1. $\left|\frac{d\vec{v}}{dt}\right|=0$ 代表了速度不变的运动，是一种匀速直线运动

而 $\frac{d|\vec{v}|}{dt}=0$，代表了速度"大小"不变的运动，这可以是曲线运动，比如说匀速圆周

2. 可以（比如斜抛就是这样的运动） 因为如果加速度（恒定）与速度方向不共线，速度方向会产生变化，运动的方向也就改变了

3. (1) 错误，比如匀速圆周运动没有切向加速度

(2) 正确 $\vec{a}=(\ddot{r}-\dot{\theta}^2 r)\hat{r}+(r\ddot{\theta}+2\dot{r}\dot{\theta})\hat{\theta}$

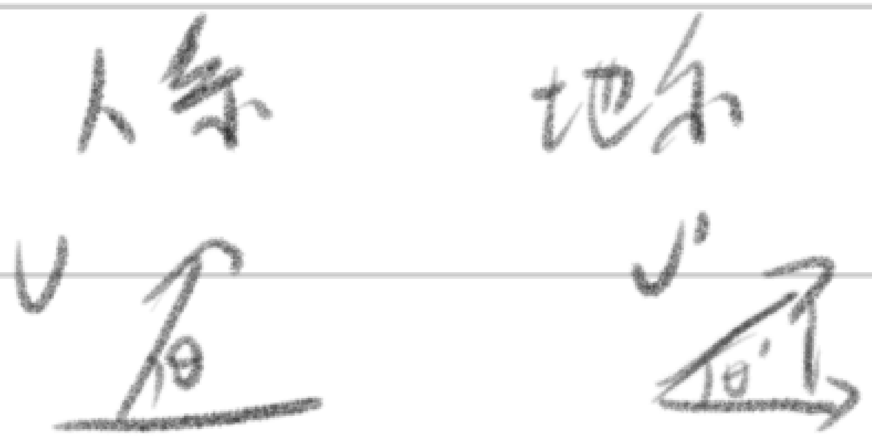
$\frac{v^2}{\rho}=\vec{a}\times\hat{e}_{\theta}=\frac{|\vec{a}\times\vec{v}|}{v}$，$\Rightarrow$ $\rho=\frac{v^3}{|\vec{a}\times\vec{v}|}$

$\vec{a}\times\vec{v}$ 表示的正是 $\vec{a}$ 在法向的投影，对曲线运动，$\rho>0$，$|\vec{a}\times\vec{v}|>0$，

则 $\frac{|\vec{a}\times\vec{v}|}{|\vec{v}|}\neq 0$，即 $\vec{a}_n\neq 0$，那么就有法向加速度

4. 重点在于"跑步投篮"说明运动员有一个自身速度（对地）v_1

在地面系下， $v'\cos\theta' = v\cos\theta + v_1$ ， $v'\sin\theta' = v\sin\theta$



水平方向的速度增量导致了 $\theta' < \theta$

要想重新投进，运动员应当要增加投篮仰角

教材习题

1.1 以甲车为参考系

$v_2 = 30 + 30 = 60\ km/h$ $\quad t_{碰} = \frac{d}{v_2} = 1h\ (d = 60km)$ 即 $t_{飞} = 1h$

（这是由于两车相碰之前鸟一直在飞）

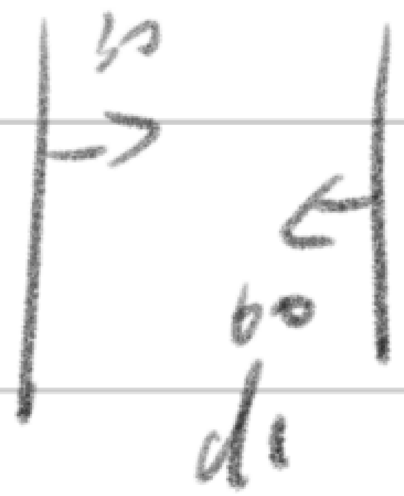
在地系看鸟， $s_{鸟} = v_{鸟} t_{飞} = 60 \times 1 = 60km$ （地面系看这个很清晰）

接下来证明会有无限次碰撞 （见下页）

证明：设小鸟某一次到达甲车处时，甲乙两车的间距为 d_1

甲车系

1.

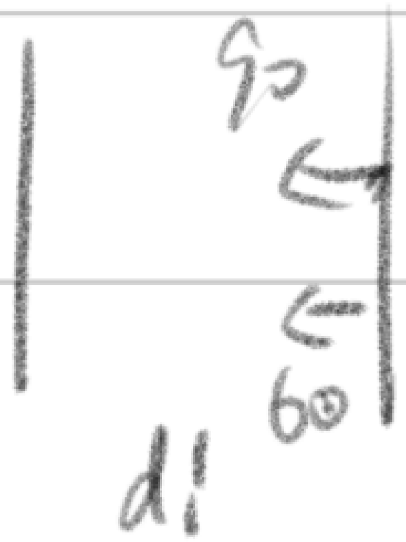


$V_{鸟对甲}=60-30=30\,km/h$

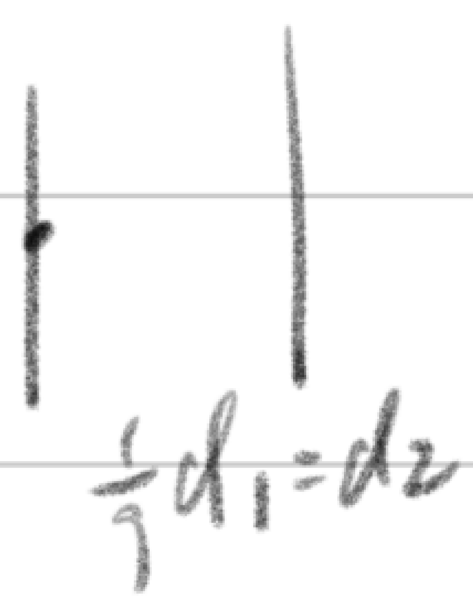
$\Delta t_1=\frac{d_1}{30+60}=\frac{d_1}{90}$ $\qquad d_1'=d_1-\frac{d_1}{90}\cdot 60=\frac{1}{3}d_1$

$V_{鸟对甲}=30+60=90\,km/h$

2.



$\Delta t_2=\frac{d_1'}{90}$ $\qquad d_2=d_1'-\frac{d_1}{3\cdot 90}\cdot 60=\frac{1}{9}d_1$



当 d_1 变为 $\frac{1}{9}d_1$ 的时候，这样的循环重新开始，

即 $d_n=60\cdot(\frac{1}{9})^{n-1}$. 这个数列每一项对应第 $(n-1)$ 次碰撞后的情况，这个数列有无限项，说明了会碰无限次

即：小鸟往返无穷多次，飞行了 1h，飞过了 60km（希望最后没有被压扁）

1.3 $x = 10t^2+6 = x(t)$

(1) $t = 3.00s$, $x = 96$, $t = 3.10s$, $x = 102.1m$, $\Delta x = 6.1m$, $\frac{\Delta x}{\Delta t} = \bar{v} = 61 m/s$

$t = 3.01s$, $x = 96.601m$, $\Delta x = 0.601m$, $\frac{\Delta x}{\Delta t} = \bar{v} = 60.1 m/s$

$t = 3.001s$, $x = 96.06001m$, $\Delta x = 0.06001$, $\frac{\Delta x}{\Delta t} = \bar{v} = 60.01 m/s$

(2) $v(t=3s) = \lim_{t\to 3s} \frac{dx}{dt} = \lim_{t\to 3s} (20t) = 60 m/s$

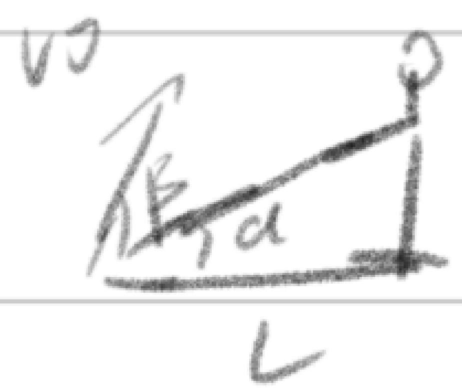
(3) $v(t) = \frac{dx}{dt} = 20t \ (m/s)$, $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 20 m/s^2$

1.25

$$\begin{cases} v_0\cos\beta\, t = L \\ v_0\sin\beta\, t - \frac{1}{2}gt^2 = L\tan\alpha \end{cases}$$

$t = \frac{L}{v_0\cos\beta}$，代入②

$v_0\sin\beta\left(\frac{L}{v_0\cos\beta}\right) - \frac{1}{2}g\left(\frac{L}{v_0\cos\beta}\right) = L\tan\alpha$



$$v_0^2 = \frac{gL}{2\cos^2\beta(\tan\beta - \tan\alpha)}$$

即 $v_0 = \sqrt{\frac{gL}{2\cos^2\beta(\tan\beta-\tan\alpha)}}$ ⇒ 这与 $v_0 = \sqrt{\frac{Lg}{2\cos\beta\sin(\beta-\alpha)}}$ 是等价的

1.31

$v_\tau(t) = 3t$，$a_\tau = 3m/s^2$　　$\ddot{r} = 0$

$a_n = \frac{(v_\tau(t))^2}{r} = \frac{9t^2}{3} = 3t^2$，当 a与半径成 $\frac{\pi}{4}$，$a_n = a_\tau$

即 $3t^2 = 3$，$t = 1s$ $(t>0)$

$s = \int_0^1 v_\tau(t)\,dt = \frac{3}{2}t^2\Big|_0^1 = \frac{3}{2}m$

即，$t=1s$，$s=1.5m$

1.33 $v_\theta = \omega_0 r = \omega_0 \cdot \frac{1}{2}at^2 \hat{\theta}$

$v_r = a_0 t \ \hat{r}$ $\qquad v = \frac{1}{2}a\omega_0 t^2 \hat{\theta} + a_0 t \hat{r}$

$\frac{dv}{dt} = \frac{1}{2}a\omega_0 \cdot 2t\hat{\theta} - \frac{1}{2}a\omega_0 t^2 \frac{d\theta}{dt}\hat{r}$

$+ a_0 \hat{r} + a_0 t \frac{d\theta}{dt}\hat{\theta} \quad = (a_0 - \frac{1}{2}a_0\omega_0^2 t^2)\hat{r} + 2a_0\omega_0 t\hat{\theta}$

补充问题

1. (a), $\vec{A} = 3\hat{i} + 4\hat{j} - 4\hat{k}$ 设 $\vec{B} = x\hat{i} + y\hat{j} + 0\hat{k}$ ($\vec{B}$在x-y平面内, $B_z=0$)

$\vec{A}\cdot\vec{B}=0$, $3x+4y=0$, 且 $x^2+y^2=1$, 其中一个解可以是

$\vec{B} = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j} + 0\hat{k}$

(b) $\vec{A}\times\vec{B} = \frac{12}{5}\hat{i} + \frac{16}{5}\hat{j} + 5\hat{k}$ (这向量垂直于$\vec{A}$,$\vec{B}$所构成的平面, 其模长为$\sqrt{41}$, $\vec{C}$是单位向量

则一个合理的$\vec{C}$可以是 $\frac{1}{\sqrt{41}}(\frac{12}{5}\hat{i} + \frac{16}{5}\hat{j} + 5\hat{k}) = \vec{C}$

(c) $\vec{B}\times\vec{C} \neq 0$, $\vec{B}$,$\vec{C}$可构成平面, $\vec{A}\perp\vec{B}$ ($\vec{A}\cdot\vec{B}=0$) 且 $\vec{A}\perp\vec{C}$, 则 $\vec{A}\perp$($\vec{B}$与$\vec{C}$所在的平面)

2. $\vec{a}\times\vec{b} = (-5,-2,1)$ $\quad (\vec{a}\times\vec{b})\cdot\vec{c} = -10-10-3 = -23$

$\vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(-12,9,7) = (-23,-19,-15)$

3. (1) $y' = 24x^2+1$ $\quad$ (2) $y=(x^2-1)\tan x$, $y' = 2x\tan x + (x^2-1)\frac{1}{\cos^2 x}$

(3) $y' = \dfrac{(9+2x)(5x+6)-5(9x+x^2)}{(5x+6)^2} = \dfrac{5x^2+12x+54}{(5x+6)^2}$

(4) $y' = \cos x - x\sin x + \dfrac{x\cos x - \sin x}{x^2}$

4. (a) $\int(3x^2+\sin x+\frac{5}{x})dx = \int 3x^2dx + \int\sin x\,dx + \int\frac{5}{x}dx = x^3 + 5\ln x - \cos x + C$

(b) $\int\sqrt{a^2-x^2}\,dx$，令 $x = a\sin\theta$， 原式 $= \int a\cos\theta\cdot a\cos\theta\,d\theta = \int a^2\cos^2\theta\,d\theta = a^2\int\left(\frac{\cos2\theta+1}{2}\right)d\theta$

$= a^2\left(\frac{1}{4}\sin2\theta + \frac{1}{2}\theta\right) + C$

5. $y=c_1\cos\omega t+c_2\sin\omega t$, $y''=-c_1\cos\omega t\cdot\omega^2-c_2\sin\omega t\cdot\omega^2=\omega^2 y$

即 $y=c_1\cos\omega t+c_2\sin\omega t$ 为 $y''+\omega^2 y=0$ 的解

6. 

(1) $a(t)=\frac{a_m}{2}\left(1-\cos\frac{2\pi t}{T}\right)\quad(0\le t\le T)$, $V_m=\int_0^T a(t)\,dt$

$$V_m=\frac{a_m}{2}t-\frac{a_m}{2}\left(\frac{T}{2\pi}\right)\left(\sin\frac{2\pi t}{T}\right)\Big|_0^T=\frac{a_m}{2}T-\frac{a_m}{2}\frac{T}{2\pi}\left(\sin(2\pi)-\sin 0\right)$$

$$=\frac{a_m}{2}T$$

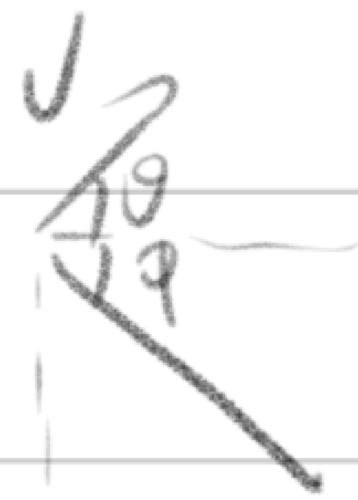
(2) 当 $t\ll T$，由 $\sin x\approx x-\frac{x^3}{6}$ $\quad V=\frac{a_m}{2}t-\frac{a_mT}{4\pi}\left(\sin\frac{2\pi t}{T}\right)$

$$=\frac{a_mT}{4\pi}\cdot\frac{1}{6}\left(\frac{2\pi t}{T}\right)^3=\frac{a_m\pi^2t^3}{3T^2}$$

(3) $\int_0^T V(t)\,dt=\int_0^T\frac{a_m}{2}t\,dt-\frac{a_m}{2}\frac{T}{2\pi}\cdot\frac{\sin\frac{2\pi t}{T}}{}dt=\left(\frac{a_m}{4}t^2+\frac{a_mT}{4\pi}\cdot\frac{T}{2\pi}\cos\frac{2\pi t}{T}\right)\Big|_0^T=\frac{a_m}{4}T^2$

$$D=2\times\frac{a_m}{4}T^2=\frac{a_m}{2}T^2$$

7.



$$\begin{cases} V\cos(\theta+\phi)t + \frac{1}{2}g\sin\phi t^2 = x \\ t = 2\cdot\frac{V\sin(\theta+\phi)}{g\cos\phi} \end{cases}$$

即 $x = V\cos(\theta+\phi)\cdot\frac{2V\sin(\theta+\phi)}{g\cos\phi} + \frac{1}{2}\sin\phi\cdot\frac{4V^2\sin^2(\theta+\phi)}{g\cos^2\phi}$

提公因式，即研究 $\frac{\sin(\theta+\phi)\cos(\theta+\phi)}{\cos\phi} + \frac{\sin^2(\theta+\phi)\sin\phi}{\cos^2\phi}$ 即可

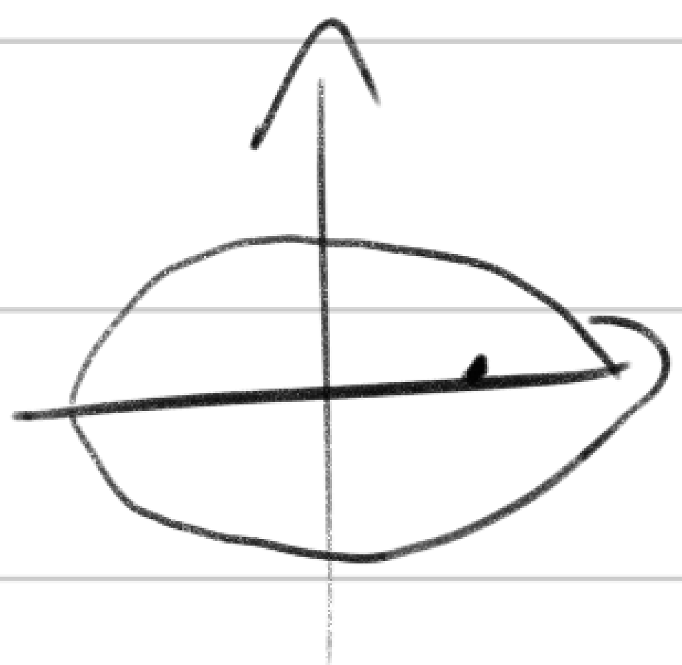
即为 $\frac{\sin(\theta+\phi)(\cos\phi\cos(\theta+\phi)+\sin\phi\sin(\theta+\phi))}{\cos^2\phi} = \frac{\sin(\theta+\phi)\cos\theta}{\cos^2\phi}$

即研究 $\sin(\theta+\phi)\cos\theta$ 即可，即对 $f(\theta) = \sin(\theta+\phi)\cos\theta$ 求导

$\cos(\theta+\phi)\cos\theta - \sin(\theta+\phi)(\sin\theta) = \cos(2\theta+\phi) = 0$

即 $2\theta+\phi = \frac{\pi}{2}$ 或 3π(舍) $\quad 2\theta+\phi = \frac{\pi}{2}$， $\theta = \frac{\pi}{4} - \frac{\phi}{2}$

8. $r=\frac{r_0}{1+e\cos\theta}$



$$A-C=\frac{r_0}{1+e} \qquad A+C=\frac{r_0}{1-e} \qquad A=\frac{1}{2}\left(\frac{r_0}{1+e}+\frac{r_0}{1-e}\right)$$

$$C=\frac{1}{2}\left(\frac{r_0}{1-e}-\frac{r_0}{1+e}\right) \qquad A^2=B^2+C^2$$

$$A=\frac{r_0}{1-e^2}, \quad C=\frac{er_0}{1-e^2}=eA, \quad B^2=(1-e^2)A^2$$

$$B=\sqrt{1-e^2}\,\frac{r_0}{1-e^2}$$

$\vec{r}=\frac{r_0}{1+e\cos\theta}\hat{r}$，其中 $\theta=\omega t$，以右焦点为原点建立极坐标系。

$$\frac{d\vec{r}}{dt}=\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}=\frac{r_0 e\omega\sin\theta}{(1+e\cos\theta)^2}\hat{r}+\underbrace{\frac{r_0\omega}{1+e\cos\theta}\hat{\theta}}_{v_\theta} \quad \text{(速度)}$$

$$\frac{d^2\vec{r}}{dt^2}=(\ddot{r}-\dot{\theta}^2 r)\hat{r}+(r\ddot{\theta}+2\dot{r}\dot{\theta})\hat{\theta}$$

$$=\left(r_0 e\omega\left(\frac{\omega\cos\omega t(1+e\cos\omega t)^2-2(1+e\cos\omega t)(-e\omega\sin^2\omega t)}{(1+e\cos\omega t)^4}\right)-\omega^2\frac{r_0}{1+e\cos\theta}\right)\hat{r}$$

$$+\underbrace{\left(0+2\frac{r_0\omega^2 e\sin\omega t}{(1+e\cos\omega t)^2}\right)\hat{\theta}}_{a_\theta}$$

9. $r = r_0 e^{\alpha\theta}\hat{e}_r \quad \rho = \frac{v^3}{|\vec{a}\times\vec{v}|}$

$\vec{v} = (\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta) = (\frac{dr}{d\theta}\dot{\theta}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$

$\vec{a} = (\ddot{r} - \dot{\theta}^2 r)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$

$\frac{dr}{d\theta} = r_0 \alpha e^{\alpha\theta}$

$v = \dot{\theta}\sqrt{(r_0\alpha e^{\alpha\theta})^2 + (r_0 e^{\alpha\theta})^2}$

$\ddot{r} = \frac{d}{dt}(\frac{dr}{d\theta}\dot{\theta}) = \frac{d(\frac{dr}{d\theta})}{d\theta}\dot{\theta}^2 + \ddot{\theta}\frac{dr}{d\theta}$

$\frac{d^2r}{d\theta^2} = r_0\alpha^2 e^{\alpha\theta}$

$= \frac{d^2r}{d\theta^2}\dot{\theta}^2 + \ddot{\theta}\frac{dr}{d\theta}$

$\dot{r} = r_0\alpha e^{\alpha\theta}\dot{\theta}$

$$\vec{a}\times\vec{v} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_z \\ \ddot{r} - \dot{\theta}^2 r & 2\dot{r}\dot{\theta} + r\ddot{\theta} & 0 \\ \dot{r} & r\dot{\theta} & 0 \end{vmatrix} \rightarrow (\ddot{r}r\dot{\theta} - \dot{\theta}^3 r^2) - (2\dot{r}^2\dot{\theta} + r\dot{r}\ddot{\theta})\hat{e}_z$$

$\ddot{r} = r_0\alpha^2 e^{\alpha\theta}\dot{\theta}^2 + \ddot{\theta} r_0 \alpha e^{\alpha\theta}$

$$= r_0^2\alpha^2(e^{\alpha\theta})^2\dot{\theta}^3 + \cancel{\alpha\ddot{\theta}\dot{\theta}^2(r_0 e^{\alpha\theta})^2} - \dot{\theta}^3(r_0 e^{\alpha\theta})^2 - 2(r_0\alpha e^{\alpha\theta})^2\dot{\theta}^3 - \cancel{\alpha(r_0 e^{\alpha\theta})^2\dot{\theta}\ddot{\theta}}$$

$$= -r_0^2\alpha^2 e^{2\alpha\theta}\dot{\theta}^3 - r_0^2 e^{2\alpha\theta}\dot{\theta}^3 = -(1+\alpha^2)r_0^2 e^{2\alpha\theta}\dot{\theta}^3$$

$$\rho = \frac{v^3}{|\vec{a} \times \vec{v}|} = \frac{\dot{\theta}^3 [(r_0^2 e^{2\alpha\theta})(1+\alpha^2)]^{\frac{3}{2}}}{\dot{\theta}^3 \cdot (1+\alpha^2)\, r_0^2 \; e^{2\alpha\theta}}$$

$$= \sqrt{r_0^2 e^{2\alpha\theta}(1+\alpha^2)} = \sqrt{1+\alpha^2}\; r_0 e^{\alpha\theta}$$