Memo No. ____________
Date 2025 / 9 / 20

1.1 解:(1) 鸟往返无数次 (1)

(2) $t = \dfrac{\frac{1}{2}s}{v_{车}} = 1h$ (2)

$s = v_{鸟} t = 60km$ (3)

1.3. 解:(1) $x = 10t^2 + 6$ (1)

$v = \dfrac{dx}{dt} = 20t$ (2)

物体做匀加速直线运动.

$\overline{v}_{3.00\sim3.10s} = v_{3.05s} = 61\,m/s$ (3)

$\overline{v}_{3.00\sim3.01s} = v_{3.005s} = 60.1\,m/s$ (4)

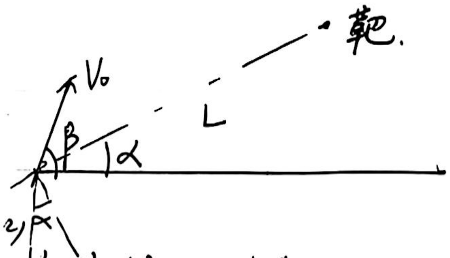
$\overline{v}_{3.00\sim3.001s} = v_{3.0005s} = 60.01\,m/s$ (5)

(2) $v_{3.00s} = 60.0\,m/s$

(3) $v = 20t$ (6)

$a = \dfrac{dv}{dt} = 20\,m/s^2$ (7)

1.25.



解: $t = \dfrac{2V_0\sin(\beta-\alpha)}{g\cos\alpha}$ (1)

$V_0\cos(\beta-\alpha)t - \frac{1}{2}g\sin\alpha\, t^2 = L$ (2)

代入: $\dfrac{2V_0^2\sin(\beta-\alpha)\cos(\beta-\alpha)}{g\cos\alpha} - \dfrac{1}{2}\,\dfrac{4V_0^2\sin^2(\beta-\alpha)\sin\alpha}{g\cos^2\alpha} = L$

$\Rightarrow \dfrac{2V_0^2}{g\cos\alpha}\left[\sin(\beta-\alpha)\cos(\beta-\alpha)\cos\alpha - \sin^2(\beta-\alpha)\sin\alpha\right] = L.$

$\Rightarrow V_0 = \sqrt{\dfrac{Lg\cos\alpha}{2\cos\beta\sin(\beta-\alpha)}}$

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1.31 解: (1) $a_n = a_\tau = \frac{V^2}{R} = \omega^2 R$ (1)

$R\omega = a_\tau t$ (2)

则 $t = 1s$ (3)

(2) $\theta = \frac{1}{2}\frac{a_\tau}{R}t^2 = 0.5\,rad$ (4)

$s = R\theta = 1.5\,m$ (5)

1.33 解: (1) $v = a_0 t\hat{r} + \frac{1}{2}a_0 t^2\omega_0\hat{\theta}$ (6)

(2) $a = \frac{dv}{dt} = a_0\hat{r} + a_0\omega_0 t\hat{\theta} + a_0 t\omega_0\hat{\theta} - \frac{1}{2}a_0\omega_0^2 t^2\hat{r}$

$= (a_0 - \frac{1}{2}a_0\omega_0^2 t_0^2)\hat{r} + 2a_0\omega_0 t\hat{\theta}$

简答题:

1. $\left|\frac{d\vec{v}}{dt}\right| = 0$ 表示匀速直线运动或静止,无加速度

$\frac{d|\vec{v}|}{dt} = 0$ 表示速率不变运动,但仍可能有加速度

(如匀速圆周运动)

2. 可改变,速度与加速度不同向

3. (1) X (匀速圆周) (2) √

(3) X (匀速圆周)

4 ①有空气阻力,球致不了篮框

② 瞄准篮板投篮。

补充:

(1) 令 $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\begin{cases} \vec{A}\cdot\vec{B} = 3x+4y-4z=0 \\ x^2+y^2+z^2=1 \end{cases}$$

取 $z=0$. 则 ~~xy~~ 有 $x=\frac{4}{5}$, $y=-\frac{3}{5}$

则 $\vec{B} = \cancel{(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})}\ (\frac{4}{5}, -\frac{3}{5}, 0)$

(2) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & -4 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{vmatrix} = (0, -\frac{3}{2}\sqrt{2}, \frac{3}{2}\sqrt{2})$

则 $\vec{C} = 0\cdot\hat{i} - \frac{3}{2}\sqrt{2}\,\hat{j} + \frac{3}{2}\sqrt{2}\,\hat{k}$

(3) $\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & -\frac{3}{2}\sqrt{2} & \frac{3}{2}\sqrt{2} \end{vmatrix} = ($

~~2. 解: (1)~~

(2) 令 $\vec{C} = (x, y, z)$.

$$\begin{cases} 3x+4y-4z=0 \\ \frac{\sqrt{2}}{2}y+\frac{\sqrt{2}}{2}z=0 \\ x^2+y^2+z^2=1 \end{cases}$$

令 ~~x=0~~ $x=-12$. 则 $y=-16$. $z=-28$

$\vec{C} = (-\frac{12}{5\sqrt{41}}, -\frac{16}{5\sqrt{41}}, \frac{-5}{\sqrt{41}})$

(3) 由于 $\vec{A}\cdot\vec{B}=0$.

$\vec{A}\cdot\vec{C}=0$ 则 $\vec{A}\perp\vec{B}\vec{C}$平面

2. ~~$\vec{a}\times\vec{b}=$~~

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 1 & -1 & 3\end{vmatrix}=(-5,-2,1)$$

$$(\vec{a}\times\vec{b})\cdot\vec{c}=-5\times2+(-2)\times5+(-3)\times1=\cancel{-3}\ -3$$

$$\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 3\\ 2 & 5 & 1\end{vmatrix}=\vec{a}\times(-16,5,7)$$

$$=-16-10+7=-19.$$

3. (a) $y'=24x^2+1.$

(b) $y'=\dfrac{x^2-1}{\cos^2x}+2x\cdot\tan x$

(c) $y'=\dfrac{(2x+9)(5x+6)-5(x^2+9x)}{(5x+6)^2}=\cancel{10x}\ \dfrac{5x^2+12x+54}{(5x+6)^2}$

(d) $y'=\cos x-x\sin x+\dfrac{x\cos x-\sin x}{x^2}$

4. (a) $\int(3x^3+\sin x+\frac{5}{x})dx=\frac{3}{4}x^4\cancel{+}-\cos x+5\ln x\cancel{+C}$

(b) $\int\sqrt{a^2-x^2}\,dx\ (a>0)$

$=\int a\sqrt{1-(\frac{x}{a})^2}\,dx$

令 $\frac{x}{a}=\cancel{k}\sin\alpha.\quad x=\cancel{a}\,a\sin\alpha$

$\cancel{1}$原式$=\int a^2\cancel{\sin}\cos\alpha\,d\sin\alpha=\int a^2\cdot\frac{1}{4}(\cos2\alpha-1)\cdot d2\alpha$

$=\frac{1}{4}a^2(\sin2\alpha-2\alpha)+C$

$=\frac{1}{4}a^2\left[\sin2\cdot(\arcsin\frac{x}{a})-2\arcsin\frac{x}{a}\right]+C$

5. $y = C_1\cos\omega t + C_2\sin\omega t$

$y'' = -C_1\omega^2\cos\omega t - \omega^2 C_2\sin\omega t$

代入：$y'' + \omega^2 y = (C_1\omega^2 - C_1\omega^2)\cos\omega t + C_2\sin\omega t(\omega^2-\omega^2)$
$=0$

则是一个解。

6.(a) $V_{max} = \int_0^T \frac{a_m}{2}(1-\cos\frac{2\pi t}{T})dt = \frac{a_m T}{2}$

(b) $V = \frac{a_m}{2}(t - \frac{T}{2\pi}\sin\frac{2\pi}{T}t)$

当 $t \ll T$ 时，$\frac{t}{T} \ll 1$ 则 $V \approx \frac{a_m}{2}(t-t)=0$

(c) $V = \frac{a_m}{2}(t - \frac{T}{2\pi}\sin\frac{2\pi}{T}t)$

$L = \int_0^t V dt = \frac{1}{4}a_m t^2 - \frac{1}{2}a_m(\frac{T}{2\pi})^2\cdot[1-\cos\frac{2\pi}{T}t]$

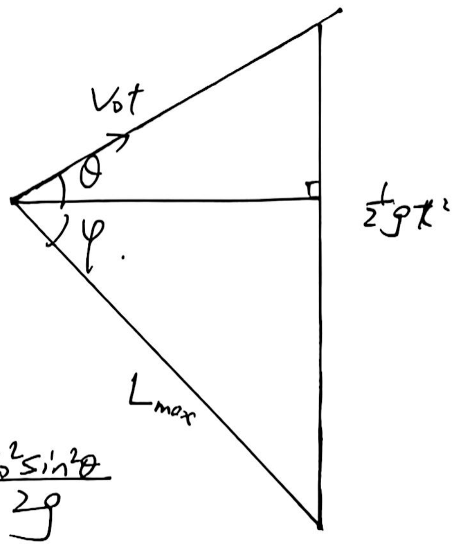
令 $t=T$，则 $L' = \frac{1}{4}a_m T^2 - \frac{1}{2}a_m(\frac{T}{2\pi})^2[1-1] = \frac{1}{4}a_m T^2$

令 $D = L + nL'$ $(n=0,1,2\cdots)$

则 $t = t + nT$

7. 如图：令初速度为 V_0

最远距离为 L_{max}



由图：$\frac{1}{2}L_{max}\cdot\cos\varphi\cdot\frac{1}{2}gt^2$

$= \frac{1}{2}V_0 t\cdot L_{max}\cdot\sin(\theta+\varphi)$

则：$t = \frac{2V_0}{g}\frac{\sin(\theta+\varphi)}{\cos\varphi}$

$t_1 = \frac{V_0}{g}\sin\theta$ $\quad h_1 = \frac{1}{2}gt^2 = \frac{V_0^2\sin^2\theta}{2g}$

$\frac{1}{2}gt_2^2 = h_1 + L_{max}\sin\varphi$

则 $t_2 = \sqrt{\frac{V_0^2\sin^2\theta}{g^2} + \frac{2L_{max}\sin\varphi}{g}}$

$t_1+t_2=t$. 则 $L_{max} = \frac{4V_0^2}{g\sin\varphi\cos\varphi}[\sin^2(\theta+\varphi) - \sin\theta\cos\varphi\sin(\theta+\varphi)]$

则令 $\frac{dL_{max}}{d\theta}=0$ 则 $2\theta+\varphi=\frac{\pi}{2}$ $\quad \theta = \frac{1}{2}(\frac{\pi}{2}-\varphi)$

8. 解：(a) 令 $\cancel{\cos\theta=}$ $\theta=0$，则 $\cos\theta=1$

$$r=\frac{r_0}{1+e}=A-C$$

令 $\theta=\pi$，则 $\cos\theta=-1$

$$r=\frac{r_0}{1-e}=A+C$$

$$A=\frac{2r_0}{1-e^2}\qquad C=\frac{2er_0}{1-e^2}$$

$$B=\sqrt{A^2-C^2}=\frac{2r_0}{\sqrt{1-e^2}}$$

(b) $V_\theta=r\dot\theta=\frac{r_0}{1+e\cos\omega t}\cdot\omega$

$$a_\theta=r\ddot\theta+2\dot r\dot\theta=2\dot r\dot\theta=2\omega\frac{r_0\omega}{(1+e\cos\omega t)^2}\sin\omega t$$

$$=\frac{2r_0\omega^2\sin\omega t}{(1+e\cos\omega t)^2}$$

$$\boxed{\begin{aligned}&9.\quad a_n=\frac{V^2}{\rho}\quad 则\ \rho=\frac{V^2}{a_n}\\&V=\dot r=r_0\alpha\theta\cdot e^{\alpha\theta}\cdot\dot\theta\\&a_n=r\ddot\theta+2\dot r\dot\theta=r_0e^{\alpha\theta}\ddot\theta+2r_0\alpha\theta\cdot e^{\alpha\theta}\dot\theta^2\\&不妨令\ \theta=\omega t\\&则\ \ddot\theta=0，则\ \rho=\frac{V^2}{a_n}=\frac{1}{2}r_0\alpha\theta e^{\alpha\theta}\end{aligned}}$$

9. $\cancel{a_n=}$ $\rho=\frac{V^3}{|V\times a|}$ $\qquad r=r_0e^{c\theta}$

令 $\ddot\theta=0$，$\dot\theta=\text{const}$

$\dot r=\frac{dr}{d\theta}\dot\theta \qquad V=r|\omega|\sqrt{1+\alpha^2}$

$\ddot r=\frac{d}{dt}(\alpha r\omega)$

则 $a_r=r\omega^2(\alpha^2-1)+\cancel{\alpha r}\ddot\theta$

$a_\theta=r\ddot\theta+2\alpha r\omega^2$

$\vec V\times\vec a=r^2\omega^3(\alpha^2+1)$

代入：$\rho(\theta)=r_0e^{\alpha\theta}\sqrt{1+\alpha^2}=r\sqrt{1+\alpha^2}$