Assignment 11 Chapter 10 Seasonal Models

10.1 Based on quarterly data, a seasonal model of the form

$$Y_t = Y_{t-4} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$$

has been fit to a certain time series.

(a) Find the first four ψ-weights for this model.

(b) Suppose that $\theta_1 = 0.5$, $\theta_2 = -0.25$, and $\sigma_e = 1$. Find forecasts for the next four quarters if data for the last four quarters are

Quarter	I	II	III	IV
Series	25	20	25	40
Residual	2	1	2	3

(c) Find 95% prediction intervals for the forecasts in part (b).

10.2 An AR model has AR characteristic polynomial

$$(1 - 1.6x + 0.7x^2)(1 - 0.8x^{12})$$

(a) Is the model stationary?

(b) Identify the model as a certain seasonal ARIMA model.

10.4 For the seasonal model $Y_t = \Phi Y_{t-4} + e_e - \theta e_{t-1}$ with $|\Phi| < 1$, find γ_0 and ρ_k.

10.5 Identify the following as certain multiplicative seasonal ARIMA models:

(a) $Y_t = 0.5Y_{t-1} + Y_{t-4} - 0.5Y_{t-5} + e_t - 0.3e_{t-1}$.

(b) $Y_t = Y_{t-1} + Y_{t-12} + Y_{t-13} + e_t - 0.5e_{t-1} - e_{t-12} + 0.25e_{t-13}$.

10.6 Verify Equations (10.2.11) on page 232.

10.7 Suppose that the process $\{Y_t\}$ develops according to $Y_t = Y_{t-4} + e_t$ with $Y_t = e_t$ for $t = 1, 2, 3$, and 4.

(a) Find the variance function for $\{Y_t\}$.

(b) Find the autocorrelation function for $\{Y_t\}$.

(c) Identify the model for $\{Y_t\}$ as a certain seasonal ARIMA model.

10.11 The quarterly earnings per share for 1960–1980 of the U.S. company Johnson & Johnson, are saved in the file named JJ.

- **(a)** Plot the time series and also the logarithm of the series. Argue that we should transform by logs to model this series.
- **(b)** The series is clearly not stationary. Take first differences and plot that series. Does stationarity now seem reasonable?
- **(c)** Calculate and graph the sample ACF of the first differences. Interpret the results.
- **(d)** Display the plot of seasonal differences and the first differences. Interpret the plot. Recall that for quarterly data, a season is of length 4.
- **(e)** Graph and interpret the sample ACF of seasonal differences with the first differences.
- **(f)** Fit the model ARIMA$(0,1,1)\times(0,1,1)_4$, and assess the significance of the estimated coefficients.
- **(g)** Perform all of the diagnostic tests on the residuals.
- **(h)** Calculate and plot forecasts for the next two years of the series. Be sure to include forecast limits.

Deadline: Before class on Dec 7, 2021.