10주차(3/3)

로지스틱 회귀 2

파이썬으로 배우는 기계학습

한동대학교
김영섭 교수

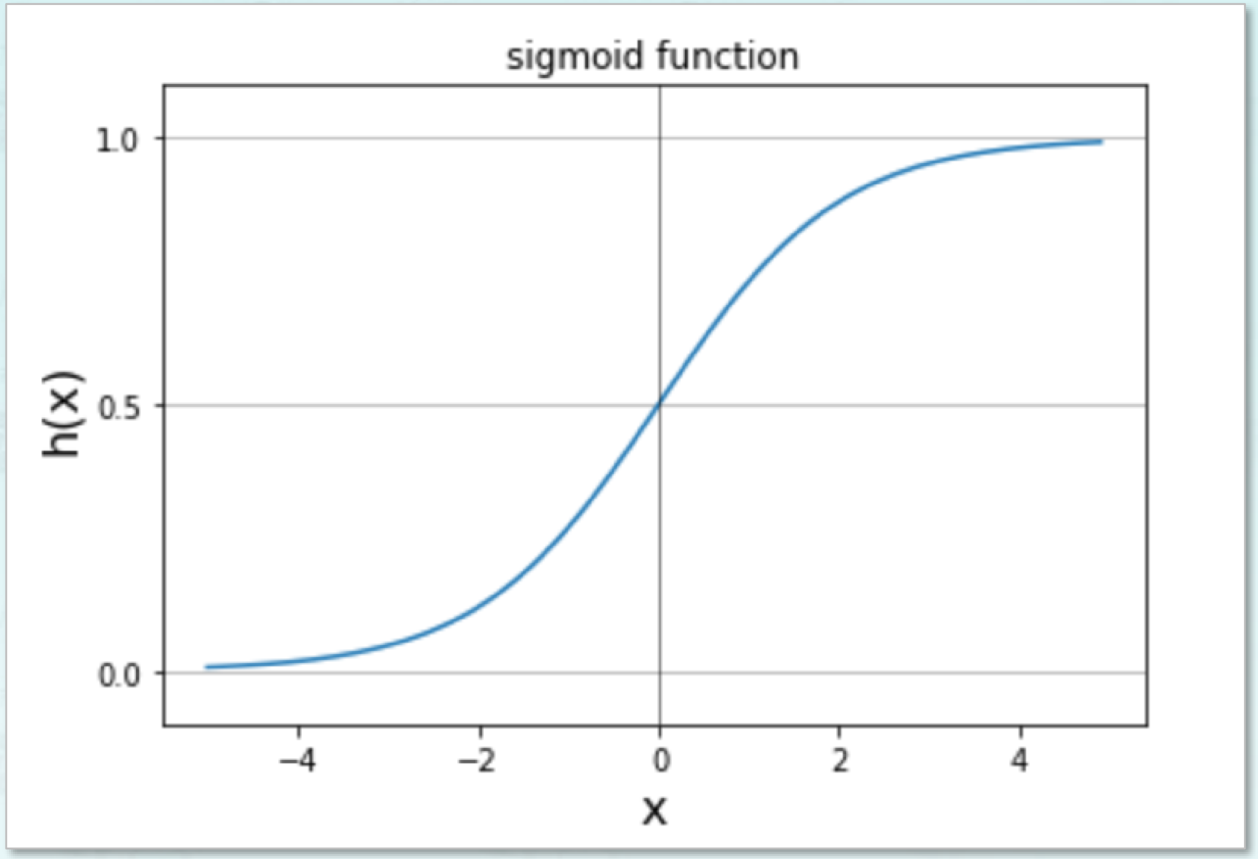
로지스틱 회귀

- 학습 목표
  - 로지스틱 함수를 이해한다.
  - 로지스틱 회귀 비용함수(교차 엔트로피)를 이해한다.
  - 로지스틱 회귀 신경망의 역전파를 계산한다.

- 학습 내용
  - 로지스틱 함수 이해
  - 로지스틱 회귀 비용함수(교차 엔트로피) 미분하기
  - 로지스틱 회귀 신경망의 역전파 계산

1. 로지스틱 함수: 시그모이드 함수

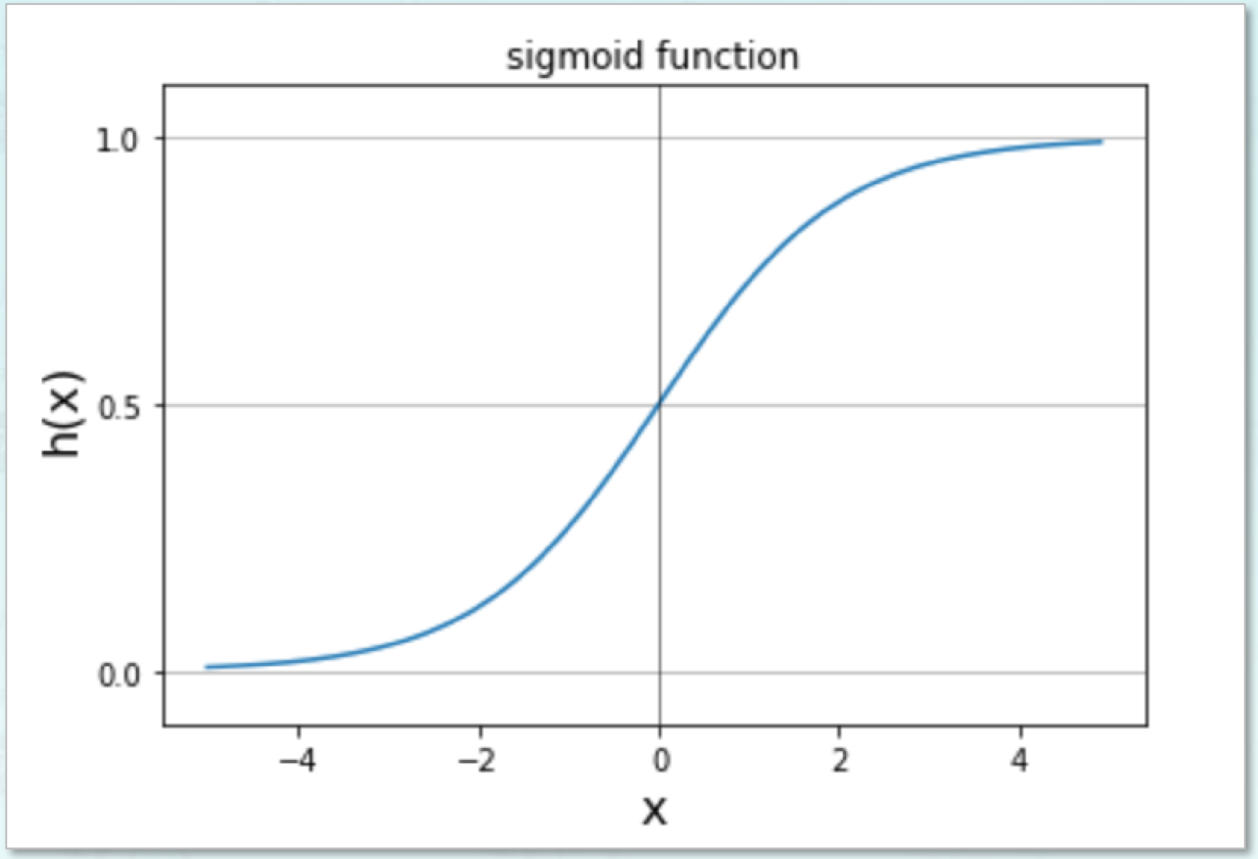
$$\sigma(x) = \frac{1}{1+e^{-x}} \quad \begin{cases} \sigma(x) \to 1 & \text{if } x \to +\infty \\ \sigma(x) \to \frac{1}{2} & \text{if } x \to 0 \\ \sigma(x) \to 0 & \text{if } x \to -\infty \end{cases}$$



1. 로지스틱 함수: 회귀 모델

$$sigmoid(x) = \sigma(x) = \frac{1}{1 + e^{-x}}$$

$$h(z) = \frac{1}{1 + e^{-Z}} = \frac{1}{1 + e^{-W \cdot X}}$$



1. 로지스틱 함수: 회귀 모델 이해하기

- $y \in \{1,0\}$

Classifier:

$$y = h(z) = \frac{1}{1+e^{-WX}} \begin{cases} y \to 1 \; if \; WX \to \infty \\ y = \frac{1}{2} \; if \; WX \to 0 \\ y \to 0 \; if \; WX \to -\infty \end{cases}$$

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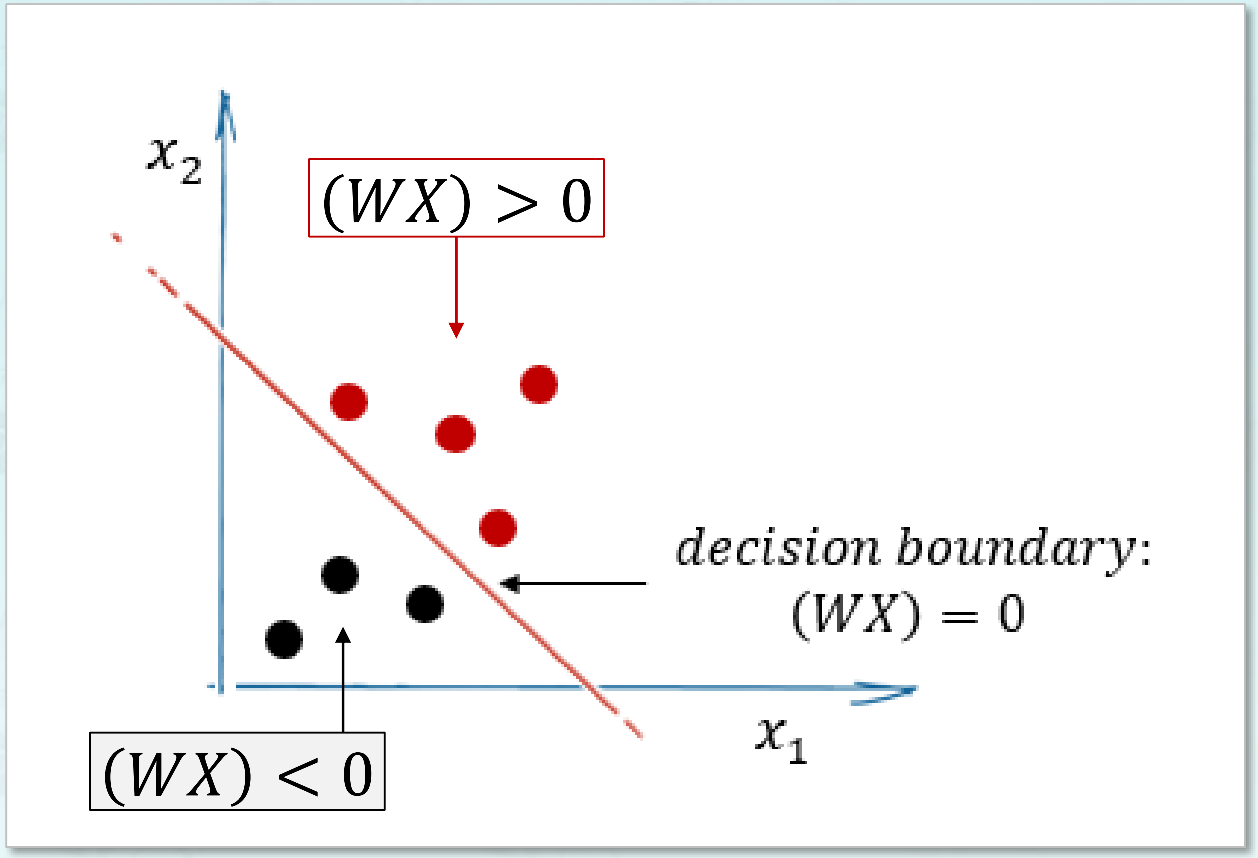
1. 로지스틱 함수: 회귀 모델 이해하기

- 결정경계선
  - **hyperplane**
  - **예시: $\mathbf{W} = [-3, 1, 1]$**

$$
\begin{aligned}
h(z) &= h(w_0 + w_1x_1 + w_2x_2) \\
&= -3 + x_1 + x_2
\end{aligned}
$$



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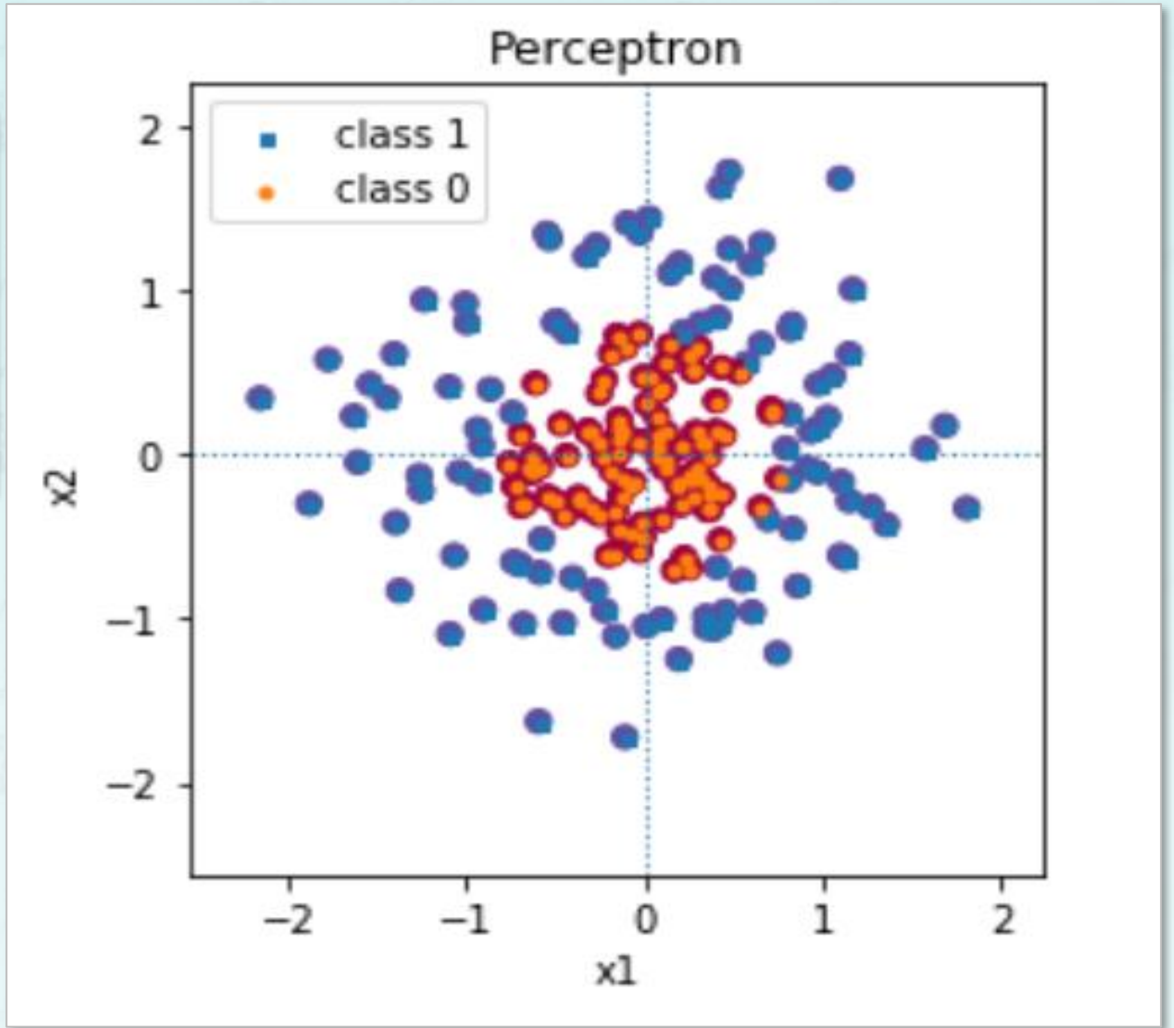

$$z = w_0 + w_1 x_1^2 + w_2 x_2^2$$

- 결정경계선
  - **hyperplane**
  - 예시: $W = [-3,1,1]$

2. 비용함수: 오차함수 E

- 오차 제곱의 합, 평균
  - **SSE: Sum of Squared Error**
  - **MSE: Mean of Squared Error**

$$SSE = \sum_{i=1}^{m} \frac{1}{2}\left(y^{(i)} - \hat{y}^{(i)}\right)^2$$

$$MSE = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(y^{(i)} - \hat{y}^{(i)}\right)^2$$

- **선형 회귀**
  - $y = WX$
- **로지스틱 회귀**
  - $y = \dfrac{1}{1 + e^{-WX}}$

3. 로지스틱 회귀: 비용함수J

$$J = \begin{cases} -\log(h(z)) & \text{if y} = 1. \\ -\log(1 - h(z)) & \text{if y} = 0. \end{cases}$$

$$h(z) = \frac{1}{1 + e^{-W \cdot X}}$$

- y: 클래스 레이블
- $h(z)$: 신경망의 출력

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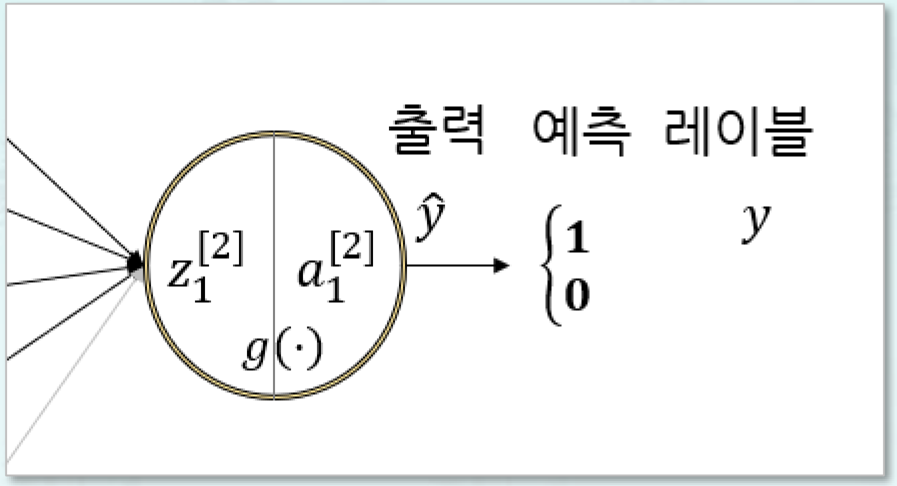
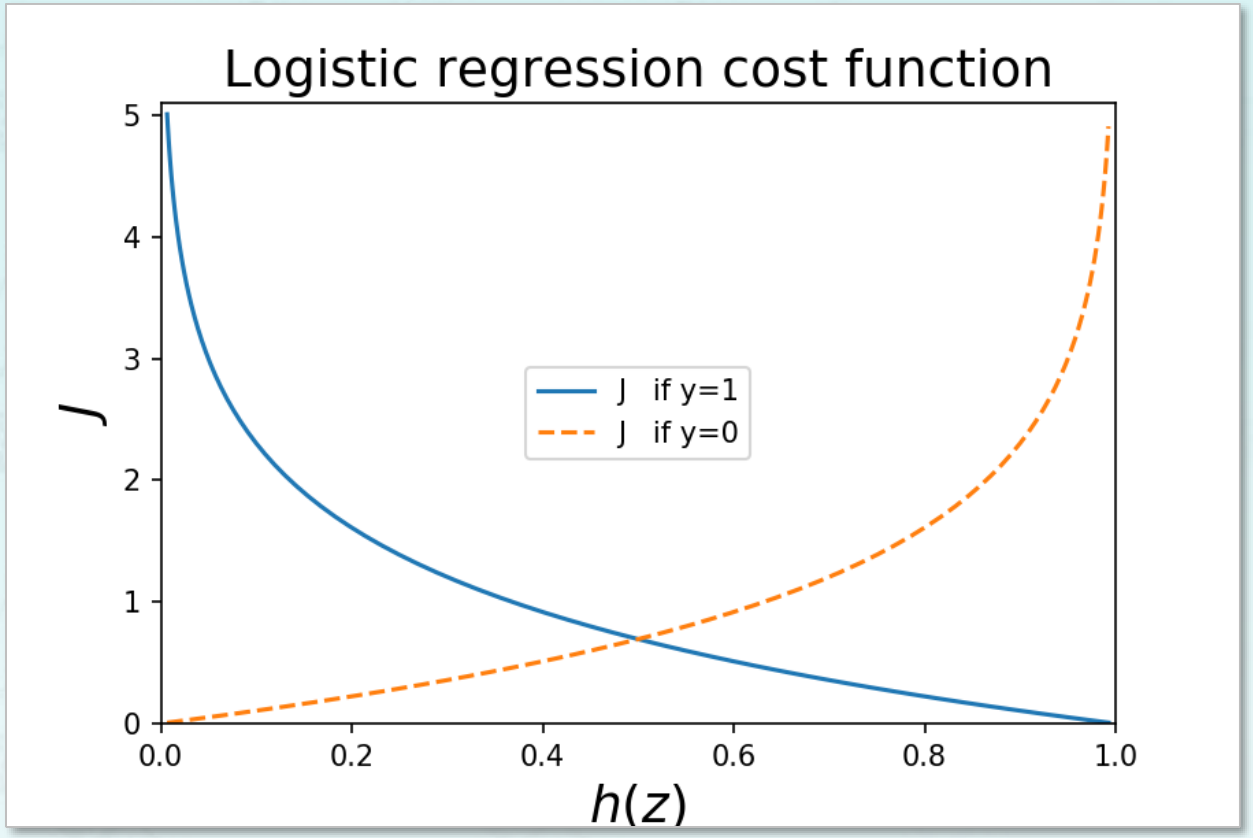
$$e^{(i)} = \begin{cases} -\log\left(a^{[2](i)}\right) & \text{if y} = 1. \\ -\log\left(1 - a^{[2](i)}\right) & \text{if y} = 0. \end{cases}$$



- $a^{[2](i)}$
  - 출력층 (i)번째 노드의 출력

3. 로지스틱 회귀: 비용함수J



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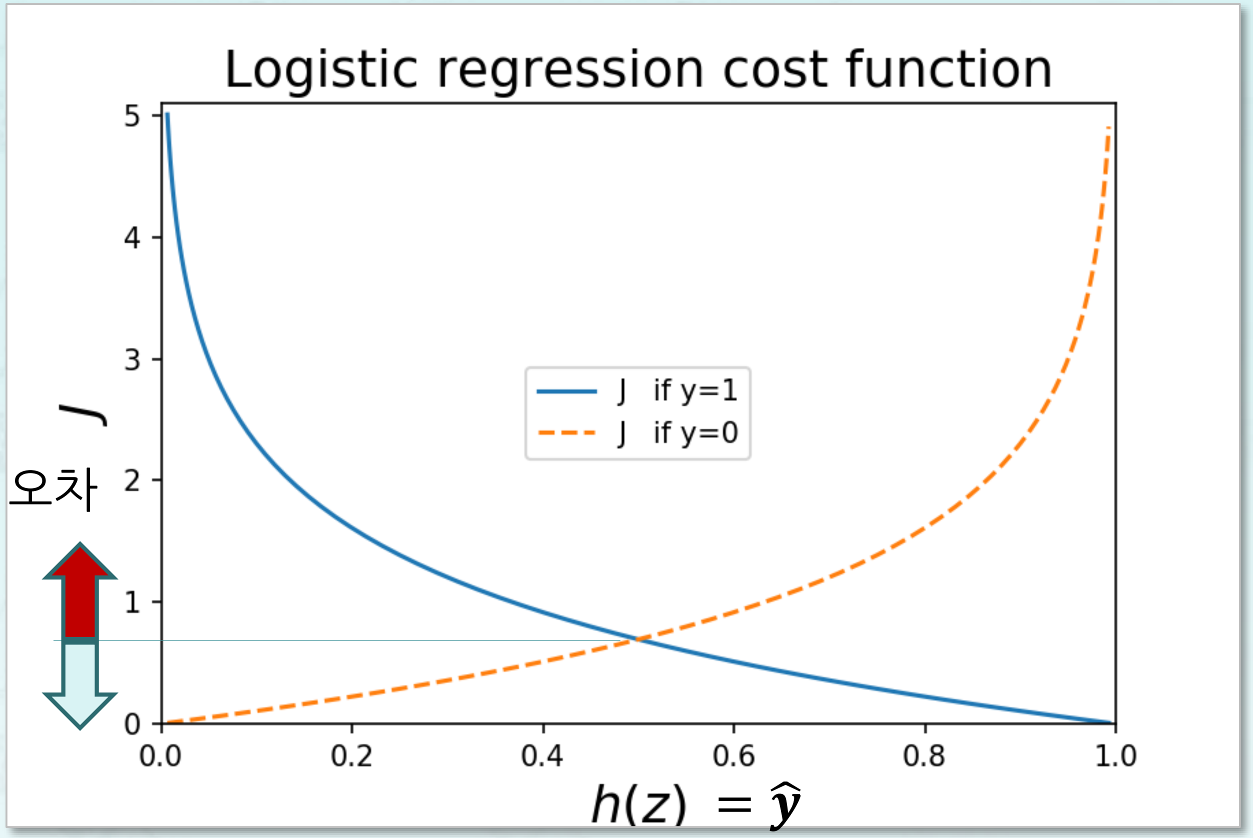
4. 비용함수J: 연습문제

$(1)\ y = 1,\ \hat{y} = 0.2$ **일 때 오차는?**

(a) 작다 **(b)** 크다

(a) 실선 **(b)** 점선



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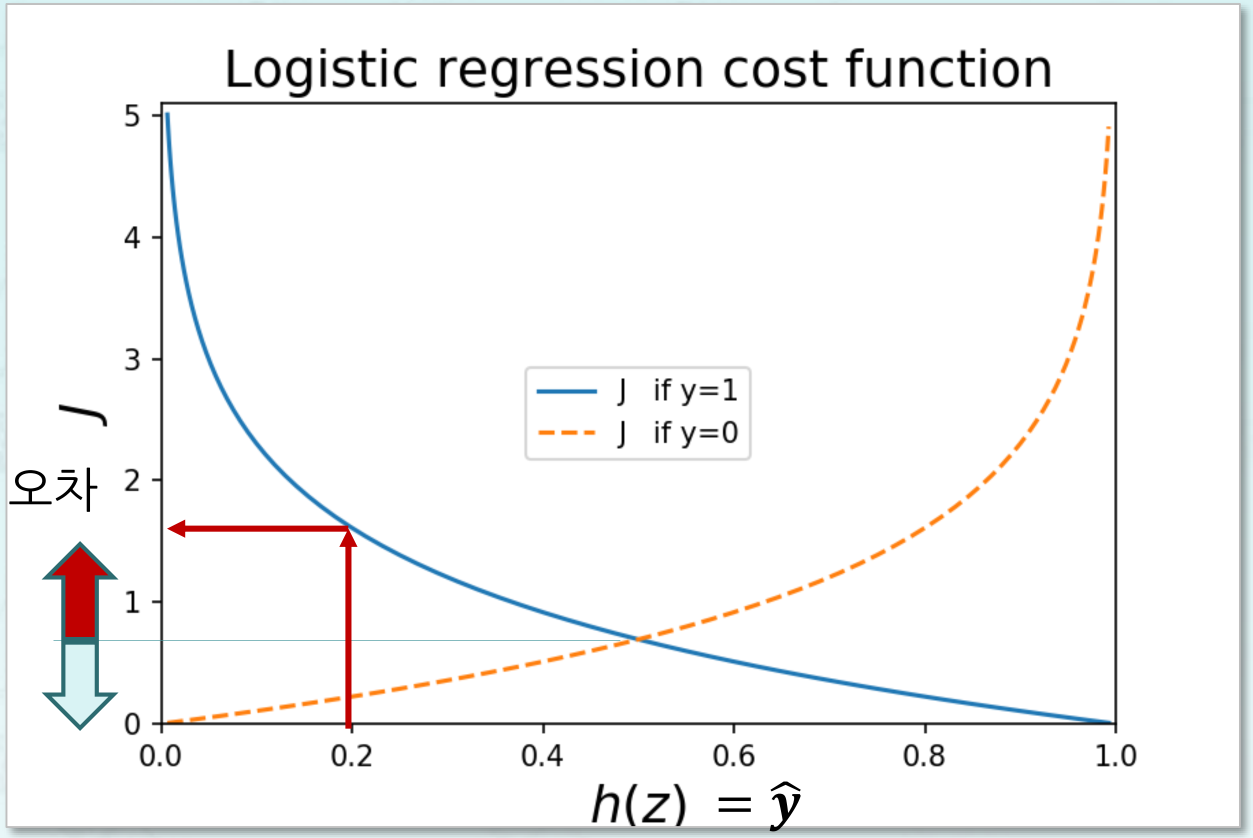

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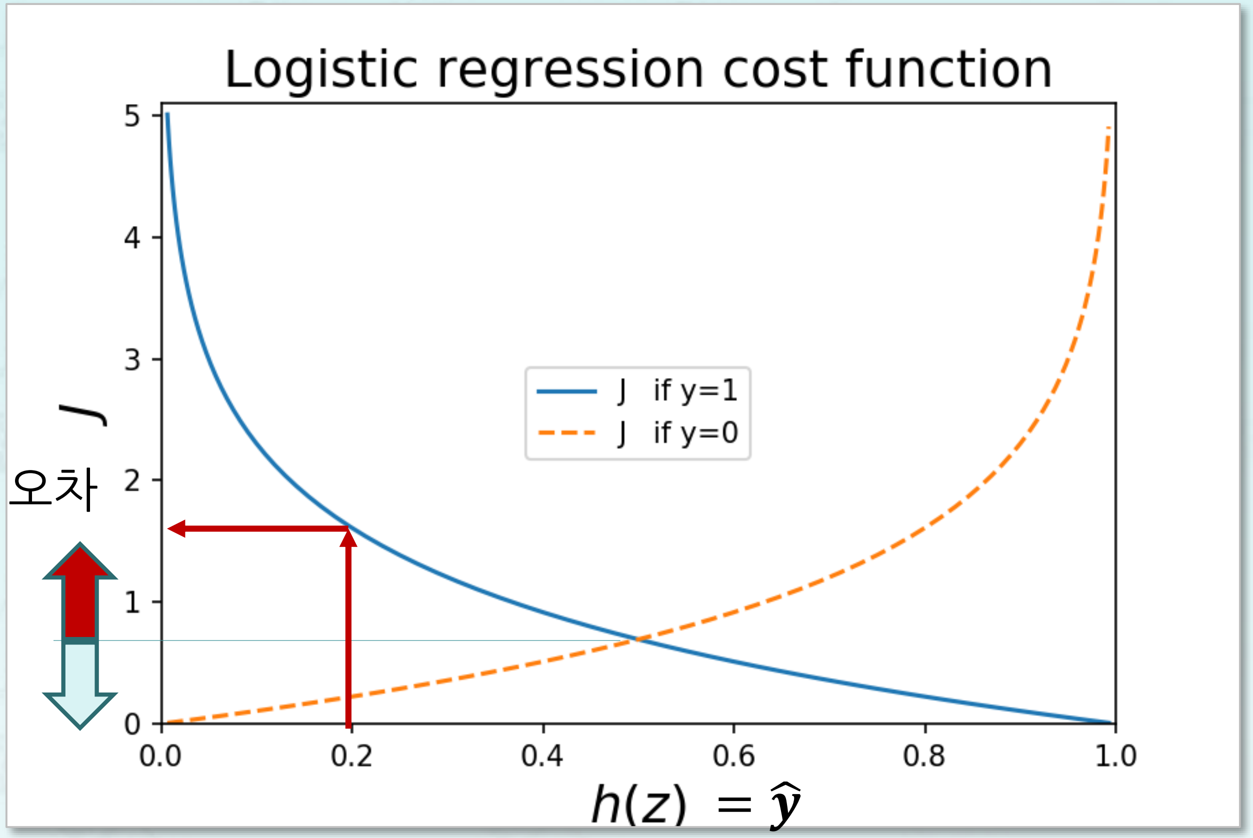
(2) $\boldsymbol{y = 1,\ \hat{y} = 0.8}$ **일 때 오차는?**

(a) 작다 (b) 크다

(a) 실선 (b) 점선



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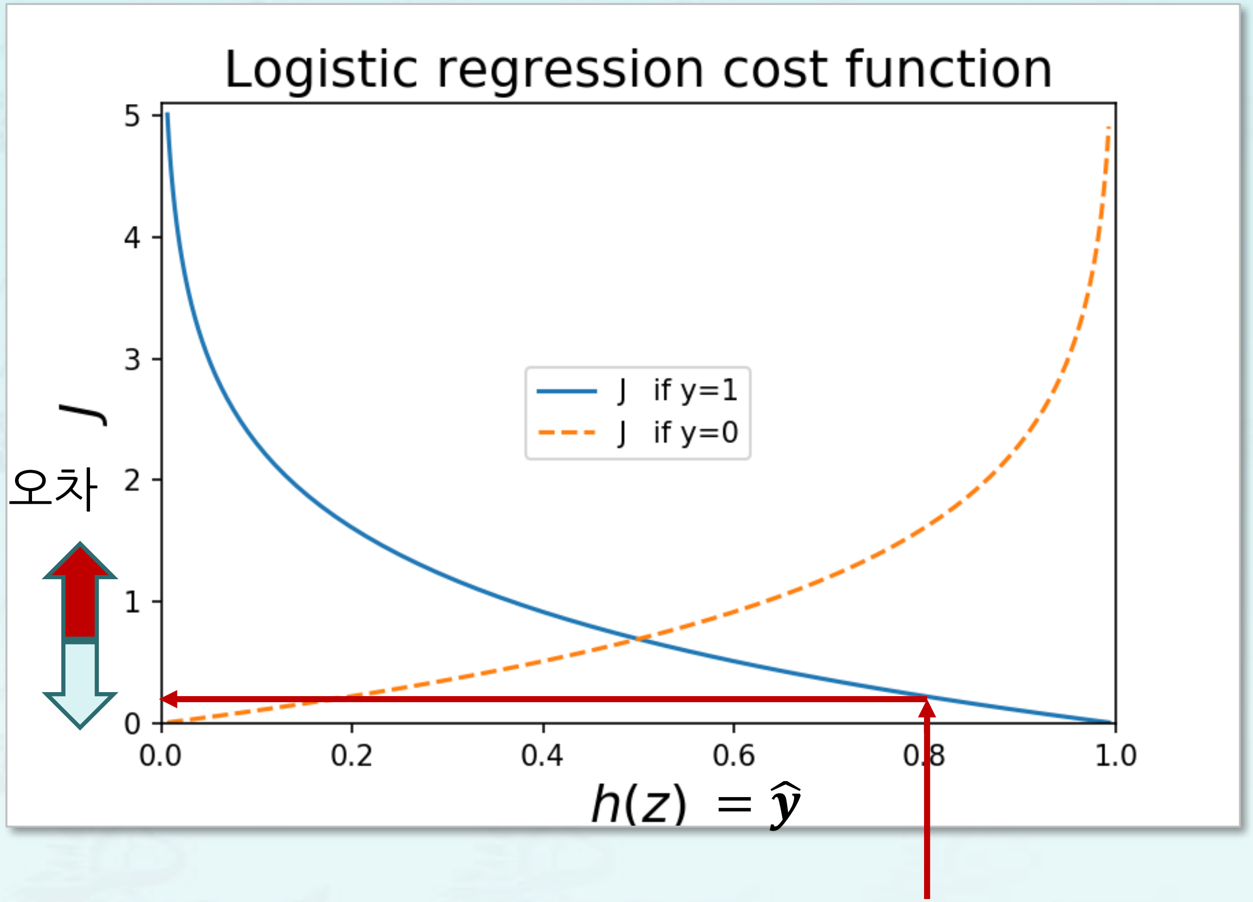

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(a) 작다 (b) 크다

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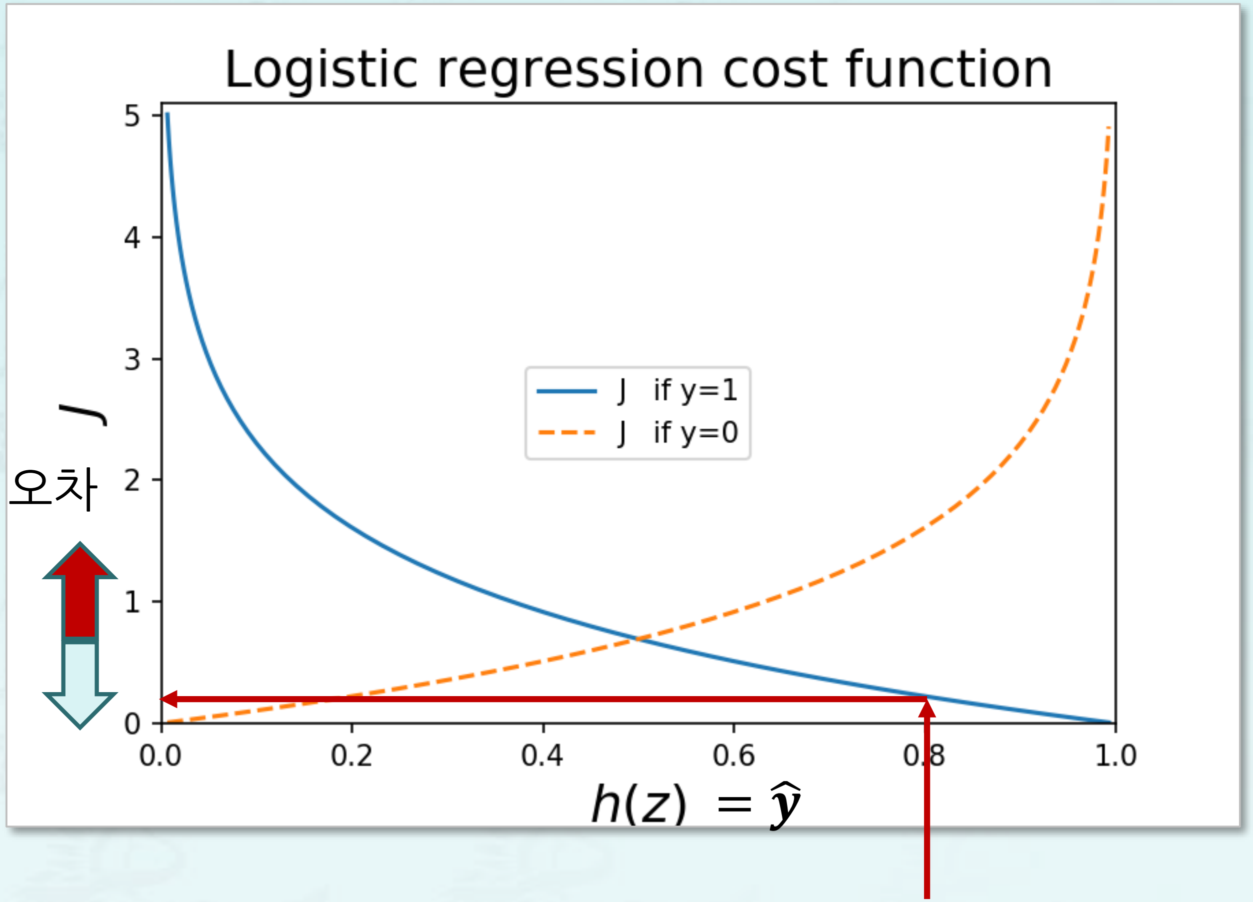
(3) $y = 0,\ \hat{y} = 0.9$ 일 때 오차는?

(a) 작다 (b) 크다

(a) 실선 (b) 점선



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(2) $y = 1,\ \hat{y} = 0.8$ 일 때 오차는?

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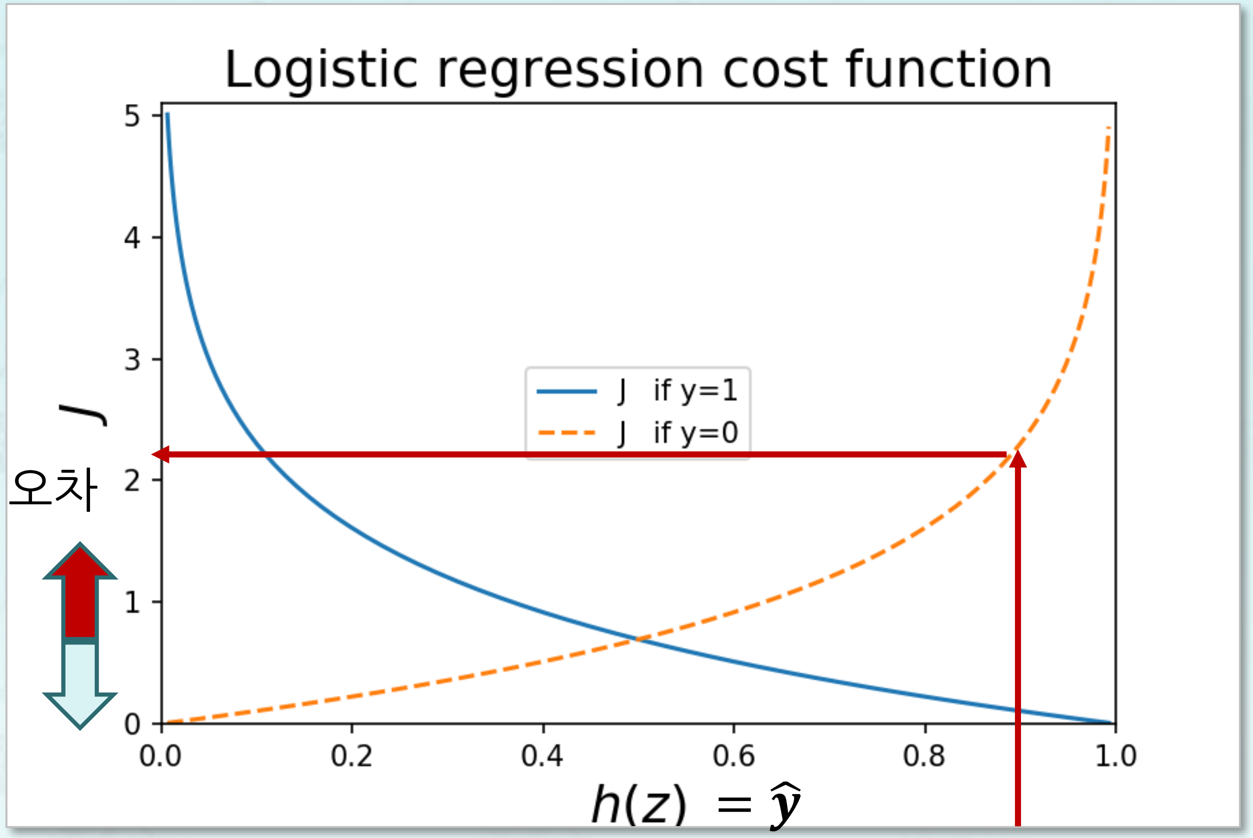
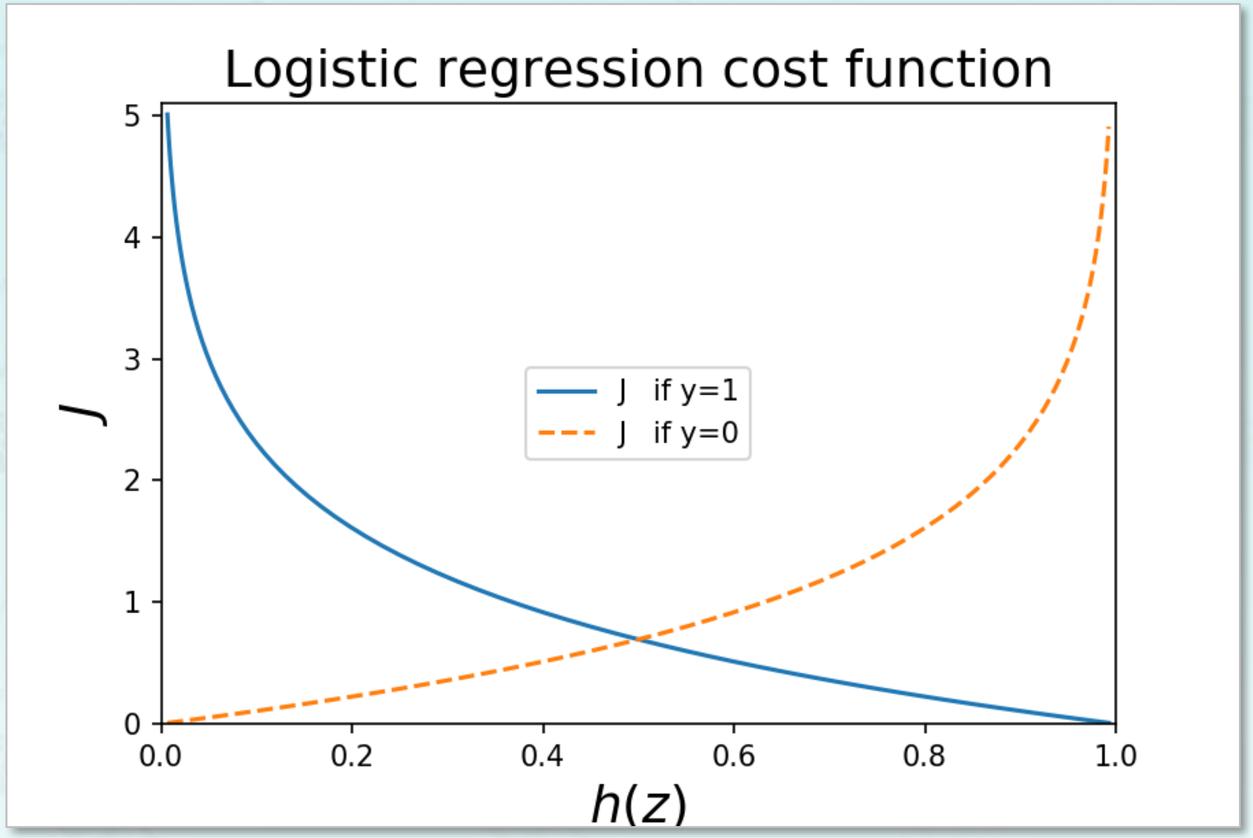


5. 비용함수 J: 하나로 결합한 식

$$J = \begin{cases} -\log(h(z)) & \text{if } y = 1. \\ -\log(1-h(z)) & \text{if } y = 0. \end{cases}$$

$$e^{(i)} = \begin{cases} -\log\left(a^{[2](i)}\right) & \text{if } y = 1. \\ -\log\left(1-a^{[2](i)}\right) & \text{if } y = 0. \end{cases}$$



- $a^{[2](i)}$
  - 출력층 (i)번째 노드의 출력

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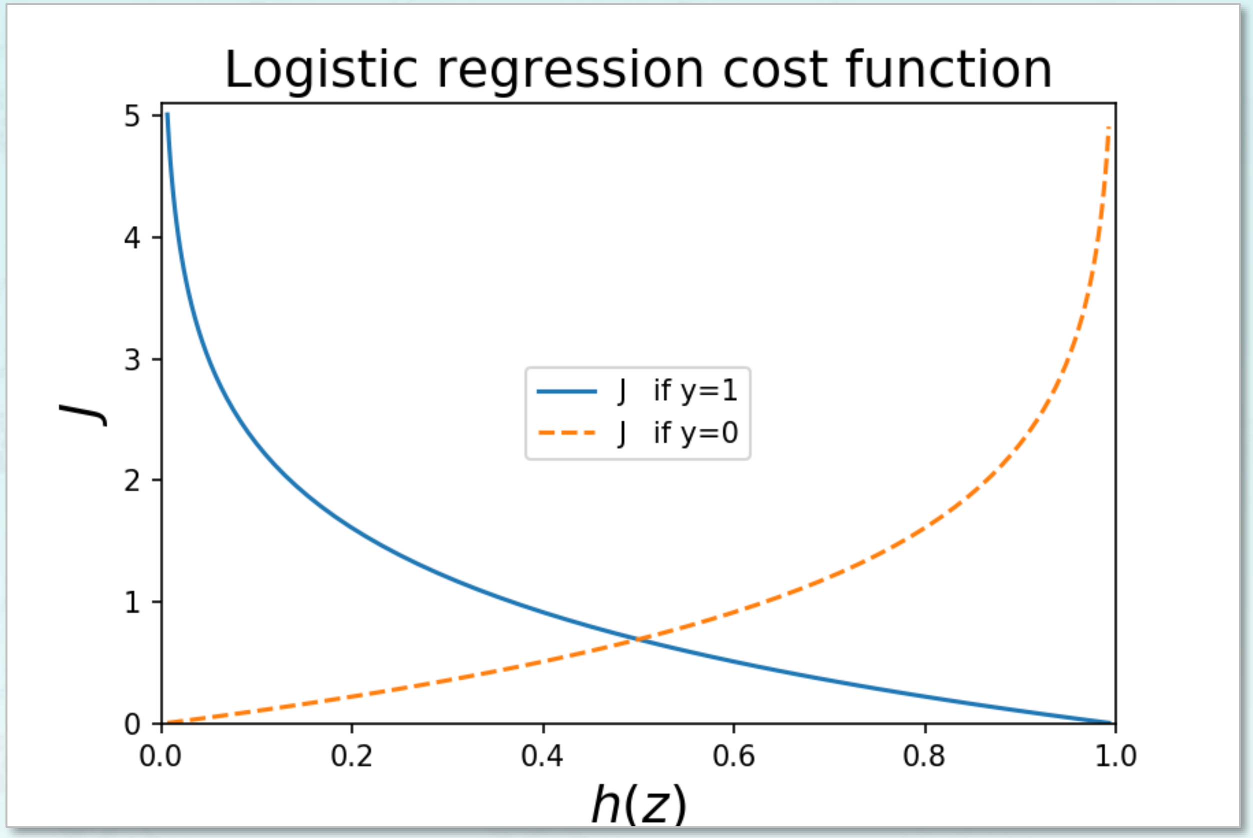

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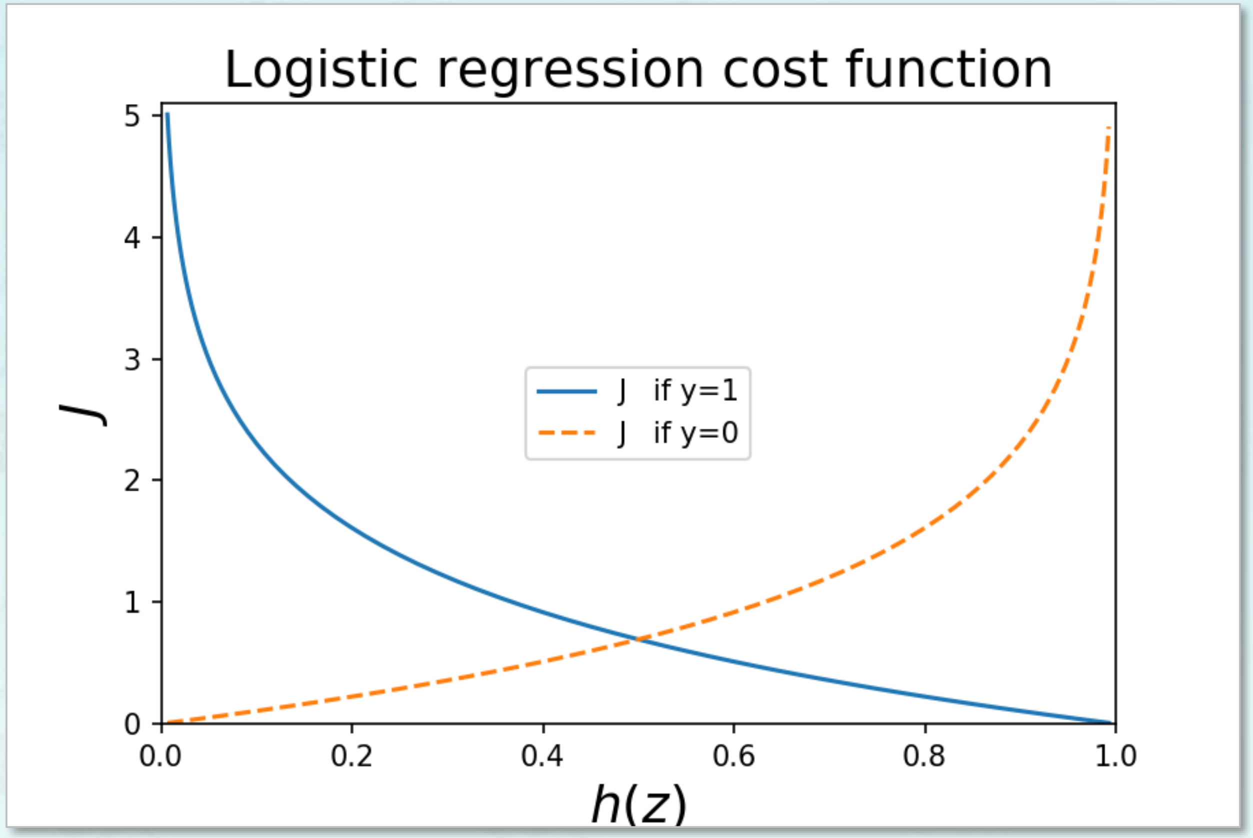

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5. 비용함수J: 로지스틱 회귀 비용함수

$$J(w) = -\frac{1}{m}\sum_{i=0}^{m}\left(y^{(i)}\log\left(a^{[2](i)}\right) + (1 - y^{(i)})\log\left(1 - a^{[2](i)}\right)\right)$$

$$\Uparrow$$

$$e^{(i)} = -y\log\left(a^{[2](i)}\right) - (1 - y)\log\left(1 - a^{[2](i)}\right)$$

6. 로지스틱 회귀 비용함수: 교차 엔트로피(Cross Entropy)

$$J(w) = -\frac{1}{m}\sum_{i=0}^{m}\left(\boxed{y^{(i)}}\log\left(\boxed{a^{[2](i)}}\right) + (1 - y^{(i)})\log\left(1 - a^{[2](i)}\right)\right)$$

$y^{(1)}$ $\hat{y}^{(1)}$

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$y^{(1)}$ $\quad$ $\hat{y}^{(1)}$ $\quad$ $y^{(2)}$ $\quad$ $\hat{y}^{(2)}$

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$$\boxed{y^{(1)}} \quad \boxed{\hat{y}^{(1)}} \quad \boxed{y^{(2)}} \quad \boxed{\hat{y}^{(2)}}$$

$$J = -\sum_{i} y^{(i)}\, log(\hat{y}^{(i)})$$

7. 로지스틱 회귀: 비용함수 미분

$$J(w) = -\frac{1}{m}\sum_{i=0}^{m}\left(y^{(i)}\log\left(a^{[2](i)}\right) + (1-y^{(i)})\log\left(1-a^{[2](i)}\right)\right)$$

y:클래스 레이블, 0 혹은 1
a:출력층의 출력, $a^{[2](i)}$
로지스틱 회귀 가설함수의 출력
z:wx

$$\frac{\partial J(w)}{\partial w_j} = \frac{\partial}{\partial w_j}\frac{-1}{m}\sum_{i=1}^{m}\left[y\log(a) + (1-y)\log(1-a)\right] \qquad \because a = h(z) = \sigma(z)$$

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$$= \frac{\partial}{\partial w_j}\frac{-1}{m}\sum_{i=1}^{m}\left[y\log(\sigma(z)) + (1-y)\log(1-\sigma(z))\right] \qquad \because \frac{d}{dx}log(x) = \frac{1}{x}$$

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$$= \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y}{\sigma(z)} - \frac{(1-y)}{1-\sigma(z)}\right]\frac{\partial\sigma(z)}{\partial w_j}$$

7. 로지스틱 회귀: 비용함수 미분

$$\frac{\partial J(w)}{\partial w_j} = \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y}{\sigma(z)} - \frac{(1-y)}{1-\sigma(z)}\right]\sigma'(z)x_j \qquad \because z = w_j x_j$$

$$= \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y(1-\sigma(z)) - \sigma(z)(1-y)}{\sigma(z)(1-\sigma(z))}\right]\sigma'(z)x_j$$

$$= \frac{-1}{m}\sum_{i=1}^{m}[y(1-\sigma(z)) - \sigma(z)(1-y)]\,x_j \qquad \because \sigma'(z) = \sigma(z)(1-\sigma(z))$$

$$= \frac{-1}{m}\sum_{i=1}^{m}[y - \sigma(z)]\,x_j$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} - \hat{y}^{(i)}\right]\,x_j^{(i)}$$

7. 로지스틱 회귀: 비용함수 미분

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$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} - \hat{y}^{(i)}\right] x_j^{(i)}$$

역전파 2: $W^{[2]}$의 오차함수 미분 – 4단계

$$\Delta w_{jk}^{[2]} = \frac{\partial E}{\partial w_{jk}} = \boxed{-(y_k - \hat{y}_k)} \cdot \boxed{g'(z_k)} \cdot \boxed{a_j}$$

$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}} = \boxed{-E^{[2]}} \cdot g'(Z^{[2]}) \cdot A^{[1]T}$$

7. 로지스틱 회귀: 비용함수 미분

$$\frac{\partial J(w)}{\partial w_j} = \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y}{\sigma(z)} - \frac{(1-y)}{1-\sigma(z)}\right]\sigma'(z)x_j \qquad \because z = w_j x_j$$

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$$= \frac{-1}{m}\sum_{i=1}^{m}\left[y(1-\sigma(z)) - \sigma(z)(1-y)\right]x_j \qquad \because \sigma'(z) = \sigma(z)(1-\sigma(z))$$

$$= \frac{-1}{m}\sum_{i=1}^{m}\left[y - \sigma(z)\right]x_j$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} - \hat{y}^{(i)}\right]x_j^{(i)}$$

$$\Delta W^{[2]} = \frac{\partial J}{\partial W^{[2]}} = -\frac{1}{m}E^{[2]}(1)A^{[1]T}$$

역전파 2: $W^{[2]}$의 오차함수 미분 – 4단계

$$\Delta w_{jk}^{[2]} = \frac{\partial E}{\partial w_{jk}} = \boxed{-(y_k - \hat{y}_k)} \cdot \boxed{g'(z_k)} \cdot \boxed{a_j}$$

$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}} = \boxed{-E^{[2]}} \cdot g'(Z^{[2]}) \cdot A^{[1]T}$$

7. 로지스틱 회귀: 비용함수 미분

$$
\begin{aligned}
\frac{\partial J(w)}{\partial w_j} &= \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y}{\sigma(z)} - \frac{(1-y)}{1-\sigma(z)}\right]\sigma'(z)x_j \qquad \because z = w_j x_j \\
&= \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y(1-\sigma(z)) - \sigma(z)(1-y)}{\sigma(z)(1-\sigma(z))}\right]\sigma'(z)x_j \\
&= \frac{-1}{m}\sum_{i=1}^{m}[y(1-\sigma(z)) - \sigma(z)(1-y)]\,x_j \qquad \because \sigma'(z) = \sigma(z)(1-\sigma(z)) \\
&= \frac{-1}{m}\sum_{i=1}^{m}[y-\sigma(z)]\,x_j \\
&= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} - \hat{y}^{(i)}\right]\,x_j^{(i)}
\end{aligned}
$$

$$\Delta W^{[2]} = \frac{\partial J}{\partial W^{[2]}} = -\frac{1}{m}E^{[2]}(1)A^{[1]T}$$

역전파 2: $W^{[2]}$의 오차함수 미분 – 4단계

$$\Delta w_{jk}^{[2]} = \frac{\partial E}{\partial w_{jk}} = \boxed{-(y_k - \hat{y}_k)} \cdot \boxed{g'(z_k)} \cdot \boxed{a_j}$$

$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}} = \boxed{-E^{[2]}} \cdot g'(Z^{[2]}) \cdot A^{[1]T}$$

7. 로지스틱 회귀: 비용함수 미분

$$\frac{\partial J(w)}{\partial w_j} = \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y}{\sigma(z)} - \frac{(1-y)}{1-\sigma(z)}\right]\sigma'(z)x_j \qquad \because z = w_j x_j$$

$$= \frac{-1}{m}\sum_{i=1}^{m}\left[\frac{y(1-\sigma(z)) - \sigma(z)(1-y)}{\sigma(z)(1-\sigma(z))}\right]\sigma'(z)x_j$$

$$= \frac{-1}{m}\sum_{i=1}^{m}[y(1-\sigma(z)) - \sigma(z)(1-y)]\,x_j \qquad \because \sigma'(z) = \sigma(z)(1-\sigma(z))$$

$$= \frac{-1}{m}\sum_{i=1}^{m}[y - \sigma(z)]\,x_j$$

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} - \hat{y}^{(i)}\right]\,x_j^{(i)}$$

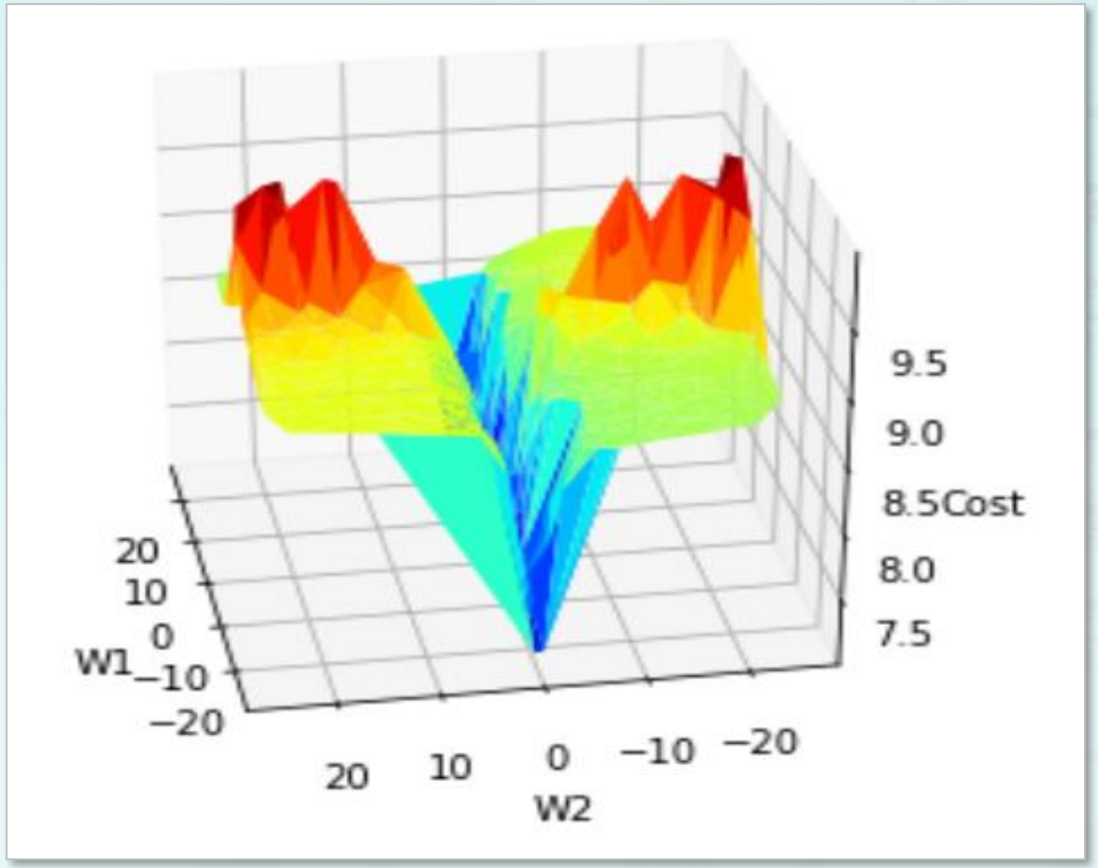
$$\Delta W^{[2]} = \frac{\partial J}{\partial W^{[2]}} = -\frac{1}{m}E^{[2]}(1)A^{[1]T}$$

역전파 2: $W^{[2]}$의 오차함수 미분 – 4단계

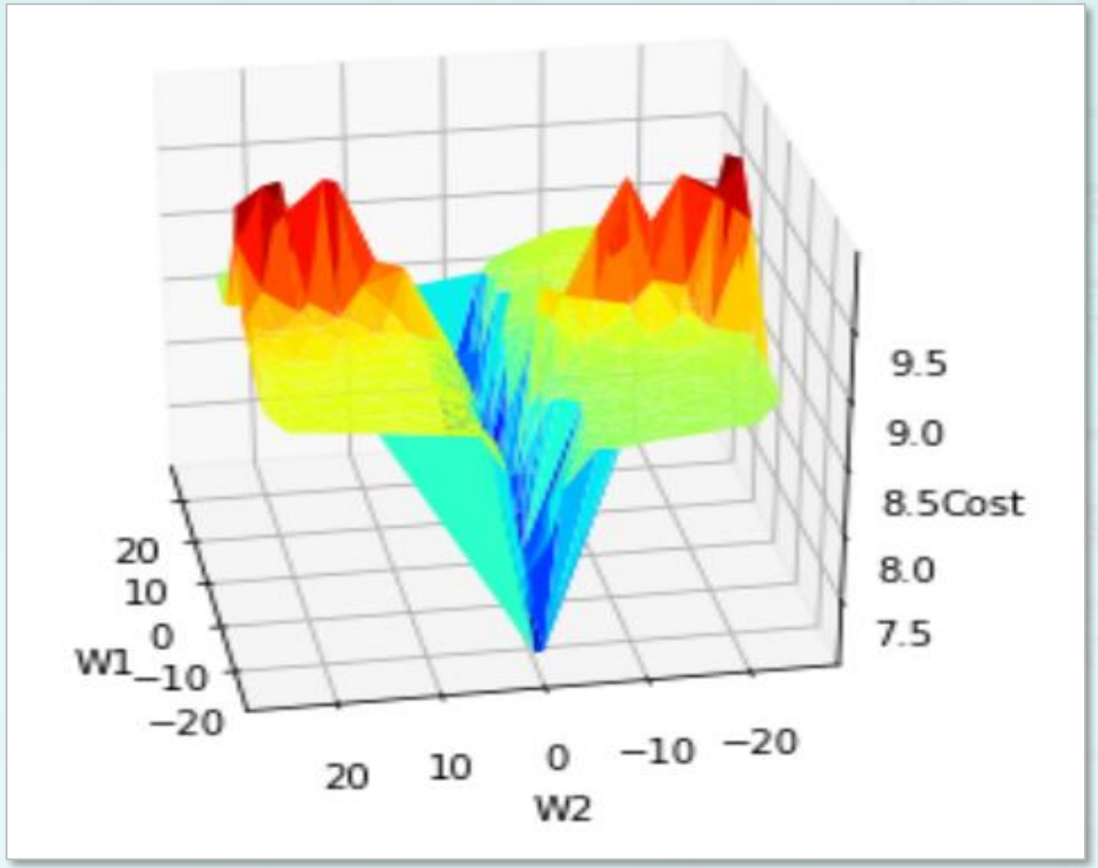
$$\Delta w_{jk}^{[2]} = \frac{\partial E}{\partial w_{jk}} = \boxed{-(y_k - \hat{y}_k)} \cdot \boxed{g'(z_k)} \cdot \boxed{a_j}$$

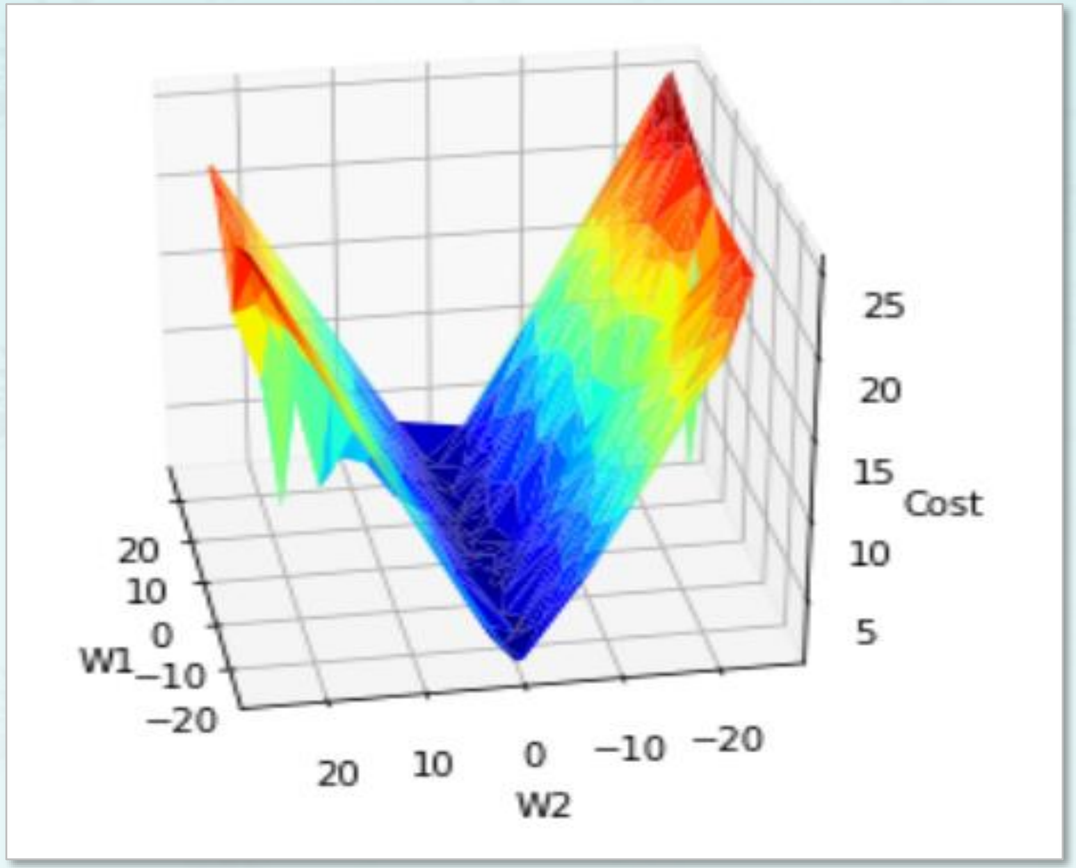
$$\Delta W^{[2]} = \frac{\partial E}{\partial W^{[2]}} = \boxed{-E^{[2]}} \cdot g'(Z^{[2]}) \cdot A^{[1]T}$$

7. 로지스틱 회귀: 제곱 합 오차(SSE)함수



7. 로지스틱 회귀: 교차엔트로피 (Cross Entropy)





로지스틱 회귀

- 학습 정리
  - 로지스틱 함수 이해하기
  - 로지스틱 회귀의 비용함수를 이해하기
  - 로지스틱 회귀 비용함수(교차 엔트로피)를 미분하기