TF4062: Diffusion Equation with Finite Difference Method

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1 Explicit method for 1d diffusion equation

1.1 Initial-boundary value problem for 1d diffusion

1d diffusion equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial t^2} + f(x,t) \tag{1}$$

Initial condition:

$$u(x,0) = I(x), \qquad x \in [0,L] \tag{2}$$

Boundary condition:

$$u(0,t) = 0, \qquad u(L,t) = 0, \qquad t > 0 \tag{3}$$

1.2 Forward Euler scheme

$$x_i = (i-1)\Delta x, i = 1, \cdot, N_x t_n = (n-1)\Delta t, n = 1, \cdot, N_t \tag{4}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \alpha \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} + f_i^n \tag{5}$$

Appendices

A Conjugate gradient for system of linear equations