SSY281 Model Predictive Control

Assignment 7

Feasibility, alternative formulations of MPC

$\boxed{\text{Due March 14 at 23:59}}$

Systems & Control

Department of Electrical Engineering

Chalmers University of Technology

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Instructions

This assignment is **individual** and must be solved according to the following rules and instructions:

- Written report:
  - For those questions where a text (motivations, explanations, observations from simulations) rather than a numerical value is asked, the answers should be concisely written in the report.
  - Figures included in the report should have legends, and axes should be labeled.
  - The report should be uploaded *before the deadline* to your project document area in PingPong.
  - Name the report as A6_XX.pdf, where XX is your *group* number.
- Code:
  - Your code should be written in the Matlab template provided with this assignment following the instructions therein.
  - Name the Matlab script as A7_XX.m where XX is your *group* number.
  - Strictly follow the instructions in the Matlab template.
- Grading:
  - This assignment if worth **15 points** in total.

1 Linear MPC design

Consider the following system

$$x(t+1) \quad = \quad \begin{bmatrix} 1.2 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{1}$$

The state and input constraints are

$$\mathcal{U} : -1 \leq u(k) \leq 1; \tag{2a}$$

$$\mathcal{X} : \begin{bmatrix} -15 \\ -15 \end{bmatrix} \leq x(k) \leq \begin{bmatrix} 15 \\ 15 \end{bmatrix}; \tag{2b}$$

Question 1 (**Points: 2**). *Consider $Q = I_2$, $R = 100$, and $N = 3$. Choose $\mathcal{X}_f = \mathbf{0}$, and a terminal weight P_f for the constraint receding horizon controller to guarantee asymptotic stability for all $x_0 \in \mathcal{X}_0$. Motivate the choice of P_f in the report and plot $\mathcal{X}_0$.*

Question 2 (**Points: 2**). *Set $x(0) = [4 \ \ -2.6]^T$ and design an MPC controller for $N = 10, \ 15, \ 20$. Provide three figures in the report where each one is simulated for N time steps and contains the state predictions at time zero and the actual states when MPC is implemented. Analyze and explain the mismatch between predicted vs closed-loop trajectories as you increase N in the MPC design.*

Question 3 (**Points: 1**). *Assume $N = 1$. Can you choose a new $\mathcal{X}_f$ so that persistent feasibility is guaranteed for all x_0 belonging to C_∞? Motivate your answer in the report.*

2 Linear MPC design with soft-constraints

An oversimplified model of building air temperature regulation is used to show the basic principles of active thermal storage.

The temperature dynamics of a given space (say a room) can be modeled by using an RC circuit analogy

$$C\dot{T} = u + P_d + (T_{oa} - T)/R, \tag{3}$$

where T is the temperature of the room, P_d is the external disturbance load generated by occupants, direct sunlight, and electrical devices, T_{oa} is the temperature of outside air, and u is the heating and cooling power input to the zone. The zone is cooled when $u \leq 0$, and heated when $u \geq 0$.

The lumped parameter R describes the thermal resistance of walls and windows isolating the zone from the outside environment, and the parameter C captures the thermal capacitance of the room components including walls, floors, and furniture.

By using Euler discretization with a sampling rate of Δt, the representation of system (3) in discrete time is

$$T(k+1) = AT(k) + Bu(k) + d(k), \tag{4}$$

where $A = 1 - \Delta t/RC$, $B = \Delta t/C$, $d = P_d \Delta t/C + T_{oa}\Delta t/RC$.

A simple model predictive control problem is formulated with objectives of minimizing total heating and cooling energy consumption, minimizing the peak power consumption, and maintaining rooms within a desired temperature range despite predicted load changes.

The predictive controller solves at each time step the following problem (call it controller C2)

$$\min_{\mathbf{U}_t,\ \underline{\varepsilon},\ \overline{\varepsilon}} \sum_{k=0}^{N-1} |u_{t+k|t}| + \kappa \max\{|u_{t|t}|, \ldots, |u_{t+N-1|t}|\} + \rho \sum_{k=1}^{N} \left(|\overline{\varepsilon}_{t+k|t}| + |\underline{\varepsilon}_{t+k|t}|\right) \tag{5a}$$

$$T_{t+k+1|t} = AT_{t+k|t} + Bu_{t+k|t} + d_{t+k|t}, \tag{5b}$$

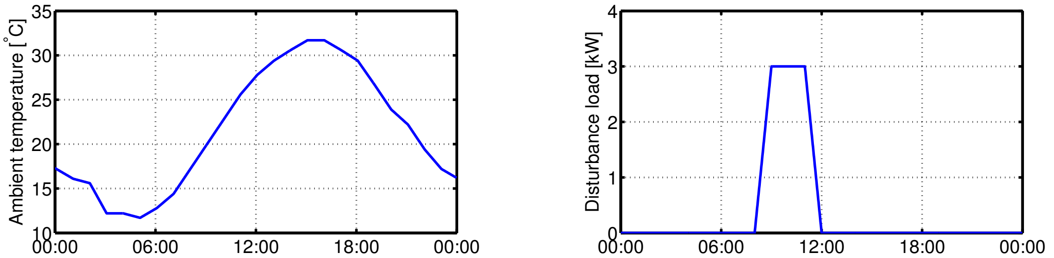
$$\underline{T} - \underline{\varepsilon}_{t+k|t} \le T_{t+k|t} \le \overline{T} + \overline{\varepsilon}_{t+k|t}, \tag{5c}$$

$$\underline{\varepsilon}_{t+k|t},\ \overline{\varepsilon}_{t+k|t} \ge 0, \tag{5d}$$

$\mathbf{U}_t = \left[u_{t|t},\ u_{t+1|t}, \ldots, u_{t+N-1|t}\right]$ is the vector of energy control inputs, $\underline{\varepsilon} = \left[\underline{\varepsilon}_{t+1|t}, \ldots, \underline{\varepsilon}_{t+N|t}\right]$ is the temperature violations from the lower bounds, $\overline{\varepsilon}$ the temperature violation from the upper bounds, $T_{t+k|t}$ is the thermal zone temperature, $d_{t+k|t}$ is the load prediction, and $\underline{T}$ and $\overline{T}$ are the lower and upper bounds on the zone temperature, respectively. ρ is the penalty on the comfort constraint violations, and κ is the penalty on peak power consumption.

Question 4 (Points: 10). *Design the MPC controller as in* (5) *with the following parameters. Thermal capacitance* $C = 9.2 \times 10^3$ *kJ/°C, thermal resistance* $R = 50$ *°C/kW, sampling rate* $\Delta t = 1$ *hour, prediction horizon* $N = 24$ *hours, and thermal comfort interval* $[\underline{T}, \overline{T}] = [21,\ 26]$ *°C.*

Assume that weather and load are periodic with a period of one day. The outside air temperature profile $T_{oa}(t)$ *and the disturbance load profile* $P_d(t)$ *used in (3) are depicted in Figures 1(b), 1(a), respectively. The corresponding .mat files are given in pingpong.*



(a) Ambient temperature (°C) (b) Disturbance load (kW)

Figure 1: Predicted Ambient Temperature and Load Profiles.

- *Design three MPC controllers by assuming that* (i) $T_{oa}(t)$ *and* $P_d(t)$ *are known and their predictions are perfect without mismatch between predictions and actual measurements,* (ii) *only the current values of* $T_{oa}(t)$ *and* $P_d(t)$ *are known and* (iii) *both* $T_{oa}(t)$ *and* $P_d(t)$ *are unknown.*

- *Simulate the three MPC controllers designed at the previous point in closed-loop with* $\rho = 1000$ *and* $\kappa = 2$ *from* $T(0) = 22°C$. *After a transient period the system will settle at steady state, i.e.,* $T(k) = T(k+24)$. *Plot a steady state closed-loop profile for one day.*

- *Implement the proportional controller (call it controller C1) designed to reject the load without predictive information:*

$$u(t) = \begin{cases} K(\overline{T} - T(t)) & T(t) \geq \overline{T}, \\ 0 & \underline{T} \leq T(t) \leq \overline{T}, \\ K(\underline{T} - T(t)) & T(t) \leq \underline{T} \end{cases} \tag{6}$$

  Simulate the proportional controller in closed-loop with $K = 400$ *and sampling time of one minute. Use* $T(0) = 22°C$. *After a transient period the system will settle at steady state, i.e.,* $T(k) = T(k+24)$. *Plot a steady state closed-loop profile for one day.*

  p.s.: A fair comparison wold require to implement the P-controller with sampling time of 1hr. Unfortunately this is too coarse for a P-controller and it does not perform well. Please use a sampling time for the P-controller of 1 minute.

- *The performance of the first MPC controller (* (i) *) and the P controller are measured by the closed loop total energy consumption*

$$J^u = \sum_{k=0}^{N-1} |u^*(k)| \Delta t, \tag{7}$$

the peak power consumption

$$J^p = \max\{|u^*(0)|, \ldots, |u^*(N-1)|\}, \tag{8}$$

and the total comfort violation

$$J^\varepsilon = \sum_{k=0}^{N} \left(|\overline{\varepsilon}^*(k)| + |\underline{\varepsilon}^*(k)|\right) \Delta t. \tag{9}$$

*All cost are computed **at steady state**.*

*Comment on the differences (**at steady state**) between the MPC controller* (i) *and the P controller simulations above by using these indices.*