Statistical Linear Models

Assignment 6

Hanbin Liu 11912410

1

(a)

For months after the murder in city A, $x_1 = x_2 = 1$. Hence, the expression is given by

$$E(y) = \beta_0 + \beta_1 + \beta_2 + \beta_3$$

(b)

For months before the murder in city B, $x_1 = x_2 = 0$. Hence, the expression is given by

$$E(y) = \beta_0$$

(c)

Full model:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

Reduced model:

$$E(y) = \beta_0 + \beta_1 x_1$$

The hypotheses:

$$H_0 : \beta_2 = \beta_3 = 0 \quad vs \quad H_1 : \text{at least one of } \ \beta_2, \beta_3 \neq 0$$

The test statistics:

$$F = \frac{SSR(x_2, x_1 x_2 | x_1)/2}{MSE(x_1, x_2, x_1 x_2)}$$

The rejection rule:

$$\text{reject } H_0 \text{ if } F = \frac{SSR(x_2, x_1 x_2 | x_1)/2}{MSE(x_1, x_2, x_1 x_2)} \geq F(0.05, 2, n-4),$$

where n is the sample size. □

2

(a)

The difference between pilot and mechanic may not be the same as the difference between mechanic and flight attendant. Also, it is unreasonable two assume that the difference between pilot and flight attendant is twice the difference between pilot and mechanic.

(b)

Introcude dummy variables:

$$x_1 = \begin{cases} 1, & \text{pilot} \\ 0, & \text{otherwise} \end{cases} \qquad x_2 = \begin{cases} 1, & \text{mechanic} \\ 0, & \text{otherwise} \end{cases}$$

Then, the model is given by

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$\square$

3

(a)

$$\beta_0 = \mu_1, \quad \beta_1 = \mu_2 - \mu_1, \quad \beta_2 = \mu_3 - \mu_1$$

(b)

The estimate of the difference between the population mean delinquent amounts for the upper- and lower-class groups is

$$\text{estimate of } \mu_3 - \mu_1 = \text{estimate of } \beta_2 = \hat{\beta}_2 = 198.2$$

(c)

Similarly, the estimate of the difference between the population mean delinquent amounts for the upper- and middle-class groups is

$$\text{estimate of } \mu_3 - \mu_2 = \text{estimate of } \beta_2 - \beta_1 = \hat{\beta}_2 - \hat{\beta}_1 = 117.9$$

(d)

The hypotheses:

$$H_0 : \mu_1 = \mu_2 = \mu_3 \quad vs \quad H_1 : \text{not all } \mu_i s \text{ equal}$$

or equaivalently,

$$H_0 : \beta_1 = \beta_2 = 0 \quad vs \quad H_1 : \text{at least one of } \beta_1, \beta_2 \text{ not equal to } 0$$

The test statistic: $F = 3.48 > F_{0.05,2,27} = 3.35$. Hence, we should reject H_0.

Conclusion: at least two of the means of these three groups are significantly different. $\square$