NOTES_Interpolasi

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TF2202 Teknik Komputasi - Interpolasi
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```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
```

```
In [2]: from IPython.display import set_matplotlib_formats
        set_matplotlib_formats("svg")
        %matplotlib inline
```

```
In [3]: import matplotlib
        #matplotlib.style.use("dark_background")
        matplotlib.style.use("default")
```

1 Interpolasi dengan polinomial Lagrange

1.1 Teori polinomial Lagrange

Polinomial Lagrange didefinisikan sebagai:

$$P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$$

Polinomial ini adalah interpolant yang memiliki derajat n dan melewati $(n+1)$ titik data atau pasangan (x_i, y_i) dan $L_i(x)$ adalah fungsi polinomial dengan bentuk:

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

Catatan: pada buku Kiusalaas persamaan ini typo. Pada buku Chapra, syarat hasil kali juga memiliki typo.

1.2 Implementasi interpolasi Lagrange

```
In [4]: def lagrange_interp(x, y, xx):

            assert len(x) == len(y)
```

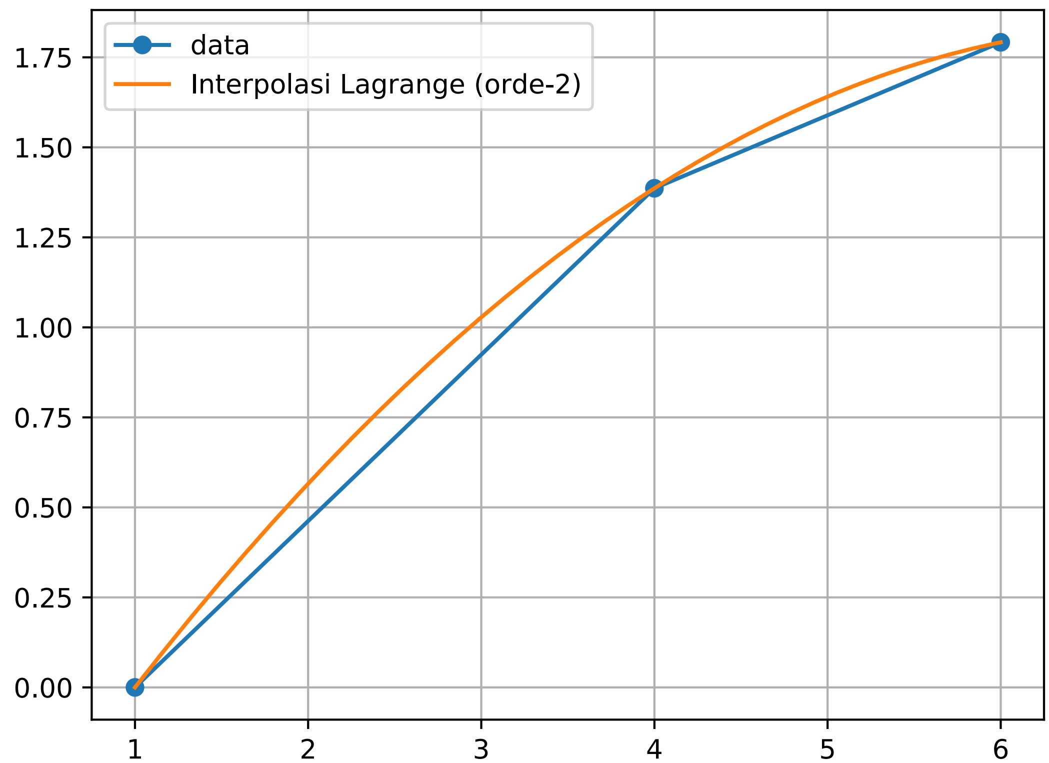
```
    # Jumlah data adalah N + 1 dan derajat polynomial adalah N
    # atau:
    # Jumlah data adalah N dan derajat polynomial adalah N - 1
    N = len(x) - 1

    yy = 0.0
    for i in range(N+1):
        # Evaluasi fungsi kardinal
        Li = 1.0 # inisialisasi ke ke 1.0
        for j in range(N+1):
            if i != j:
                Li = Li * (xx - x[j])/(x[i] - x[j])
        yy = yy + y[i]*Li
    return yy
```

1.2.1 Contoh

Sebagai contoh, diberikan data sebagai berikut:

x_i	y_i
1	0
4	1.386294
6	1.791760

```
In [5]: x = np.array([1.0, 4.0, 6.0])
        y = np.array([0, 1.386294, 1.791760])

In [6]: lagrange_interp(x, y, 2.0)

Out[6]: 0.5658439999999999

In [7]: NptsPlot = 1000
        xx = np.linspace(x[0], x[-1], NptsPlot)
        yy = np.zeros(NptsPlot)
        for i in range(NptsPlot):
            yy[i] = lagrange_interp(x, y, xx[i])

In [8]: plt.clf()
        plt.plot(x, y, marker="o", label="data")
        plt.plot(xx, yy, label="Interpolasi Lagrange (orde-2)")
        plt.grid()
        plt.legend()

Out[8]: <matplotlib.legend.Legend at 0x7fb4f20c5390>
```



1.3 Interpolasi dengan polinomial Newton

1.3.1 Teori polinomial Newton

Polinomial Newton memiliki bentuk sebagai berikut:

$$P_n = a_0 + (x - x_0)a_1 + (x - x_0)(x - x_1)a_2 + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})a_n$$

Koefisien a_n dapat dihitung dengan:

$$\begin{aligned} a_0 &= f(x_0) \\ a_1 &= f[x_1, x_0] \\ a_2 &= f[x_2, x_1, x_0] \\ &\vdots \\ a_n &= f[x_n, x_{n-1}, \ldots, x_1, x_0] \end{aligned}$$

di mana fungsi dengan tanda kurung siku merupakan beda terbagi hingga (*finite divided differences*).

Beda terbagi hingga pertama didefinisikan sebagai:

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

Beda terbagi hingga kedua didefinisikan sebagai:

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, f_k]}{x_i - x_k}$$

Secara umum, untuk beda terbagi hingga ke-n adalah:

$$f[x_n, x_{n-1}, \ldots, x_1, x_0] = \frac{f[x_n, x_{n-1}, \ldots, x_1] - f[x_{n-1}, x_{n-2}, \ldots, x_0]}{x_n - x_0}$$

1.4 Implementasi interpolasi Newton

```
In [9]: def create_newton_polynom(x, y):
            Ndata = len(x) # jumlah data
            coefs = np.copy(y)
            for k in range(1,Ndata):
                coefs[k:Ndata] = (coefs[k:Ndata] - coefs[k-1])/(x[k:Ndata] - x[k-1])
            return coefs

        def eval_newton_polynom(coefs, x, xo):
            N = len(x) - 1 # derajat polinom
            p = coefs[N]
            for k in range(1,N+1):
                p = coefs[N-k] + (xo - x[N-k])*p
            return p
```

1.4.1 Contoh penggunaan interpolasi Newton

```
In [10]: x = np.array([1.0, 4.0, 6.0])
         y = np.array([0, 1.386294, 1.791760])

In [11]: coefs = create_newton_polynom(x, y)
         coefs

Out[11]: array([ 0.       ,  0.462098, -0.051873])

In [12]: eval_newton_polynom(coefs, x, 2.0)

Out[12]: 0.5658439999999999
```

1.4.2 Aplikasi interpolasi Newton pada fungsi cos

```
In [13]: def func_01(x):
             return np.cos(2*x)

In [14]: N = 5
         A = 0.0
         B = 2*np.pi
         x_sample = np.linspace(A, B, N)
         y_sample = func_01(x_sample)
```

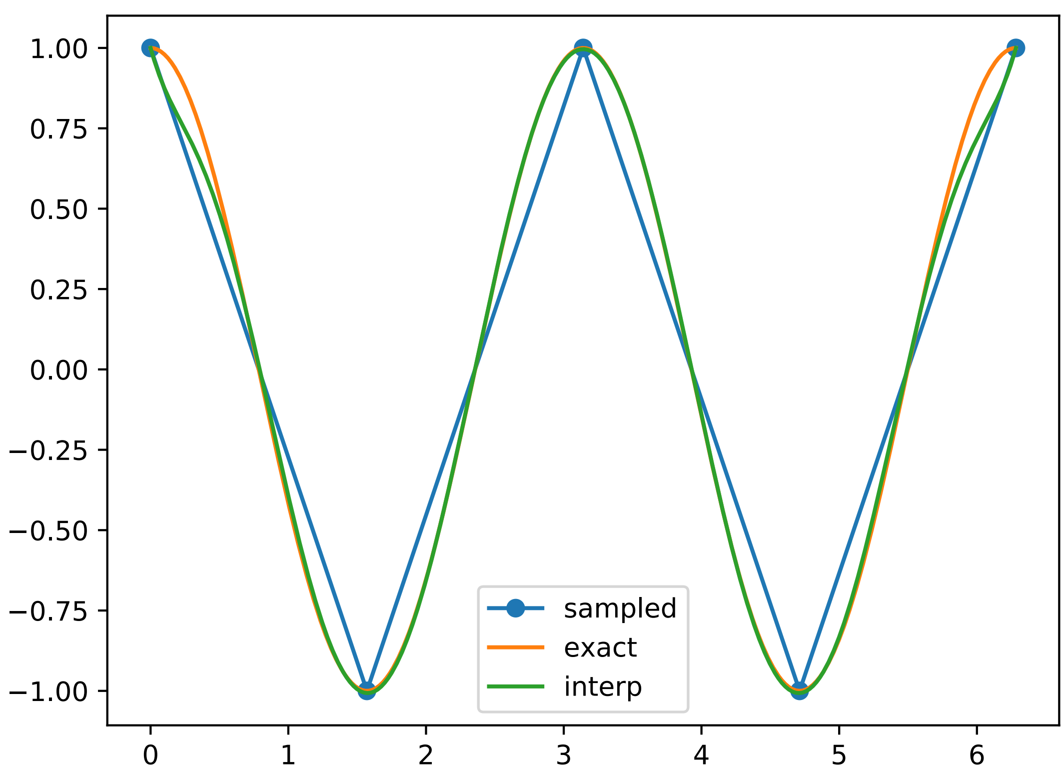
```
NptsPlot = 500
x_dense = np.linspace(A,B,NptsPlot)
y_dense = func_01(x_dense)

Ninterp = 10
x_interp = np.linspace(A,B,Ninterp)
y_interp = func_01(x_interp)
coefs = create_newton_polynom(x_interp, y_interp)

x_interp_plt = np.linspace(A,B,NptsPlot)
y_interp_plt = np.zeros(NptsPlot)
for i in range(NptsPlot):
    y_interp_plt[i] = eval_newton_polynom(coefs, x_interp, x_interp_plt[i])
```

In [15]:
```
x_sample, y_sample
```

Out[15]:
```
(array([0.        , 1.57079633, 3.14159265, 4.71238898, 6.28318531]),
 array([ 1., -1.,  1., -1.,  1.]))
```

In [16]:
```
plt.clf()
plt.plot(x_sample, y_sample, marker="o", label="sampled")
plt.plot(x_dense, y_dense, label="exact")
plt.plot(x_interp_plt, y_interp_plt, label="interp")
plt.legend()
```

Out[16]:
```
<matplotlib.legend.Legend at 0x7fb4f1e8c2e8>
```



2 Metode Neville

Metode ini pada dasarnya merupakan bentuk alternatif dari polinomial Newton.

2.1 Implementasi metode Neville

```
In [17]: def neville_interp(x, y_, xx):
             m = len(x)
             y = np.copy(y_)
             for k in range(1,m):
                 y[0:m-k] = ((xx- x[k:m])*y[0:m-k] + (x[0:m-k] - xx)*y[1:m-k+1])/(x[0:m-k] - x
             return y[0]
```

2.1.1 Contoh penggunaan metode Neville

```
In [18]: x = np.array([1.0, 4.0, 6.0])
         y = np.array([0, 1.386294, 1.791760])
         neville_interp(x, y, 2.0)

Out[18]: 0.5658439999999999
```

3 Interpolasi dengan fungsi rasional

Aproksimasi ini baik digunakan untuk fungsi yang memiliki pole, akan tetapi pada beberapa jenis data atau fungsi interpolasi ini tidak stabil.

3.0.1 Implementasi dalam Python

```
In [19]: def rational_interp(x, y, xx):
             m = len(x)
             r = np.copy(y)
             rOld = np.zeros(m)
             #SMALL = np.finfo("float64").eps
             SMALL = 1e-8
             for k in range(m-1):
                 for i in range(m-k-1):
                     if (abs(xx - x[i+k+1]) < SMALL):
                         return y[i+k+1]
                     else:
                         c1 = r[i+1] - r[i]
                         c2 = r[i+1] - rOld[i+1]
                         c3 = (xx - x[i])/(xx - x[i+k+1])
                         r[i] = r[i+1] + c1/(c3*(1.0 - c1/c2) - 1.0)
                         rOld[i+1] = r[i+1]
             return r[0]
```

3.0.2 Contoh penggunaan interpolasi fungsi rasional

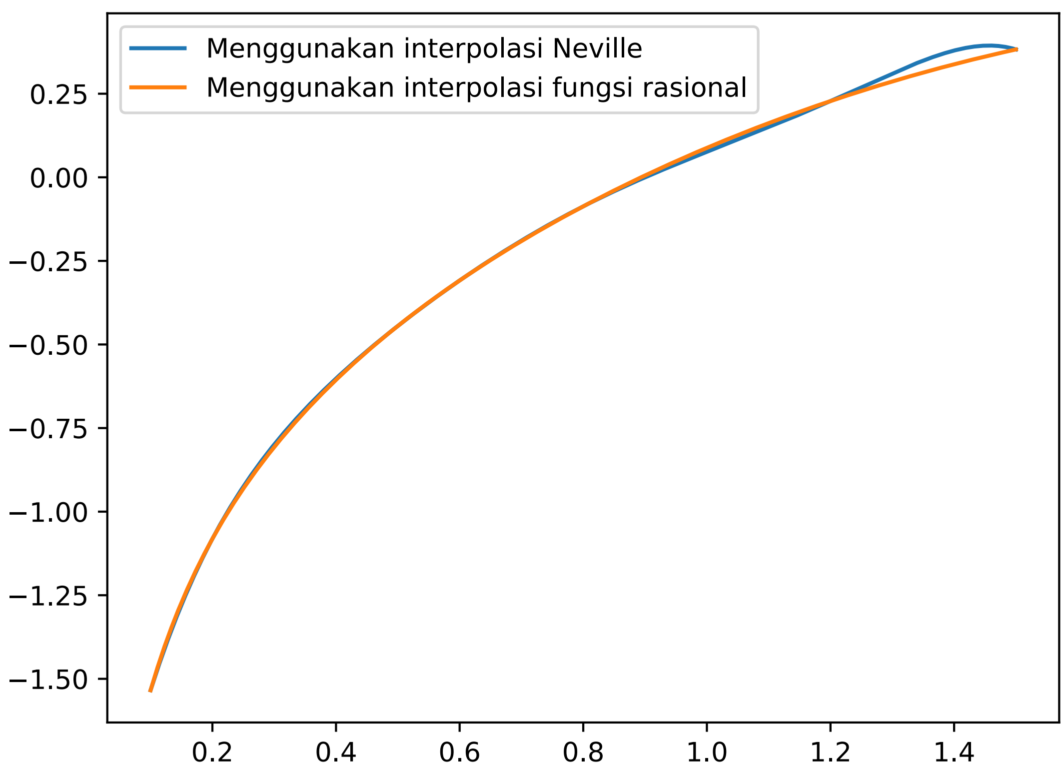
In [20]:
```
x = np.array([0.0, 0.6, 0.8, 0.95])
y = np.array([0.0, 1.3764, 3.0777, 12.7062])
```

In [21]:
```
rational_interp(x, y, 0.5)
```

Out[21]:
```
1.0131205116558464
```

In [22]:
```
x = np.array([0.1, 0.2, 0.5, 0.6, 0.8, 1.2, 1.5])
y = np.array([-1.5342, -1.0811, -0.4445, -0.3085, -0.0868, 0.2281, 0.3824])
```

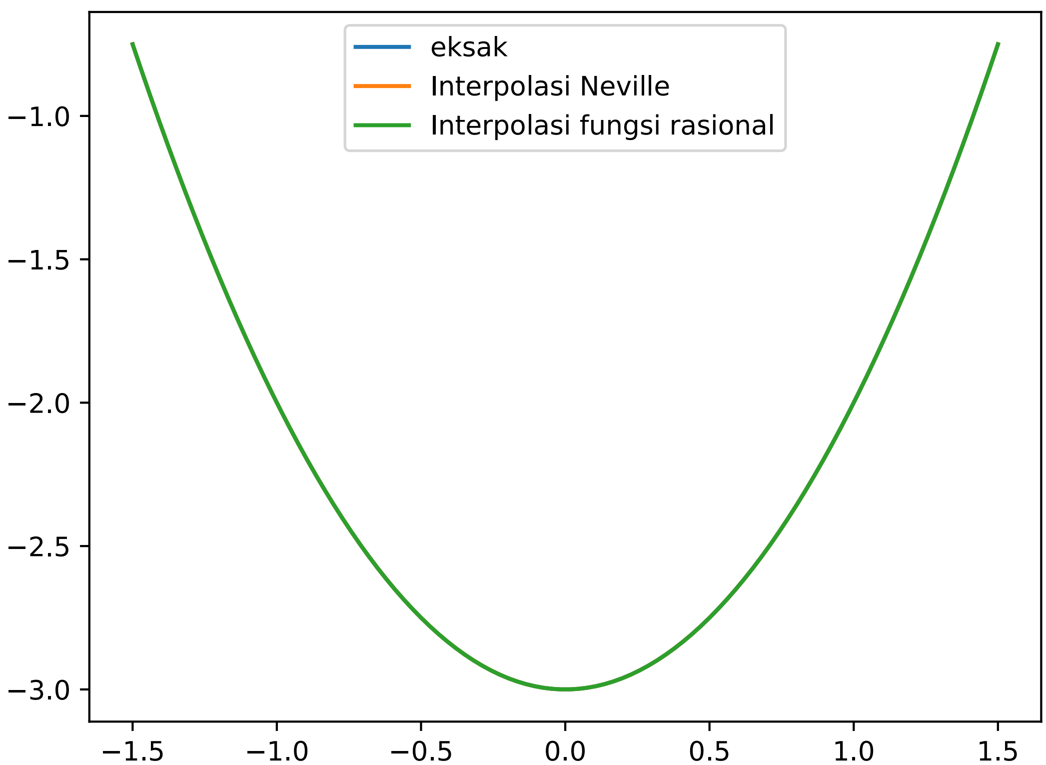
In [29]:
```
NptsPlot = 500
A = 0.1
B = 1.5
x_plot = np.linspace(0.1, 1.5, NptsPlot)
y_neville = np.zeros(NptsPlot)
y_rational = np.zeros(NptsPlot)
for i in range(NptsPlot):
    y_neville[i] = neville_interp(x, y, x_plot[i])
    y_rational[i] = rational_interp(x, y, x_plot[i])
plt.clf()
plt.plot(x_plot, y_neville, label="Menggunakan interpolasi Neville")
plt.plot(x_plot, y_rational, label="Menggunakan interpolasi fungsi rasional")
plt.legend();
```



3.0.3 Tes fungsi x^2

```
In [24]: def func_02(x):
             return x**2 - 3.0
```

```
In [30]: A = -1.5
         B = 1.5

         Nsample = 5
         x_sample = np.linspace(A, B, Nsample)
         y_sample = func_02(x_sample)

         NptsPlot = 100
         x_plot = np.linspace(A, B, NptsPlot)
         y_exact = func_02(x_plot)
         y_neville = np.zeros(NptsPlot)
         y_rational = np.zeros(NptsPlot)
         for i in range(NptsPlot):
             y_neville[i] = neville_interp(x_sample, y_sample, x_plot[i])
             y_rational[i] = rational_interp(x_sample, y_sample, x_plot[i])
         plt.clf()
         plt.plot(x_plot, y_exact, label="eksak")
         plt.plot(x_plot, y_neville, label="Interpolasi Neville")
         plt.plot(x_plot, y_rational, label="Interpolasi fungsi rasional")
         plt.legend();
```

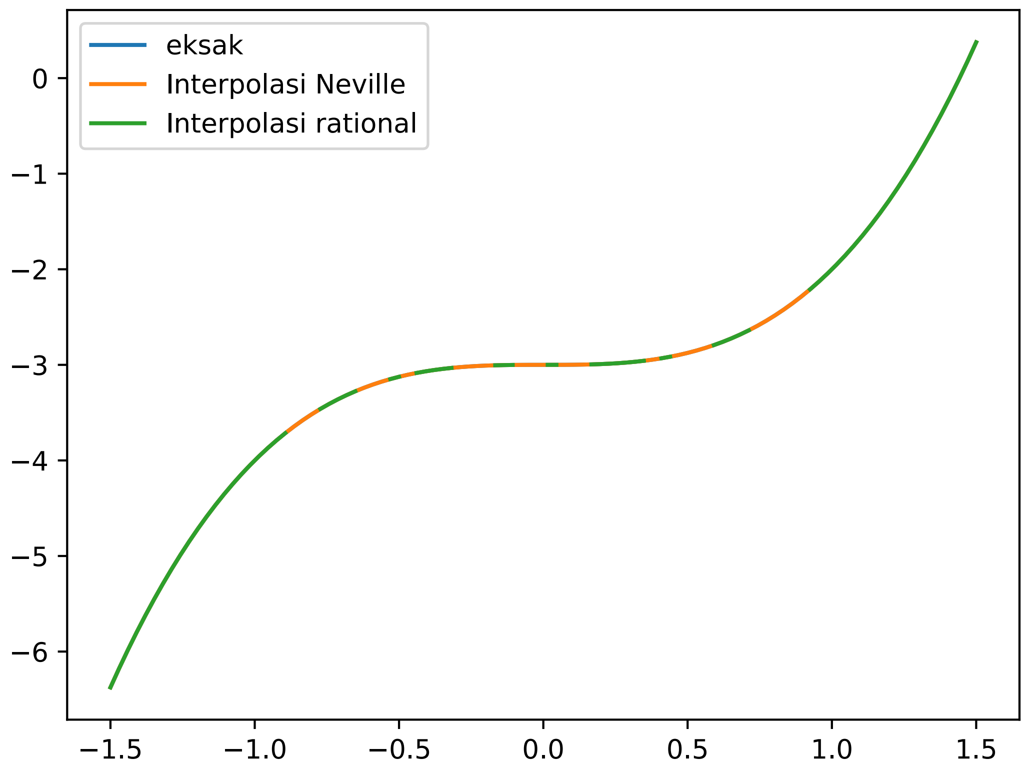


3.0.4 Tes fungsi x^3

```
In [26]: def func_03(x):
             return x**3 - 3.0
```

```
In [31]: A = -1.5
         B = 1.5

         Nsample = 10
         x_sample = np.linspace(A, B, Nsample)
         y_sample = func_03(x_sample)

         NptsPlot = 100
         x_plot = np.linspace(A, B, NptsPlot)
         y_exact = func_03(x_plot)
         y_neville = np.zeros(NptsPlot)
         y_rational = np.zeros(NptsPlot)
         for i in range(NptsPlot):
             y_neville[i] = neville_interp(x_sample, y_sample, x_plot[i])
             y_rational[i] = rational_interp(x_sample, y_sample, x_plot[i])
         plt.clf()
         plt.plot(x_plot, y_exact, label="eksak")
         plt.plot(x_plot, y_neville, label="Interpolasi Neville")
         plt.plot(x_plot, y_rational, label="Interpolasi rational")
         plt.legend();
```

```
/home/efefer/miniconda3/lib/python3.7/site-packages/ipykernel_launcher.py:15: RuntimeWarning: d
  from ipykernel import kernelapp as app
/home/efefer/miniconda3/lib/python3.7/site-packages/ipykernel_launcher.py:15: RuntimeWarning: i
  from ipykernel import kernelapp as app
```



4 Interpolasi dengan spline

TODO.

Merupakan metode yang dianggap paling robus dan banyak digunakan dalam aplikasi. Implementasinya cukup rumit untuk dilakukan.

5 Menggunakan pustaka Python

Module `scipy.interpolate` menyediakan banyak fungsi untuk melakukan interpolasi. Informasi lebih lengkap dapat dilihat pada dokumentasinya.

Beberapa fungsi yang sering dipakai adalah: - fungsi `interp1d` - `splrep` dan `splev` (menggunakan metode B-spline)

```
In [41]: import scipy.interpolate
```

Buat data dengan menggunakan fungsi yang sudah kita ketahui bentuknya.

```
In [39]: A = 0.0
         B = np.pi
         Nsample = 7
         x_sample = np.linspace(A,B,Nsample)
         y_sample = func_01(x_sample)
```

Buat interpolant, dengan menggunakan spline linear, kuadratic, dan kubik.

```
In [48]: f_slinear = scipy.interpolate.interp1d(x_sample, y_sample, kind="slinear")
         f_quadratic = scipy.interpolate.interp1d(x_sample, y_sample, kind="quadratic")
         f_cubic = scipy.interpolate.interp1d(x_sample, y_sample, kind="cubic")
```

```
In [49]: NptsPlot = 500
         x_plot = np.linspace(A,B,NptsPlot)
         y_slinear = f_slinear(x_plot)
         y_quadratic = f_quadratic(x_plot)
         y_cubic = f_cubic(x_plot)
```

```
In [54]: plt.clf()
         plt.plot(x_sample, y_sample, marker="o", label="sample")
         plt.plot(x_plot, y_slinear, label="slinear")
         plt.plot(x_plot, y_quadratic, label="squadratic")
         plt.plot(x_plot, y_cubic, label="cubic")
         plt.legend();
```