SSY281 - Model Predictive Control
Micro-Assignment MA06 - Optimization Basics and Convexity

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Question 1 *For a generic unconstrained optimization problem, what is a necessary condition for x to be a solution?*

For a generic unconstrained optimization problem, there exist a solution x* if the problem is bounded below. That means that: $\inf f(x) > -\infty$

Question 2 *Are the KKT conditions necessary and sufficient optimality conditions for any type of constrained optimization problem?*

No, the KKT conditions are only necessary and sufficient for convex problems, since the local optimum will be also the global optimum.

Question 3 *Write the Lagrangian for problem 40 in slide 69.*

The Lagrangian is simply given by:

$$\mathcal{L}(x, \mu, \lambda) = f(x) + \mu^T g(x) + \lambda^T h(x) \tag{1}$$

Question 4 *Write the KKT conditions and show that they hold at the solution you have found with the graphical method.*

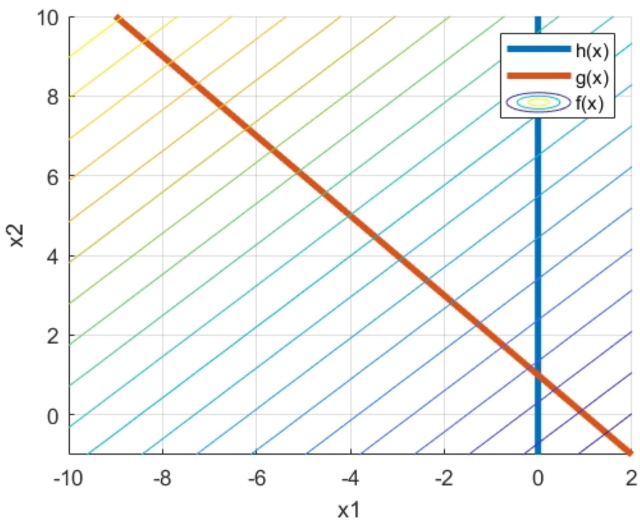

Figure 1: *Solution of the constrained minimization problem*

As can be verified in the picture, the optimal solution is given by $x^* = [0, 1]$.

The KKT conditions are given by:

$$\nabla f(x^*) + \nabla g(x^*)\mu^* + \nabla h(x^*)\lambda^* = 0 \tag{2}$$

$$\begin{bmatrix} 2 * x1^* \\ 2 * x2^* \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mu^* + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \lambda^* = 0 \tag{3}$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mu^* + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \lambda^* = 0 \tag{4}$$

$$\mu^* + \lambda^* = 0 \quad \text{and} \quad 2 + \lambda^* = 0 \tag{5}$$

What implies that $\lambda^* = -2$ and $\mu^* = 2$, which satisfies all the KKT conditions, in particular $\mu^* \geq 0$