Two_Arch2: An Improved Two-Archive Algorithm for Many-Objective Optimization

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Outline of The Presentation

- Many-objective Optimization: Introduction
- Non-dominated sorting
- Objective Reduction
- Alternative Dominance Relationship
- Two-Archive Algorithm
- Improved Two-Archive Algorithm (Two_Arch2)
- Experimental Studies of Two_Arch2
- Conclusions and Future Work

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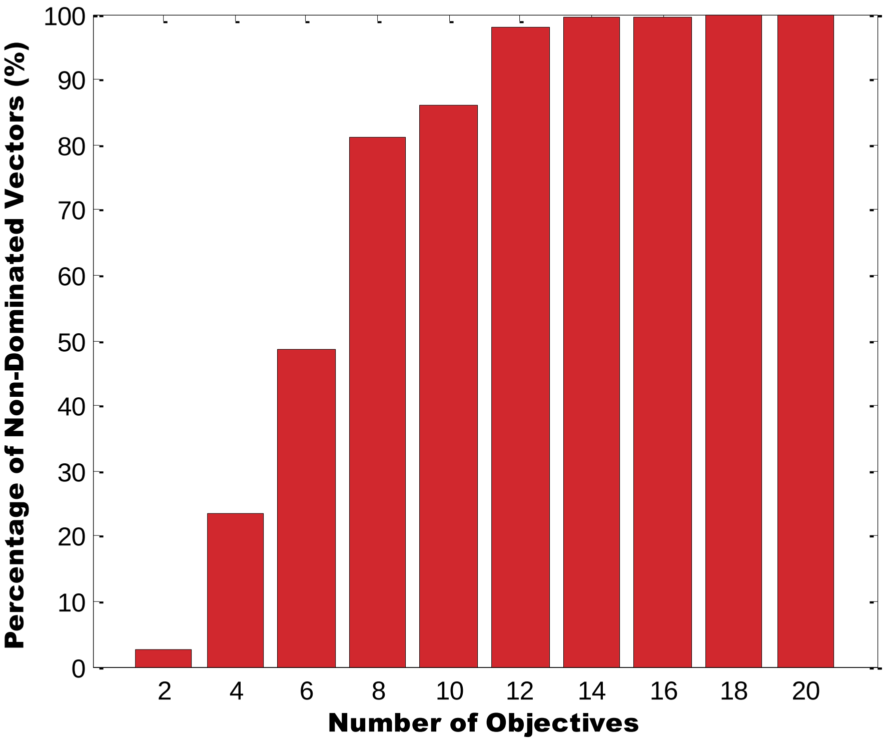
Introduction

Many-objective optimization problems (MaOPs) is a subset of multi-objective optimization problems (MOPs) with more than three objectives.



Weakness:

- **Convergence: ineffective Pareto-based non-dominated sort**
- **Diversity: similarity in a high-dimensional space**
- **Visualization**

OK, MaOPs are difficult. Existing MOEAs take a long time to run.

How can we make existing algorithms faster?

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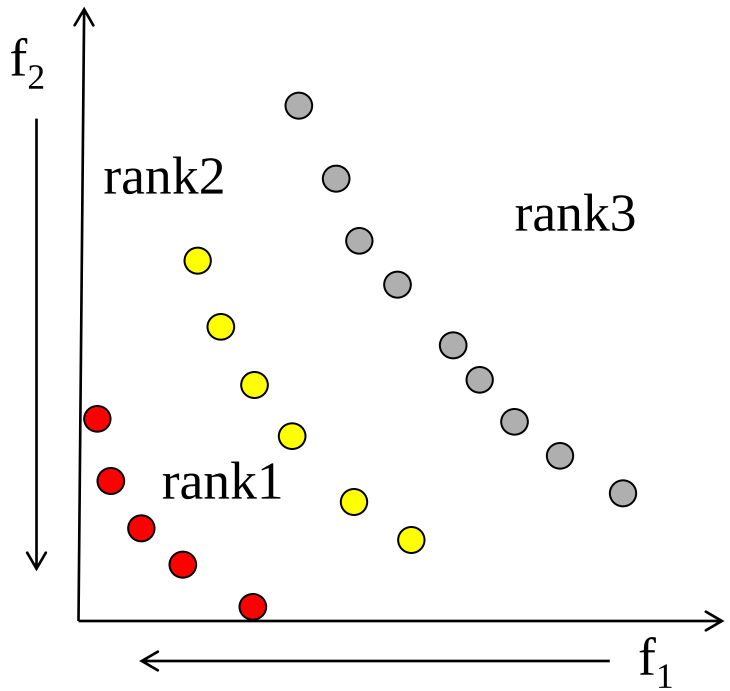
Non-Dominated Sort

- Non-dominated sort is one of the most important part of Pareto-based multi-objective optimization evolutionary algorithms (MOEAs).



Existing Non-Dominated Sorts

$O(mN^3)$ **Computational time complexity for the dataset of N solutions with m objectives.**

$O(mN^2)$ **Fast non-dominated sort needs the dominance relation of every two solutions.**

$O(mN^2)$ **Non-dominated rank sort ignores the dominated solutions during sorting of the current rank.**

$O(mN^2)$ **Deductive sort ignores some comparisons between two solutions that can be inferred, such as A and D in** $A \prec B \prec D$

$O(mN^2)$ ***Corner sort is designed for many-objective optimization problems***

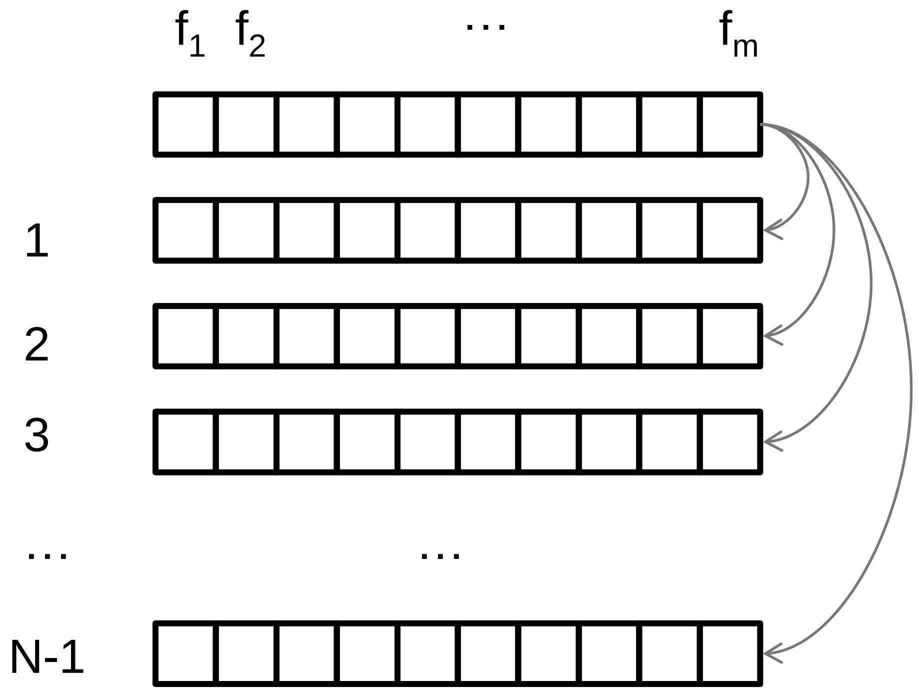
H. Wang and X. Yao, "Corner Sort for Pareto-Based Many-Objective Optimization," IEEE Transactions on Cybernetics, vol.44, no.1, pp.92-102, 2014.

Simple Idea

Obtain a non-dominated solution

Ignore the dominated solutions

Obtain a non-dominated solution

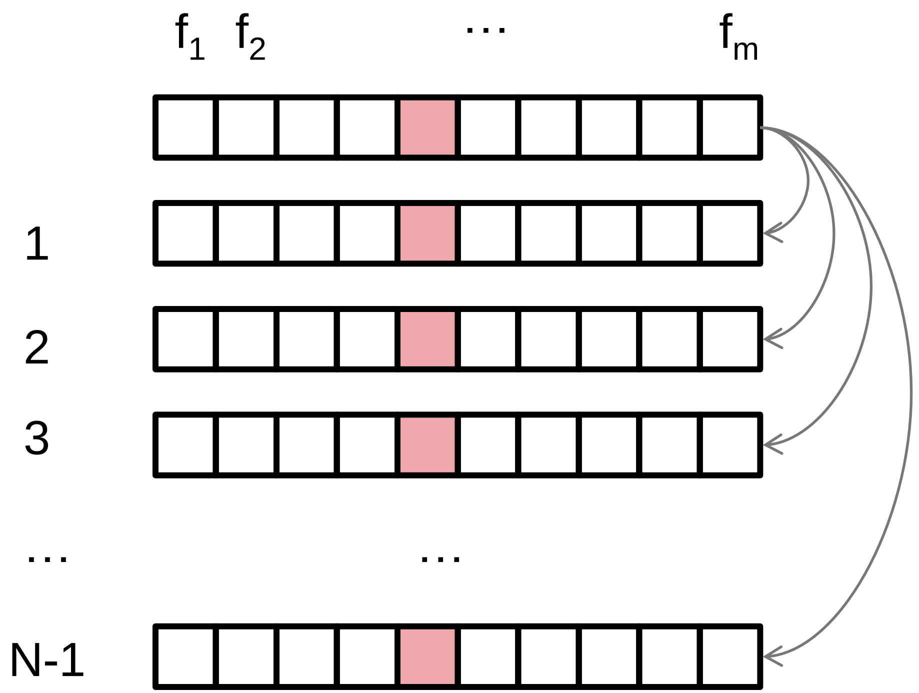


m(N-1) objective comparisons

Is there any method using fewer objective comparisons to obtain a non-dominated solution?

An Observation

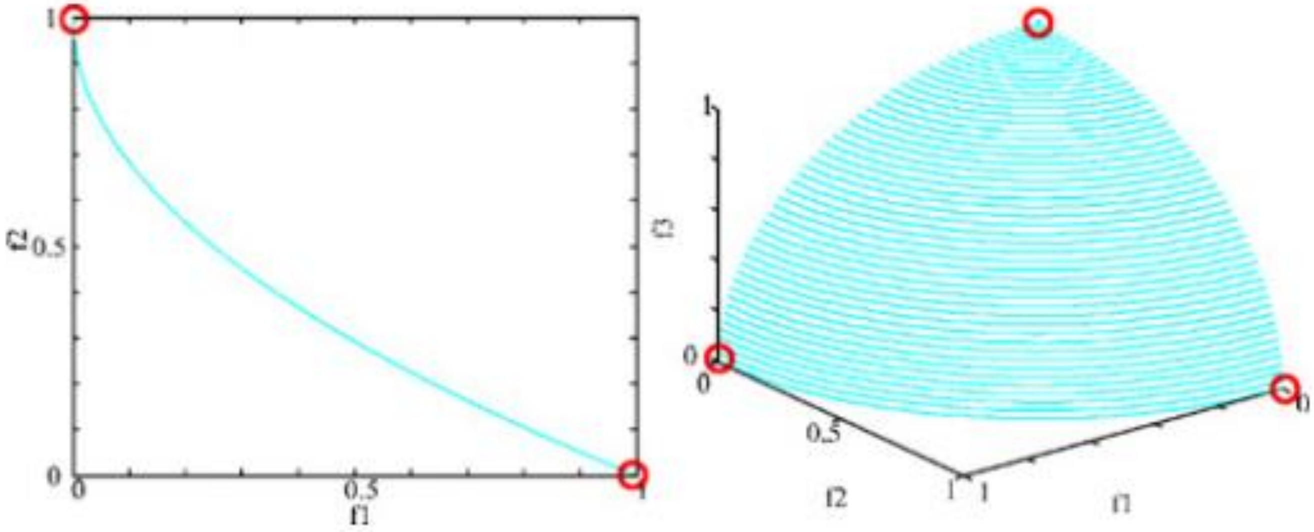
- **The solutions with the best objective values are always non-dominated.**



N-1 objective comparisons

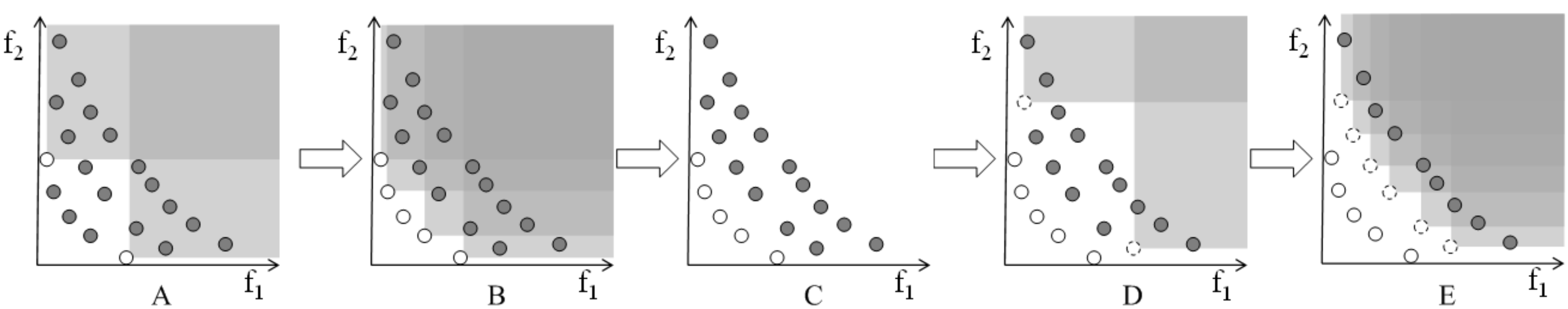
Corner Solutions

- **Corner solutions are the "optimal" solutions without considering all the objectives.**



Corner Sort

- From a corner solution, a non-dominated solution can be found using a smaller number of comparisons.
- Once a non-dominated solution is found, all its dominated solutions can be ignored.



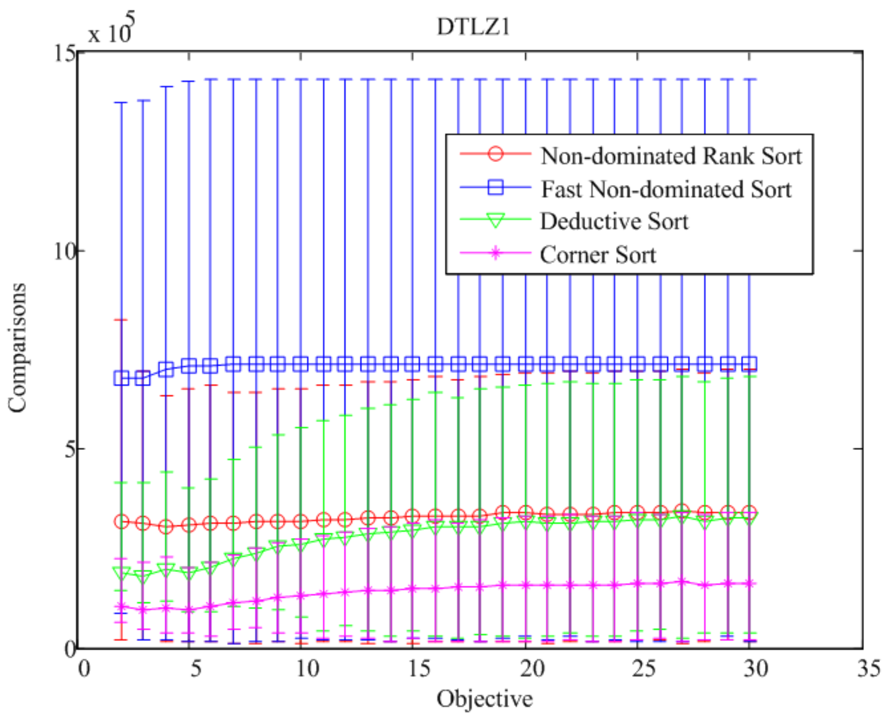
Corner sort

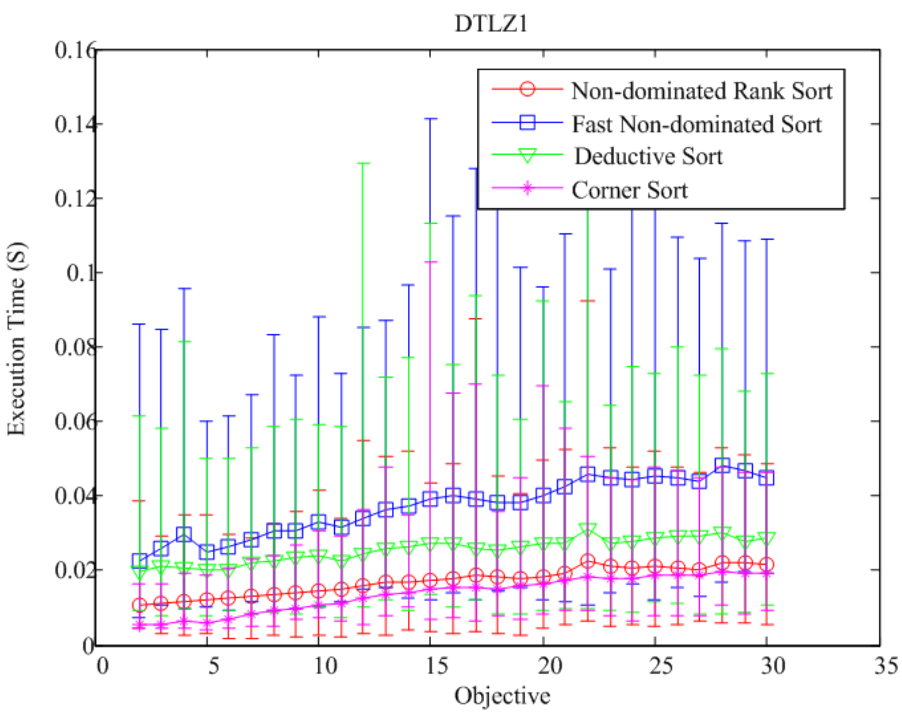
Parameters: P-Solution set, N-The number of solutions, $Rank$-Rank result, m-Number of objectives.

```
Rank[1 : N] = 0
i = 1
Do
    Unmark all the unranked solutions(whose ranks are 0), j = 1
    Do
        Find solution P[q] of the best objective f among the unmarked ones
        mark q, Rank[q] = i
        j = (j + 1)%m + 1 // Loop objectives
        For k = 1 : N
            If P[k] is unmarked and P[q] ≺ P[k]
                Mark P[k]
            End
        End
    Until all the solutions in P are marked
    i = i + 1
Until all the solutions in P are ranked
```

Experiment on DTLZ1 Data Using NSGA-II

It costs fewer objective comparisons than other existing sorts on many-objective optimization problems.





Discussions on Corner Sort

- Computationally efficient in comparison with other sorting methods.
- Can be used with any MOEAs

Corner Sort performed well in comparison with all other non-dominated sort, especially on MaOPs.

However, it does not tackle the key challenge of MaOPs.

Can we simplify MaOPs into MOPs?

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Objective Reduction

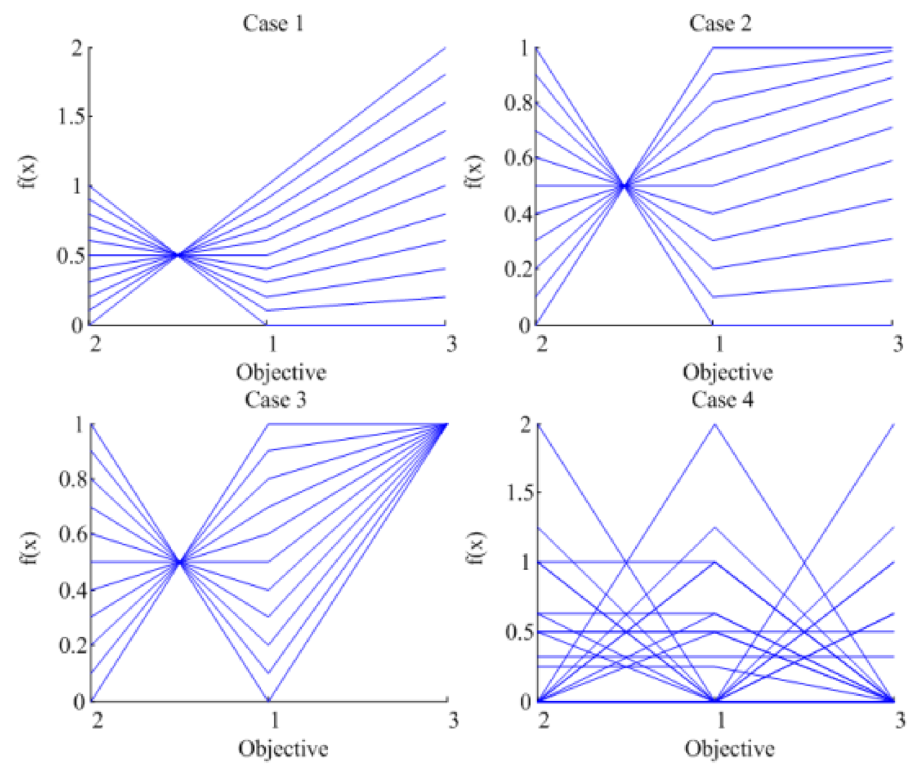
For some MaOPs where there is redundancy among objectives, objective reduction can be an effective approach to convert a MaOP to a MOP so that the existing MOEAs could be used.

- Dominance structure changes: k-EMOSS
- Feature selection
- Dimension reduction: PCA and MVU
- Pareto corner: PCSEA

Reducible MOPs

- The correlation between redundant objectives might be **either linear or non-linear**.
- Most existing approaches use **linear tools** to reduce objectives.

Case 1	$f_1 = x_1$ $f_2 = 1 - x_1$ $f_3 = 2f_1$ $PF: f_1 + f_2 = 1, f_3 = 2f_1$
Case 2	$f_1 = x_1$ $f_2 = 1 - x_1$ $f_3 = \sin(0.5\pi f_1)$ $PF: f_1 + f_2 = 1, f_3 = \sin(0.5\pi f_1)$
Case 3	$f_1 = x_1$ $f_2 = 1 - x_1$ $f_3 = 1$ $PF: f_1 + f_2 = 1, f_3 = 1$
Case 4	$f_1 = x_1 x_2 (1 + x_3{}^2)$ $f_2 = x_1 (1 - x_2)(1 + x_3{}^2)$ $f_3 = \begin{cases} (1 - x_1)(1 - x_2)(1 + x_3{}^2), x_3 \neq 0 \\ 0, x_3 = 0 \end{cases}$ $PF: f_1 + f_2 = 1, f_3 = 0$



Nonlinear Correlation Information Entropy (NCIE)

- **NCIE is a different kind of entropy measure.**
- **NCIE firstly divides variables *X* and *Y* into *b*b* uniform rank grids. Then, the probabilities can be sampled by the counts in those grids. Thus, p_{ij} in the *ij*-th grid can be calculated by the number of solutions dropping in *ij*-th grid (n_{ij}/N).**
- **Parameter *b* is set as *N^0.5*.**

$$H^r(X) = -\sum_{i=1}^{b} \frac{n_i}{N} log_b(\frac{n_i}{N})$$

$$H^r(X,Y) = -\sum_{i=1}^{b}\sum_{j=1}^{b} \frac{n_{ij}}{N} log_b(\frac{n_{ij}}{N})$$

$$NCIE(X,Y) = H^r(X) + H^r(Y) - H^r(X,Y)$$

Objective Reduction Based on NCIE

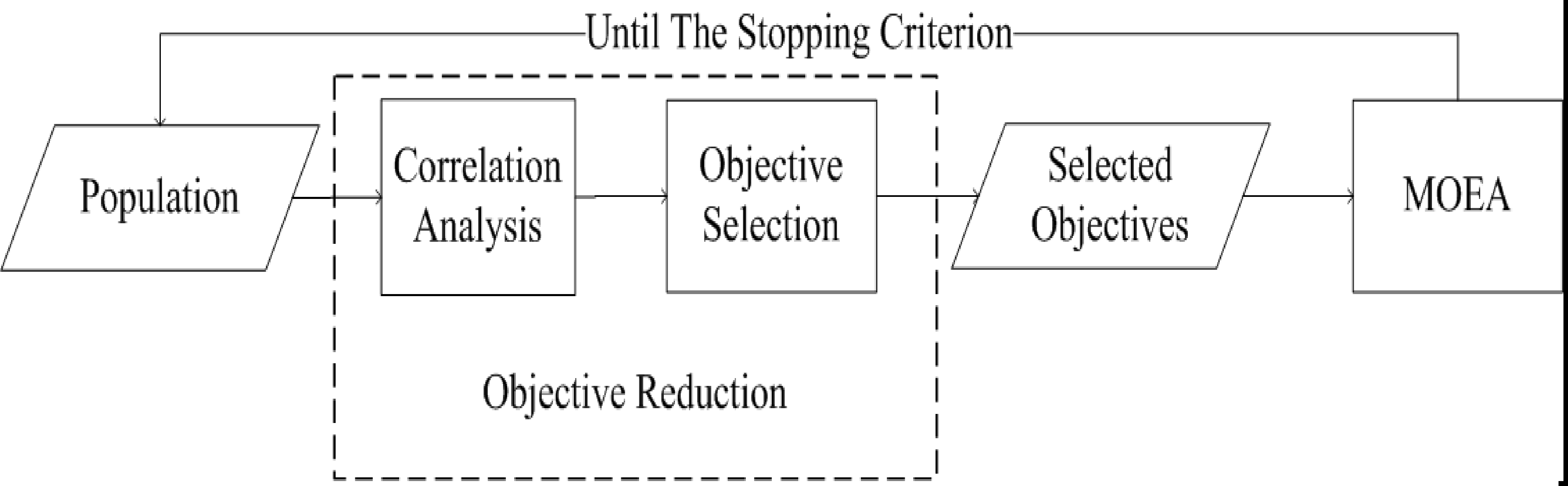
- **Correlation analysis is based on the matrix of modified NCIE R^N of *the non-dominated population*.**

$$R^N = \{Sgn(cov_{ij})NCIE_{ij}\}, (1 \le i, j \le m)$$

- **Objective selection aims to choose the most conflicting objectives.**
  - **Our approach is applied in every generation of MOEAs to update the correlation information among objectives.**

H. Wang and X. Yao, "Objective Reduction Based on Nonlinear Correlation Information Entropy," *Soft Computing*, DOI: 10.1007/s00500-015-1648-y, 2015.

MOEAs with NCIE



Objective Selection

- Select and omit the most conflicting objective
- Remove the objectives that are positively correlated to the selected objective

	f_1	f_2	f_3	f_4	f_5
f_1	1.0000	0.4959	0.4244	0.5348	-0.3552
f_2	0.4959	1.0000	0.3972	0.4686	-0.3381
f_3	0.4244	0.3972	1.0000	0.4765	-0.4352
f_4	0.5348	0.4686	0.4765	1.0000	-0.4488
f_5	-0.3552	-0.3381	-0.4352	-0.4488	1.0000
$\sum NCIE < 0$	-0.3552	-0.3381	-0.4352	-0.4488	-1.5773

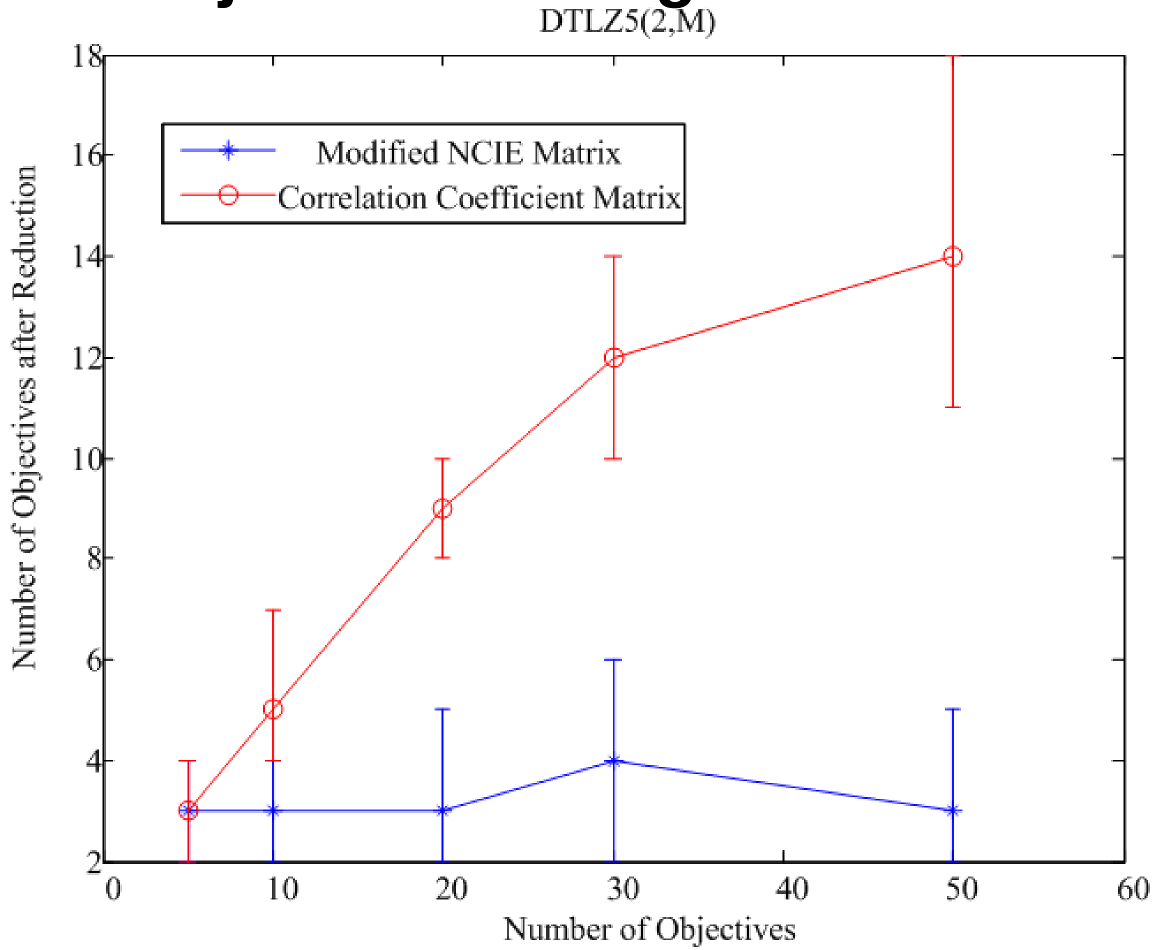
- ✓ f_5 is selected, because it has the most conflicting degree with other objectives.
- ✓ There is no objective positively correlated to f_5, thus, there is not a redundant objective with f_5 in the remaining objectives.
- ✓ f_4 is selected, because it has the largest absolute sum of NCIEs to other objectives. f_1 , f_2, and f_3 are omitted, they are all positively correlated to f_4
- ✓ Output $\{f_5, f_4\}$

Omitting Positively Correlated Objectives

- In the process of omitting objectives, a threshold *T* is applied to determine whether two objectives are positively correlated.
- This is actually done by a clustering algorithm, based on which the *T* is set at the least dense point.

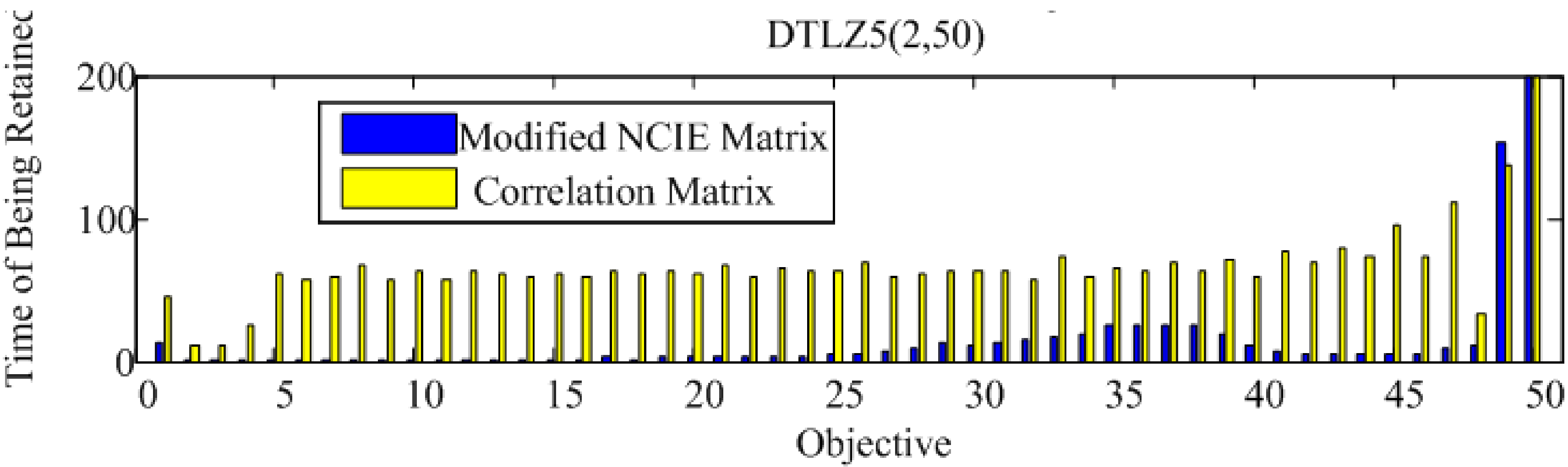
NCIE vs. Correlation

- **The modified NCIE matrix performs better than the correlation matrix on keeping objectives when the number of objectives is large.**



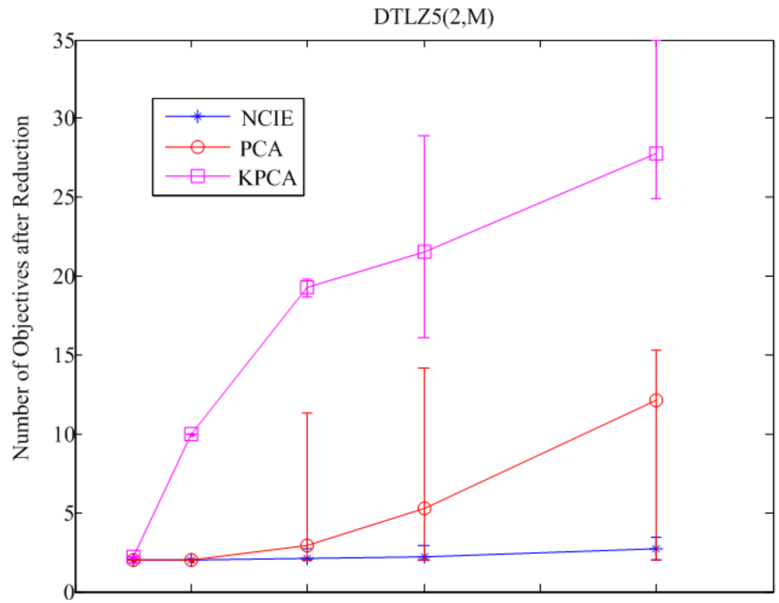
NCIE vs. Correlation

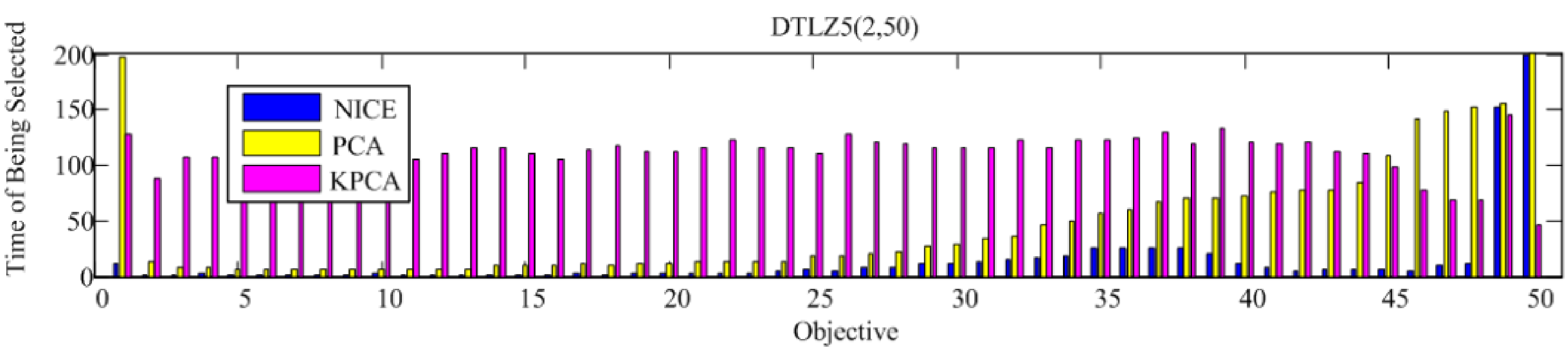
- **When the number of objectives increases to 50, the chance of retaining $\{f_1,f_M\}$ by the approach based on the modified NCIE matrix decreases, but that of retaining $\{f_{M-1},f_M\}$ increases.**



NCIE vs. PCA

- NCIE has advantages over non-linear correlation.
- In KPCA, the Kernel function has to be chosen in advance, which affects its performance significantly.





IGD of the NSGA-II with NCIE and the original NSGA-II on DTLZ5(I,M)

I	M	NSGA-II(NCIE)	NSGA-II	p-value
2	5	**0.0042±0.0000**	0.0042±0.0000	0.0256
2	10	**0.0042±0.0000**	1.8023±1.5792	0.0000
2	20	**0.4613±0.2211**	-	-
2	30	**0.1086±0.2108**	-	-
2	50	**0.3886±0.3688**	-	-
3	5	0.1155±0.1684	0.0550±0.0009	0.4735
3	10	**0.0886±0.1068**	3.4976±7.2347	0.0000
3	20	**0.1192±0.1706**	-	-
5	10	**20.2226±38.9470**	70.7366±47.8311	0.0003
5	20	**134.9047±62.8815**	-	-
7	10	93.9852±58.3535	**67.9335±33.8466**	0.0315
7	20	**153.1658±54.6619**	-	-

IGD of the IBEA with NCIE and the original IBEA on DTLZ5(I,M)

I	M	IBEA(NCIE)	IBEA	p-value
2	5	0.6529±0.1694	0.5784±0.1953	0.0764
2	10	0.6324±0.1789	**0.5453±0.2052**	0.0439
2	20	0.6734±0.1457	**0.4877±0.1922**	0.0003
2	30	0.6083±0.1835	0.5603±0.1601	0.2503
2	50	0.7018±0.1159	0.6298±0.1997	1.0000
3	5	0.7876±0.1536	0.7506±0.1633	0.9676
3	10	0.8257±0.1953	0.7815±0.1771	0.3369
3	20	0.7874±0.2094	0.8227±0.1836	0.9461
5	10	0.9366±0.1882	0.9312±0.2111	0.7764
5	20	0.8839±0.2212	0.8099±0.1878	0.4903
7	10	1.1193±0.1839	1.0622±0.2286	0.8817
7	20	1.1482±0.1584	1.0911±0.2212	0.7353

IGD of the NSGA-IIs with NCIE and random objectives reduced and the original NSGA-II on DTLZ1-4

DTLZ	M	NSGA-II(NCIE)	NSGA-II(Random)	NSGA-II
1	5	**3.5257±5.4437**	29.7310±18.8620	28.1565±14.7707
1	15	**15.8986±17.1624**	36.6916±23.5109	-
1	25	17.7863±15.8425	27.6838±21.9433	-
2	5	0.5188±0.2108	1.2836±0.3187	0.4893±0.0741
2	15	2.1166±0.2912	2.2857±0.2768	-
2	25	**2.4970±0.3190**	2.7165±0.1811	-
3	5	129.0159±62.0312	209.6908±20.1481	160.7862±24.2639
3	15	**216.4849±15.3258**	232.9653±13.5670	-
3	25	**230.2662±17.5408**	277.0580±107.0687	-
4	5	0.7899±0.3023	1.1632±0.0114	0.5651±0.0613
4	15	**1.2916±0.0634**	1.3412±0.0399	-
4	25	**1.3417±0.0602**	1.4126±0.0750	-

Summary of NCIE

- Our approach improves Pareto-based MOEAs (NSGA-II) on reducible problems (DTLZ5 and WFG3) but cannot improve the performance of indicator-based MOEAs (IBEA).
- Our approach also improves the performance of Pareto-based MOEAs on the irreducible problems (DTLZ1-4) slightly, because the difficulty of the original problems decreases locally, which promotes convergence.
  - The NCIE-based correlation analysis is based on the non-dominated population in every generation, thus, the conflict between objectives are local rather than global.

Objective reduction can remove objective redundancy, but does not address the key challenges of MaOPs --- the ineffectiveness of Pareto dominance.

Can we use alternative dominance relationships?

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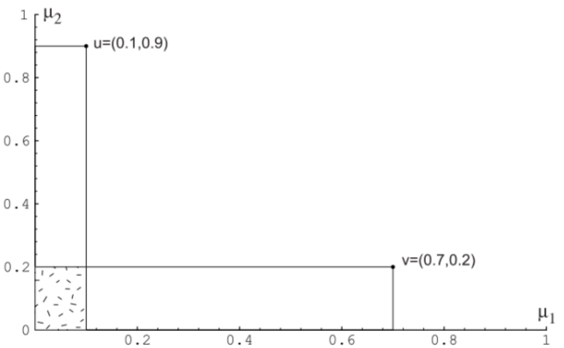
Different Dominance Definitions



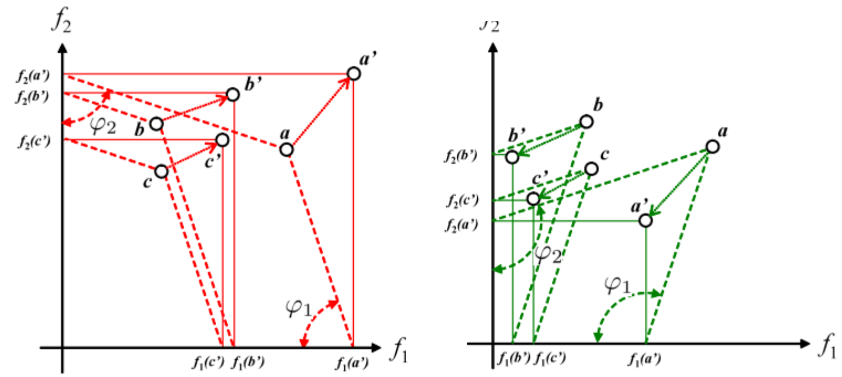
■ Fuzzy-Pareto-dominance

$$\mu_a(\boldsymbol{a}, \boldsymbol{b}) = \frac{\prod_i \min(a_i, b_i)}{\prod_i a_i}$$

■ Ranking-Dominance

$$R_{sum}(\vec{x}_j) = \sum_{i=1}^{M} rank\left(f_i(\vec{x}_j)\right)$$

$$R_{min}(\vec{x}_j) = \min_{i=1,\ldots,M} rank\left(f_i(\vec{x}_j)\right).$$



(b) $S_1 = S_2 < 0.5$ (c) $S_1 = S_2 > 0.5$

■ Controlling Dominance Area

$$f'_i(x) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \qquad (i = 1, 2, \cdots, m)$$

■ (1 − k)-Dominance

$$\begin{aligned} n_b(\mathbf{v_1}, \mathbf{v_2}) &=_{\text{def}} \left|\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v_1}) < f_i(\mathbf{v_2})\}\right| \\ n_e(\mathbf{v_1}, \mathbf{v_2}) &=_{\text{def}} \left|\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v_1}) = f_i(\mathbf{v_2})\}\right| \\ n_w(\mathbf{v_1}, \mathbf{v_2}) &=_{\text{def}} \left|\{i \in \mathbb{N} | i \leq M \wedge f_i(\mathbf{v_1}) > f_i(\mathbf{v_2})\}\right| \end{aligned}$$

$$\begin{cases} n_e < M \\ n_b \geq \dfrac{M - n_e}{k+1}, \end{cases}$$

Many Others

- Indicators mentioned by Dr. Emmerich yesterday can be regarded as alternative dominance relationships too.
- Here is one of the latest additions --- Θ-dominance.

Θ-dominance --- Intuition

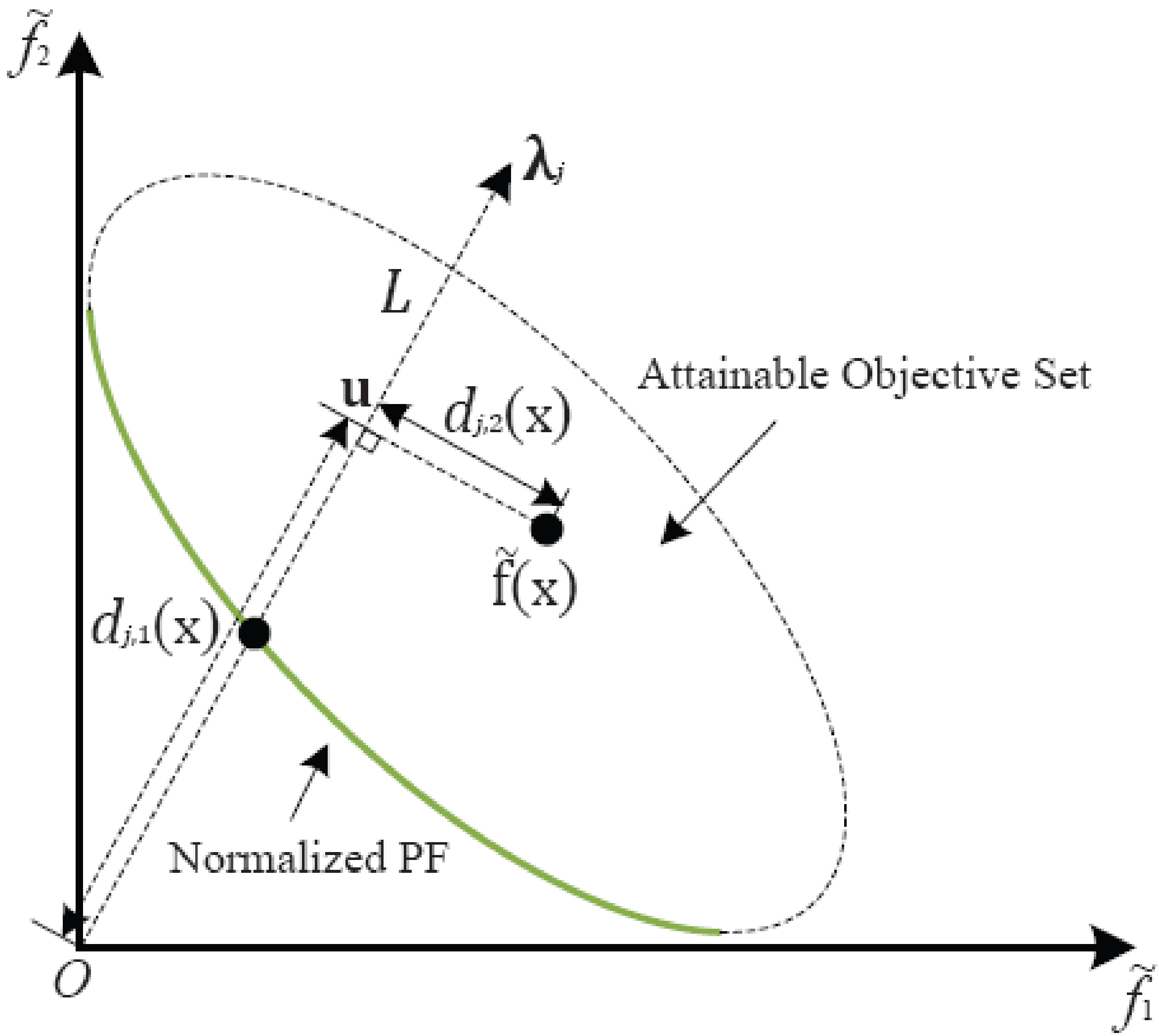


Fig. 3. Illustration of distances $d_{j,1}(\mathbf{x})$ and $d_{j,2}(\mathbf{x})$.

***f*'s are normalised fitness functions.**
λ is the reference direction (point).

Y. Yuan, H. Xu, B. Wang and X. Yao, "A New Dominance Relation Based Evolutionary Algorithm for Many-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, DOI: 10.1109/TEVC.2015.2420112, 2015.

Θ-dominance --- Definition

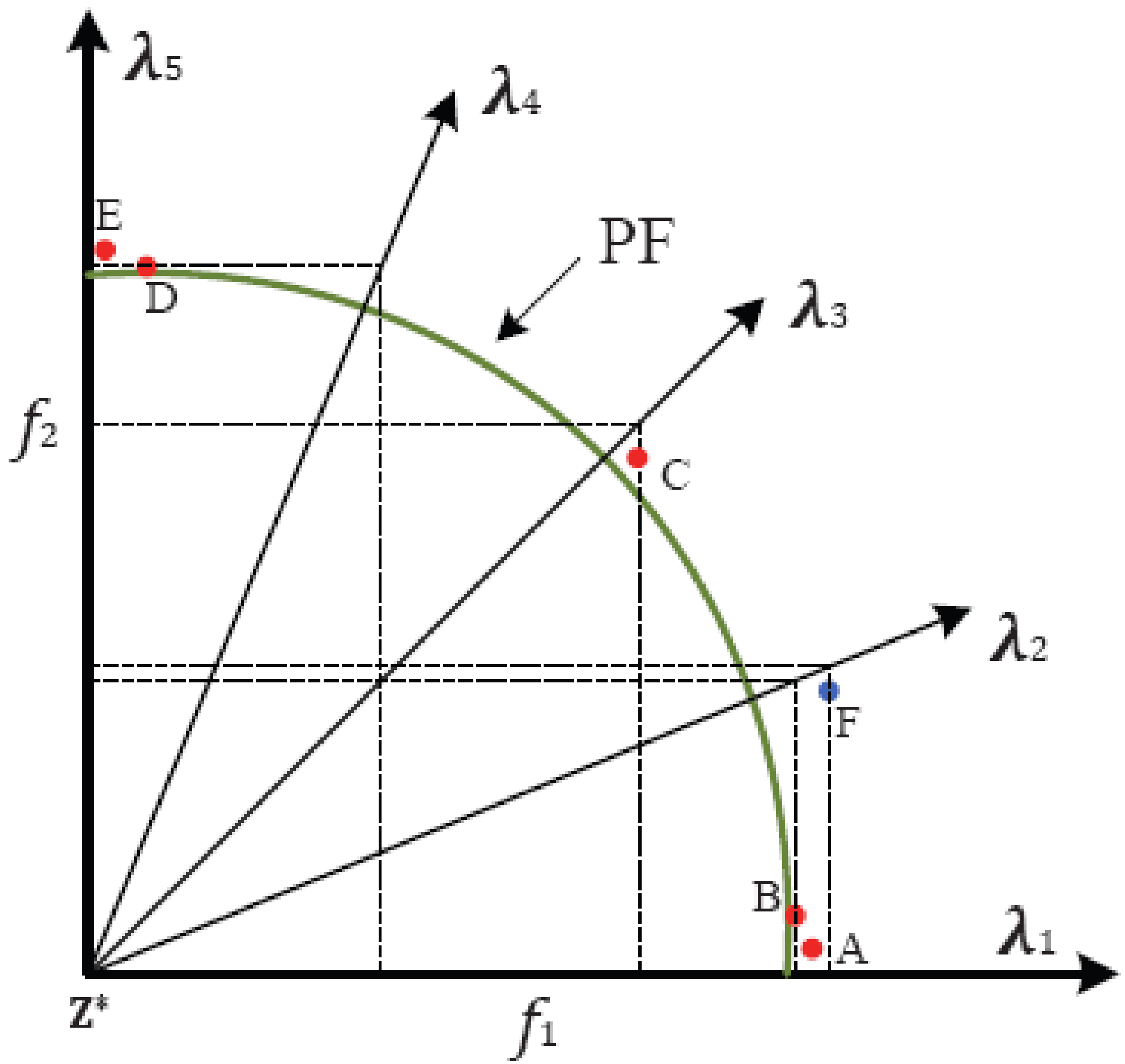
Definition 7: Given two solutions $\mathbf{x}, \mathbf{y} \in S_t$, $\mathbf{x}$ is said to θ-dominate $\mathbf{y}$, denoted by $\mathbf{x} \prec_\theta \mathbf{y}$, iff $\mathbf{x} \in C_j$, $\mathbf{y} \in C_j$, and $\mathcal{F}_j(\mathbf{x}) < \mathcal{F}_j(\mathbf{y})$, where $j \in \{1, 2, \ldots, N\}$.

$$\mathcal{F}_j(\mathbf{x}) = d_{j,1}(\mathbf{x}) + \theta d_{j,2}(\mathbf{x})$$

Balancing Convergence and Diversity

- The form of $F_j(x)$ indicates that the balance between convergence and diversity is even more important in MaOEAs.
- Why not manipulating the balance explicitly?
  - Y. Yuan, H. Xu, B. Wang, B. Zhang and X. Yao, "Balancing Convergence and Diversity in Decomposition-Based Many-Objective Optimizers," *IEEE Transactions on Evolutionary Computation*, DOI: 10.1109/TEVC.2015.2443001, 2015.

An Example of an Existing Problem



What if alternative dominance relationships still do not provide a satisfactory solution to a MaOPs?

We have to consider new algorithms.

New MaOEAs

Before we develop new MaOEAs, we have to understand the state-of-the-art:

1. **Presentations you have heard so far at the Summer School.**

2. **The latest literature survey:**

   B. Li, J. Li, K. Tang and X. Yao, "Many-Objective Evolutionary Algorithms: A Survey," *ACM Computing Surveys*, 35 pages, 2015. (http://www.cs.bham.ac.uk/~xin/papers/surveyfinalV10unmarked.pdf)

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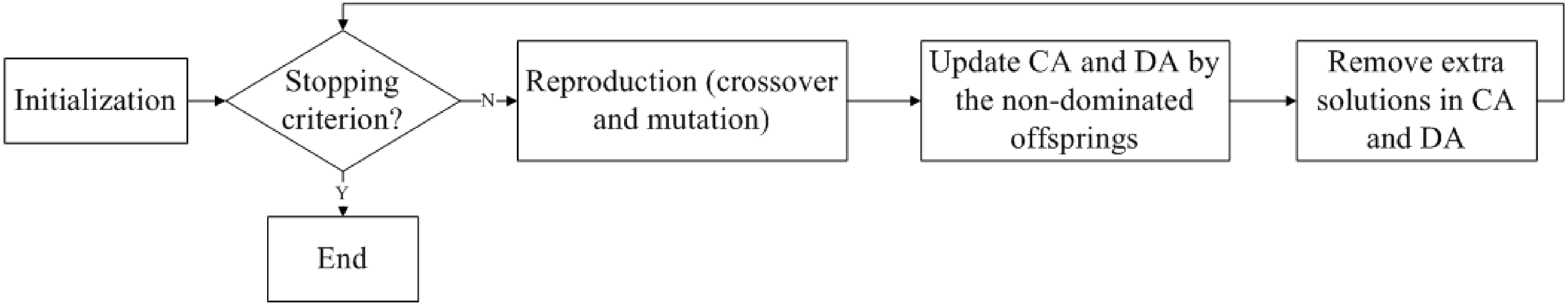
Two-Archive Algorithm

- Two-Archive algorithm (Two_Arch) maintains two archives (CA and DA) to promote convergence and diversity separately.
  - K. Praditwong and X. Yao, "A New Multi-objective Evolutionary Optimisation Algorithm: The Two-Archive Algorithm," *Proc. of the 2006 International Conference on Computational Intelligence and Security (CIS'2006)*, 3-6/11/2006, Ramada Pearl Hotel, Guangzhou, China. IEEE Press, Volume 1, pp.286-291.



Update CA and DA

- The non-dominated offspring that dominate any solution in either CA or DA (the non-dominated solution with domination) are added to CA.

- The non-dominated offspring that dominate no solution in both CA and DA (the non-dominated solution without domination) are added to DA.

- Two_Arch removes extra solutions from DA according to their distances to CA.

Strengths and Drawbacks

- CA encourages the convergence, and DA maintains the diversity. Almost no additional complexity is introduced.
- Two_Arch is a Pareto-based MOEA and ineffective in handling MaOPs with a large number of objectives.
- There is no diversity maintenance within CA. Two_Arch might be stuck without any update of CA and DA, when CA is full of solutions on the true PF. The diversity of CA might be less satisfactory.

Can we do better?

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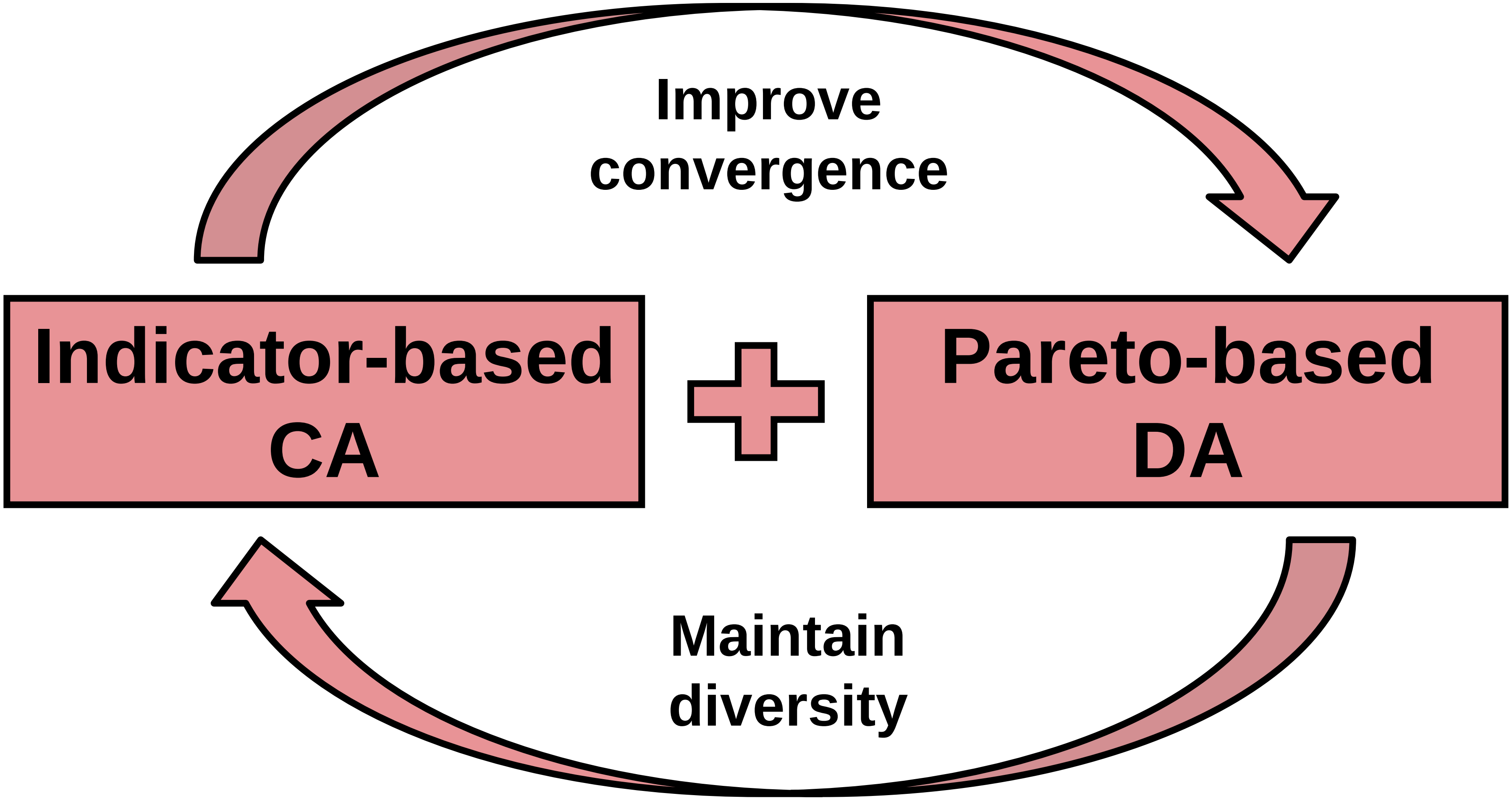
Main Idea


Improve convergence
Indicator-based CA
Pareto-based DA
Maintain diversity

Main Steps

- The quality indicator $I_{\varepsilon+}$ can encourage convergence.
- The Pareto dominance can promote diversity.
  - H. Wang, L. Jiao and X. Yao, "An Improved Two-Archive Algorithm for Many-Objective Optimization," *IEEE Transactions on Evolutionary Computation*, DOI: 10.1109/TEVC.2014.2350987, 2015.

Step 1: Initialization.
Step 2: Output DA if satisfy the stopping criterion, otherwise continue.
Step 3: Generate new solutions from CA and DA by crossover and mutation.
Step 4: Update CA and DA separately, go Step 2.

Convergence Archive (CA)

- **The quality indicator $I_{\varepsilon+}$ in IBEA as the selection principle for CA in Two_Arch2. $I_{\varepsilon+}$ is an indicator that describes the minimum distance that one solution needs to dominate another solution in the objective space.**

$$I_{\varepsilon+}(x_1, x_2) = min_{\varepsilon}(f_i(x_1) - \varepsilon \leq f_i(x_2), 1 \leq i \leq m)$$

- **The fitness is assigned as below, the solution with the smallest fitness is removed from CA first.**

$$F(x_1) = \sum_{x_2 \in P \setminus \{x_1\}} -e^{-I_{\varepsilon+}(x_2, x_1)/0.05}$$

Diversity Archive (DA)

- Update DA
  - When DA overflows, boundary solutions (solutions with maximal or minimal objective values) are firstly selected.
  - In the iterative process, the most different solution to DA is added until reaching the size.
- L_p-norm distance is adopted as the similarity measure in DA.
- DA with good diversity is used as the final output of Two_Arch2.

Degraded Euclidean Distance (Distance Concentration) in High-Dimensional Space

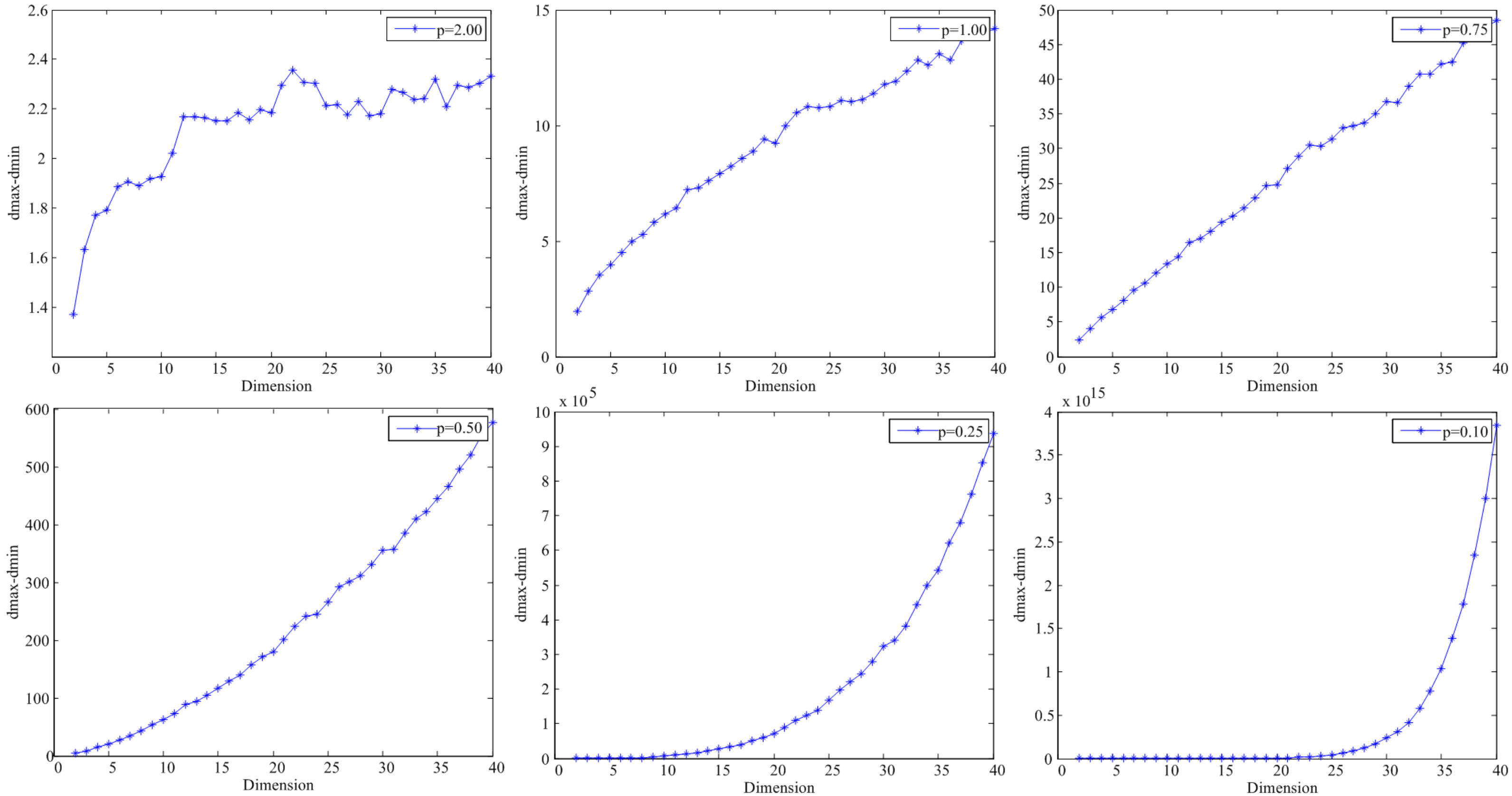
- **The Euclidean distance (L_2-norm) degrades its similarity indexing performance in a high-dimensional space.**

- **Most of existing diversity maintenance methods use the Euclidean distance to measure similarity among solutions for MaOPs.**

C. C. Aggarwal, A. Hinneburg and D. A. Keim, "On the surprising behavior of distance metrics in high dimensional space." Springer, 2001.

Similarity in High-Dimensional Space

- The fractional distances (L_p-norm, $p<1$) perform better in a high-dimensional space.
- **$L_{1/m}$-norm** is employed in Two_Arch2.



Outline of The Presentation

- Many-objective Optimization: Introduction
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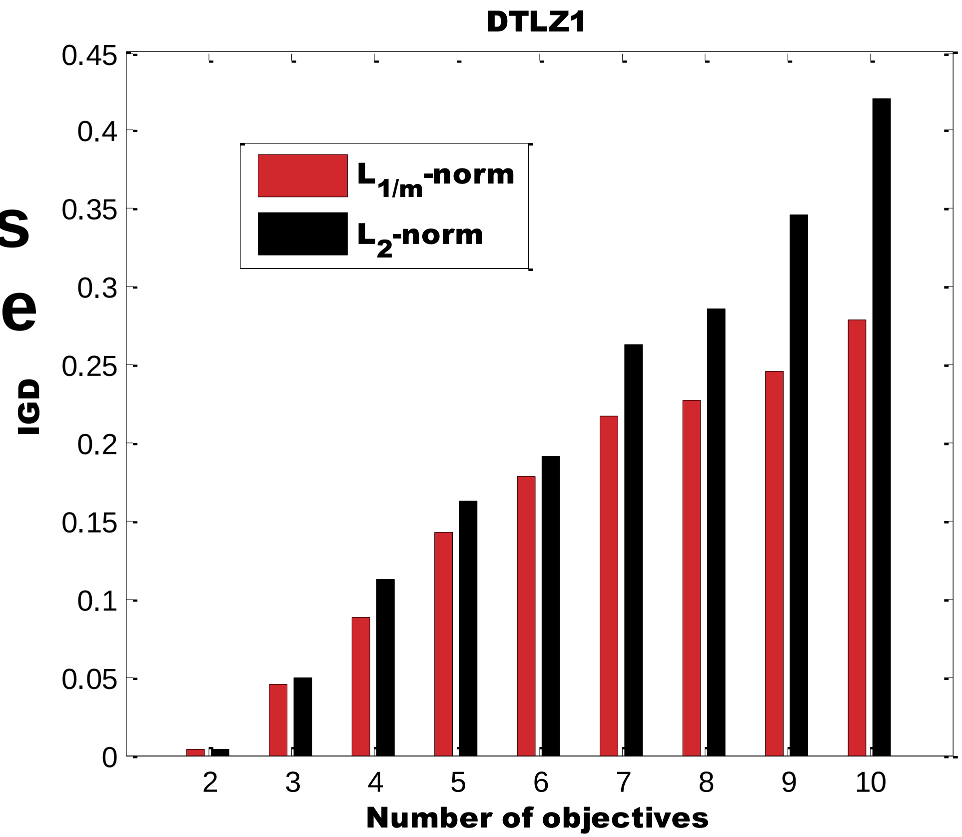
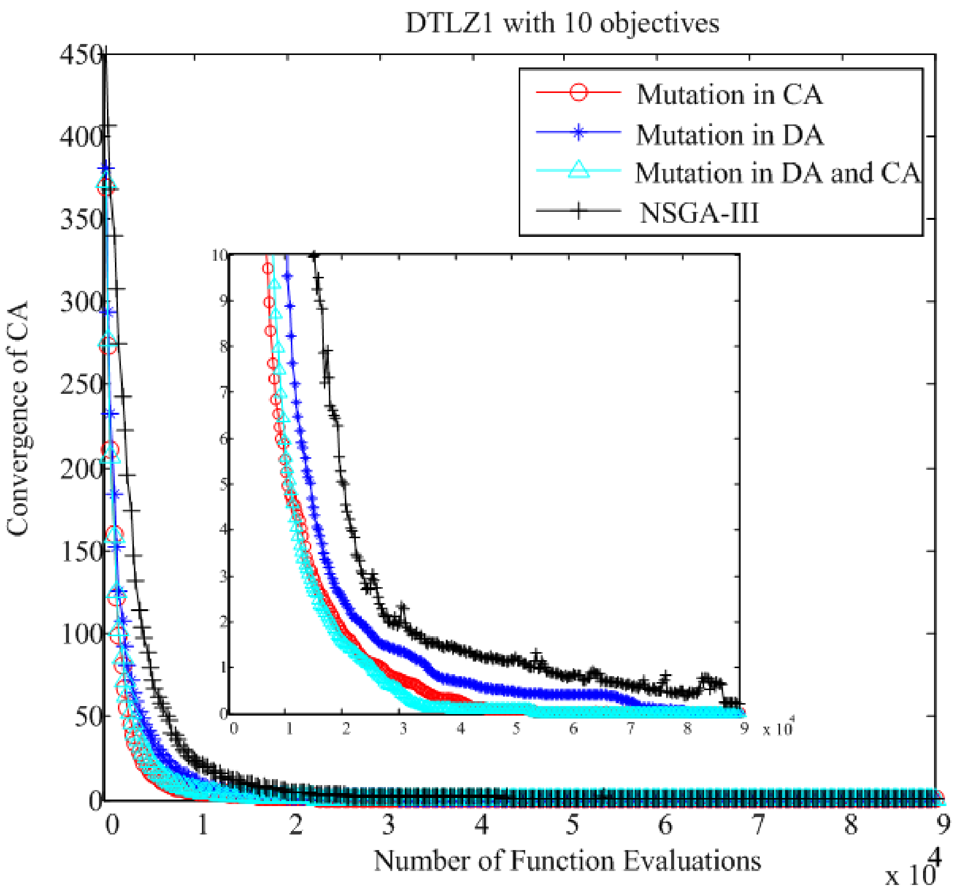
L_p-norm-based distances for Diversity Archive Maintenance

- The fractional distances performs better than the Euclidean distance.
- P=1/m in Two_Arch2



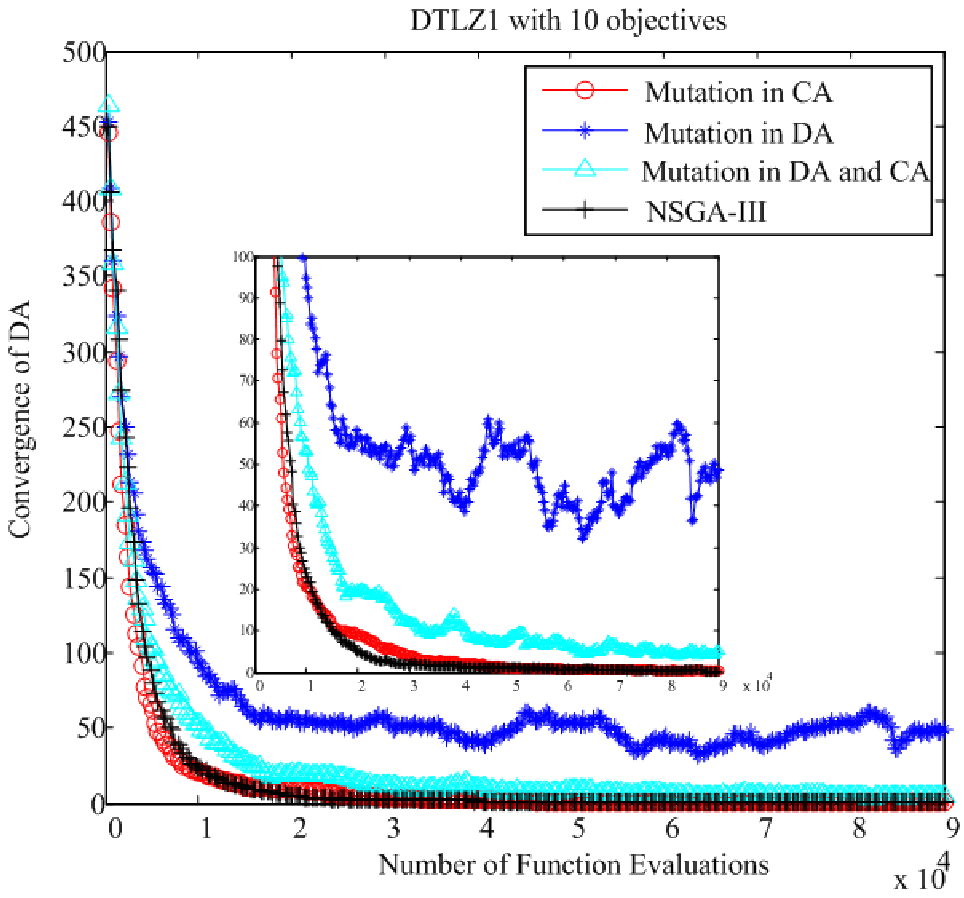
Interaction between CA and DA: Mutation



- The mutation applied to DA only cannot provide a faster convergence speed.
- The mutation on CA can prevent prematurity.

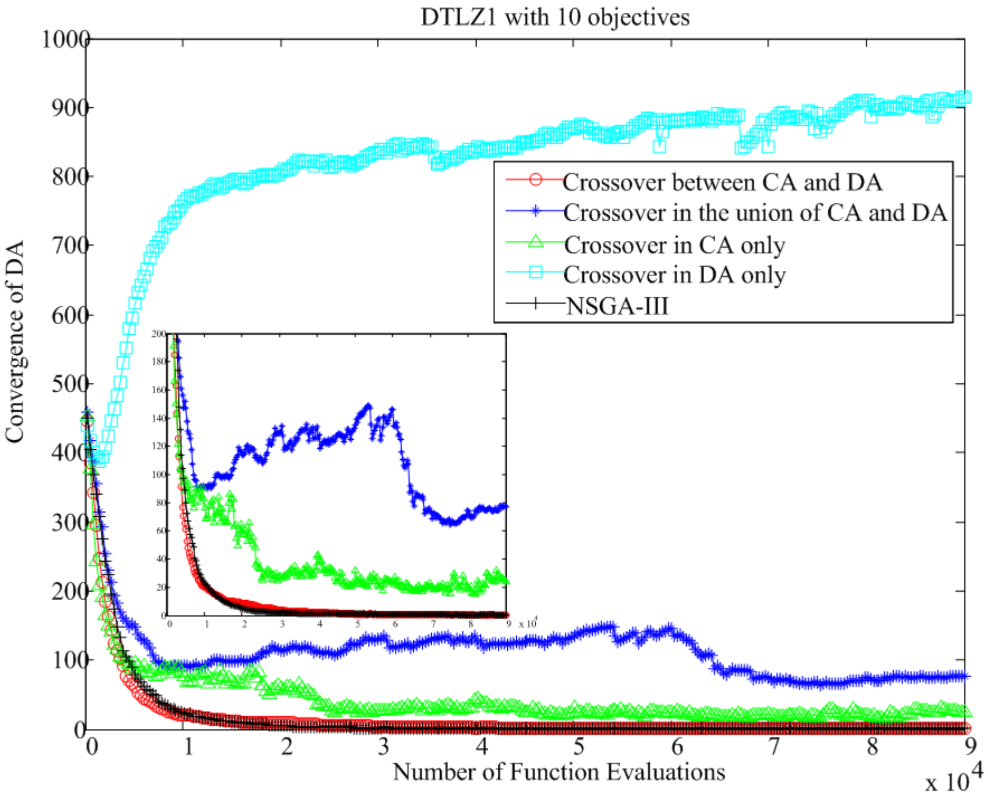
Interaction between CA and DA: Mutation

- The mutation for some members of DA disturbs the guidance of CA to DA.
- The mutation in CA only is applied in Two_Arch2.



CA is the guidance of convergence

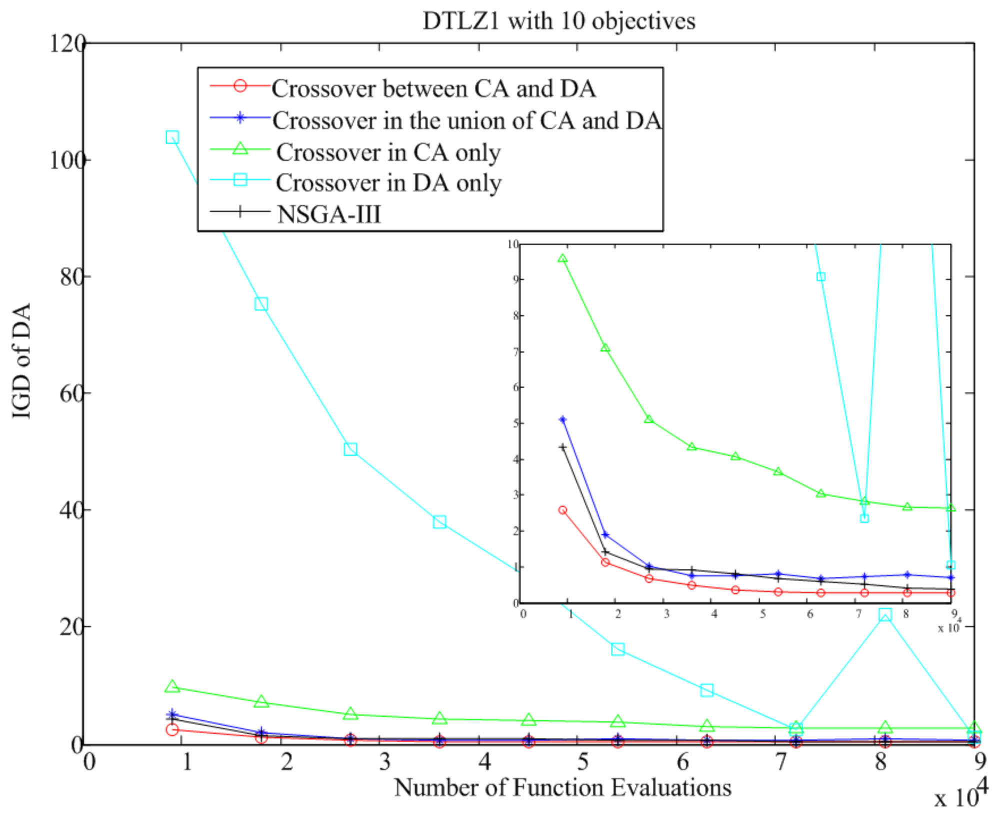
Interaction between CA and DA: Crossover



- **The crossover between CA and DA has the fastest convergence speed.**

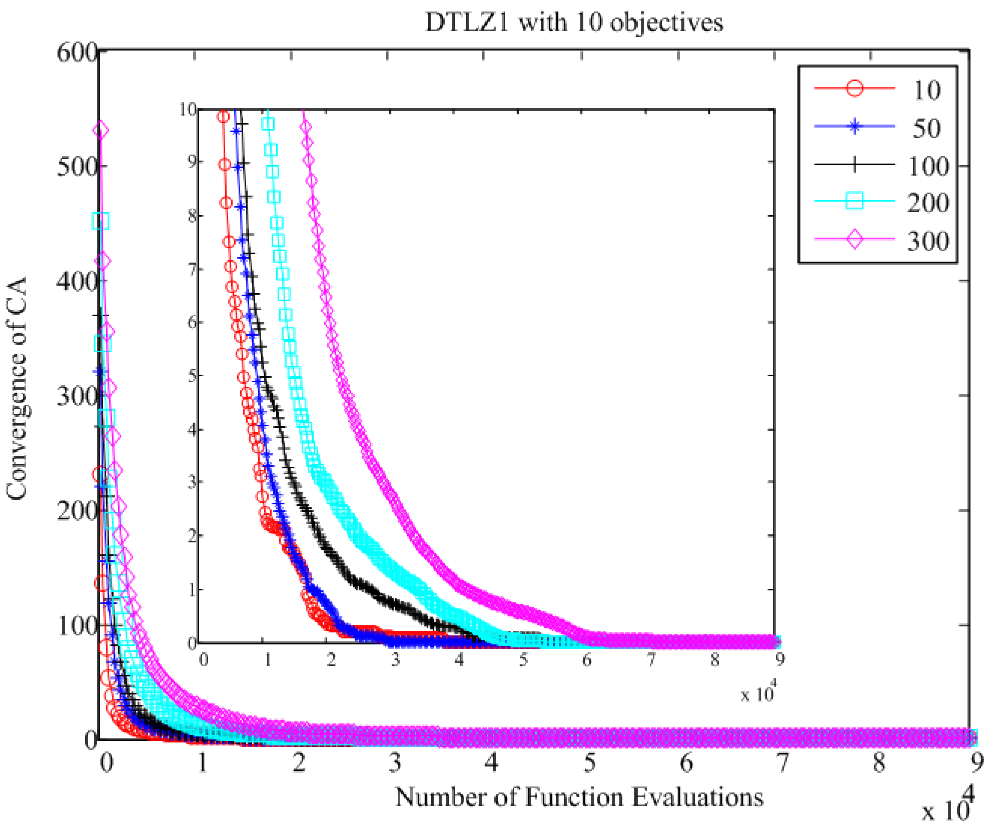
Interaction between CA and DA: Crossover

- The diversity of CA is too poor to improve IGD.
- The crossover between CA and DA is employed in Two_Arch2



The crossover between CA and DA passes the good convergence from CA to DA.

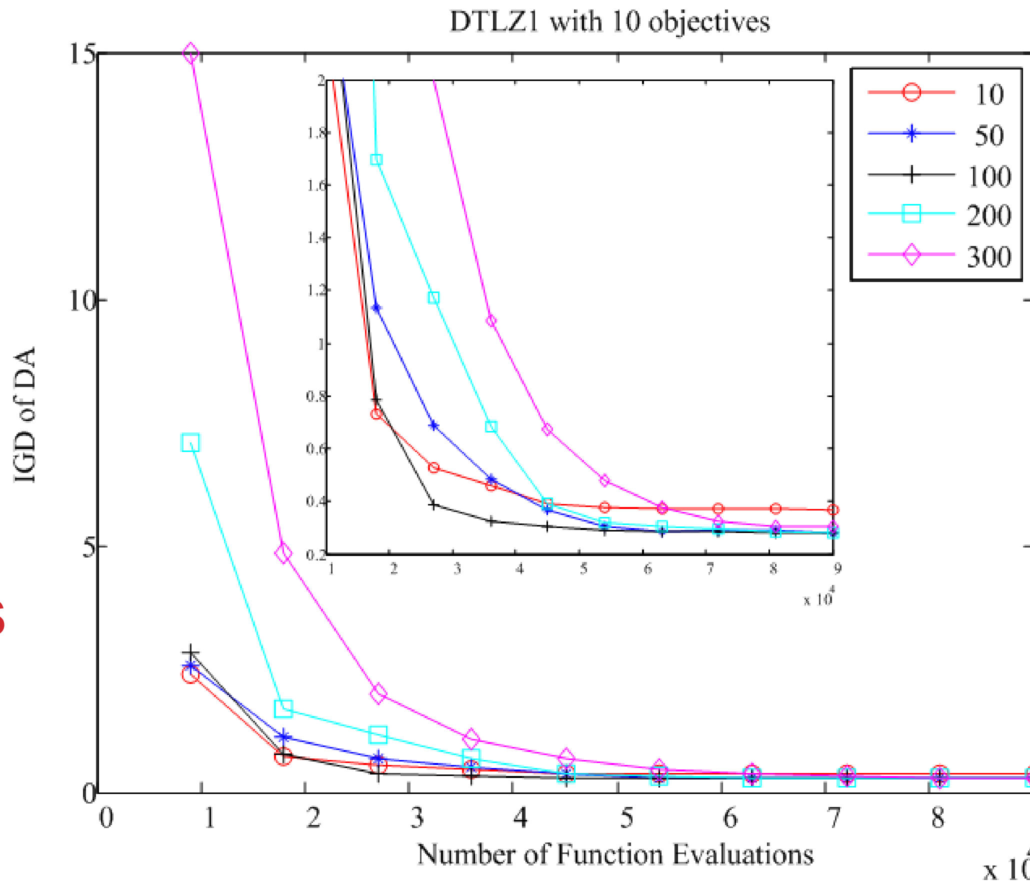
Archive Size of CA: Effect on CA



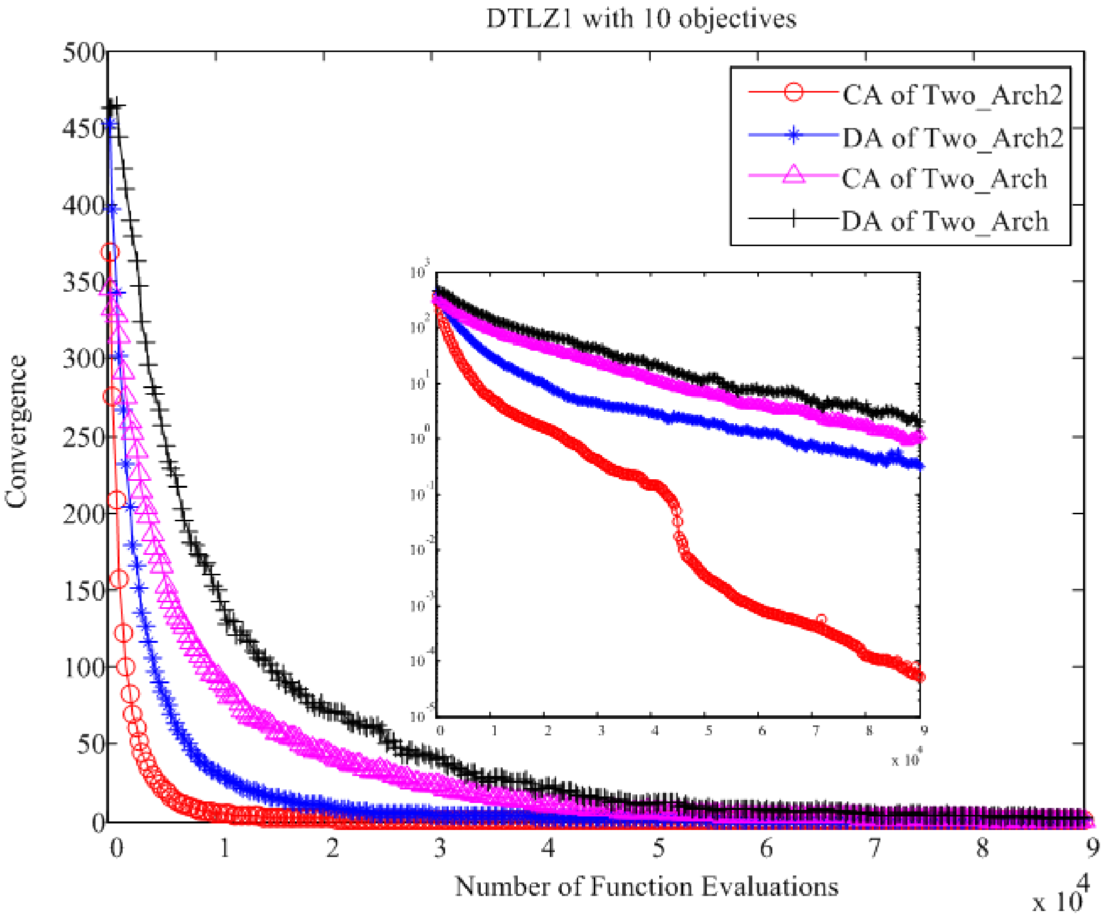
- A smaller size of CA can increase the convergence speed. Thus, the search focuses on a small number of good solutions.

Archive Size of CA: Effect on DA

- However, CA with a small size cannot result in good diversity in DA.
- CA with 100 solutions is set in Two_Arch2.



Improvement on Two_Arch



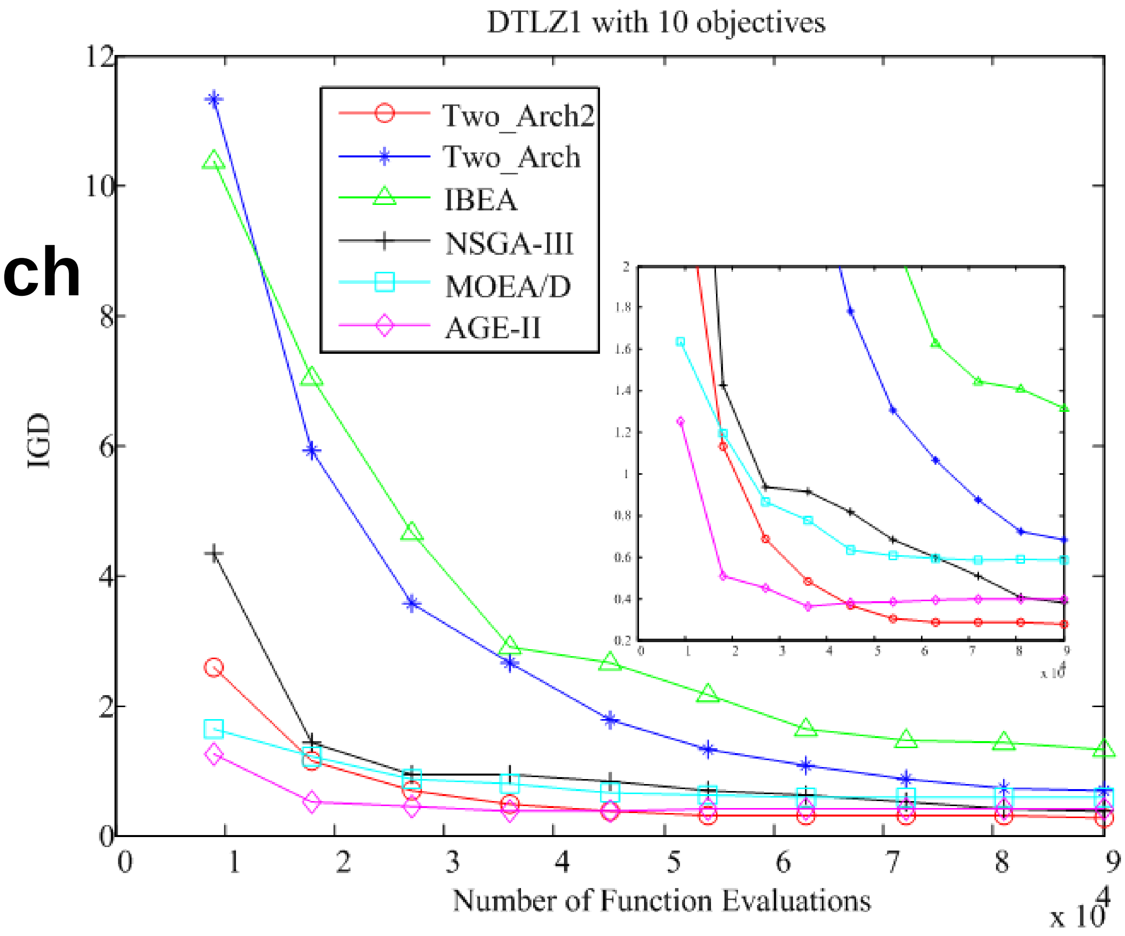
- CA in Two_Arch2 converge faster than CA in Two_Arch.
- DA (Pareto-based) in Two_Arch2 converge faster than CA in Two_Arch.
- CA passes good convergence to DA in Two_Arch2.

Comparison of MOEAs

- **Two_Arch**: a reference to show the improvement of Two_Arch2 on MaOPs
- **IBEA**: indicator-based ($I_{\varepsilon+}$) MOEA with good convergence but poor diversity
- **NSGA-III**: newly-proposed MOEA with reference points for MaOPs
- **MOEA/D**: aggregation function-based MOEA
- **AEG-II**: Pareto-based MOEA with the ε-grid approximation in the objective space

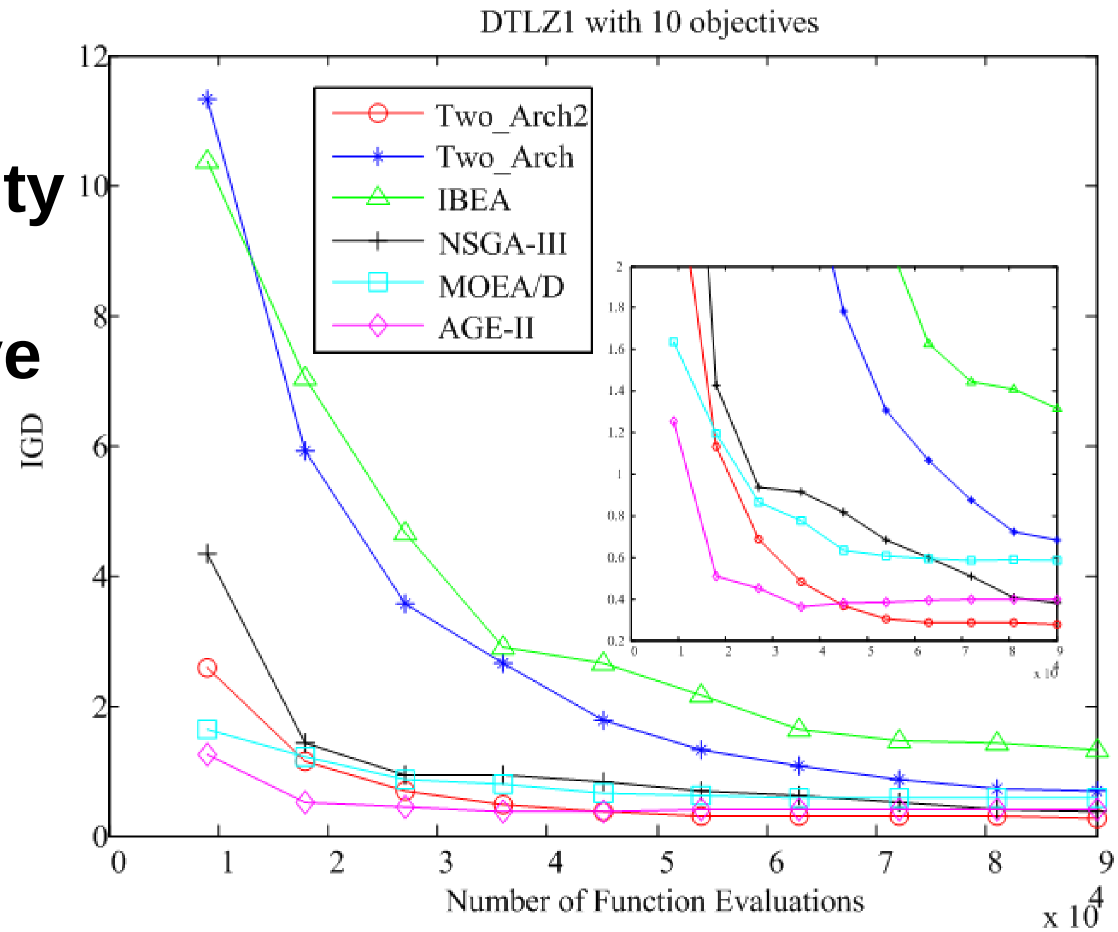
DTLZ1 with 10 Objectives

- IBEA focuses on convergence too much to maintain a good diversity.



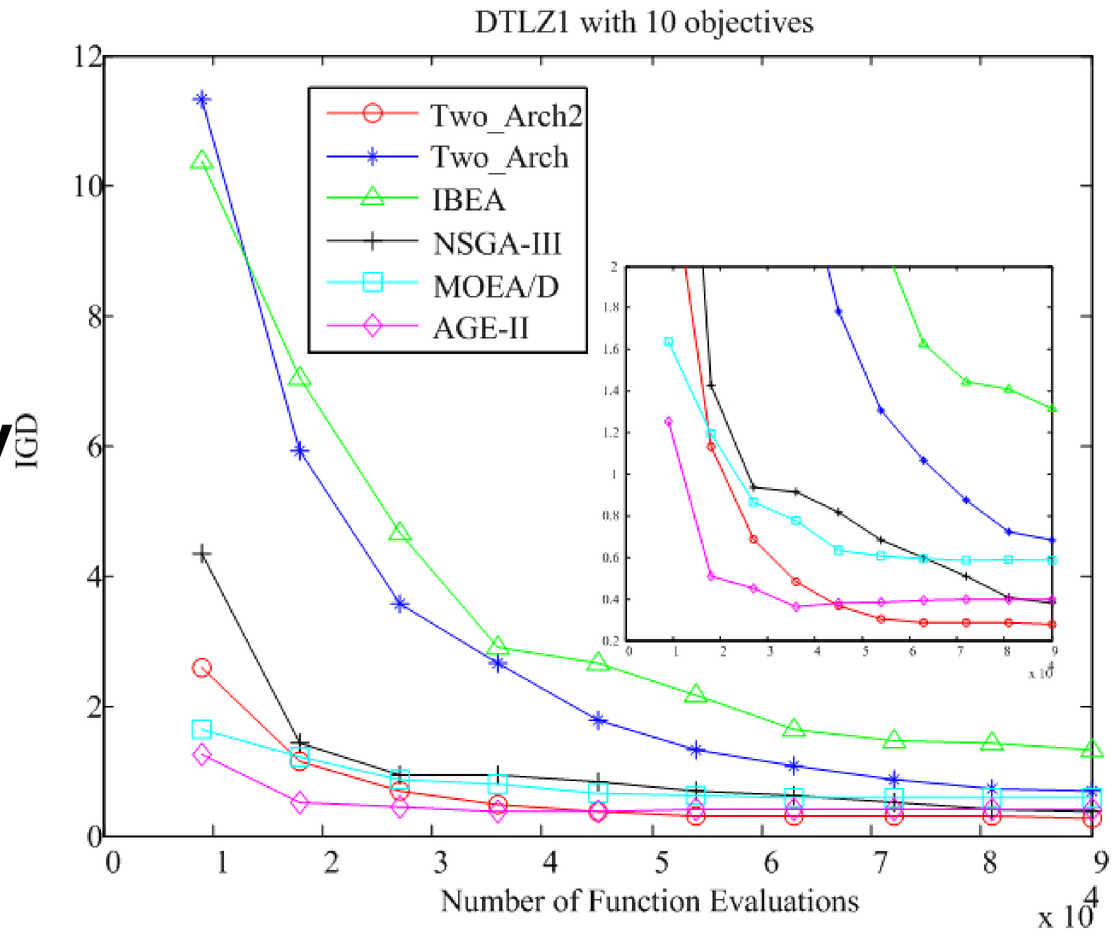
DTLZ1 with 10 Objectives

- MOEA/D cannot achieve good diversity in the high-dimensional objective space.



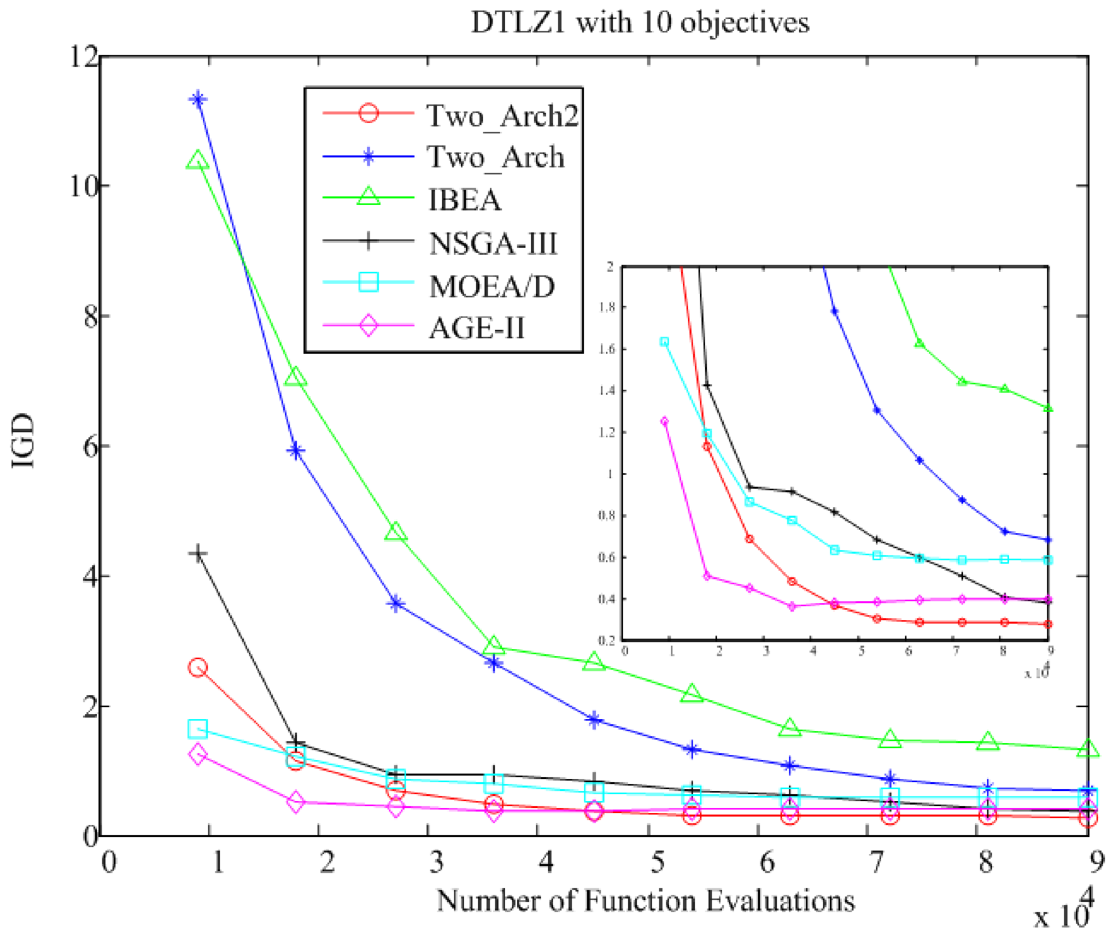
DTLZ1 with 10 Objectives

- **AGE-II: the ε-grid approximation can reduce the disadvantage of the Pareto dominance by lowering the conflicting degree among objectives.**



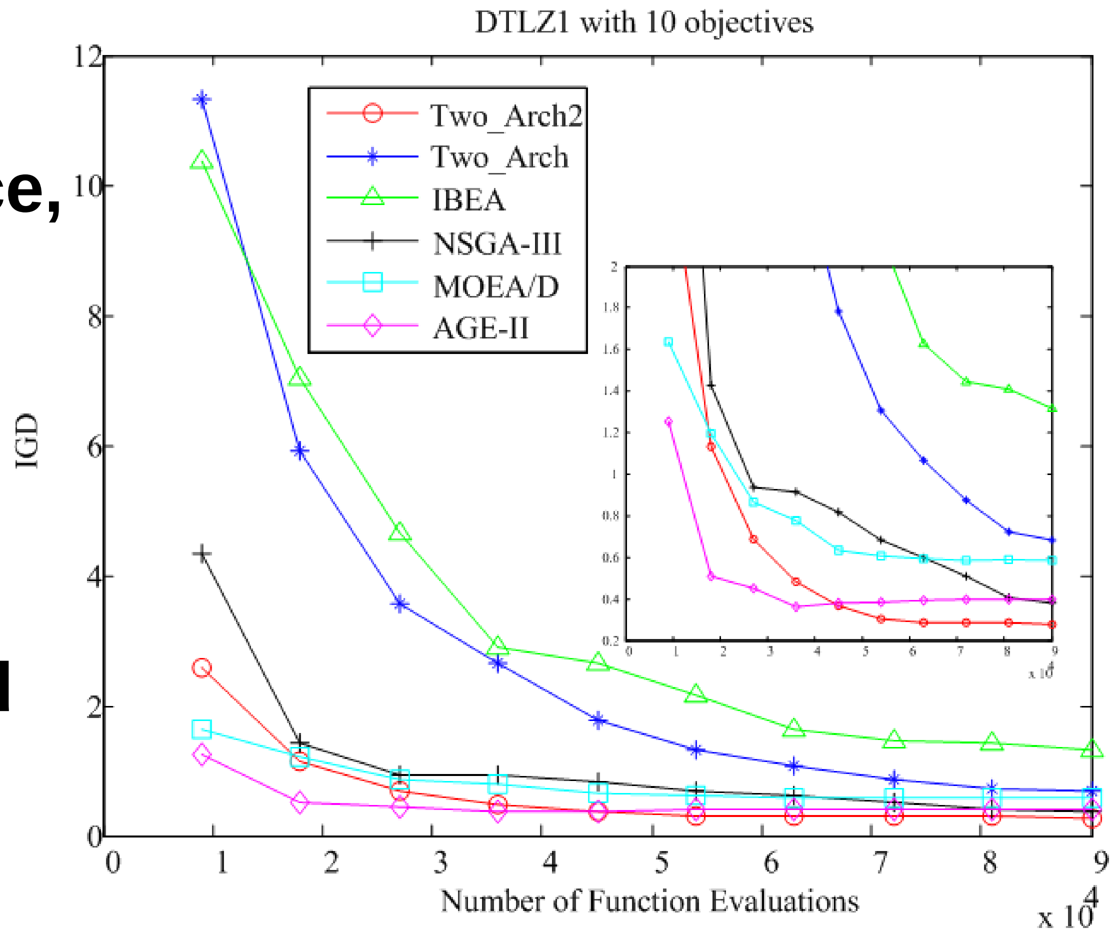
DTLZ1 with 10 Objectives

- Two_Arch: as a Pareto-based MOEA, its convergence on MaOPs is unsatisfactory.



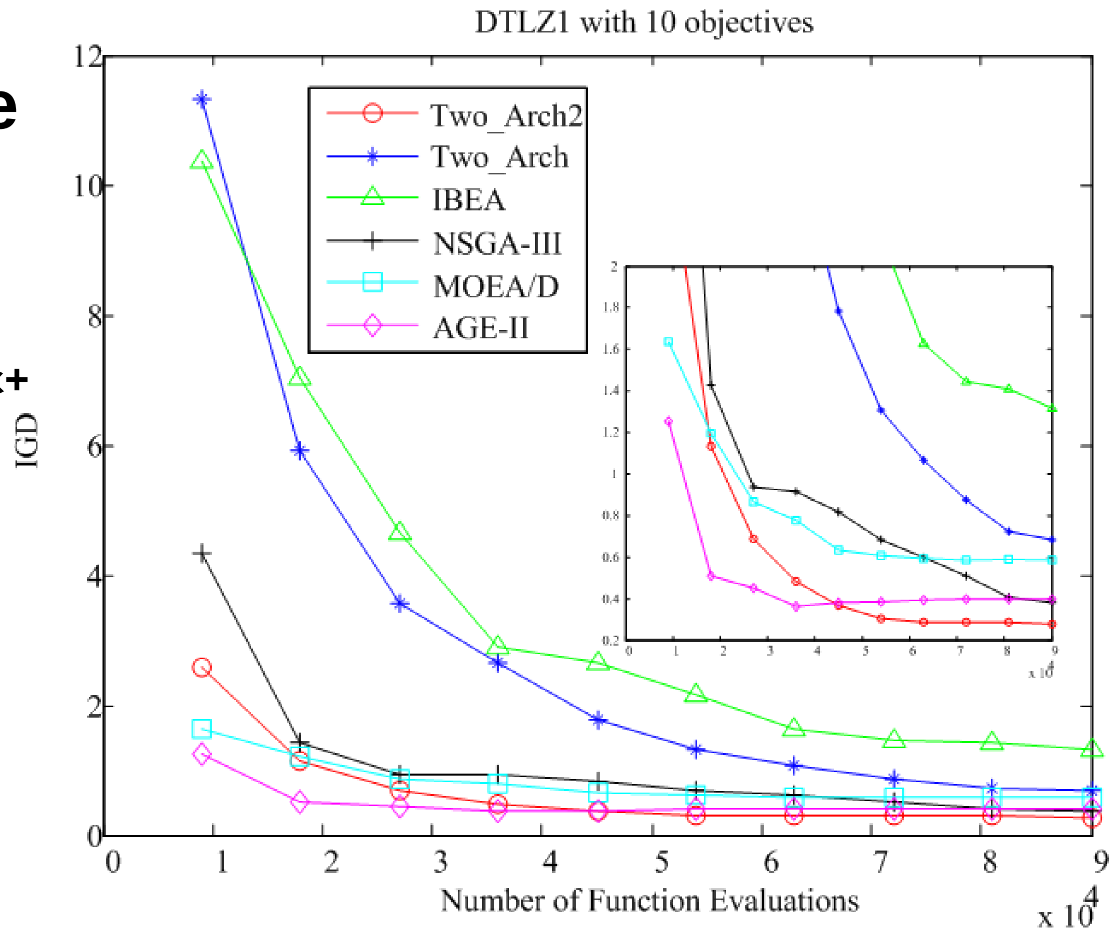
DTLZ1 with 10 Objectives

- NSGA-III: Similar to AGE-II in performance, with a reasonable balance between convergence (guaranteed by non-dominated sort) and diversity (guaranteed by reference points).



DTLZ1 with 10 Objectives

- Two_Arch2: Could be seen as a “hybrid” MOEA, takes the advantages of both $I_{\varepsilon+}$ and the $L_{1/m}$-norm-based distance.

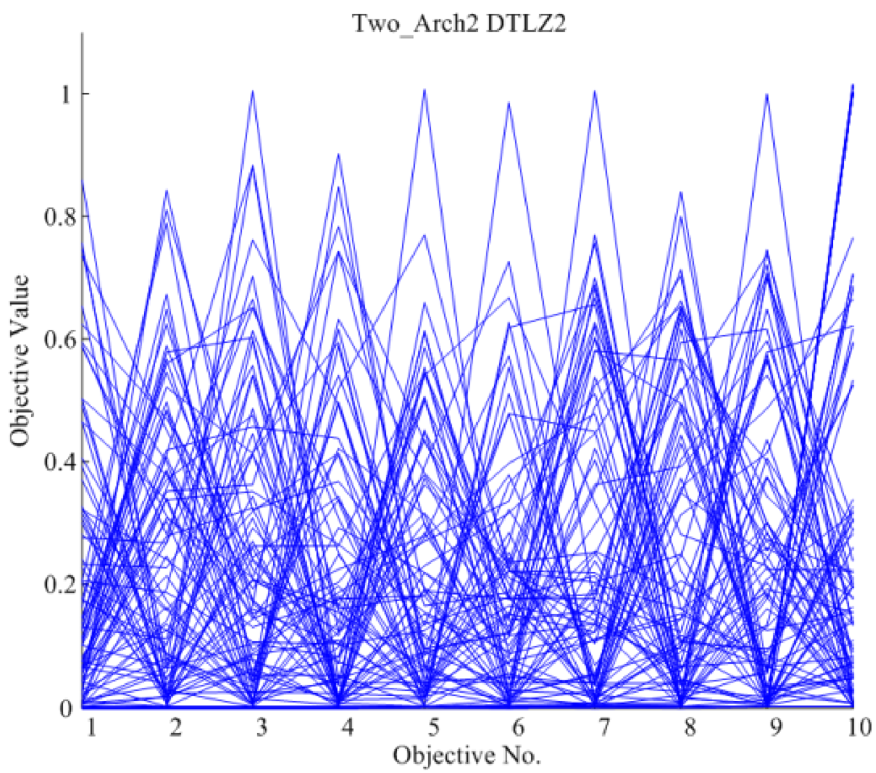


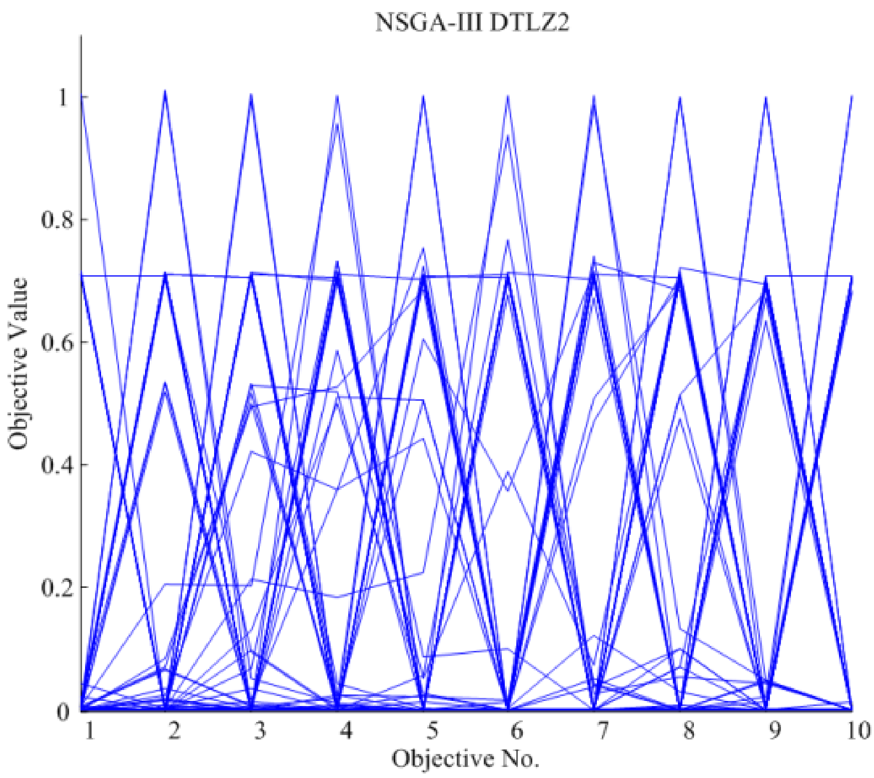
Computational Complexity Analysis

Algorithm	Convergence (Pareto-based)	Diversity Maintenance	Indicator-based Selection	Total
Two_Arch 2	$O(Nlog^{m-2}N)$	$O(mN^2)$	$O(N^2)$	$\max\{O(Nlog^{m-2}N), O(mN^2)\}$
Two_Arch	$O(mN^2)$	$O(mN^2)$	NA	$O(mN^2)$
IBEA	NA	NA	$O(N^2)$	$O(N^2)$
NSGA-III	$O(Nlog^{m-2}N)$	$O(mN^2)$	NA	$\max\{O(Nlog^{m-2}N), O(mN^2)\}$
MOEA/D	$O(mN^2)$	$O(mNT)$	NA	$O(mN^2)$
AGE-II	$O(Nlog^{m-2}N)$	$O(mN)$	NA	$\max\{O(Nlog^{m-2}N), O(mN)\}$

Two_Arch2 vs. NSGA-III on DTLZ2 with 10 Objectives

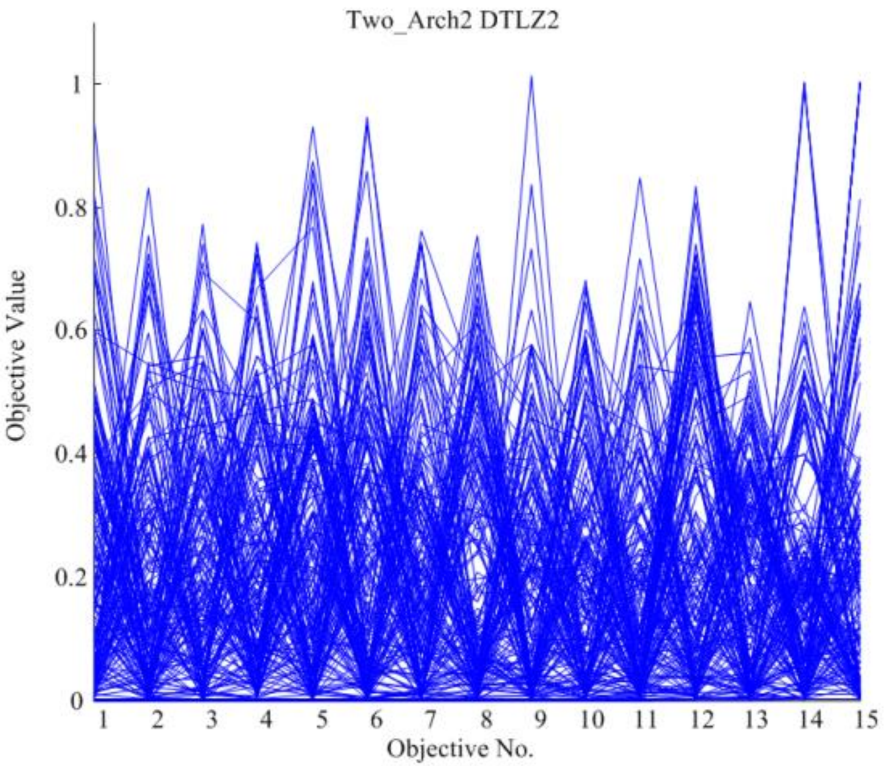
	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Fair
NSGA-III	Good	Fair	Good

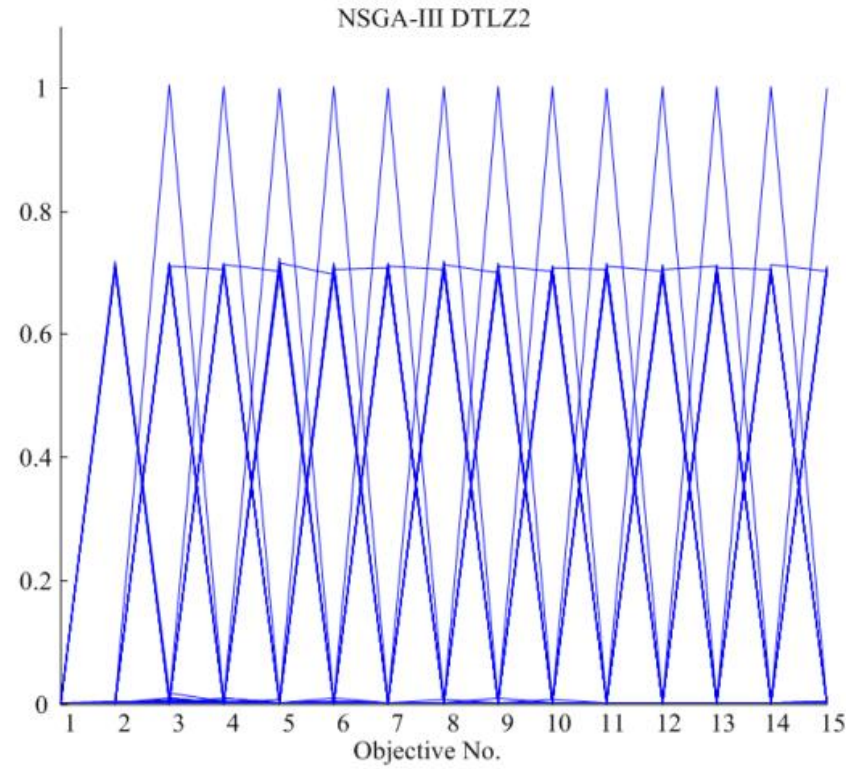




Two_Arch2 vs. NSGA-III on DTLZ2 with 15 Objectives

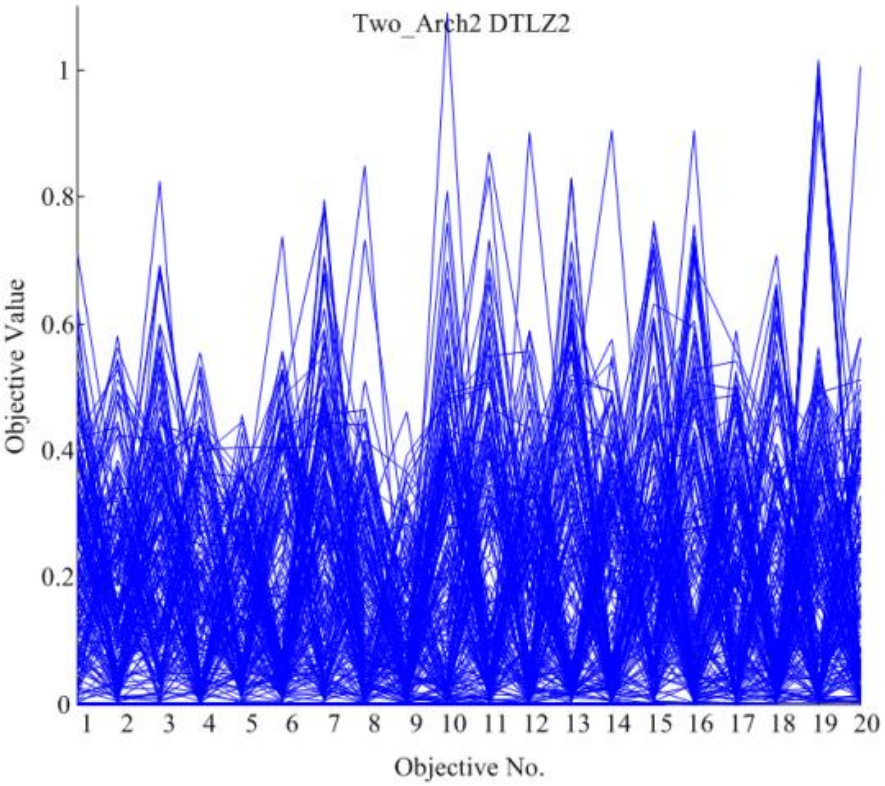
	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Poor
NSGA-III	Good	Fair	Good

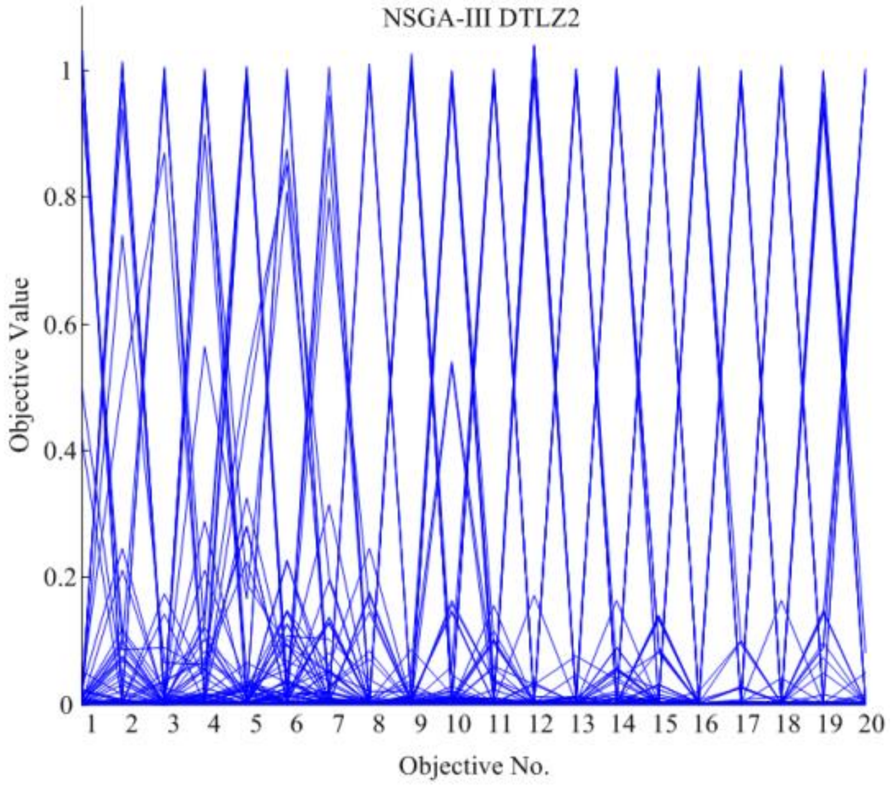




Two_Arch2 vs. NSGA-III on DTLZ2 with 20 Objectives

	Convergence	Diversity	Extreme point
Two_Arch2	Good	Good	Poor
NSGA-III	Good	Fair	Good





Two_Arch2 vs. NSGA-III

	Two_Arch2	NSGA-III
Convergence methodology	$I_{\varepsilon+}$	Pareto dominance
Convergence degeneration	No	No
Diversity maintenance	$L_{1/m}$-norm-based distance	Minimal perpendicular distances to reference points
Diversity degeneration	No	Increase with the dimension of objective space
Manual Settings	None	Reference points

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Conclusions and Future Work

- Two_Arch2 uses both an indicator and Pareto dominance.
- Such "Hybrid" MOEAs can solve MaOPs better than other existing MaOEAs.
- L_p-norm-based distances ($p<1$) work well for the diversity maintenance of MaOPs.

Future work:

- ☐ Other "hybrid" MaOEAs for MaOPs.
- ☐ Improve the extreme point maintenance in Two_Arch2.

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