SSY281 Model Predictive Control

Assignment 8

Finite Time Control

$\boxed{\text{Due March 20 at 23:59}}$

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* to your project document area in PingPong.
  - Name the report as A8_XX.pdf, where XX is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab script as A8_XX.m where XX is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **20 points** in total.

1 Bonus Question

Question 1 (**Points: 20**). *Consider the model of the DC-servo below*

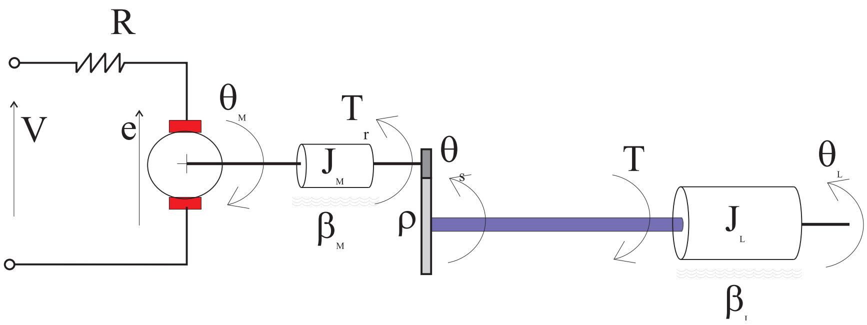

Figure 1: **DC-servomechanism**

with the following parameters

Table 1: Model parameters

Symbol	Value (MKS)	Meaning
L_S	1.0	shaft length
d_S	0.02	shaft diameter
J_S	negligible	shaft inertia
J_M	0.5	motor inertia
β_M	0.1	motor viscous friction coefficient
R	20	resistance of armature
k_T	10	motor constant
ρ	20	gear ratio
k_θ	1280.2	torsional rigidity
J_L	$50J_M$	nominal load inertia
β_L	25	load viscous friction coefficient

By setting $x_p = [\theta_L \ \dot{\theta}_L \ \theta_M \ \dot{\theta}_M]^\top$, *the model can be described by the fol-*

lowing state-space form

$$
\begin{aligned}
\dot{x}_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{R J_M} \end{bmatrix} V \\
T &= \begin{bmatrix} k_\theta & 0 & -\frac{k_\theta}{\rho} & 0 \end{bmatrix} x_p
\end{aligned}
$$

with the torsional torque T subject to the constraint

$$|T| \leq 157\ Nm.$$

Moreover, the input DC voltage V has to be constrained within the range

$$|V| \leq 200\ V$$

By using the commands in MPT, design an optimal controller to brake the rotating shafts, i.e., $\dot{\theta}_L = \dot{\theta}_M = 0$, in minimum time when starting from the initial state $x_p(0) = [0\ 2.5\ 0\ 75]^\top$.

NOTE. *You have to design your own minimum time controller by implementing the algorithm we studied. Only afterward, if you want, you can compare your solution with the MPT minimum time controller.*