SSY281 - Model Predictive Control
Micro-Assignment MA10 - Feasibilty

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Question 1 *Given a linear dynamical system $x+ = f(x; u)$, along with state and input constraints $x \in X$; $u \in U$, a terminal set constraint $x \in X_f$ and the stage and terminal costs $l(x; u)$; $Vf(x)$, defining a cost function, what does the feasibility of the resulting RH controller depend on?*

The feasibility of the resulting RH controller still requires that the initial state x belongs to the feasible set if initial states X_N, i.e. $x_0 \in X_N$.

Question 2 *Mathematically define the feasibility set X_N for the RH controller at question a).*

$$\mathcal{X}_N = \{x_0 \in \mathbb{X} | \mathcal{U}_N(x_0) \neq \emptyset\} \tag{1}$$

Question 3 *Provide the mathematical definition of an invariant set and concisely explain its meaning.*

The set $\mathbb{S}$ is invariant if:

$$x(0) \in \mathcal{S} \quad \Rightarrow \quad x(k) \in \mathcal{S}, \quad \forall k \in \mathbb{N}_+ \tag{2}$$

In other words, if $x \in \mathbb{S}$, then it is possible to stay in $\mathbb{S}$ for all the next time steps evolving with feasible solutions.

Question 4 *What is the difference between an invariant and a control invariant set?*

The concept behind the two definitions are the same. However, invariant sets are defined for autonomous systems and control invariant sets for system with input. Both definitions imply that if $x \in \mathbb{C}$, then it is possible to stay in $\mathbb{C}$ for all the next time steps with feasible solutions.

Question 5 *If Xf is a control invariant set, which property does hold for the feasibility set X_N?*

If Xf is a control invariant set, then $\mathbb{X}_f \subset \mathcal{X}_N = K_N(\mathbb{X}_f) \subset \mathbb{X}$, i.e. the feasible set set exists and is non-empty.