SSY281 - Model Predictive Control
Micro-Assignment MA07 - Solving QP problems

Lucas Rath - **Group 09**

Question 1

First we write the Lagrangian $\mathcal{L}$ and the dual cost function q:

$$\mathcal{L}(x, \lambda, \mu) = x1^2 + 2x2^2 + \lambda(x1 - 2x2 - 1) + \mu(x1 - 2) \tag{1}$$

$$q(\mu, \lambda) = \inf_x \mathcal{L}(x, \lambda, \mu) \quad \Rightarrow \begin{cases} x1^* = -\lambda/2 - \mu/2 \\ x2^* = \lambda/2 \end{cases} \tag{2}$$

$$\tag{3}$$

$$\Rightarrow q(\mu, \lambda) = \begin{bmatrix} \lambda \\ \mu \end{bmatrix}^T \begin{bmatrix} -3/4 & -1/4 \\ -1/4 & -1/4 \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix}^T \begin{bmatrix} \lambda \\ \mu \end{bmatrix} \tag{4}$$

So the dual problem reduces to:

$$\max_{\lambda,\mu} \quad q(\lambda, \mu) \tag{5}$$

$$s.t. \quad \mu \geq 0 \tag{6}$$

Question 2

$$\mathcal{L}(x, \lambda, \mu) = x1 + x2 + \lambda(x1 + 2x2 - 5) + \mu * (x1 - 1) \tag{7}$$

$$q(\mu, \lambda) = \inf_x \mathcal{L}(x, \lambda, \mu) = \begin{cases} -5\lambda - \mu & \text{if} \quad \lambda + \mu + 1 = 0 \quad \text{and} \quad 2\lambda + 1 = 0 \\ -\infty, & \text{otherwise} \end{cases} \tag{8}$$

Therefore, the dual problem reduces to:

$$\max_{\lambda,\mu} \quad q(\lambda, \mu) = -5\lambda - \mu \tag{9}$$

$$s.t. \quad \begin{matrix} \mu \geq 0 \\ \lambda + \mu + 1 = 0 \\ 2\lambda + 1 = 0 \end{matrix} \tag{10}$$

Question 3

To prove the strong duality, one can simply solve the primal and the dual problem and check if the primal optimal value p^0 and the dual optimal value d^0 are the same:

```
% Primal formulation
H = 2*[1 0; 0 2];
f = [0 ; 0];
A = [1 0];
b = 2;
Aeq = [1 -2];
beq = 1;
[varprimal,p0,exitflag,output] = quadprog(H,f,A,b,Aeq,beq);

% Dual formulation
H = 2*[-3/4 -1/4; -1/4 -1/4];
f = [-1 ; -2];
A = [0 -1];
b = 0;
% -H and -f in order to convert the problem to minimization
[vardual,d0,exitflag,output] = quadprog(-H,-f,A,b);
```

which resulted in the same optimal value:

$$p_0 = d_0 = 0.3333 \tag{11}$$

$$x^* = \begin{bmatrix} 0.3333 \\ -0.3333 \end{bmatrix} \tag{12}$$

meaning that the strong duality holds.

Question 4

The advantage of the active set method is that all the iterations produce feasible solutions, in case one wants to stop simulation before total convergence. On the other hand, the interior point method does not guarantee that the solution is feasible before convergence.