

## 简答题

1. ①  $|\frac{d\vec{v}}{dt}| = 0$  表示加速度为 0 的运动，即速度不变的运动  
为静止或匀速直线运动

②  $\frac{d|\vec{v}|}{dt} = 0$  表示速率大小不变的运动

2. 可以发生改变，若加速度矢量方向与速度矢量方向  
不共线，则速度矢量方向会发生变化，运动方向改变

3. (1) 错误 反例：匀速圆周运动

(2) 正确

(3) 错误 反例：匀速圆周运动

4. 因为自身在运动

应当根据本身运动情况

适当调整瞄准方向

# 补充习题

1. (a)  $\vec{A} = 3\hat{i} + 4\hat{j} - 4\hat{k}$

令  $\vec{B} = a\hat{i} + b\hat{j}$ , 若  $\vec{B} \perp \vec{A}$ , 则  $\vec{B} \cdot \vec{A} = 0$

则  $3a + 4b = 0$

由于  $\vec{B}$  单位向量, 则  $|\vec{B}| = 1$

即  $\sqrt{a^2 + b^2} = 1$ , 即  $a^2 + b^2 = 1$

取  $a = -\frac{4}{5}$ ,  $b = \frac{3}{5}$ . 则  $\vec{B} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

(b) 令  $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$  ( $|\vec{c}| = 1$ )

$$\vec{c} \perp \vec{A}, \vec{c} \perp \vec{B} \Rightarrow \begin{cases} 3x + 4y - 4z = 0 \\ -\frac{4}{5}x + \frac{3}{5}y = 0 \Rightarrow -4x + 3y = 0 \end{cases}$$

令  $x = 3$ , 则  $y = 4$ ,  $z = \frac{25}{4}$

即  $\vec{c} = 3\hat{i} + 4\hat{j} + \frac{25}{4}\hat{k}$

(c)  $\vec{A} \perp \vec{B}, \vec{c}$  所在平面  $\alpha \Leftrightarrow \forall \vec{D} \in \alpha, \vec{A} \perp \vec{D}$

由于  $\vec{D} \in \alpha$ , 则  $\vec{D} = m\vec{B} + n\vec{c}$

则  $\vec{A} \cdot \vec{D} = m\vec{B} \cdot \vec{A} + n\vec{c} \cdot \vec{A} = 0$ , 即  $\vec{A} \perp \vec{D}$

可得  $\vec{A} \perp \vec{B}, \vec{c}$  所在平面  $\alpha$

$$2. \vec{a} = (1, -2, 1), \vec{b} = (1, -1, 3), \vec{c} = (2, 5, -3)$$

$$\textcircled{1} \quad \vec{a} \times \vec{b} = (1, -2, 1) \times (1, -1, 3) = (-2 \times 3 - 1 \times (-1), 1 \times 1 - 3 \times 1, 1 \times (-1)) \\ = (-5, -2, 1)$$

$$\textcircled{2} \quad (\vec{a} \times \vec{b}) \cdot \vec{c} = (-5, -2, 1) \cdot (2, 5, -3) = -10 - 10 - 3 = -23$$

$$\textcircled{3} \quad (\vec{b} \times \vec{c}) = (1, -1, 3) \times (2, 5, -3) \\ = (-1 \times (-3) - 3 \times 5, 3 \times 2 - (-3) \times 1, 1 \times 5 - 2 \times (-1)) \\ = (-12, 9, 7)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (1, -2, 1) \times (-12, 9, 7) \\ = (-2 \times 7 - 1 \times 9, 1 \times (-12) - 7 \times 1, 1 \times 9 - (-12) \times (-2)) \\ = (-23, -19, -15)$$

$$3. (a) y' = 24x^2 + 1$$

$$(b) y = \frac{(x^2 - 1) \sin x}{\cos x}$$

$$\begin{aligned} y' &= \frac{[2x \sin x + \cos x (x^2 - 1)] \cos x + \sin^2 x (x^2 - 1)}{\cos^2 x} \\ &= \frac{2x \sin x (\cos x + x^2 - 1)}{\cos^2 x} = \frac{x \sin 2x + x^2 - 1}{\cos^2 x} \end{aligned}$$

$$(c) y = \frac{9x + x^3}{5x + 6} = \frac{x(9+x^2)}{5x+6}$$

$$\begin{aligned} y' &= \frac{[9+x+x^3](5x+6) - 5x(9+x^2)}{(5x+6)^2} \\ &= \frac{5x^2 + 12x + 54}{(5x+6)^2} \end{aligned}$$

$$(d) y = x \cos x + \frac{\sin x}{x}$$

$$y' = \cos x - x \sin x + \frac{x \cos x - \sin x}{x^2}$$

$$4. (a) \int (3x^3 + 8\ln x + \frac{5}{x}) dx$$

$$= \frac{3}{4}x^4 - 8x + 5\ln|x| + C$$

$$(b) \int \sqrt{a^2 - x^2} dx$$

$$\because x = a\sin\theta, \text{ 则 } \sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2\theta} = a\cos\theta, \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\text{则 } \int \sqrt{a^2 - x^2} = \int a\cos\theta dx = \int a\cos\theta da\sin\theta$$

$$= \int a\cos\theta \cdot a\cos\theta \cdot d\theta = a^2 \int \cos^2\theta \cdot d\theta$$

$$= a^2 \int \left( \frac{1 + \cos 2\theta}{2} \right) d\theta$$

$$= \frac{1}{2}a^2 \int (1 + \cos 2\theta) d\theta$$

$$= \frac{1}{2}a^2 \int d(\theta + \frac{1}{2}\sin 2\theta)$$

$$= \frac{1}{2}a^2 (\theta + \frac{1}{2}\sin 2\theta) + C$$

$$\text{故 } \int \sqrt{a^2 - x^2} dx$$

$$= \frac{1}{2}a^2 \left[ \arcsin\left(\frac{x}{a}\right) + \frac{x\sqrt{1-x^2}}{a^2} \right] + C$$

$$\therefore x = a\sin\theta$$

$$\therefore \frac{x}{a} = \sin\theta$$

$$\therefore \theta = \arcsin\left(\frac{x}{a}\right), \cos\theta = \sqrt{1 - \frac{x^2}{a^2}} = \frac{\sqrt{1-x^2}}{a}$$

$$\therefore \frac{1}{2}\sin 2\theta = \frac{x\sqrt{1-x^2}}{a^2}$$

5.  $y = C_1 \cos nt + C_2 \sin nt$  为  $y'' + n^2 y = 0$  的解

证明：易得. 方程特解  $y_0 = 0$

令  $y(t) = Ce^{\lambda t}$  ( $C$  为任意常数)

则  $y'' = \lambda^2 Ce^{\lambda t}$   $y = Ce^{\lambda t}$

代入，得：  $\lambda^2 Ce^{\lambda t} + n^2 Ce^{\lambda t} = 0$

即  $\lambda^2 + n^2 = 0 \Rightarrow \lambda = \pm \sqrt{-n^2} = \pm ni$

则  $y = C_3 e^{nit} + C_4 e^{-nit}$

由欧拉公式，得

$$\begin{aligned} y &= C_3 (\cos nt + i \sin nt) + C_4 (\cos nt - i \sin nt) \\ &= C_1 \cos nt + C_2 \sin nt \end{aligned}$$

( $C_1$  满足  $C_1 = C_3 + C_4$ ,

$C_2$  满足  $C_2 = (C_3 - C_4)i$ )

证毕

6. (a) 由題意，得：當  $0 \leq t \leq T$  時：

$$a(t) = (a_m/2)(1 - \cos(2\pi t/T)) \quad (0 \leq t \leq T)$$

$$dV(t) = a(t) \cdot dt \Rightarrow V(t) = \int a(t) \cdot dt = \int d[(a_m/2)$$

$$(t - \frac{1}{2\pi} \sin(2\pi t/T))] = (a_m/2)(t - \frac{1}{2\pi} \sin(2\pi t/T)) + C$$

而  $V(0) = 0$ ，則  $C = 0$ ，可得： $V(t) = (a_m/2)(t - \frac{1}{2\pi} \sin(2\pi t/T))$   
 $(0 \leq t \leq T)$

當  $T \leq t \leq 2T$  時：

$$a(t) = -(a_m/2)(1 - \cos(2\pi t/T))$$

$$dV(t) = a(t) \cdot dt \Rightarrow V(t) = \int a(t) \cdot dt = \int d[-(a_m/2)$$

$$(t - \frac{1}{2\pi} \sin(2\pi t/T))] = -(a_m/2)(t - \frac{1}{2\pi} \sin(2\pi t/T)) + C$$

而  $V(T) = (a_m/2)T = -(a_m/2)T + C$

R.I.  $C = a_m \cdot T \Rightarrow V(t) = -(a_m/2)(t - \frac{1}{2\pi} \sin(2\pi t/T))$   
+  $a_m \cdot T$

總上：當  $0 \leq t \leq T$  時： $a(t) \geq 0$

當  $T \leq t \leq 2T$  時： $a(t) \leq 0$

R.I.  $V_{max} = V(T) = \frac{a_m}{2}T$

(b) 当  $t \ll T$  时：

由 Taylor 展开 相关知识，得：

$$\begin{aligned} V(t) &= (a_m/2) \left( t - \frac{T}{2\pi} \sin(2\pi t/T) \right) \\ &\approx (a_m/2) \left[ t - \frac{T}{2\pi} \left( \frac{2\pi t}{T} - \frac{1}{6} \cdot \left( \frac{2\pi t}{T} \right)^3 \right) \right] \\ &= (a_m/2) \left[ \frac{T}{2\pi} \cdot \frac{8\pi^3 t^3}{6T^3} \right] \\ &= \frac{\pi^2 t^3 a_m}{3T^2} \quad (t \ll T) \end{aligned}$$

(C) 由題意得：

$0 \leq t \leq T$  時：

$$V(t) = (am/2)(t - \frac{T}{2\pi} \sin(2\pi t/T))$$

$$S(t) = \int_0^t V(t') \cdot dt' = \int_0^t d(am/2)(\frac{1}{2}t'^2 + \frac{T^2}{4\pi^2} \cos(2\pi t'/T))$$

$$= \frac{am}{2} \cdot (\frac{1}{2}t^2 + \frac{T^2}{4\pi^2} \cos(2\pi t/T)) - \frac{am}{2} \cdot \frac{T^2}{4\pi^2}$$

$$S(T) = \frac{am}{4}T^2$$

觀察，得：

$$\text{若 } 0 \leq t \leq T, \text{ 則 } a(t) = (am/2)(1 - \cos(2\pi t/T))$$

$$a(2T-t) = -(am/2)(1 - \cos(2\pi(2T-t)/T))$$

$$= -(am/2)(1 - \cos(4\pi - \frac{2\pi t}{T}))$$

$$= -(am/2)(1 - \cos(\frac{2\pi t}{T}))$$

$$= -a(t)$$

$$\text{則 } V(t) = V(2T-t)$$

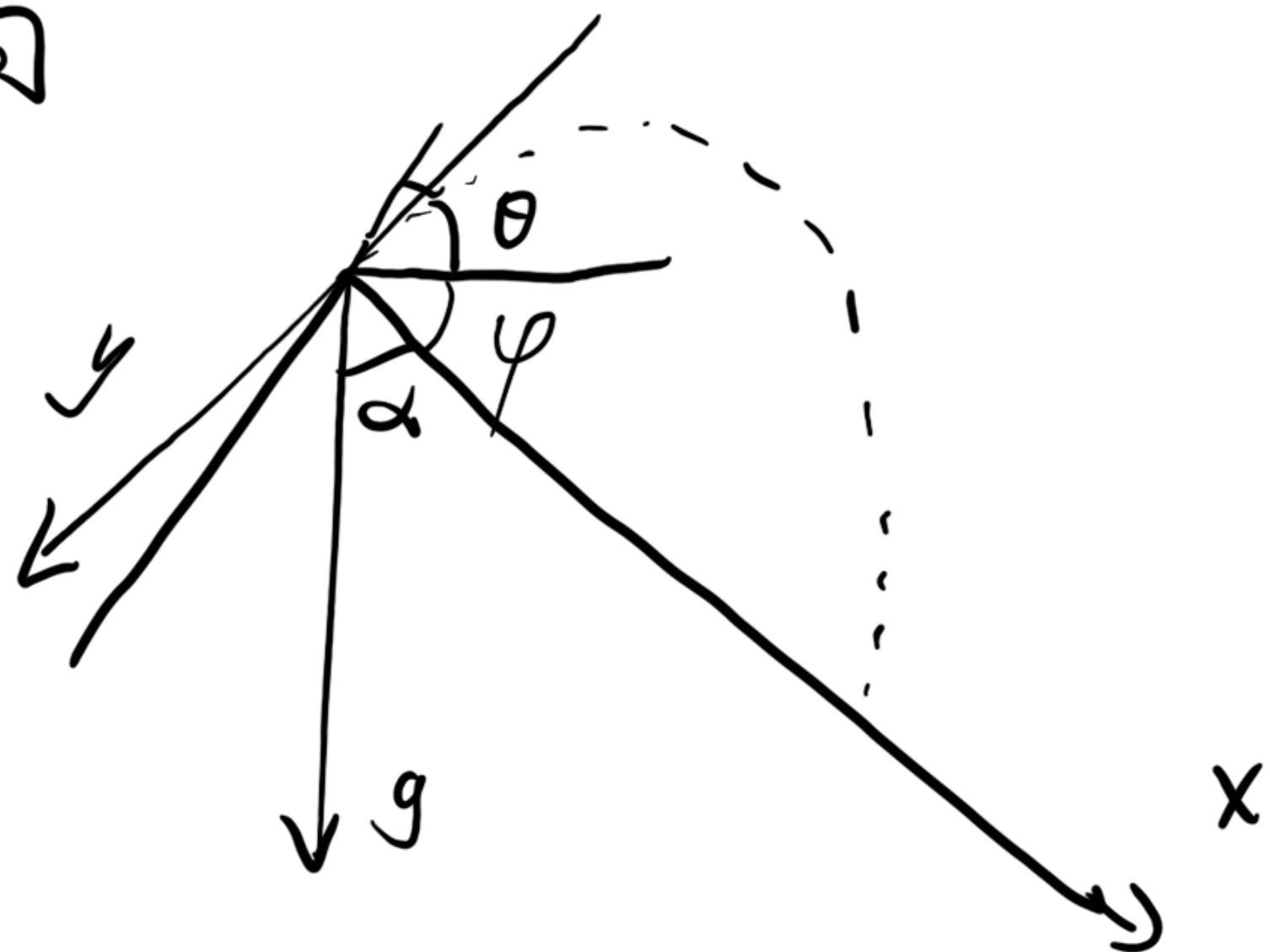
↓

$$S(2T) = 2S(T) = \frac{am}{2}T^2 = D$$

$$T = \sqrt{\frac{2D}{am}} \Rightarrow \text{故有因} 2T = 2\sqrt{\frac{2D}{am}}$$

7. 由题意得

令沿斜面向下为正方向，垂直斜面向下也为正方向



$$a_x = g \cdot \cos \alpha = g \cdot \cos\left(\frac{\pi}{2} - \varphi\right) = g \sin \varphi$$

$$a_y = g \cos \varphi$$

初速度为  $V_0$

$$V_{x0} = -\cos(\pi - (\theta + \varphi)) \cdot V_0 = \cos(\theta + \varphi) \cdot V_0$$

$$V_{y0} = -\sin(\theta + \varphi) \cdot V_0$$

$$(Rv) V_x(t) = \cos(\theta + \varphi) \cdot V_0 + g \sin \varphi t$$

$$V_y(t) = -\sin(\theta + \varphi) \cdot V_0 + g \cos \varphi t$$

$$(Rv) x(t) = \int_0^t V_x(t) \cdot dt = \int_0^t d \left( \cos(\theta + \varphi) V_0 t + \frac{1}{2} g \sin \varphi t^2 \right)$$

$$= \cos(\theta + \varphi) V_0 t + \frac{1}{2} g \sin \varphi t^2$$

$$y(t) = \int_0^t V_y(t) \cdot dt = \int_0^t d \left( -\sin(\theta + \varphi) V_0 t + \frac{1}{2} g \cos \varphi t^2 \right)$$

$$= -\sin(\theta + \varphi) V_0 t + \frac{1}{2} g \cos \varphi t^2$$

当落回斜面时：

$$y(t_1) = -\sin(\theta + \varphi)V_0 t_1 + \frac{1}{2}g \cos \varphi t_1^2 = 0$$

$$t_1 = \frac{2 \sin(\theta + \varphi) V_0}{g \cos \varphi}$$

$$\begin{aligned} \text{此时: } X(t_1) &= (\cos(\theta + \varphi) V_0 t_1 + \frac{1}{2} g \sin \varphi t_1^2 \\ &= \frac{2 \sin(\theta + \varphi) (\cos(\theta + \varphi) V_0)^2}{g \cos \varphi} + \frac{4 \sin^2(\theta + \varphi) V_0^2}{g^2 \cos^2 \varphi} \cdot \frac{1}{2} g \sin \varphi \\ &= \frac{\sin(2\theta + 2\varphi) V_0^2}{g \cos \varphi} + \frac{2 \sin^2(\theta + \varphi) V_0^2 \sin \varphi}{g \cos^2 \varphi} \\ &= \frac{2 \sin(\theta + \varphi) V_0^2 (\cos(\theta + \varphi) + \sin(\theta + \varphi) \cdot \sin \varphi)}{g \cos \varphi} \\ &= \frac{V_0^2 [\sin(2\theta + \varphi) + \sin^2 \varphi]}{g \cos^2 \varphi} \end{aligned}$$

当  $2\theta + \varphi = \frac{\pi}{2}$  时，即  $\theta = \frac{\pi}{4} - \frac{\varphi}{2}$

$$X_{\max} = \frac{V_0^2 + \sin^2 \varphi V_0^2}{g \cos^2 \varphi}$$

8.

(a) 当  $\theta = 0$  时：

$$r = A - \sqrt{A^2 - B^2} = \frac{r_0}{e+1}$$

当  $\theta = \frac{\pi}{2}$  时：

$$r = A + \sqrt{A^2 - B^2} = \frac{r_0}{1-e}$$

则  $\frac{1-e}{1+e} = \frac{A - \sqrt{A^2 - B^2}}{A + \sqrt{A^2 - B^2}} \Rightarrow e = \frac{\sqrt{A^2 - B^2}}{A}$

$$\text{故 } r_0 = \frac{B^2}{A}$$

(b)  $\theta = wt \Rightarrow r = \frac{r_0}{1+e \cos \theta}$

$$\vec{V}_\theta = r \cdot \dot{\theta} \cdot \hat{\theta} = \frac{r_0}{1+e \cos \theta} \cdot w \cdot \hat{\theta} = \frac{r_0 w}{1+e \cos \theta} \cdot \hat{\theta}$$

$$= \frac{B^2 w}{A(1 + \frac{\sqrt{A^2 - B^2}}{A} \cos \theta)} = \frac{B^2 w}{A + \sqrt{A^2 - B^2} \cos \theta}$$

而  $\vec{a}_\theta = (2r\dot{\theta} + r\ddot{\theta})\hat{\theta}$

则  $\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{e \sin \theta r_0 w}{(1+e \cos \theta)^2}$

$$\vec{a}_\theta = (2r\dot{\theta} + r\ddot{\theta})\hat{\theta} = \frac{2w^2 e \sin \theta r_0}{(1+e \cos \theta)^2} \cdot \hat{\theta}$$

$$= \frac{2w^2 \frac{\sqrt{A^2 - B^2}}{A} \cdot \sin \theta \cdot \frac{B^2}{A}}{\left(\frac{A + \sqrt{A^2 - B^2} \cos \theta}{A}\right)^2} \hat{\theta}$$

$$\hat{\theta} = \frac{2w^2 B^2 \sin \theta \sqrt{A^2 - B^2}}{(A + \sqrt{A^2 - B^2} \cos \theta)^2} \hat{\theta}$$

$$9. r = r_0 e^{a\theta}$$

$$V_r = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\therefore \theta = \omega t$$

$$\text{即 } V_r = r_0 \cdot e^{a\theta} \cdot \omega = a r \omega$$

$$V_\theta = r \cdot \dot{\theta} = r \omega$$

↓

$$V_{\text{合}} = \sqrt{V_r^2 + V_\theta^2} = \sqrt{a^2 + 1} r \omega$$

可得：

$$a\theta = 2 \frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \frac{d^2\theta}{(dt)^2}$$

$$= 2 \cdot \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{(dt)^2}$$

$$= 2 \cdot r_0 a \cdot e^{a\theta} \cdot \omega^2 + 0$$

$$= 2 a \omega^2 r$$

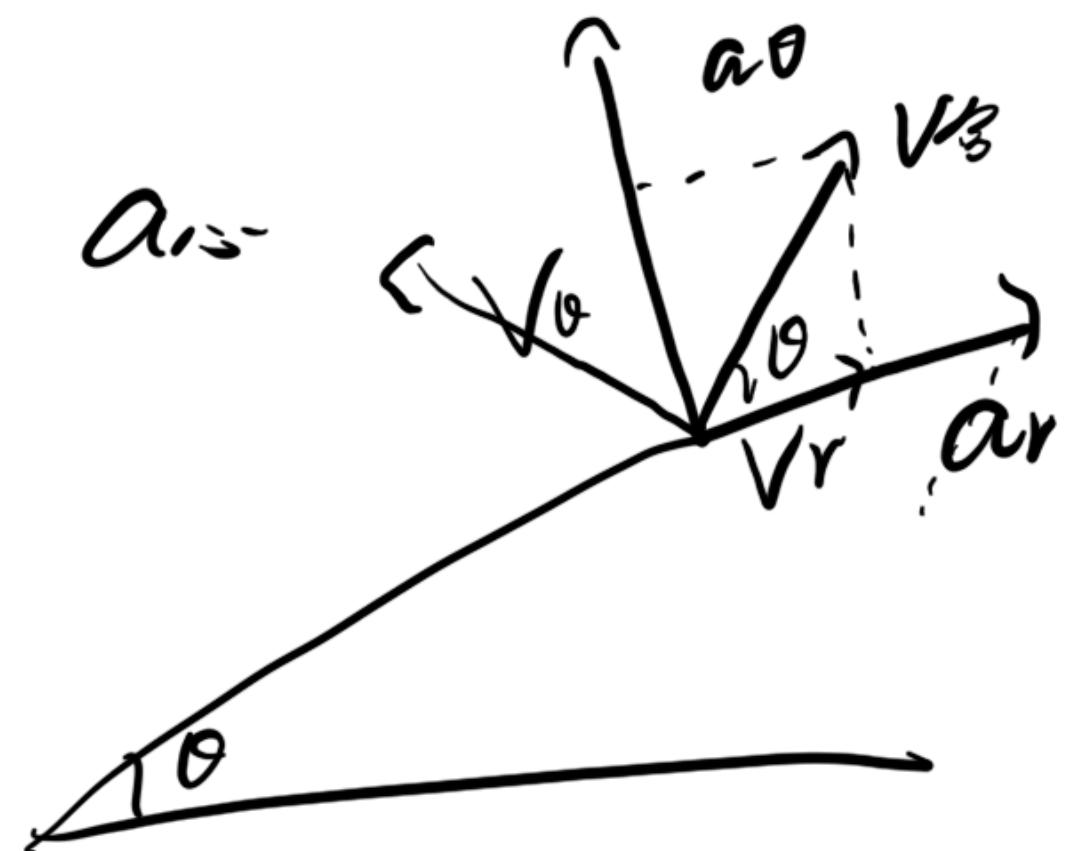
$$ar = \frac{d^2r}{(dt)^2} - r \left( \frac{d\theta}{dt} \right)^2 = \frac{d^2r}{(d\theta)^2} \cdot \frac{(d\theta)^2}{(dt)^2} - r \left( \frac{d\theta}{dt} \right)^2$$

$$= a^2 r \omega^2 - r \omega^2$$

$$= (a^2 - 1) r \omega^2$$

$$\text{如图: } \sin\theta = \frac{V_\theta}{V_{\text{合}}} = \frac{1}{\sqrt{a^2 + 1}}, \cos\theta = \frac{a}{\sqrt{a^2 + 1}}$$

$$\begin{aligned} \text{即 } a\theta &= ar \cdot \sin\theta - a\theta \cdot \cos\theta = \frac{a^2 - 1}{\sqrt{a^2 + 1}} \cdot r \omega^2 - \frac{2a^2}{\sqrt{a^2 + 1}} r \omega^2 \\ &= \frac{-a^2 - 1}{\sqrt{a^2 + 1}} r \omega^2 = -\sqrt{a^2 + 1} r \omega^2 \end{aligned}$$



则曲率半径

$$\begin{aligned} \rho &= \frac{V_{\text{合}}^2}{1 a \sin 1} \\ &= \frac{(a^2 + 1) \omega^2 r}{\sqrt{a^2 + 1} r \omega^2} \\ &= \sqrt{a^2 + 1} \cdot r \end{aligned}$$

教材：

1-1 答案：(1) 易得，无穷多次

(2)

$$t = \frac{d}{V_p + V_u} = \frac{60\text{km}}{60\text{km/h}} = 1\text{h}$$

$$S_B = V_S t = 60 \text{ km}$$

1.3

$$\text{Bsp: (1) } x = 10t^2 + 6$$

$$V(t) = \dot{x}(t) = 20t$$

3.00 s ~ 3.10 s:

$$\bar{V}_1 = \frac{V_1 + V_2}{2} = \frac{60 \text{ m/s} + 62 \text{ m/s}}{2} = 61 \text{ m/s}$$

3.00 ~ 3.01 s:

$$\bar{V}_2 = \frac{V_1 + V_3}{2} = \frac{60 \text{ m/s} + 60.2 \text{ m/s}}{2} = 60.1 \text{ m/s}$$

3.000 ~ 3.001 s:

$$\bar{V}_3 = \frac{V_1 + V_4}{2} = \frac{60 \text{ m/s} + 60.02 \text{ m/s}}{2} = 60.01 \text{ m/s}$$

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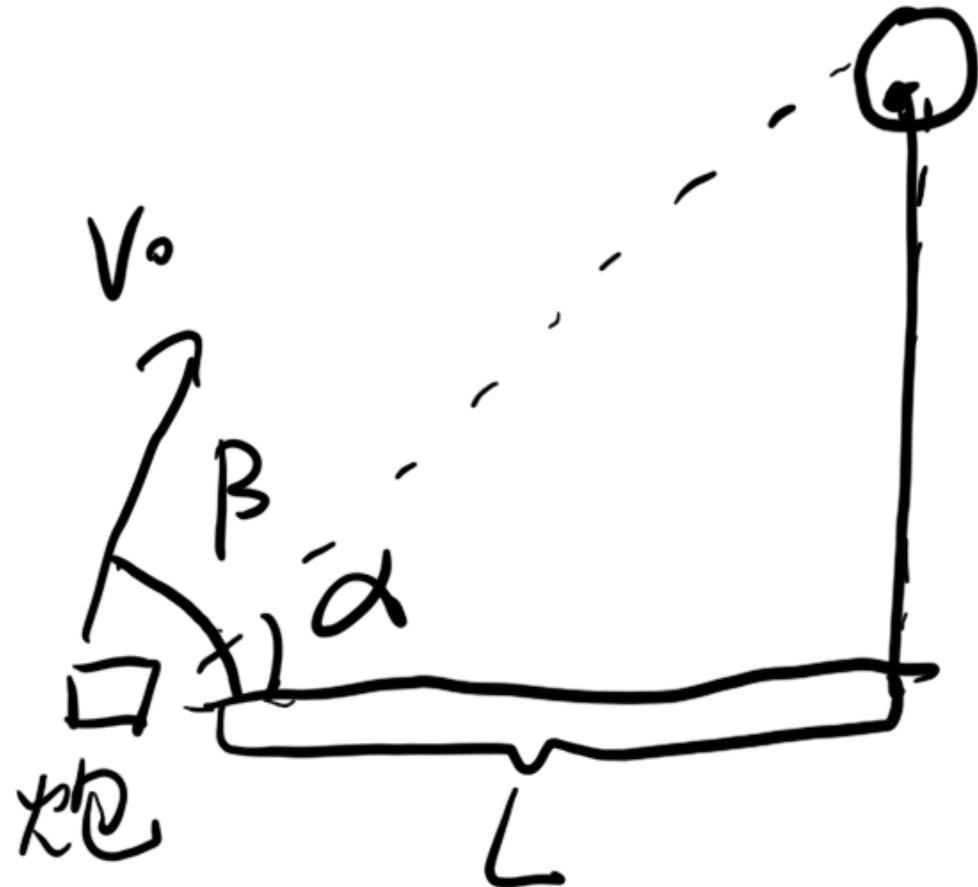
$$V_1 = (3 \times 20) \text{ m/s} = 60 \text{ m/s}$$

13)

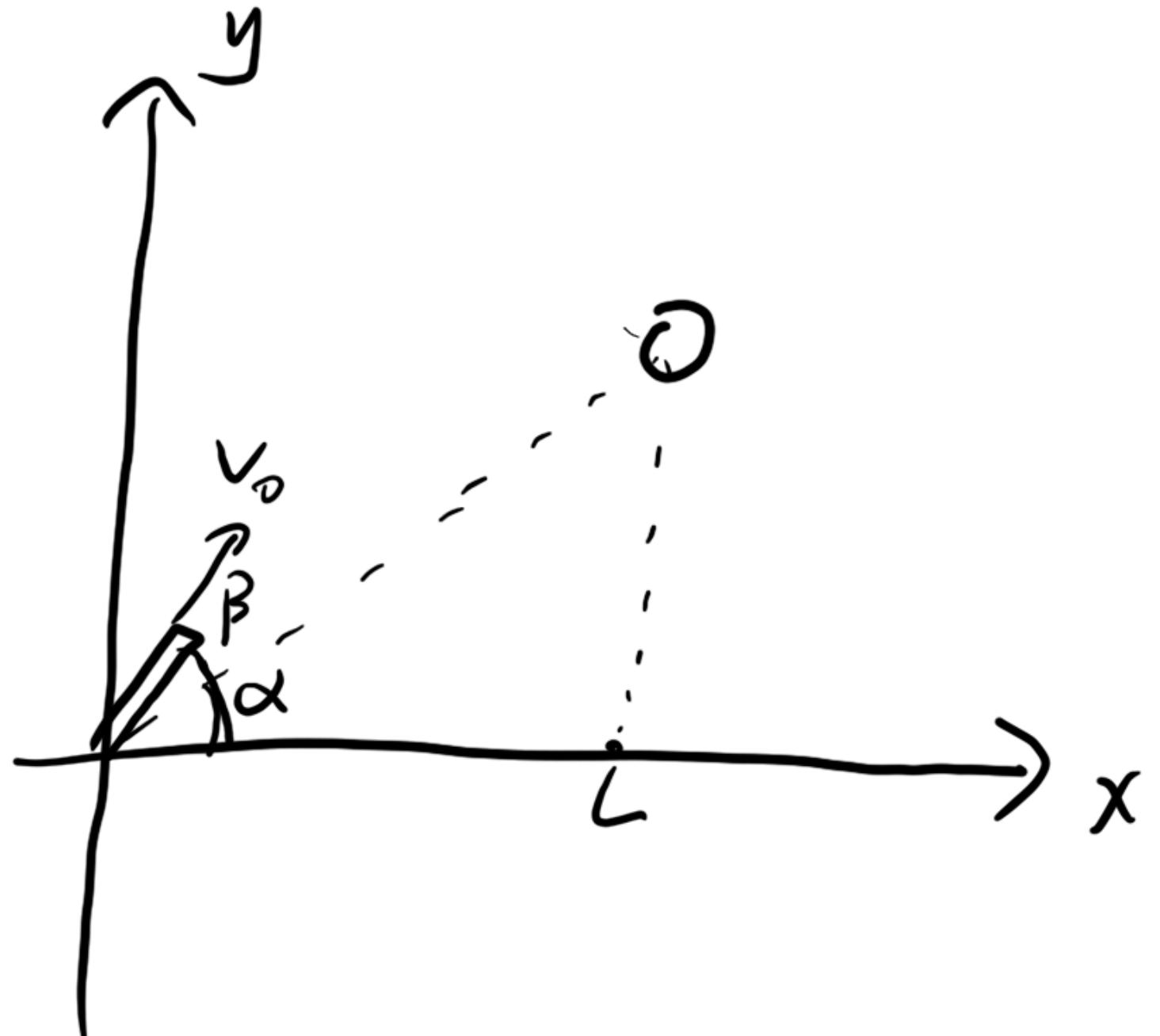
$$V(t) = \frac{dX(t)}{dt} = \dot{x}(t) = 20t$$

$$a(t) = \frac{d(V(t))}{dt} = \ddot{v}(t) = 20$$

1.25



$\Rightarrow$



由题意得：

$$a_x = 0, \text{ 则 } V_x = V_0 \cos \beta.$$

$$x(t) = V_0 \cos \beta t$$

$$a_y = -g, \text{ 则 } V_y = V_0 \sin \beta - gt$$

$$y(t) = \int V_y \cdot dt = V_0 \sin \beta t - \frac{1}{2} g t^2$$

当击中靶时：

$$x(t_0) = V_0 \cos \beta t_0 = L \Rightarrow t_0 = \frac{L}{V_0 \cos \beta}$$

$$y(t_0) = \tan \beta L - \frac{1}{2} g \frac{L^2}{V_0^2 \cos^2 \beta} = \tan \alpha \cdot L$$

$$\text{则 } \tan \beta - \frac{gL}{2V_0^2 \cos^2 \beta} = \tan \alpha$$

$$\text{则 } V_0 = \sqrt{\frac{gL \cos \alpha}{2 \cos^2 \beta \sin(\beta - \alpha)}}$$

1.31

(1) 由题意得

$$\vec{a} = \vec{a}_t + \vec{a}_R$$

而  $\langle \vec{a}, \vec{a}_R \rangle = 45^\circ$ ,  $\vec{a}_t \perp \vec{a}_R$

由几何关系得

$$|\vec{a}| = \sqrt{2} |\vec{a}_t| = \sqrt{2} |\vec{a}_R|$$

则  $V(t) = a_t \cdot t = (3t) \text{ m/s}$

$$a_R = \frac{\overset{\circ}{V(t)}}{R} = \frac{9t^2}{R} = (3t^2) \text{ m/s}^2$$

则  $3t^2 = 3 \text{ m/s}^2 \Rightarrow t = 1 \text{ s}$

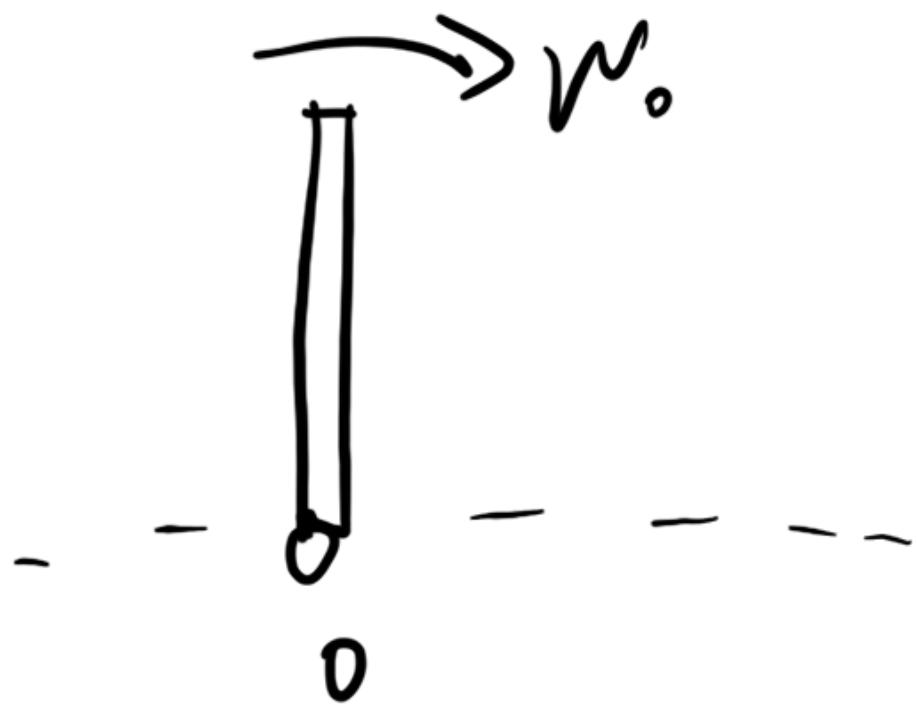
(2)

对物体本身建立自然坐标系

则  $s(t) = \int v(t) \cdot dt = \int d\left(\frac{3}{2}t^2\right) = \frac{3}{2}t^2$

当  $t=1 \text{ s}$  时,  $s(1) = \frac{3}{2} \text{ m}$

1. 33



$$\vec{V}_r = a_0 t \hat{r}$$

$$\vec{V}_\theta = w_0 r \hat{\theta}$$

$$\text{而 } r = \frac{1}{2} a_0 t^2$$

$$\text{则 } \vec{V}_\theta = \frac{1}{2} a_0 w_0 t^2 \hat{\theta}$$

$$\text{所以 } \vec{V}_{\text{总}} = \vec{V}_\theta + \vec{V}_r = \frac{1}{2} a_0 w_0 t^2 \hat{\theta} + a_0 t \hat{r}$$

$$\vec{a}_{\text{总}} = \frac{d\vec{V}_{\text{总}}}{dt} = \frac{1}{2} a_0 w_0 \cdot 2t \cdot \hat{\theta} - \frac{1}{2} a_0 w_0 t^2 \hat{r} w_0$$

$$+ a_0 \hat{r} + a_0 t \hat{\theta} w_0$$

$$= 2a_0 t w_0 \hat{\theta} + (a_0 - \frac{1}{2} a_0 w_0^2 t^2) \hat{r}$$