LING570: Shallow Processing Techniques
University of Washington
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Homework 1

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This homework answers the problem set sequentially.

1. *See the attached scripts.*

2. Let X be the number of heads I get out of 5 flips. Then

$$\begin{aligned} P(X \geq 4) &= P(X = 4) + P(X = 5) \\ &= \binom{5}{4} \cdot 0.8^4 \cdot 0.2 + \binom{5}{5} \cdot 0.8^5 \\ &= \frac{2304}{3125} \end{aligned}$$

3. (a) $P(X = 0) = \sum_y P(X = 0, Y = y) = 0.6$,
$P(X = 1) = \sum_y P(X = 1, Y = y) = 0.4$.
Therefore, the distribution of X is:

$$P(X = x) = \begin{cases} 0.6 & \text{if } x = 0 \\ 0.4 & \text{if } x = 1 \end{cases}$$

(b) $P(Y = 0) = \sum_x P(X = x, Y = 0) = 0.75$,
$P(Y = 1) = \sum_x P(X = x, Y = 1) = 0.25$.
Therefore, the distribution of X is:

$$P(Y = y) = \begin{cases} 0.75 & \text{if } y = 0 \\ 0.25 & \text{if } y = 1 \end{cases}$$

(c) $P(Y \mid X) = \frac{P(X,Y)}{P(X)}$. Plugging in different values for X and Y, we have:

$$P(Y \mid X) = \begin{cases} \frac{5}{6} & \text{if } x = 0, y = 0 \\ \frac{1}{6} & \text{if } x = 0, y = 1 \\ \frac{5}{8} & \text{if } x = 1, y = 0 \\ \frac{3}{8} & \text{if } x = 1, y = 1 \end{cases}$$

(d) Because $P(Y \mid X) \neq P(Y)$, X and Y are not independent.

4. (a) Let C be the coin chosen, let H be the event that I get a head.
Then

$$\begin{aligned} P(H) &= \sum_c P(H \mid C = c)P(C = c) \\ &= 0.1 \times 0.2 + 0.4 \times 0.5 + 0.7 \times 0.3 \\ &= 0.43 \end{aligned}$$

(b) By Bayes' Theorem,

$$\begin{aligned} P(C = c_1 \mid H) &= \frac{P(H \mid C = c_1)P(C = c_1)}{P(H)} \\ &= \frac{0.1 \times 0.2}{0.43} \\ &= \frac{2}{43} \end{aligned}$$

5. (a) No.
We have two coins, one is fair and the other has 0.9 probability of coming up heads. We first choose a coin, and toss it twice. Let X be the event that the first toss comes up heads, and let Y be the event that the second toss comes up heads. Let Z be the event that the fair coin is chosen. Then X and Y are independent given Z, but are clearly not independent.

(b) No.
We toss a fair coin twice. Let X be the event that the first toss comes up heads, and let Y be the event that the second toss comes up heads. Clearly X and Y are independent. Let Z be the event that both tosses are heads or both tosses are tails. X and Y are clearly not independent given Z.

6. (a) $L \to aL \mid cB$
$B \to \epsilon \mid bB$

(b) (Assuming $n \geq 0$): $L \to c \mid aLb$

(c) $L \to \epsilon \mid xLx \mid yLy \mid zLz$

7. (a)

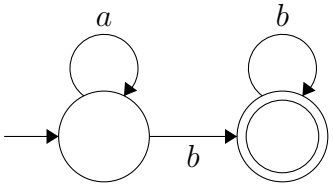

Figure 1: FSA

(b)

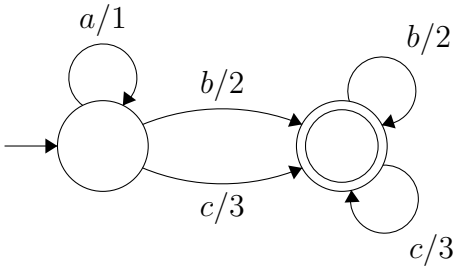

Figure 2: FST

(c) Yes, as in all language that can be represented with regular grammar can also be expressed with an FSA, and vice versa.

(d) Yes.

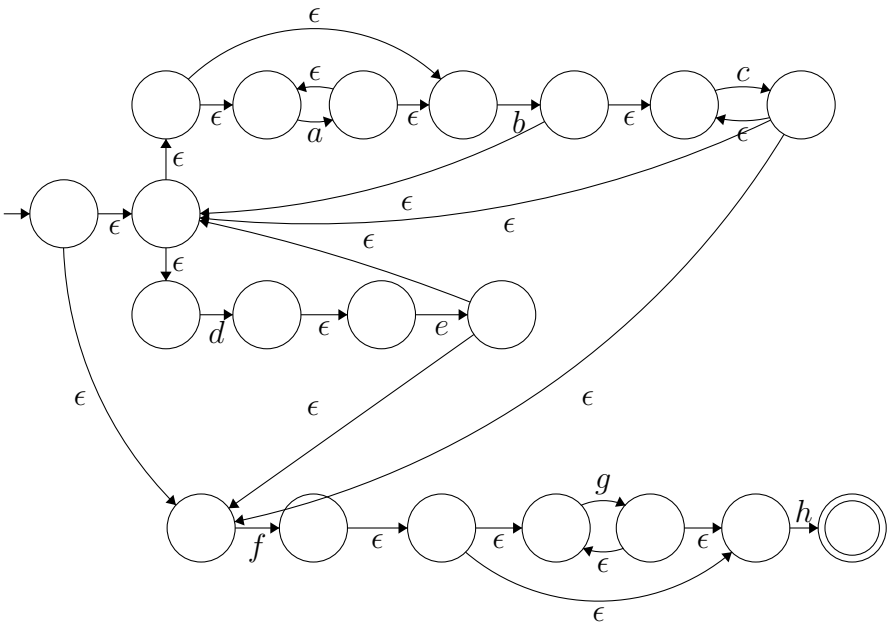

Figure 3: Non-deterministic FSA