SSY281 Model Predictive Control

# Assignment 6

## MPT and Persistent Feasibility

$\boxed{\text{Due March 6 at 23:59}}$

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

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## Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* to your project document area in PingPong.
  - Name the report as A6_XX.pdf, where XX is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab script as A6_XX.m, Pre_XX.m, Reach_XX.m, ShortestN_XX.m, and RHCXf_XX.m where XX is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **15 points** in total.

# 1 MPT

After installing the Multi-Parametric Toolbox 3 (MPT) in Matlab, run `mpt_demo_sets1` and `mpt_demo2` in MATLAB to study the two demos of the MPT package.

**Question 1** (**Points: 1**). *Use the commands in MPT to find the V- and the H-representation of the following polyhedron, plot them and explain the differences in the report.*

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -1 & -1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

**Question 2** (**Points: 1**). *Use the commands in MPT to compute the Minkowski sum and the Pontryagin difference of the following polytopes (i.e. $P+Q$ and $P-Q$) and plot them.*

$$P = \{x : A_1 x \leq b_1, \ x \in \mathbb{R}^2\}$$

$$Q = \{x : A_2 x \leq b_2, \ x \in \mathbb{R}^2\}$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

# 2 Persistent Feasibility

**Question 3** (**Points: 3**). *Consider the system* (1).

$$x^+ = Ax, \quad A = \begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix}. \tag{1}$$

*Show that the set $S$ is positively invariant for the system* (1).

$$S := \{x : \ A_{in} x \leq b_{in}, \ x \in \mathbb{R}^2\},$$

$$A_{in} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \\ 1 & 1 \\ 1 & -1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix}, \quad b_{in} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}.$$

**Question 4** (**Points: 3**). *Consider the system* (2)

$$x^+ = Ax + Bu, \quad A = \begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

*with* $-1 \leq u \leq 1$.

*In the Matlab script* `Reach_XX.m` *write a function calculating the set of states reachable from the set* $S$ *in the previous question in one step. Show the set* $S$ *and the reachable set you have calculated in the same plot.*

***Hint.*** *You can use* `alpha` *in your plot option to adjust transparency of the polyhedrons in the figure.*

***Note.*** *In this question you cannot use the* `reachableSet` *command.*

**Question 5** (**Points: 3**). *n the Matlab script* `Pre_XX.m` *write a function calculating the set "Pre" of initial states of the system* (2) *entering the set* $S$ *in one step. Show the set* $S$ *and the "Pre" you have calculated in the same plot.*

***Hint.*** *You can use* `projection` *command in MPT.*

**Question 6** (**Points: 4**). *Consider the following system.*

$$x^+ = Ax + Bu, \quad A = \begin{bmatrix} 0.9 & 0.4 \\ -0.4 & 0.9 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3}$$

*Design a RH controller with* $P = 0, Q = I_{2\times2}, R = 1$, *and* $\mathcal{X}_f = \mathbf{0}_{2\times1}$. *Let* $x(0) = \begin{bmatrix} 2 & 0 \end{bmatrix}^\top$ *be the initial condition and consider the following constraints.*

$$|x_i(k)| \leq 3, \; |u(k)| \leq 0.1 \;\; \forall k \in \{0, 1, 2, \ldots\}, \; \forall i \in \{1, 2\}$$

*1. Find the shortest prediction horizon $N$ such that the RH controller is feasible until convergence to the origin.*

*2. Set $N = 2$ and choose $\mathcal{X}_f$ as maximal control invariant set. Is the RH controller still feasible until convergence to the origin?*

   ***Hint.*** *You can use `invariantSet` command in MPT.*

*3. Plot the set of feasible initial states for the controllers designed at the previous two points. How do you explain the difference? What is the size of the optimization problems (number of optimization variables and constraints) underlying the two RH controllers?*

   ***Hint.*** *You can use `reachableSet` command in MPT or `Pre_XX.m` to calculate feasible initial states.*