PB25512014 马楠盛

简答题

1. $\left|\frac{d\vec{v}}{dt}\right|=0$ 代表无加速度的运动（静止或匀速直线运动）

$\frac{d|\vec{v}|}{dt}=0$ 代表速率不变的运动（如匀速~~直线~~圆周运动）（可有加速度）

区别明显（有无加速度）

2. 可以，只要加速度有垂直于当前速度方向的分量，即可在下一时刻发生速度方向的改变，即运动方向的改变。

3. (1) ~~作曲线~~ 错，例如匀速圆周运动

(2) 对，因为运动方向发生了改变，即有法向速度变化量，也即有法向加速度。

(3) 错，如匀速圆周运动

4. 在运动员参考系中，出手瞬间当成静止状态预测轨迹并出手，然而球出手后仍以速度v向运动员运动，这就导致篮框偏离最终落~~地~~点。

想要投准，使用V_y确定抛体时间，以此估算篮球最终位置，并计算所需水平速度即可

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第一次力学作业

1.1 (1) $+\infty$

(2) ~~$t=\Delta x$~~ $t_z = t_{车相遇} = \frac{\Delta x}{V_p+V_z} = 1(h)$

$S_{鸟z} = V_{鸟} t_z = 60\,km.$

~~同~~

1.3 (1) $\bar{V} = \frac{x(t_f)-x(t_i)}{t_f - t_i}$

∴ 3.00 ~ 3.10 s 时 $\bar{V} = 61.0(m/s)$

3.00 – 3.01 s 时 $\bar{V} = 60.10(m/s)$

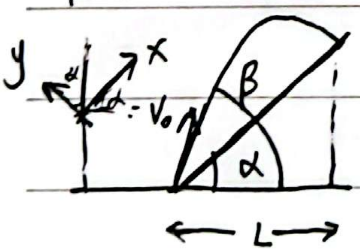
3.000 – 3.001 s 时 $\bar{V} =$ ~~60(m/s)~~ $60.01\ (m/s)$

(2) $V(t=3.00s) = \frac{dx}{dt}\Big|_{t=3.00s} = 20t\Big|_{t=3.00s} = 60\ (m/s).$

(3) $V(t) = \frac{dx}{dt} = 20t\ (m/s^2)$

$a(t) = \frac{dV}{dt} = 20\ (m/s^2)$

1.25



$g_y = g\cos\alpha$. ~~$g_x = g\sin\alpha$~~. $g_x = -g\sin\alpha$

在y方向作上抛运动 $V_{y0} = V_0\sin(\beta-\alpha)$

总时长 $t_0 = \frac{2V_{y0}}{g_y} = \frac{2V_0\sin(\beta-\alpha)}{g\cos\alpha}$.

x与炮靶连线平行 ~~x=~~ $V_{x0} = V_0\cos(\beta-\alpha)$

y与炮靶连线垂直 $x_{max} = V_{x0}t_0 + \frac{1}{2}g_x t_0^2 = \frac{L}{\cos\alpha}$.

$\therefore \frac{2V_0^2\sin(\beta-\alpha)}{g\cos\alpha}\left[\cos(\beta-\alpha) - \tan\alpha\sin(\beta-\alpha)\right] = \frac{L}{\cos\alpha}$.

$\Rightarrow V_0 = \sqrt{\frac{gL}{2\sin(\beta-\alpha)\cos\beta}}$

1.31 (1) $a_n = \frac{v_\tau^2}{R}$, 45°角时 $\tan\varphi = \frac{a_n}{a_\tau} = 1$ $\therefore \frac{v_\tau^2}{R} = a_\tau$

解得 $v_\tau = 3(m/s)$ $\Rightarrow t = \frac{v_\tau}{a_\tau} = 1(s)$

(2) $dS = v_t dt$, $v_\tau = a_t t$ $\therefore ds = at dt$

$\therefore \Delta S = \frac{1}{2}at^2 = 1.5(m)$

1.33 $\vec{r}(t) = r\hat{r}$

$\vec{V}(t) = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \frac{dr}{dt}\hat{r} + r\omega\hat{\theta}$

$\vec{a}(t) = \frac{d\vec{V}(t)}{dt} = \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\omega\hat{\theta} + \frac{dr}{dt}\omega\hat{\theta} + r\frac{d^2\theta}{dt^2}\hat{\theta} - r\omega^2\hat{r}$

$= \frac{d^2r}{dt^2}\hat{r} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)\hat{\theta} + r\frac{d^2\theta}{dt^2}\hat{\theta} - \cancel{r\omega^2} r\left(\frac{d\theta}{dt}\right)^2\hat{r}$

$\because a_0 = \frac{d^2r}{dt^2}$ $\therefore \frac{dr}{dt} = a_0 t$, $\frac{d\theta}{dt} = \omega_0$, $r = \frac{1}{2}a_0 t^2$

代入得 $\vec{V} = a_0 t\hat{r} + \frac{1}{2}a_0\omega_0 t^2\hat{\theta}$

$\vec{a} = (a_0 - \frac{1}{2}a_0\omega_0^2 t^2)\hat{r} + 2a_0\omega_0 t\hat{\theta}$

补充习题

1. 设有矢量 $\vec{A}=3\vec{i}+4\vec{j}-4\vec{k}$

(a) 在 x-y 平面找一 $\hat{B}$，使 $\hat{B}\perp\vec{A}$

解：$\hat{B}=\cos\theta\,\vec{i}+\sin\theta\,\vec{j}+0\vec{k}$

$\hat{B}\cdot\vec{A}=3\cos\theta+4\sin\theta=0 \Rightarrow \tan\theta=-\frac{3}{4}$，$\sin\theta=\pm\frac{3}{5}$

$\cos\theta=\mp\frac{4}{5}$

∴ $\hat{B}=\frac{3}{5}\vec{i}-\frac{4}{5}\vec{j}$ 或 $-\frac{3}{5}\vec{i}+\frac{4}{5}\vec{j}$

2. (b) 设 $\hat{C}=a\vec{i}+b\vec{j}+c\vec{k}$

$\hat{C}\cdot\vec{A}=0 \Rightarrow 3a+4b-4c=0$

$\hat{C}\cdot\hat{B}=0 \Rightarrow \frac{3}{5}a-\frac{4}{5}b=0$ 或 $-\frac{3}{5}a+\frac{4}{5}b=0$

① 若 $\hat{B}=\frac{3}{5}\vec{i}-\frac{4}{5}\vec{j}$，则 $\begin{cases}3a+4b-4c=0\\ \frac{3}{5}a-\frac{4}{5}b=0\\ a^2+b^2+c^2=1\end{cases} \Rightarrow \hat{C}=$

② 若 $\hat{B}=-\frac{3}{5}\vec{i}+\frac{4}{5}\vec{j}$，则 $\begin{cases}3a+4b-4c=0\\ -\frac{3}{5}a+\frac{4}{5}b=0\\ a^2+b^2+c^2=1\end{cases} \Rightarrow \hat{C}=$

(c) 证明：$\hat{B}\perp\vec{A}$，$\hat{C}\perp(\vec{A},\hat{B}$所在平面$)\Rightarrow \hat{C}\perp\vec{A}$

∴ $\vec{A}\perp(\hat{B},\hat{C}$所在平面$)$

(2) 设 $\vec{a}=(1,-2,1)$, $\vec{b}=(1,-1,3)$ $\vec{c}=(2,5,-3)$. 求: $\vec{a}\times\vec{b}$, $(\vec{a}\times\vec{b})\cdot\vec{c}$, $\vec{a}\times(\vec{b}\times\vec{c})$

① $\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ 1 & -1 & 3\end{vmatrix}=(-5,-2,1)$

② $(\vec{a}\times\vec{b})\cdot\vec{c}=-5\times2-2\times5-3\times1=-23$

③ $\vec{b}\times\vec{c}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ -12 & 9 & 7\end{vmatrix}=-12\vec{i}+9\vec{j}+7\vec{k}$

$\therefore \vec{a}\times(\vec{b}\times\vec{c})=(-23,-19,-15)$

3、求下列函数的导数.

(a) $y=8x^3+x+7 \Rightarrow y'=24x^2+1$

(b). $y=(x+1)(x-1)\tan x. \Rightarrow y'=2x\tan x+(x^2-1)\frac{1}{\cos^2 x}$

(c) $y=\frac{9x+x^2}{5x+6} \Rightarrow y'=\frac{(9+2x)(5x+6)-5(9x+x^2)}{(5x+6)^2}$

(d) $y=x\cos x+\frac{\sin x}{x} \Rightarrow y'=\cos x-x\sin x+\frac{\cos x}{x}-\frac{\sin x}{x^2}$

4、求不定积分

(a) $\int(3x^3+\sin x+\frac{5}{x})dx=\frac{3}{4}x^4-\cos x+5\ln x+\text{Const}$

(b) $\int\sqrt{a^2-x^2}\,dx\ (a>0)=a^2\int\sqrt{1-(\frac{x}{a})^2}\,d(\frac{x}{a})\quad(a>0)$

$=a^2\int\sqrt{1-\sin^2\theta}\,d\sin\theta\quad(a>0,\theta=\frac{x}{a})$

$=a^2\int\cos^2\theta\,d\theta=\frac{a^2}{2}\int(1+\cos2\theta)d\theta$

$= \frac{a^2}{2}(\theta + \frac{1}{2}\sin 2\theta) + Const$

$= \frac{a^2}{2}[\frac{x}{a} + \frac{1}{2}\sin(\frac{2x}{a})] + Const$

~~另解：$\int \sqrt{a^2-x^2}\,dx = \int \sqrt{}$~~

5. 验证函数 $y = C_1\cos wt + C_2\sin wt$（其中 C_1. C_2 是任意常数）是微分方程 $y'' + w^2 y = 0$ 的解.

证明：$y'' = -w^2 C_1 \cos wt - w^2 C_2 \sin wt$

$\therefore y'' + w^2 y = 0$

6. 电梯有 $a(t) = \begin{cases} (a_m/2)(1-\cos(2\pi t/T)), & 0 \le t \le T \\ -(a_m/2)(1-\cos(2\pi t/T)), & T \le t \le 2T \end{cases}$

(a) 电梯运行的最大速度是多少？

依题意，$0 \le t \le T$ 时，$V(t) = \int_0^t a(t)\,dt$

$= (a_m/2)[t - \frac{T}{2\pi}\sin(\frac{2\pi t}{T})]\Big|_0^t$

$= (a_m/2)[t - \frac{T}{2\pi}\sin(\frac{2\pi t}{T})]$

~~$V(t) = V(t)_{max}$ 时 $a(t) = 0 \Rightarrow t = 0$ 或 T~~

$0 \le t \le T$ 时，由于 $1-\cos(\frac{2\pi t}{T}) \ge 0$ $\therefore a(t) \ge 0$

$T \le t \le 2T$ 时，由于 $-[1-\cos(\frac{2\pi t}{T})] \le 0$ $\therefore a(t) \le 0$

$\therefore V(t)_{max} = V(t)\big|_{t=T} = \frac{a_m T}{2}$

(b) $t \ll T \Rightarrow V(t) \cancel{\approx [a(t)|_{t=0}]\,t} = (a_m/2)[t - \frac{T}{2\pi}\sin(\frac{2\pi t}{T})]$

$\approx (a_m/2)[t - t] \approx 0$

(c) $0\le t\le T$ 时. $\cancel{x(t)}\ x_1=\int_0^T V(t)dt=\frac{a_m}{2}\int_0^T[t-\frac{T}{2\pi}\sin(\frac{2\pi t}{T})]dt$

$$=\frac{a_m}{2}\left[\frac{1}{2}t^2+\frac{T^2}{2\pi^2}\cos(\frac{2\pi t}{T})\right]\Big|_0^T$$

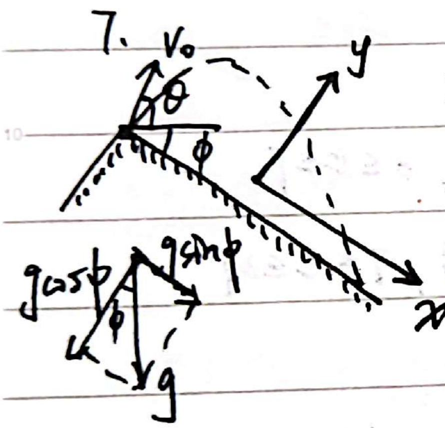
$$=\frac{a_m}{4}T^2$$

由运动的对称性知，$T\le t\le 2T$ 时，$x_2=x_1$

$\therefore D=x_1+x_2=\frac{a_m}{2}T^2$

$\therefore t_{总}=2T=2\sqrt{\frac{2D}{a_m}}$

7.



设初速度 V_0 给定.

以抛出点为原点，沿斜面向下为x正方向.

~~沿斜~~ 垂直斜面向上的y轴正方向.

$a_x=g\sin\phi \qquad a_y=-g\cos\phi$

$V_{x0}=V_0\cos(\theta+\phi) \qquad V_{y0}=V_0\sin(\theta+\phi)$

石头在y方向的运动为上抛运动.

$\therefore$ 从抛出到落地总用时 $t_{总}=\frac{2V_{y0}}{|a_y|}$

$\therefore t_{总}=\frac{2V_0\sin(\theta+\phi)}{g\cos\phi}$

$$x=V_{x0}t_{总}-\frac{1}{2}a_x t_{总}^2$$

$$=\frac{2V_0^2}{g}\left[\frac{\cos(\theta+\phi)\sin(\theta+\phi)}{\cos\phi}-\frac{\sin\phi}{\cos^2\phi}\sin^2(\theta+\phi)\right]$$

$$\frac{dx}{d\theta}=\frac{2v_0^2}{g}\cdots$$ 整理得：

$$x=\frac{1}{2\cos^2\phi}\sin(2\theta+3\phi)-\frac{1}{2}\frac{\sin\phi}{\cos^2\phi}$$

$\therefore x_{max}=\frac{1-\sin\phi}{2\cos^2\phi}$，当仅 $2\theta+3\phi=\frac{\pi}{2}+2k\pi$ 时.

对应 $\theta=\frac{\frac{\pi}{2}-3\phi+2k\pi}{2}=\frac{\pi}{4}-\frac{3}{2}\phi+k\pi$

且 $\theta<\frac{\pi}{2}$.

8、 $c=\sqrt{A^2-B^2}$

(a) 取 $\theta=0$ 时 $\frac{r_0}{1+e}=A-c=A-\sqrt{A^2-B^2}$

取 $\theta=\pi$ 时 $\frac{r_0}{1-e}=A+c=A+\sqrt{A^2-B^2}$.

作商 $\Rightarrow \frac{1-e}{1+e}=\frac{A-\sqrt{A^2-B^2}}{A+\sqrt{A^2-B^2}}\Rightarrow e=\frac{\sqrt{A^2-B^2}}{A}$

$$r_0=(A-\sqrt{A^2-B^2})(1+e)=(A-\sqrt{A^2-B^2})\left(1+\frac{\sqrt{A^2-B^2}}{A}\right)$$

$$=\frac{A^2-(A^2-B^2)}{A}=\frac{B^2}{A}$$

(b) $v_\theta=r\frac{d\theta}{dt}=r\omega=\frac{r_0\omega}{1+e\cos\omega t}=\frac{r_0\omega}{1+e\cos\theta}$.

$$a_\theta=r\frac{d^2\theta}{dt^2}+2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)$$

$$=2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right)=2\omega\left(\frac{dr}{dt}\right)=2\omega^2\left(\frac{dr}{d\theta}\right)=2\omega^2\cdot\frac{r_0e\sin\theta}{(1+e\cos\theta)^2}$$

$$=\frac{2\omega^2r_0e\sin\theta}{(1+e\cos\theta)^2}$$

9. $v_\theta = r\frac{d\theta}{dt}$. $\cancel{\rho = v_\theta}$ $\frac{v_\theta^2}{\rho} =$

设螺线上(θ, r)处对应的曲率圆圆心在(θ_1, r_1)处

则 $\theta_1 = \theta_1(\theta)$, $r_1 = r_1(\theta)$

由曲率圆的定义，知此时 $\frac{d}{d\theta}\sqrt{r_1^2+r^2-2rr_1\cos(\theta-\theta_1)}=0$

$$\frac{d}{d\theta}\sqrt{r_1^2+r^2-2rr_1\cos(\theta-\theta_1)}=0$$

$$\Rightarrow \frac{2r_1\frac{dr_1}{d\theta}+2r(\alpha r_0 e^{\alpha\theta})-\frac{d}{d\theta}[2rr_1\cos(\theta-\theta_1)]}{2\sqrt{r_1^2+r^2-2rr_1\cos(\theta-\theta_1)}}=0$$

$$\Rightarrow 2r_1\frac{dr_1}{d\theta}+2\alpha r^2-2\alpha rr_1\cos(\theta-\theta_1)-2r\frac{dr_1}{d\theta}\cos(\theta-\theta_1)+2rr_1\sin(\theta-$$

不太会解?