SSY281 - Model Predictive Control
Assignment A03 - MPC practice and Kalman Filter

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Question 1

For this case, y_s can be tracked to the desired set-point, since the number of inputs u and outputs y_{sp} to be tracked are the same ($p = m$). Therefore it is possible to solve exactly the following system of equations for x_s and u_s:

$$\begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ y_{sp} \end{bmatrix} \tag{1}$$

which resulted in:

```
xs1 =
    2.5000
   -1.5000
    1.2500
   -2.2500

us1 =
    2.5000
   -1.5000
```

It is possible now to feed equation (53) again with the values of x_s and u_s, which resulted in:

```
y_sp =
     1
    -1
```

Which proves that our system can be tracked to the desired set-point.

Question 2

Since there are more outputs to be tracked than manipulated inputs ($p > m$), there is no way to track all the outputs y_{sp} to the desired set-points. Therefore, one possible solution is to minimize the weighted norm of the difference between y_{sp} and the set-points:

$$\min_{x_s, u_s} \quad |Cx_s - y_{sp}|^2 \tag{2}$$

$$s.t. \quad [I - A \quad -B] \begin{bmatrix} x_s \\ u_s \end{bmatrix} = 0 \tag{3}$$

which resulted in:

```
xs2 =
    0.4000
         0
    0.2000
         0

us2 =
    0.4000
```

Question 3

Since there there are more inputs than controlled outputs ($p < m$), then y_s can be tracked to the desired set-point and there are still remaining flexibility to bring some inputs close to a desired set-point. Therefore, we can solve equation 1 and at the same time minimize the two norm of the input:

$$\min_{x_s, u_s} \quad |u_s|^2 \tag{4}$$

$$s.t. \quad \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ y_{sp} \end{bmatrix} \tag{5}$$

```
xs3 =
    0.5000
    0.5000
    0.2500
    0.7500

us3 =
    0.5000
    0.5000
```

Question 4

The augmented system is given by

$$A_e = \begin{bmatrix} A & Bd \\ 0 & I \end{bmatrix} \qquad B_e = \begin{bmatrix} B \\ 0 \end{bmatrix} \qquad C_e = \begin{bmatrix} C & C_d \end{bmatrix} \tag{6}$$

where the new state variables are $\xi = [x \; d]^T$. Moreover, the detectability can be checked by verifying two conditions:

$$rank\,(obsv(A, C)) = n \qquad \text{and} \qquad rank \begin{bmatrix} I - A & -Bd \\ C & C_d \end{bmatrix} = n + n_d \tag{7}$$

These two conditions are only valid for systems (1./a) and (3./c), which are therefore the detectable models.

Question 5

The Kalman filter (filter case) can be designed by solving the stationary discrete-time algebraic Riccati equations (DAREs). Since we want to predict the disturbances, the extended matrices presented in the last question should be used. In this way, the Kalman filter can be obtained using the following code:

```
Q = eye(n+nd);
R = eye(p);
[P,lambda,L] = dare(Ae',Ce',Q,R);

% Kalman Filter (FILTERING CASE)
Le = Ae\L'
```

Since the covariance matrices Q and R were not provided, identity matrices of appropriate dimensions have been chosen.

Question 6

Following equation (56) of the lecture notes, the steady state target can be calculated as follows, once the estimation of the disturbance is known:

$$\begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} I-A & -B \\ HC & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ y_{sp} \end{bmatrix} + \underbrace{\begin{bmatrix} I-A & -B \\ HC & 0 \end{bmatrix}^{-1} \begin{bmatrix} B_d \\ -HC_d \end{bmatrix}}_{M_{ss}} \hat{d}(k) \tag{8}$$

Question 7

Figures 1 and 2 show the system response for the detectable models A and C respectively. The controller is only able to remove the disturbance for the proposed model **C**.

As can be verified in the plots, the model A is not able to predict the states x and the disturbance d correctly. As a result the reference signal z_{sp} (`error zsp`) and $\tilde{x}$ (`estimation error x`) do not go to zero, as desired. This happens because the model A was not a good proposal for the disturbance, which is actually a load disturbance and it was modelled as measurement disturbance.

On the other hand, the model C is able to predict the applied disturbance, as can be seen in the plot (`d hat`). Therefore, the states x and d are predicted correctly and the (`error zsp`) goes to zero as desired.

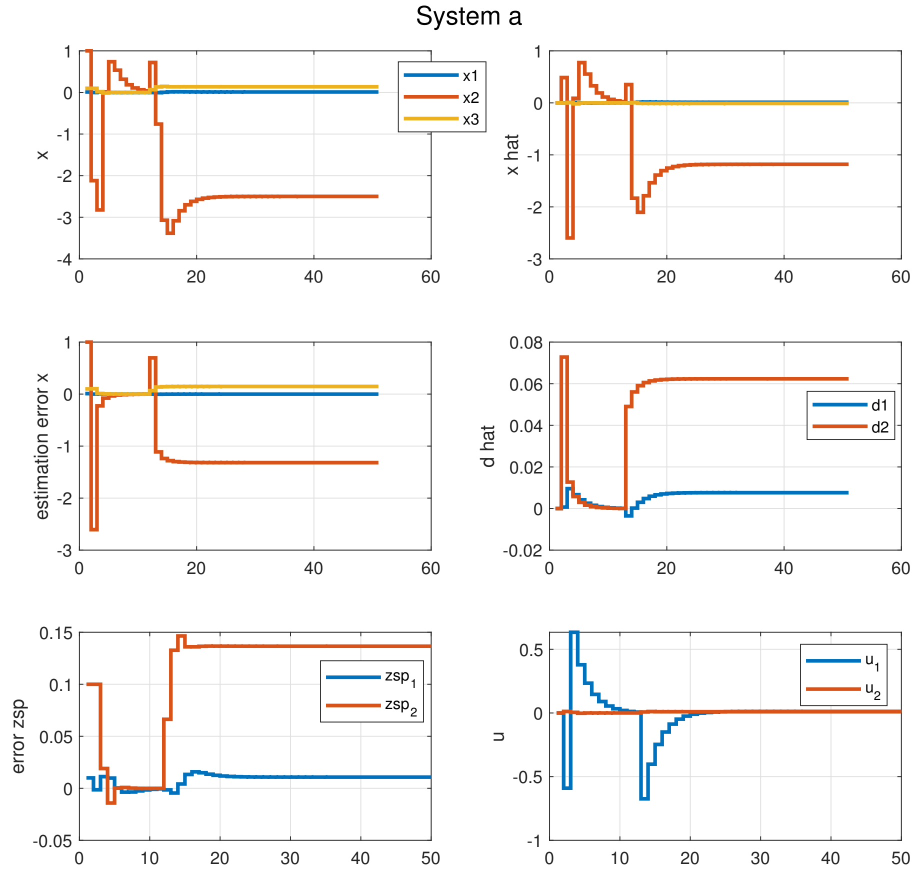

Figure 1: *System response for model a*

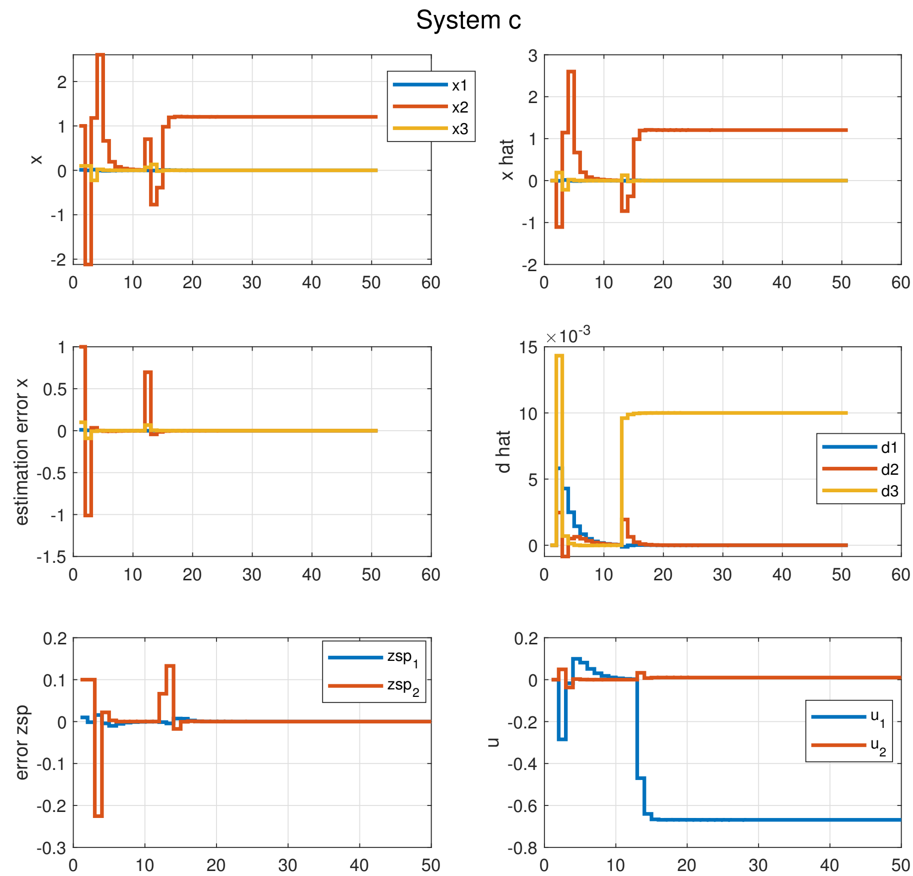

Figure 2: *System response for model c*