Chap4: 4.4-4.5 The Mixed Autoregressive Moving Average Model and Invertibility

4.10 Sketch the autocorrelation functions for each of the following ARMA models:
(a) ARMA(1,1) with $\phi = 0.7$ and $\theta = 0.4$.
(b) ARMA(1,1) with $\phi = 0.7$ and $\theta = -0.4$.

4.11 For the ARMA(1,2) model $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$, show that
(a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$.
(b) $\rho_2 = 0.8\rho_1 + 0.6\sigma_e^2/\gamma_0$.

4.22 Show that the statement "The roots of $1 - \phi_1 x - \phi_2 x^2 - \cdots - \phi_p x^p = 0$ are greater than 1 in absolute value" is equivalent to the statement "The roots of $x^p - \phi_1 x^{p-1} - \phi_2 x^{p-2} - \cdots - \phi_p = 0$ are less than 1 in absolute value." (Hint: If G is a root of one equation, is $1/G$ a root of the other?)

4. Consider the following time series: $Y_t = 0.2\,Y_{t-1} + 0.63\,Y_{t-2} + w_t + 1.2w_{t-1} + 0.35w_{t-2}$,

   w_t is a white noise process with variance σ_w^2.

   1) Write down the reduced form of this model if it can be simplified.
   2) Is this model invertible and stationary?
   3) If invertible, transform this model into MA(∞) with the representation $Y_t = w_t + \sum_{j=1}^{\infty} \psi_j w_{t-j}$.

      Please write the first four terms of ψ_j and the general term of ψ_j

Pls submit your assignments before class on (Tuesday) 12 OCT