Problem Set 1

1. 简答题

1. $\left|\frac{d\vec{v}}{dt}\right|=0$ 表示加速度大小为0，速度的大小和方向均不变，即静止或匀速直线运动

$\frac{d|\vec{v}|}{dt}=0$ 表示速度大小不变，而方向可能改变

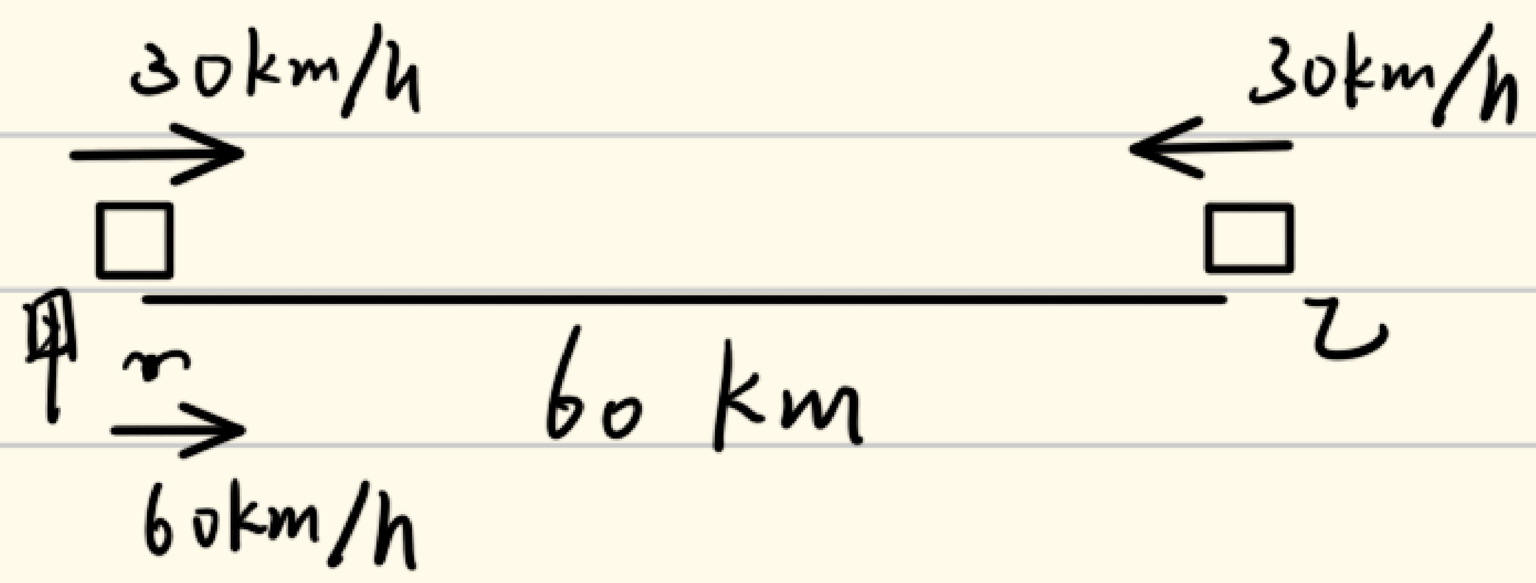
2. 可以，当物体的加速度方向与初速度方向不共线时，物体运动方向即会改变

3. (1) 错误，如匀速圆周运动
(2) 正确
(3) 错误，如匀速圆周运动

4. 因为篮球在出手前已经具有一定初速度，应该在瞄准篮所需的出手速度上减去初速度再出手

2. 教材习题

1.1



(1) 无穷多次

(2) $t=\frac{x}{v_{甲乙}}=1h$，$S=v_0\cdot t=60km$

1.3 (1) $t_1=3.0s$时，$x_1=96m$

$t_2=3.10s$时，$x_2=102.1m$

$t_3=3.01s$时，$x_3=96.601m$

$t_4=3.001s$时，$x_4=96.06001$

$\bar{v}_1=\frac{x_2-x_1}{t_2-t_1}=61m/s$ $\quad \bar{v}_2=\frac{x_3-x_1}{t_3-t_1}=60.1m/s$

$\bar{v}_3=\frac{x_4-x_1}{t_4-t_1}=60.01m/s$ $\quad \vec{v}_1,\vec{v}_2,\vec{v}_3$ 方向均沿x正向

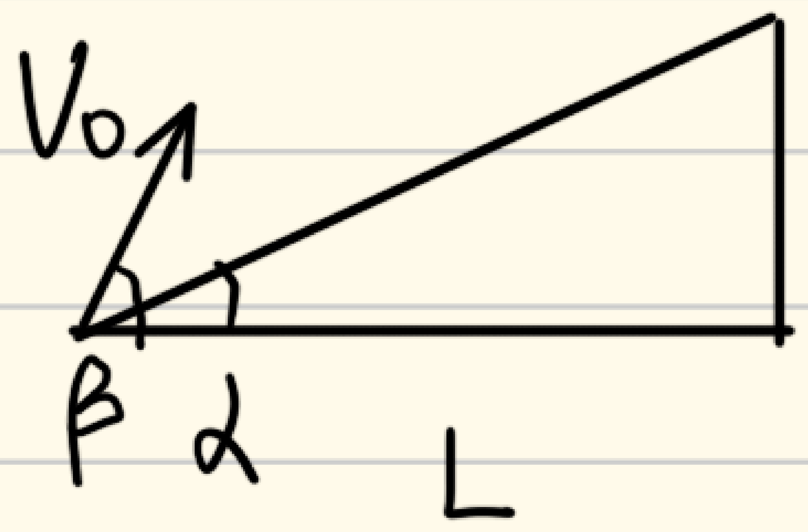
(2) $V = \dot{x} = \frac{dx}{dt} = 20t$, $t = 3.0s$ 时, $V = 60m/s$

$\vec{v}$ 方向沿 x 正向

(3) $V = \dot{x} = \frac{dx}{dt} = \frac{20t\,dt}{dt} = 20t$ m/s

$a = \dot{v} = \ddot{x} = 20\ m/s^2$

1.25



$t = \frac{L}{\cos\beta \cdot V_0}$

$\frac{1}{2} g t^2 = L(\tan\beta - \tan\alpha)$

$\frac{gL^2}{2V_0^2\cos^2\beta} = L \cdot \left(\frac{\sin\beta}{\cos\beta} - \frac{\sin\alpha}{\cos\alpha}\right)$

$\frac{gL}{2V_0^2} = \frac{\sin(\beta-\alpha)\cos\beta}{\cos\alpha}$

$V_0^2 = \frac{gL}{2} \frac{\cos\alpha}{\sin(\beta-\alpha)\cos\beta}$

$\therefore V_0 = \sqrt{\frac{gL\cos\alpha}{2\sin(\beta-\alpha)\cos\beta}}$

1.31 (1) 即 $a_n = a_t = 3m/s^2$

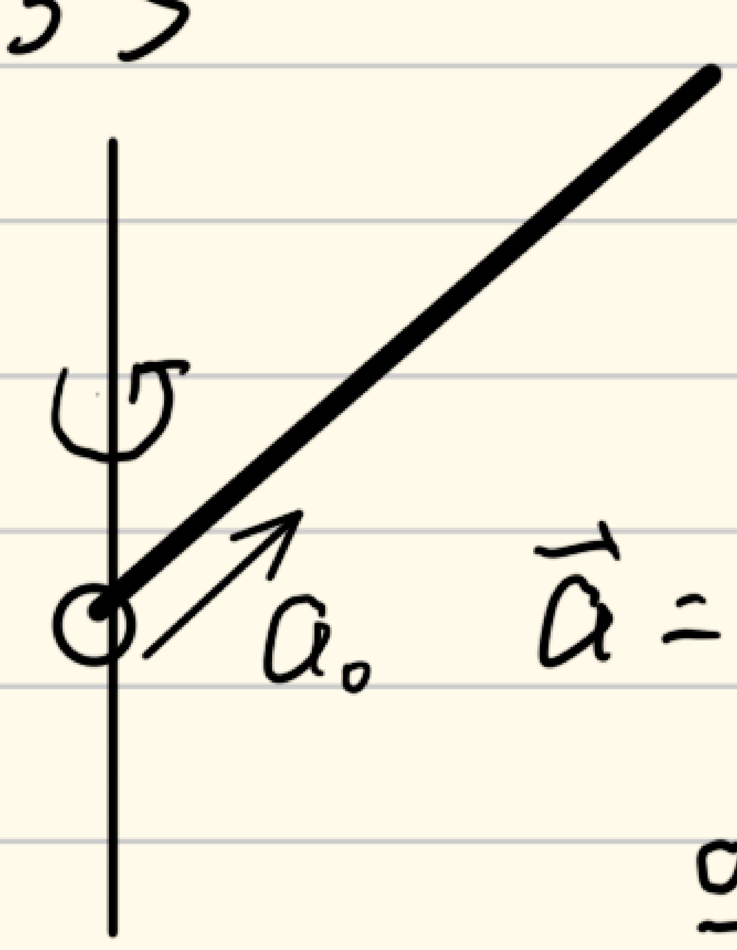
$a_n = \frac{v^2}{R} = 3\ m/s^2 \quad \therefore V = 3m/s$

$t = \frac{v}{a_t} = 1\ s$

(2) $S = \frac{1}{2} a_t \cdot t^2 = 1.5\ m$

1.33

以 O 为原点，杆所在射线为径向建立极坐标系



$\therefore \vec{V} = \vec{V_n} + \vec{V_t} = a_0 t\,\hat{r} + \frac{1}{2} a_0 t^2 w_0 \hat{\theta}$

$\vec{a} = \dot{\vec{v}} = a_0 \hat{r} + a_0 t \frac{d\hat{r}}{dt} + a_0 w_0 t \hat{\theta} + \frac{1}{2} a_0 w_0 t^2 \frac{d\hat{\theta}}{dt}$

$\frac{d\hat{\theta}}{dt} = w_0 \cdot \hat{r} \qquad \frac{d\hat{r}}{dt} = w_0 \cdot \hat{\theta}$

$\therefore \vec{a} = (a_0 + \frac{1}{2} a_0 w_0^2 t^2)\,\hat{r} + 2a_0 w_0 t\,\hat{\theta}$

3. 补充习题

1. $\vec{A}=(3,4,-4)$

(a) 设 $\vec{B}=(a,b,0)$ $\begin{cases}\vec{B}\cdot\vec{A}=3a+4b=0\\ a^2+b^2=1\end{cases}$

$\therefore a=\frac{4}{5},\ b=-\frac{3}{5}$ $\quad\therefore \vec{B}=(\frac{4}{5},-\frac{3}{5},0)$

(b) $\vec{C}=(x,y,z)$

$\begin{cases}\vec{C}\cdot\vec{A}=3x+4y-4z=0\\ \vec{C}\cdot\vec{B}=\frac{4}{5}x-\frac{3}{5}y=0\end{cases}$ $\quad 3y=4x$

取 $\vec{C}=(3,4,\frac{25}{4})$

(c) $\vec{A}\perp\vec{B}$, $\vec{A}\perp\vec{C}$, $\vec{B},\vec{C}$ 起点均为O点,

$\therefore \vec{A}$ 垂直于 $\vec{B},\vec{C}$ 所在的平面

2. ① $\vec{a}=(1,-2,1)$, $\vec{b}=(1,-1,3)$, $\vec{c}=(2,5,-3)$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{e}_x & \vec{e}_y & \vec{e}_z\\ 1 & -2 & 1\\ 1 & -1 & 3\end{vmatrix}=\begin{vmatrix}-2 & 1\\ -1 & 3\end{vmatrix}\vec{e}_x-\begin{vmatrix}1 & 1\\ 1 & 3\end{vmatrix}\vec{e}_y+\begin{vmatrix}1 & -2\\ 1 & -1\end{vmatrix}\vec{e}_z=(-5,-2,1)$$

② $(\vec{a}\times\vec{b})\cdot\vec{c}=(-5,-2,1)\cdot(2,5,-3)=-23$

③ $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}\cdot(\vec{a}\cdot\vec{b})$

$=-11\times(1,-1,3)-6\times(2,5,-3)$

$=(-11,11,-33)-(12,30,-18)=(-23,-19,-15)$

3. (a) $y'=24x^2+1$

(b) $y'=(x-1)\tan x+(x+1)\tan x+(x^2-1)\frac{1}{\cos^2 x}=2x\tan x+\frac{x^2-1}{\cos^2 x}$

(c) $y'=\frac{(2x+9)(5x+6)-5(9x+x^2)}{(5x+6)^2}=\frac{10x^2+57x+54-45x-5x^2}{(5x+6)^2}$

$=\frac{5x^2+12x+54}{25x^2+60x+36}$

$f'(x)\,dx=df(x)$

(d) $y'=\cos x-x\sin x+\frac{x\cos x-\sin x}{x^2}$

4. (a) $\int(3x^2+\sin x+\frac{5}{x})dx=\int 3x^2dx+\int\sin x\,dx+\int\frac{5}{x}dx$

$=x^3-\cos x+5\ln|x|$

(b) $\int\sqrt{a^2-x^2}\,dx\underset{(a>0)}{=}a\int\sqrt{1-\sin^2x}\,da\sin x=a\int\sqrt{1-\sin^2x}\,a\cos x\,dx$

$$=a^2\int \cos^2 x\, dx =$$

5. 代入： $y' = -C_1 w \sin wt - C_2 w \sin wt$

$y'' = -C_1 w^2 \cos wt - C_2 w^2 \sin wt$

$\therefore y'' + w^2 y = 0$

6. (a) $V_m = \int_0^T a_{(t)}\, dt = \frac{a_m}{2}\int_0^T (1-\cos\frac{2\pi}{T}t) = \frac{a_m}{2}(t - \frac{T}{2\pi}\sin\frac{2\pi}{T}t)\Big|_0^T = \frac{a_m T}{2}$

(b) 在 $\tau \ll T$ 时间内，电梯可视为做匀加速运动

$\therefore V_\tau = \int_0^\tau a_{(t)}\, dt = \frac{a_m}{2}(t - \frac{T}{2\pi}\sin\frac{2\pi}{T}t)\Big|_0^\tau = \frac{a_m}{2}(\tau - \frac{T}{2\pi}\sin\frac{2\pi}{T}\cdot\tau)$

$\sin\frac{2\pi}{T}\tau \doteq \frac{2\pi}{T}\cdot\tau \quad \therefore V_\tau \doteq 0$

(c) $V_{(t)} = \int_0^t a_{(t)}\, dt = \frac{a_m}{2}(t - \frac{T}{2\pi}\sin\frac{2\pi}{T}t)$

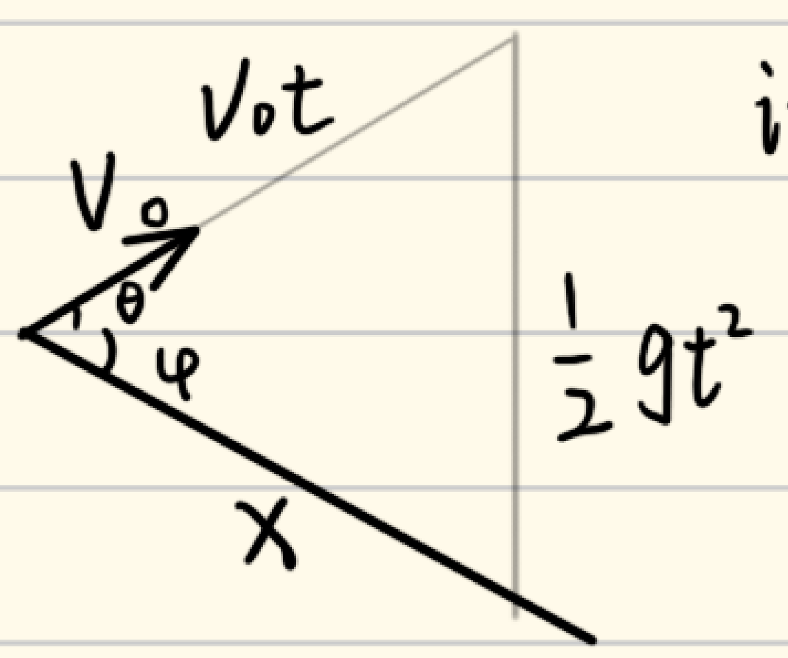
$X_{(t)} = \int_0^t V_{(t)} = \frac{a_m}{2}(\frac{1}{2}t^2 + \frac{T^2}{4\pi^2}\cos\frac{2\pi}{T}t)$

则 0-T 时间内电梯运行距离为 $X_{(T)} = \frac{a_m}{2}(\frac{1}{2}T^2 - \frac{T^2}{4\pi^2})$

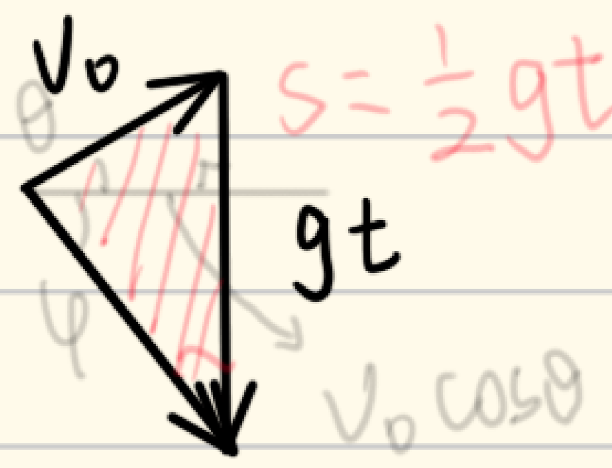
则 $D = X_{总} = 2X_{(T)} = \frac{2\pi^2-1}{4\pi^2}a_m T^2 \quad \therefore T = \sqrt{\frac{4\pi^2 D}{(2\pi^2-1)a_m}}$

$\therefore t_1 = 2T = 4\pi\sqrt{\frac{D}{(2\pi^2-1)a_m}}$

7.



设初速度为 V_0，运动时间为 t



$s = \frac{1}{2}gt\, v_0\cos\theta$

$v_0 t\cos\theta = \frac{2s}{g}$

S取最大值时，位移最远

$\theta + \varphi = \frac{1}{2}(\pi - (\frac{\pi}{2} - \varphi)) = \frac{1}{2}(\frac{\pi}{2} + \varphi) = \frac{\pi}{4} + \frac{\varphi}{2}$

$\therefore \theta = \frac{\pi}{4} - \frac{\varphi}{2}$

8. (a) $r = \frac{r_0}{1+e\cos\theta}$, $\theta = 0$, $r_1 = \frac{r_0}{1+e}$; $\theta = \pi$, $r_2 = \frac{r_0}{1-e}$

$r_1 + r_2 = r_0(\frac{1}{1+e} + \frac{1}{1-e}) = \frac{2r_0}{1-e^2} = 2A \quad \therefore r_0 = (1-e^2)A$, $e = \frac{\sqrt{A^2-B^2}}{A}$

(b) $\vec{r}_{(\theta)} = \frac{r_0}{1+e\cos\theta}\hat{r}$ $\qquad \vec{V}_\theta = \dot{\vec{r}}_{(\theta)} = \frac{(1+e\cos\theta + e\sin\theta\, r_0)w}{(1+e\cos\theta)^2}\hat{r} + \frac{r_0}{1+e\cos\theta}\dot{\hat{r}}$

$= \frac{(1+e\cos\theta + e\sin\theta\, r_0)w}{(1+e\cos\theta)^2}\hat{r} + \frac{r_0 w}{1+e\cos\theta}\hat{\theta}$

$V_\theta = \sqrt{(\frac{(1+e\cos\theta + e\sin\theta\, r_0)w}{(1+e\cos\theta)^2})^2 + (\frac{r_0 w}{1+e\cos\theta})^2}$

$\vec{a}_\theta = \dot{\vec{v}}_\theta =$

9. $\vec{r} = r_0 e^{\alpha\theta}\,\hat{r}$

$\vec{v} = \dot{\vec{r}} = r_0 \alpha e^{\alpha\theta} \cdot \dot{\theta}\,\hat{r} + r_0 e^{\alpha\theta}\,\dot{\hat{r}}$ $\qquad \hat{v} = \dfrac{\alpha\hat{r} + \hat{\theta}}{\sqrt{\alpha^2+1}} \qquad \hat{n} = \dfrac{\hat{r} - \alpha\hat{\theta}}{\sqrt{\alpha^2+1}}$

$= r_0 \alpha e^{\alpha\theta}\dot{\theta}\,\hat{r} + r_0 e^{\alpha\theta}\dot{\theta}\,\hat{\theta}$

$\vec{a} = \dot{\vec{v}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$

$= [r\dot{\theta}^2(\alpha^2-1) + \alpha r\ddot{\theta}]\,\hat{r} + [r\ddot{\theta} + 2\alpha r\dot{\theta}^2]\,\hat{\theta}$

$a_n = \vec{a}\cdot\hat{n} = \dfrac{1}{\sqrt{\alpha^2+1}}\{[r\dot{\theta}^2(\alpha^2-1) + \alpha r\ddot{\theta} - \alpha(r\ddot{\theta} + 2\alpha r\dot{\theta}^2)\}$

$= \dfrac{1}{\sqrt{\alpha^2+1}}[-r\dot{\theta}^2(\alpha^2+1)]$ $\qquad \therefore |a_n| = r\dot{\theta}^2\sqrt{\alpha^2+1}$

$v^2 = r^2\dot{\theta}^2(\alpha^2+1)$

$\rho = \dfrac{v^2}{|a_n|} = \dfrac{r^2\dot{\theta}^2(\alpha^2+1)}{r\dot{\theta}^2\sqrt{\alpha^2+1}} = r\sqrt{1+\alpha^2} = \sqrt{1+\alpha^2}\, r_0 e^{\alpha\theta}$