SSY281 - Model Predictive Control
Assignment A07 - Feasibility, alternative formulations of MPC

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Question 1

The stability of the MPC controller for this linear system can be ensured for all $x \in \mathcal{X}_N$ by choosing the terminal cost $\mathbb{V}_f$ as the value function for the unconstrained LQ problem:

$$V_f(x) = x^\mathsf{T} P x \tag{1}$$

where P is the solution of the stationary discrete Riccati equation `dare`, which is:

```
P =
   16.0929   32.9899
   32.9899   93.9396
```

If follows from Theorem 10.10 from the lecture notes that for the linear quadratic; MPC with linear constraints; Q and R positive definite matrices; V_f chosen as the value function of the corresponding unconstrained infinite horizon LQ controller; terminal constraint set Xf chosen as control invariant; then the origin will be asymptotically stable with a region of attraction $\mathcal{X}_N$ for the controlled system $x+ = Ax + BN(x)$.

Moreover, the feasibility set $\mathcal{X}_N$ must be chosen as 3-step controllable set that is equal to the three-step backward reachability of the terminal state constraint:

$$\mathcal{X}_N = \mathcal{K}_3(\mathbb{X}_f) = Pre^3(\mathbb{X}_f) \tag{2}$$

leading to the following feasibility set $\mathcal{X}_N$:

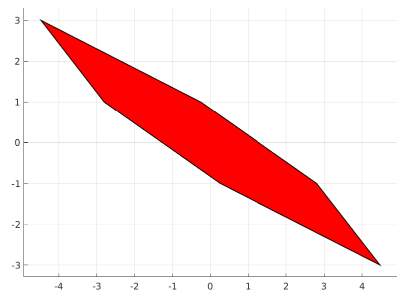
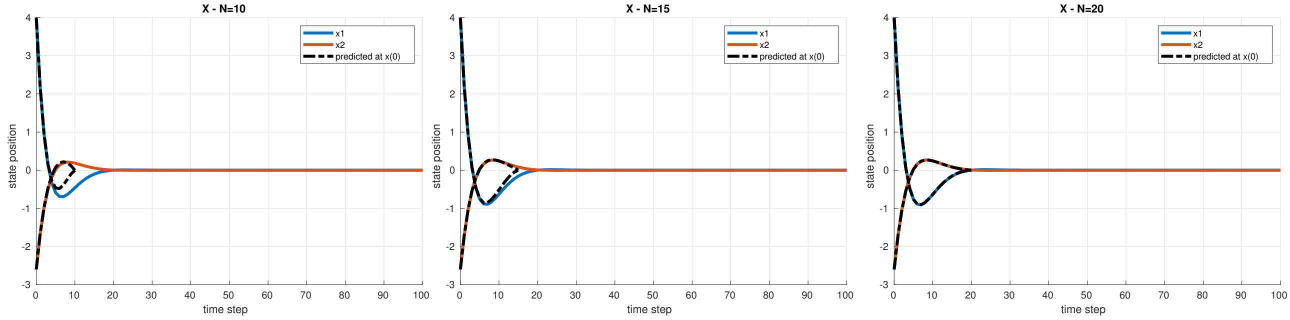


Figure 1: *Feasibility set $\mathcal{X}_N$ of the MPC controller*

Question 2

After designing the three MPC controllers with N=10,15,20, they were simulated and the results are given in the figure 2. Clearly, the smaller N, the worse is the predicted trajectory. For N=20, the controller is able to predict the trajectory with a very good precision, since in 20 time-steps the system was already settled to zero. The controllers with small horizon length fail to predict the right trajectory because future states beyond the predicted horizon changed the cost function and therefore changed the optimal control input to be applied in the next time steps, which also changed the trajectory.



(a) *N=10* ***(b)*** *N=15* ***(c)*** *N=20*

Figure 2: *Different controllers for the temperature regulation problem*

Therefore, one can conclude that a good starting point for choosing a horizon length N is the number of steps for which the system takes to go to zero from a given initial feasible position.

Question 3

First, the recursive feasibility is ensured if $\mathbb{X}_f$is subset of C_∞. In addition, since it is given that $\mathcal{X}_N$=C_∞, we need to be able to reach $\mathbb{X}_f$in one step. A trivial choice would then be:

$$\mathbb{X}_f = Reach(\mathcal{X}_N) \cap C_\infty \tag{3}$$

We then ensure recursive feasibility since $\mathbb{X}_f \subseteq C_\infty$ and we ensure that the given feasibility set $\mathcal{X}_N$=$C_\infty$is feasible since it can always reach $\mathbb{X}_f$ in N=1 steps.

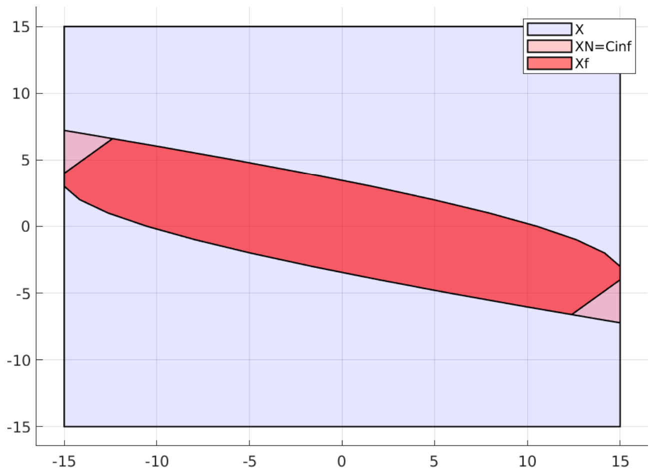


Figure 3: *Feasibility set $\mathcal{X}_N$ of the MPC controller*

Question 4

The MPC controllers were implemented with help of the MPT3 and its sub-toolbox YALMIP for Matlab, as can be verified in the Matlab code. The costs of each controller are available in table 1 and the plot of the steady-state solution is given in figure 4.

As can be verified, the first MPC controller presents a far better performance on peak power J^p, does not violate the comfort constraints $J^\epsilon = 0$ and has almost the same total energy consumption J^u in comparison to the other controllers. This happens because this controller is able to predict the disturbances in the system for entire horizon and also is modelled with the exact dynamics.

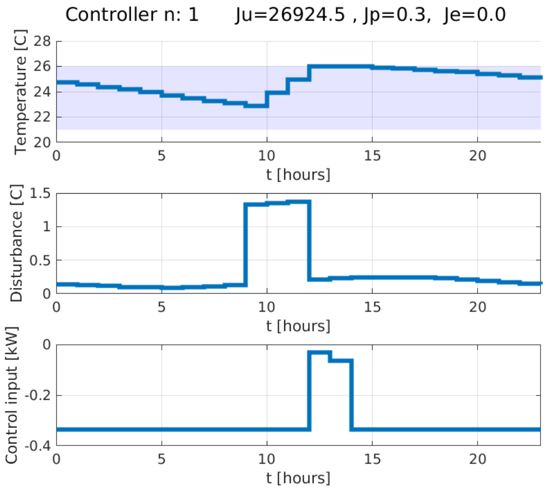
On the other hand, the MPC controller that has no information about T_{oa} and P_d performs really bad, since it was not expecting such behaviour of the system, given the inputs. The resulting cost function is not optimal and the controller performs really bad.

The proportional controller is able to keep the temperature most of the time in the comfort zone ($J^e = 63$) but needs a sampling time 60 smaller than the MPC controller and presents a very high peak power consumption.

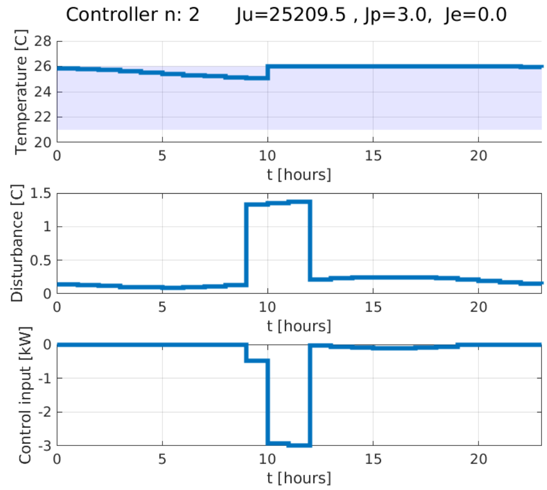
In sum, the MPC controllers are able to overcome the proportional controller in all the aspects. The MPC controller with one-step prediction of the disturbance performs really well and could be implemented in a real application, since we could try to predict the disturbance including it as a state variable and predicting it with an observer.

Table 1: *Performance indices for different controllers*

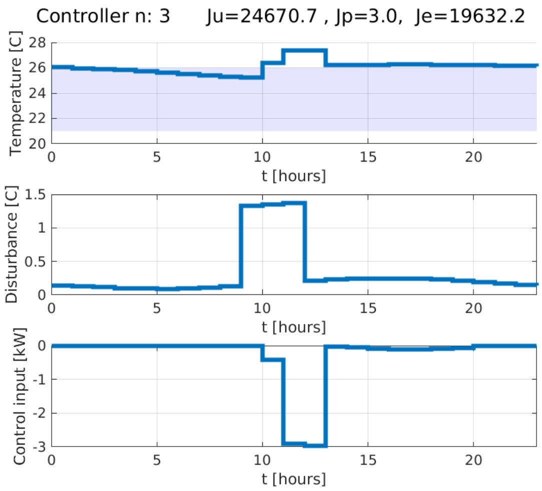
	J^u	J^p	J_ϵ
MPC 1	26924	0.3	0
MPC 2	25209	3.0	0
MPC 3	24670	3.0	19632
P controller	25201	7.5	63



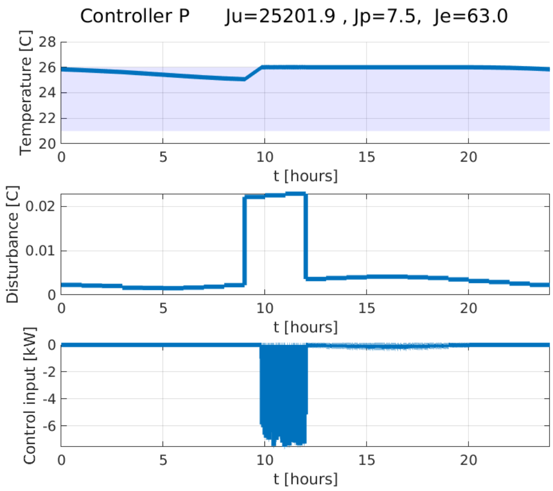
(a) *MPC 1 - all T_{oa} and P_d known*



(b) *MPC 2 - only current T_{oa} and P_d known*



(c) *MPC 3 - T_{oa} and P_d unknown*



(d) *Proportional controller*

Figure 4: *Different controllers for the temperature regulation problem. Here showing the steady state response of the system for a period of one day*