Introduction to Finite Element Method

TF40XX

Fadjar Fathurrahman

Program Studi Teknik Fisika
Institut Teknologi Bandung

1d Diffusion

Governing PDE

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + H$$

Initial conditions:

$$T(x, t = 0) = 0 \ \forall x \in [0, L_x]$$

Boundary conditions:

$$T(x = 0, t) = 0 \text{ and } T(x = L_x, t) = 0$$

Discretized equations

$$\mathbf{L}\mathbf{T}^{n+1} = \mathbf{R}\mathbf{T}^{n} + \mathbf{F}$$

$$\mathbf{L} = \frac{\mathbf{M}}{\Delta t} + \mathbf{K}$$

$$\mathbf{M} = \begin{bmatrix} \frac{\Delta x}{3} & \frac{\Delta x}{6} \\ \frac{\Delta x}{6} & \frac{\Delta x}{3} \end{bmatrix}$$

$$\mathbf{R} = \frac{\mathbf{M}}{\Delta t}$$

Stiffness matrix

$$\mathbf{K} = \kappa \begin{bmatrix} \frac{1}{\Delta x} & -\frac{1}{\Delta x} \\ -\frac{1}{\Delta x} & \frac{1}{\Delta x} \end{bmatrix}$$

Load vector

$$\mathbf{F} = H \begin{bmatrix} \frac{\Delta x}{2} \\ \frac{\Delta x}{2} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

Sistem global (menggunakan matriks dan vektor global):

$$\tilde{\mathbf{L}}\tilde{\mathbf{T}}^{n+1} = \tilde{\mathbf{R}}\ \tilde{\mathbf{T}}^{n} + \tilde{\mathbf{F}} = \tilde{\mathbf{b}}$$