THE OPTIMALITY OF A* REVISITED*

Rina Dechter and Judea Pearl

Cognitive Systems Laboratory
Computer Science Department
University of California, Los Angeles, CA 90024

ABSTRACT

This paper examines the optimality of A^*, in the sense of expanding the least number of distinct nodes, over three classes of algorithms which return solutions of comparable costs to that found by A^*. We first show that A^* is optimal over those algorithms guaranteed to find a solution at least as good as A^*'s for every heuristic assignment h. Second, we consider a wider class of algorithms which, like A^*, are guaranteed to find an optimal solution (i.e., admissible) if all cost estimates are optimistic (i.e., $h \le h^*$). On this class we show that A^* is not optimal and that no optimal algorithm exists unless h is also consistent, in which case A^* is optimal. Finally we show that A^* is optimal over the subclass of *best-first* algorithms which are admissible whenever $h \le h^*$.

1. INTRODUCTION AND PRELIMINARIES

1.1 A* and Informed Best-First Strategies

Of all search strategies used in problem solving, one of the most popular methods of exploiting heuristic information to cut down search time is the *informed best-first* strategy. The general philosophy of this strategy is to use the heuristic information to assess the "merit" latent in every candidate search avenue, then continue the exploration along the direction of highest merit. Formal descriptions of this strategy are usually given in the context of path searching problems, a formulation which represents many combinatorial problems such as routing, scheduling, speech recognition, scene analysis, and others. Given a weighted directional graph G with a distinguished start node s and a set of goal nodes Γ, the *optimal path problem* is to find a lowest cost path from s to Γ where the cost of the path may, in general, be an arbitrary function of the weights assigned to the nodes and branches along that path.

By far, the most studied version of informed best-first strategies is the algorithm A^* (Hart, Nilsson and Raphael, 1968) which was developed for *additive cost measures*, i.e, where the cost of a path is defined as the sum of the costs of its arcs. To match this cost measure, A^* employs a special additive form of the evaluation function f made up from the sum $f(n)=g(n)+h(n)$, where $g(n)$ is the cost of the currently evaluated path from s to n and h is a heuristic estimate of the cost of the path remaining between n and some goal node. A^* constructs a tree T of selected paths of G using the elementary operation of *node expansion*, i.e., generating all successors of a given node. Starting with s, A^* selects for expansion that leaf node of T which has the lowest f value, and only maintains the lowest-g path to any given node. The search halts as soon as a node selected for expansion is found to satisfy the goal conditions. It is known that if $h(n)$ is a lower bound to the cost of any continuation path from n to Γ, then A^* is *admissible*, that is, it is guaranteed to find the optimal path.

1.2 Previous Works

The *optimality* of A^*, in the sense of expanding the *least number of distinct nodes*, has been a subject of some confusion. The well-known property of A^* which predicts that decreasing errors h^*-h can only improve its performance (Nilsson, 1980, result 6) has often been interpreted to reflect some supremacy of A^* over other search algorithms of equal information. Consequently, several authors have assumed that A^*'s optimality is an established fact (e.g., Nilsson, 1971; Martelli, 1977; Méro, 1981; Barr and Feigenbaum, 1982). In fact, all this property says is that some A^* algorithms are better than other A^* algorithms depending on the heuristics which guide them. It does not indicate whether the additive rule $f=g+h$ is better than other ways of combining g and h (e.g., $f=g+h^2/(g+h)$); neither does it assure us that expansion policies based only on g and h can do as well as more sophisticated best-first policies using the entire information gathered along each path (e.g., $f(n)=\max\{f(n') \mid n'$ is on the path to $n\}$). These two conjectures will be examined in this paper, and will be given a qualified confirmation.

Gelperin (1978) has correctly pointed out that in any discussion of the optimality of A^* one should compare it to a wider class of equally informed algorithms, not merely those guided by $f=g+h$, and that the comparison class should include, for example, algorithms which adjust their h in accordance with the information gathered during the search. His analysis, unfortunately, falls short of considering the entirety of this extended class, having to follow an over-restrictive definition of *equally-informed*. Gelperin's interpretation of the statement "an algorithm B is *never more informed* than A" not only restricts B from using information unavailable to A, but also forbids B from processing common information in a better way than A does.

*Supported in part by NSF Grant No. MSC 81-142209