Numerical Methods

Ordinary Differential Equation

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Examples

Newton's equation of motion

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{F}{m}$$

Fourier's heat law:

$$q = -k\frac{\mathrm{d}T}{\mathrm{d}x}$$

Fick's law of diffusion:

$$J = -D\frac{\mathrm{d}c}{\mathrm{d}x}$$

Faraday's law:

$$\Delta V_L = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

General form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y) \tag{1}$$

A simple method:

$$y_{i+1} = y_i + \phi h$$

y_i: old value

y_{i+1}: new value

ϕ: slope estimate

Euler's method

Use the 1st derivative as direct estimate of slope at x_i:

$$\phi = f(x_i, y_i)$$

or

$$y_{i+1} = y_i + f(x_i, y_i)h$$

Example

Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2x^3 + 12x^2 - 20x + 8.5$$

from $x = 0$ to $x = 4$ with a step size $h = 0.5$. Initial condition: $y(x = 0) = 1$.

Comparing with general form of ODE (1), we have:

$$f(x, y) = -2x^3 + 12x^2 - 20x + 8.5$$

Note that in this case $f(x, y)$ does not depend of y.

Compare the obtained numerical solution with exact solution:

$$y(x) = -0.5 * x^4 + 4x^3 - 10x^2 + 8.5x + 1$$

Let's make define several functions that we will use later.

First the one-step application of Euler's method.

This code fragment implements:

$$y_{i+1} = y_i + f(x_i, y_i)h$$

```
# One-step application of Euler's method for ODE
def ode_euler_1step(dfunc, xi, yi, h):
    return yi + dfunc(xi,yi)*h

# initial cond
x0 = 0.0
y0 = 1.0
```

The left hand side of dy/dx=... (in general depends on x and y) In the present case it only depends on x.

```
def deriv(x, y):
    return -2*x**3 + 12*x**2 - 20*x + 8.5
```

and the exact solution:

```
def exact_sol(x):
    return -0.5*x**4 + 4*x**3 - 10*x**2 + 8.5*x + 1
```

Example code: `Code`

Frame with reduced font size

Test

- Nunc sed pede. Praesent vitae lectus.
- Nunc sed pede. Praesent vitae lectus.