MSDS 710/MSBD 710 Midterm Examination part 1

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Question 1 (10)

Prove that

$$1 + 4 + 7 + \ldots + (3n - 1) = \frac{(n+1)(3n+2)}{2}$$

Identify the sequence:

$$S = 1 + 4 + 7 + \ldots + (3n + 1)$$

After plugging in the common difference between terms, d is 3, and the first term a is 1 into the general terms formula, we get:

$$a_k = a + (k - 1)d$$

$$a_k = 3k - 2$$

Then we find the sum of the first $n + 1$ terms:

$$S = \sum_{k=0}^{n}(3k + 1)$$

And break this down as:

$$S = \sum_{k=0}^{n} 3k + \sum_{k=0}^{n} 1$$

Separately, we get:

$$\sum_{k=0}^{n} 3k = 3\sum_{k=0}^{n} k = 3 \cdot \frac{n(n+1)}{2}$$

and

$$\sum_{k=0}^{n} 1 = n + 1$$

We combine these and have:

$$S = 3 \cdot \frac{n(n+1)}{2} + (n + 1)$$

Factor out $(n+1)$:

$$S = (n+1)\left(\frac{3n}{2} + 1\right)$$

Simplify within the parentheses:

$$S = (n+1)\left(\frac{3n+2}{2}\right)$$

Then the sum is:

$$S = \frac{(n+1)(3n+2)}{2}$$

This proves that:

$$1 + 4 + 7 + \ldots + (3n+1) = \frac{(n+1)(3n+2)}{2}$$

Question 2 (10)

Assume

$$h(g_1, g_2) = 2g_1 - 3g_2$$

$$g_1(x, y) = 4x + 5y$$

$$g_2(x, y) = x - 2y$$

Apply the Chain Rule, calculate $\nabla_{x,y} h$.
First, identify partial derivatives of h:

$$\frac{\partial h}{\partial g_1} = 2 \quad \text{and} \quad \frac{\partial h}{\partial g_2} = -3$$

Now, calculate the partial derivatives of g_1 and g_2:

$$\frac{\partial g_1}{\partial x} = 4, \quad \frac{\partial g_1}{\partial y} = 5$$

$$\frac{\partial g_2}{\partial x} = 1, \quad \frac{\partial g_2}{\partial y} = -2$$

Applying the Chain Rule, find the partial derivatives of h in relation to x and y:
For $\frac{\partial h}{\partial x}$:

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial g_1} \cdot \frac{\partial g_1}{\partial x} + \frac{\partial h}{\partial g_2} \cdot \frac{\partial g_2}{\partial x}$$

$$\frac{\partial h}{\partial x} = 2 \cdot 4 + (-3) \cdot 1 = 8 - 3 = 5$$

For $\frac{\partial h}{\partial y}$:

$$\frac{\partial h}{\partial y} = \frac{\partial h}{\partial g_1} \cdot \frac{\partial g_1}{\partial y} + \frac{\partial h}{\partial g_2} \cdot \frac{\partial g_2}{\partial y}$$

$$\frac{\partial h}{\partial y} = 2 \cdot 5 + (-3) \cdot (-2) = 10 + 6 = 16$$

Therefore, the gradient of h in relation to x and y is:

$$\nabla_{x,y} h = (5, 16)$$

Question 3 (10)

You toss two coins two times and record the numbers of heads in each toss.

1. List the sample space. (Hint: The member of the sample space should look like **12** which means the first toss gets 1 head and the second toss gets 2 heads)

$$\{00, 01, 02, 10, 11, 12, 20, 21, 22\}$$

2. List the elements that make up the following events:

   (a) $A =$ The first toss gets **at least** one head.

   $$A = \{01, 02, 10, 11, 12, 20, 21, 22\}$$

   (b) $B =$ The second toss gets **at most** one head.

   $$B = \{00, 01, 10, 11, 20, 21\}$$

3. List the elements of the following events:

   (a) A^c

   $$A^c = \{00\}$$

   (b) $A \cap B$

   $$A \cap B = \{01, 10, 11, 20, 21\}$$