

SSY281 Model Predictive Control

Assignment 4

Optimization basics and QP problems

Due February 23 at 23:59

Systems & Control
Department of Electrical Engineering
Chalmers University of Technology

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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
 - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
 - Figures included in the report should have legends, and axes should be labeled.
 - The report should be uploaded *before the deadline* to your project document area in PingPong.
 - Name the report as A4_XX.pdf, where XX is your *group* number.
- Code:
 - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
 - Name the Matlab script as A4_XX.m where XX is your *group* number.
 - Strictly follow the instructions in the Matlab template.
- Grading:
 - This assignment is worth **15 points** in total.

1 Optimization Basics

Consider the optimization problem

$$\begin{aligned} & \min_x f(x) \\ \text{s.t. } & g(x) \leq 0 \\ & h(x) = 0 \end{aligned} \tag{1}$$

where

$$\begin{aligned} f &: \mathbb{R}^n \rightarrow \mathbb{R} & g &: \mathbb{R}^n \rightarrow \mathbb{R}^m \\ h &: \mathbb{R}^n \rightarrow \mathbb{R}^q & x &\in \mathbb{R}^n \end{aligned}$$

Unless otherwise stated, assume that f, g and h are general nonlinear functions.

Question 1 (Points: 3). Consider the optimization problem (1). Let x_1 and x_2 be feasible points.

- (a) Can you find simple conditions such that $z = x_1 + x_2$ is a feasible solution?
- (b) Can you find conditions such z is never worse than both x_1 and x_2 ? If this conditions holds, can it happen that z is better than both of them?
- (c) Can you do the same for a point $z = x_1 + x_2$?

Question 2 (Points: 3). Which of the following sets are convex? Motivate your answers.

- (a) A slab, i.e., $\{x \in \mathbb{R}^n | \alpha \leq a^\top x \leq \beta\}$

- (b) The set

$$M = \{x | \|x - y\| \leq f(y) \text{ for all } y \in S\},$$

where $S \subseteq \mathbb{R}^n$.

- (c) A set of points closer to a given point than to a given set, i.e.,

$$\{x | \|x - x_0\|_2 \leq \|x - y\|_2 \text{ for all } y \in S\},$$

where $S \subseteq \mathbb{R}^n$.

Question 3 (Points: 3). *Linear programs have the general form:*

$$\begin{aligned} \min_x & b^\top x \\ \text{s.t.} & Fx \leq g \end{aligned}$$

Show that these problems can be rewritten as linear programs.

The 1-norm objective is given by:

$$\begin{aligned} \min_x & \|Ax\|_1 \\ \text{s.t.} & Fx \leq g \end{aligned}$$

The ∞ -norm objective is given by:

$$\begin{aligned} \min_x & \|Ax\|_\infty \\ \text{s.t.} & Fx \leq g \end{aligned}$$

Question 4 (Points: 3). *Solve the following linear regression problems*

- (a) *Fill in the function `N1.m` which takes A and b as inputs and returns the x^* that minimizes $\|Ax^* - b\|_1$.*
- (b) *Fill in the function `Ninf.m` which takes A and b as inputs and returns the x^* that minimizes $\|Ax^* - b\|_\infty$.*

2 QP Problems

Question 5 (Points: 3). *Consider the optimization problem:*

$$\begin{aligned} \min_{x,u} f(x,u) &= \frac{1}{2}(x_1^2 + x_2^2 + u_0^2 + u_1^2) \\ \text{s.t.} \quad & 2.5 \leq x_1 \leq 5 \\ & -1 \leq x_2 \leq 1 \\ & -2 \leq u_0 \leq 2 \\ & -2 \leq u_1 \leq 2 \end{aligned} \tag{2}$$

resulting from a finite-time constrained optimal control problem for the SISO process: $x_{k+1} = 0.5x_k + u_k$, with initial state $x_0 = 2$.

- (a) *Solve the QP using MATLAB.*

- (b) Do the KKT conditions hold at the solution found at point (a)? Which constraints are active? (Hint: The Lagrangian multipliers are calculated by the MATLAB command `quadprog`).
- (c) What would happen in the optimization problem if we remove lower bound on x_1 , and what if we remove the upper bound on x_1 ? Why?