

简答题

1. ① $|\frac{d\vec{v}}{dt}|=0$ 表示加速度为0的运动, 即速度不变的运动
为静止或匀速直线运动

② $\frac{d|\vec{v}|}{dt}=0$. 表示速率大小不变的运动

2. 可以发生改变, 若加速度矢量方向与速度矢量方向不共线, 则速度矢量方向会发生变化. 运动方向改变

3. (1) 错误 反例: 匀速圆周运动

(2) 正确

(3) 错误 反例: 匀速圆周运动

4. 因为自身在运动

应当根据本身运动情况

适当调整瞄准方向

补充习题

1. (a) $\vec{A} = 3\hat{i} + 4\hat{j} - 4\hat{k}$

令 $\vec{B} = a\hat{i} + b\hat{j}$. 若 $\vec{B} \perp \vec{A}$, 则 $\vec{B} \cdot \vec{A} = 0$

则 $3a + 4b = 0$

由于 \vec{B} 单位向量, 则 $|\vec{B}| = 1$

即 $\sqrt{a^2 + b^2} = 1$, 即 $a^2 + b^2 = 1$

取 $a = -\frac{4}{5}$, $b = \frac{3}{5}$. 则 $\vec{B} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

(b) 令 $\vec{C} = x\hat{i} + y\hat{j} + z\hat{k}$ ($|\vec{C}| = 1$)

$$\vec{C} \perp \vec{A}, \vec{C} \perp \vec{B} \Rightarrow \begin{cases} 3x + 4y - 4z = 0 \\ -\frac{4}{5}x + \frac{3}{5}y = 0 \Rightarrow -4x + 3y = 0 \end{cases}$$

令 $x = 3$, 则 $y = 4$, $z = \frac{25}{4}$

即 $\vec{C} = 3\hat{i} + 4\hat{j} + \frac{25}{4}\hat{k}$

(c) $\vec{A} \perp \vec{B}$, \vec{C} 所在平面 $\alpha \Leftrightarrow \forall \vec{D} \in \alpha, \vec{A} \perp \vec{D}$

由于 $\vec{D} \in \alpha$, 则 $\vec{D} = m\vec{B} + n\vec{C}$

则 $\vec{A} \cdot \vec{D} = m\vec{B} \cdot \vec{A} + n\vec{C} \cdot \vec{A} = 0$, 即 $\vec{A} \perp \vec{D}$

可得出 $\vec{A} \perp \vec{B}, \vec{C}$ 所在平面 α

$$2. \vec{a} = (1, -2, 1), \vec{b} = (1, -1, 3), \vec{c} = (2, 5, -3)$$

$$\begin{aligned} \textcircled{1} \vec{a} \times \vec{b} &= (1, -2, 1) \times (1, -1, 3) = (-2 \times 3 - 1 \times (-1), 1 \times 1 - 3 \times 1, 1 \times (-1) \\ &\quad - (-2) \times 1) \\ &= (-5, -2, 1) \end{aligned}$$

$$\textcircled{2} (\vec{a} \times \vec{b}) \cdot \vec{c} = (-5, -2, 1) \cdot (2, 5, -3) = -10 - 10 - 3 = -23$$

$$\begin{aligned} \textcircled{3} (\vec{b} \times \vec{c}) &= (1, -1, 3) \times (2, 5, -3) \\ &= (-1 \times (-3) - 3 \times 5, 3 \times 2 - (-3) \times 1, 1 \times 5 - 2 \times (-1)) \\ &= (-12, 9, 7) \end{aligned}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (1, -2, 1) \times (-12, 9, 7)$$

$$= (-2 \times 7 - 1 \times 9, 1 \times (-12) - 7 \times 1, 1 \times 9 - (-12) \times (-2))$$

$$= (-23, -19, -15)$$

$$3. (a) y' = 24x^2 + 1$$

$$(b) y = \frac{(x^2-1) \sin x}{\cos x}$$

$$y' = \frac{[2x \sin x + \cos x (x^2-1)] \cos x + \sin^2 x (x^2-1)}{\cos^2 x}$$

$$= \frac{2x \sin x \cos x + x^2 - 1}{\cos^2 x} = \frac{x \sin 2x + x^2 - 1}{\cos^2 x}$$

$$(c) y = \frac{9x + x^2}{5x + 6} = \frac{x(9+x)}{5x+6}$$

$$y' = \frac{[9+x+x](5x+6) - 5x(9+x)}{(5x+6)^2}$$

$$= \frac{5x^2 + 12x + 54}{(5x+6)^2}$$

$$(d) y = x \cos x + \frac{\sin x}{x}$$

$$y' = \cos x - x \sin x + \frac{x \cos x - \sin x}{x^2}$$

$$4. \star (a) \int (3x^3 + \sin x + \frac{5}{x}) dx$$

$$= \frac{3}{4}x^4 - \cos x + 5\ln|x| + C$$

$$(b) \int \sqrt{a^2 - x^2} dx$$

$$\star \frac{1}{2} \quad \text{令 } x = a \sin \theta, \text{ 则 } \sqrt{a^2 - x^2} = \sqrt{a^2 \cos^2 \theta} = a \cos \theta, \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\text{则 } \int \sqrt{a^2 - x^2} = \int a \cos \theta dx = \int a \cos \theta da \sin \theta$$

$$= \int a \cos \theta \cdot a \cos \theta d\theta = a^2 \int \cos^2 \theta \cdot d\theta$$

$$= a^2 \int \left(\frac{1 + \cos 2\theta}{2} \right) d\theta$$

$$= \frac{1}{2} a^2 \int (1 + \cos 2\theta) d\theta$$

$$= \frac{1}{2} a^2 \int d(\theta + \frac{1}{2} \sin 2\theta)$$

$$= \frac{1}{2} a^2 (\theta + \frac{1}{2} \sin 2\theta) + C$$

$$\therefore x = a \sin \theta$$

$$\therefore \frac{x}{a} = \sin \theta$$

$$\therefore \theta = \arcsin\left(\frac{x}{a}\right), \cos \theta = \sqrt{1 - \frac{x^2}{a^2}} = \frac{\sqrt{1-x^2}}{a}$$

$$\therefore \frac{1}{2} \sin 2\theta = \frac{x \sqrt{1-x^2}}{a^2}$$

$$\star \frac{1}{2} \quad \text{故 } \int \sqrt{a^2 - x^2} dx$$

$$= \frac{1}{2} a^2 \left(\arcsin\left(\frac{x}{a}\right) + \frac{x \sqrt{1-x^2}}{a^2} \right) + C$$

5. $y = C_1 \cos \omega t + C_2 \sin \omega t$ 为 $y'' + \omega^2 y = 0$ 的解

证明: 易得. 方程特解 $y_0 = 0$

令 $y(t) = Ce^{\lambda t}$ (C 为任意常数)

则 $y'' = \lambda^2 Ce^{\lambda t}$ $y = Ce^{\lambda t}$

代入, 得: $\lambda^2 Ce^{\lambda t} + \omega^2 Ce^{\lambda t} = 0$

即 $\lambda^2 + \omega^2 = 0 \Rightarrow \lambda = \pm \sqrt{-\omega^2} = \pm \omega i$

则 $y = C_3 e^{\omega i t} + C_4 e^{-\omega i t}$

由欧拉公式, 得

$$\begin{aligned} y &= C_3 (\cos \omega t + i \sin \omega t) + C_4 (\cos \omega t - i \sin \omega t) \\ &= C_1 \cos \omega t + C_2 \sin \omega t \end{aligned}$$

(C_1 满足 $C_1 = C_3 + C_4$.)

(C_2 满足 $C_2 = C_3 i - C_4 i$)

证毕

6. (a) 由题意, 得: 当 $0 \leq t \leq T$ 时:

$$a(t) = (a_m/2)(1 - \cos(2\pi t/T)) \quad (0 \leq t \leq T)$$

$$dv(t) = a(t) \cdot dt \Rightarrow v(t) = \int a(t) \cdot dt = \int d[(a_m/2)$$

$$(t - \frac{T}{2\pi} \sin(2\pi t/T))] = (a_m/2)(t - \frac{T}{2\pi} \sin(2\pi t/T)) + C$$

$$\text{而 } v(0) = 0, \text{ 则 } C = 0, \text{ 可得: } v(t) = (a_m/2)(t - \frac{T}{2\pi} \sin(2\pi t/T)) \\ (0 \leq t \leq T)$$

当 $T \leq t \leq 2T$ 时:

$$a(t) = -(a_m/2)(1 - \cos(2\pi t/T))$$

$$dv(t) = a(t) \cdot dt \Rightarrow v(t) = \int a(t) \cdot dt = \int d[-(a_m/2)$$

$$(t - \frac{T}{2\pi} \sin(2\pi t/T))] = -(a_m/2)(t - \frac{T}{2\pi} \sin(2\pi t/T)) + C$$

$$\text{而 } v(T) = (a_m/2)T = -(a_m/2)T + C$$

$$\text{则 } C = a_m \cdot T \Rightarrow v(t) = -(a_m/2)(t - \frac{T}{2\pi} \sin(2\pi t/T)) \\ + a_m \cdot T$$

综上: 当 $0 \leq t \leq T$ 时: $a(t) \geq 0$

当 $T \leq t \leq 2T$ 时: $a(t) \leq 0$

$$\text{则 } V_{\max} = v(T) = \frac{a_m}{2} T$$

(b) 当 $t \ll T$ 时:

由 Taylor 展开相关知识, 得:

$$v(t) = (a_m/2) \left(t - \frac{T}{2\pi} \sin(2\pi t/T) \right)$$

$$\approx (a_m/2) \left[t - \frac{T}{2\pi} \left(\frac{2\pi t}{T} - \frac{1}{6} \left(\frac{2\pi t}{T} \right)^3 \right) \right]$$

$$= (a_m/2) \left[\frac{T}{2\pi} \cdot \frac{8\pi^3 t^3}{6T^3} \right]$$

$$= \frac{\pi^2 t^3 a_m}{3T^2} \quad (t \ll T)$$

CC) 由题表, 得:

$0 \leq t \leq T$ 时:

$$v(t) = (a_m/2) \left(t - \frac{T}{2\pi} \sin(2\pi t/T) \right)$$

$$S(t) = \int_0^t v(t) \cdot dt = \int_0^t d \left(a_m/2 \right) \left(\frac{1}{2} t^2 + \frac{T^2}{4\pi^2} \cos(2\pi t/T) \right)$$

$$= \frac{a_m}{2} \cdot \left(\frac{1}{2} t^2 + \frac{T^2}{4\pi^2} \cos(2\pi t/T) \right) \rightarrow \frac{a_m}{2} \cdot \frac{T^2}{4\pi^2}$$

$$S(T) = \frac{a_m}{4} T^2$$

观察, 得:

$$\text{若 } 0 \leq t \leq T, \text{ 则 } a(t) = (a_m/2) (1 - \cos(2\pi t/T))$$

$$a(2T-t) = -(a_m/2) (1 - \cos(2\pi(2T-t)/T))$$

$$= -(a_m/2) (1 - \cos(4\pi - \frac{2\pi t}{T}))$$

$$= -(a_m/2) (1 - \cos(\frac{2\pi t}{T}))$$

$$= -a(t)$$

$$\text{则 } v(t) = v(2T-t)$$

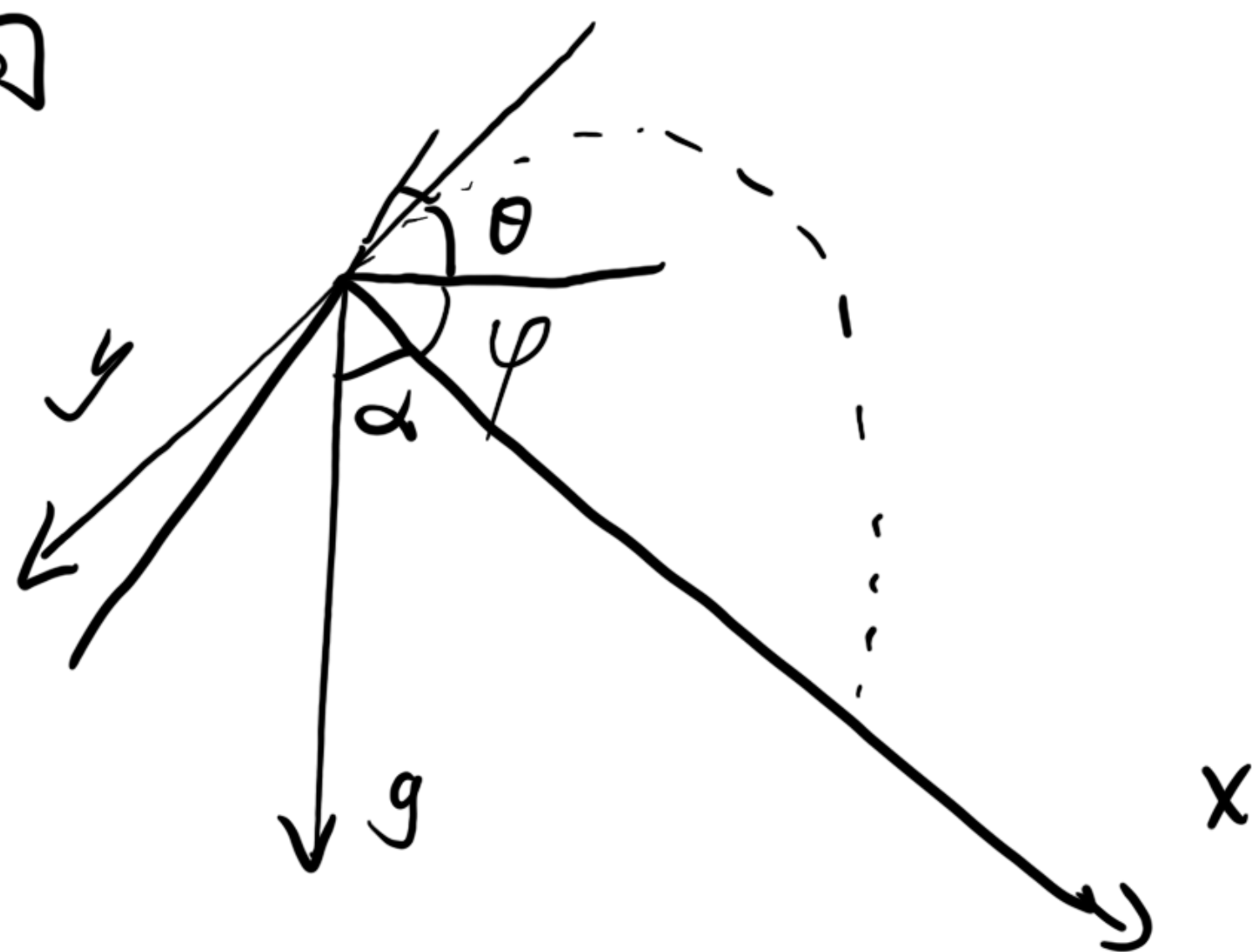
\Downarrow

$$S(2T) = 2S(T) = \frac{a_m}{2} T^2 = D$$

$$T = \sqrt{\frac{2D}{a_m}} \Rightarrow \text{总时间 } 2T = 2\sqrt{\frac{2D}{a_m}}$$

7. 由题意得

令沿斜面向下为正方向，垂直斜面向下也为正方向



$$a_x = g \cdot \cos \alpha = g \cdot \cos \left(\frac{\pi}{2} - \varphi \right) = g \sin \varphi$$

$$a_y = g \cos \varphi$$

初速度为 V_0

$$V_{x0} = -\cos(\pi - (\theta + \varphi)) \cdot V_0 = \cos(\theta + \varphi) \cdot V_0$$

$$V_{y0} = -\sin(\theta + \varphi) \cdot V_0$$

$$\text{则 } V_x(t) = \cos(\theta + \varphi) \cdot V_0 + g \sin \varphi t$$

$$V_y(t) = -\sin(\theta + \varphi) \cdot V_0 + g \cos \varphi t$$

$$\text{则 } x(t) = \int_0^t V_x(t) \cdot dt = \int_0^t d(\cos(\theta + \varphi) V_0 t + \frac{1}{2} g \sin \varphi t^2)$$

$$= \cos(\theta + \varphi) V_0 t + \frac{1}{2} g \sin \varphi t^2$$

$$y(t) = \int_0^t V_y(t) \cdot dt = \int_0^t d(-\sin(\theta + \varphi) V_0 t + \frac{1}{2} g \cos \varphi t^2)$$
$$= -\sin(\theta + \varphi) V_0 t + \frac{1}{2} g \cos \varphi t^2$$

当落回斜面时:

$$y(t_1) = -\sin(\theta + \varphi) V_0 t_1 + \frac{1}{2} g \cos \varphi t_1^2 = 0$$

$$t_1 = \frac{2 \sin(\theta + \varphi) V_0}{g \cos \varphi}$$

$$\begin{aligned} \text{此时: } x(t_1) &= \cos(\theta + \varphi) V_0 t_1 + \frac{1}{2} g \sin \varphi t_1^2 \\ &= \frac{2 \sin(\theta + \varphi) \cos(\theta + \varphi) V_0^2}{g \cos \varphi} + \frac{4 \sin^2(\theta + \varphi) V_0^2}{g \cos^2 \varphi} \cdot \frac{1}{2} g \sin \varphi \\ &= \frac{\sin(2\theta + 2\varphi) V_0^2}{g \cos \varphi} + \frac{2 \sin^2(\theta + \varphi) V_0^2 \cdot \sin \varphi}{g \cos^2 \varphi} \\ &= \frac{2 \sin(\theta + \varphi) V_0^2 \left(\cos(\theta + \varphi) + \frac{\sin(\theta + \varphi) \cdot \sin \varphi}{\cos \varphi} \right)}{g \cos \varphi} \\ &= \frac{V_0^2 [\sin(2\theta + 2\varphi) + \sin \varphi]}{g \cos^2 \varphi} \end{aligned}$$

$$\text{当 } 2\theta + 2\varphi = \frac{\pi}{2} \text{ 时, 即 } \theta = \frac{\pi}{4} - \frac{\varphi}{2} \text{ 时}$$

$$x_{\max} = \frac{V_0^2 + \sin \varphi V_0^2}{g \cos^2 \varphi}$$

8.

(a) 当 $\theta = 0$ 时:

$$r = A - \sqrt{A^2 - B^2} = \frac{r_0}{e+1}$$

当 $\theta = \frac{\pi}{2}$ 时:

$$r = A + \sqrt{A^2 - B^2} = \frac{r_0}{1-e}$$

$$\text{则} \frac{1-e}{1+e} = \frac{A - \sqrt{A^2 - B^2}}{A + \sqrt{A^2 - B^2}} \Rightarrow e = \frac{\sqrt{A^2 - B^2}}{A}$$

$$\text{故 } r_0 = \frac{B^2}{A}$$

$$(b) \theta = \omega t \Rightarrow r = \frac{r_0}{1 + e \cos \omega t}$$

$$\vec{V}_\theta = r \cdot \dot{\theta} \cdot \hat{\theta} = \frac{r_0}{1 + e \cos \theta} \cdot \omega \cdot \hat{\theta} = \frac{r_0 \omega}{1 + e \cos \theta} \hat{\theta}$$

$$= \frac{B^2 \omega}{A(1 + \frac{\sqrt{A^2 - B^2}}{A} \cos \theta)} = \frac{B^2 \omega}{A + \sqrt{A^2 - B^2} \cos \theta}$$

$$\text{而 } \vec{a}_\theta = (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$\text{则 } \dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{e \sin \theta r_0 \omega}{(1 + e \cos \theta)^2}$$

$$\vec{a}_\theta = (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} = \frac{2\omega^2 e \sin \theta r_0}{(1 + e \cos \theta)^2} \hat{\theta}$$

$$= \frac{2\omega^2 \cdot \frac{\sqrt{A^2 - B^2}}{A} \cdot \sin \theta \cdot \frac{B^2}{A}}{\left(\frac{A + \sqrt{A^2 - B^2} \cos \theta}{A} \right)^2} \hat{\theta} = \frac{2\omega^2 B^2 \sin \theta \sqrt{A^2 - B^2}}{(A + \sqrt{A^2 - B^2} \cos \theta)^2} \hat{\theta}$$

9. $r = r_0 e^{a\theta}$

$$V_r = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$$

令 $\theta = \omega t$

則 $V_r = a r_0 \cdot e^{a\theta} \cdot \omega = a \omega r$

$$V_\theta = r \cdot \dot{\theta} = \omega r$$

↓

$$V_{\text{合}} = \sqrt{V_r^2 + V_\theta^2} = \sqrt{a^2 + 1} \omega r$$

可得：

$$a_\theta = 2 \frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \frac{d^2\theta}{(dt)^2}$$

$$= 2 \cdot \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{(dt)^2}$$

$$= 2 \cdot r_0 a \cdot e^{a\theta} \cdot \omega^2 + 0$$

$$= 2 a \omega^2 r$$

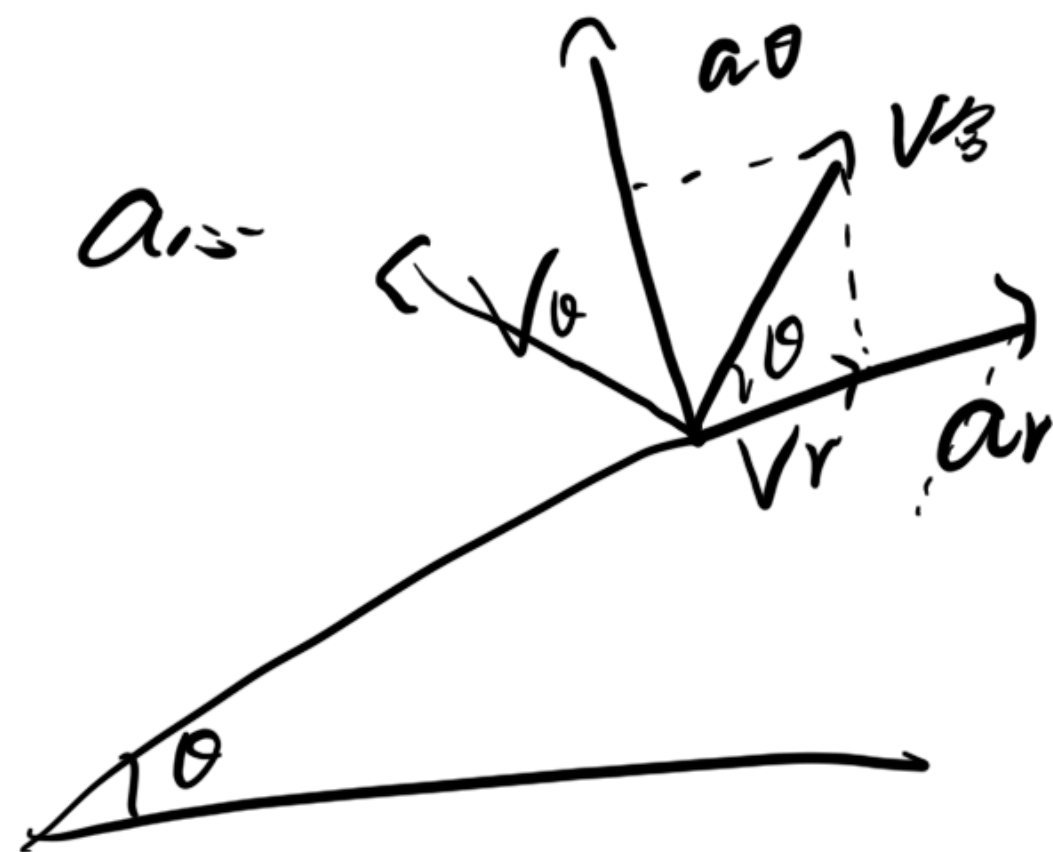
$$a_r = \frac{d^2r}{(dt)^2} - r \left(\frac{d\theta}{dt} \right)^2 = \frac{d^2r}{(d\theta)^2} \cdot \left(\frac{d\theta}{dt} \right)^2 - r \left(\frac{d\theta}{dt} \right)^2$$

$$= a^2 r \omega^2 - \omega^2 r$$

$$= (a^2 - 1) r \omega^2$$

如圖： $\sin\theta = \frac{V_\theta}{V_{\text{合}}} = \frac{1}{\sqrt{a^2 + 1}}$, $\cos\theta = \frac{a}{\sqrt{a^2 + 1}}$

則 $a_{\text{心}} = a_r \cdot \sin\theta - a_\theta \cdot \cos\theta = \frac{a^2 - 1}{\sqrt{a^2 + 1}} \cdot r \omega^2 - \frac{2a^2}{\sqrt{a^2 + 1}} r \omega^2$
 $= \frac{-a^2 - 1}{\sqrt{a^2 + 1}} r \omega^2 = -\sqrt{a^2 + 1} r \omega^2$



則曲率半徑

$$\rho = \frac{V_{\text{合}}^2}{|a_{\text{心}}|} = \frac{(a^2 + 1) \omega^2 r^2}{\sqrt{a^2 + 1} r \omega^2} = \sqrt{a^2 + 1} \cdot r$$

教材:

1-1 解: (1) 易得, 无穷多次

(2)

$$t = \frac{d}{V_甲 + V_乙} = \frac{60\text{km}}{60\text{km/h}} = 1\text{h}$$

$$S_B = V_B t = 60\text{km}$$

1.3

804: (1) $x = 10t^2 + 6$

$$V(t) = \dot{x}(t) = 20t$$

3.00s ~ 3.10s:

$$\bar{V}_1 = \frac{V_1 + V_2}{2} = \frac{60\text{m/s} + 62\text{m/s}}{2} = 61\text{m/s}$$

3.00 ~ 3.01s:

$$\bar{V}_2 = \frac{V_1 + V_3}{2} = \frac{60\text{m/s} + 60.2\text{m/s}}{2} = 60.1\text{m/s}$$

3.000 ~ 3.001s:

$$\bar{V}_3 = \frac{V_1 + V_4}{2} = \frac{60\text{m/s} + 60.02\text{m/s}}{2} = 60.01\text{m/s}$$

(2)

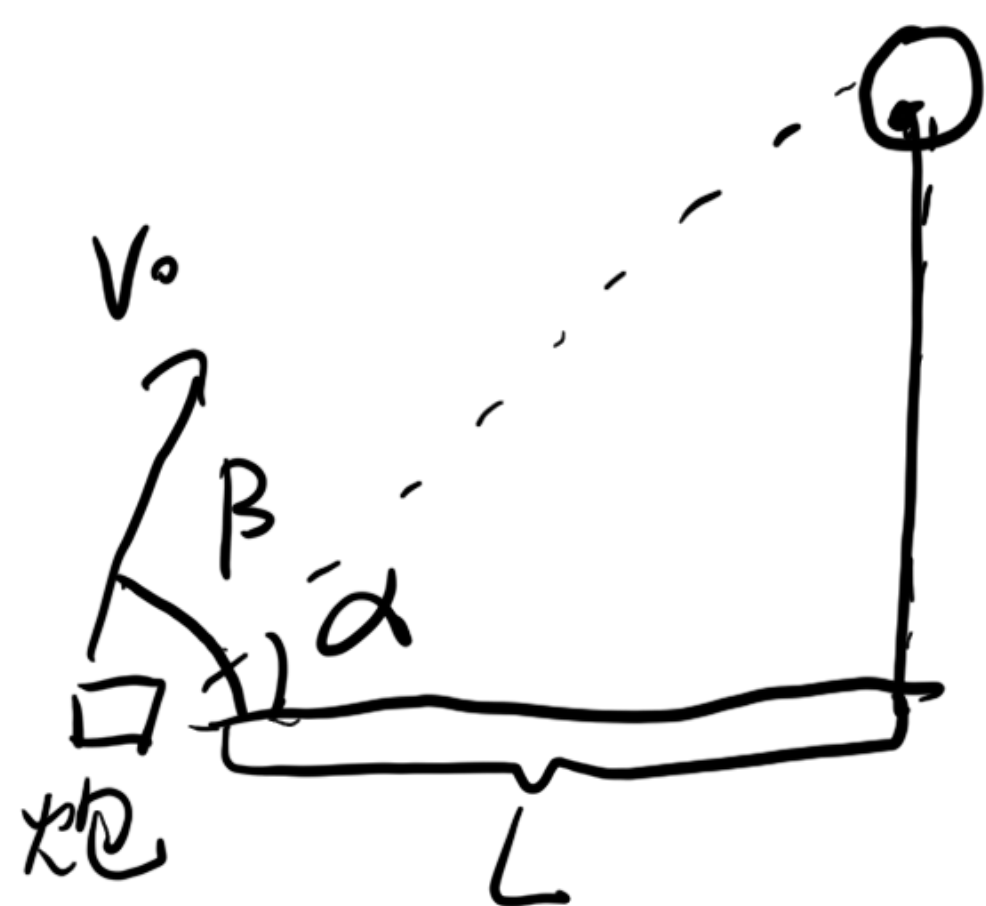
$$V_1 = (3 \times 20)\text{m/s} = 60\text{m/s}$$

(3)

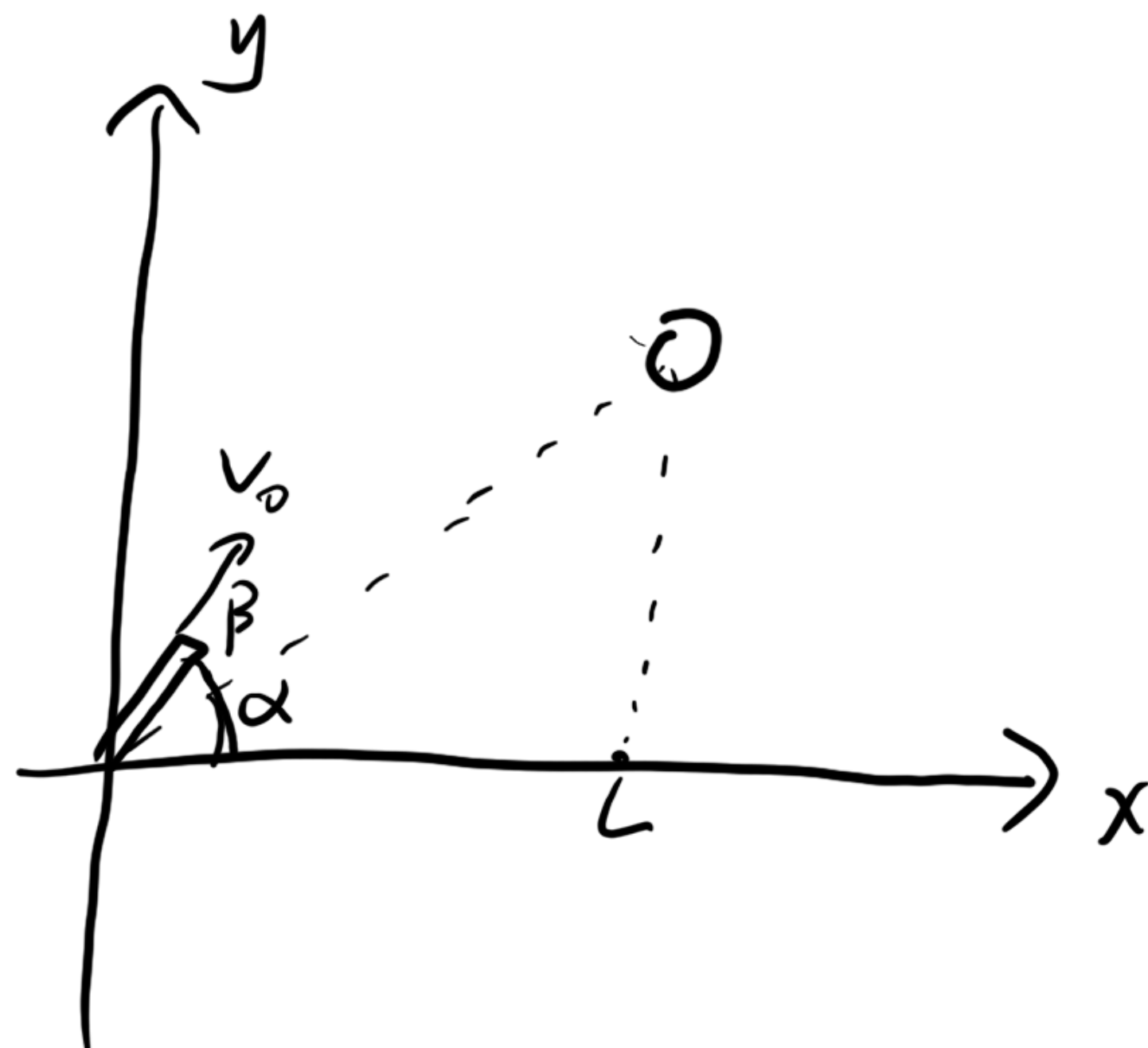
$$V(t) = \frac{dx(t)}{dt} = \dot{x}(t) = 20t$$

$$a(t) = \frac{dV(t)}{dt} = \dot{V}(t) = 20$$

1.25



\Rightarrow



由题意得:

$$a_x = 0, \text{ 则 } V_x = V_0 \cos \beta.$$

$$x(t) = V_0 \cos \beta t$$

$$a_y = -g, \text{ 则 } V_y = V_0 \sin \beta - gt$$

$$y(t) = \int V_y \cdot dt = V_0 \sin \beta t - \frac{1}{2} g t^2$$

当击中靶时:

$$x(t_0) = V_0 \cos \beta t_0 = L \Rightarrow t_0 = \frac{L}{V_0 \cos \beta}$$

$$y(t_0) = \tan \beta L - \frac{1}{2} g \frac{L^2}{V_0^2 \cos^2 \beta} = \tan \alpha \cdot L$$

$$\text{则 } \tan \beta - \frac{gL}{2V_0^2 \cos^2 \beta} = \tan \alpha$$

$$\text{则 } V_0 = \sqrt{\frac{gL \cos \alpha}{2 \cos \beta \sin(\beta - \alpha)}}$$

1.31

(1) 由题意, 得

$$\vec{a} = \vec{a}_t + \vec{a}_R$$

$$\text{而 } \langle \vec{a}, \vec{a}_R \rangle = 45^\circ, \quad \vec{a}_t \perp \vec{a}_R$$

由几何关系, 得

$$|\vec{a}| = \sqrt{2} |\vec{a}_t| = \sqrt{2} |\vec{a}_R|$$

$$\text{则 } V(t) = a_t \cdot t = (3t) \text{ m/s}$$

$$a_R = \frac{V(t)^2}{R} = \frac{9t^2}{R} = (3t^2) \text{ m/s}^2$$

$$\text{则 } 3t^2 = 3 \text{ m/s}^2 \Rightarrow t = 1 \text{ s}$$

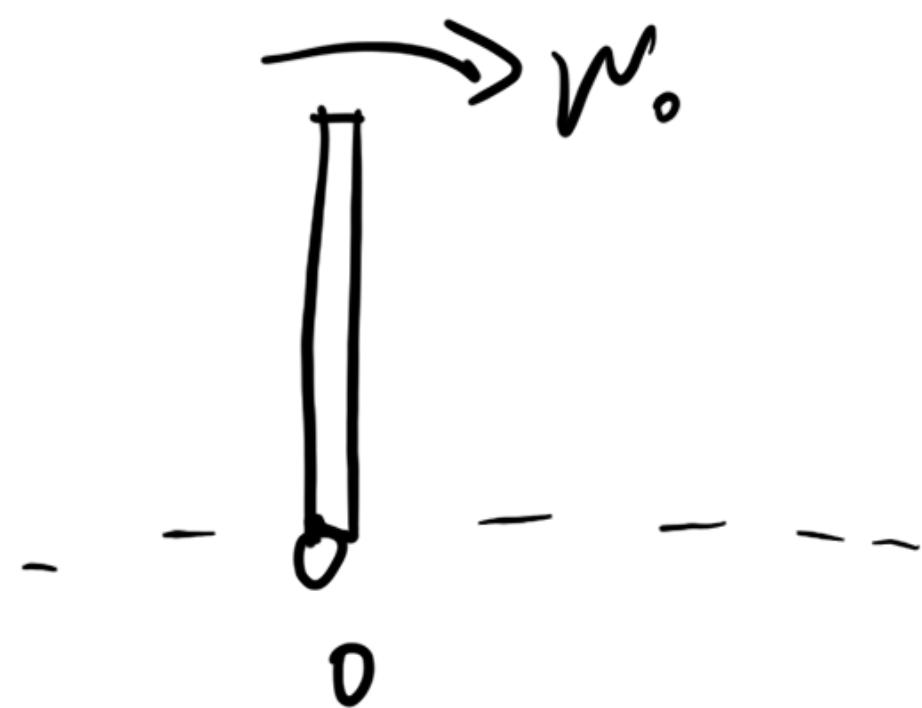
(2)

对物体本身建立自然坐标系

$$\text{则 } S(t) = \int V(t) \cdot dt = \int d\left(\frac{3}{2}t^2\right) = \frac{3}{2}t^2$$

$$\text{当 } t = 1 \text{ s 时, } S(1) = \frac{3}{2} \text{ m}$$

1. 33



$$\vec{V}_r = a_0 t \hat{r}$$

$$\vec{V}_\theta = \omega_0 r \hat{\theta}$$

$$\text{而 } r = \frac{1}{2} a_0 t^2$$

$$\text{则 } \vec{V}_\theta = \frac{1}{2} a_0 \omega_0 t^2 \hat{\theta}$$

$$\text{则 } \vec{V}_{\text{总}} = \vec{V}_\theta + \vec{V}_r = \frac{1}{2} a_0 \omega_0 t^2 \hat{\theta} + a_0 t \hat{r}$$

$$\vec{A}_{\text{总}} = \frac{d\vec{V}_{\text{总}}}{dt} = \frac{1}{2} a_0 \omega_0 \cdot 2t \cdot \hat{\theta} - \frac{1}{2} a_0 \omega_0 t^2 \cdot \hat{r} \omega_0$$

$$+ a_0 \hat{r} + a_0 t \hat{\theta} \omega_0$$

$$= 2a_0 t \omega_0 \hat{\theta} + (a_0 - \frac{1}{2} a_0 \omega_0^2 t^2) \hat{r}$$