

NAME ZHENYUAN FENG

MATRICULATION NUMBER S1613213

**Low complexity Spatial-division
Multiplexing in Few Mode
optical fibre**

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Mission Statement

Project Definition

The spatial division multiplexing in MMF using SM-MIMO techniques

Main Tasks

1. Applying the SM-MIMO techniques into random coupling MMF.
2. Making trade-off between receiver complexity , BER performance and spectral efficiency.

Background Knowledge

The background knowledge contain the characteristic of multi-mode optical fibre, traditional linear MIMO techniques and the principle of SM-MIMO techniques.

Abstract

All the SM-MIMO techniques are used into the random coupling MMF system to show the BER performance based on different detection method. The receiver complexity of applying different SM-MIMO techniques and traditional linear MIMO techniques is shown to demonstrate that the SM has 80% to 90% receiver complexity reduction over MMSE-MIMO and ZF-MIMO. The complexity of GSM is larger than SM but still has 20% to 80% receiver complexity reduction over linear MIMO in different cases. The trade-off between receiver diversity, BER performance and spectral efficiency in MMF is shown based on some particular cases and receiver complexity order figure. The GSM has a good performance when the number transmit modes is lower than 14. A active antenna excitation method and high utilization efficiency method for GSM is delivered to make a trade-off between BER performance, transmit diversity gain and spectral efficiency. A possible mode switch design is shown based on low loss photonic lantern.

Declaration of Originality

I declare that this thesis is my
original work except where stated.

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Statement of Achievement

1. All the SM-MIMO techniques are used into the random coupling MMF system to show the BER performance based on different detection method.
2. The receiver complexity of applying different SM-MIMO techniques and traditional linear MIMO techniques is shown.
3. The trade-off between receiver diversity, BER performance and spectral efficiency in MMF is shown.
4. A active antenna excitation method and high utilization efficiency method for GSM is delivered to make a trade-off between BER performance, transmit diversity gain and spectral efficiency. This two methods are also suitable in MMF transmission
5. A possible mode switch design is shown based on low loss photonic lantern.

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Chapter 1

Introduction

1.1 Project Motivation

The capacity of single mode fibre (SMF) will reach its Shannon limit in the next few years [1]. And the time division multiplexing (TDM), wavelength division multiplexing (WDM) and polarization division multiplexing (PDM) have been deeply explored in the past thirty years. The spatial dimension is the only field that has been left to multiply the channel capacity [1]. However, from 2011, the number of the simultaneous transmitting modes was increased to 30 with a high complexity in the receiver [2] because of processing high rank matrix using multi-input and multi-output (MIMO) digital signal processing (DSP). Therefore, the motivation of this project is to deeply reduce the the complexity of receiver while still maintaining a relatively high spectral efficiency for few-mode-fibre (FMF) using spatial modulation (SM) to make it more feasible and promising compared to the traditional MIMO DSP in the future.

1.2 The aim and organization

The aim of the paper is to apply SM-MIMO techniques in optical fibre to reduce the complexity of receiver and make trade-off among the spectral efficiency , receiver complexity and BER peformance from shift space keying (SSK), generalized space shift keying (GSSK), spatial modulation (SM), generalized spatial modulation (GSM) and linear MIMO tecniques in FMF using spatial division multiplexing (SDM).

The rest of the paper is organized as follows. The Chapter 2 is to illustrate the background and literature review of SDM in optical fibre, In Chapter 3, SM-MIMO tecniques and conventional MIMO techniques are provided. The

combining of FMF and SM-MIMO is shown in Chapter 5. And we conclude the paper and pave the way for future work at Chapter 6.

Chapter 2

Principle and reference of SDM optical fibre

The following notations and assumptions are used throughout the paper. Bold and lower letters '**h**' denote vectors. Bold and upper letters '**H**' denote matrix. We denote $(\cdot)^\dagger$ for conjugate transpose, $(\cdot)^*$ and $(\cdot)^T$ for complex conjugate and transpose. $(:)$ denote binomial coefficient. $\|\cdot\|_F$ denote Frobenius norm. We use $(\cdot)^+$ for pseudoinverse, $|\cdot|$ for absolute value, $Re\{\cdot\}$ and $Im\{\cdot\}$ for real part operators and imaginary part operators. $\lfloor \cdot \rfloor$ and $\lfloor \cdot \rfloor$ denote floor function and round function(round variable to the closest integer). And $(\cdot)^{-1}$ is the inverse of a square matrix. In this paper, low SNR region/scenario denotes the SNR range from 0dB to 10dB, which is 10 to 30dB for moderate region/scenario and over 30dB for high SNR region.

2.1 The optical fibre

2.1.1 The history of optical fibre

The optical fibre takes advantage of total reflection principle to propagate along silica or plastic material with high information rate [3]. In 1960s, researchers successfully realized high data transmission but with a high attenuation about 1000dB/km [4]. Then, in the next 20 years, the main work focused on reducing the loss of optical fibre. K. C. Kao [5] advised on the idea of dielectric fiber and cut the attenuation down to around 20dB/km. Afterward, W. G. French [6] made breakthrough on the manufacture process and further reduced the attenuation to 0.2dB/km [7] which approached the limit of silica fibre.

Then, the researchers changed their emphasis on the capacity of transmission. Huang Hung-Chin [8] promoted coupled mode theory to greatly increase the

system capacity in single mode fiber (SMF) in 1980s. And in the next ten years, the optical communication system witnessed a slow increasing ratio of bit rate by using the TDM in SMF , before seeing a dramatic increase in system capacity from 1Gb/s to 1Tb/s [9]when WDM began to be applied in optical fibre transmission.

However, after 2000, the increase ratio of system data rate started to slow down and SMF is to reach its capacity Shannon limit in the next few years [1]. That's why few mode fibre (FMF) appeared in recent years to get multiplication of system capacity and paves the way for coming 5G era.

2.1.2 Multi-core fibre (MCF) and Multi-mode fibre (MMF)

In this paper, the so called few mode fibre (FMF) has no distinct difference with multi mode fibre (MMF). In spatial division multiplexing (SDM) system, different modes are distributed on various spatial paths in the MMF or MCF with crosstalk and mode coupling .

MCF transmits one mode on one core. The more the cores incorporated in a MCF, the more modes it can maintain at the same time [10]. However, under restrictions of the crosstalk between different modes and the size of traditional optical fibre, the MCF can not accommodate too many cores, which makes it a main disadvantage on the number of available modes compared with MMF. For example, if the diameter of a fibre is longer than $200\mu\text{m}$, the fibre is thick , making it susceptible and increasing the possibility of fracture and breakage in long haul optical transmission system. In addition to the diameter of fibre, the space between two cores is also very important. The distance between two cores should be larger than $40\mu\text{m}$ to obtain a low crosstalk less than -25dB/km [1]. K. Igarashi promoted a 19 core fibre with a spectral efficiency over 345bit/s/Hz in 2015 [11]. But the large crosstalk confined the transmission distance to only 9.8km.

MMF can maintain many modes on the only one big core, it outperforms MCF on the point that it can contain more modes simultaneously. However, the disadvantage is that there is unavoidable crosstalk and mode coupling between different paths. Effective algorithm should be derived to detect different modes. MIMO techniques and spatial modulation are two useful ways to process multi signals in wireless system. They can also be applied in MMF, and the paper will discuss about it in the later part.

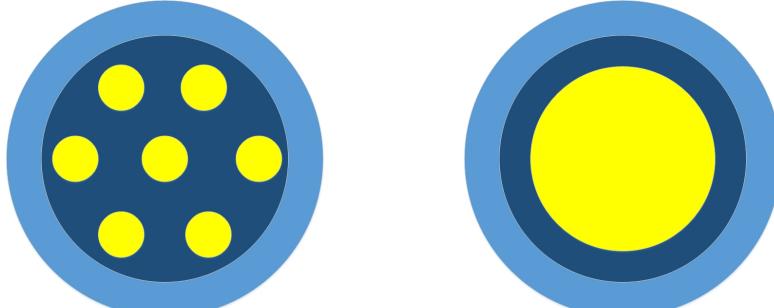


Figure 2.1: Left:Multicore fibre Right: Multimode fibre

2.1.3 Linear effect of optical fibre

The linear effect is dependent on the wavelength of signal and contributes to the main loss of optical fiber transmission. The linear effect is divided into optical signal loss and dispersion.

Optical loss

We always term the optical loss as *attenuation*. If the P_T is signal input power and P_R is signal output power. The key parameter for the optical signal loss *attenuation constant* is defined below (L is transmission distance):

$$P_R = P_T \exp(-\alpha L) \quad (2.1)$$

$$\alpha_{dB} = -\frac{10}{L} \log\left(\frac{P_R}{P_T}\right) \quad (2.2)$$

Both T. Li [12] and T. Miya [7] showed that the minimum attenuation of optical silica fiber is around 0.2dB/km at the traditional wavelength 1550nm in 1979.

Dispersion

In optical communication system, the phenomenon that phase velocity greatly depends on the signal frequency is termed as dispersion. Dispersion is divided into chromatic dispersion (CD) and modal dispersion (MD).

CD is the property that an electromagnetic wave interact with the bound electrons of a dielectric [13]. CD exists in SMF and MMF. However, MD only ap-

pears in MMF. The first order MD is called group delay (GD) and GD has closely relationship with the length of the MIMO equalizer which makes it a significant design index in optical fibre transmission. The second order MD is mode-dependent loss or gain (MDL). MDL is not important in short distance transmission. But if the number of segment in the system is large. The MDL will accumulate with cascaded amplifiers in each segment and become an performance limiting factor. The three or higher order of MD is related to nonlinearity of optical fibre. The paper will discuss it in the next part.

2.1.4 Non-linear effect of optical fibre

Different from linear effect which is dependent on the frequency of signal, the nonlinear effect mainly depends on the intensity (power) of the signal. In general, the larger the input power, the larger the nonlinear effect. Although compared with linear effect, nonlinear effect is much smaller. It is still a benefit of SM-MIMO techniques in MMF because we transmit less number of modes using SM. I will mention this benefit in Chapter 4.

Stimulated Scattering (SS)

SS consists of Stimulated Raman Scattering (SRS) and Stimulated Brillouin Scattering (SBS). Both SRS and SBS result from vibrational excitation modes of silica. However, the SRS generates optical phonon and moves the light forward and SBS generates acoustic phonon and moves the light backwards. The pump power threshold for them are given below [13]:

$$P_{thre}(SBS) \approx \frac{21A_{eff}}{ga_B L_{eff}} \quad (2.3)$$

$$P_{thre}(SRS) \approx \frac{16A_{eff}}{ga_R L_{eff}} \quad (2.4)$$

$$A_{eff} \approx \pi r_{core}^2 \quad (2.5)$$

$$L_{eff} = 1/\alpha(1 - e^{-\alpha l}) \quad (2.6)$$

A_{eff} is effective region of fibre and L_{eff} is effective length of fibre. In practical condition, the *Raman gain* ga_R is about $1 \times 10^{-13} m/W$ and *Brillouin gain* ga_B is around $5 \times 10^{-11} m/W$. And *attenuation constant* α takes the optimal value 0.2dB/km [7]. The core radius $r_{core} \approx 4 \times 10^{-6}$. Then, $P_{th}(SRS) \approx 400 mW$ and $P_{th}(SBS) \approx 1 mW$.

SRS and SBS will appear in the optical fibre only if the input signal power is larger than the threshold. Therefore, SRS and SBS causes unavoidable crosstalk between different pair of modes. If we reduce the input power to be lower than threshold. There is no SRS and SBS at the expense of smaller number of modes. But we want to transmit more modes at the same time to increase the spectral efficiency. Therefore, there should be an optimization work in the number of modes and nonlinear effect.

2.2 Literature Review of SDM in FMF

Compared with MCF, the SDM in FMF has more complexity in the detector because there is inevitable spatial overlap (coupling) between different pair of modes. This part will illustrate the literature review of SDM in FMF using MIMO techniques.

2.2.1 Mode division multiplexing (MDM) based SDM transmission

In this paper, the term MDM means that different modes transmit in the same fibre with different position of spatial cross section. The research of MDM based SDM transmission began from linear polarized (LP) modes LP_{01} and LP_{11} which are stable, simple and have low loss.

N. Hanzawa [14] proposed paper on MDM transmission two mode fibre over 10km and got low penalty of signal to noise ratio (SNR) at receiver using maximum likelihood (ML) detection. The information rate is sharply increased by A. Li to 107Gbps via improving the receiver [15] and transmitter [16] on two mode fibre over 4.5km. Afterwards, the property of dual-spatial mode (degenerate modes LP_{11a} and LP_{11b}) began to be explored on FMF transmission. Transmission [17] and reception [18] using 4×4 MIMO-OFDM are operated on two mode fibre over 10km. Then, Koebele [19] paves the way for three mode fibre by combining the LP_{01} , LP_{11a} and LP_{11b} and successfully invented prototype FMF over 40km. The number of modes and transmission distance are increasing continuously in the next few years. R. Ryf [20] experimented on 6×6 MIMO three mode fibre over 10km with a non ideal penalty of 8dB. The data was transmitted reliably on 6 orthogonal channels for different wavelength. Further research [21] reduced the penalty to 1.2dB on MDM based FMF over 96km using MIMO DSP. The same team presented 12×12 MIMO transmission FMF over 130km using the combination of low loss photonic lantern (PL) and 3D waveguide mode multiplexers [22] in 2012. And in 2015, 20×20 MIMO transmission [23] and 30×30 MIMO transmission [2] was finished by J. V. Weerdenburg and

NK Fontaine with low loss respectively.

2.2.2 Partial MIMO techniques

However, the number of modes peaked at 30 with a high complexity at detector and receiver using off-line MIMO DSP. Researchers started to focus on other road to reduce the complexity using partial MIMO or MIMO free techniques to make it more practical. For partial MIMO, elliptical core fibre is illustrated as a way to obtain negligible mode coupling even in degenerate modes group [24]. Ezra lp [25] experimentally demonstrated the first real time elliptical core FMF (eFMF) over 0.5km with -26dB crosstalk between degenerate modes LP_{11a} and LP_{11b} . Giovanni Milione [26] transmitted error free (<-22dB crosstalk) data stream to more modes LP_{01} , LP_{11a} and LP_{11b} . Then further study [27] dramatically increased the data rate to 1.2Tb/s over 1km eFMF with 125 μ m cladding. The weakly coupling property of eFMF is promising for reducing receiver complexity. However, it is difficult to build the system model of eFMF, most papers are based on the experiment and specific core ellipticity with different kinds of definitions and assumptions. Therefore, it is still a long way to go for eFMF. This paper demonstrates another direction to reduce the receiver complexity using spatial modulation in MMF and will discuss it in Chapter 4.

2.3 MIMO techniques

MIMO communication techniques have developed a lot in recent 30 years benefited from wireless system with high speed and broadband employing multiple transmit and receive antennas [28] [29]. Different MIMO techniques focus on different scenarios and conditions in wireless system. The Bell Labs Layered Space Time (BLAST) system achieves high spectral efficiency in wireless transmission [30]. Note that BLAST system only need fully or partial channel state information (CSI) at receiver. On the other side, the CSI is available for transmitter , at least partially , such as frequency division duplex (FDD) system and time division duplex (TDD) systems. Hence, the corresponding CSI dependent DSP schemes e.g., linear pre-coding and decoding (LPD) [31]. The performance gains of LPD is achieved by power allocation over multiple transmit antennas with partial or fully CSI at transmitter. Space time block coding (STBC) is another scheme to realize full diversity and reliable transmission in wireless communication system [32] [33].

This paper will compare the bit error rate (BER), spectral efficiency and receiver complexity of MIMO with maximum likelihood (ML) detection and other linear

detector such as zero forcing MIMO (ZF-MIMO) and minimum mean square error MIMO (MMSE-MIMO) in FMF.

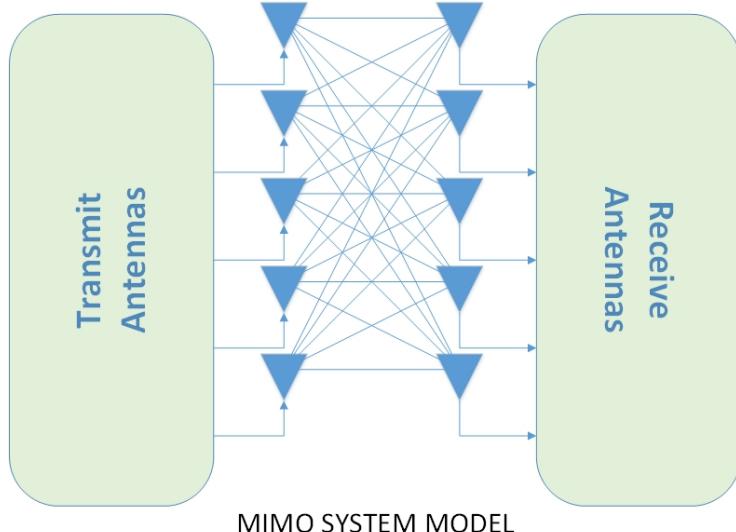


Figure 2.2: 5x5 MIMO system model

2.4 Modeling of SDM system in Few mode Fibre

The following assumptions are used throughout this chapter. We assume to neglect the effect of nonlinearity , group delay ,fibre optical loss in the fibre. Hence, there is no equalizer in the end of a segment of the fibre. We assume uniform random coupling between different pair of modes and full rank channel matrix. The noise is added in the receiver with value σ^2 . And we have full CSI at receiver.

According to the assumptions above, the system model can be written as:

$$y = \sqrt{E_0} \sqrt{L} \mathbf{H} \mathbf{x} + \mathbf{n} \quad (2.7)$$

The channel matrix \mathbf{H} is full rank unitary matrix generated from[34] using QR decomposition with normalized eigenvectors in each column and normalized eigenvalues. The element in the \mathbf{H} is gaussian distributed to ensure a Gaussian channel [35]. Note that, the column vectors are orthogonal to each other. Then, the modes transmitted in the channel are orthogonal. The number of modes equal or smaller than the rank of square channel matrix. Therefore, the computation complexity of MIMO DSP at receiver increases with the rank of the channel. \mathbf{n} is additive white gaussain noise (AWGN). The reason for

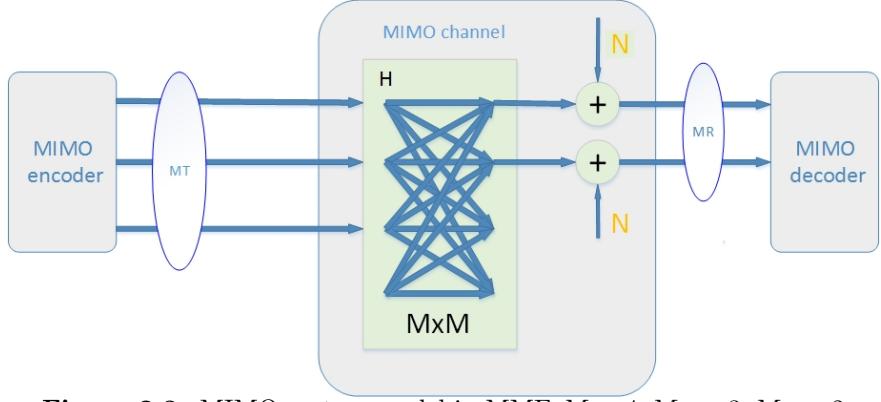


Figure 2.3: MIMO system model in MMF $M = 4, M_T = 3, M_R = 2$

choosing AWGN is that ML detector has a good performance for AWGN to make it easier to compare ML detector with other linear detectors. E_0 is average signal power per symbol. Then, if we normalized the power of noise \mathbf{n} to 1. E_0 is also the SNR. M_T, M_R and M is the number of transmit antennas, receive antennas and the rank of matrix, respectively. M_T and M_R satisfy $\text{maximum}(M_T, M_R) \leq M$. And $\|\mathbf{x}\|^2 = M_T$ if assumed normalized.

L is mormalized mode-dependent loss or gain and λ_i is the eigenvalue of channel matrix:

$$L = \frac{1}{M} \text{tr}\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^\dagger\} = \frac{1}{M} \sum_{i=1}^M \tilde{\lambda}_i \quad (2.8)$$

In this model, the CSI is known only to the receiver. The signal on each modes and noise on each receiver is uncorrelated with each other and have the same E_0 and σ^2 . Hence, the BLAST capacity[30] can be written as:

$$C = \sum_{i=1}^r \log_2\left(1 + \lambda_i \frac{E_0 L}{N_0}\right) \quad (2.9)$$

Different from wireless communication, the channels here in MMF do not have fading and we use [34] to generate frequency selective channel because flat channels and slow fading channels are sources of unreliability and has bad outage capacity [36]. Outage probability is an important parameter for frequency selective channel, it is a probability that the capacity of the channel is lower than a threshold so that the channel is seen as a "bad" channel. For frequency-fast fading channel, the differential modal delay is large, the signal will ultimately see the average capacity \bar{C} . The definition of outage probability is shown below:

$$P_{out} = \int_0^{C_T} p_C(C) \quad (2.10)$$

$p_C(C)$ is the probability density function (PDF) of channel capacity C . The P_{out} should be lower than 10^{-5} for good performance in practical life.

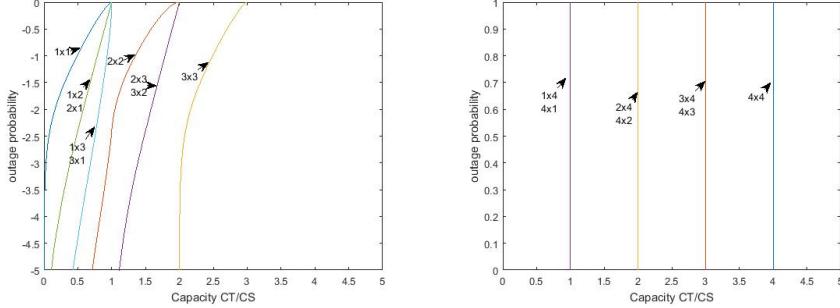


Figure 2.4: The relationship between the outage probability and relative capacity different combination of M_T and M_R . (SNR=20dB)

The simulation is based on 10^5 realizations of channel matrix because the degree of accuracy in the paper can reach 10^{-5} . Figure (2.4) demonstrates the relationship between P_{out} and relative capacity r , r is defined as:

$$C \leq rC_s = r\log_2(1 + SNR) \quad (2.11)$$

C_s is single mode capacity. Hence, the r is something like multiplexing gain in MIMO wireless system (The multiplexing gain is defined as $\min(M_T, M_R)$.) And the Figure (2.4) shows that $\max(r) = \min(M_T, M_R)$. The maximum multiplexing gain in MMF is the same as the case in wireless system. Note that if $\max(M_T, M_R) = M$, r is stable and equals to $\min(M_T, M_R)$ because there is no power loss after transmission. The reason behind it is that if $\max(M_T, M_R) = M$, $\text{tr}\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^\dagger\} = \min(M_T, M_R)$ without attenuation and loss.

From the left one in figure (2.5), the relative capacity is $r = (2.0, 2.1, 2.2, 2.4)$ for $SNR = (10, 20, 30, 40)$ dB with $P_{out} = 10^{-3}$ in 3×3 which means that the MMF system capacity increases with SNR but still can not have opportunity for performance boost without full CSI both in transmitter and receiver [36]. Right figure exhibits the average capacity of 10^5 highly frequency-selective channel realizations. Only the $M_T = M_R = 4$, the system can get full capacity because the signal power randomly couples to other modes.

From Figure (2.6), we can find that MMF shows great potential for increasing the system capacity. For fixed modes number $M \in \{2, 4, 8, 16, 32, 64, 128\}$, the

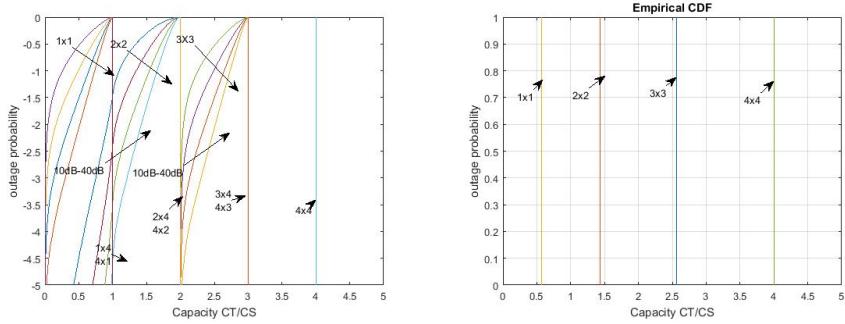


Figure 2.5: Left: The relationship between the outage probability and relative capacity different combination of M_T and M_R . (SNR=10,20,30,40dB). Right: Relative capacity of highly frequency selective channel(SNR=20)

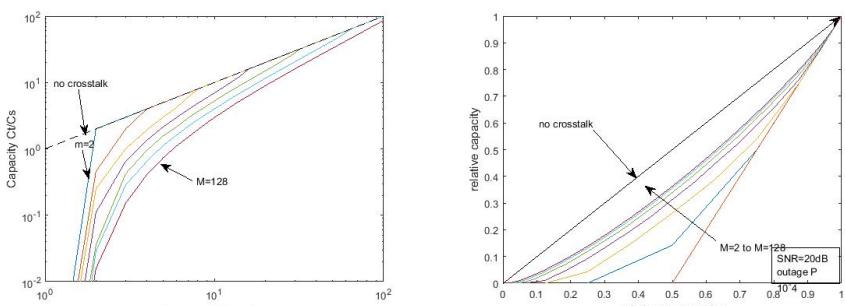


Figure 2.6: Left: The Transponder modes(M_T) $P_{out} = 10^{-4}$ $SNR = 20$. Right: Relative transponder modes(M_T/M) $P_{out} = 10^{-4}$ $SNR = 20$.X-axis denote the number of transmitting modes M_T with fixed modes $M \in \{2, 4, 8, 16, 32, 64, 128\}$ Y-axis is the relative system capacity The dashed line is no crosstalk line in single mode case

system capacity increases with the number of transmitting modes M_T . Because even the power of modes will couple to other modes, the rank of channel matrix is M_T and all the power will come out at receiver without loss.

Inspired by the transponder modes, the generalized spatial modulation MIMO in MMF (GSM-MIMO) will choose the optimal number of modes that are activated (i.e. active modes) and explore the potential of spectral efficiency without sacrificing the bit error ratio (BER).

2.5 Modeling of fibre link with distributed noise loading

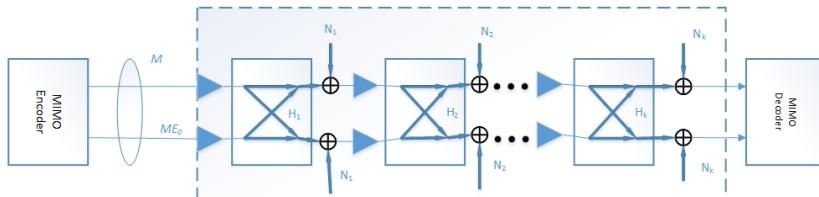


Figure 2.7: System model with distributed noise

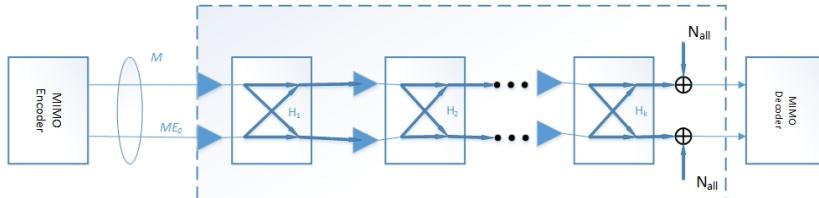


Figure 2.8: System model with noise pump at receiver

We now consider the spatial division multiplexing system with $M \times M$ channel matrix of K concatenated segments. The AWGN is added to the end of each segment. If the noise at each equalizer is identical individual distributed (i.i.d.). The noise at the receiver is:

$$\mathbf{n} = N_1 \mathbf{H}_k \mathbf{H}_{k-1} \cdots \mathbf{H}_2 + N_2 \mathbf{H}_k \mathbf{H}_{k-1} \cdots \mathbf{H}_3 + \cdots + N_{k-1} \mathbf{H}_k + N_k \quad (2.12)$$

The correlation matrix \mathbf{Rn} for the noise is :

$$\mathbf{Rn} = N_1 \mathbf{H}_k \mathbf{H}_{k-1} \cdots \mathbf{H}_2 \mathbf{H}_2^\dagger \mathbf{H}_3^\dagger \cdots \mathbf{H}_k^\dagger + \cdots + N_{k-1} \mathbf{H}_k \mathbf{H}_k^\dagger + N_k \mathbf{H}_M \quad (2.13)$$

And \mathbf{I} is the $M \times M$ identical matrix. The channel matrix \mathbf{H} is unitary matrix.

Hence, $\mathbf{H}\mathbf{H}^\dagger = \mathbf{I}$. After some mathematic calculation, \mathbf{Rn} is:

$$\mathbf{Rn} = \sum_{i=1}^K N_i \mathbf{I} = N_0 \mathbf{I} \quad (2.14)$$

From equation(2.14), adding noise to each segment is equal to adding noise to the end of the concatenated segments of channel matrix with power N_0 . It means that directly doing noise loading at receiver do not impact the system capacity if the noise power is the sum of all the segment noise power. The paper will show the BER difference between directly receiver noise loading and distributed noise loading at each segment to compare the performance and receiver complexity in Chapter 4.

If the number of segment become very large. It is time consuming and high computational complexity to calculate $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$ for each segment to go through the whole system. Equation (2.14) provides a simplified way to pump the noise at the receiver and promotes a distributed nature of optical noise.

Chapter 3

Principle and reference of SM-MIMO system

In this chapter, the modulated signal from different schemes is transmitted $M_t \times M_r$ wireless channel. The receive signal is written as:

$$\mathbf{y} = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{n} \quad (3.1)$$

Note that only the receiver has CSI and the receiver experiences M_r dimension AWGN [$n_1 \ n_2 \ \dots \ n_{M_r}$]. ρ is signal to noise ratio at each receive antenna. We make assumption that all the noise including demodulator noise, receiver noise and detector noise is added as \mathbf{n} in the formula. The entries of \mathbf{H} and \mathbf{n} is individual identical distributed with respect to $\mathcal{CN}(0,1)$. The signal \mathbf{x} experience *Rayleigh* fading in the channel. And all the simulation results come from Monte Carlo simulation [37] [38] [39] [40]. This chapter is to illustrate different SM-MIMO techniques and compare them with different key parameters.

3.1 Literature Review of Spatial Modulation

Spatial modulation MIMO (SM-MIMO) technique is a promising transmission concept in wireless communication system [41]. In this paper, the SM-MIMO techniques is different from SM-MIMO. SM-MIMO techniques contain different type of spatial modulation including (SM, GSM, SSK and GSSK). But the SM-MIMO is just the particular type SM in MIMO system. The concept for SM-MIMO techniques is that one or several transmit antennas are activated at one time, which is termed as active antennas. The transmitter will switch to different antennas instead of utilizing all antennas at all time in traditional MIMO system. In 2006, R. Y. Mesleh [42] use the term "spatial modulation"

for the first time to identify this encoding scheme [43]. Three years later in 2009, J. Jeganathan [44] proposed shift keying modulation (SSK) in MIMO in which only the spatial constellation diagrams deliver information bits and only one antenna is activated at one time. In [45], the same author further investigated in the scheme that more than one transmit antennas are activated which is named as generalized shift keying modulation (GSSK). Inspired by SSK-MIMO and GSSK-MIMO, the SM-MIMO [46] and GSM-MIMO [47] have been proposed with various kinds of modulation types such as PSK, ASK and QAM.

There are several advantages of SM [41]. First, smaller number of active transmit antenna greatly reduces the complexity of receiver and detector. Second, it is more energy efficient than MIMO because it only uses several transmit antennas at the same time. Third, SM-MIMO uses smaller number of RF chains which costs much less than traditional MIMO in wireless communication system. In a nutshell, the SM-MIMO techniques meets the energy efficient and sustainable development concept of future technology.

In MMF, using smaller number of laser source will reduce the cost in MMF using SM-MIMO (the mode switch design is shown in chapter 4). And it will meet the conditions and scenarios of the first and second advantage and this paper will show it in Chapter 4.

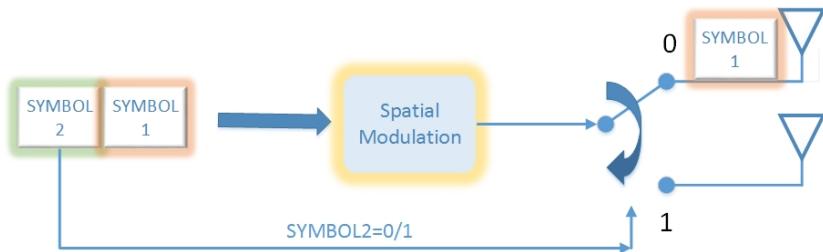


Figure 3.1: Spatial Modulation

3.2 Conventional MIMO technique

Maximum likelihood(ML) detector has the best performance for MIMO system experiencing AWGN noise [48]. However, it has a large computational complexity. Then, many linear MIMO detectors are derived [49] [50] to reduce the complexity.

3.2.1 Linear Zero Forcing Detector

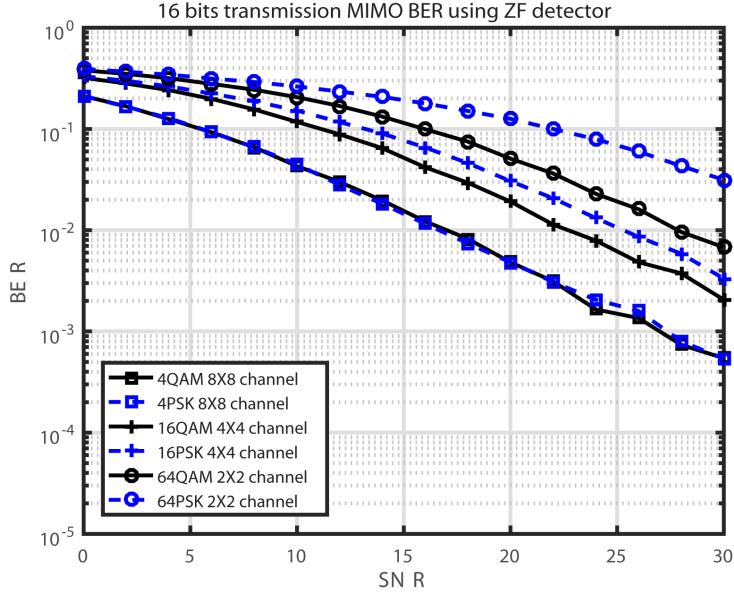


Figure 3.2: The ZF-MIMO detector performance with different QAM PSK constellation and channel size in 16 bit transmission Solid line refers to QAM and dashed line refers to PSK

A linear detector is a filter \mathbf{F} that multiplies the receiver signal vector. And it is a parallel decision on different layers. Zero-forcing refers to the detector which perfectly suppresses the interference between layers which is defined as the Moore-Penrose pseudo-inverse of the channel matrix [51].

$$\mathbf{F}_{ZF} = \mathbf{H}^+ = (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \quad (3.2)$$

Here, \mathbf{H} is randomly generated Rayleigh Fading channel matrix and is full rank. After filtered by \mathbf{F}_{ZF} . the receive signal \mathbf{y} becomes :

$$\bar{\mathbf{x}}_{ZF} = \mathbf{F}_{ZF} \mathbf{y} = \mathbf{x} + (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{n} \quad (3.3)$$

Then, the covariance matrix of the estimation error is :

$$\Phi_{ZF} = E\{(\bar{\mathbf{x}} - \mathbf{x})(\bar{\mathbf{x}} - \mathbf{x})^\dagger\} = \sigma_n^2 (\mathbf{H}^\dagger \mathbf{H})^{-1} \quad (3.4)$$

Therefore, in ZF-MIMO, the power of error is linearly increased with the power of noise. Hence the ZF-MIMO detector is not suitable to the high noise power scenario. And if the eigenvalue of \mathbf{H} is small. The noise will be greatly amplified

by $(\mathbf{H}^\dagger \mathbf{H})^{-1}$. According to the random matrix theory [52], if $M_t = M_r$ and tends infinity , the noise amplification will tend to infinity.

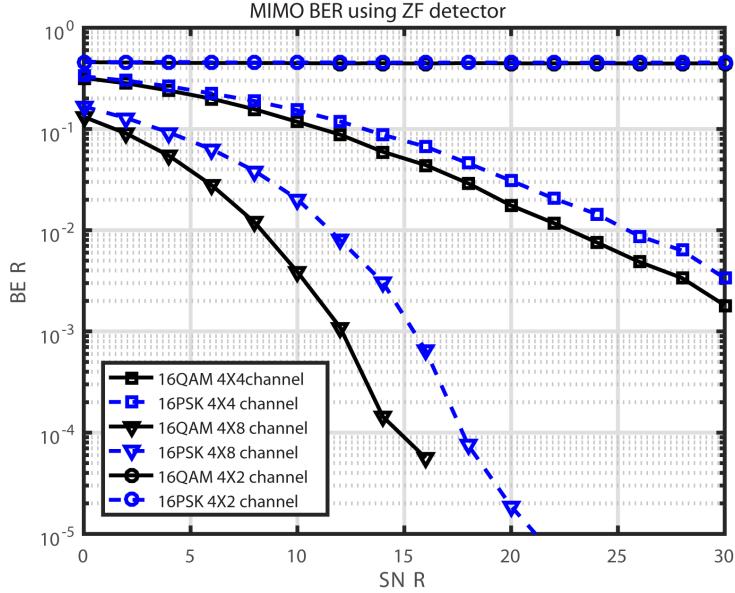


Figure 3.3: The ZF-MIMO detector performance with different number of transmit antennas. Solid line refers to QAM and dashed line refers to PSK

From Fig (3.2), 4QAM has the best performance. Even 64QAM and 16QAM outperform 64PSK and 16PSK. The difference between 4QAM and 4PSK is very small due to similar constellation distribution. The performance become worse with the increase of the number of constellation symbol as the distance between two closest symbol become shorter. From Fig (3.3), the ZF-MIMO detector is more suitable for downlink transmission when the number of transmit antennas is smaller than that of receive antennas. Big number of receive antennas means high dimension receive noise which improves the BER performance of system. In general, the number of receive antennas should be larger than transmit antennas to ensure full column rank of channel matrix \mathbf{H} .

3.2.2 Minimum Mean Square Error detection

The noise is enhanced in the ZF-MIMO detection. The trade-off between interference suppression and noise enhancement is made by MMSE detection.

The linear MMSE detector filter matrix is [51]:

$$\mathbf{F}_{MMSE} = (\mathbf{H}^\dagger \mathbf{H} + \sigma_n^2 \mathbf{I}_{M_t})^{-1} \mathbf{H}^\dagger \quad (3.5)$$

The output of the filter is:

$$\bar{\mathbf{x}}_{MMSE} = (\mathbf{H}^\dagger \mathbf{H} + \sigma_n^2 \mathbf{I}_{Mt})^{-1} \mathbf{H}^\dagger \mathbf{x} \quad (3.6)$$

the error covariance matrix become:

$$\Phi_{MMSE} = \sigma_n^2 (\mathbf{H}^\dagger \mathbf{H} + \sigma_n^2 \mathbf{I}_{Mt})^{-1} \quad (3.7)$$

From Fig(3.3) and the formula of MMSE-MIMO detector. When the noise power is small, the formula tends to ZF-MIMO. In the large noise scenario, Φ_{MMSE} tends to zero and should have a better performance than ZF-MIMO theoretically.

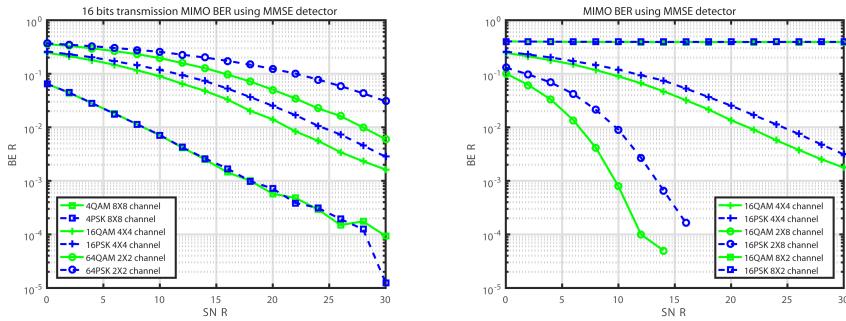


Figure 3.4: Left: The MMSE-MIMO detector performance with different QAM PSK constellation and channel size in 16 bit transmission Solid line refers to QAM and dashed line refers to PSK. Right: The MMSE-MIMO detector performance with different number of transmit antennas. Solid line refers to QAM and dashed line refers to PSK

3.2.3 Comparison between ZF-MIMO and MMSE-MIMO

The comparison between the two detectors is based on the spectral efficiency and channel size, in both Fig(4.5)(4.6), the MMSE-MIMO outperforms ZF-MIMO in all the conditions which proves the theory above.

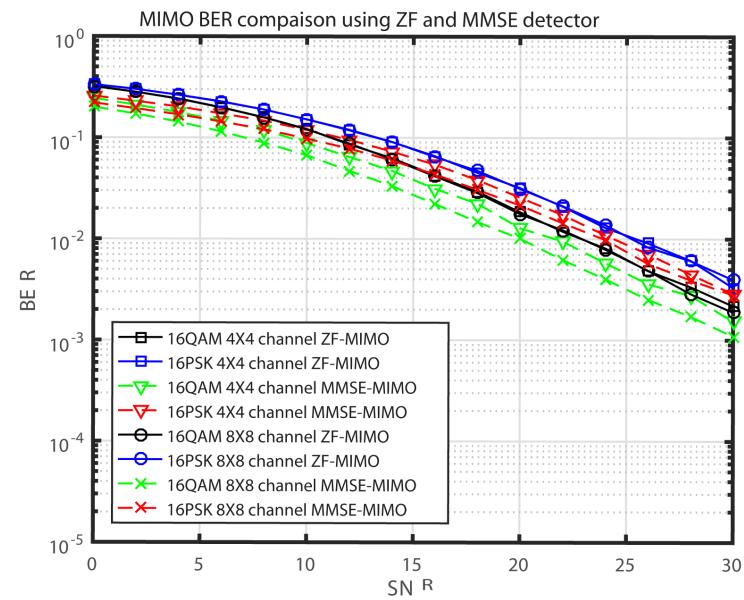


Figure 3.5: The performance comparison with different QAM PSK constellation and channel size . Solid line refers to ZF and dashed line refers to MMSE

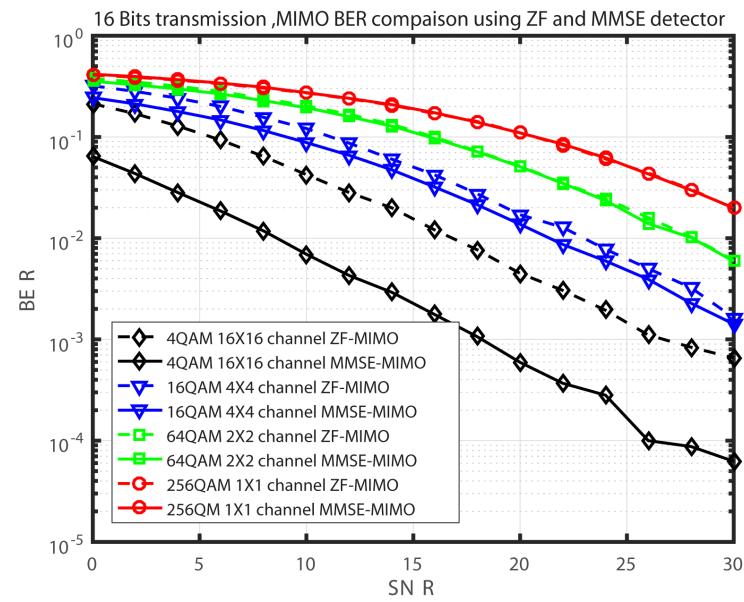


Figure 3.6: The performance comparison with different QAM constellation and channel size in 16 bit transmission Solid line refers to MMSE and dashed line refers to ZF

3.3 Space Shift Keying modulation in MIMO system

In traditional MIMO system, we use M_t transmit antennas simultaneously. However, in shift space keying (SSK) MIMO system, only one antenna is activated (i.e. active antenna). SSK modulation consists of m_b bits to map the transmission symbol x_j where $x_j \in (0, 1)$ and $E[\mathbf{x}^\dagger \mathbf{x}] = 1$. The x_j equals 1 only at the position of the active antenna. Then \mathbf{x}_j has the following form:

$$\mathbf{x}_j = [0 \cdots 0 \underset{j^{th} position}{1} 0 \cdots 0] \quad (3.8)$$

Then, the receive vector at the receiver is $\mathbf{y} = \sqrt{\rho} \mathbf{h}_j + \mathbf{n}$. The \mathbf{h}_j is the j th column of corresponding channel matrix \mathbf{H} . Because the active antenna position bits are randomly generated. The specified column of the channel matrix can be seen as random constellation points for space shift keying modulation[44]. Maximum likelihood detector is the optimal detector for SSK modulation. The optimal detector is written as:

$$\hat{j} = \underset{j}{\operatorname{argmax}} p_Y(\mathbf{Y} | \mathbf{x}_j, \mathbf{H}) = \underset{j}{\operatorname{argmin}} \|\mathbf{y} - \sqrt{\rho} \mathbf{h}_j\|_F^2 \quad (3.9)$$

After some simplification, it becomes:

$$\hat{j} = \underset{j}{\operatorname{argmax}} \operatorname{Re}\left\{(\mathbf{y} - \frac{\sqrt{\rho}}{2} \mathbf{h}_j)^H \mathbf{h}_j\right\} \quad (3.10)$$

Table 3.1: Table I
SSK mapping rule m=3bits

$\mathbf{b} = [b_1 \ b_2 \ b_3]$	symbol	antenna index j	$\mathbf{x} = [x_1 \ \cdots \ x_8]^T$
[0 0 0]	0	1	[1 0 0 0 0 0 0 0]
[0 0 1]	1	2	[0 1 0 0 0 0 0 0]
[0 1 0]	2	3	[0 0 1 0 0 0 0 0]
[0 1 1]	3	4	[0 0 0 1 0 0 0 0]
[1 0 0]	4	5	[0 0 0 0 1 0 0 0]
[1 0 1]	5	6	[0 0 0 0 0 1 0 0]
[1 1 0]	6	7	[0 0 0 0 0 0 1 0]
[1 1 1]	7	8	[0 0 0 0 0 0 0 1]

The entries of the channel matrix \mathbf{H} is randomly generated and can be seen as random constellation points. It is meaningful because the constellation space is much larger than ASK, PSK and QAM in which the value of symbols are fixed and have orientation. Note that using the antenna index as the information source actually deliver the channel information which is known at the

receiver. However, the number of the transmit antennas must be power of 2 and the spectral efficiency is very low compared to traditional MIMO system. The spectral efficiency of SSK-MIMO and traditional MIMO are $\log_2 M_t$ and M_t with the 0/1 as the transmission symbol.

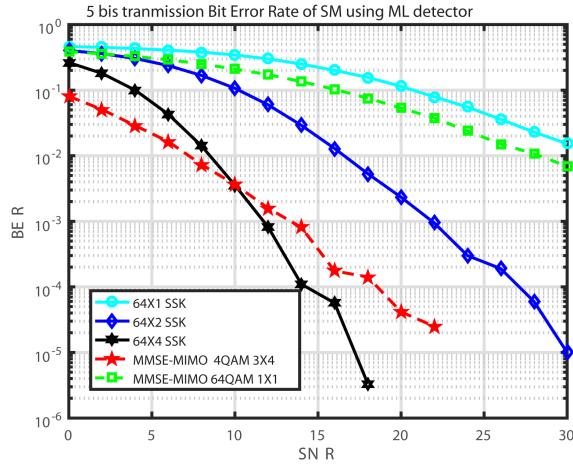


Figure 3.7: The performance comparison 6 bits transmission . Solid curve refers to SSK and dashed curve refers to MMSE-MIMO

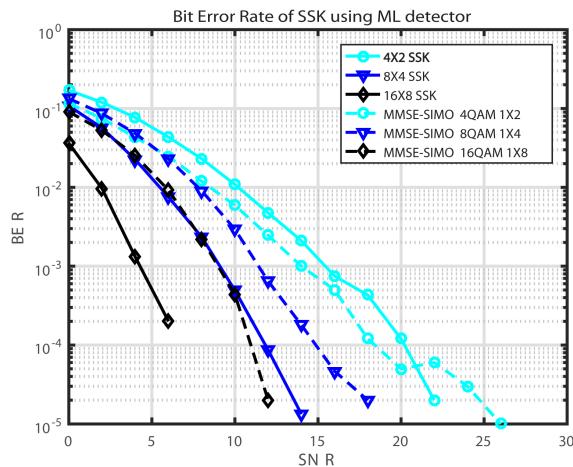


Figure 3.8: The performance comparison with same receive antennas and possibilities of constellation symbols. Solid curve refers to SSK and dashed curve refers to MMSE-MIMO

From Fig (3.7), adding more receive antennas increases the BER performance. The SSK-MIMO outperforms the MMSE-MINO at 10dB and exhibits

great potential at multi transmit antennas scenario. Fig (3.8) is meaningful, because it demonstrates advantages of space constellation symbols over PAM constellation symbols. SSK-MIMO has more spatial efficiency than PAM with random phase directions and amplitudes. SSK-MIMO has better performance than other modulation types when the number of transit antennas is over 4. Even though MMSE-MIMO has better BER at 4 QAM in low SNR condition, SSK-MIMO still outperforms it at high SNR (around 25dB). The reason that SSK-MIMO has better performance is that SSK has larger distance between two symbols and more trasmit antennas to increase the spatial multiplexing gain. In addition, in traditional MIMO, we need to make sure that the number of transmit antenna smaller than that of receive antenna to get good performance. From Fig (3.8), it is clear that increasing the number of transmit antennas actually reduces the BER because only one antenna is activated at one time so that we don't need to worry about the rank problems of that in traditional MIMO as the "active antenna" is always one in SSK.

3.4 Generalized space shift keying

The realization of generalized space shift keying (GSSK) modulation comes from the antenna swithcing [45]. If we want to transmit information bits via SSK by the antenna index. The antenna index need to change and maintain a switching state . However, because of the process of pulse shaping, the transmitted pulse will go through several periods of the transmitted symbol and restrict the RF chain if only one active antennas. Then, we need the number of the active antennas to equal the number of symbol periods that the shaped pulse extends. And active antennas need equal number of RF chains which contributes to the main cost of wireless system. However, the number of RF chains in GSSK-MIMO and SSK-MIMO is much smaller than traditional MIMO [41].

In GSSK-MIMO, more than one antennas is activated each time. We make the assumption that M_c is the number of active antenna. There are $M = \binom{M_t}{M_c}$ possible constellation symbol combination. For, example, we have $M_t = 9, M_c = 3$ and then $M = 84$. However, the number of possible combination must be the power of 2 because we are transmitting binary bits. The maximum number of $\hat{M} = 64$ in the above case. The number of bits that used to transmit the position information is:

$$m = \log_2 \hat{M} = \lfloor \log_2 M \rfloor = \lfloor \log_2 \binom{M_t}{M_c} \rfloor \quad (3.11)$$

For the consideration of spectral efficiency, the largest number of m of fixed M is:

$$m = \lfloor \log_2 \left(\frac{M_t}{\lfloor \frac{M_t}{2} \rfloor} \right) \rfloor \quad (3.12)$$

Then, GSSK-MIMO still need to satisfy the signal power restriction $E[\mathbf{x}^\dagger \mathbf{x}] = 1$. The vector \mathbf{x}_j has the following form:

$$\mathbf{x}_j = [\underbrace{\frac{1}{\sqrt{M_c}} \quad 0 \quad \cdots \quad 0}_{M_c \text{ non-zero value}} \quad \underbrace{\frac{1}{\sqrt{M_c}} \quad \cdots \quad \frac{1}{\sqrt{M_c}} \quad 0}_{}]^T \quad (3.13)$$

The Table II shows the example of $M_t = 5$, $M_c = 2$ and the 2 active antenna is randomly chosen because of randomly generated information bits. Then, the receive vector become:

$$\mathbf{y} = \sqrt{\rho} \mathbf{h}_{\mathbf{j}, M_c} + \mathbf{n} \quad (3.14)$$

Here, $\hat{\rho} = \frac{\rho}{M_c}$ and $\mathbf{h}_{\mathbf{j}, M_c} = \mathbf{h}_{\mathbf{j}, 1} + \mathbf{h}_{\mathbf{j}, 2} + \cdots + \mathbf{h}_{\mathbf{j}, M_c}$ $\mathbf{j}(\cdot)$ is the column index of channel matrix \mathbf{H} . $\mathbf{h}_{\mathbf{j}, M_c}$ is the sum of them. Then, we also use ML detector for GSSK-MIMO which has similar form as that in SSK-MIMO:

$$\hat{j} = \underset{j}{\operatorname{argmax}} p_Y(\mathbf{Y} \mid \mathbf{x}_{\mathbf{j}, M_c}, \mathbf{H}) = \underset{j}{\operatorname{argmin}} \|\mathbf{y} - \sqrt{\rho} \mathbf{h}_{\mathbf{j}, M_c}\|_F^2 \quad (3.15)$$

After some simplification, it becomes:

$$\hat{j} = \underset{j}{\operatorname{argmax}} \operatorname{Re}\{(\mathbf{y} - \frac{\sqrt{\rho}}{2} \mathbf{h}_{\mathbf{j}, M_c})^H \mathbf{h}_{\mathbf{j}, M_c}\} \quad (3.16)$$

$$p_Y(\mathbf{Y} \mid \mathbf{x}_{\mathbf{j}, M_c}, \mathbf{H}) = \frac{1}{\pi^{M_r}} \exp(-\|\mathbf{y} - \sqrt{\rho} \mathbf{H} \mathbf{x}_{\mathbf{j}}\|_F^2) \quad (3.17)$$

Table 3.2: Table II
GSSK mapping rule m=3bits 2 active antennas 5 transmit antennas

$\mathbf{b} = [b_1 \quad b_2 \quad b_3]$	symbol	antenna index j	$\mathbf{x} = [x_1 \quad \cdots \quad x_5]^T$
[0 0 0]	0	(1,2)	$[\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0 \quad 0]$
[0 0 1]	1	(1,3)	$[\frac{1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0]$
[0 1 0]	2	(1,4)	$[\frac{1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0]$
[0 1 1]	3	(1,5)	$[\frac{1}{\sqrt{2}} \quad 0 \quad 0 \quad 0 \quad \frac{1}{\sqrt{2}}]$
[1 0 0]	4	(2,3)	$[0 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0]$
[1 0 1]	5	(2,4)	$[0 \quad \frac{1}{\sqrt{2}} \quad 0 \quad \frac{1}{\sqrt{2}} \quad 0]$
[1 1 0]	6	(2,5)	$[0 \quad \frac{1}{\sqrt{2}} \quad 0 \quad 0 \quad \frac{1}{\sqrt{2}}]$
[1 1 1]	7	(3,4)	$[0 \quad 0 \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 0]$

The simulation results are based on ML detector and iterative ratio combining detector(i-MRC). The principle of i-MRC will be illustrated in the spatial modulation part.

The development from SSK-MIMO to GSSK-MIMO has several advantages. Firstly, it reduces the number of transmit antennas and the number of transmit antennas does not need to be the power of 2 any more. Secondly, activating more antennas can meet the requirement of the shaped pulse period to increase the time utilizing rate. Third, GSSK-MIMO has higher transmit diversity gain than SSK-MIMO.

The simulation shows GSSK-MIMO performance versus SSK-MIMO and MMSE-MIMO with spectral efficiency of 3bits,4bits and 6bits, respectively. Note that all the BER performance experience same dimension noise $M_r = 4$ and come from 50000 realizations.In the simulation figures. (M_t, M_c) denotes the number of transmit antennas and the number of active antennas.

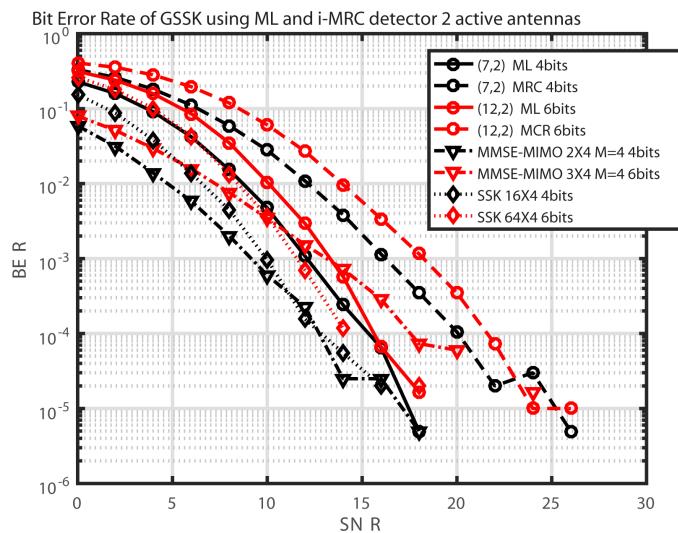


Figure 3.9: The performance comparison among GSSK ,SSK and MMSE-MIMO for spectral efficiency of 4 and 6 bits with black and red curves respectively. Solid circle curve denotes GSSK using ML detector. Dashed circle curve denotes GSSK using MRC detector. Dashdot triangle curve denotes MMSE-MIMO method and dotted diamond curve denotes SSK using ML method.

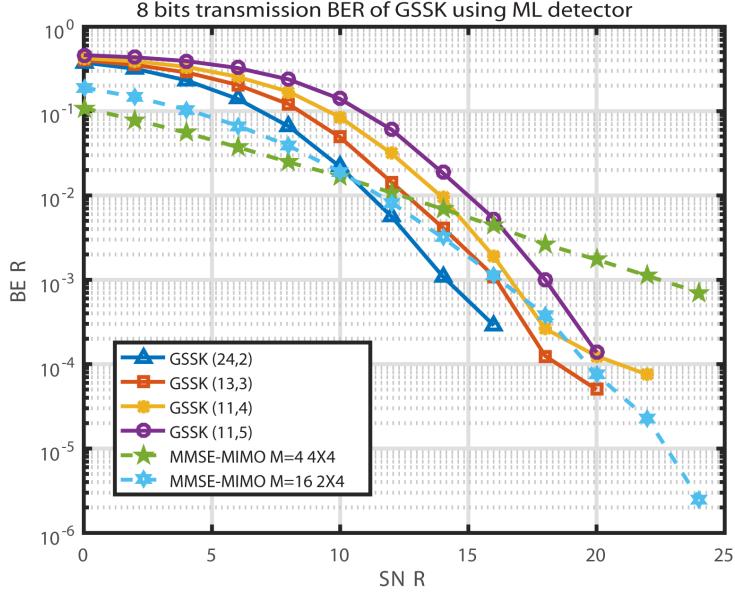


Figure 3.10: The 8 bits transmission between GSSK with different combination of (transmit, active). Solid curves denotes GSSK-MIMO. Dashed curves denote MMSE-MIMO.

From Fig(3.9), the ML detector has better performance in GSSK-MIMO than MCR as expected. The GSSK-MIMO of combination of (5,2) outperforms MMSE-MIMO when SNR is bigger than around 14.5 dB at spectral efficiency of 3bits which is 18dB and 14dB for combination of (7,2) and (12,2) at spectral efficiency of 4 and 6 bits. In general ,SSK-MIMO has the best performance when SNR is higher than 10dB at the expense of large number of transit antennas and MMSE-MIMO has the best performance when SNR is lower than 10dB. However, GSSK-MIMO has the fastest falling speed than all other schemes and demonstrates great potential at high SNR scenarios.

From Fig(3.10), the system performance degrades with the increase of the number of active antennas because of the interference between different transmit antennas which is the same in MMSE-MIMO. Increasing the active antennas will reduce the average distance between two transmit symbol constellation points($0/\sqrt{M_c}$) which will cause degradation to the system as expected. Note that, GSSK-MIMO (24,2) gains 2dB and 8dB in SNR over MMSE-MIMO at the BER of 10^{-3} which is 0.1dB and 6dB for GSSK-MIMO (13,3). The (11,4) and (11,5) outperform MMSE-MIMO at SNR of 17dB and 22dB.

3.5 Spatial Modulation

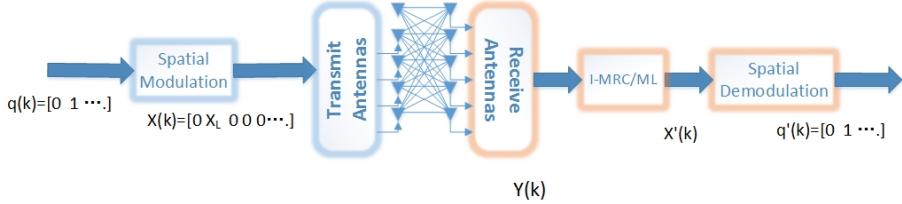


Figure 3.11: Spatial Modulation system model.

The difference between SSK-MIMO and spatial modulation (SM) MIMO is that SSK-MIMO just transmits symbol 0/1 from each antennas. But SM-MIMO transmits symbols with different constellation symbols values of corresponding modulation types .The number of transmit antennas still need to be power of 2. Compared with SSK-MIMO, the spatial modulation is a spectral efficiency enhancing technique [43]. Because it increase the number of constellation symbols to decrease the transmit antennas. The mapping rule can be seen in Table III and IV for 4QAM and BPSK. The number of bits that can be transmit-

Table 3.3: Table III
SM mapping rule $m=3$ bits $M_t = 2 \quad M = 4$

$\mathbf{b} = [b_1 \quad b_2 \quad b_3]$	symbol	antenna index j	$\mathbf{x} = [x_1 \quad x_2]^T$
[0 0 0]	+1+j	1	[+1+j 0]
[0 0 1]	-1+j	1	[-1+j 0]
[0 1 0]	-1-j	1	[-1-j 0]
[0 1 1]	+1-j	1	[+1-j 0]
[1 0 0]	+1+j	2	[0 +1+j]
[1 0 1]	-1+j	2	[0 -1+j]
[1 1 0]	-1-j	2	[0 -1-j]
[1 1 1]	+1-j	2	[0 +1-j]

ted on each OFDM subchannel in a system which utilizes a M-ary modulation constellation points diagram is:

$$\hat{m} = \log_2 (M_t) + \log_2 (M) \quad (3.18)$$

3.5.1 iterative maximum ratio combining detection

Then, because of the use of different constellation symbols values, the scenarios becomes similar in SIMO system. Then, the ML detector has high complexity in SM. In order to solve this high complexity problem, a detection algorithm called *iterative maximum ratio combining* (i-MRC) is shown. In this algorithm,

at one specified time, only one transmit antenna is transmitting information bits. The channel vectors between each transmit antenna and the number of receive antenna are considered separately at receiver. The receiver iteratively computes the MRC results between the channel paths from each transmit antenna to the corresponding receive antennas [43]. We make the assumption that receiver has full CSI and receiver will use transmit antenna index which has the biggest correlation value with the channel column vectors. The process of i-MRC detection can be written as:

$$g_j = \mathbf{h}_j^H \mathbf{y}, \text{For } j = 1 : Mt \quad (3.19)$$

$$\mathbf{g} = [g_1 \ g_2 \ \cdots \ g_{M_t}]^T \quad (3.20)$$

$$\hat{l} = \underset{j}{\operatorname{argmax}} |\mathbf{g}| \quad (3.21)$$

$$\hat{\mathbf{x}}_l = Q(\mathbf{g}_{j=\hat{l}}) \quad (3.22)$$

$Q(\cdot)$ is the constellation slicing function.

3.5.2 Optimal detection

The signal vector of SM-MIMO still meet the normalized power constraint $E[\mathbf{x}^\dagger \mathbf{x}] = 1$. The constellation vector has the following form:

$$\mathbf{x}_j = [0 \cdots 0 \underset{j^{th} position}{x_q} 0 \cdots 0] \quad (3.23)$$

Table 3.4: Table IV
SM mapping rule m=3 bits $M_t = 4 \ M = 2$

$\mathbf{b} = [b_1 \ b_2 \ b_3]$	symbol	antenna index j	$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$
[0 0 0]	-1	1	[-1 0 0 0]
[0 0 1]	+1	1	[+1 0 0 0]
[0 1 0]	-1	2	[0 -1 0 0]
[0 1 1]	+1	2	[0 +1 0 0]
[1 0 0]	-1	3	[0 0 -1 0]
[1 0 1]	+1	3	[0 0 +1 0]
[1 1 0]	-1	4	[0 0 0 -1]
[1 1 1]	+1	4	[0 0 0 +1]

The x_q is scalar in modulation constellation diagram. The output of the channel transmitting x_q on j^{th} antenna is:

$$\mathbf{y} = \sqrt{\rho} \mathbf{h}_j x_q + \mathbf{n} \quad (3.24)$$

\mathbf{h}_j is the j^{th} column of \mathbf{H} . The optimal detector is based on the ML principle:

$$[\hat{j}_{ML}, \hat{q}_{ML}] = \underset{j,q}{\operatorname{argmax}} p_Y(\mathbf{Y} | \mathbf{x}_{jq}, \mathbf{H}) \quad (3.25)$$

$$[\hat{j}_{ML}, \hat{q}_{ML}] = \underset{j,q}{\operatorname{argmin}} \sqrt{\rho} \|\mathbf{g}_{jq}\|_F^2 - 2 \operatorname{Re}\{\mathbf{y}^H \mathbf{g}_{jq}\}, \quad (3.26)$$

$$p_Y(\mathbf{Y} | \mathbf{x}_{jq}, \mathbf{H}) = \frac{1}{\pi^{M_r}} \exp(-\|\mathbf{y} - \sqrt{\rho} \mathbf{H} \mathbf{x}_{jq}\|_F^2) \quad (3.27)$$

The ML detection has a joint detection of constellation symbols values and antenna position index. It has better performance but still experiences a relatively higher complexity than i-MRC detector. But it is much smaller than MMSE-MIMO and ZF-MIMO. The complexity comparison will be shown at chapter 4. The disadvantage of SM-MIMO is that the increase of spectral efficiency is based on the two logarithm which is much smaller than that of traditional MIMO with large transmit antenna number.

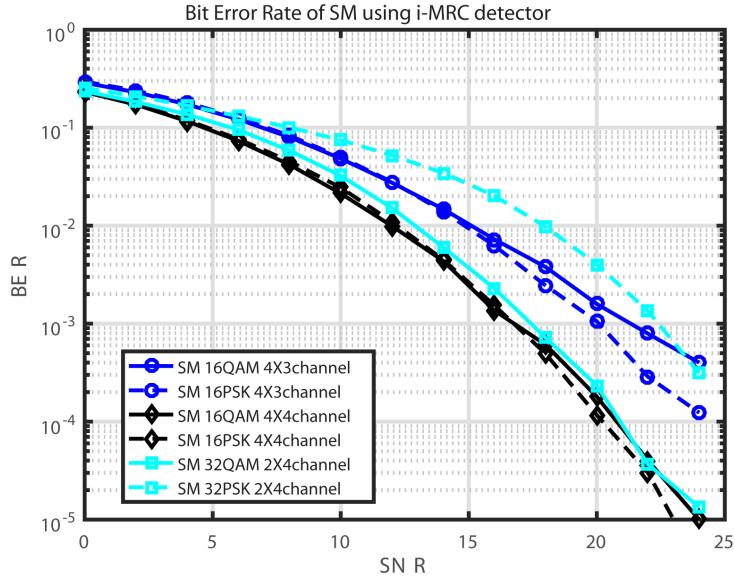


Figure 3.12: The 8 bits transmission between GSSK with different combination of (transmit, active). Solid curves denotes GSSK-MIMO. Dashed curves denote MMSE-MIMO.

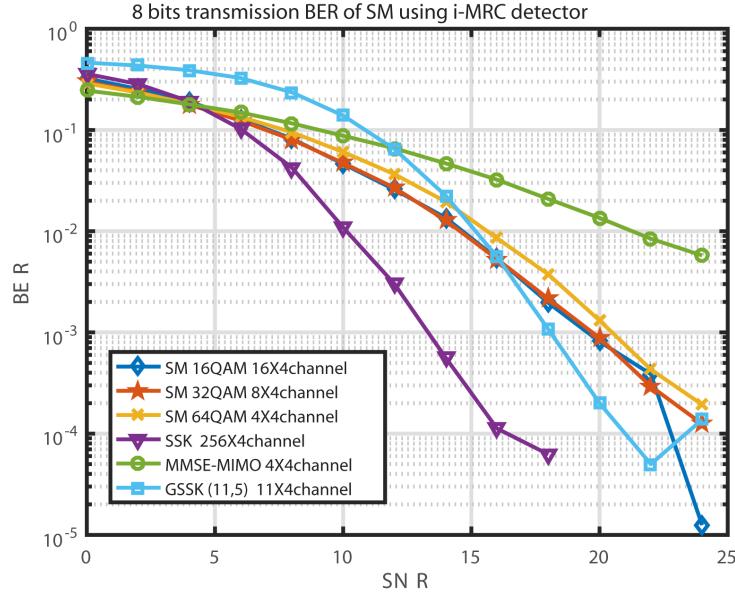


Figure 3.13: The 8 bits transmission between GSSK with different combination of (transmit, active). Solid curves denotes GSSK-MIMO. Dashed curves denote MMSE-MIMO.

From Fig(3.12), QAMs outperform PSKs for 6dB and 7.5 dB in SNR at 32-ary and 64-ary with a BER of $10^{(-2)}$. However, for 16-ary, the PSK have better performance (around 0.5dB) than QAM. Increasing the number of transmit and receive antennas in spatial modulation will increase the performance. That's why 32QAM 2X4 has similar BER to 16QAM 4X4. The 32QAM 2X4 is a little bit better at high SNR region. With the same channel size, larger M-ary QAM and PSK degrades the system performance, resulting in a 3dB gap between 16QAM and 64QAM.

Fig (3.13) is clear because there are some cross points in the figure. The first one is on the SNR of around 5dB. SSK outperform all other schemes in BER and MMSE-MIMO becomes the worst one when SNR is larger than 12 dB. The second one is around 16dB when GSSK-MIMO outperforms all the SM-MIMO cases and gains 3dB in SNR at BER of 10^{-3} . SM-MIMO exhibits great performance in the moderate SNR region.

3.6 Generalized spatial modulation

3.6.1 The idea to choose most efficient combination

The combination here is the same as GSSK-MIMO with style of (M_t, M_c) . In this part ,we will demonstrate a new efficient way to choose combinations for fixed transmit antennas and fixed mapping bits m .

For fixed number of transmit antennas M_t , the choice of active antenna should base on the spectral efficiency , transmit diversity gain and BER. For M -ary QAM modulation type, the choice of the number of active number should be $\frac{M_t}{2}$ for even number and $\frac{M_t+1}{2}$ for odd number. For example, M is very large (e.g. 256) and M_t is 11. $\lfloor \log_2 \left(\binom{11}{4} \right) \rfloor = \lfloor \log_2 \left(\binom{11}{5} \right) \rfloor = \lfloor \log_2 \left(\binom{11}{6} \right) \rfloor = 8$. We should choose 6 to mostly increase the transmit diversity gain and maintain high spectral efficiency. For consideration of BER, smaller number of active antennas is better. Then, we choose 4 in this case. Then if we want to make a trade-off between spectral efficiency , transmit diversity gain and BER. The 5 active antennas is the best one. If the number of transmit antennas is very large (e.g. 9999), the choice of 4999,5000,5001, and 5002 has the same spectral efficiency. However, the transmit diversity is large enough for each one of them. In this scenario, BER performance is more important. Hence we choose 4999 or smaller one with the same spectral efficiency.The choice table is shown below:

Table 3.5: Table V

GSM active antenna excitation method with fixed spectral efficiency . Column one denotes the number of transmit antennas. 'For diversity gain' in Column two denotes the number of active antennas that have maximum diversity gain.

'For BER' in Column 3 denotes the number of active antennas that have maximum BER performance.

No. of M_t	For diversity gain	For BER	$\lfloor \log_2 \left(\binom{M_t}{M_c} \right) \rfloor$ bits/s/Hz
11	6	4	8
12	7	5	9
13	8	5	10
14	8	6	11
15	9	6	12
16	9	7	13
17	10	7	14
18	10	8	15

On the other side, for fixed number of mapping bits m , we define the utilization efficiency η as:

$$\eta = \frac{2^m}{\binom{M_t}{M_c}} \times 100\% \quad (3.28)$$

And in the above formula $m = \lfloor \log_2 \left(\binom{M_t}{M_c} \right) \rfloor$. For example, if $m = 3, 2^m = 8$

and $\binom{M_t}{M_c} > 8$ but still closest to 8. Then we consider 9 or 10. By simple factorization, 9 is impossible and $10 = \binom{5}{2} = \binom{5}{3}$. Then, we choose one from $\binom{M_t}{M_c}$ for consideration of diversity gain and BER performance. The table below show the choice for highest efficiency (the choice of the combination tends to the half of the transmit antenna for least number of transmit antennas):

Table 3.6: Table VI

GSM high utilization efficiency method //Column one denotes the number of bits that use to transmit the position. Column 2 is the best choice of tranmit antenna in this method . 'For diversity gain' in Column 3 denotes the number of active antennas that have maximum diversity gain. 'For BER' in Column 4 denotes the number of active antennas that have maximum BER performance.

No. of m	M_t	For diversity gain M_c	For BER M_c	efficiency η
3	5	3	2	80%
4	6	3	3	93%
5	7	4	3	91%
6	8	4	4	91%
7	10	6	4	61%
8	11	7	4	78%
9	12	7	5	65%
10	13	8	5	80%

From table VI, the high utilization efficiency method has average η of 80% and will be higher when m is large as the increasing ratio of combination number is smaller than exponential growth. Discussing the utilization efficiency here is meaningful. Because when we apply GSM to MIMO system, we don't want that some antennas will not work or just work for once or twice in a long time. What we hope is to explore the usage frequency of every antenna to maintain a green and sustainable future without wasting too many transmit antennas.

3.6.2 GSM-MIMO system model

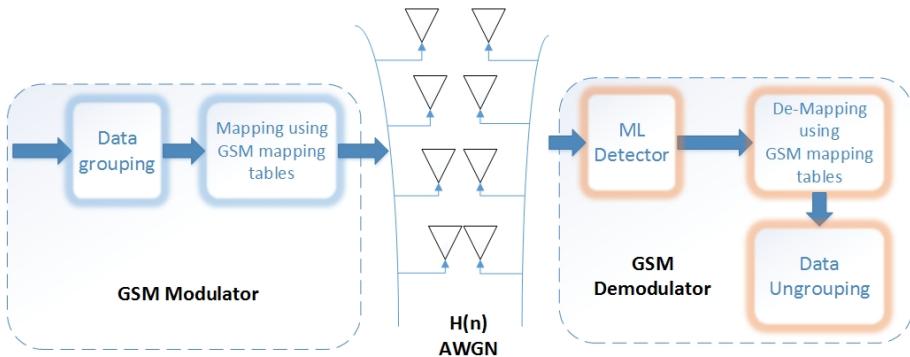


Figure 3.14: Generalized Spatial Modulation system model.

Even though the two logarithmic increase the spectral efficiency of SM-MIMO. The number of antennas must be a power of 2 and sometimes it is not practical in practical life. The relationship between generalized spatial modulation (GSM) MIMO and SM-MIMO is similar to that between GSSK-MIMO and SSK-MIMO. At a time, more than one transmit antennas are activated to transmit information bits simultaneously. And to achieve the same spectral efficiency, GSM only need no more than a half of the antennas that SM-MIMO needs. By transmitting replicas of signals, GSM-MIMO offers spatial diversity gain [47] which enhances the system feasibility and reliability . Then , the active antennas should be synchronised to avoid ISI. ML detector and i-MRC detector are used to estimates the antenna position index and the value of the constellation symbol.

The same as GSSK-MIMO, GSM-MIMO takes advantage of more one active antennas to send equal complex symbol. For example, in the BPSK modulation where $M = 2$ for combination of (5,2) is used. If the group bits isg(n)[0 0 0 0], it will be mapped to $\mathbf{x}(n) = [+1 + 1 0 0 0]$. The mapping table is shown below: In general, the number of bits transmitted using GSM-MIMO is:

$$m = m_l + m_s = \lfloor \log_2 \left(\frac{M_t}{M_c} \right) \rfloor + \log_2 M \quad (3.29)$$

The receiver vector can be written as :

$$\mathbf{y} = \mathbf{h}_l s + \mathbf{n} \quad (3.30)$$

$s \in M$ -ary modulation types. At the receiver, using the ML principle for detection:

$$[\hat{l}, \hat{s}] = \underset{l,s}{\operatorname{argmax}} p_y(\mathbf{y} \mid \mathbf{x}_{ls}, \mathbf{H}) = \underset{l,s}{\operatorname{argmax}} \sum_{i=1}^{M_r} |y_i - h_{l,s}s|^2 \quad (3.31)$$

where

$$p_Y(\mathbf{y} \mid s, l, \mathbf{H}) = \frac{1}{(\pi \sigma_n^2)^{M_r}} \exp\left(-\frac{\|\mathbf{y} - \sqrt{\rho} \mathbf{h}_l s\|_F^2}{\sigma_n^2}\right) \quad (3.32)$$

In GSM-MIMO, arbitrary number of transmit antennas are used. And GSM-MIMO achieve higher spectral efficiency with much lower number of transmit antennas. In addition, GSM-MIMO also avoids the inter channel interference which is the key advantage of SM-MIMO. in addition, spatial multiplexing gain is also obtained by GSM-MIMO. But these achievements are based on the increase of the complexity. However, the complexity increase is outweighed by the significant reduction of the number of transmit antennas.

Table 3.7: Table VII
GSM mapping rule m=4bits 2 active antennas 5 transmit antennas

$\mathbf{b} = [b_1 \ b_2 \ b_3 \ b_4]$	symbol	antenna index j	$\mathbf{x} = [x_1 \ \dots \ x_5]^T$
[0 0 0 0]	-1	(1,2)	$[\frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ 0]$
[0 0 0 1]	1	(1,2)	$[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[0 0 1 0]	-1	(1,3)	$[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[0 0 1 1]	1	(1,3)	$[\frac{1}{\sqrt{2}} \ 0 \ 0 \ 0 \ \frac{1}{\sqrt{2}}]$
[0 1 0 0]	-1	(1,4)	$[0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[0 1 0 1]	1	(1,4)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0]$
[0 1 1 0]	-1	(1,5)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ \frac{1}{\sqrt{2}}]$
[0 1 1 1]	1	(1,5)	$[0 \ 0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0]$
[1 0 0 0]	-1	(2,3)	$[\frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ 0]$
[1 0 0 1]	1	(2,3)	$[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[1 0 1 0]	-1	(2,4)	$[\frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[1 0 1 1]	1	(2,4)	$[\frac{1}{\sqrt{2}} \ 0 \ 0 \ 0 \ \frac{1}{\sqrt{2}}]$
[1 1 0 0]	-1	(2,5)	$[0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0 \ 0]$
[1 1 0 1]	1	(2,5)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ \frac{1}{\sqrt{2}} \ 0]$
[1 1 1 0]	-1	(3,4)	$[0 \ \frac{1}{\sqrt{2}} \ 0 \ 0 \ \frac{1}{\sqrt{2}}]$
[1 1 1 1]	1	(3,4)	$[0 \ 0 \ \frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0]$

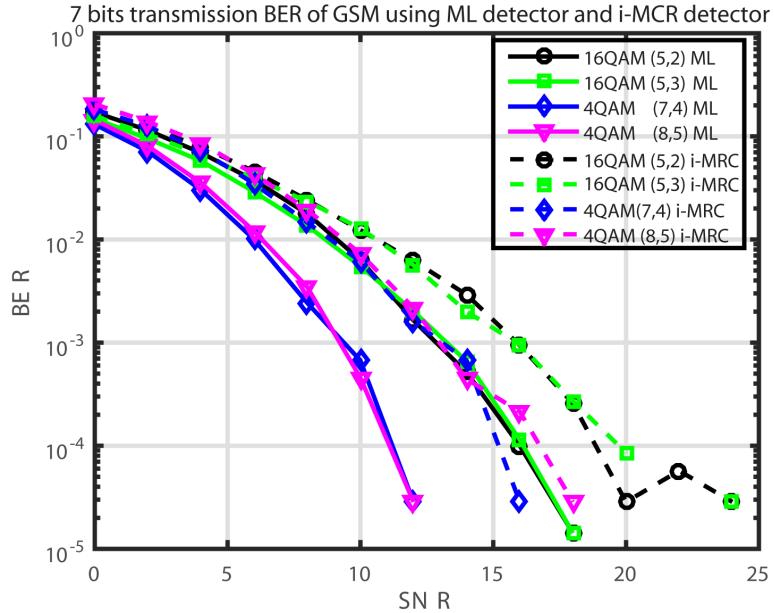


Figure 3.15: 7bits transmission between two detector in GSM. Solid line refers to ML and dashed line refer to i-MRC

From Fig(3.15), in 7 bits transmission, ML detector has better performance than i-MRC detector as expected and the gaps become wider with the increase of active antennas and transmit antennas. Note that, for fixed number of spectral efficiency , more transmit antennas and active antennas will reduce the requirement for the M in M -ary QAM which decrease the distance between two constellation symbol and has better BER performance. The similar performance between (5,2) and (5,3) demonstrates the key property of GSM-MIMO system to overcome the ICI between different transmit streams. And the similar performance between (8,5) and (7,4) seems more important because it illustrates the potential of GSM-MIMO in increasing transmit diversity and BER performance while still maintain a high spectral efficiency in moderate SNR region.

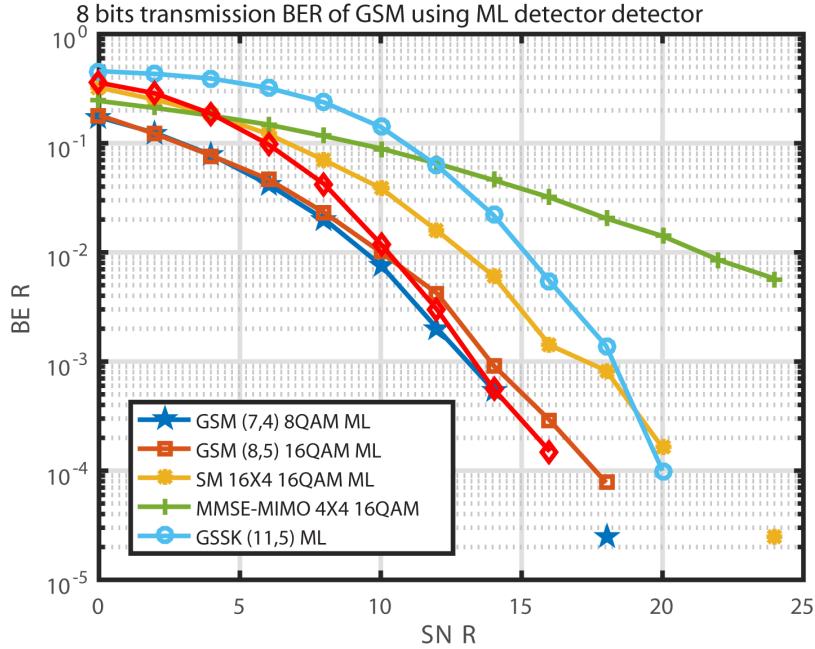


Figure 3.16: 8bits transmission over SM,GSM,SSK and GSSK.

From Fig (3,16),in low SNR region , GSM (7,4) and (8,5) have the best performance and still maintain relative low BER at moderate and high SNR region which exhibits great potential at reducing the transmit antennas compared with SM,SSK and GSSK. SSK outperforms all other schemes at SNR of 14dB at the expense of too many transmit antennas(256). GSM combinations gains around 5dB over GSSK and 3dB over SM at BER of 10^{-3} .

In a sum, for SSK,GSSK,SM and GSM, if we have infinite number of transmit antennas,SSK will be the best choice considering the BER and transmit diversity because of the spatial multiplexing gain from the logarithmic of M_t . At finite number of transmit antenna scenarios, GSSK is not better than GSM because for same spectral efficiency, GSM has lower number of active antennas to obtain shorter distance between different constellation symbol. SM seems to be a trade-off between GSSK and GSM for the complexity and BER performance in moderate SNR region. GSM exhibits great potential in reducing the number of transmit antennas.

Chapter 4

The spatial division multiplexing in MMF using SM-MIMO techniques

In this chapter, we make the same assumption as Chapter 3 that the MMF experience uniform random coupling between different modes. The channel matrix are generated from [34]. We assume to neglect the effect of nonlinearity , group delay and fibre optical loss in the fibre. Hence, there is no equalizer in the end of a segment of the fibre. We make the assumption that there exists a efficient ideal switch that can quickly switch from one mode to another. In this Chapter, the term SSK, GSSK, SM and GSM refer to SSK-MIMO, GSSK-MIMO, SM-MIMO and GSM-MIMO respectively.

4.1 The difference between wireless and fibre transmission environment

The difference between RF wireless system and MMF system in this case is that there is no fading in MMF in our assumption here. And the size of channel satisfies $M_t = M_r$. Because there is no fading in MMF, the BER performance exhibits exponential decrease with the increase of SNR in the Gaussian channel . However, in wireless communication system, they decrease linearly in Rayleigh fading channel.

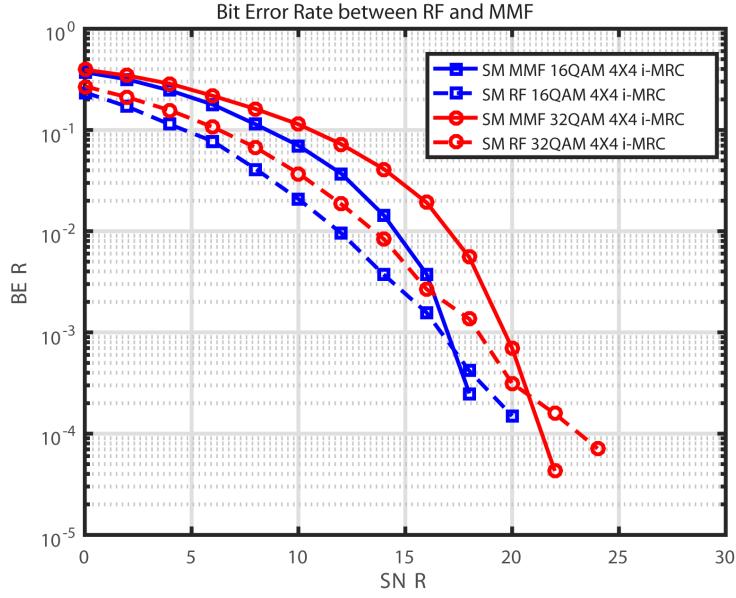


Figure 4.1: The BER difference between RF and MMF in SSK,GSSK,SM and GSM. Solid curve denotes MMF and dashed curve denotes RF.

Fig (4.1) proves the words above, the cross points are clear on the red curve , blue curve (SM in 4×4 channel). In general, the BER of MMF transmission exhibits a exponential property and wireless communication shows a linear property in BER curves. The differences between fibre transmission and wireless transmission do not have significant physical meaning in practical use. Hence ,this paper will not discuss it later.

4.2 A possible design for switch of SM-MIMO system in MMF

Applying the SM-MIMO techniques in MMF need to solve a key problem: mode switch. This mode switch can arrange the input signal to the desire mode. However, the diameter of the fibre is always in micrometer scale. Hence it is not easy to design it. This part is to show a possible design for the mode switch based on photonic lantern (PL) [53] [54] [55] [56].

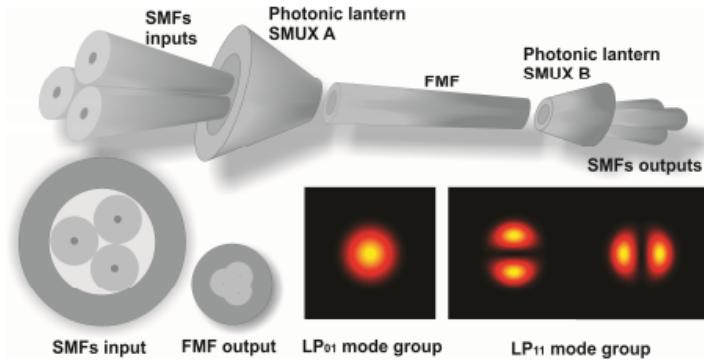


Figure 4.2: The model of photonic lantern spatial multiplexer 3-mode fibre system. The mode group is shown in black box [57]

PL is a new spatial multiplexer (SMUX) in few mode fibre. The SMUX is finished by utilizing the uncoupled cores of the MCF as independent channels. Different modes is transmitted on different SMF and than couple to a MMF by PL SMUX shown in Fig (4.2).The spatial demultiplexer (SDEMUX) is just the inverse of SMUX as is shown in Fig (4.2). Note that one key property of SM-MIMO techniques is that they successfully avoid the inter channel interference (ICI) so that the channels in SM-MIMO system is independent to each other. And the uncoupled cores in PL meet the requirement of ICI suppression in SM-MIMO. In addition, the head of the PL consists of several SMFs and each SMF connects to the same light laser source . Then, the design of mode switch is simplified to transmit different modes to each SMF each time. This laser source in Fig (4.3) is a all-mode source. It can generate light with different desired modes according to the input control pattern. This control pattern also control the full reflection rotating device on the right with specified angle to reflect the light to the corresponding SMF. Then, this control pattern also satisfies the logarithmic property of SM-MIMO techniques.

This design greatly reduce the number of laser source which reduce the potential cost of the whole system. The synchronization of the rotation device and laser source is very important to ensure the performance of the SM-MIMO techniques in MMF system.

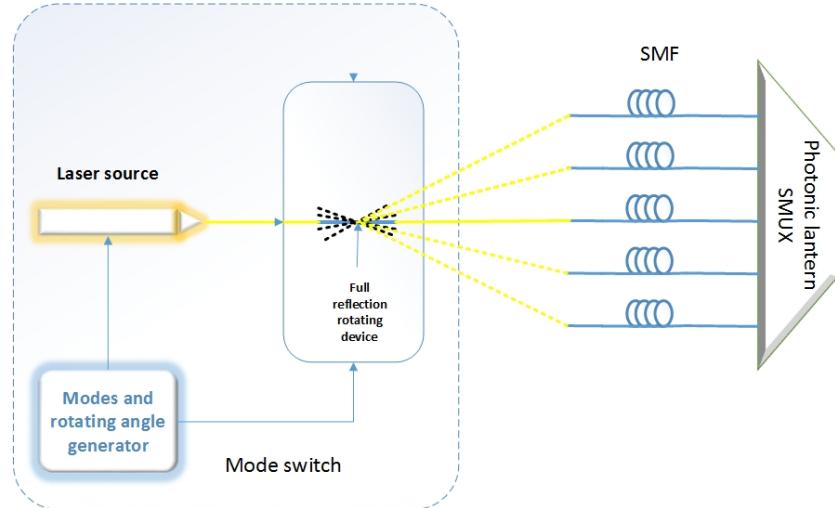


Figure 4.3: The model of mode switch with 5-mode group.

4.3 SSK and GSSK modulation in MMF for one segment

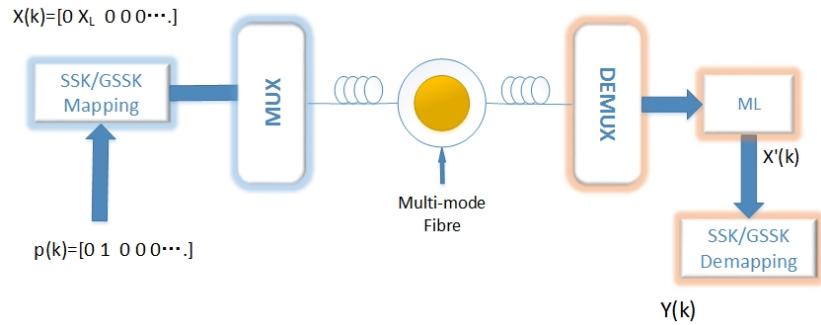


Figure 4.4: The system model of SSK/GSSK system model in MMF.

Applying SSK/GSSK into MMF is similar with that into wireless communication. We need to use laser source to generate light signal on 0/1 symbol in SSK and $0/\frac{1}{\sqrt{M_c}}$ and use mode and rotation generator to realize the SSK/GSSK mapping. Hence, The spectral efficiency is $\log_2 M_t$ for SSK and $\lfloor \log_2 \binom{M_t}{M_c} \rfloor$ for GSSK.

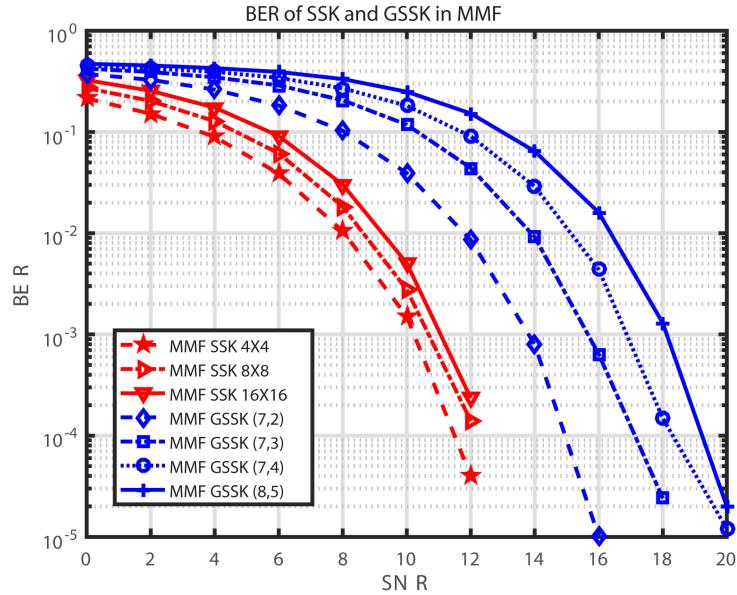


Figure 4.5: The BER difference between RF and MMF in SSK,GSSK,SM and GSM. Solid curve denotes MMF and dashed curve denotes RF.

From Fig(4.5), in general,SSK and GSSK still have a good BER performance of $10^{(-5)}$ at SNR of 12 dB and 20dB. different from that in wireless communication system. Increasing the number of transmit modes will increase the receive modes accordingly with the full rank assumption and signal experiences more dimension of AWGN which causes degradation to system in gaussian channel. However, from the three SSK curves in Fig (4.5) and Fig (4.8)in which the size of channel is increased to 256×256 , the SSK still can get to error free at the SNR of 12dB. We can conclude that SSK is really suitable to low SNR region and greatly reduce the receiver complexity compared with traditional MIMO at the expense of more transmit modes. Different from wireless communication system that SSK need large number of transmit antenna to get large spectral efficiency. SSK/GSSK in MMF only need large core diameter optical fibre to accommodate more modes which cost much less than transmit antennas which is one motivation of applying SM-MIMO techniques to MMF.

4.4 SM and GSM modulation in MMF for one segment

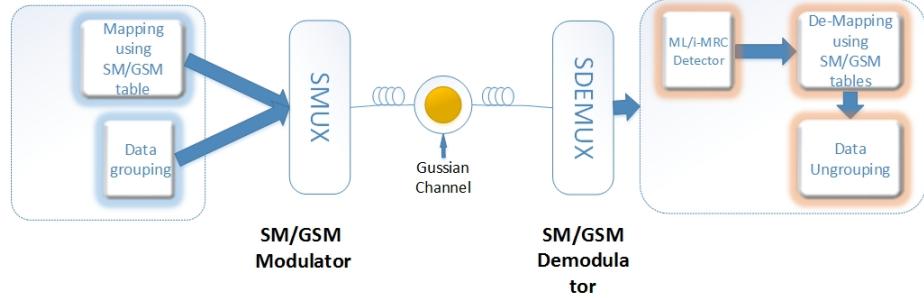


Figure 4.6: The system model of SM/GSM system model in MMF.

The system model of SM/GSM is based on the mode switch model designed in 5.2.

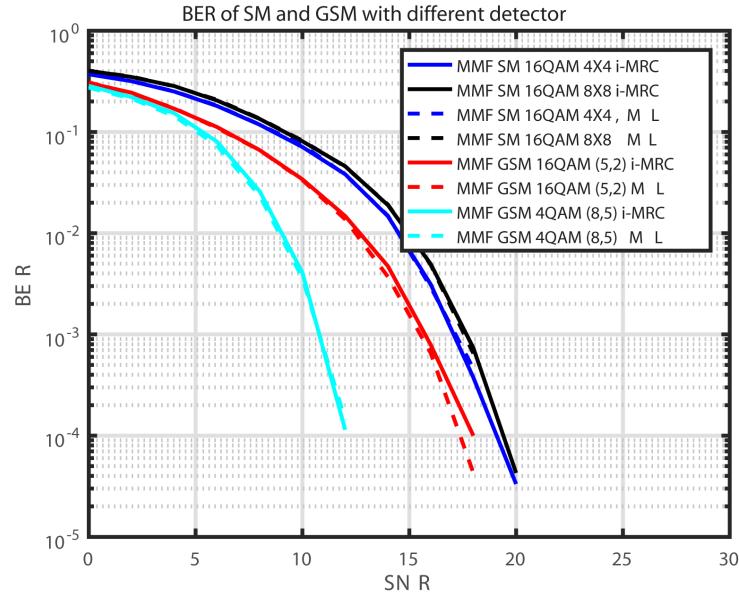


Figure 4.7: The BER difference between i-MRC and ML detector in SSK,GSSK,SM and GSM. Solid curve denotes i-MRC detection performance and dashed curve denotes ML detection performance.

Fig (4.7) is very important. Recalling the Fig (3.15), ML gains more than 3dB over i-MRC detector in wireless communication. However, the GSM and SM in MMF demonstrate similar performance between ML and i-MRC. ML

gains no more than 0.5 dB over i-MRC which greatly reduce the receiver complexity at the same time. Because of the great performance of i-MRC, in the fibre link part, we will only use i-MRC in SM/GSM and compare the complexity of this linear detector with linear detector in MMSE-MIMO and ZF-MIMO in the 5.7 and 5.8.

4.5 Comparison between SSK,GSSK,SM and GSM

The 8 bits transmission is shown below in Fig (4.8), it is clear that MMSE-MIMO using 4QAM has the best BER performance which gains 3dB in SNR over SSK and around 6dB over other schemes because it has the largest distance between different symbols. Hence, if we compare the performance between GSM and MMSE with 16QAM, GSM outperform MMSE-MIMO for around 1dB at BER of 10^{-2} at the expense of more transmit modes. GSSK outperforms SM at 14dB on SNR and exhibits a faster decreasing speed than GSM at high SNR region. In general, MMSE-MIMO has the best performance in low SNR region and SSK has the fastest decreasing ratio. If the distance of the constellation symbol is constraint to a little range and spectral efficiency is fixed, GSM shows the best BER performance at low SNR region SM 256×256 has the best BER performance at moderate SNR scenarios. However, in practical use, it is difficult to transmit 256 modes simultaneously.

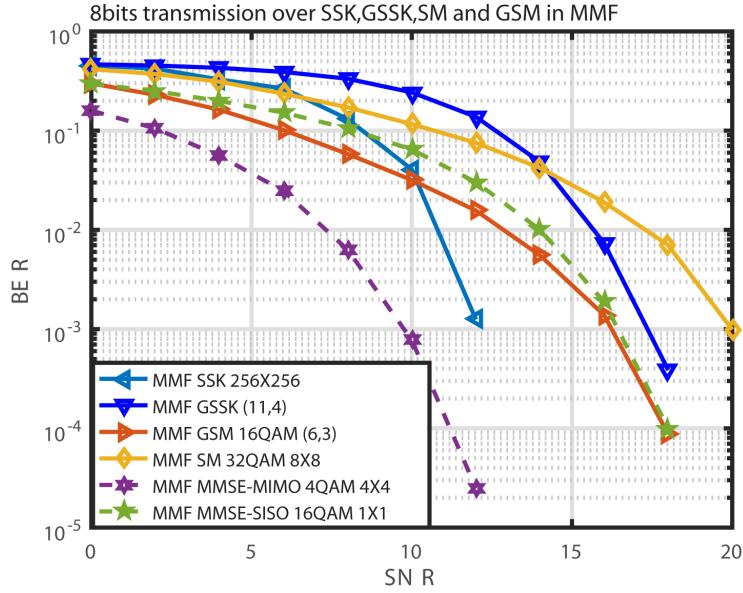


Figure 4.8: 8 bits transmission among SSK,GSSK,SM , GSM and MMSE-MIMO in MMF. Solid curve denotes SM-MIMO techniques and dashed curve denotes MMSE-MIMO .

4.6 Fibre link analysis

This part will illustrate the performance of SM-MIMO techniques based on the framework shown in Chapter 2 (i.e. Fig (2.7) and Fig (2.8)). Fig (2.7) applies distributed AWGN at the end of each segment and Fig (2.8) pumps the noise at the last segment with noise in equation (2.14). This part will compare the results of these two framework based on $10^4 - 10^5$ realizations.

4.6.1 SSK and GSSK modulation in MMF for fibre link

Fig (4.9) below demonstrates that the pumped noise method is effective in SSK and GSSK which proves the correctness of equation (2.14). After transmitting into several segments, the BER curves still maintain the exponential property and obtain error free result at high SNR scenarios. By comparing the black curves and blue curves, more number of segments will degrade system performance because of the noise accumulation. From the blue and light blue curves, even though increasing the transmit and receive modes will cause degradation to the system which is shown in the previous part, the impact of accumulating noise has relatively smaller impact on system performance (around 0.5 dB better at BER of 10^{-3}). This property demonstrates a possible advantage of

SSK/GSSK in MMF that system performance is robust to the noise accumulation. In addition, the gap between 1 segment dotted curves (green and red) at the BER of $10^{(-3)}$ is roughly the same as that for 10 segment scenarios. This constant gap can also be seen in the Fig (4.10)

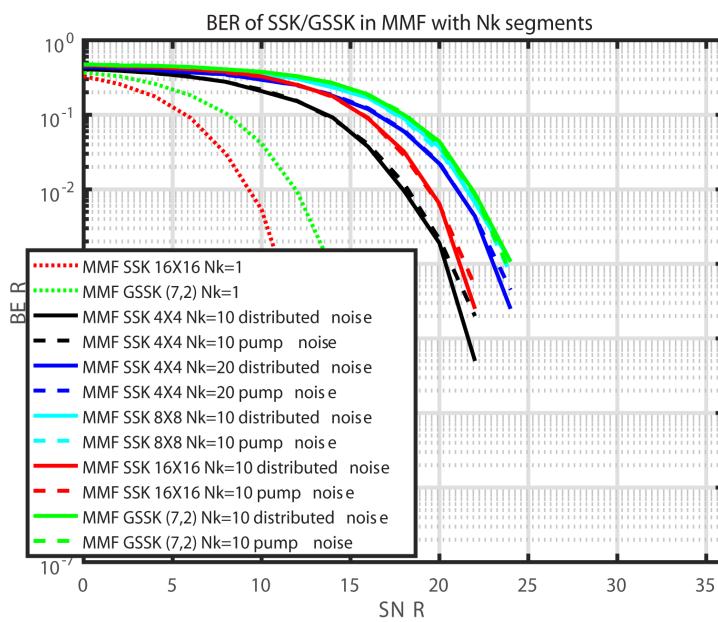


Figure 4.9: SSK/GSSK based transmission with different number of segments N_k . Solid curves denote performance with distributed noise in each segment. Dashed curves denote performance with noise pumped into the receiver. Dotted curves are reference curves

4.6.2 SM and GSM in MMF for fibre link

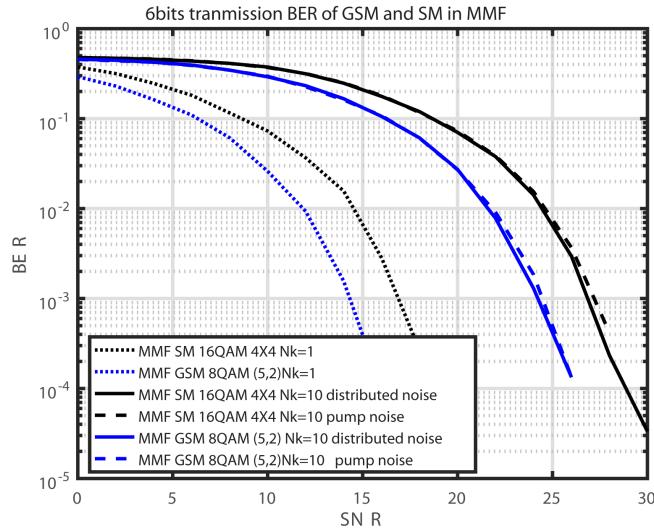


Figure 4.10: SM/GSM based transmission with different number of segments N_k . Solid curves denote performance with distributed noise in each segment. Dashed curves denote performance with noise pumped into the receiver. Dotted curves are reference curves

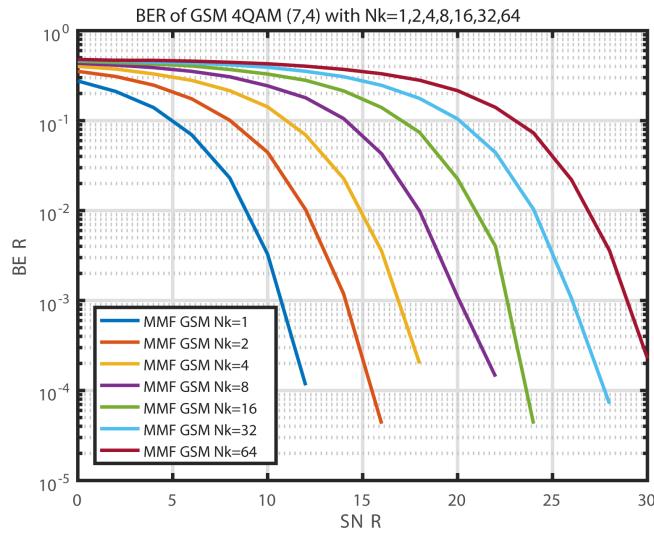


Figure 4.11: BER performance of 1,2,4,8,16,32,64 segment in GSM

Fig (4.10) has the same conclusion with that in SSK/GSSK above. Fig(4.11) illustrates a constant gap (3dB) between two adjacent curves as the amount of

noise (accumulated noise power) of one curve is 2 times to the previous one. And $10 \times \log_{10} 2 = 3dB$ which is shown in the Fig (4.10). The 64 segments performance get to a BER of $10^{(-5)}$ at around 30dB in SNR. And in practical use, one segment refers to 40 to 80km optical fibre length and 64 segment can still maintain good BER performance . This ensures the relatively good performance of SM-MIMO techniques in long haul transmission in MMF.

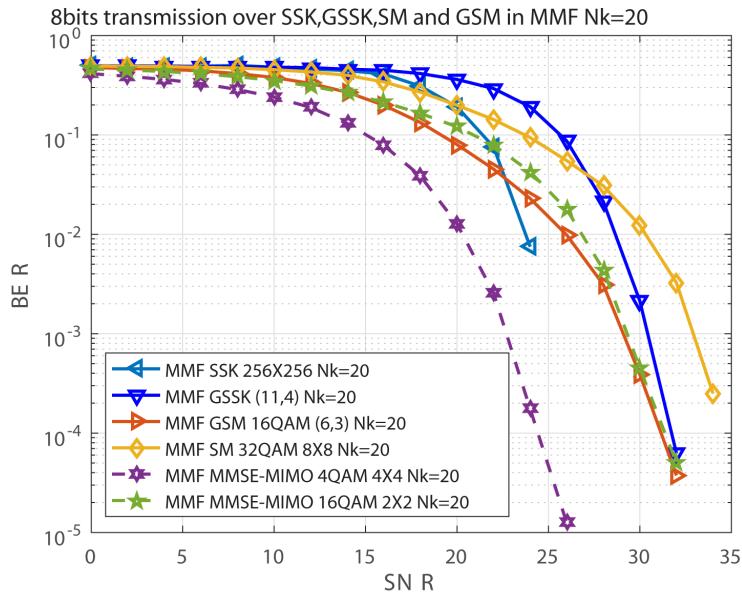


Figure 4.12: 8 bits transmission BER performance of SSK, GSSK,SM and GSM. Solid curves denote SM-MIMO techniques and dashed curves linear MIMO techniques

Compared with Fig (4.8). Fig (4.12) is just the right translation of left one with a distance of roughly 13dB. It is 13dB because $10 \log_{10} 20 = 13$ as the noise power is amplified 20 times after accumulation.

4.7 Receiver complexity of different SM-MIMO techniques

The receiver complexity N_r is divided into detection complexity N_d and demodulation complexity N_m . And $N_t = N_d + N_m$. Consider only multiplication and addition of a complex numbers as an operation. Note that in MMF, the $M_t = M_r = N$.

From the equation (3.2) and (3.5) for ZF-MIMO and MMSE-MIMO. For ZF criterion, a pseudo inverse need to be calculated. We make the assumption

that transposes are not counted. The first matrix multiplication ($\mathbf{H}^\dagger \mathbf{H}$) needs N^3 and matrix inverse($\mathbf{H}^\dagger \mathbf{H}$) $^{-1}$ need $4N^3$. The second matrix multiplication ($\mathbf{H}^\dagger \mathbf{H}$) $^{-1} \mathbf{H}^\dagger$ takes N^3 . The total N transmit stream cost $N \times (N^3 + 4N^3 + N^3) = 6N^4$ for detection complexity. MMSE-MIMO has one more addition ($\mathbf{H}^\dagger \mathbf{H} + \sigma_n^2 \mathbf{I}$) compared with ZF-MIMO. Then, the detection complexity is $6N^4 + N^3$ in MMSE-MIMO. For the demodulation complexity (e.g. in M -ary QAM), calculating the distance with M constellation points need M addition and M multiplication. Then finding the minimum distance need M more additions. Note that the demodulation complexity only appears in the linear detector. In maximum likelihood detection, we don't need to calculate the demodulation because ML detection process the data as a whole.

In a sum, following the above rule, the receiver complexity is demonstrated below in table [47] [44] [45] [46](In the table , $N_c = 2^{\lfloor \log_2 (\frac{N}{M_c}) \rfloor}$) M_c the number of active modes:

Table 4.1: Table VII
Receiver complexity in full rank MMF

<i>SM and MIMO</i>	detector	N_d	N_m	receiver N_t
ML-MIMO	ML	NM^N	—	NM^N
ZF-MIMO	ZF	$6N^4$	$3M$	$6N^4 + 3M$
MMSE-MIMO	MMSE	$6N^4 + N^3$	$3M$	$6N^4 + N^3 + 3M$
SSK Optimal	ML	$(3N - 1)N$	—	$(3N - 1)N$
GSSK Optimal	ML	$(2N + NM_c - 1)N_c$	—	$(2N + NM_c - 1)N_c$
SM Optimal	ML	$(3N + 1)NM$	—	$(3N + 1)NM$
SM Linear	i-MRC	$2N^2 - N$	$3M$	$2N^2 - N + 3M$
GSM Optimal	ML	$NM(M_c + 2)N_c$	—	$NM(M_c + 2)N_c$
GSM Linear	i-MRC	$(M_c N + N - 1)N_c$	$3M$	$(M_c N + N - 1)N_c + 3M$

In general, ML-MIMO has the highest complexity for all the techniques that use ML detection. For linear detector, the order of MMSE-MIMO and ZF-MIMO is 4. And for SM it is 2 . The order of i-MRC GSM depends on the practical value.

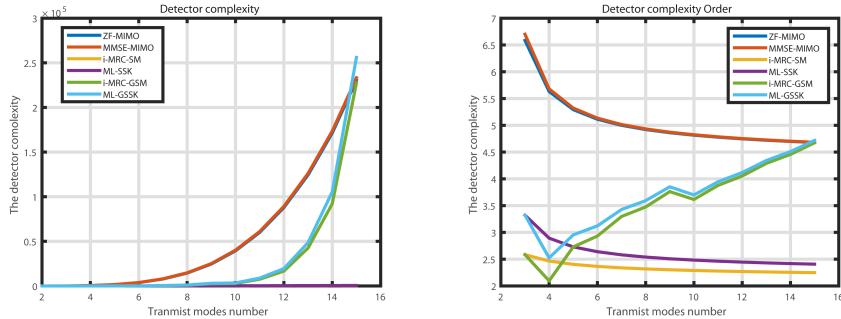


Figure 4.13: Left: Detector complexity. Right: Detector complexity order

The detection complexity make the assumption that the number of active modes is half or near half of the transmit modes. If the complexity is C , then, the order is calculated using $\log_N C$. N is the number of transmit modes. From Fig (4.13), we find that the complexity of linear MIMO is larger than other schemes when transmit modes is fewer than 14. The order of GSSK/GSM go beyond other schemes when transmit modes is bigger than 14 which means that GSM/GSSK modulation still have a large complexity and need larger M-ary QAM to reduce the requirement of transmit modes at high spectral efficiency region. However, SSK/SM maintain a low complexity order and is very promising in the high transmit modes MMF scenarios. Therefore, when the number of transmit modes is fewer than 14, we can use i-MRC detector and when we have larger number of transmit modes. The ML detector is better because i-MRC detector has larger complexity than linear MIMO and we choose ML detection for best BER performance.

4.8 Trade-off between complexity ,spectral efficiency and BER

It is very difficult to judge the receiver complexity order of GSSK and GSM because of combination calculation there. The tables below show the detail value of detection complexity N_d and receiver complexity N_t in the two bottom line.

Table 4.2: Table VIII
Detector and Receiver complexity for 4-b/s/Hz transmission in MMF

MMSE-MIMO	ZF-MIMO	i-MRC-SM	i-MRC-SM	i-MRC-GSM	i-MRC-GSM
2×2	2×2	4×4	2×2	(4,2)	(5,2)
4QAM	4QAM	4QAM	8PSK	4QAM	BPSK
40	32	28	6	44	112
52	44	40	30	56	118

Table 4.3: Table IX
Detector and Receiver complexity for 6-b/s/Hz transmission in MMF

MMSE-MIMO	ZF-MIMO	i-MRC-SM	i-MRC-SM	i-MRC-GSM	i-MRC-GSM
3×3	3×3	16×16	4×4	(6,3)	(4,2)
4QAM	4QAM	4QAM	16QAM	4QAM	16QAM
513	486	496	28	368	44
525	498	508	76	380	92

Table 4.4: Table X
Detector and Receiver complexity for 12-b/s/Hz transmission in MMF

MMSE-MIMO	ZF-MIMO	i-MRC-SM	i-MRC-SM	i-MRC-GSM	i-MRC-GSM
6×6	6×6	64×64	16×16	(10,5)	(6,3)
4QAM	4QAM	16QAM	64QAM	32QAM	256QAM
7992	7776	8128	496	7552	368
8004	7788	8176	688	7648	1136

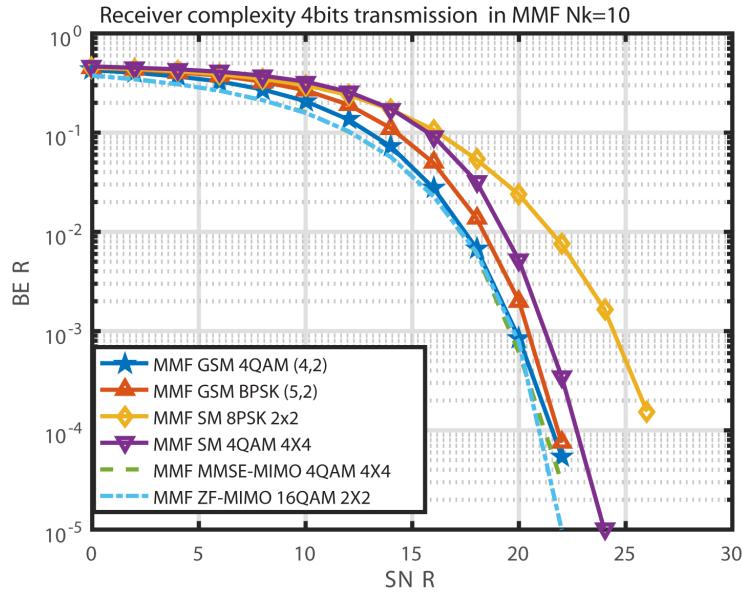


Figure 4.14: 4 bits transmission with similar receiver complexity. Solid curves denote linear SM-MIMO techniques and dashed curves denote traditional MIMO

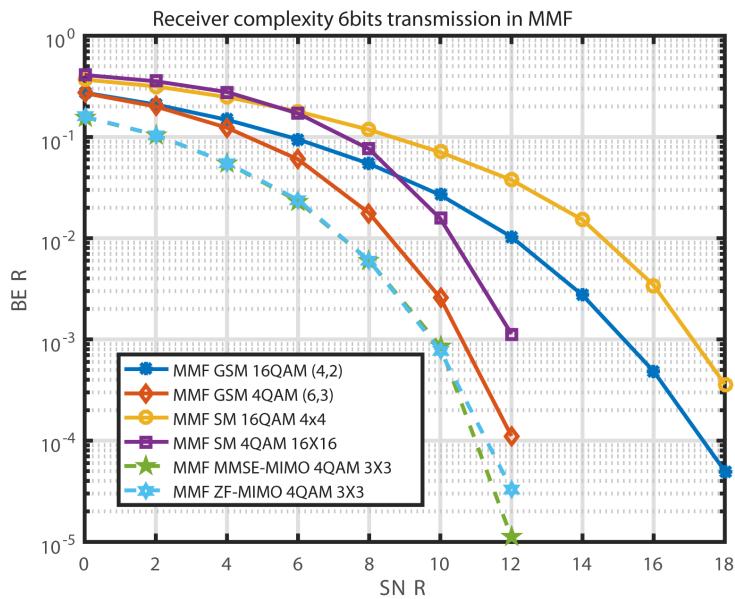


Figure 4.15: 6 bits transmission with similar receiver complexity. Solid curves denote linear SM-MIMO techniques and dashed curves denote traditional MIMO

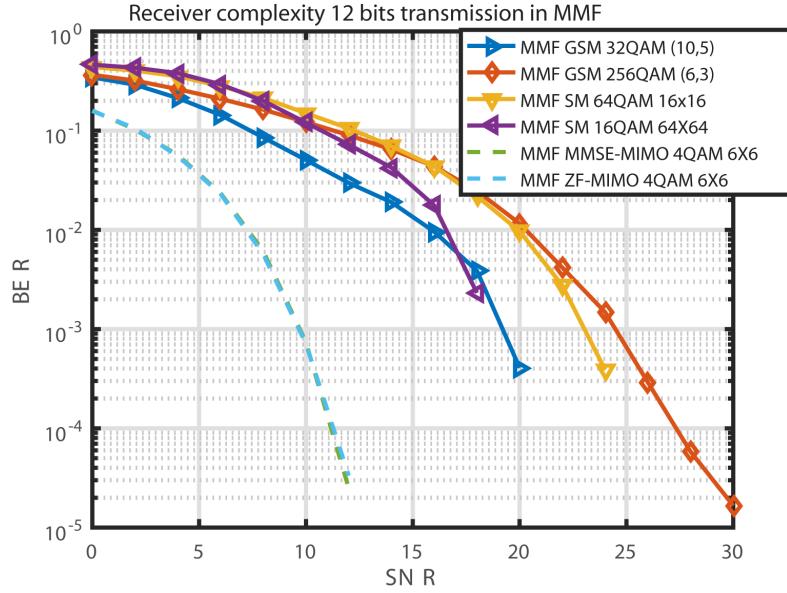


Figure 4.16: 12 bits transmission with similar receiver complexity. Solid curves denote linear SM-MIMO techniques and dashed curves denote traditional MIMO

From Fig (4.14) in 4 bits transmission, the linear MIMO gains around 0.5-1dB over GSM (4,2) and GSM (5,2). The SM curves which have low detection complexity is 1.5dB and 4dB worse than linear MIMO. From Fig (4.15), the gap between linear MIMO and linear SM/GSM become larger. But GSM (6,3) and (4,2) which has lower and much lower complexity still have a good performance. The linear MIMO just gains 1dB over GSM(6,3) and around 2dB over GSM (4,2). From Fig (4.16), the 12 bits transmission demonstrates a larger gap between schemes with similar receiver complexity. Linear MIMO have better performance 7dB over SM-MIMO and 8dB over GSM-MIMO and around 95% more detection complexity.

In a sum, linear MIMO has the best BER performance in perspective of fixed spectral efficiency. The SM has 80% to 90% receiver complexity reduction over MMSE-MIMO and ZF-MIMO. The complexity of GSM is larger than SM but still has 20% to 80% receiver complexity over linear MIMO in different cases. But the performance of GSM is around 3 dB better than SM and is very close to linear MIMO (no more than 2dB) in the above cases. The trade-off is shown below:

- 1: When the number of transmit modes is smaller than 14, it is better to choose GSM-MIMO to get small and moderate spectral efficiency and relatively low receiver complexity.

- 2: When the number of transmit modes is smaller than 14, and the requirement of spectral efficiency is high (e.g. 20 b/s/Hz). It is better to choose linear MIMO for the consideration of BER performance.
- 3: When the number of transmit modes is larger than 14, both the linear MIMO and GSM-MIMO have very high receiver complexity. SM-MIMO is the best one to reduce the receiver complexity while still maintain a high spectral efficiency at the expense of more transmit modes and high M-ary modulation types.

4.9 The discussion about transmitter complexity

Conventional OFDM-MIMO system has obtained great attention because of the their good performance in time selective channels. But the transmitter in space frequency division block coding (SFBC) MIMO-OFDM has a high computational complexity [58] because it needs to perform the inverse Fast Fourier transform (IFFT) operation antennas. In this part, we divide the transmitter complexity into two part. The first one is SFBC encoding process. The second one is the computational complexity after the encoder. The encoding process is the same for every symbol. Therefore, the complexity is linearly increased with the number of the symbols. For SM-MIMO techniques, the active modes transmit same symbol. Therefore, whatever the number of active modes, the complexity of encoding is just for one transmit mode and then same symbol is transmitted on different active mode. For example, there are 4 transmit modes and one symbol is transmitted in each time block. Then, the encoding complexity of one symbol is Q which is for SM-MIMO techniques. And for conventional MIMO is $4Q$. The reduction is 75%.

The computational complexity has the same principle. From the transmitter architecture of [58], the transmitter complexity consists of $\frac{MN}{2} \log_2(N)$ multiplications and $MN \log_2(N)$ additions. Note that M is number of transmit modes and N is the number of sub-carriers[58]. We find that ,same as encoding process, the computational complexity after the encoder is still proportional to the number of transmit modes. Therefore, the transmitter complexity reduction of the SM-MIMO techniques compared with conventional MIMO is $\frac{M-1}{M} \times 100\%$. The reduction proportion is increased with the number of transmit modes.

4.10 Some discussion about nonlinearity using SM-MIMO techniques in MMF

As is shown in chapter 2, the nonlinear effect in MMF is dependent on the optical intensity (i.e. signal power). And for stimulated scattering effect, the threshold

for causing SRS nonlinear effect is 400mw. For example, if we have a 8×8 channel, we want to transmit 8 signals, each signal has 100mw signal power. If we use traditional MIMO techniques to transmit 8 signals simultaneously. The total signal power is 800mw which is larger than 400mw and causes SRS effect. However, in SM and GSM (8,2) (8,3) and (8,4), total signal power is not larger than threshold and will not cause SRS effect. In general, using SM/GSM techniques installs less signal power than tradition MIMO with the same channel size and can sometimes avoid nonlinear effect to some extent. If both the SM-MIMO signals and traditional MIMO signals exceed the power threshold. The larger the signal power, the larger the nonlinear effect on the whole. Reducing the nonlinear effect is another benefit of SM techniques.

Chapter 5

Conclusion and future work

5.1 Conclusion

In this paper, I introduce the spatial division multiplexing in MMF using SM-MIMO techniques. First, the project motivation is to reduce the large receiver complexity from traditional MIMO techniques by spatial modulation. Second, the background of optical fibre is delivered based on the optical fibre characteristic, the literature review of spatial division multiplexing in optical fibre, MIMO techniques and SM-MIMO techniques. Third, the modeling of SDM system in MMF with distributed noise loading and pumped noise loading is illustrated. Fourth, the comparison of MMSE-MIMO, ZF-MIMO, SSK,GSSK,SM and GSM in wireless communication system is demonstrated. Finally, applying SM-MIMO techniques into the uniform random coupling MMF transmission system with different number of segments is shown , making trade-off between the receiver diversity, BER performance , spectral efficiency and different detection methods. Then, a short discussion about nonlinearity in MMF using SM-techniques is provided based on the

The main contribution of this paper is that:

1. All the SM-MIMO techniques are used into the random coupling MMF system to show the BER performance based on different detection method.
2. The receiver complexity of applying different SM-MIMO techniques and traditional linear MIMO techniques is shown to demonstrate that the SM has 80% to 90% receiver complexity reduction over MMSE-MIMO and ZF-MIMO. The complexity of GSM is larger than SM but still has 20% to 80% receiver complexity over linear MIMO in different cases.
3. The trade-off between receiver diversity, BER performance and spectral efficiency in MMF is shown based on some particular cases and receiver complexity order figure. The GSM has a good performance when the number transmit

modes is lower than 14.

4. A active antenna excitation method and high utilization efficiency method for GSM is delivered to make a trade-off between BER performance, transmit diversity gain and spectral efficiency. This two methods are also suitable in MMF transmission
5. A possible mode switch design is shown to in Fig (4.3) based on low loss photonic lantern.

5.2 Future work

Motivation:

1. SM-MIMO techniques in MMF reduce the cost by decreasing the number of laser source. In MMF , changing the material from silica fibre to plastic fibre can further reduce the cost.
2. As is shown in chapter 4, the receiver complexity order of GSM is very high when the transmit modes is larger than 14 using the i-MRC method. A lower complexity detection algorithm should be derived to reduce the detection complexiy when the number of transmit modes is large.

In the future work, I aim to apply the SM-MIMO techniques into the few modes plastic optical fibre (POF) [59] [60] to reduce the cost. Comparing the complexity of receiver and transmitter, the spectral efficiency, capacity and energy efficiency between classical silicon fibre and plastic fibre, I will make a trade-off between coherent detection and direct detection because the POF can only be used in short distance. Then, I will derive efficient algorithm to reduce the bit error rate (BER) of transmission and suitable algorithm to further reduce the complexity of GSM receiver. Afterwards, the nonlinearity of optical fibre will be considered to increase the capacity of transmission at high signal power. The future work will be beneficial to the short distance data centre application. For example, in a building, the requirement of the length of optical fibre is not very long (typical less than 1km). The few modes plastic fibre is promising because of its low cost. And spatial modulation provides a low complexity and spectral efficiency enhancing transmission to ensure the good performance. Therefore, it will have a good future for company, school and hospital inner-network.

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