7주차(2/3)

# 순방향 신경망 예제

파이썬으로배우는기계학습

한동대학교 김영섭교수

### 순방향 신경망 예제

- 학습 목표
  - 예제를 통해 순방향 신경망을 깊이 있게 이해한다
- 학습 내용
  - MNIST 자료 이해 하기
  - 다층 신경망 설계 하기
  - 순방향 신경망 신호처리
  - 순방향 신경망 예제 구현 하기

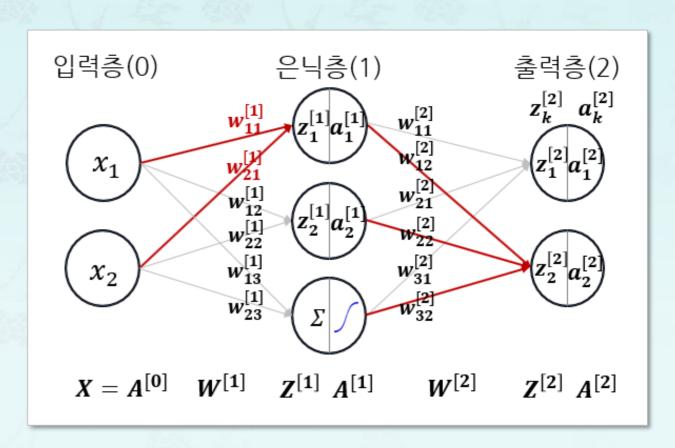
# 1. 다층 신경망: 입력 자료

### 1. 다층 신경망: 입력 자료

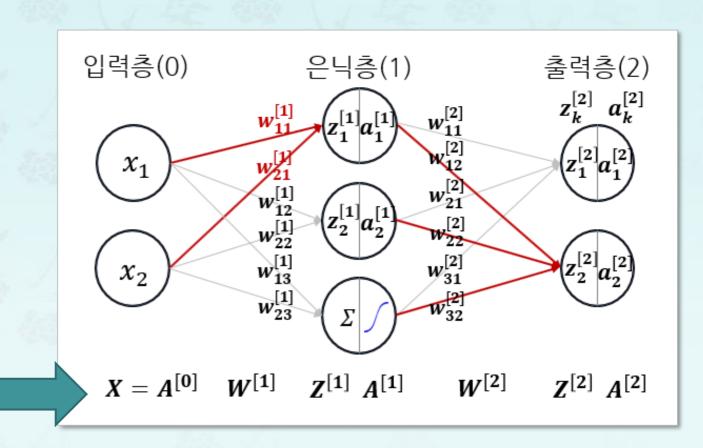
MNIST



# 1. 다층 신경망: 신호 표기(복습)



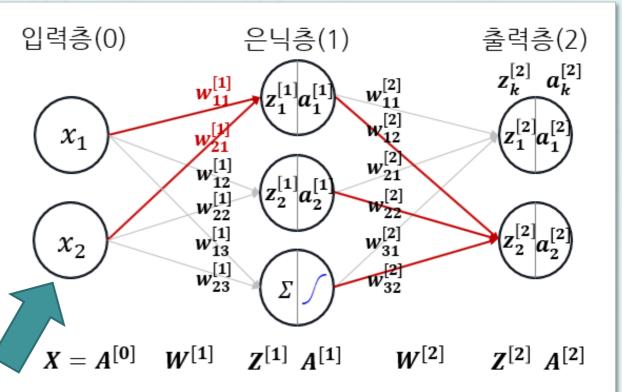
# 1. 다층 신경망: 신호 표기(복습)



### 1. 다층 신경망: 입력 자료

MNIST

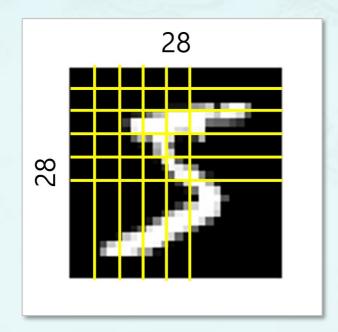




### 1. 다층 신경망: 입력 자료

- MNIST
  - 28x28

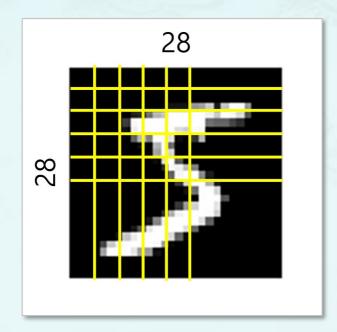




- MNIST
  - 28x28

■ 입력층 : 784(28x28)

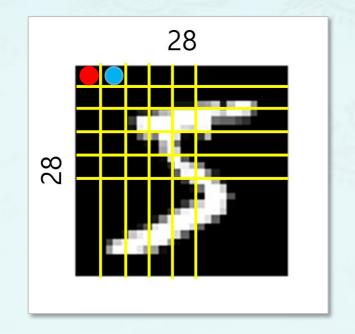


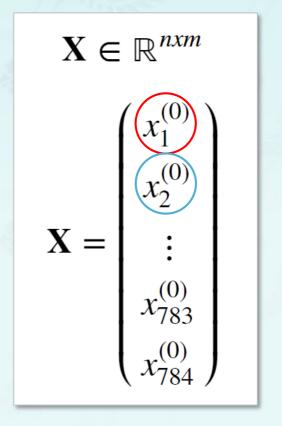


- MNIST
  - 28x28

■ 입력층: 784(28x28)

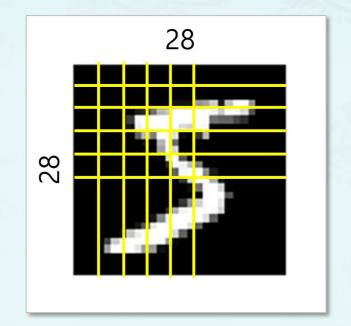






- MNIST
  - 28x28

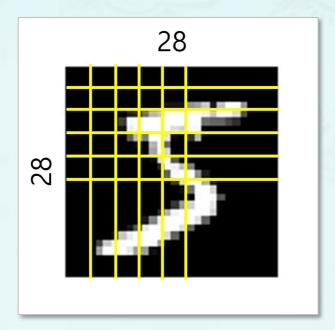




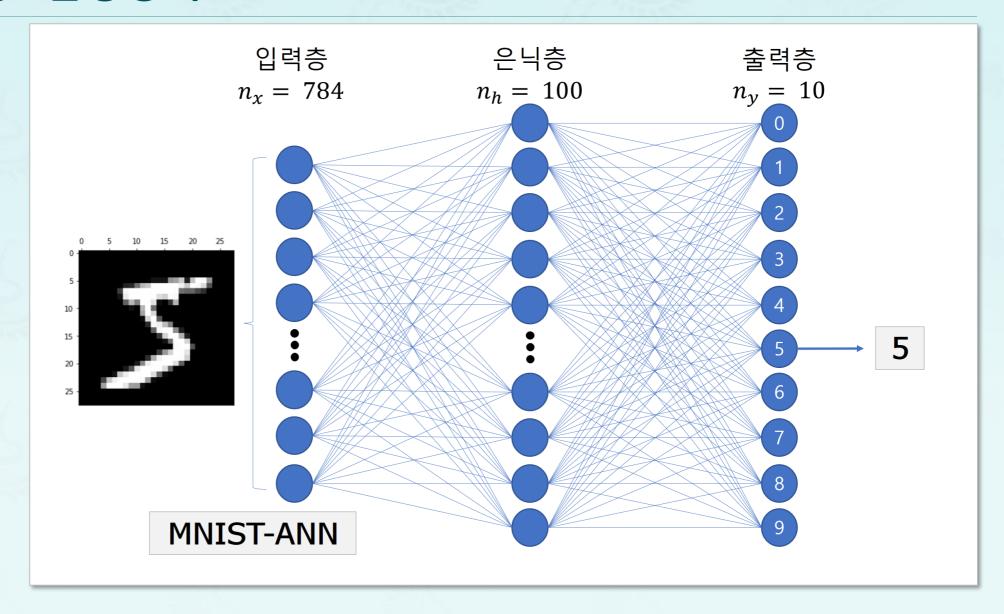
- 입력층 : 784(28x28)
- 은닉층:100

- MNIST
  - 28x28

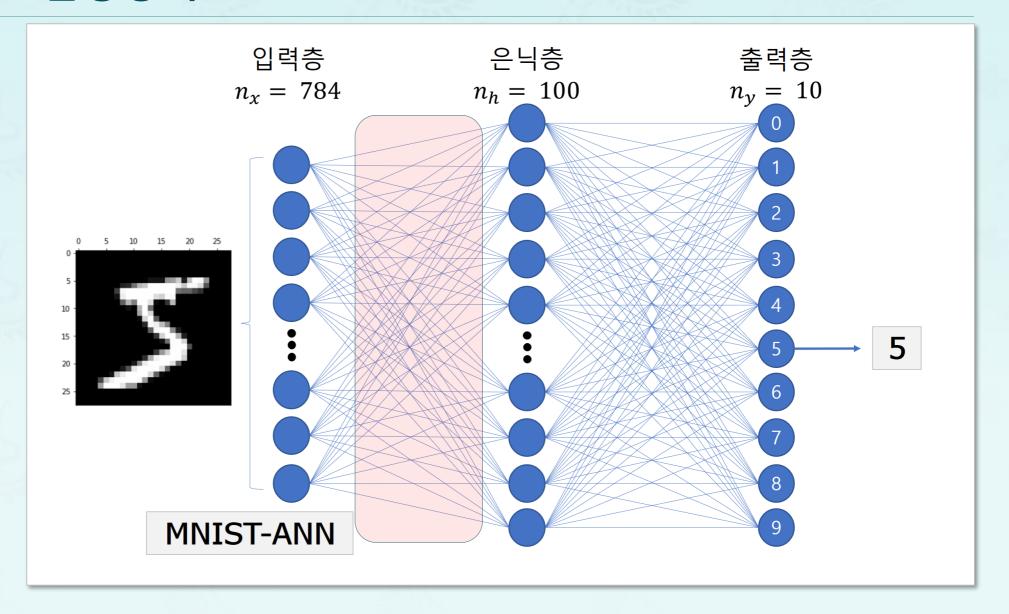




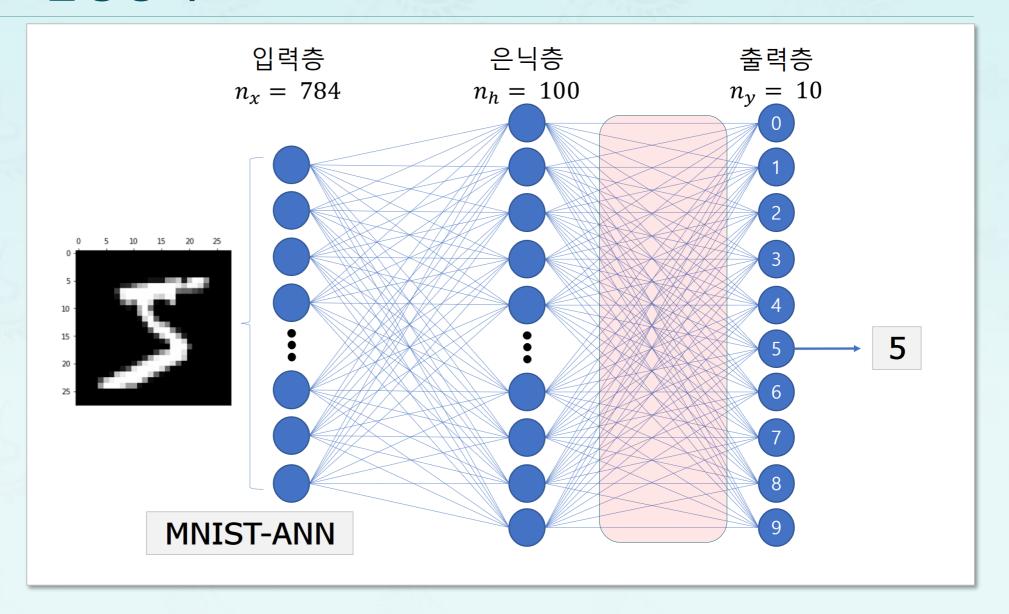
- 입력층 : 784(28x28)
- 은닉층:100
- 출력층:10



- 가중치
  - **784x100**

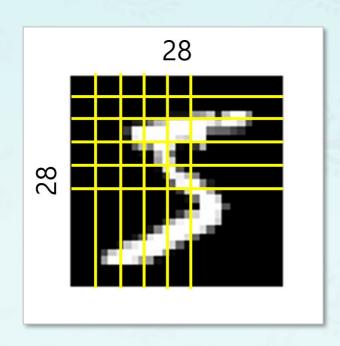


- 가중치
  - **784x100**
  - 100x10



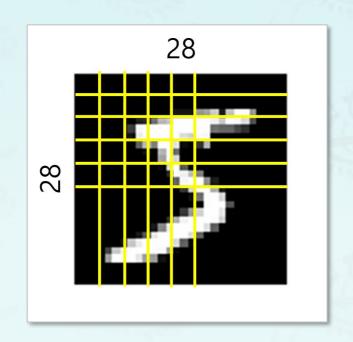
# 2. 숫자 인식 인공 신경망 구현: 입력 자료

- Xnxm
- n = 784, m = 1



# 2. 숫자 인식 인공 신경망 구현: 입력 자료

- Xnxm
- n = 784, m = 1

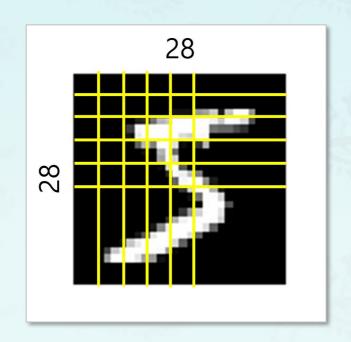


$$\mathbf{X} \in \mathbb{R}^{nxm}$$

$$\begin{pmatrix} x_1^{0} \\ x_2^{0} \\ \vdots \\ x_{783}^{0} \\ x_{784}^{0} \end{pmatrix}$$

# 2. 숫자 인식 인공 신경망 구현: 입력 자료

- Xnxm
- n = 784, m = 1



$$\mathbf{X} \in \mathbb{R}^{n \times m}$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_{1}^{(0)} \\ \mathbf{x}_{2}^{(0)} \\ \vdots \\ \mathbf{x}_{783}^{(0)} \\ \mathbf{x}_{784}^{(0)} \end{pmatrix}$$

- 사전에 학습된 가중치
- 96% 성능

•  $W^{[1]} = 100x784$ 

$$W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}$$

•  $W^{[1]} = 100x784$ 

```
W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}
```

!type data/w\_xh.txt

# 2. 숫자 인식 인공 신경망 구현: 은닉층 순입력 계산

# 2. 숫자 인식 인공 신경망 구현: 은닉층 순입력 계산

$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$

### 2. 숫자 인식 인공 신경망 구현: 은닉층 순입력 계산

$$\mathbf{Z}^{[1]} = W^{[1]} A^{[0]}$$

$$= \begin{pmatrix} w_{11}^{(1)} & w_{21}^{(1)} \\ w_{12}^{(1)} & w_{22}^{(1)} \\ w_{13}^{(1)} & w_{23}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= \begin{pmatrix} z_1^{(1)} \\ z_2^{(1)} \\ z_3^{(1)} \end{pmatrix}$$

$$\mathbf{W}^{[1]}\mathbf{X}^{[0]} = \begin{pmatrix} w_{1}^{(1)} & w_{1}^{(2)} & \cdots & w_{1}^{(783)} & w_{1}^{(784)} \\ w_{2}^{(1)} & w_{2}^{(2)} & \cdots & w_{2}^{(783)} & w_{2}^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix} \begin{pmatrix} x_{1}^{(0)} \\ x_{2}^{(0)} \\ \vdots \\ x_{783}^{(0)} \\ x_{783}^{(0)} \\ x_{784}^{(0)} \end{pmatrix}$$

$$\mathbf{A}^{[1]} = sigmoid(\mathbf{Z}^{[1]})$$

# 2. 숫자 인식 인공 신경망 구현: 출력층 계산

A<sup>[1]</sup>: 100x1

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

# 2. 숫자 인식 인공 신경망 구현: 출력층 계산

A<sup>[1]</sup>: 100x1

W<sup>[2]</sup>: 10x100

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

### 2. 숫자 인식 인공 신경망 구현: 출력층 계산

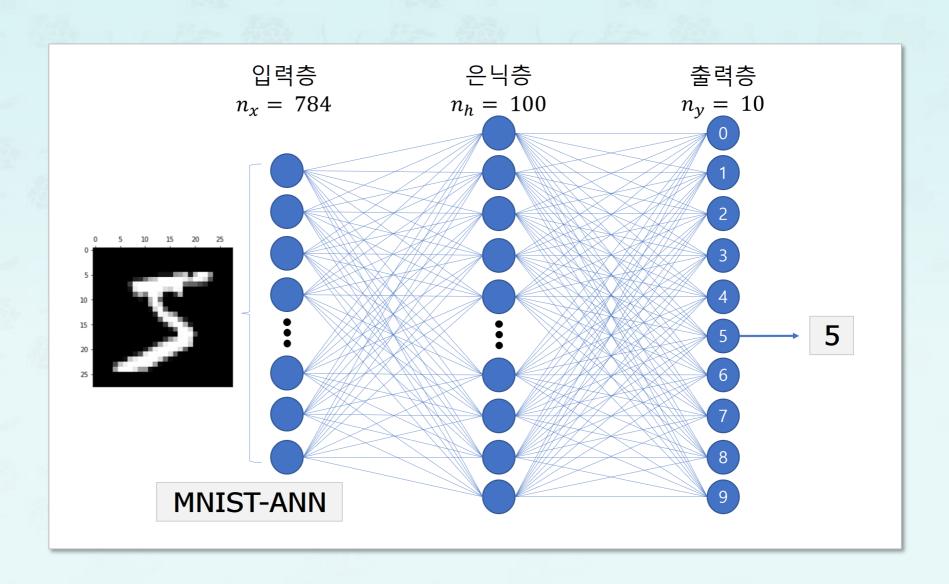
A<sup>[1]</sup>: 100x1

W<sup>[2]</sup>: 10x100

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

$$\mathbf{A}^{[2]} = sigmoid(\mathbf{Z}^{[2]})$$

# 2. 숫자 인식 인공 신경망 구현: 인공 신경망 구조



#### 2. 숫자 인식 인공 신경망 구현

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

# 2. 숫자 인식 인공 신경망 구현: 라이브러리 추가

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

# 2. 숫자 인식 인공 신경망 구현: 라이브러리 추가

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

### 2. 숫자 인식 인공 신경망 구현: 함수 생성

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

### 2. 숫자 인식 인공 신경망 구현: 입력 자료 불러오기

■ 입력 자료 불러오기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```



#### 2. 숫자 인식 인공 신경망 구현: 입력 자료 불러오기

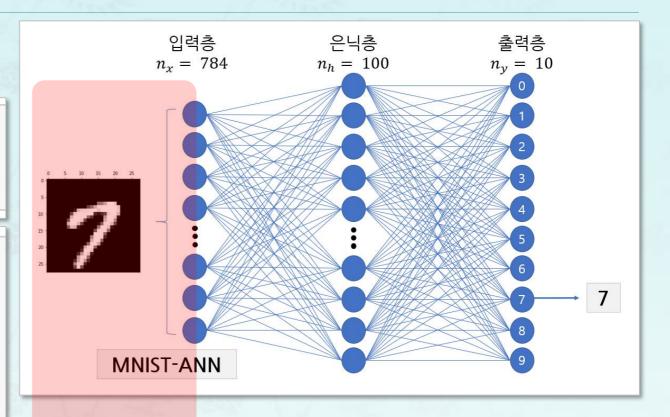
■ 입력 자료 불러오기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



• 가중치 불러오기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}$$

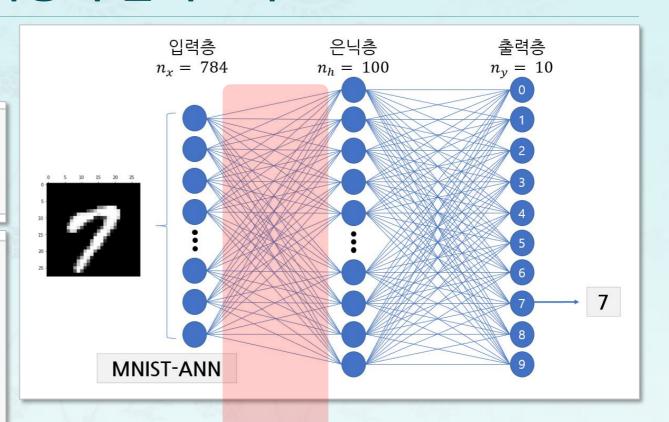
• 가중치 불러오기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



## 2. 숫자 인식 인공 신경망 구현: 순입력 계산하기

• 순입력 계산

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}$$

# 2. 숫자 인식 인공 신경망 구현: 순입력 계산하기

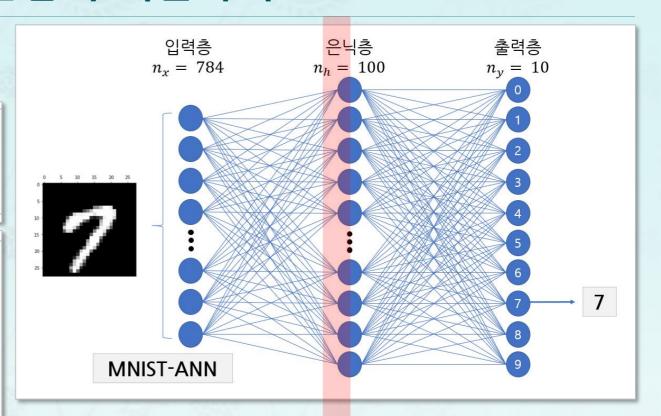
• 순입력 계산

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



### 2. 숫자 인식 인공 신경망 구현: 은닉층 계산하기

■ 은닉층 계산

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(783)} & w_1^{(784)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(783)} & w_2^{(784)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(783)} & w_{100}^{(784)} \end{pmatrix}$$

$$\mathbf{A}^{[1]} = sigmoid(\mathbf{Z}^{[1]})$$

## 2. 숫자 인식 인공 신경망 구현: 은닉층 계산하기

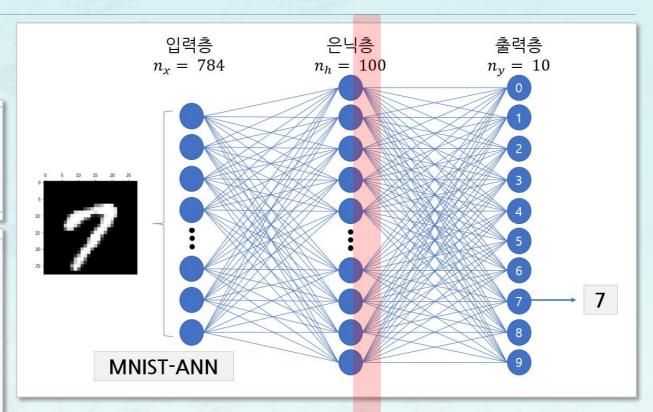
■ 은닉층 계산

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



## 2. 숫자 인식 인공 신경망 구현: 가중치 불러오기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[2]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & W_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & W_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{100}^{(1)} & w_{100}^{(2)} & \cdots & w_{100}^{(99)} & w_{100}^{(100)} \end{pmatrix}$$

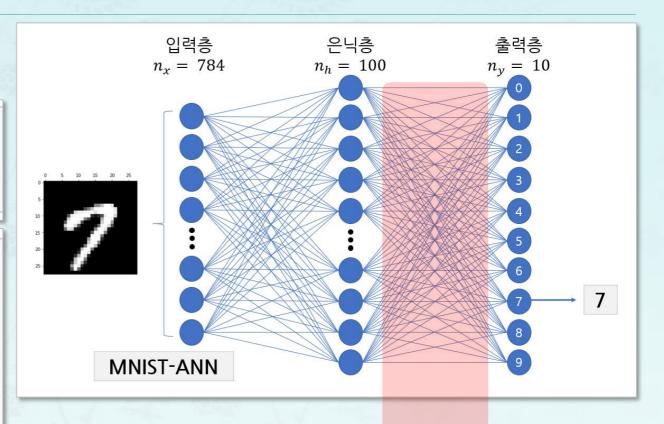
#### 2. 숫자 인식 인공 신경망 구현

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



### 2. 숫자 인식 인공 신경망 구현: 은닉층 계산

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

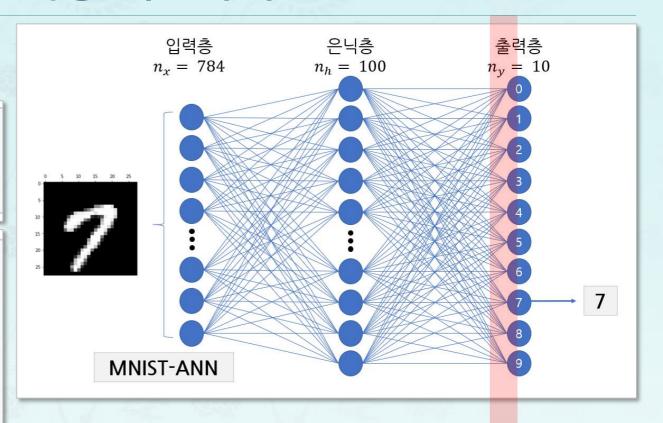
## 2. 숫자 인식 인공 신경망 구현: 은닉층 계산하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



## 2. 숫자 인식 인공 신경망 구현: 출력층 계산하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

$$W^{[2]}A^{[1]} = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(99)} & w_1^{(100)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(99)} & w_2^{(100)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{10}^{(1)} & w_{10}^{(2)} & \cdots & w_{10}^{(99)} & w_{10}^{(100)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ \vdots \\ a_{99}^{(1)} \\ a_{100}^{(1)} \end{pmatrix}$$

$$\mathbf{A}^{[2]} = sigmoid(\mathbf{Z}^{[2]})$$

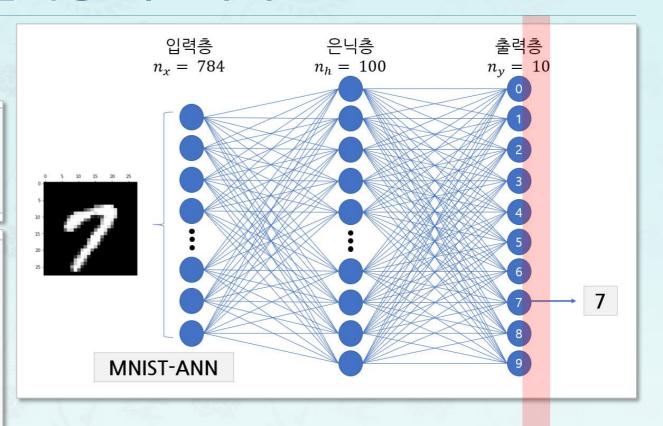
## 2. 숫자 인식 인공 신경망 구현: 출력층 계산하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



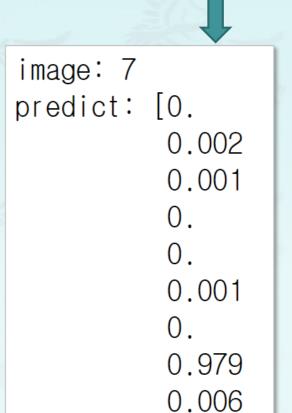
### 2. 숫자 인식 인공 신경망 구현: 예측하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



0.003]

### 2. 숫자 인식 인공 신경망 구현: 예측하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```

```
image: 7
predict: [0.
           0.002
           0.001
           0.
           0.
           0.001
           0.
           0.979
           0.006
           0.003]
```

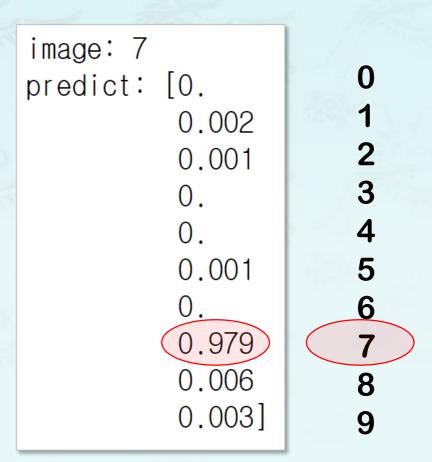
### 2. 숫자 인식 인공 신경망 구현: 예측하기

```
import joy
import numpy as np
g = lambda x : 1 / (1 + np.exp(-x))
```

```
(X, y) = joy.load_mnist_num(7)
W1 = joy.load_mnist_weight('data/w_xh.weights')
Z1 = np.dot(W1, X)
A1 = g(Z1)

W2 = joy.load_mnist_weight('data/w_hy.weights')
Z2 = np.dot(W2, A1)
yhat = g(Z2)

print('image:', y)
print('predict:', np.round_(yhat, 3))
```



#### 순방향 신경망 예제

- 학습 정리
  - 예제를 통해 순방향 신경망을 깊이 있게 이해하기
  - 자료의 특성 이해하기
  - 순방향 신경망 설계
  - 가중치 불러오기

- 차시 예고
  - 7-3 아달라인 경사하강법 소개

7주차(2/3)

# 순방향 신경망 예제

파이썬으로배우는기계학습

한동대학교 김영섭교수

여러분 곁에 항상 열려 있는 K-MOOC 강의실에서 만나 뵙기를 바랍니다.