Supplementary Material of "Performance Indicator based Adaptive Model Selection for Offline Data-Driven Multi-Objective Evolutionary Optimization"

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S.I. Computational Complexity Analysis

Many data-driven EAs consider that their own complexities are much smaller than the expensive fitness evaluation. However, their remaining complexities still need to be analyzed. Thus, the computational complexities of various algorithms are compared including IBEA-MS, two state-of-the-art offline data-driven evolutionary algorithms (AK-IBEA and NSGA-II-GP) and four online data-driven evolutionary algorithms (Par-EGO, MOEA/D-EGO, K-RVEA and CSEA) in Table S.I. All the algorithms are considered to optimize an MOP with M objectives and with N_{ini} initial training data points. These algorithms are supposed to maintain N individuals in one population and terminate in the same number of iterations G_{Ite}.

Building M Kriging models and radial basis function networks (RBFN) both needs to calculate the inverse matrix, whose complexity is $O(MN_{ini}^3)$ [1]. However, the RBFNs select $\sqrt{N_{ini}}$ individuals as centers and reduce the complexity to O(MN_{ini}^2). Thus, initial model building of IBEA-MS costs $O(MN_{ini}^3)$. The Kriging models need $O(MN)$ for the approximated evaluations in the population, which is larger than $O(M\sqrt{N})$ of the RBFNs. In order to assess the reliability of the Kriging models, IBEA-MS costs $O(MN^2)$ in each generation to calculate the objective differences and the sum values of root mean squared errors (RMSEs) for all objectives. Finally, one single objective is selected to calculate $I_{\epsilon+}$ for environmental selection costing $O(MN^2)$ [2] in each generation. Similar to IBEA-MS, AK-IBEA uses the Kriging models to assist IBEA. Therefore, AK-IBEA shares the same complexity in initial model building and model evaluation with IBEA-MS. However, in AK-IBEA, no model management is implemented and the environmental selection in NSGA-II is performed in the end for all the individuals evaluated by the Kriging models during the evolutionary process. Thus, the complexity of environmental selection in AK-IBEA is $O(MN^2G_{Ite}^2)$. NSGA-II-GP employs the second-order polynomial regression and Kriging models to assist NSGA-II. The second-order polynomial regression models are set to be the initial models costing $O(MD^6)$ [3], where D is the number of decision dimension. The Kriging models are used to evaluate the individuals and they are re-built every g_{itr} iterations costing $O(\frac{MN^3G_{Ite}}{g_{itr}})$. In addition, NSGA-II-GP performs the environmental selection in NSGA-II costing $O(MN^2)$ [4] in each generation.

The complexities of initial models building in Par-EGO, K-RVEA and MOEA/D-EGO are $O(MN_{ini}^3)$ owing to the Kriging models. The complexities of their model evaluations are $O(MN)$ in each generation. All of these three algorithms cost total $O(\frac{MN^3G_{Ite}}{g_{itr}})$ for re-building the Kriging models in model management during the evolutionary process. The only difference exists in their different environmental selection strategies, where Par-EGO using the augmented Tchebycheff function reduces the complexity to $O(MN)$. CSEA employs a feedforward neural network (FNN) [5] to classify the individuals. Building the initial FNN costs $O(N_FN_{ini}^2)$ and N_F represents the number of the iterations for the back-

TABLE S.I
Computational Complexity of IBEA-MS, AK-IBEA, NSGA-II-GP, Par-EGO, K-RVEA, MOEA/D-EGO and CSEA.

Algorithms	**Initial Model Building**	**Evaluation Using Model**	**Model Management**	**Environmental Selection**
IBEA-MS	$O(MN_{ini}^3)$	$O(MNG_{Ite})$	$O(MN^2G_{Ite})$	$O(MN^2G_{Ite})$
AK-IBEA	$O(MN_{ini}^3)$	$O(MNG_{Ite})$	$O(0)$	$O(MN^2G_{Ite}^2)$
NSGA-II-GP	$O(MD^6)$	$O(MNG_{Ite})$	$O(\frac{MN^3G_{Ite}}{g_{itr}})$	$O(MN^2G_{Ite})$
Par-EGO	$O(MN_{ini}^3)$	$O(MNG_{Ite})$	$O(\frac{MN^3G_{Ite}}{g_{itr}})$	$O(MNG_{Ite})$
K-RVEA	$O(MN_{ini}^3)$	$O(MNG_{Ite})$	$O(\frac{MN^3G_{Ite}}{g_{itr}})$	$O(MN^2G_{Ite})$
MOEA/D-EGO	$O(MN_{ini}^3)$	$O(MNG_{Ite})$	$O(\frac{MN^3G_{Ite}}{g_{itr}})$	$O(MN^2G_{Ite})$
CSEA	$O(N_FN_{ini}^2)$	$O(NG_{Ite})$	$O(\frac{N_FN^2G_{Ite}}{g_{itr}})$	$O(MN^2G_{Ite})$

propagation algorithm [6]. In each generation, classifying the individuals by FNN costs $O(N)$ and managing the model in CSEA costs $O(\frac{N_F N^2 G_{Ite}}{g_{itr}})$ in total. In addition, the complexity of environmental selection of CSEA is $O(MN^2)$ in each generation.

The most time-consuming part of IBEA-MS is initial model building, whose complexity is $O(MN_{ini}^3)$. AK-IBEA spends the most of the time in environmental selection costing $O(MN^2G_{Ite}^2)$, which is bigger than $O(MN_{ini}^3)$ generally. Other five evolutionary algorithms have almost the same complexity with IBEA-MS in the first part except NSGA-II-GP. However, their model management strategies costing $O(\frac{MN^3G_{Ite}}{g_{itr}})$ or $O(\frac{N_F N^2 G_{Ite}}{g_{itr}})$ $(N_F \geq MN)$ consume the most of the time during the evolutionary process, where IBEA-MS only costs $O(MN^2G_{Ite})$. Other components in these algorithms cannot decide the total complexity in general. Thus, IBEA-MS has the lowest computational complexity among these seven algorithms.

TABLE S.II
SIX VARIANTS USING DIFFERENT MOEAS AND DIFFERENT SURROGATE MODELS, RESPECTIVELY, WHERE MOEAS ARE THE MAIN OPTIMIZER THESE VARIANT USE AND SURROGATES REPRESENT THEIR SURROGATE MODELS.

Algorithm	MOEAs	Surrogates
K-MOEA/D	MOEA/D	Kriging model
R-MOEA/D	MOEA/D	RBFN model
K-NSGA-II	NSGA-II	Kriging model
R-NSGA-II	NSGA-II	RBFN model
K-IBEA	IBEA	Kriging model
R-IBEA	IBEA	RBFN model

TABLE S.III
CHARACTERISTICS OF THE SELECTED PROBLEMS.

Problem	Characteristic
DTLZ1	Multimodal
DTLZ2	Unimodal
DTLZ6	Degenerate curve
DTLZ7	Disconnected PFs

S.II. EFFECTS OF DIFFERENT MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS AND SURROGATE MODELS

To discuss the effects of different multi-objective evolutionary algorithms (MOEAs) on the proposed algorithm, six variants defined in Table S.II are compared, which use MOEA/D [7], NSGA-II [4] and IBEA$_{\epsilon+}$ [8] as the main optimizers and the Kriging models and the RBFNs as the surrogate models directly. DTLZ1, DTLZ2, DTLZ6 and DTLZ7 problems with two and five objectives are selected for the experiment, where the characteristics of these problems [9] are listed in Table S.III.

The comparative results are displayed in Table S.IV, which is analyzed by the Friedman test with the Bergman-Hommel post-hoc test [10] and their Friedman ranking is listed in the last line. The average results of the independent runs are shown in the cells and their variances are written in the following brackets. Moreover, the p-values of their Friedman ranking are shown in Table S.V.

As shown in Table S.IV, R-IBEA wins the overall ranking among these six algorithms on the selected problems and K-IBEA ranks the second place. Comparing with different MOEAs assisted by the same surrogate models, K-IBEA and R-IBEA win the ranking in their respective groups. Thus, IBEA$_{\epsilon+}$ can be considered to have advantages for solving offline data-driven multi-objective optimization problems (MOPs) over MOEA/D and NSGA-II. These experimental results verify our hypothesis that IBEA$_{\epsilon+}$ is suitable for solving offline data-driven optimization problems. In addition, the performance of K-IBEA and R-IBEA are also compared in Table S.IV, which are analyzed by the Wilcoxon rank sum test (0.05 significance level) [10]. The relationship between K-IBEA and R-IBEA are expressed by the symbols '+', '−' and '=', which represent performing significantly better, significantly worse and no statistically difference, respectively. The better results between such two algorithms are marked in bold. In the comparison of K-IBEA with R-IBEA in Table S.IV, it can be observed that there are obvious differences between the two algorithms on different problems. These differences provide a sufficient motivation for our adaptive model selection mechanism.

TABLE S.IV
COMPARISON OF THREE DIFFERENT BASELINE MOEAS: MOEA/D, NSGA-II AND IBEA ASSISTED BY THE KRIGING MODELS AND THE RBFNS , WHERE THE RESULTS ARE ANALYZED BY THE FRIEDMAN TEST AND THEIR AVERAGE RANKS ARE SHOWN IN THE LAST LINE.

Problem	M	D	K-MOEA/D	K-NSGA-II	K-IBEA	R-MOEA/D	R-NSGA-II	R-IBEA
DTLZ1	2	10	1.315e+2(3.54e+1)	1.200e+2(2.91e+1)	**1.072e+2(2.22e+1)+**	1.373e+2(4.39e+1)	1.362e+2(3.90e+1)	1.354e+2(4.57e+1)
	5	10	4.709e+1(1.66e+1)	7.952e+1(2.17e+1)	**5.434e+1(1.74e+1)=**	5.557e+1(1.46e+1)	8.637e+1(2.37e+1)	5.831e+1(1.60e+1)
DTLZ2	2	10	4.216e-1(1.11e-1)	5.299e-1(1.89e-1)	6.001e-1(2.10e-1)-	4.156e-2(1.60e-2)	4.534e-2(2.00e-2)	**3.859e-2(1.46e-2)**
	5	10	6.107e-1(5.23e-2)	8.110e-1(1.32e-1)	8.234e-1(1.75e-1)-	4.925e-1(9.03e-2)	6.202e-1(7.71e-2)	**4.863e-1(8.43e-2)**
DTLZ6	2	10	4.479e+0(9.45e-1)	3.228e+0(9.99e-1)	2.533e+0(1.19e+0)=	3.790e+0(7.74e-1)	2.846e+0(1.06e+0)	**2.452e+0(9.76e-1)**
	5	10	2.507e+0(4.18e-1)	3.591e+0(5.31e-1)	2.440e+0(6.92e-1)-	1.576e+0(6.19e-1)	2.474e+0(5.08e-1)	**1.619e+0(5.67e-1)**
DTLZ7	2	10	1.621e-2(3.39e-3)	1.111e-2(3.80e-3)	**1.021e-2(1.25e-3)+**	4.486e-2(1.45e-1)	9.640e-2(1.80e-1)	3.164e-2(9.70e-2)
	5	10	6.168e-1(6.69e-2)	4.847e-1(3.53e-2)	**4.548e-1(9.09e-2)+**	6.813e-1(1.12e-1)	7.711e-1(1.41e-1)	6.379e-1(3.98e-1)
Average Rank			3.500	3.875	2.750	3.625	4.625	2.625

TABLE S.V
P-VALUES OF SIX VARIANTS ALGORITHMS ANALYZED BY THE FRIEDMAN TEST WITH THE BERGMAN-HOMMEL POST-HOC, WHERE K1-3 REPRESENT K-MOEA/D, K-NSGA-II, AND K-IBEA AND R1-3 REPRESENT R-MOEA/D, R-NSGA-II, AND R-IBEA.

\	K1	K2	K3	R1	R2	R3
K1	NA	0.688	0.423	0.894	0.229	0.350
K2	0.688	NA	0.229	0.789	0.423	0.181
K3	0.423	0.229	NA	0.350	**0.045**	0.894
R1	0.894	0.789	0.350	NA	0.285	0.285
R2	0.229	0.423	**0.045**	0.285	NA	**0.033**
R3	0.350	0.181	0.894	0.285	**0.033**	NA

S.III. Effect of Surrogate Model Settings

In order to analyze the impact of the model settings on the performance of the proposed algorithm, five variants of the proposed algorithm (denoted by V1-5) with different surrogate model parameters or structures have been examined on 3-objective DTLZ1, DTLZ2, DTLZ6 and DTLZ7 problems with 10 and 30 decision variables in this experiment. The models settings of V1-5 are different from IBEA-MS with only one model parameter, respectively, including the hyper-parameter range of the Kriging model, the order of polynomial regression of the Kriging model, the hyper-parameter α of the RBFN, and the number of hidden layer nodes of the RBFN. These settings are listed in Table S.VI.

The performance of V1-5 is compared with that of IBEA-MS and the results on different problems analyzed by the Wilcoxon rank sum test (0.05 significance level) symbolized via '$+$', '$-$' and '$\approx$' are presented in Table S.VII. The times of 'win', 'loss', and 'draw' of V1-5 are summed in the last row of the tables and the best results among the compared algorithms on each problem are highlighted in bold. Table S.VII shows that the performance of V2 and V5 is slightly worse than that of IBEA-MS, while the performance of V1, V3 and V4 is similar to that of IBEA-MS. The results of V1, V4 and V5 indicate that the parameter settings of the Kriging model hyper-parameter range changed from $[10^{-3}, 10^3]$ to $[0.1, 10]$ and the number of hidden layer nodes of RBFN changed from $\sqrt{N_{ini}}$ to $\sqrt{N_{ini}}*2$ or $\sqrt{N_{ini}}/2$ have little effect on the results.

TABLE S.VI
THE DIFFERENT SETTING BETWEEN FIVE VERSIONS WITH ORIGINAL IBEA-MS, WHERE $\boldsymbol{D}$ IS THE DISTANCE MATRIX OF THE CENTERS DISTANCE PAIRS OF RBFN AND N_{ini} IS THE NUMBER OF THE OFFLINE DATA.

Algorithm	Parameter Setting
V1	Hyper-parameter range of the Kriging model: $[0.1, 10]$
V2	Polynomial regression of the Kriging model: zero order
V3	Hyper-parameter α of the RBFN: $mean(\boldsymbol{D})$
V4	Number of hidden layer nodes of the RBFN: $\sqrt{N_{ini}}*2$
V5	Number of hidden layer nodes of the RBFN: $\sqrt{N_{ini}}/2$

Thus, the default values in [11] and [12] are adopted for these two parameter settings. When the number of the RBFN hidden layer nodes increases to $\sqrt{N_{ini}}*2$, V4 performs similarly to that of IBEA-MS. However, fewer nodes will lead to a lower computational complexity, which is the reason of using $\sqrt{N_{ini}}$ in IBEA-MS. Through comparing V2 with IBEA-MS, it can be concluded that the first order polynomial regression can better fit the offline data-driven problems than the zero-order polynomial regression. In addition, the offline data quantity requirement of the second order polynomial regression makes it impossible to construct high-quality models for the high-dimensional problems. Finally, the results of V3 indicate that a larger α will lead to better performance, because the problems that the Kriging models fail to optimize are complex and a larger α resulting in a smoother fitting curve can better capture the global features of the objective function. In summary, the performance of the proposed algorithm is relatively insensitive to the parameter settings of the surrogate models.

S.IV. Effects of Offline Data

In order to verify the robustness of IBEA-MS to the distribution of the offline data, two different methods Latin hypercube sampling (LHS) and random sampling strategy (RS) are adopted to generate offline data separately. All algorithms under comparison, NSGA-II-GP, IBEA-MS, and three variants of AK-IBEA are run with offline data sampled by two sampling strategies on 3-objective DTLZ1, DTLZ2, DTLZ6 and DTLZ7 problems with 10, 20 and 30 decision variables. The Wilcoxon rank sum test (0.05 significance level) is performed to compare the performance of each algorithm with different sampling strategies on each test problems. In other words, the results are compared pairwise in Table S.VIII. In addition, NSGA-II-GP cannot solve 20- and 30-dimensional problems due to the data amount requirement by the low-order polynomial regression models. Thus, NSGA-II-GP is only tested on the problems with 10 decision variables.

It can be found in Table S.VIII that there is little difference in performance of all algorithms under different sampling strategies. These results indicate that the two offline data sampling strategies have no significant impact on all algorithms on the test problem. It can also be observed in Table S.VIII that there is no significant difference in the performance of IBEA-MS on all problems when using these two different sampling strategies. Therefore, IBEA-MS is fairly insensitive to the offline data.

S.V. Comparative Experiments

Various of test problems are chosen to compare the performance of IBEA-MS with other comparative algorithms including DTLZ1-7, ZDT1-4 and ZDT6, inverse DTLZ(IDTLZ1-2), WFG1-9, UF1-12 and MaF1-9 problems. DTLZ and ZDT problems are also designed with high decision dimensions and objective numbers to further test algorithm performance. Considering the amount requirement for the offline data of the low-order polynomial regression, NSGA-II-GP cannot deal with high-dimensional problems. Thus, all the compared algorithms are tested on the DTLZ and ZDT problems with 10 decision

TABLE S.VII
EXPERIMENT OF THE EFFECT WITH DIFFERENT SURROGATE MODELS PARAMETERS AND STRUCTURES, WHICH IS TESTED ON 3-OBJECTIVE DTLZ1, DTLZ2, DTLZ6 AND DTLZ7 PROBLEMS WITH 10 AND 30 DECISION VARIABLES. V1-5 ARE COMPARED WITH IBEA-MS AND ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL).

	M	D	V1	V2	V3	V4	V5	IBEA-MS
DTLZ1	3	10	9.376e+1 (3.13e+1)=	9.862e+1(2.71e+1)=	1.162e+2(1.84e+1)-	**7.546e+1 (2.14e+1)=**	1.094e+2 (2.64e+1)-	8.442e+1(2.44e+1)
	3	30	6.336et2(4.55e+1)=	6.426e+2(4.77e+1)=	6.093e+2 (5.47e+1)=	**5.998e+2(6.56e+1)=**	6.221e+2 (4.59e+1)=	6.302e+2(4.67e+1)
DTLZ2	3	10	1.556e-1(4.50e-2)=	1.519e-1 (5.82e-2)=	1.355e-1(2.40e-2)=	2.880e-1 (1.71e-1)-	2.324e-1(3.23e-2)-	**1.439e-1(4.84e-2)**
	3	30	2.176e-1(3.82e-2)=	2.204e-1(3.53e-2)=	1.829e-1(2.95e-2)+	**1.781e-1(5.17e-2)+**	3.482e-1 (5.70e-2)-	2.125e-1(3.83e-2)
DTLZ6	3	10	2.074e+0 (7.08e-1)=	2.191et0(9.75e-1)=	2.743e+0 (5.29e-1)-	2.371e10(6.31e-1)=	**1.524e+0(5.46e-1)+**	2.374e+0(8.63e-1)
	3	30	1.626e+1 (1.27e+0)=	1.616e+1 (1.37e+0)=	2.132e+1(1.36e+0)=	1.614e+1(1.45e+0)=	1.723e+1 (1.66e+0)=	**1.580e+1(1.32e+0)**
DTLZ7	3	10	1.317e-1(8.42e-3)-	1.734e-1(1.62e-1)-	1.200e-1(4.77e-2)=	1.135e-1(2.14e-2)=	1.225e-1(4.79e-2)=	**1.099e-1(1.63e-2)**
	3	30	1.303e-1(7.38e-3)=	3.163e10(1.60e10)-	1.328e-1(7.85e-3)=	1.321e-1(5.96e-3)=	2.584e-1 (7.01e-2)=	**1.330e-1(8.55e-3)**
+/-/=			0/1/7	0/2/6	1/2/5	1/1/6	1/3/4	- -

TABLE S.VIII
COMPARISON EXPERIMENTS OF LHS AND RANDOM SAMPLING STRATEGY ON 3-OBJECTIVE DTLZ1, DTLZ2, DTLZ6 AND DTLZ7 PROBLEMS WITH 10, 20, 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL).

Problem	M	D	Sampling Strategy	NSGA-II-GP	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	3	10	LHS	1.554e+2(5.90e+1)=	1.109e+2(2.47e+1)=	1.146e+2(2.18e+1)=	7.651e+1(1.06e+1)=	9.459e+1(2.85e+1)=
			RS	1.583e+2 (7.40e+1)	1.090e+2(2.22e+1)	1.127e+2(1.84e+1)	7.698e+1 (1.74e+1)	8.722e+1(2.49e+1)
	3	20	LHS	- -	3.878e+2(4.42e+1)=	3.882e+2(4.02e+1)=	2.666e+2(4.79e+1)=	3.751e+2(4.31e+1)=
			RS	- -	4.098e+2(4.25e+2)	3.904e+2(5.66e+2)	2.785e+2(5.99e+2)	3.868e+2(2.70e+2)
	3	30	LHS	- -	6.761e+2(7.03e+1)=	7.012e+2(5.97e+1)=	6.475e+2(1.47e+2)=	6.292e+2(4.52e+1) =
			RS	- -	6.898e+2(5.80e+2)	6.591e+2(7.19e+2)	6.038e+2(1.08e+3)	6.305e+2(3.34e+2)
DTLZ2	3	10	LHS	4.205e-1(2.69e-2)=	3.931e-1(5.35e-2)=	4.089e-1(5.24e-2)=	8.206e-1(1.26e-1)=	1.402e-1(4.80e-2)=
			RS	4.198e-1(3.23e-2)	3.824e-1(4.69e-2)	4.020e-1(4.37e-2)	8.079e-1 (1.12e-1)	1.508e-1(6.33e-2)
	3	20	LHS	- -	9.018e-1(8.59e-2)=	9.105e-1(6.29e-2)=	1.558e+0(1.73e-1)=	1.866e-1(6.70e-2)=
			RS	- -	9.092e-1(6.63e-3)	9.104e-1(8.20e-3)	1.564e+0(1.52e-2)	1.751e-1(3.56e-3)
	3	30	LHS	- -	1.464e+0(9.89e-2)=	1.450e+0(9.51e-2)=	2.196e+0(2.19e-1)=	2.248e-1(3.66e-2)=
			RS	- -	1.499e+0(1.16e-2)	1.484e+0(1.13e-2)	2.172e+0(2.45e-2)	2.330e-1(4.10e-3)
DTLZ6	3	10	LHS	4.871e+0(7.75e-1)=	5.710e+0(4.00e-1)=	5.427e+0(5.30e-1)=	3.173e+0(8.16e-1)=	2.134e+0(6.18e-1)=
			RS	4.731e+0(7.41e-1)	5.489e+0 (4.22e-1)	5.481e+0(5.14e-1)	3.243e+0(8.28e-1)	2.099e+0 (5.67e-1)
	3	20	LHS	- -	1.266e+1(7.55e-1)=	1.280e+1(1.08e+0)=	1.159e+1(1.35e+0)=	8.109e+0(1.02e+0)=
			RS	- -	1.296e+1(8.61e-2)	1.287e+1(8.29e-2)	1.126e+1(1.12e+1)	8.091e+0(8.62e-2)
	3	30	LHS	- -	1.694e+1(5.69e-1)+	1.721e+1(7.13e-1)=	1.631e+1(8.22e-1)+	1.326e+1(1.26e+0)=
			RS	- -	2.132e+1(1.09e+1)	2.080e+1(1.10e+1)	2.048e+1(9.36e-2)	1.589e+1(144e+1)
DTLZ7	3	10	LHS	9.782e-1(5.21e-1)=	6.408e-1(1.65e-1)=	4.933e-1(1.87e-1)=	1.307e+0(5.26e-1)=	1.126e-1(1.71e-2)=
			RS	9.317e-1 (4.29e-1)	7.461e-1(3.05e-1)	4.818e-1(2.33e-1)	1.584e+0 (9.04e-1)	1.128e-1(1.73e-2)
	3	20	LHS	- -	9.118e-1(3.59e-1)=	9.239e-1(5.28e-1)-	2.849e+0(1.70e+0)=	1.287e-1(6.52e-3)=
			RS	- -	9.320e-1(3.46e-2)	8.799e-1(3.65e-2)	2.941e+0(1.16e+1)	1.291e-1(7.36e-4)
	3	30	LHS	- -	1.115e+0(3.47e-1)=	1.147e+0(3.84e-1)=	4.348e+0(2.13e+0)=	1.364e-1(2.63e-2)=
			RS	- -	1.178e+0(5.27e-2)	1.068e+0(4.03e-2)	4.781e+0(2.13e+1)	1.341e-1(9.72e-4)

variables. AK-IBEA variants are compared with IBEA-MS on the problems with high decision variables then. In addition, the results of the DTLZ and ZDT problems are also evaluated by the hyper-volume (HV) performance indicator for extended verification. Other experiment results are evaluated by the inverted generational distance (IGD) performance indicator. All the results on different problems are analyzed by the Wilcoxon rank sum test (0.05 significance level) symbolized by '+', '$-$' and '='. The '$+ \backslash - \backslash =$' values of the competitor algorithms are summed in the last line of the tables and the best results among compared algorithms on every problem are marked in bold. The comparative experiment results are displayed from Table S.IX to S.XVIII.

The experiment results of DTLZ and ZDT problems are shown in the Table S.IX, S.X and S.XI and the experiment results are discussed in the Section IV-B of the manuscript. The re-evaluated results of the DTLZ and ZDT problems group by the HV performance indicator are displayed in the Table S.XII, S.XIII and S.XIV. It can be observed that the results evaluated by the HV indicator are similar to the tests evaluated by the IGD indicator. Such results also further confirm the superiority of the proposed algorithm. The results of the experiments on the DTLZ and ZDT problems with high decision variants and objectives are presented in Table S.XV. In this group, IBEA-MS is compared with AK-IBEA variants on the three-objective DTLZ1-7 problems with 40 and 50

TABLE S.IX
COMPARATIVE EXPERIMENT BETWEEN NSGA-II-GP, AK-IBEA VARIANTS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 10 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	NSGA-II-GP	IBEA-MS
DTLZ1	2	10	1.36e+02(3.33e+01)=	1.52e+02(2.38e+01)-	**8.20e+01(1.08e+01)+**	1.41e+02(2.69e+01)-	1.30e+02(4.13e+01)
	3	10	1.11e+02(2.47e+01)-	1.15e+02(2.18e+01)-	**7.65e+01(1.06e+01)+**	1.21e+02(2.44e+01)-	9.66e+01(3.10e+01)
	5	10	6.12e+01(2.19e+01)=	6.00e+01(1.99e+01)=	6.19e+01(9.45e+00)=	6.99e+01(1.70e+01)-	**5.96e+01(1.43e+01)**
	10	10	**7.30e-01(4.41e-01)=**	8.35e-01(6.33e-01)=	2.51e+00(2.20e+00)-	5.38e+00(1.81e-01)-	1.31e+00(1.96e+00)
DTLZ2	2	10	3.07e-01(7.41e-02)-	3.12e-01(4.92e-02)-	7.74e-01(2.14e-01)-	3.36e-01(4.01e-02)-	**4.03e-02(2.75e-02)**
	3	10	3.93e-01(5.35e-02)-	4.09e-01(5.24e-02)-	8.21e-01(1.26e-01)-	3.78e-01(2.69e-02)-	**1.39e-01(3.34e-02)**
	5	10	4.86e-01(2.63e-02)=	5.09e-01(3.01e-02)-	8.97e-01(1.18e-01)-	**4.42e-01(3.31e-02)=**	4.66e-01(6.78e-02)
	10	10	6.70e-01(2.94e-02)+	6.47e-01(2.45e-02)+	9.88e-01(6.61e-02)+	**6.35e-01(2.95e-02)+**	1.05e+00(1.02e-01)
DTLZ3	2	10	3.34e+02(6.57e+01)-	3.53e+02(7.19e+01)-	**2.32e+02(3.54e+01)=**	3.04e+02(5.85e+02)-	2.50e+02(6.49e+01)
	3	10	3.10e+02(5.90e+01)-	2.77e+02(6.93e+01)-	**1.99e+02(4.34e+00)+**	2.97e+02(2.73e+02)-	2.07e+02(3.20e+01)
	5	10	2.04e+02(4.80e+01)-	1.62e+02(4.35e+01)=	1.61e+02(2.23e+01)-	1.96e+02(4.45e+01)-	**1.49e+02(9.57e+00)**
	10	10	1.82e+00(5.92e-01)+	1.85e+00(6.81e-01)+	5.81e+00(5.11e+00)=	**1.62e+00(5.44e-01)+**	1.04e+01(1.12e+01)
DTLZ4	2	10	6.17e-01(1.73e-01)=	6.79e-01(1.52e-01)=	9.14e-01(3.58e-01)-	**6.10e-01(1.26e-01)=**	6.63e-01(9.46e-02)
	3	10	**6.78e-01(1.03e-01)+**	6.85e-01(9.93e-02)+	9.81e-01(1.31e-01)-	6.91e-01(1.66e-01)+	8.11e-01(9.19e-02)
	5	10	**7.03e-01(6.98e-02)+**	7.39e-01(7.00e-02)+	1.03e+00(9.85e-02)-	7.86e-01(5.15e-02)+	8.97e-01(8.07e-02)
	10	10	**6.86e-01(1.85e-02)=**	6.89e-01(1.86e-02)=	7.51e-01(4.13e-02)-	7.38e-01(2.06e-02)-	6.94e-01(3.11e-02)
DTLZ5	2	10	3.21e-01(6.21e-02)-	2.99e-01(5.04e-02)-	7.79e-01(1.98e-01)-	3.44e-01(3.46e-02)-	**3.61e-02(1.67e-02)**
	3	10	2.86e-01(5.19e-02)-	3.00e-01(5.95e-02)-	6.49e-01(1.16e-01)-	2.91e-01(3.59e-02)-	**5.78e-02(2.49e-02)**
	5	10	2.12e-01(3.76e-02)-	2.39e-01(4.05e-02)-	5.50e-01(6.12e-02)-	1.90e-01(2.32e-02)-	**9.97e-02(6.37e-02)**
	10	10	4.02e-02(7.04e-03)+	7.16e-02(2.02e-02)+	3.54e-01(2.80e-02)-	**2.04e-02(2.65e-03)=**	2.17e-01(7.04e-02)
DTLZ6	2	10	4.63e+00(1.18e+00)-	4.68e+00(7.09e-01)-	5.66e+00(1.24e+00)-	5.02e+00(7.80e-01)-	**2.23e+00(9.52e-01)**
	3	10	5.71e+00(4.00e-01)-	5.43e+00(5.30e-01)-	3.17e+00(8.16e-01)-	5.31e+00(6.75e-01)-	**2.23e+00(8.66e-01)**
	5	10	4.43e+00(2.35e-01)-	4.16e+00(4.13e-01)-	2.14e+00(6.52e-01)=	4.25e+00(4.88e-01)-	**1.80e+00(7.64e-01)**
	10	10	5.25e-01(1.77e-01)-	4.51e-01(1.59e-01)=	4.76e-01(1.38e-01)=	7.04e-01(5.20e-02)-	**4.25e-01(1.11e-01)**
DTLZ7	2	10	2.51e-01(1.60e-01)-	1.34e-01(5.76e-02)-	1.23e+00(7.06e-01)-	5.48e-01(2.13e-01)-	**1.04e-02(1.22e-03)**
	3	10	6.41e-01(1.65e-01)-	4.93e-01(1.87e-01)-	1.31e+00(5.26e-01)-	8.27e-01(2.21e-01)-	**1.16e-01(1.58e-02)**
	5	10	2.03e+00(8.78e-01)-	1.79e+00(5.62e-01)-	1.29e+00(2.71e-01)-	1.30e+00(3.68e-01)-	**4.42e-01(8.57e-02)**
	10	10	1.99e+00(2.81e-01)=	1.93e+00(2.24e-01)=	**1.39e+00(1.27e-01)+**	1.91e+00(3.35e-01)=	2.12e+00(5.48e-01)
ZDT1	2	10	4.77e-01(2.78e-01)-	3.40e-01(2.38e-01)-	9.68e+00(6.79e+00)-	122e+00(9.10e-01)-	**4.85e-03(6.13e-04)**
ZDT2	2	10	1.50e+00(1.15e+00)-	1.10e+00(5.60e-01)-	1.83e+01(4.37e+00)-	2.35e+00(1.34e+00)-	**3.75e-01(1.40e-01)**
ZDT3	2	10	6.39e-01(3.67e-01)-	5.76e-01(4.63e-01)-	4.95e+00(3.71e+00)-	1.40e+00(6.34e-01)-	**2.22e-02(5.60e-03)**
ZDT4	2	10	**7.90e+01(1.07e+01)+**	7.30e+01(1.29e+01)+	2.06e+02(2.19e+01)-	7.90e+01(1.22e+01)+	1.01e+02(2.97e+01)
ZDT6	2	10	5.61e+00(1.81e+00)-	4.91e+00(1.58e+00)-	9.24e+00(8.14e-01)-	5.74e+00(1.37e+00)-	**5.21e-01(2.06e-01)**
+/-/=			6/20/7	6/20/7	5/23/5	7/22/4	- -

decision variables for checking the scalability of the IBEA-MS with respect to the number of decision variables. The results indicate that IBEA-MS is better than AK-IBEA variants on almost all tested problems. However, the performance of IBEA-MS significantly deteriorates on DTLZ1 and DTLZ3 problems with 40 and 50 decision variables. This may be attributed to the unreliable predictions of the Kriging models when the dimension of the decision space is over 30. These results imply that IBEA-MS works well for problems less than 30-dimensional. IBEA-MS is also compared with AK-IBEA variants on the 15- and 25- objective DTLZ problems with 30 decision variables for checking the scalability of the IBEA-MS with respect to the number of objectives. From observing the results, These results indicate that IBEA-MS scales well when the number of objectives increases. IBEA-MS has better performance than AK-IBEA variants on all of the tested problems except for DTLZ7. IBEA-MS prefers the Kriging model during the optimization process on the DTLZ7 problem, while AK-IBEA3 adopts the EI values to substitute the original objective values, which could further reduce the effect of prediction errors in fitness assignment in IBEA. In addition, the performance of IBEA-MS does not significantly deteriorate even if the objective number rises to 25. An important reason is that the fitness assignment in IBEA will not be negatively influenced as the number of objectives increases, while the Pareto-based non-dominated sort will become incompetent. The inverse DTLZ (IDTLZ1 and IDTLZ2) and WFG1-9 problems are tested by IBEA-MS comparing with AK-IBEA variants in Table S.XVI. All of the problems are performed with three objectives and 30 decision variables. IBEA-MS has the overall best performance comparing with AK-IBEA variants on the tested problems. However, the performance of IBEA-MS on the WFG1 problems is not outstanding relatively. Thus, the optimized population of AK-IBEA1, AK-IBEA2

TABLE S.X
COMPARATIVE EXPERIMENT BETWEEN AK-IBEAS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 20 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	2	20	4.37e+02(5.65e+01)=	4.38e+02(5.58e+01)=	**1.80e+02(1.93e+01)+**	4.54e+02(4.62e+01)
	3	20	4.06e+02(3.95e+01)-	3.77e+02(4.00e+01)=	**2.75e+02(4.74e+01)+**	3.82e+02(3.08e+01)
	5	20	3.20e+02(3.65e+01)-	3.25e+02(4.85e+01)-	3.84e+02(7.96e+01)-	**2.72e+02(3.21e+01)**
	10	20	1.76e+02(2.50e+01)-	1.72e+02(2.57e+01)=	2.37e+02(3.90e+01)-	**1.59e+02(2.59e+01)**
DTLZ2	2	20	8.54e-01(5.72e-02)-	8.70e-01(7.53e-02)-	1.81e+00(3.24e-01)-	**7.18e-02(2.20e-02)**
	3	20	8.95e-01(6.75e-02)-	9.08e-01(7.30e-02)-	1.49e+00(2.12e-01)-	**1.64e-01(3.96e-02)**
	5	20	9.94e-01(6.86e-02)-	1.04e+00(7.62e-02)-	1.50e+00(1.39e-01)-	**4.86e-01(4.78e-02)**
	10	20	1.07e+00(4.48e-02)-	1.09e+00(5.55e-02)-	1.47e+00(1.01e-01)-	**9.82e-01(8.11e-02)**
DTLZ3	2	20	1.06e+03(1.72e+02)-	1.07e+03(1.77e+02)-	**4.80e+02(6.88e+00)+**	7.39e+02(2.87e+02)
	3	20	1.01e+03(1.48e+02)-	1.02e+03(1.51e+02)-	**6.13e+02(1.34e+02)=**	6.28e+02(2.38e+02)
	5	20	9.69e+02(1.10e+02)-	9.99e+02(1.09e+02)-	8.60e+02(1.37e+02)-	**6.15e+02(1.59e+02)**
	10	20	5.74e+02(8.16e+01)-	5.76e+02(9.41e+01)-	5.70e+02(1.17e+02)-	**3.40e+02(8.70e+01)**
DTLZ4	2	20	1.09e+00(1.15e-01)-	1.11e+00(1.41e-01)-	2.96e+00(1.26e+00)-	**7.34e-01(2.89e-02)**
	3	20	1.26e+00(1.04e-01)-	1.19e+00(1.12e-01)-	1.73e+00(1.81e-01)-	**9.36e-01(5.69e-02)**
	5	20	1.30e+00(9.87e-02)-	1.29e+00(1.05e-01)-	1.76e+00(1.36e-01)-	**1.11e+00(3.12e-02)**
	10	20	1.18e+00(5.16e-02)-	1.18e+00(5.80e-02)-	1.49e+00(6.38e-02)-	**1.14e+00(3.35e-02)**
DTLZ5	2	20	8.50e-01(6.73e-02)-	8.56e-01(7.44e-02)-	1.73e+00(3.76e-01)-	**7.87e-02(2.23e-02)**
	3	20	8.38e-01(7.87e-02)-	8.53e-01(8.04e-02)-	1.42e+00(1.66e-01)-	**9.83e-02(3.40e-02)**
	5	20	6.99e-01(1.02e-01)-	7.66e-01(9.83e-02)-	1.23e+00(1.34e-01)-	**1.41e-01(3.52e-02)**
	10	20	5.22e-01(6.58e-02)-	5.32e-01(5.87e-02)-	9.18e-01(1.51e-01)-	**2.69e-01(8.41e-02)**
DTLZ6	2	20	1.31e+01(1.09e+00)-	1.27e+01(9.89e-01)-	1.02e+01(1.76e+00)-	**8.23e+00(1.02e+00)**
	3	20	1.28e+01(1.00e+00)-	1.28e+01(1.07e+00)-	1.18e+01(1.16e+00)-	**8.41e+00(9.69e-01)**
	5	20	1.18e+01(7.86e-01)-	1.21e+01(6.89e-01)-	1.11e+01(8.00e-01)-	**8.18e+00(1.15e+00)**
	10	20	8.35e+00(5.52e-01)-	8.51e+00(4.49e-01)-	7.44e+00(6.21e-01)-	**5.91e+00(9.39e-01)**
DTLZ7	2	20	4.06e-01(1.32e-01)-	3.48e-01(1.10e-01)-	2.87e+00(1.47e+00)-	**1.27e-02(7.14e-04)**
	3	20	7.63e-01(1.44e-01)-	7.77e-01(2.82e-01)-	2.39e+00(1.27e+00)-	**1.29e-01(8.21e-03)**
	5	20	3.43e+00(1.29e+00)-	3.57e+00(1.30e+00)-	1.67e+00(1.25e+00)-	**4.91e-01(8.39e-02)**
	10	20	1.02e+01(2.15e+00)-	9.83e+00(2.34e+00)-	**1.69e+00(1.19e-01)+**	2.12e+00(4.80e-01)
ZDT1	2	20	4.81e+00(2.24e+00)-	5.64e+00(2.64e+00)-	3.27e+01(1.05e+01)-	**2.22e-02(2.61e-02)**
ZDT2	2	20	9.20e+00(2.77e+00)-	9.41e+00(2.86e+00)-	5.14e+01(1.13e+01)-	**5.56e-01(5.61e-02)**
ZDT3	2	20	4.15e+00(2.38e+00)-	4.24e+00(2.12e+00)-	2.64e+01(9.33e+00)-	**2.85e-02(1.27e-02)**
ZDT4	2	20	2.16e+02(1.43e+01)=	**2.13e+02(1.59e+01)=**	4.59e+02(9.96e+00)-	2.15e+02(1.56e+01)
ZDT6	2	20	8.33e+00(1.03e+00)+	**7.95e+00(1.15e+00)+**	1.28e+01(6.39e-01)-	9.14e+00(3.33e+00)
+/-/=			1/30/2	1/28/4	4/28/1	- -

and IBEA-MS are drawn in the Fig.S.1, respectively. The figure indicates that the optimized population of these three algorithms have the similar convergence performance. It could be considered that both the Kriging and RBFN models have the similar performance on the WFG1 problem. The diversity of the optimized population in AK-IBEA1 and AK-IBEA2 is slightly better than IBEA-MS. The reason could be AK-IBEA variants add the evaluated individuals to an archive and use the environmental selection operation in NSGA-II for the archives to select their final outputs. The results of UF1-12 problems are displayed in the Table S.XVII. The UF1-7 problems are tested with two objectives and UF8-10 are tested with three objectives. Five-objective UF11-12 problems are also performed in the experiment. All of the UF problems have 30 decision variables. The results indicate that IBEA-MS has significant advantages over AK-IBEA on the tested problems. However, AK-IBEA1 and AK-IBEA2 exhibit better performance than IBEA-MS on the UF4 problems, which have the deceptive Pareto fronts. These results imply that the proposed model selection mechanism may fail to work properly on some particular test problem, and further improving the selection mechanism will be our future work. Finally, the three-objective MaF1-9 problems are tested and the results are shown in the Table S.XVIII. MaF1-7 are performed with 30 decision variables and MaF8-9 are tested with 2 decision variables. The results indicate that IBEA-MS obtains overall better performance. However, both the AK-IBEA variants and IBEA-MS perform poorly on the MaF3 problem. It can also be observed that AK-IBEA1 and AK-IBEA2 are significantly better than IBEA-MS only on the MaF3 problem. The reason may be that MaF3 has a large number of local fronts. Therefore, AK-IBEA1 and AK-IBEA2 using the Kriging models can obtain better performance than IBEA-MS on the MaF3 problem.

TABLE S.XI

COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	2	30	7.54e+02(4.73e+01)=	7.66e+02(5.17e+01)=	**4.10e+02(8.08e+01)+**	7.64e+02(4.64e+01)
	3	30	6.76e+02(7.03e+01)-	7.01e+02(5.97e+01)-	6.48e+02(1.47e+02)=	**6.32e+02(3.57e+01)**
	5	30	5.75e+02(5.37e+01)-	5.71e+02(5.45e+01)-	7.19e+02(1.13e+02)-	**5.09e+02(4.29e+01)**
	10	30	3.69e+02(4.09e+01)=	3.62e+02(4.46e+01)=	5.20e+02(8.32e+01)-	**3.59e+02(4.57e+01)**
DTLZ2	2	30	1.41e+00(1.22e-01)-	1.47e+00(1.06e-01)-	2.43e+00(3.10e-01)-	**1.19e-01(2.82e-02)**
	3	30	1.46e+00(9.89e-02)-	1.45e+00(9.51e-02)-	2.20e+00(2.19e-01)-	**2.31e-01(3.63e-02)**
	5	30	1.57e+00(1.19e-01)-	1.60e+00(1.40e-01)-	2.22e+00(1.78e-01)-	**5.56e-01(6.44e-02)**
	10	30	1.56e+00(1.11e-01)-	1.60e+00(7.85e-02)-	2.03e+00(1.31e-01)-	**1.14e+00(1.28e-01)**
DTLZ3	2	30	1.83e+03(1.71e+02)=	1.87e+03(2.05e+02)=	**9.43e+02(1.73e+02)+**	1.63e+03(4.79e+02)
	3	30	1.89e+03(1.85e+02)-	1.84e+03(1.99e+02)-	**1.40e+03(2.35e+02)=**	1.52e+03(3.26e+02)
	5	30	1.86e+03(1.69e+02)-	1.86e+03(1.56e+02)-	1.87e+03(2.03e+02)-	**1.62e+03(1.92e+02)**
	10	30	1.40e+03(1.87e+02)-	1.48e+03(1.26e+02)-	1.47e+03(1.36e+02)-	**1.10e+03(1.88e+02)**
DTLZ4	2	30	1.61e+00(1.48e-01)-	1.65e+00(1.67e-01)-	4.32e+00(1.52e+00)-	**7.50e-01(1.04e-02)**
	3	30	1.82e+00(1.22e-01)-	1.76e+00(1.38e-01)-	2.54e+00(3.59e-01)-	**9.54e-01(1.63e-02)**
	5	30	1.85e+00(9.73e-02)-	1.93e+00(1.28e-01)-	2.42e+00(2.17e-01)-	**1.12e+00(3.43e-02)**
	10	30	1.77e+00(7.87e-02)-	1.78e+00(6.31e-02)-	2.09e+00(1.43e-01)-	**1.27e+00(2.98e-02)**
DTLZ5	2	30	1.45e+00(1.12e-01)-	1.43e+00(9.93e-02)-	2.71e+00(3.42e-01)-	**1.03e-01(2.27e-02)**
	3	30	1.38e+00(1.50e-01)-	1.41e+00(1.48e-01)-	2.06e+00(2.97e-01)-	**1.45e-01(4.46e-02)**
	5	30	1.25e+00(1.62e-01)-	1.38e+00(1.39e-01)-	1.87e+00(1.86e-01)-	**2.43e-01(4.40e-02)**
	10	30	1.07e+00(1.53e-01)-	1.08e+00(1.29e-01)-	1.48e+00(1.81e-01)-	**4.29e-01(1.31e-01)**
DTLZ6	2	30	2.12e+01(1.03e+00)-	2.11e+01(1.04e+00)-	1.87e+01(1.37e+00)-	**1.60e+01(1.43e+00)**
	3	30	2.10e+01(1.07e+00)-	2.13e+01(6.97e-01)-	1.99e+01(1.13e+00)-	**1.58e+01(1.26e+00)**
	5	30	2.06e+01(1.09e+00)-	2.04e+01(1.30e+00)-	1.95e+01(1.09e+00)-	**1.62e+01(1.41e+00)**
	10	30	1.69e+01(5.69e-01)-	1.72e+01(7.13e-01)-	1.63e+01(8.22e-01)-	**1.39e+01(1.16e+00)**
DTLZ7	2	30	6.24e-01(2.86e-01)-	5.89e-01(2.14e-01)-	3.91e+00(1.32e+00)-	**1.80e-02(1.87e-03)**
	3	30	1.12e+00(3.47e-01)-	1.15e+00(3.84e-01)-	4.35e+00(2.13e+00)-	**1.31e-01(6.54e-03)**
	5	30	4.50e+00(1.50e+00)-	4.62e+00(1.41e+00)-	4.19e+00(2.50e+00)-	**5.15e-01(9.56e-02)**
	10	30	1.60e+01(2.17e+00)-	1.52e+01(2.83e+00)-	**2.20e+00(5.65e-01)=**	2.29e+00(6.23e-01)
ZDT1	2	30	1.30e+01(3.18e+00)-	1.36e+01(3.29e+00)-	6.69e+01(1.58e+01)-	**1.15e-01(5.14e-02)**
ZDT2	2	30	2.14e+01(5.55e+00)-	2.14e+01(4.09e+00)-	8.56e+01(1.12e+01)-	**6.66e-01(5.49e-02)**
ZDT3	2	30	1.39e+01(5.03e+00)-	1.53e+01(4.48e+00)-	6.34e+01(1.40e+01)-	**1.50e-01(6.28e-02)**
ZDT4	2	30	3.62e+02(1.72e+01)=	**3.53e+02(2.03e+01)=**	7.04e+02(1.02e+01)-	3.82e+02(8.60e+01)
ZDT6	2	30	1.07e+01(7.27e-01)+	**1.06e+01(7.01e-01)+**	1.47e+01(5.31e-01)-	1.26e+01(1.48e+00)
+/-/=			1/28/4	1/28/4	2/28/3	- -

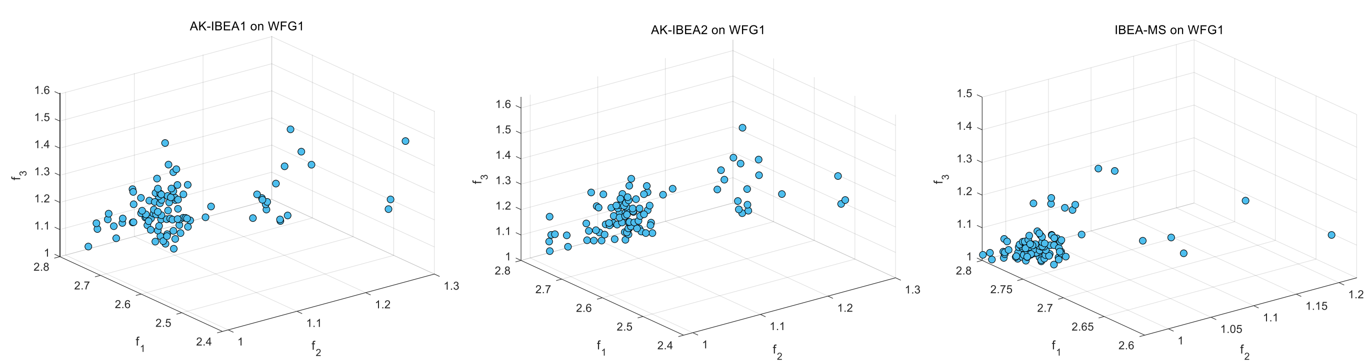

Fig. S.1. The optimized objective values of AK-IBEA1, AK-IBEA2 and IBEA-MS on the WFG1 problem. The objective values are calculated by the mean values of 30 independent runs.

TABLE S.XII
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE HV PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	NSGA-II-GP	IBEA-MS
DTLZ1	2	10	9.56e-01(1.71e-04)=	9.49e-01(1.34e-04)=	9.52e-01(5.23e-04)=	**9.59e-01(1.05e-04)=**	9.57e-01(6.20e-04)
	3	10	9.88e-01(2.34e-05)-	9.89e-01(1.55e-05)-	9.37e-01(1.32e-03)-	9.88e-01(8.88e-06)-	**9.91e-01(3.02e-05)**
	5	10	9.91e-01(1.72e-05)=	**9.95e-01(6.68e-06)=**	9.23e-01(1.11e-03)-	9.94e-01(7.84e-06)=	9.91e-01(4.50e-05)
	10	10	9.99e-01(6.14e-07)+	9.99e-01(1.37e-06)+	9.91e-01(7.23e-05)-	**1.00e+00(1.77e-07)+**	9.95e-01(5.50e-05)
DTLZ2	2	10	8.67e-01(9.89e-04)-	8.67e-01(7.73e-04)-	7.59e-01(6.09e-03)-	8.66e-01(7.82e-04)-	**9.22e-01(2.39e-04)**
	3	10	9.28e-01(2.91e-04)-	9.28e-01(2.58e-04)-	7.99e-01(2.99e-03)-	9.44e-01(1.51e-04)-	**9.59e-01(1.33e-04)**
	5	10	9.27e-01(2.59e-04)=	**9.29e-01(2.78e-04)=**	7.73e-01(4.77e-03)-	9.57e-01(1.56e-04)+	9.25e-01(4.34e-04)
	10	10	7.28e-01(9.27e-04)+	7.16e-01(7.95e-04)+	5.08e-01(3.69e-03)+	**7.72e-01(5.53e-04)+**	4.16e-01(1.73e-02)
DTLZ3	2	10	9.40e-01(2.23e-04)-	9.42e-01(2.82e-04)-	9.52e-01(8.27e-04)-	9.61e-01(1.03e-04)-	**9.76e-01(3.59e-04)**
	3	10	9.71e-01(8.95e-05)-	9.77e-01(1.44e-04)-	9.23e-01(1.87e-03)-	9.81e-01(3.49e-05)-	**9.96e-01(3.53e-05)**
	5	10	9.76e-01(1.42e-04)-	9.88e-01(5.96e-05)-	8.83e-01(1.06e-03)-	9.89e-01(2.09e-05)-	**9.96e-01(7.05e-05)**
	10	10	9.97e-01(3.73e-06)=	9.98e-01(1.56e-06)+	9.85e-01(2.74e-04)-	**9.99e-01(3.84e-07)+**	9.93e-01(1.34e-04)
DTLZ4	2	10	7.70e-01(6.29e-03)=	7.45e-01(4.82e-03)=	6.50e-01(1.42e-02)-	**7.97e-01(3.26e-03)+**	7.50e-01(1.33e-03)
	3	10	**8.70e-01(2.24e-03)+**	8.77e-01(2.07e-03)+	7.79e-01(4.15e-03)=	8.62e-01(4.49e-03)+	7.66e-01(2.89e-03)
	5	10	**9.55e-01(8.08e-04)+**	9.46e-01(1.13e-03)+	8.60e-01(3.33e-03)+	9.11e-01(1.78e-03)+	8.04e-01(5.22e-03)
	10	10	9.25e-01(5.00e-04)=	**9.26e-01(4.51e-04)=**	8.63e-01(1.83e-03)-	8.11e-01(2.26e-03)-	9.11e-01(1.33e-03)
DTLZ5	2	10	8.79e-01(2.41e-04)-	8.82e-01(3.07e-04)-	7.82e-01(4.36e-03)-	8.77e-01(2.06e-04)-	9.31e-01(7.36e-05)
	3	10	8.45e-01(3.36e-04)-	8.45e-01(3.04e-04)-	7.67e-01(1.20e-03)-	8.46e-01(2.51e-04)-	8.89e-01(1.13e-04)
	5	10	7.48e-01(3.38e-04)-	7.45e-01(5.32e-04)-	6.58e-01(8.98e-04)-	7.61e-01(2.23e-04)-	7.78e-01(4.19e-04)
	10	10	2.95e-01(1.02e-05)+	2.92e-01(3.93e-05)+	2.66e-01(4.48e-04)=	2.98e-01(5.76e-06)+	2.35e-01(3.57e-03)
DTLZ6	2	10	6.58e-01(5.18e-03)-	6.61e-01(2.65e-03)-	5.80e-01(1.89e-02)-	6.53e-01(1.58e-03)-	**9.11e-01(2.72e-03)**
	3	10	6.49e-01(7.98e-04)-	6.64e-01(1.30e-03)-	7.63e-01(7.39e-03)-	6.99e-01(1.11e-03)-	**9.29e-01(1.09e-03)**
	5	10	6.32e-01(5.22e-04)-	6.69e-01(1.07e-03)-	7.58e-01(7.17e-03)-	7.01e-01(4.76e-04)-	**9.26e-01(8.85e-04)**
	10	10	4.86e-01(9.38e-03)-	5.74e-01(6.44e-03)+	**5.79e-01(3.33e-03)+**	4.50e-01(4.28e-03)-	5.55e-01(9.21e-04)
DTLZ7	2	10	8.22e-01(2.32e-04)-	8.33e-01(1.48e-04)-	6.04e-01(1.93e-02)-	7.99e-01(3.67e-04)-	**8.46e-01(7.64e-05)**
	3	10	8.12e-01(6.84e-04)-	8.36e-01(3.04e-04)-	5.45e-01(1.93e-02)-	7.93e-01(4.99e-04)-	**8.69e-01(5.42e-05)**
	5	10	7.52e-01(4.21e-03)-	7.75e-01(2.41e-03)-	5.83e-01(1.72e-02)-	6.61e-01(1.28e-03)-	**8.59e-01(2.03e-05)**
	10	10	5.27e-01(2.28e-02)-	5.44e-01(3.13e-02)-	6.51e-01(4.55e-02)-	1.09e-01(4.67e-04)-	**8.42e-01(2.10e-04)**
ZDT1	2	10	9.82e-01(8.89e-05)-	9.88e-01(1.56e-05)-	7.68e-01(2.84e-02)-	9.64e-01(3.58e-04)-	**9.96e-01(7.41e-08)**
ZDT2	2	10	9.71e-01(2.66e-04)-	9.77e-01(7.12e-05)-	7.20e-01(3.64e-03)-	9.54e-01(2.83e-04)-	**9.87e-01(1.59e-06)**
ZDT3	2	10	9.66e-01(1.95e-04)-	9.74e-01(6.96e-05)-	7.44e-01(4.21e-02)-	9.47e-01(1.87e-04)-	**9.89e-01(4.17e-07)**
ZDT4	2	10	7.40e-01(1.75e-03)+	**7.53e-01(1.27e-03)+**	3.39e-01(1.00e-02)-	7.37e-01(2.04e-03)+	6.61e-01(9.21e-03)
ZDT6	2	10	3.64e-01(5.65e-03)+	**3.84e-01(8.23e-03)+**	1.72e-01(9.53e-04)-	3.42e-01(2.79e-03)+	2.73e-01(1.29e-02)
			7/20/6	9/19/5	3/27/3	10/21/2	

S.VI. BAYESIAN SIGNED-RANK TESTS

In this section, we show the additional results of the Bayesian signed-rank tests of the Section IV.B in the manuscript. First, the Bayesian signed-rank test is performed between IBEA-MS and AK-IBEA variants on all the test problems based on the HV performance. All the comparison settings are same as the Bayesian signed-rank tests of the Section IV.B in the manuscript. The results are shown in Fig. S.2. It can be observed that the comparison results are similar to those based on the IGD performance. The points are located in the bottom-right region of the triangle in all comparisons. This means that IBEA-MS outperforms the AK-IBEA variants at a high probability. The proportion of the location of the points in Fig. S.2 is shown in Table S.XIX. In addition, in order to explore the Bayesian signed-rank tests results on each problem, we construct a 30*4 matrix of the experimental results from 30 independent runs of AK-IBEA1, AK-IBEA2, AK-IBEA3 and IBEA-MS for each problem. The Bayesian signed-rank test is performed by comparing the data column of IBEA-MS and other columns. Because of the page limit, we only show the Bayesian signed-rank test results of the compared algorithm based on IGD and HV performances on the 30-dimensional three-objective DTLZ1-7 problems in Figs. S.3 to S.16, respectively, where the results on these problems cover the most situations of Bayesian signed-rank tests on other problems. We have uploaded the complete results on each problem in the Github [1]. It is worth noting that if the difference between the compared algorithms is large and the sampling points of the Bayesian signed-rank test will be distributed on the sides of the triangle. After performing Bayesian signed-rank test on each problem, we can find that IBEA-MS outperforms the compared algorithms

[1] The complete Bayesian signed-rank test results can be available at https://github.com/HandingWangXDGroup/Bayesin-Signed-Rank-Tests-Results-

TABLE S.XIII
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE HV PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	2	20	8.94e-01(3.23e-04)=	8.93e-01(2.10e-04)=	**9.00e-01(6.48e-04)=**	8.91e-01(2.75e-04)
	3	20	9.57e-01(7.41e-05)-	9.60e-01(5.32e-05)=	8.37e-01(3.07e-03)-	**9.64e-01(3.93e-05)**
	5	20	9.79e-01(7.87e-05)-	9.78e-01(7.26e-05)=	7.85e-01(7.77e-03)-	**9.83e-01(3.63e-05)**
	10	20	9.77e-01(6.81e-05)-	9.74e-01(4.77e-05)-	8.99e-01(6.67e-04)-	**9.84e-01(8.33e-05)**
DTLZ2	2	20	9.18e-01(3.86e-05)-	9.18e-01(6.50e-05)-	7.81e-01(9.87e-03)-	**9.75e-01(1.24e-05)**
	3	20	9.63e-01(1.84e-05)-	9.62e-01(2.51e-05)-	9.00e-01(1.65e-03)-	**9.87e-01(2.14e-05)**
	5	20	9.65e-01(5.94e-05)-	9.65e-01(1.55e-04)-	9.11e-01(1.93e-03)-	**9.81e-01(4.23e-05)**
	10	20	9.24e-01(3.24e-04)-	9.19e-01(3.39e-04)-	8.15e-01(6.52e-03)-	**9.56e-01(8.36e-05)**
DTLZ3	2	20	8.59e-01(2.05e-03)-	8.57e-01(1.77e-03)-	8.79e-01(5.27e-04)-	**9.26e-01(2.95e-03)**
	3	20	9.43e-01(4.59e-04)-	9.42e-01(3.14e-04)-	8.07e-01(1.81e-03)-	**9.82e-01(5.78e-04)**
	5	20	9.56e-01(2.99e-04)-	9.51e-01(2.89e-04)-	7.14e-01(3.64e-03)-	**9.91e-01(1.46e-04)**
	10	20	9.42e-01(1.68e-04)-	9.47e-01(3.12e-04)-	8.29e-01(3.16e-03)-	**9.88e-01(3.98e-04)**
DTLZ4	2	20	8.22e-01(9.40e-04)-	8.19e-01(1.68e-03)-	4.53e-01(4.59e-02)-	8.56e-01(5.86e-04)
	3	20	9.27e-01(9.00e-04)+	**9.32e-01(6.23e-04)+**	8.25e-01(2.39e-03)-	8.88e-01(1.79e-03)
	5	20	**9.72e-01(2.87e-04)+**	9.70e-01(3.96e-04)+	8.89e-01(2.34e-03)=	9.05e-01(1.30e-03)
	10	20	9.89e-01(1.06e-04)+	**9.90e-01(1.04e-04)+**	9.63e-01(3.33e-04)=	9.47e-01(1.12e-03)
DTLZ5	2	20	9.16e-01(1.08e-04)-	9.15e-01(1.21e-04)-	8.00e-01(7.20e-03)-	**9.74e-01(1.49e-05)**
	3	20	9.16e-01(6.65e-05)-	9.16e-01(3.56e-05)-	8.54e-01(1.00e-03)-	**9.58e-01(3.14e-05)**
	5	20	8.97e-01(1.36e-04)-	8.96e-01(1.11e-04)-	8.24e-01(2.43e-03)-	**9.35e-01(3.88e-05)**
	10	20	8.00e-01(1.19e-03)-	8.04e-01(6.42e-04)-	6.78e-01(2.06e-03)-	**8.50e-01(4.01e-04)**
DTLZ6	2	20	5.45e-01(3.72e-03)-	5.64e-01(3.28e-03)-	6.62e-01(1.67e-02)-	**8.02e-01(1.89e-03)**
	3	20	6.41e-01(1.78e-03)-	6.43e-01(1.67e-03)-	4.72e-01(6.63e-03)-	**8.73e-01(8.22e-04)**
	5	20	6.87e-01(1.62e-03)-	6.88e-01(8.13e-04)-	4.76e-01(8.60e-03)-	**8.99e-01(1.75e-03)**
	10	20	5.97e-01(1.39e-03)-	5.93e-01(1.08e-03)-	6.10e-01(6.71e-03)-	**7.83e-01(8.03e-03)**
DTLZ7	2	20	8.06e-01(1.59e-04)-	8.14e-01(8.17e-05)-	5.22e-01(4.99e-03)-	**8.47e-01(7.02e-06)**
	3	20	7.74e-01(8.93e-04)-	7.86e-01(3.37e-04)-	4.64e-01(1.19e-02)-	**8.50e-01(2.65e-05)**
	5	20	6.50e-01(4.61e-03)-	6.80e-01(3.11e-03)-	4.62e-01(9.34e-03)-	**8.31e-01(1.50e-04)**
	10	20	4.03e-01(1.55e-02)-	4.23e-01(1.52e-02)-	3.74e-01(4.40e-02)-	**7.61e-01(2.07e-03)**
ZDT1	2	20	9.42e-01(5.28e-04)-	9.40e-01(5.78e-04)-	6.57e-01(1.34e-02)-	**9.97e-01(4.52e-08)**
ZDT2	2	20	9.23e-01(4.60e-04)-	9.21e-01(4.91e-04)-	5.75e-01(8.45e-03)-	**9.92e-01(9.49e-08)**
ZDT3	2	20	9.46e-01(4.95e-04)-	9.53e-01(2.27e-04)-	6.68e-01(2.58e-02)-	**9.95e-01(8.37e-08)**
ZDT4	2	20	**6.60e-01(4.63e-04)+**	6.57e-01(4.34e-04)+	2.55e-01(3.75e-03)-	6.23e-01(1.43e-03)
ZDT6	2	20	2.42e-01(2.54e-03)+	**2.81e-01(2.66e-03)+**	1.51e-01(8.87e-04)-	2.10e-01(2.46e-03)
			5/27/1	5/24/3	0/31/2	

on most problems. For example, the sampling points distribute on the right of the bottom of the triangle in Fig. S.3 on the 30-dimensional three-objective DTLZ1 problem. This indicates that IBEA-MS can perform better than AK-IBEA variants at a high probability, and the possibility of the similar performance between these algorithms is small based on the IGD performance. In Fig. S.4, the results of the DTLZ1 problem are tested on the HV performance and the sampling points are basically concentrated on the right side of the triangle. This shows that IBEA-MS outperforms AK-IBEA variants with a higher probability, but the possibility of no significant difference between them is increased comparing with the results in terms of the IGD performance. However, the results of DTLZ4 problem in Figs. 10(a) and 10(b) indicate that AK-IBEA1 and AK-IBEA2 have better performance than IBEA-MS based on the HV performance, where the results are completely opposite to the results based on the IGD performance in Figs. 9(a) and 9(b). It means that the performance of IBEA-MS may be different according to different performance indicators. The decision maker may select the final solutions according to his or her own preference over the performance indicators. This also proves the necessity of Bayesian signed-rank tests based on the HV performance. We can also observe that the test results of the Bayesian signed-rank tests are similar to those of the Wilcoxon rank sum tests on most problems. In addition, the Bayesian signed-rank test can show which algorithm is slightly better by observing the distribution of the sampling points, even if no significant difference can be observed between the compared algorithms according to the Wilcoxon rank sum test. For example, on the 30-dimensional three-objective DTLZ3 problem, although AK-IBEA3 and IBEA-MS have no significant difference in Table S.XI based on the Wilcoxon rank sum test, the sampling points are biased towards IBEA-MS slightly in Fig. 7(c).

TABLE S.XIV
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE DTLZ AND ZDT PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE HV PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	2	30	**8.60e-01(1.12e-04)=**	8.60e-01(1.76e-04)=	8.23e-01(2.37e-03)-	8.58e-01(1.59e-04)
	3	30	9.45e-01(7.25e-05)-	9.43e-01(6.41e-05)-	7.44e-01(5.78e-03)-	**9.54e-01(5.61e-05)**
	5	30	9.74e-01(3.26e-05)-	9.76e-01(2.78e-05)-	8.03e-01(7.50e-03)-	**9.78e-01(3.34e-05)**
	10	30	9.57e-01(1.17e-04)-	9.61e-01(1.17e-04)-	8.71e-01(2.80e-03)-	**9.78e-01(4.89e-05)**
DTLZ2	2	30	9.27e-01(5.04e-05)-	9.25e-01(4.48e-05)-	8.13e-01(6.40e-03)-	**9.84e-01(3.25e-05)**
	3	30	9.62e-01(1.43e-05)-	9.62e-01(1.85e-05)-	9.03e-01(1.08e-03)-	**9.89e-01(3.03e-05)**
	5	30	9.65e-01(9.36e-05)-	9.64e-01(7.60e-05)-	9.02e-01(2.63e-03)-	**9.85e-01(4.12e-05)**
	10	30	9.20e-01(6.78e-04)-	9.22e-01(5.68e-04)-	8.26e-01(6.21e-03)-	**9.69e-01(1.25e-04)**
DTLZ3	2	30	8.20e-01(6.78e-04)=	8.14e-01(1.04e-03)=	8.08e-01(1.36e-03)=	**8.50e-01(5.20e-03)**
	3	30	9.00e-01(5.40e-04)-	9.03e-01(3.79e-04)-	6.79e-01(2.54e-03)-	**9.42e-01(1.08e-03)**
	5	30	9.27e-01(3.75e-04)-	9.23e-01(3.45e-04)-	6.43e-01(4.42e-03)-	**9.64e-01(3.53e-04)**
	10	30	9.02e-01(1.01e-03)-	8.98e-01(8.35e-04)-	7.93e-01(4.46e-03)-	**9.72e-01(3.67e-04)**
DTLZ4	2	30	8.59e-01(1.03e-03)-	8.57e-01(1.36e-03)-	4.63e-01(4.05e-02)-	9.02e-01(3.97e-04)
	3	30	9.28e-01(4.84e-04)=	**9.34e-01(7.25e-04)+**	8.29e-01(4.85e-03)-	9.18e-01(5.66e-04)
	5	30	**9.71e-01(1.93e-04)+**	9.67e-01(3.41e-04)+	8.80e-01(2.81e-03)-	9.41e-01(7.21e-04)
	10	30	9.88e-01(1.28e-04)+	**9.88e-01(5.92e-05)+**	9.63e-01(3.31e-04)-	9.74e-01(2.61e-04)
DTLZ5	2	30	9.18e-01(6.39e-05)-	9.18e-01(1.12e-04)-	7.60e-01(5.44e-03)-	**9.84e-01(2.59e-05)**
	3	30	9.26e-01(3.44e-05)-	9.24e-01(5.70e-05)-	8.73e-01(1.44e-03)-	**9.70e-01(9.39e-05)**
	5	30	9.05e-01(1.44e-04)-	9.05e-01(1.09e-04)-	8.31e-01(3.33e-03)-	**9.53e-01(4.82e-05)**
	10	30	8.40e-01(7.73e-04)-	8.34e-01(8.13e-04)-	7.12e-01(3.13e-03)-	**9.09e-01(2.43e-04)**
DTLZ6	2	30	5.10e-01(1.39e-03)-	5.21e-01(1.33e-03)-	5.54e-01(1.18e-02)-	**6.99e-01(2.23e-03)**
	3	30	6.24e-01(1.36e-03)-	6.20e-01(5.71e-04)-	4.16e-01(2.34e-03)-	**8.00e-01(1.87e-03)**
	5	30	6.70e-01(7.12e-04)-	6.82e-01(9.46e-04)-	4.16e-01(6.63e-03)-	**8.35e-01(2.62e-03)**
	10	30	6.07e-01(5.31e-04)-	6.02e-01(5.22e-04)-	4.30e-01(4.98e-03)-	**7.37e-01(8.54e-03)**
DTLZ7	2	30	7.71e-01(3.04e-04)-	7.78e-01(1.75e-04)-	4.21e-01(5.94e-03)-	**8.31e-01(1.43e-05)**
	3	30	7.50e-01(1.23e-03)-	7.61e-01(7.33e-04)-	4.54e-01(8.38e-03)-	**8.47e-01(1.42e-04)**
	5	30	6.08e-01(7.92e-03)-	6.06e-01(5.03e-03)-	3.84e-01(2.02e-02)-	**8.12e-01(3.37e-04)**
	10	30	3.42e-01(9.77e-03)-	3.16e-01(1.28e-02)-	3.96e-01(3.72e-02)-	**7.34e-01(2.47e-03)**
ZDT1	2	30	9.12e-01(3.67e-04)-	9.07e-01(5.53e-04)-	5.33e-01(1.58e-02)-	**9.97e-01(9.51e-08)**
ZDT2	2	30	8.83e-01(8.39e-04)-	8.83e-01(4.88e-04)-	5.21e-01(3.82e-03)-	**9.94e-01(1.09e-07)**
ZDT3	2	30	9.01e-01(9.31e-04)-	9.02e-01(6.17e-04)-	5.81e-01(1.06e-02)-	**9.96e-01(8.58e-07)**
ZDT4	2	30	6.12e-01(7.60e-04)+	**6.16e-01(6.90e-04)+**	2.03e-01(4.67e-03)-	5.29e-01(1.12e-02)
ZDT6	2	30	2.16e-01(1.52e-03)+	**2.43e-01(2.17e-03)+**	1.30e-01(6.95e-04)-	1.50e-01(5.87e-04)
			4/26/3	5/26/2	0/32/1	

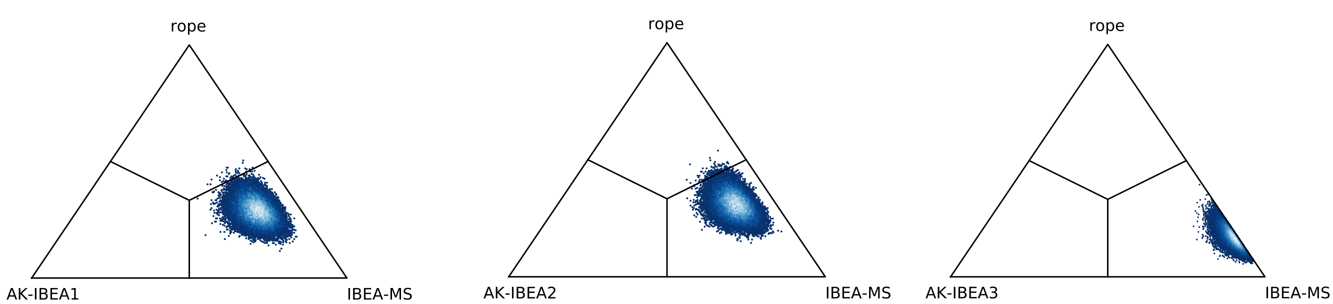

Fig. S.2. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on all the tested problems based on the HV performance.

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TABLE S.XV
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE 3- OBJECTIVE DTLZ1-7 PROBLEMS WITH 40 AND 50 DECISION VARIABLES AND THE 15- AND 25- OBJECTIVE DTLZ1-7 PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
DTLZ1	3	40	9.87e+02(7.13e+03)-	9.91e+02(6.40e+03)-	9.99e+02(1.47e+04)-	**9.18e+02(4.33e+03)**
	3	50	1.30e+03(6.08e+03)-	1.29e+03(7.72e+03)-	1.40e+03(5.23e+04)-	**1.20e+03(2.86e+03)**
	15	30	2.71e+02(9.39e+02)=	2.53e+02(1.30e+03)=	3.74e+02(5.49e+03)-	**2.53e+02(1.11e+03)**
	25	30	6.34e+01(2.63e+02)-	5.71e+01(3.56e+02)=	9.24e+01(6.10e+02)-	**4.94e+01(2.19e+02)**
DTLZ2	3	40	2.10e+00(2.40e-02)-	2.08e+00(1.94e-02)-	2.91e+00(9.62e-02)-	**2.98e-01(1.46e-03)**
	3	50	2.72e+00(3.09e-02)-	2.68e+00(3.45e-02)-	3.61e+00(7.37e-02)-	**3.92e-01(5.14e-03)**
	15	30	1.47e+00(9.10e-03)-	1.49e+00(4.98e-03)-	1.81e+00(1.14e-02)-	**1.31e+00(1.10e-02)**
	25	30	1.21e+00(7.54e-04)=	1.20e+00(8.43e-04)=	1.37e+00(2.15e-03)-	**1.20e+00(1.07e-03)**
DTLZ3	3	40	2.80e+03(2.87e+04)-	2.82e+03(3.34e+04)-	**2.52e+03(1.12e+05)=**	2.58e+03(5.07e+04)
	3	50	3.77e+03(4.57e+04)-	3.76e+03(4.85e+04)-	3.67e+03(1.43e+05)=	**3.61e+03(5.47e+04)**
	15	30	1.01e+03(7.28e+03)-	1.03e+03(1.10e+04)-	1.02e+03(1.41e+04)-	**8.22e+02(2.44e+04)**
	25	30	1.87e+02(4.03e+03)=	**1.76e+02(2.10e+03)=**	2.57e+02(3.50e+03)-	1.82e+02(1.27e+03)
DTLZ4	3	40	2.46e+00(2.89e-02)-	2.37e+00(1.58e-02)-	3.33e+00(1.04e-01)-	**9.58e-01(1.11e-03)**
	3	50	3.04e+00(2.02e-02)-	3.00e+00(3.41e-02)-	4.12e+00(1.15e-01)-	**9.73e-01(4.12e-04)**
	15	30	1.54e+00(4.40e-03)-	1.55e+00(2.33e-03)-	1.71e+00(8.26e-03)-	**1.30e+00(3.76e-03)**
	25	30	1.04e+00(5.61e-04)-	1.04e+00(5.77e-04)-	1.18e+00(1.52e-03)-	**9.95e-01(3.64e-04)**
DTLZ5	3	40	2.00e+00(1.91e-02)-	1.93e+00(3.74e-02)-	2.76e+00(7.37e-02)-	**2.06e-01(9.31e-04)**
	3	50	2.64e+00(3.91e-02)-	2.61e+00(2.01e-02)-	3.54e+00(9.12e-02)-	**2.94e-01(2.72e-03)**
	15	30	7.74e-01(4.67e-03)-	8.11e-01(1.51e-02)-	1.16e+00(1.94e-02)-	**4.31e-01(2.36e-02)**
	25	30	2.96e-01(2.35e-03)-	2.95e-01(1.76e-03)-	6.36e-01(1.50e-02)-	**2.26e-01(3.16e-03)**
DTLZ6	3	40	2.97e+01(1.32e+00)-	2.93e+01(1.20e+00)-	2.86e+01(1.89e+00)-	**2.47e+01(2.04e+00)**
	3	50	3.78e+01(2.64e+00)-	3.78e+01(1.13e+00)-	3.71e+01(2.68e+00)-	**3.34e+01(1.93e+00)**
	15	30	1.29e+01(5.16e-01)-	1.31e+01(2.59e-01)-	1.21e+01(4.77e-01)-	**1.00e+01(1.09e+00)**
	25	30	4.44e+00(2.17e-01)-	4.38e+00(1.59e-01)-	4.29e+00(2.18e-01)-	**3.34e+00(3.02e-01)**
DTLZ7	3	40	1.67e+00(3.14e-01)-	1.59e+00(5.00e-01)-	5.37e+00(2.55e+00)-	**1.41e-01(3.45e-04)**
	3	50	1.91e+00(3.53e-01)-	1.95e+00(4.09e-01)-	6.37e+00(2.73e+00)-	**1.62e-01(4.00e-04)**
	15	30	2.58e+01(1.35e+01)-	2.51e+01(1.80e+01)-	**4.95e+00(7.19e+00)+**	7.12e+00(1.89e+00)
	25	30	2.84e+01(5.58e+01)-	2.73e+01(2.68e+01)-	**3.40e+00(5.05e-01)+**	1.69e+01(2.75e+00)
+/-/=			0/26/3	0/25/4	2/25/2	- -

TABLE S.XVI
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE IDTLZ1-2 AND WFG1-9 PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL).

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
IDTLZ1	3	30	1.17e+03(1.29e+04)=	1.17e+03(1.78e+04)=	**1.029e+02(1.24e+02)+**	1.14e+03(7.83e+03)
IDTLZ2	3	30	1.39e+00(3.42e-02)-	1.41e+00(2.93e-02)-	2.99e+00(3.89e-02)-	**1.13e-01(4.65e-04)**
WFG1	3	30	2.25e+00(3.77e-03)+	**2.22e+00(5.65e-03)+**	2.39e+00(2.56e-03)-	2.28e+00(2.15e-03)
WFG2	3	30	9.18e-01(2.79e-03)-	9.44e-01(5.44e-03)-	1.30e+00(2.51e-02)-	**8.64e-01(1.57e-03)**
WFG3	3	30	8.01e-01(8.08e-04)-	7.99e-01(7.49e-04)-	1.25e+00(5.62e-02)-	**6.91e-01(1.06e-03)**
WFG4	3	30	7.07e-01(1.71e-03)-	6.97e-01(1.48e-03)-	8.83e-01(1.92e-02)-	**6.30e-01(1.29e-03)**
WFG5	3	30	8.07e-01(7.29e-04)-	7.84e-01(4.74e-04)-	1.59e+00(2.64e-01)-	**7.16e-01(9.32e-03)**
WFG6	3	30	9.33e-01(7.90e-04)=	9.37e-01(6.08e-04)=	1.61e+00(3.88e-02)-	**9.24e-01(6.81e-04)**
WFG7	3	30	8.05e-01(9.27e-04)-	7.99e-01(9.71e-04)-	1.73e+00(2.76e-02)-	**7.50e-01(1.92e-03)**
WFG8	3	30	**8.37e-01(6.86e-04)+**	8.45e-01(7.36e-04)=	1.57e+00(3.86e-02)-	8.60e-01(1.23e-03)
WFG9	3	30	1.02e+00(2.34e-03)=	1.01e+00(1.67e-03)=	1.50e+00(8.31e-02)-	**9.94e-01(1.44e-03)**
+/-/=			2/6/3	1/6/4	0/10/1	- -

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TABLE S.XVII
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE UF1-12 PROBLEMS WITH 30 DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
UF1	2	30	1.17e+00(1.28e-02)=	1.21e+00(1.01e-02)-	1.79e+00(1.58e-01)-	**1.16e+00(1.06e-02)**
UF2	2	30	5.94e-01(2.10e-03)-	6.03e-01(9.98e-04)-	1.15e+00(8.61e-02)-	**2.54e-01(2.35e-03)**
UF3	2	30	1.11e+00(3.45e-03)-	1.09e+00(7.63e-03)-	2.17e+00(5.24e-01)-	**6.48e-01(1.04e-03)**
UF4	2	30	**1.79e-01(6.22e-05)+**	1.80e-01(2.82e-05)+	3.56e-01(3.09e-03)-	2.39e-01(1.45e-03)
UF5	2	30	4.97e+00(1.33e-01)=	4.99e+00(9.66e-02)=	6.30e+00(6.01e-01)-	**4.93e+00(7.56e-02)**
UF6	2	30	**5.14e+00(2.51e-01)=**	5.37e+00(2.26e-01)=	8.72e+00(3.87e+00)-	5.22e+00(2.25e-01)
UF7	2	30	**1.27e+00(1.48e-02)=**	1.28e+00(1.13e-02)=	1.65e+00(6.95e-02)-	1.29e+00(5.81e-03)
UF8	3	30	2.73e+00(1.06e-01)-	2.67e+00(1.09e-01)-	3.83e+00(3.87e-01)-	**1.20e+00(2.59e-01)**
UF9	3	30	2.79e+00(9.96e-02)-	2.71e+00(7.54e-02)-	4.44e+00(3.45e-01)-	**1.42e+00(2.10e-01)**
UF10	3	30	1.43e+01(2.64e+00)-	1.39e+01(2.71e+00)-	3.78e+01(1.11e+02)-	**8.11e+00(5.12e+00)**
UF11	5	30	5.67e+00(3.72e-01)-	5.57e+00(2.48e-01)-	6.67e+00(7.13e-01)-	**4.45e+00(3.90e-01)**
UF12	5	30	3.27e+03(5.09e+04)-	3.32e+03(9.50e+04)-	3.33e+03(1.15e+05)-	**3.03e+03(7.92e+04)**
+/-/=			1/7/4	1/8/3	0/12/0	- -

TABLE S.XVIII
COMPARATIVE EXPERIMENT BETWEEN AK-IBEA VARIANTS AND IBEA-MS ON THE MAF1-7 PROBLEMS WITH 30 DECISION VARIABLES AND MA8-9 WITH TWO DECISION VARIABLES ANALYZED BY THE WILCOXON RANK SUM TEST (0.05 SIGNIFICANCE LEVEL). ALL OF THE RESULTS ARE EVALUATED BY THE IGD PERFORMANCE INDICATOR.

Problem	M	D	AK-IBEA1	AK-IBEA2	AK-IBEA3	IBEA-MS
MaF1	3	30	1.76e+00(2.97e-02)-	1.74e+00(3.31e-02)-	3.08e+00(3.28e-01)-	**1.19e-01(3.21e-04)**
MaF2	3	30	1.56e-01(5.04e-05)-	1.56e-01(2.96e-05)-	3.43e-01(6.75e-04)-	**6.42e-02(6.15e-05)**
MaF3	3	30	3.39e+06(1.01e+12)+	**3.19e+06(1.10e+12)+**	2.68e+11(6.38e+23)-	5.44e+08(7.25e+18)
MaF4	3	30	6.75e+03(3.76e+05)=	7.09e+03(9.47e+05)-	7.20e+03(2.26e+06)=	**6.47e+03(3.56e+05)**
MaF5	3	30	6.49e+00(1.02e+00)-	6.64e+00(1.08e+00)-	1.20e+01(2.78e+01)-	**4.79e+00(1.36e-01)**
MaF6	3	30	1.20e+02(3.26e+02)-	1.23e+02(1.55e+02)-	4.22e+02(2.43e+04)-	**4.58e+00(1.62e+00)**
MaF7	3	30	1.15e+00(2.11e-01)-	1.19e+00(1.23e-01)-	5.11e+00(3.67e+00)-	**1.33e-01(4.80e-05)**
MaF8	3	2	5.45e+02(9.13e+04)-	6.08e+02(1.22e+05)-	2.01e+03(1.89e+07)-	**3.94e+02(7.53e+04)**
MaF9	3	2	1.02e+03(7.99e+05)-	1.01e+03(4.56e+05)-	3.10e+03(1.63e+07)-	**1.66e+02(1.13e+04)**
+/-/=			1/7/1	1/8/0	0/7/1	- -

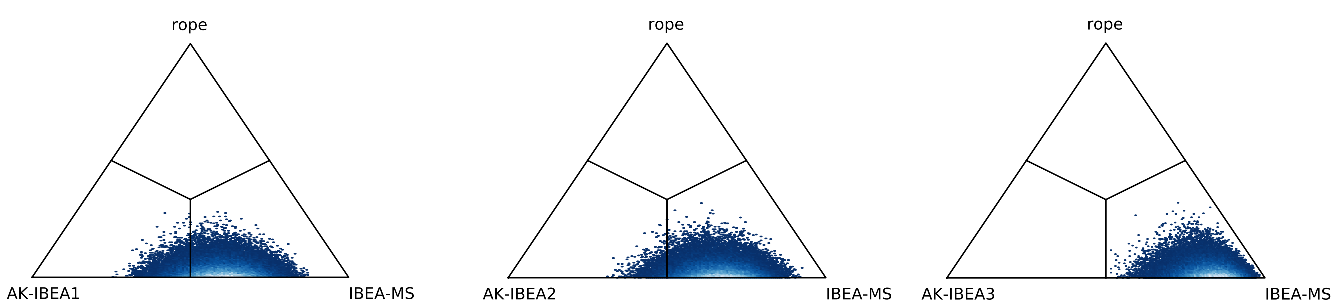

Fig. S.3. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ1 problem based on the IGD performance.

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TABLE S.XIX
THE PROPORTION OF THE LOCATION OF THE POINT IN FIG. S.2, WHERE THE VALUES IN THE THREE REGIONS REPRESENT THE PROBABILITY THAT THE AK-IBEA VARIANTS IS BETTER THAN, EQUIVALENT TO AND WORSE THAN IBEA-MS

Compared Algorithms	$[-\infty, rope]$	$[-rope, rope]$	$[rope, \infty]$
AK-IBEA1	0	0.0073	0.9927
AK-IBEA2	0	0.02194	0.97806
AK-IBEA3	0	0	1.0

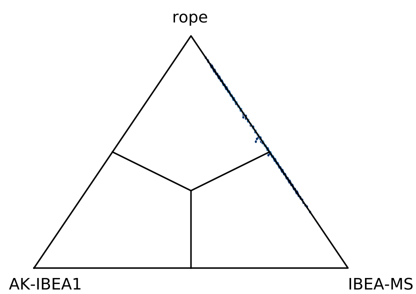

Fig. S.4. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ1 problem based on the HV performance.

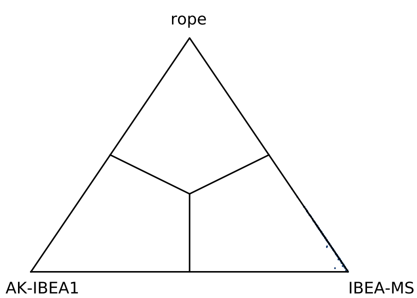

Fig. S.5. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ2 problem based on the IGD performance.

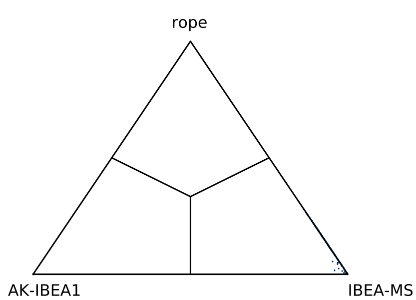

Fig. S.6. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ2 problem based on the HV performance.

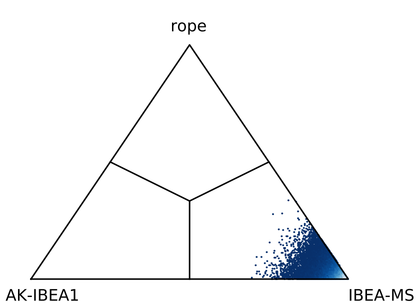

Fig. S.7. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ3 problem based on the IGD performance.

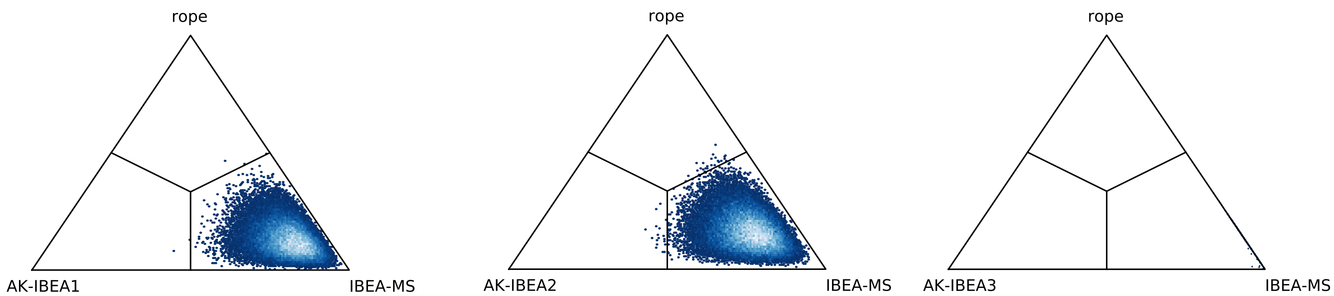

Fig. S.8. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ3 problem based on the HV performance.

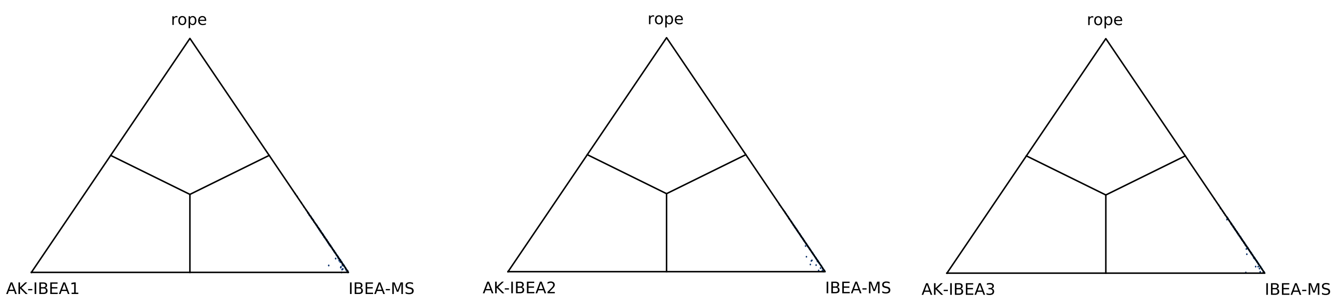

Fig. S.9. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ4 problem based on the IGD performance.

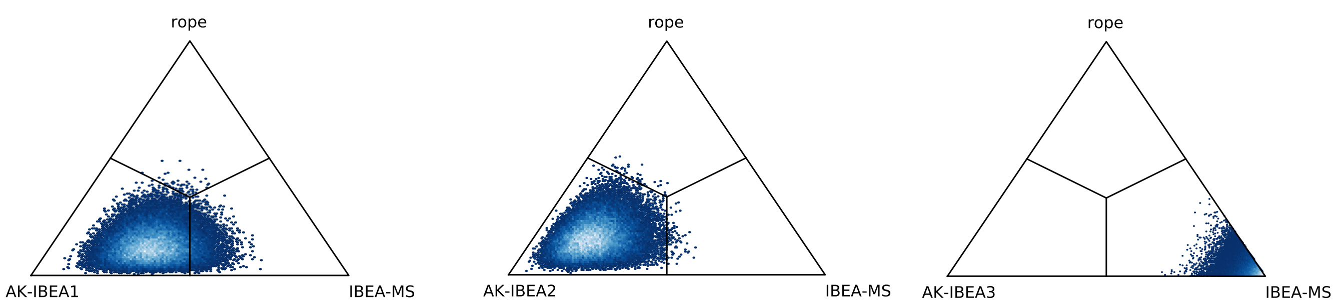

Fig. S.10. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ4 problem based on the HV performance.

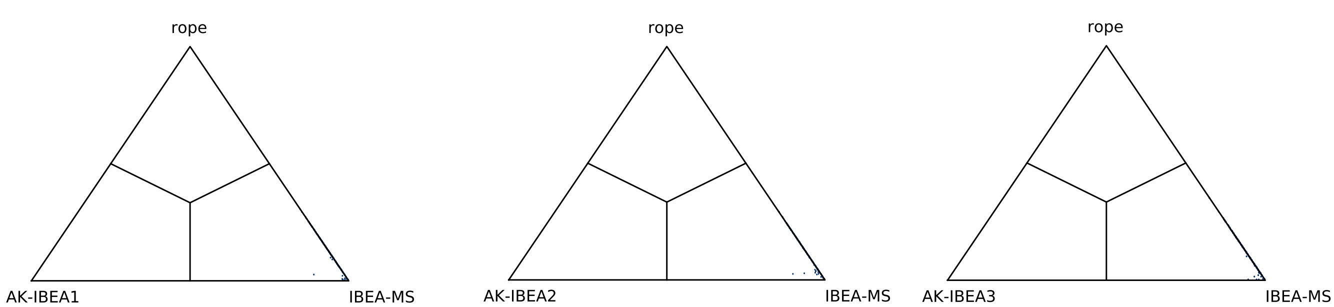

Fig. S.11. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ5 problem based on the IGD performance.

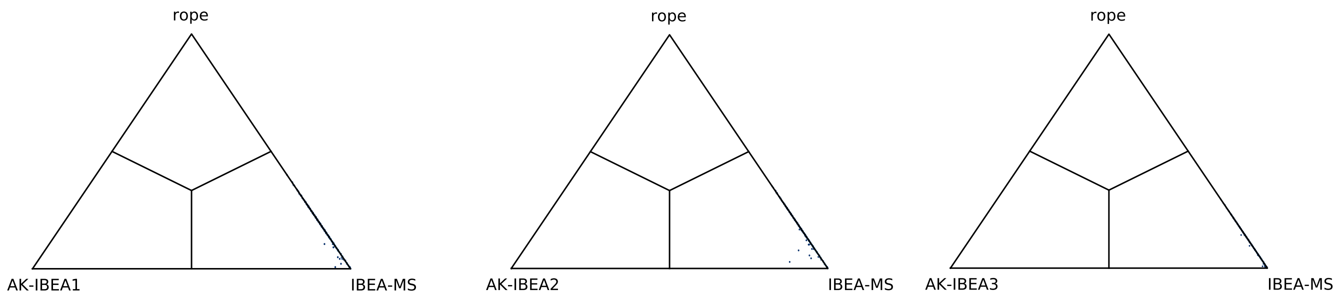

Fig. S.12. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ5 problem based on the HV performance.

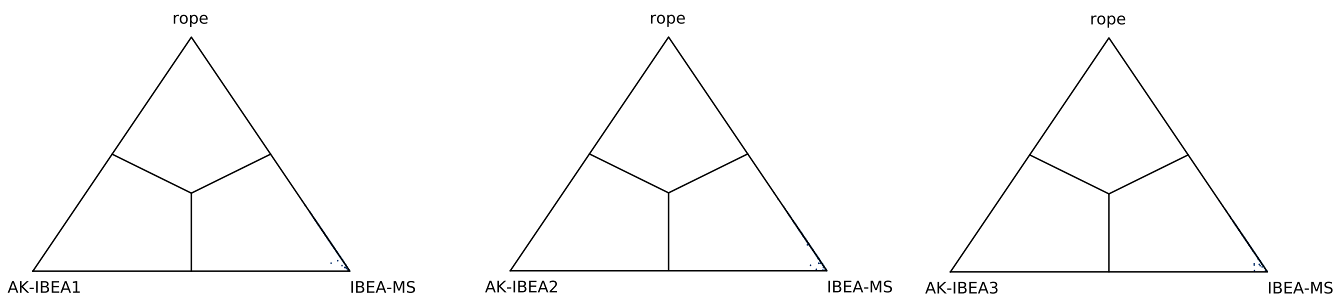

Fig. S.13. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ6 problem based on the IGD performance.

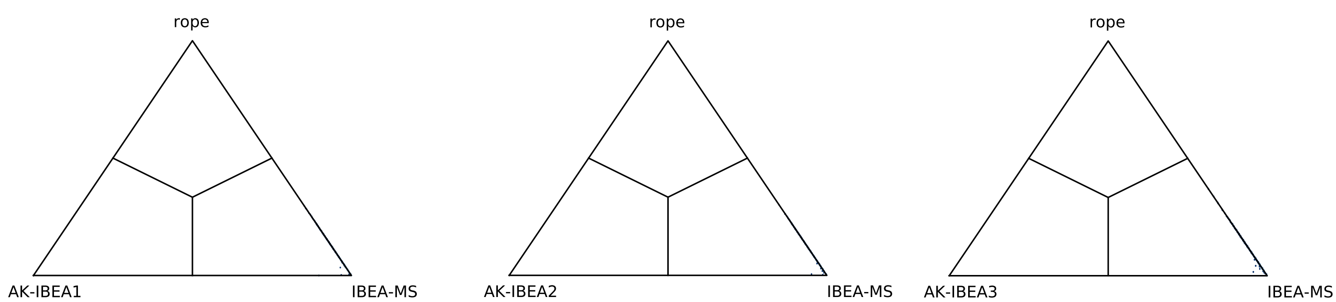

Fig. S.14. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ6 problem based on the HV performance.

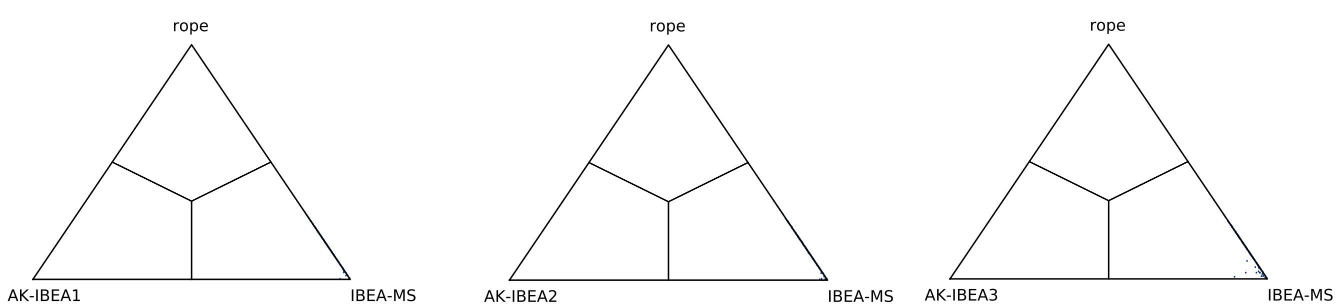

Fig. S.15. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ7 problem based on the IGD performance.

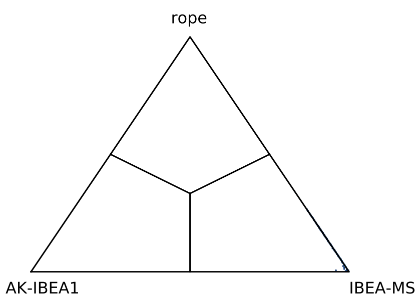

Fig. S.16. The Bayesian Signed-Rank test between AK-IBEA variants and IBEA-MS on 30-dimensional three-objective DTLZ7 problem based on the HV performance.