

1 a) To get init condition for circular orbit:

① Set $\boxed{r_0 = 1.0}$

② $a_c = \frac{v_0^2}{r_0} = v_0^2$

③ From previous Lectures / Hws, we know

$$a \approx \frac{4\pi^2}{r^2} \approx 4\pi^2 = v_0^2$$

$$\Rightarrow v_0 = \pm (2\pi)$$

choose $\boxed{v_0 = 2\pi}$

□

1b) Reasoning from lecture:

$$\mu r'' = -\frac{dV}{dr} + \frac{L^2}{\mu r^3}$$

$$\frac{dV_{\text{eff}}}{dr} = 0 = -F(r) = \mu r''$$

$$r'' = 0$$

$$V(r) = -\frac{\alpha}{r}$$

$$\frac{d\phi}{dt} = \frac{L^2}{\mu r^2} \quad | \quad r = r_{\min} \quad \text{const.}$$

$$K = \frac{1}{2} \mu r'^2 + \frac{1}{2} r^2 \phi'^2$$

at $r = r_{\min} = \text{constant}$

1c) also from lecture:

$$\frac{dA}{dt} = \frac{L}{2\mu} = \frac{1}{2\mu} |\vec{r} \times \vec{p}|$$

$$\vec{p} = \mu \vec{v}$$

$$\frac{L}{2\mu} = \frac{1}{2} |\vec{r} \times \vec{v}|$$

... Set $\mu = 1$...

$$L = |\vec{r} \times \vec{v}|$$

If L constant, then L conserved.
See ipynb.

(d) (e) analysis: see below code cells for each part.

$$2a) \quad x_{EJ} = x_E - x_J$$

* add α if needed.

$$r_{EJ} = \sqrt{(x_E - x_J)^2 + (y_E - y_J)^2}$$

$$F_{E-J_x} = -\frac{G \cdot M_J \cdot M_E}{(r_{EJ})^3} x_{E-J} \quad , \text{add to original } F$$

$$a_{Ex} = -\frac{GM_\odot}{|r|^3} x_E - \underbrace{\frac{GM_J}{|r_{EJ}|^3} (x_E - x_J)}_{\text{new}}$$

$$= -\frac{4\pi^2 x_E}{|r_E|^3} - 4\pi^2 \left(\frac{M_J}{M_\odot}\right) \frac{(x_E - x_J)}{|r_{EJ}|^3}$$

In general: $a_E = \boxed{-\frac{4\pi r_E}{|r_E|^3} - 4\pi^2 \left(\frac{M_J}{M_\odot}\right) \frac{(r_E - r_J)}{|r_{EJ}|^3}}$

Similarly, for the motion of Jupiter:

$$\boxed{a_J = \frac{-4\pi^2}{|r_J|^3} - 4\pi^2 \left(\frac{M_E}{M_\odot}\right) \frac{(r_J - r_E)}{|r_{JE}|^3}}$$

All distances are in AU.

2b) See analysis below 2b) code cells.

2c), scratch, notes.

$$3D: F_z = -\frac{GM_{\odot}M_E}{(\sqrt{x^2+y^2+z^2})^3} \cdot z \quad r_{abs}$$

$$\frac{dv_z}{dt} = a = -\frac{GM_E}{|r|^3}$$

$$\frac{dz}{dt} = v_z$$

same for x and y .

\Rightarrow Total of 6 diff eqs. $(\begin{matrix} x & y & z \\ v_x & v_y & v_z \end{matrix})$

Friday Apr 8th

$$M_J = 1.9 \times 10^{27} \text{ kg}$$

$$F_{E-J_x} = -\frac{G \cdot M_J \cdot M_E}{(r_{EJ})^3} x_{E-J}$$

r radius between EJ

$$x_{EJ} = x_E - x_J$$

$$r_{EJ} = \sqrt{(x_E - x_J)^2 + (y_E - y_J)^2}$$

$$\frac{F_{Ex}}{M_E} = \frac{dV_{Ex}}{dt} = -\frac{GM_{\odot}}{|r|^3} x_E$$

$$-\frac{GM_J}{|r_{EJ}|^3} (x_E - x_J)$$

Then do y as well.

a received by
Earth from
J.

$$\text{Note: } GM_{\odot} = 4\pi^2, \text{ and } a = -\frac{4\pi^2}{|r|^3} \cdot r$$

\downarrow Change To

$$a_{Ex} = -\frac{4\pi^2 x_E}{|r|^3} - 4\pi^2 \left(\frac{M_J}{M_{\odot}} \right) \frac{(x_E - x_J)}{|r_{EJ}|^3}$$

Sum-up:

$$x_{EJ} = x_E - x_J$$

$$r_{EJ} = \sqrt{(x_E - x_J)^2 + (y_E - y_J)^2}$$

$$F_{E-J_x} = -\frac{G \cdot M_J \cdot M_E}{(r_{EJ})^3} x_{E-J}, \text{ add to original } F$$

$$a_{Ex} = -\frac{GM_{\odot}}{|r|^3} x_E - \frac{G M_J}{|r_{EJ}|^3} (x_E - x_J)$$

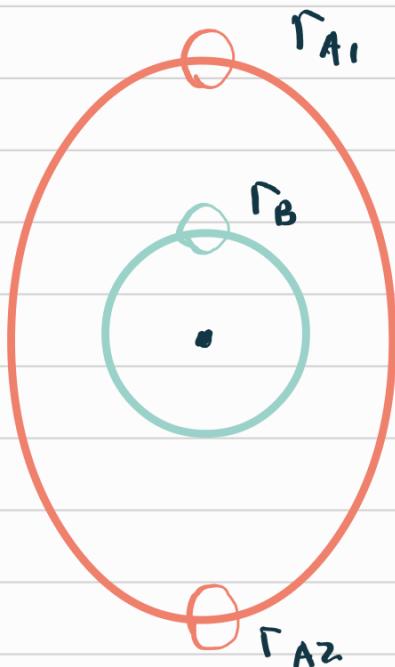
use \downarrow

$$= -\frac{4\pi^2 x_E}{|r|^3} - 4\pi^2 \left(\frac{M_J}{M_{\odot}} \right) \frac{(x_E - x_J)}{|r_{EJ}|^3}$$

new \downarrow



Relationships:



Calculated F sign	F Should Be
$r_B - r_{A1} = -$	+F
$r_B - r_{A2} = +$	-F
$r_O - r_B = -r_B = -$	-F

Wait...

If set sign in front as -, then all match up.

Newton's 3rd Law.

* rabs Planet-Planet. Sun already done.

1 E	7 <input checked="" type="checkbox"/> EJ, E ...
2 J	6 <input checked="" type="checkbox"/> JMerz, JV, JMars, JS, JU, JN
3 Merc	5 <input checked="" type="checkbox"/> MercV, MercMars, MercS, MercU, MercN
4 V	4 <input checked="" type="checkbox"/> VMars, VS, VU, VN
5 Mars	3 <input checked="" type="checkbox"/> MarsS, MarsU, MarsN
6 S	2 <input checked="" type="checkbox"/> SU, SN
7 U	1 <input checked="" type="checkbox"/> UN
8 N	

Change:

instead of update with $t[i+1]$, do $i += 1$

then on the bottom becomes

$$t[i+1] = t[i] + \Delta t$$

$$t[i] = t[i-1] + \Delta t$$

rabs

rabs update

vac

vac update

Done ?

Problem left: Sign ...

See "Relationship" above.

Done.



Thank you for grading this mess

and have a great day! :)

Karen