Assignment : 1

Calculate the mean, median, mode and Standard deviation for the problem Statement 1 & 2

① The marks awarded for an assignment set for a year 8 class of 20 students were as follows:

6, 7, 5, 7, 7, 8, 7, 6, 9, 7, 4, 10, 6, 8, 8, 9, 5, 6, 4, 8

$$\text{Mean} = \frac{6+7+5+7+7+8+7+6+9+7+4+10+6+8+8+9+5+6+4+8}{20}$$

$$= \frac{137}{20} = \underline{\underline{6.85}}$$

Mode = 7

Median = 4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10

$$\text{Median} = \frac{7+7}{2} = \underline{\underline{7}}$$

Standard deviation

$$\text{Variance } (\sigma^2) = \frac{\Sigma(x_i - M)^2}{N}$$

$$= \left(2\times(4-6.85)^2 + 2\times(5-6.85)^2 + 4\times(6-6.85)^2 + 5\times(7-6.85)^2 + 4\times(8-6.85)^2 + 2\times(9-6.85)^2 + (10-6.85)^2 \right) \Big/ 20$$

$$= 2.5275$$

$$\text{Standard deviation } (\sigma) = \sqrt{2.5275} = \underline{\underline{1.589}}$$

② The number of calls from motorists per day for roadside Service was recorded for a particular month:

28, 122, 217, 130, 120, 86, 80, 90, 140, 120, 70, 40, 145, 113, 90, 68, 174, 194, 170, 100, 75, 104, 97, 75, 123, 100, 75, 104, 97, 75, 123, 100, 89, 120, 109.

$n = 35$

$$\text{Mean} = \frac{\Sigma x_i}{n} = \frac{3763}{35} = \underline{\underline{107.5}}$$

Median:

28, 40, 68, 70, 75, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97, 100, 100, 100, 104, 104, 109, ~~10~~ 113, 120, 120, 120, 122, 123, 123, 123, 130, 140, 145, 170, 174, 194 + 217

Median = 10

Mode = 75

Variance (σ^2) = 1503.33

Standard deviation $(\sigma) = \sqrt{1503.33} = \underline{38.77}$

③ The number of times I go to them gym in week days are given below along with its associated probability.

X =	0	1	2	3	4	5
f(x) =	0.09	0.15	0.40	0.25	0.10	0.01

Calculate mean no: of work outs in a week. Also calculate Variance.

$$\text{Mean} = \Sigma x p$$

$$= 0 \times 0.09 + 1 \times 0.15 + 2 \times 0.40 + 3 \times 0.25 + 4 \times 0.10 + 5 \times 0.01$$

$$= \underline{\underline{2.15}}$$

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Variance $= \sum x^2 p - \mu^2$

$= 5.85 - (2.15)^2$

$= \underline{\underline{1.2275}}$

⑤ A company manufactures LED bulbs with the faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the std deviation associated with it.

Binomial

$p = 30\% = 0.3$

$q = 70\% = 0.7$ $N = 6,\ n = 2$

$$P(n/N) = \frac{N!}{n!\,(N-n)!}\, p^n q^{N-n}$$

$$P(n=2) = \frac{6!}{2!\,4!} \times 0.3^2 \times 0.7^4$$

$= \underline{\underline{0.3241}}$

Mean $(\mu) = n * p = 2 \times 0.03 = 0.6$

$\sigma = \sqrt{n * p * q} = \sqrt{2 \times 0.3 \times 0.7} = 0.648$

⑥ Gaurav and Barakha are both preparing for entrance exams - Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correlation rate of 45%. What is the probability that each of them will solve

5 questions correctly ?. What happens in case of 4 and 6 Correct sol^ns ? What do you infer from it. What are the two main governing factors affecting their ability to Solve questions correctly ?

Gaurav

$N=8,\ n=3,\ P=0.75,\ q=0.25$

$P(n=4) = \frac{8!}{4!\,4!}\ 0.75^4 \times 0.2^4$

$= \underline{\underline{0.865}}$

$P(n=5) = \frac{8!}{5!\,3!}\ 0.75^5 \times 0.25^3$

$= \underline{\underline{0.207}}$

$P(n=6) = \frac{8!}{6!\,2!}\ 0.75^6 \times 0.25^2$

$= \underline{\underline{0.311}}$

Barakha

$N=12,\ n=5,\ P=0.45,\ q=0.55$

$P(n=4) = \frac{12!}{4!\,8!}\ 0.45^4 \times 0.55^8$

$= \underline{\underline{0.1699}}$

$P(n=5) = \frac{12!}{5!\,7!}\ 0.45^5 \times 0.55^7$

$= \underline{\underline{0.2138}}$

$P(n=6) = \frac{12!}{6!\,6!}\ 0.45^6 \times 0.55^6$

$= \underline{\underline{0.212}}$

⑦ Customers arrive at a rate of 72 / hr to my shop. What is the probability of K customers arriving in 4 minutes ?

a) 5 customers , b) not more than 3 customers , c) more than 3 customers ?

P(customer arriving in 4 min) :

$1hr = 72$; $1min = \frac{72}{60} = 1.2$

$4 min = 1.2 \times 4 = \underline{4.8}$

$\mu = 4.8$

$$p = \frac{e^{-\mu} \mu^{x}}{x!}$$

(a) $P(x=5)$:

$$P(x=5) = \frac{e^{-4.8} \times 4.8^{5}}{5!} = \frac{20.96}{5!} = \underline{0.174}$$

(b) $P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= \frac{e^{-4.8} \times 4.8^{0}}{0!} + \frac{e^{-4.8} \times 4.8^{1}}{1!} + \frac{e^{-4.8} \times 4.8^{2}}{2!} + \frac{e^{-4.8} \times 4.8^{3}}{3!}$$

$$= 0.0082 + 0.0395 + 0.094 + 0.15$$

$$= \underline{\underline{0.29}}$$

(c) $P(x > 3) = 1 - \{P(x=0) + P(x=1) + p(x=2) + P(x=3)\}$

$$= 1 - 0.29$$

$$= \underline{\underline{0.706}}$$

(8) I work as a data analyst in Aeon Learning Pvt Ltd. After analysing data, I make reports, when I have the efficiency of entering 77 words / min with 6 errors/hr. What is the probability that I will commit 2 errors in a 455 word financial report? What happens when the no. of words increases / decreases (in case of 1000 words?, 255 words)?

77 words / min $\longrightarrow$ 6 error / hr

77 words $\rightarrow$ 1 min $\qquad \mu = \frac{6}{60} = 0.1$

455 words $\rightarrow \frac{455}{77} = 5.90$ min

Average error $= 5.9 \times 0.1 = \underline{\underline{0.59}}$ (μ)

~~P(x≥2)~~:

455 words

$\mu = 0.59$

$P(x=2) = \frac{e^{-0.59} \times 0.59^2}{2!}$

$= \underline{\underline{0.096}}$

1000 words

$\frac{1000}{77} = 12.98$

$\mu = 12.98 \times 0.1$

$= \underline{\underline{1.298}}$

~~$P(x=2) = e^{-0.59}$~~

$P(x=2) = \frac{e^{-1.298} \times (1.298)^2}{2!}$

$= \underline{\underline{0.23}}$

255 words

$255/77 = 3.31$

$\mu = 3.31 \times 0.1 = 0.331$

$P(x=2) = \frac{e^{-0.33} \times 0.33^2}{2!}$

$= 0.0782/2$

$= \underline{\underline{0.039}}$

255 words $\rightarrow \mu = 0.331 \rightarrow P = 0.039$

455 words $\rightarrow \mu = 0.59 \rightarrow P = 0.096$

1000 words $\rightarrow \mu = 1.298 \rightarrow P = 0.23$

(10) Compute the following

a) $P(Z > 1.26)$, $P(Z < -0.86)$ $P(Z > -1.37)$ $P(-1.25 < Z < 0.37)$
$P(Z \leq -4.6)$.

b) Find the value of z such that $P(Z > z) = 0.05$

c) Find the value of z such that $P(-z < Z < z) = 0.99$.

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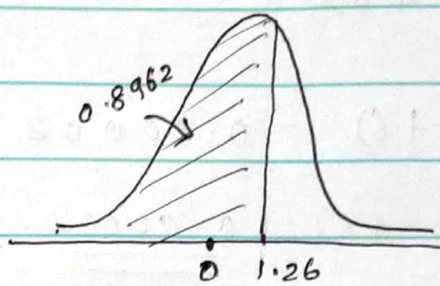
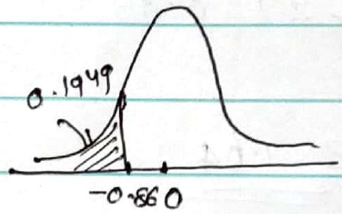
a) ① $P(Z > 1.26)$



$P(Z \le 1.26) = 0.8962$

$P(Z > 1.26) = 1 - 0.8962 = \underline{\underline{0.1038}}$

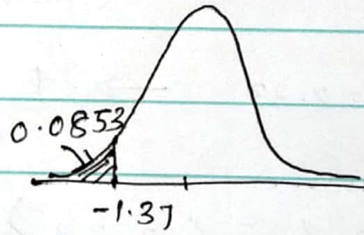
② $P(Z < -0.86)$



$Z(-0.86) = 0.1949$

$P(Z < -0.86) = \underline{\underline{0.1949}}$

③ $P(Z > -1.37)$

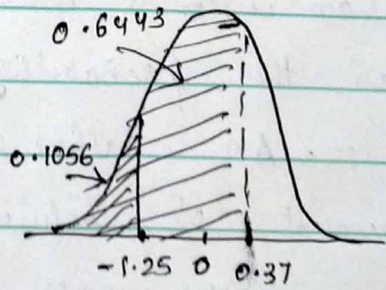


$Z(-1.37) = 0.0853$

$P(Z > -1.37) = 1 - 0.0853$

$= \underline{\underline{0.9147}}$

④ $P(-1.25 < Z < 0.37)$



$Z(-1.25) = 0.1056$

$Z(0.37) = 0.6443$

~~P(Z<~~ $P(-1.25 < Z < 0.37) = 0.6443 - 0.1056$

$= \underline{\underline{0.5387}}$

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⑤ $P(Z \leq -4.6)$

$$Z(-4.6) = 0.00002$$

$$P(Z \leq -4.6) = \underline{\underline{0.00002}}$$

(b) $P(Z > z) = 0.05$

$$\underline{\underline{Z = 1.64}}.$$

(c) $P(-z < Z < z) = 0.99$

$$Z = 2.33 \longrightarrow P = 0.99.$$

$P(-2.33 \leq Z \leq 2.33)$.

$$\underline{\underline{Z = \pm 2.33}}$$

⑪ The current flow in a copper wire follow a normal distribution with a mean of 10 mA and Variance of $4 (mA)^2$ What is the probability that a current measurement will exceed 13 mA? What is the probability that a current measurement is between 9 and 11 mA? Determine the current measurement which has a probability of 0.98?

$\mu = 10$, $\sigma^2 = 4$; $\sigma = 2$

$$Z = \frac{X - \mu}{\sigma}$$

$P(x > 13)$

$$Z = \frac{13-10}{2} = 1.5$$

$$\underset{\text{table}}{Z(1.5)} = 0.9332$$

$$P(x > 1.5) = 1 - 0.9332 = \underline{0.0668}$$

(b) $P(9 < x < 11)$

$$Z_1 = \frac{9-10}{2} = -0.5 \qquad Z_2 = \frac{11-10}{2} = 0.5$$

$$P(-0.5 < Z < 0.5) = 0.6915 - 0.3085$$
$$= \underline{0.383}$$

(c) $P(x) = 0.98$

$Z_{val} = 2.06$

$$\frac{X - 10}{2} = 2.06$$

$X = 14.1$

Current = $\underline{14.1\,mA}$

(12) The shaft in a piston has its diameter normally distributed with a mean of 0.2508 inch and std deviation of 0.0005 inch. The specification of the shaft are 0.2500 ± 0.0015 inch. What proportion of shafts are in sync with the specifications. If the process is centered so that the mean

is equal to the target value of 0.2500. What proportion of shafts conform to new specifications?

Specification $\rightarrow$ 0.2500 ± 0.0015 $\mu = 0.2508$

$\rightarrow 0.2515 \,,\, 0.2485$ $\sigma = 0.0005$

$0.2485 \leq X \leq 0.2515$.

$$Z_1 = \frac{X-\mu}{\sigma} = \frac{0.2485 - 0.2508}{0.0005}$$

$$= \underline{\underline{-4.6}}$$

$$Z_2 = \frac{0.2515 - 0.2508}{0.0005} = \underline{\underline{1.4}}$$

$P(-4.6 \leq z \leq 1.4) = 0.91924 - 0.002$

$\downarrow$ 0.0002 $\downarrow$ 0.91924

$= \underline{\underline{0.91904}}$

(2) If $\mu = 0.2500$

$$Z_1 = \frac{0.2485 - 0.2500}{0.0005} = -3$$

$$Z_2 = \frac{0.2515 - 0.2500}{0.005} = 3$$

$P(-3 \leq z \leq 3) = 0.99865 - 0.00135 = \underline{\underline{0.9973}}$

$\downarrow$ 0.00135 $\downarrow$ 0.99865

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