gaussian

```

```

## [1] 6.00000 6.00000 13.03889 13.03889 15.70783 11.28350 25.80378 25.80378
## [9] 25.80378 22.57818 22.57818 21.85378 21.85378 21.85378 21.85378 35.04991
## [17] 35.04991 35.04991 35.04991 49.78746 49.78746 49.78746 49.78746 33.94889
## [25] 33.94889 33.94889 36.07304 36.07304 41.38204 41.38204 41.38204 63.77348
## [33] 63.77348 63.77348 63.77348 50.48694 50.48694 50.48694 50.39379 50.39379

```

```
## [41] 50.39379 50.39379 50.39379 65.69288 57.97006 93.43566 93.43566 93.43566
## [49] 93.43566 85.83190
```

Local polynomial model:

```
## model with span=0.15: fitted y
fitted(tt1) # == predict(tt1)
```

```
## [1] 5.97671 5.97671 13.00000 13.00000 16.00000 10.00000 26.00000 26.00000
## [9] 26.00000 22.50000 22.50000 21.50000 21.50000 21.50000 21.50000 35.00000
## [17] 35.00000 35.00000 35.00000 50.50000 50.50000 50.50000 50.50000 33.33333
## [25] 33.33333 33.33333 36.00000 36.00000 40.66667 40.66667 40.66667 64.50000
## [33] 64.50000 64.50000 64.50000 50.00000 50.00000 50.00000 50.40000 50.40000
## [41] 50.40000 50.40000 50.40000 66.00000 74.50000 93.75000 93.75000 93.75000
## [49] 93.75000 85.00000
```

```
## model with span=0.5: fitted y
fitted(tt2) # == predict(tt2)
```

```
## [1] 6.124161 6.124161 12.329738 12.329738 14.801415 16.733767 18.990452
## [8] 18.990452 18.990452 22.667608 22.667608 29.208622 29.208622 29.208622
## [15] 29.208622 35.383851 35.383851 35.383851 35.383851 41.588031 41.588031
## [22] 41.588031 41.588031 40.553445 40.553445 40.553445 43.519416 43.519416
## [29] 47.479712 47.479712 47.479712 55.016200 55.016200 55.016200 55.016200
## [36] 54.434758 54.434758 54.434758 54.012981 54.012981 54.012981 54.012981
## [43] 54.012981 64.322532 73.542693 85.271671 85.271671 85.271671 85.271671
## [50] 99.770407
```

Compare:

```
nw_box_error <- dist - nw_fitted_box
nw_box_error
```

```
## [1] -4.0000000 4.0000000 -9.0000000 9.0000000 0.0000000 0.0000000
## [7] -8.0000000 0.0000000 8.0000000 -5.5000000 5.5000000 -7.5000000
## [13] -1.5000000 2.5000000 6.5000000 -9.0000000 -1.0000000 -1.0000000
## [19] 11.0000000 -24.5000000 -14.5000000 9.5000000 29.5000000 -13.3333333
## [25] -7.3333333 20.6666667 -4.0000000 4.0000000 -8.6666667 -0.6666667
## [31] 9.3333333 -22.5000000 -8.5000000 11.5000000 19.5000000 -14.0000000
## [37] -4.0000000 18.0000000 -18.4000000 -2.4000000 1.6000000 5.6000000
## [43] 13.6000000 0.0000000 0.0000000 -23.7500000 -1.7500000 -0.7500000
## [49] 26.2500000 0.0000000
```

```
nw_gaussian_error <- dist - nw_fitted_gaussian
nw_gaussian_error
```

```
## [1] -4.0000000 4.0000000 -9.0388879 8.9611121 0.2921668 -1.2835007
## [7] -7.8037832 0.1962168 8.1962168 -5.5781768 5.4218232 -7.8537816
## [13] -1.8537816 2.1462184 6.1462184 -9.0499096 -1.0499096 -1.0499096
## [19] 10.9500904 -23.7874619 -13.7874619 10.2125381 30.2125381 -13.9488853
## [25] -7.9488853 20.0511147 -4.0730417 3.9269583 -9.3820379 -1.3820379
```

```
## [31] 8.6179621 -21.7734842 -7.7734842 12.2265158 20.2265158 -14.4869446
## [37] -4.4869446 17.5130554 -18.3937940 -2.3937940 1.6062060 5.6062060
## [43] 13.6062060 0.3071219 -3.9700566 -23.4356563 -1.4356563 -0.4356563
## [49] 26.5643437 -0.8318986
```

```
local_error1 <- dist - fitted(tt1)
local_error1
```

```
## [1] -3.976710e+00 4.023290e+00 -9.000000e+00 9.000000e+00 3.552714e-15
## [6] 5.329071e-15 -8.000000e+00 7.105427e-15 8.000000e+00 -5.500000e+00
## [11] 5.500000e+00 -7.500000e+00 -1.500000e+00 2.500000e+00 6.500000e+00
## [16] -9.000000e+00 -1.000000e+00 -1.000000e+00 1.100000e+01 -2.450000e+01
## [21] -1.450000e+01 9.500000e+00 2.950000e+01 -1.333333e+01 -7.333333e+00
## [26] 2.066667e+01 -4.000000e+00 4.000000e+00 -8.666667e+00 -6.666667e-01
## [31] 9.333333e+00 -2.250000e+01 -8.500000e+00 1.150000e+01 1.950000e+01
## [36] -1.400000e+01 -4.000000e+00 1.800000e+01 -1.840000e+01 -2.400000e+00
## [41] 1.600000e+00 5.600000e+00 1.360000e+01 0.000000e+00 -2.050000e+01
## [46] -2.375000e+01 -1.750000e+00 -7.500000e-01 2.625000e+01 0.000000e+00
```

```
local_error2 <- dist - fitted(tt2)
local_error2
```

```
## [1] -4.1241610 3.8758390 -8.3297383 9.6702617 1.1985851 -6.7337672
## [7] -0.9904521 7.0095479 15.0095479 -5.6676077 5.3323923 -15.2086223
## [13] -9.2086223 -5.2086223 -1.2086223 -9.3838512 -1.3838512 -1.3838512
## [19] 10.6161488 -15.5880314 -5.5880314 18.4119686 38.4119686 -20.5534451
## [25] -14.5534451 13.4465549 -11.5194163 -3.5194163 -15.4797119 -7.4797119
## [31] 2.5202881 -13.0162002 0.9837998 20.9837998 28.9837998 -18.4347581
## [37] -8.4347581 13.5652419 -22.0129809 -6.0129809 -2.0129809 1.9870191
## [43] 9.9870191 1.6774684 -19.5426926 -15.2716715 6.7283285 7.7283285
## [49] 34.7283285 -14.7704067
```

```
## SSE & MSE & std.error
```

```
nw_box_sse <-
  sum(nw_box_error ^ 2)
nw_gaussian_sse <- sum(nw_gaussian_error ^ 2)
local_sse1 <- sum(local_error1 ^ 2)
local_sse2 <- sum(local_error2 ^ 2)
nw_box_mse <- nw_box_sse / 50
nw_gaussian_mse <- nw_gaussian_sse / 50
local_mse1 <- local_sse1 / 50
local_mse2 <- local_sse2 / 50
Model <-
  c(
    "Nadaraya-Watson with box kernel",
    "Nadaraya-Watson with gaussian kernel",
    "local polynomial with span=0.15",
    "local polynomial with span=0.5"
  )
SSE <- c(nw_box_sse, nw_gaussian_sse, local_sse1, local_sse2)
MSE <- c(nw_box_mse, nw_gaussian_mse, local_mse1, local_mse2)
std.error <-
```

```

c(sd(nw_box_error),
  sd(nw_gaussian_error),
  sd(local_error1),
  sd(local_error2))
data.frame(Model, SSE, MSE, std.error)

```

```

##
## 1      Nadaraya-Watson with box kernel 6764.783 135.2957 11.74976
## 2 Nadaraya-Watson with gaussian kernel 6791.637 135.8327 11.77257
## 3      local polynomial with span=0.15 7185.034 143.7007 12.10217
## 4      local polynomial with span=0.5 9299.113 185.9823 13.76285

```

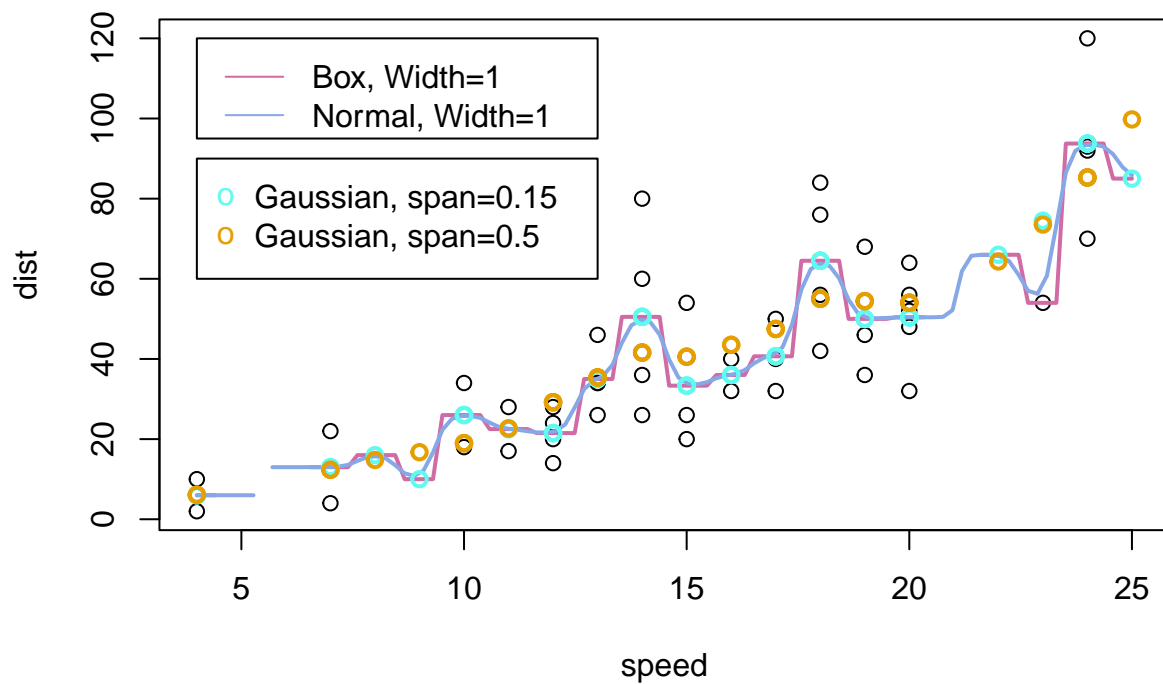
Nadaraya-Watson kernel regression model with box kernel fits better.

(d)

```

plot(speed, dist)
lines(nw_box, col = '#D16BA5', lwd = 2)
lines(nw_gaussian, col = '#86A8E7', lwd = 2)
points(tt1$x, fitted(tt1), col = '#5FFBF1', lwd = 2)
points(tt2$x, fitted(tt2), col = '#E69F00', lwd = 2)
legend(
  x = c(4, 13),
  y = c(95, 120),
  c("Box, Width=1", "Normal, Width=1"),
  col = c('#D16BA5', '#86A8E7'),
  lwd = 1
)
legend(
  x = c(4, 13),
  y = c(60, 90),
  c("Gaussian, span=0.15", "Gaussian, span=0.5"),
  col = c('#5FFBF1', '#E69F00'),
  pch = 'o'
)

```



```
detach(cars)
```

### 3.2

```
library(MASS)
data(galaxies)
```

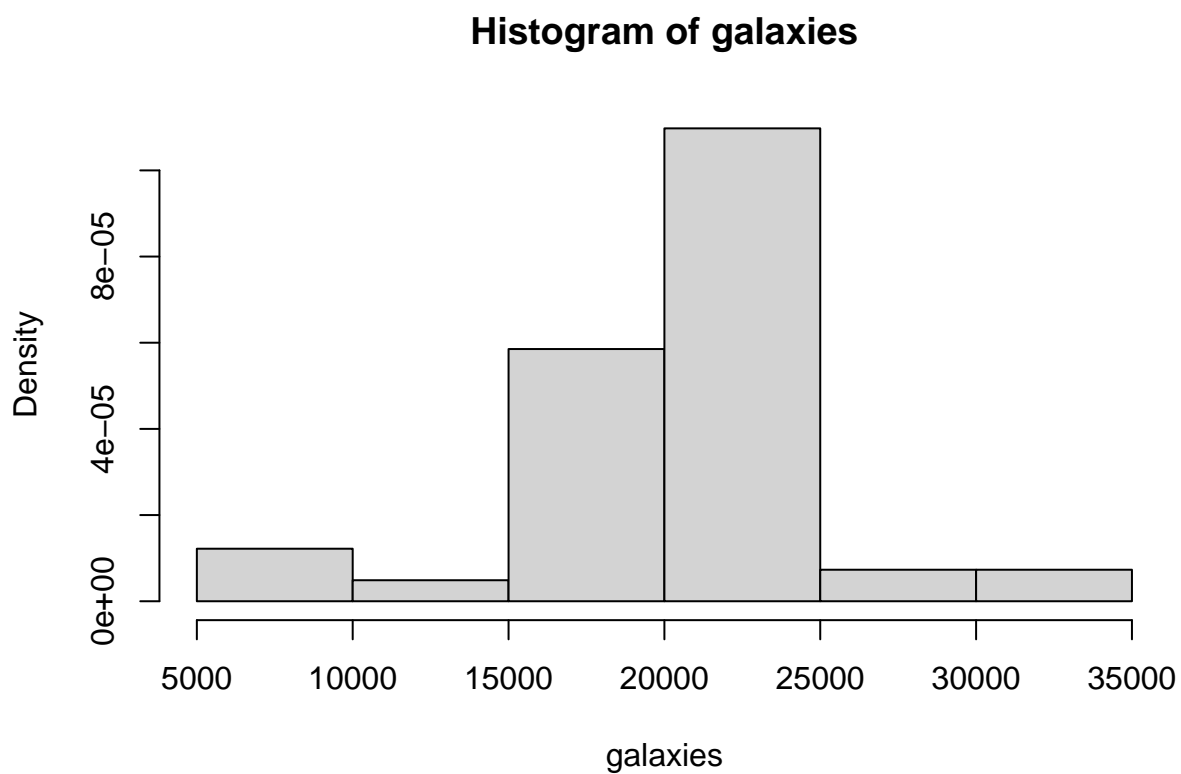
#### Histogram Smoothing:

$s$  is the sample standard deviation and  $n$  is the sample size, then

$$h^* = 3.491 s n^{-1/3}$$

```
n <- length(galaxies)
s <- sd(galaxies)
iqr <- IQR(galaxies)
h1 <- 3.491 * s * n ^ {
  -1 / 3
}
nobreks <- (max(galaxies) - min(galaxies)) / h1
hist(galaxies,
     breaks = round(nobreks),
     probability = TRUE)
```

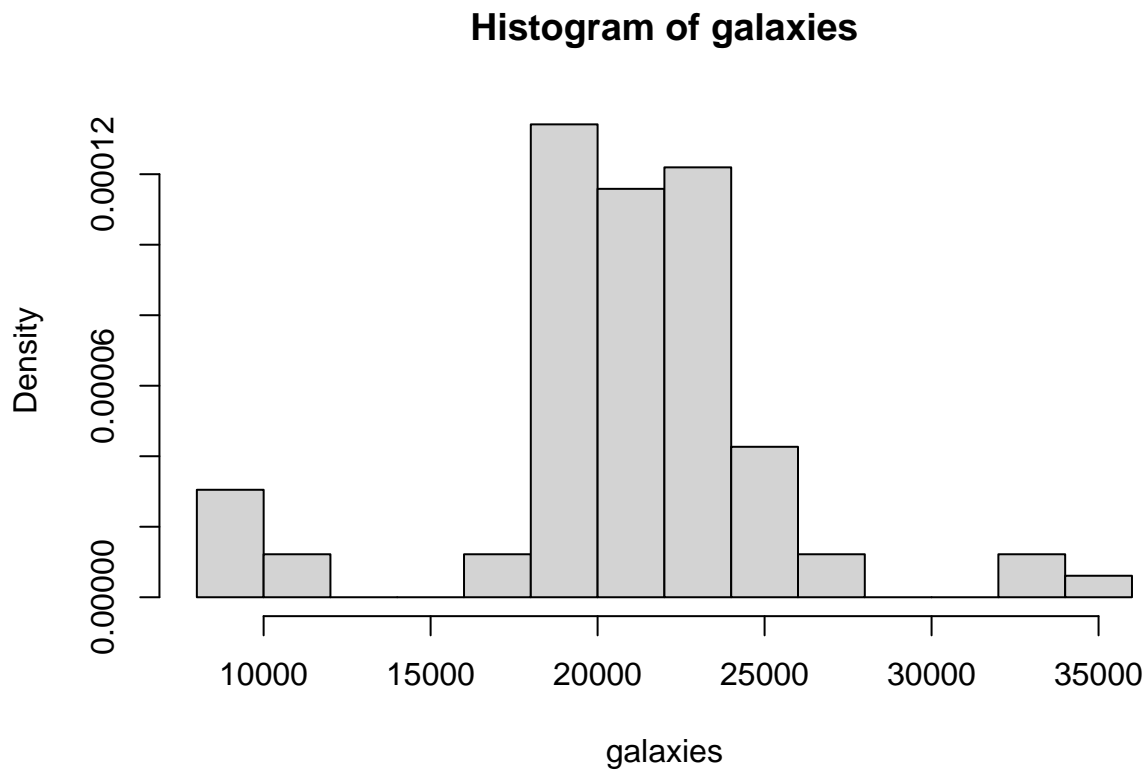




Another sensible estimate is obtained by replacing  $s$  by the inter-quantile range, IQR, that is

$$h^* = 2.6IQR \times n^{-1/3}$$

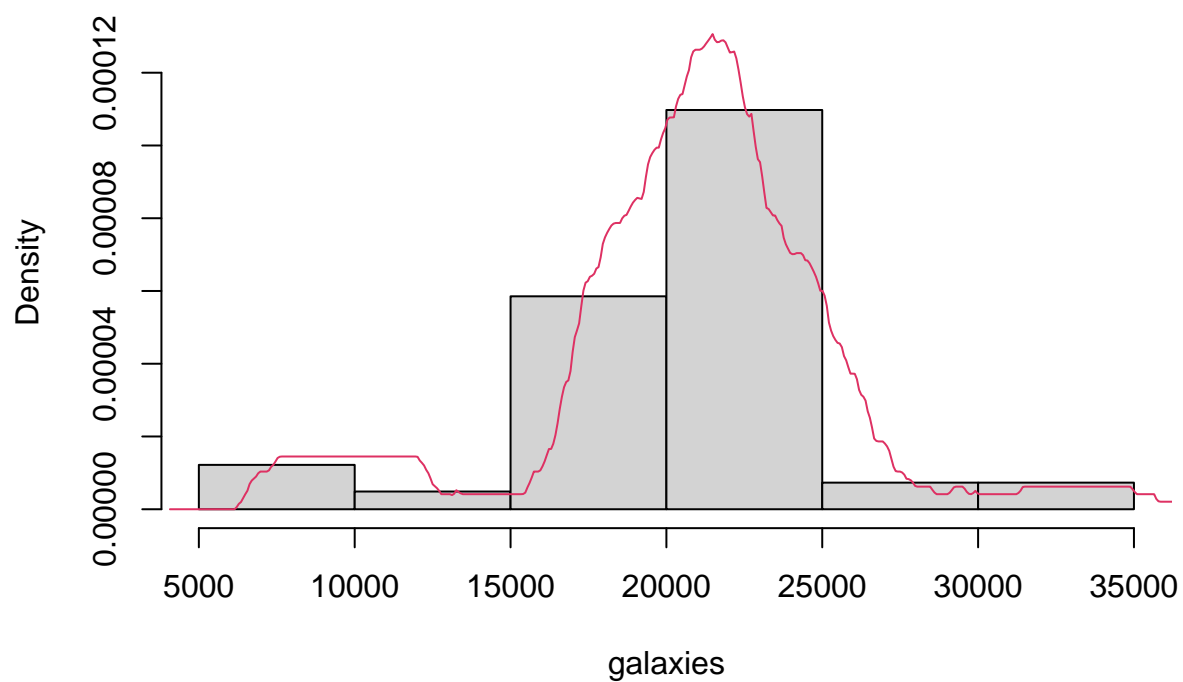
```
h2 <- 2.6 * iqr * n ^ {  
  -1 / 3  
}  
nobreaks2 <- (max(galaxies) - min(galaxies)) / h2  
hist(galaxies,  
      breaks = round(nobreaks2),  
      probability = TRUE)
```



Kernel Smoothing:

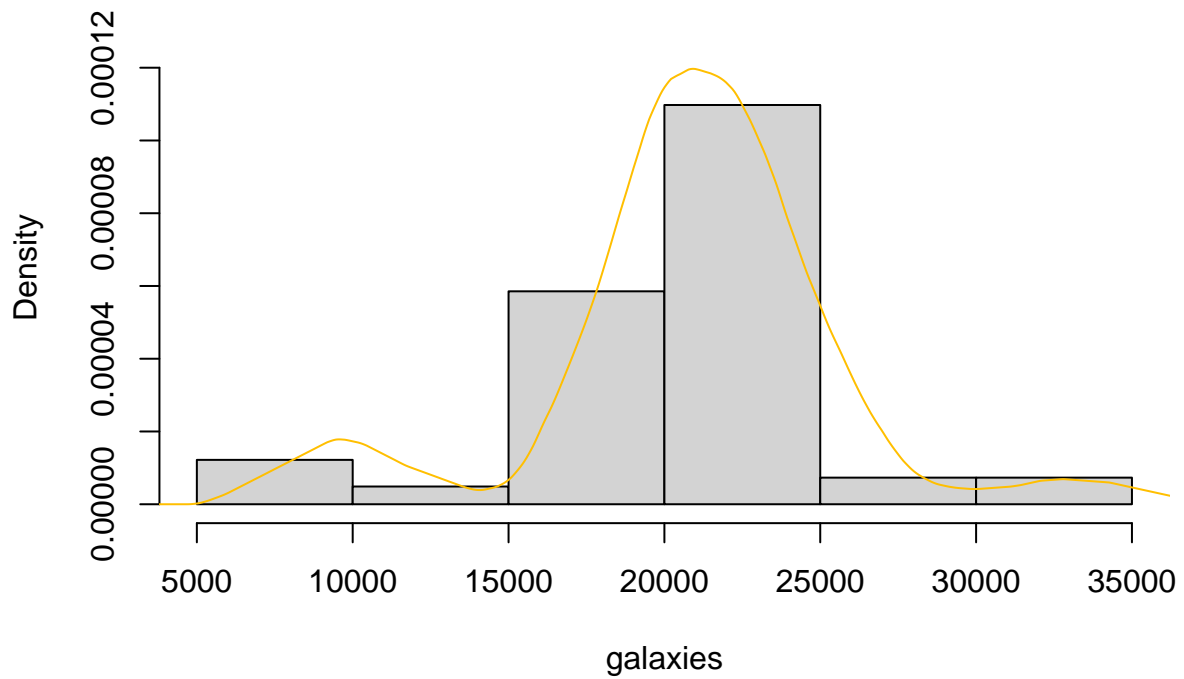
```
hist(  
  galaxies,  
  breaks = round(nobreaks),  
  probability = TRUE,  
  ylim = c(0, 13e-05),  
  main = 'Uniform Kernel with h=1700'  
)  
lines(density(galaxies, kernel = 'rectangular', bw = 1700), col = '#DE3163')
```

## Uniform Kernel with h=1700



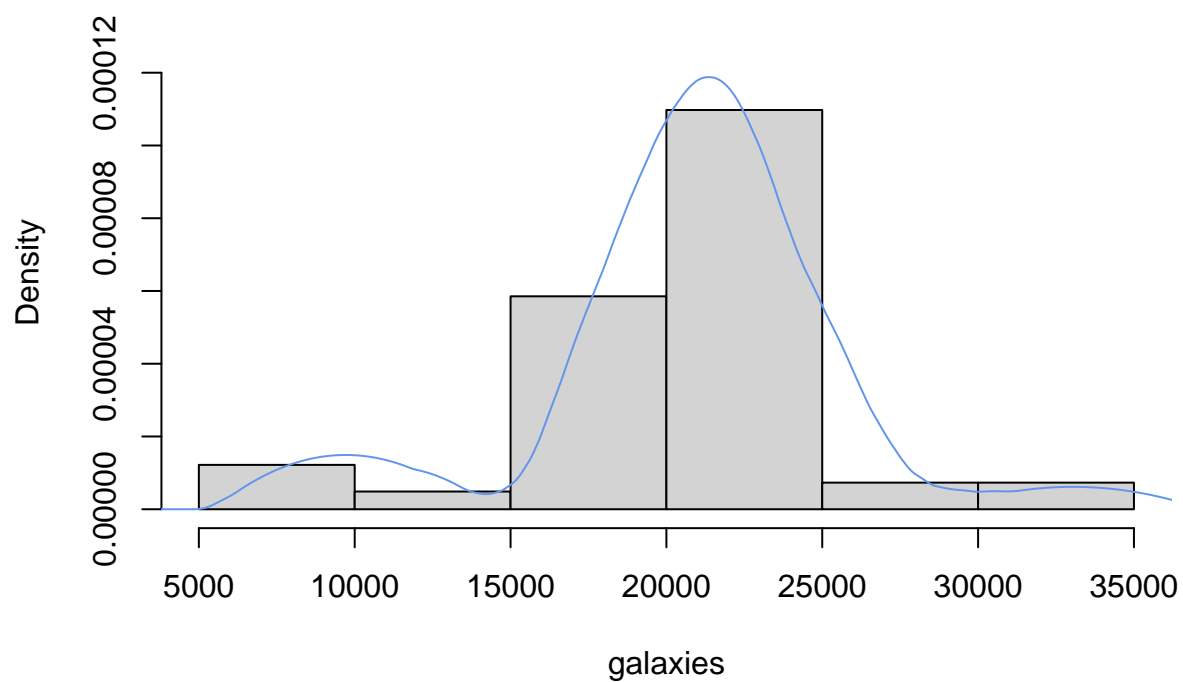
```
hist(  
  galaxies,  
  breaks = round(nobreaks),  
  probability = TRUE,  
  ylim = c(0, 13e-05),  
  main = 'Triangle Kernel with h=1800'  
)  
lines(density(galaxies, kernel = 'triangular', bw = 1800), col = '#FFBF00')
```

## Triangle Kernel with h=1800



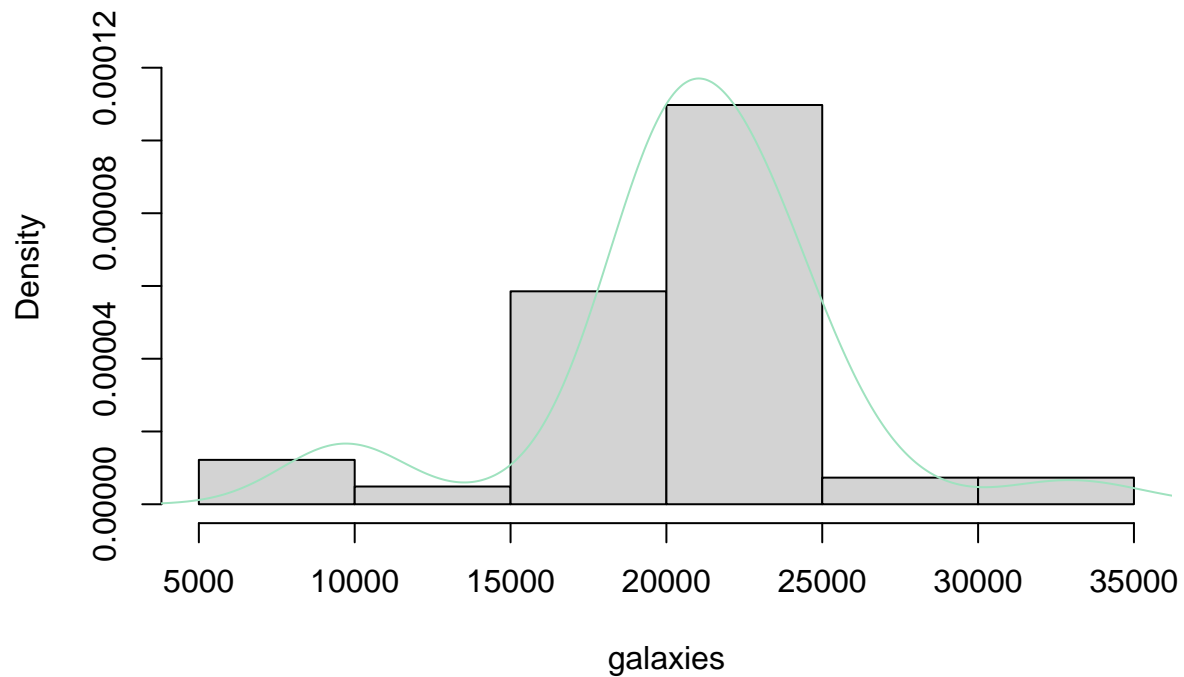
```
hist(  
  galaxies,  
  breaks = round(nobreaks),  
  probability = TRUE,  
  ylim = c(0, 13e-05),  
  main = 'Epanechnikov Kernel with h=1900'  
)  
lines(density(galaxies, kernel = 'epanechnikov', bw = 1900), col = '#6495ED')
```

## Epanechnikov Kernel with h=1900



```
hist(  
  galaxies,  
  breaks = round(nobreaks),  
  probability = TRUE,  
  ylim = c(0, 13e-05),  
  main = 'Gaussian Kernel with h=2000'  
)  
lines(density(galaxies, kernel = 'gaussian', bw = 2000), col = '#9FE2BF')
```

## Gaussian Kernel with $h=2000$



There are at least 2 peaks of the distribution of velocities. Thus, the multimodality of the distribution of velocities implies the existence of superclusters.

### 3.3

```
library(HSAUR3)
```

```
##      tools
```

```
data(foster)
attach(foster)
```

(a)

```
table(litgen, motgen)
```

```
##      motgen
## litgen A B I J
##      A 5 3 4 5
##      B 4 5 4 2
##      I 3 3 5 3
##      J 4 3 3 5
```

```
# group means
aggregate(weight, by = list(litgen, motgen), FUN = mean)
```

```
##      Group.1 Group.2      x
## 1         A      A 63.68000
## 2         B      A 52.32500
## 3         I      A 47.10000
## 4         J      A 54.35000
## 5         A      B 52.40000
## 6         B      B 60.64000
## 7         I      B 64.36667
## 8         J      B 56.10000
## 9         A      I 54.12500
## 10        B      I 53.92500
## 11        I      I 51.60000
## 12        J      I 54.53333
## 13        A      J 48.96000
## 14        B      J 45.90000
## 15        I      J 49.43333
## 16        J      J 49.06000
```

```
# group standard deviations
aggregate(weight, by = list(litgen, motgen), FUN = sd)
```

```
##      Group.1 Group.2      x
## 1         A      A  3.273683
## 2         B      A  5.533158
## 3         I      A 18.103315
## 4         J      A  5.325098
## 5         A      B  9.374433
## 6         B      B  5.647389
## 7         I      B  7.124839
## 8         J      B  3.351119
## 9         A      I  5.321889
## 10        B      I  5.114277
## 11        I      I  8.624964
## 12        J      I  8.376953
## 13        A      J  8.760594
## 14        B      J  7.636753
## 15        I      J  5.372461
## 16        J      J  5.335541
```

(b)

```
library(HH)
```

```
##      lattice
```

```
##      grid
```

```
##      latticeExtra

##
##      'latticeExtra'

## The following object is masked from 'package:ggplot2':
##
##      layer

##      multcomp

##      mvtnorm

##      survival

##      TH.data

##
##      'TH.data'

## The following object is masked from 'package:HSAUR3':
##
##      birds

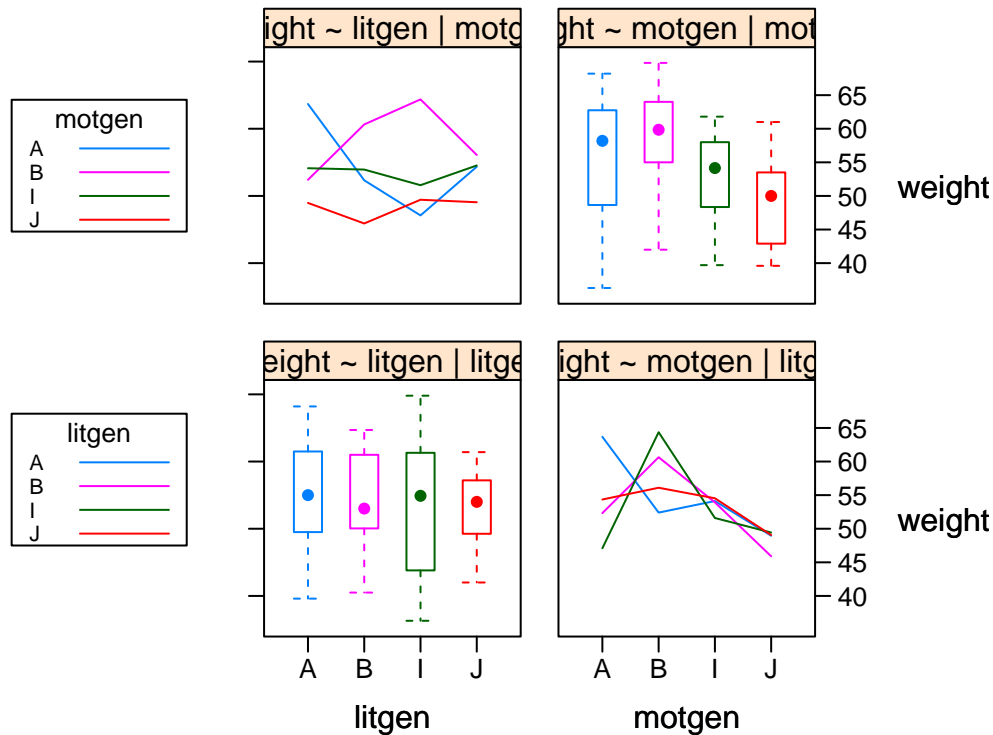
## The following object is masked from 'package:MASS':
##
##      geyser

##      gridExtra
```



```
interaction2wt(weight ~ litgen * motgen)
```

## weight: main effects and 2-way interactions



From the plot in the upper left corner, the slope of different *motgen* from one *litgen* type to another *litgen* type is different. Similarly, from the plot in the lower right corner, the slope of different *litgen* with respect to *motgen* is not the same. Therefore, there seems to exist some interaction between *litgen* and *motgen*.

(c)

```
fit1 <- aov(weight ~ litgen * motgen)
fit2 <- aov(weight ~ motgen * litgen)
fit3 <- aov(weight ~ litgen + motgen)
fit4 <- aov(weight ~ motgen + litgen)
summary(fit1)
```

```
##               Df Sum Sq Mean Sq F value    Pr(>F)
## litgen         3   60.2    20.05    0.370  0.77522
## motgen         3  775.1   258.36    4.763  0.00574 **
## litgen:motgen   9  824.1    91.56    1.688  0.12005
## Residuals     45 2440.8    54.24
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(fit2)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## motgen        3   771.6   257.20    4.742 0.00587 **
## litgen        3    63.6    21.21    0.391 0.76000
## motgen:litgen  9   824.1    91.56    1.688 0.12005
## Residuals     45  2440.8    54.24
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(fit3)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## litgen        3     60    20.05    0.332 0.80247
## motgen        3   775    258.36    4.273 0.00886 **
## Residuals     54   3265    60.46
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(fit4)
```

```
##              Df Sum Sq Mean Sq F value    Pr(>F)
## motgen        3   772    257.20    4.254 0.00905 **
## litgen        3     64    21.21    0.351 0.78870
## Residuals     54   3265    60.46
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All the results indicate that the *motgen* main effect is significant and the *litgen* main effect is not significant. For analysis with interaction term, the results indicate that there exists some interaction between *motgen* and *litgen*, but this interaction is not significant( $p=0.12005$ ).

(d)

The dependent variable is assumed to be normally distributed, and have equal variance in each group. outlier

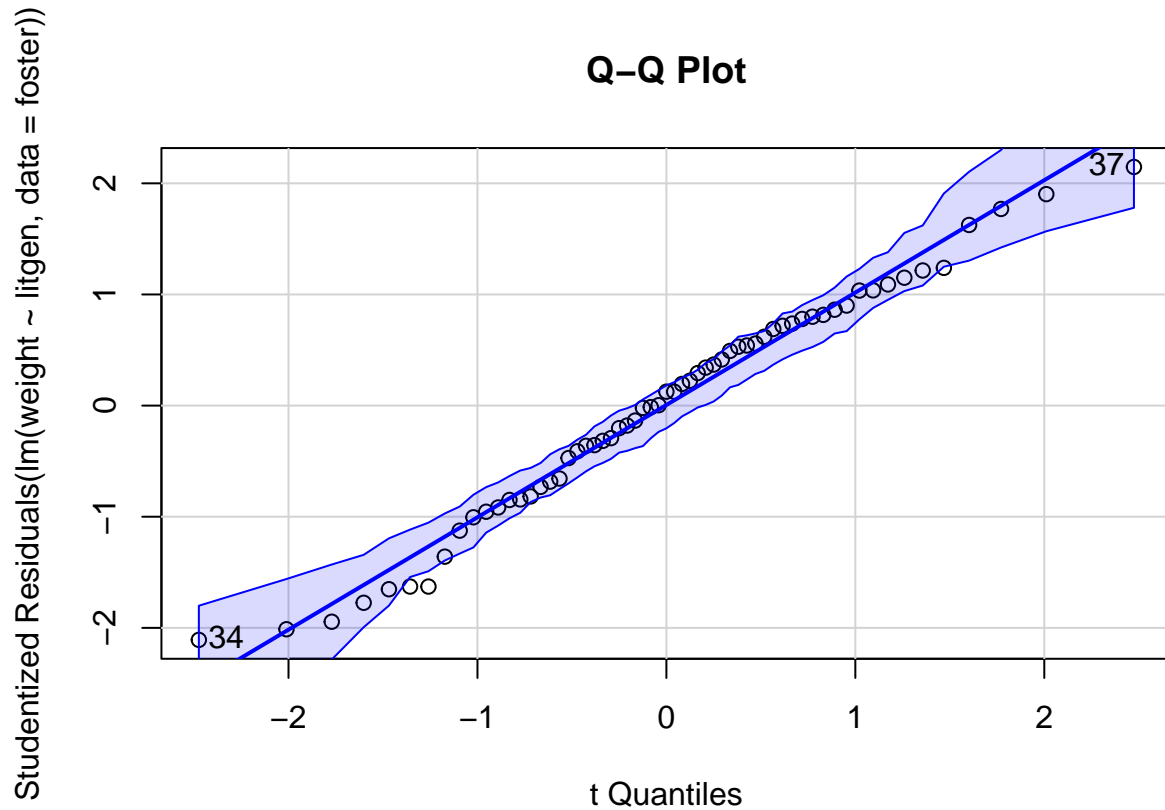
```
# Normally distributed:
library(car)
```

```
##      carData

##
##      'car'

## The following objects are masked from 'package:HH':
##
##      logit, vif
```

```
qqPlot(
  lm(weight ~ litgen, data = foster),
  simulate = TRUE,
  main = 'Q-Q Plot',
  labels = FALSE
)
```



```
## [1] 34 37
```

```
# equality of variances
bartlett.test(weight ~ litgen, data = foster)
```

```
##
## Bartlett test of homogeneity of variances
##
## data: weight by litgen
## Bartlett's K-squared = 6.1503, df = 3, p-value = 0.1045
```

```
# outlier
fit <- aov(weight ~ litgen)
outlierTest(fit)
```

```
## No Studentized residuals with Bonferroni p < 0.05
## Largest |rstudent|:
```

```
##      rstudent unadjusted p-value Bonferroni p
## 37 2.147241          0.036118          NA
```

Yes. These assumptions are satisfied.

(e)

```
library(lmPerm)
set.seed(1234)
aovobject <- aovp(weight ~ litgen * motgen, data = foster, perm = "Prob")
```

```
## [1] "Settings: unique SS "
```

```
summary(aovobject)
```

```
## Component 1 :
##           Df R Sum Sq R Mean Sq Iter Pr(Prob)
## litgen      3  27.66   9.219  114  0.9737
## motgen      3 671.74 223.913 5000 0.0084 **
## litgen:motgen 9  824.07  91.564 2158 0.1348
## Residuals   45 2440.82  54.240
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
### result in (c)
```

```
summary(fit1)
```

```
##           Df Sum Sq Mean Sq F value Pr(>F)
## litgen      3   60.2   20.05   0.370 0.77522
## motgen      3  775.1  258.36   4.763 0.00574 **
## litgen:motgen 9  824.1   91.56   1.688 0.12005
## Residuals   45 2440.8   54.24
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Compared to the result in (c), the *litgen* main effect, the *motgen* main effect and the interaction are all less significant.

```
detach(foster)
```

### 3.4

```
library(ISLR)
data(Default)
```

(a)

```
summary(Default)
```

```
## default      student      balance      income
## No :9667      No :7056      Min.   : 0.0      Min.   : 772
## Yes: 333      Yes:2944      1st Qu.: 481.7    1st Qu.:21340
##                                     Median : 823.6    Median :34553
##                                     Mean   : 835.4    Mean   :33517
##                                     3rd Qu.:1166.3    3rd Qu.:43808
##                                     Max.   :2654.3    Max.   :73554
```

```
logit_fit <-
  glm(default ~ student + balance + income,
       family = binomial(link = "logit"),
       data = Default)
summary(logit_fit)
```

```
##
## Call:
## glm(formula = default ~ student + balance + income, family = binomial(link = "logit"),
##      data = Default)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -2.4691  -0.1418  -0.0557  -0.0203   3.7383
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.087e+01  4.923e-01 -22.080  < 2e-16 ***
## studentYes  -6.468e-01  2.363e-01  -2.738  0.00619 **
## balance       5.737e-03  2.319e-04  24.738  < 2e-16 ***
## income       3.033e-06  8.203e-06   0.370  0.71152
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 2920.6  on 9999  degrees of freedom
## Residual deviance: 1571.5  on 9996  degrees of freedom
## AIC: 1579.5
##
## Number of Fisher Scoring iterations: 8
```

The estimated standard error for the estimated coefficient of studentYes is  $2.363 \times 10^{-1}$ , while the estimated standard error for the estimated coefficient of balance and income are  $2.319 \times 10^{-4}$  and  $8.203 \times 10^{-6}$  respectively.

(b)

```
boot.fn <- function(formula, data, indices) {
  d <- data[indices, ]
```

```

fit <- glm((formula),
          family = binomial(link = "logit"),
          data = d)
return(coef(fit))
}

```

(c)

```
library(boot)
```

```

##
##      'boot'

## The following object is masked from 'package:car':
##
##      logit

## The following object is masked from 'package:HH':
##
##      logit

## The following object is masked from 'package:survival':
##
##      aml

## The following object is masked from 'package:lattice':
##
##      melanoma

```

```

set.seed(1234)
results <-
  boot(
    data = Default,
    statistic = boot.fn,
    R = 1000,
    formula = default ~ student + balance + income
  )
print(results)

```

```

##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = Default, statistic = boot.fn, R = 1000, formula = default ~
##      student + balance + income)
##
##
## Bootstrap Statistics :

```

##		original	bias	std. error
## t1*	-1.086905e+01	-2.846982e-02	5.022490e-01	
## t2*	-6.467758e-01	-1.151472e-02	2.390398e-01	
## t3*	5.736505e-03	1.953943e-05	2.330627e-04	
## t4*	3.033450e-06	-1.787413e-07	8.595409e-06	

The estimated standard error for the estimated coefficient of studentYes is  $2.390398 \times 10^{-1}$ , while the estimated standard error for the estimated coefficient of balance and income are  $2.330627 \times 10^{-4}$  and  $8.595409 \times 10^{-6}$  respectively.

(d)

The standard errors obtained by the bootstrap appear to be a quite close to those obtained using the statistical formulas underlying the `glm()` function. This suggests that the data satisfies the underlying assumptions of a logistic regression model: the responses  $Y_i$  are independent random variables coming from Bernoulli distributions with probabilities  $P_i$ , and the log-odds corresponding to  $P_i$  is a linear combination of the predictors.