SSY281 Model Predictive Control

Assignment 5

MPC Stability

$\boxed{\text{Due February 27 at 23:59}}$

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

January 2019

Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* to your project document area in PingPong.
  - Name the report as A5_XX.pdf, where XX is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab script as A5_XX.m and Pf_XX.m where XX is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **15 points** in total.

1 Stability

Question 1 (Points: 1). *Consider the discrete-time system*

$$x(k+1) = \begin{bmatrix} 0.5 & 1 \\ -0.1 & 0.2 \end{bmatrix} x(k) \tag{1}$$

where $x(k) \in \mathbb{R}^2$ and $k \geq 0$. Find a Lyapunov function $V(x(k)) = x(k)^\top S x(k)$.

Question 2 (Points: 1). *Consider the discrete-time system*

$$x(k+1) = Ax(k) + Bu(k), \tag{2}$$

with

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ k \geq 0.$$

Show that $V(u, x_0)$ in (3) with the parameters in (4) is a Lyapunov function for the system (2) controlled with the controller obtained by minimizing $V(u, x_0)$.

$$V(u, x_0) = \sum_{i=0}^{\infty} \big(x(i)^T Q x(i) + u(i)^T R u(i)\big), \tag{3}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ R = 1. \tag{4}$$

Question 3 (Points: 3). *Consider the system (2), the cost function (5), and the parameters (4), (6).*

$$V_N(x) = x(N)^T P_f x(N) + \sum_{i=0}^{N-1} \big(x(i)^T Q x(i) + u(i)^T R u(i)\big), \tag{5}$$

$$P_f = Q. \tag{6}$$

(a) *Find the shortest N such that the RH controller designed with the cost (5) and the system and weighting matrices in (4) stabilizes the system.*

(b) *What is the effect of Q on the stability when $N = 1$?*

(c) *Find a P_f for which the system is stable with $N = 1$. Fill in the function* `Pf.m` *which takes A, B, Q, and R as inputs and returns P_f for which the system is stable with $N = 1$. Concisely motivate your answer in the report.*

(d) *Consider the parameters* (4)*, find another* R *for which the system is stable with* $N = 1$ *and* P_f *as in* (6)*.*

Question 4 (**Points: 4**)**.** *Consider the following parameters for the system* (2)*.*

$$A = \begin{bmatrix} \bar{A}_{(n-1)\times n} \\ 0_{1\times n} \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \; P_f = diag(\begin{bmatrix} p_1 & p_2 & \dots & p_n \end{bmatrix}), \; N = 1$$

where $\bar{A}$ *is an arbitrary matrix,* P_f *is a positive definite diagonal matrix,* (A, B) *are controllable, and* A *is unstable. Show that a RH controller cannot stabilize the system with the cost function* (5) *regardless of* Q *and* R*.*

Question 5 (**Points: 4**)**.** *Consider system* (2)*, the cost function* (5)*, and parameters* (4)*.*

(a) *Analytically find the range of* R *for which the system is stable with* $N = 1$*.*

(b) *Analytically find the range of* R *for which the system is stable with* $N = 2$*.*

Question 6 (**Points: 2**)**.** *Show that the system* (2) *is stable when controlled with an infinite-time LQ controller. Note that* R *and* Q *are positive definite. Hint: use* $V(x) = x^T P x$ *as the Lyapunov function where matrix* P *is the solution to the Riccati equation.*