Unveiling the Interplay between Central Bank Digital Currency and Bank Deposits*

Hanfeng Chen[1] Maria Elena Filippin[1,*]

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Abstract

We extend the Real Business Cycle model in Niepelt (2022) to analyze the risk to financial stability following the introduction of a central bank digital currency (CBDC). CBDC competes with commercial bank deposits as households' source of liquidity. We consider different degrees of substitutability between payment instruments and review the equivalence result in Niepelt (2022) by introducing a collateral constraint banks must respect when borrowing from the central bank. When CBDC and deposits are perfect substitutes, the central bank can offer loans to banks that render the introduction of CBDC neutral to the real economy. We show that the optimal level of the central bank's lending rate depends on the restrictiveness of the collateral constraint: the tighter it is, the lower the loan rate the central bank needs to post. However, when CBDC and deposits are imperfect substitutes, the central bank cannot make banks indifferent to the competition from CBDC. It follows that the introduction of CBDC has real effects on the economy.

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[1]Uppsala University and Center for Monetary Policy and Financial Stability (CeMoF).

*E-mail: mariaelena.filippin@nek.uu.se

1 Introduction

Digital currencies have been around for a while, but their potential significance in the global economy has increased recently due to a growing demand for digital payment methods for retail purposes and the gradual decline of the use of cash for transactions in many economies. Besides the private digital means of payment currently in circulation, many central banks have been investigating the possibility of launching a central bank digital currency (CBDC). CBDCs are central bank liabilities denominated in an existing unit of account that serves as a medium of exchange and a store of value.[1] Were CBDCs to be issued to retail customers (i.e., households), they would likely be a digital form of cash that shares features with central bank reserves, as they are universally accessible, like banknotes, but in a digital form.

Alongside an intense policy debate, a growing academic literature on the broader economic implications of CBDCs has emerged. A primary concern for central banks when considering the issuance of a CBDC is the risk to financial stability as a potential cause of financial crises. In this work, the risk of financial instability is intended as the risk of CBDC disintermediating the banking sector, potentially leading to a financial crisis. Introducing a CBDC will likely alter the equilibrium in the real economy, as it will represent a novel payment option alternative to cash and commercial bank deposits. The macroeconomic consequences of introducing a CBDC will impact individuals and financial institutions. This paper analyses the implications of introducing a retail CBDC, particularly concerning its relationship with bank deposits.

In recent work, Niepelt (2022) provides a macroeconomic analysis of the introduction of a retail CBDC. The paper addresses several normative questions, from optimality in terms of the monetary system and policy to the effect of the introduction of a CBDC on equilibrium outcomes. In particular, the author extends findings in Brunnermeier and Niepelt (2019) and establishes an equivalence result between different funding sources.[2] Introducing CBDC has no real consequences if the private and the public sectors are equally efficient in operating payment systems. For this to happen, the central bank must refinance banks at a lending interest rate that supports the banks' original portfolio position so that central bank funding exactly replaces the lost deposits for banks.

In Niepelt's framework, central bank loans are extended against no collateral. The collateral requirement imposed by the central bank is important for how introducing a CBDC may affect the banking sector and the real economy.[3] Central banks' lending to commercial banks (i.e., discount window lending) is a monetary policy instrument meant to support the

[1] As defined by the Committee on Payments and Market Infrastructures (CPMI) of the Bank of International Settlements.

[2] The two authors consider the simplified scenario without reserves and resource costs of providing liquidity.

[3] See, e.g., Burlon et al. (2022) and Williamson (2022).

liquidity and stability of the banking system. The liquidity provided by central banks helps financial institutions to manage their liquidity risks efficiently. These loans are issued at an administered discount rate and must be collateralized to the satisfaction of the issuing central bank. In the euro area, banks can obtain loans resorting to the marginal lending facility, which enables banks to obtain overnight liquidity from the European Central Bank against the presentation of sufficient eligible assets. In the United States, the Federal Reserve offers different types of discount window credit from the regional Federal Reserve Banks. The lending rates are the same across all Reserve Banks, and all discount window loans must be collateralized. The discount window mechanism has become increasingly important after the Global Financial Crisis.

Additionally, Niepelt's equivalence analysis only considers CBDC and deposits that are perfect substitutes. However, CBDC will likely not be a perfect substitute for bank deposits [see, e.g., Bacchetta and Perazzi (2022)]. In what follows, we will extend the equivalence result considering different degrees of substitutability between CBDC and deposits.

This work addresses the potential risk to financial stability following the introduction of a CBDC and investigates the impact of the substitutability between CBDC and bank deposits on this risk. Specifically, we investigate if the issuance of CBDC leads to disruptions in financial markets, thereby positing a risk to financial stability due to bank disintermediation rather than bank runs. To address this concern, we develop a model with a CBDC and deposits issued by banks subject to a collateral requirement when borrowing from the central bank. The framework builds on Niepelt (2022) and is an extension of the model by Sidrauski (1967) that embeds banks, deposits, government bonds, reserves, and a CBDC into the baseline Real Business Cycle model. Households value goods, leisure, and the liquidity services deposits and CBDC provide. Non-competitive banks invest in capital, reserves, and government bonds and fund themselves through either deposits or borrowing from the central bank. Firms produce using labor and physical capital. Finally, the consolidated government collects taxes, pays deposit subsidies, invests in capital, lends to banks against collateral, and issues CBDC and reserves.

In the first part of the paper, we review the equivalence analysis between operating payment systems by introducing a collateral constraint for the central bank lending to banks. We study the problem under different degrees of substitutability between CBDC and bank deposits. First, we consider the case of perfect substitutes, as in Niepelt (2022), and then the more general case of imperfect substitutability between different means of payments.

Our findings reveal that when CBDC and deposits are perfect substitutes, the introduction of CBDC has no real consequences as long as (1) the resource cost per unit of effective real balances is the same for CBDC and deposits and (2) the central bank offers a loan rate that induces non-competitive banks to maintain the same balance sheet positions as before the introduction of CBDC. Our equivalent loan rate is lower than the one obtained in Niepelt (2022) because of the collateral requirement banks must respect when borrowing from the central bank. In particular, when the collateral constraint facing banks becomes more restrictive, the equivalent loan rate must be lower. Below a threshold, the collateral constraint becomes so restrictive that the central bank needs to pay banks to take these loans. This scenario is unlikely, as the haircut is too large to be seen.

However, when CBDC and deposits are imperfect substitutes, the central bank cannot make banks indifferent to the competition from CBDC. This is because there is no loan rate that the central bank can offer such that it leaves unchanged banks' profits. It follows that the new policy does not guarantee the same equilibrium allocations as before, implying that the introduction of CBDC has real effects on the economy.

In the second part of the paper, we explore how an increase in the demand for CBDC affects the real economy and if it could potentially lead to the crowding out of deposits. To do so, we study the economy's responses to changes in the households' relative preferences for CBDC over bank deposits. We consider an economy where households hold CBDC and deposits, and we assume that the payment instruments are imperfect substitutes. First, we look at how the economy reacts to a positive shock to the liquidity share of CBDC and then to a negative shock to the substitutability between payments. In both cases, we find that the demand for CBDC increases without crowding out deposits, but banks' profit drops due to the lower market power of banks.

Related Literature The contribution of our work to the existing literature is threefold. First, we complement the recent literature examining the impact of the introduction of CBDC on commercial banks.

For instance, Chiu et al. (2019) develop a micro-founded general equilibrium model calibrated to the U.S. economy and find that CBDC expands bank intermediation when the price of CBDC falls within a certain range while leading to disintermediation if its interest rate exceeds the upper limit of that range.

In another study, Burlon et al. (2022) construct a quantitative euro area dynamic stochastic general equilibrium (DSGE) model, where banks must post government bonds as collateral to borrow from the central bank. They investigate the transmission channels of the issuance of CBDC to bank intermediation, finding a bank disintermediation effect with central bank financing replacing deposits and government bonds displacing reserves and loans.

Along a parallel trajectory, Assenmacher et al. (2021) use a DSGE model to investigate the macroeconomic effects of CBDC when the central bank administrates the CBDC rate and collateral and quantity requirements. Their findings indicate that a more ample supply of CBDC reduces bank deposits, while stricter collateral or quantitative constraints reduce welfare but can potentially contain bank disintermediation. The latter effect is particularly true when the elasticity of substitution between bank deposits and CBDC is low.

Williamson (2022), on the other hand, explores the effects of the introduction of CBDC using a model of multiple means of payment. In his model, CBDC is a more efficient payment instrument than cash, but it lengthens the central bank's balance sheet, creating collateral scarcity in the economy.

Differently from these works, our study investigates the implications of CBDC issuance on bank intermediation using a Real Business Cycle model. Specifically, we extend the framework proposed in Niepelt (2022) by incorporating a collateral constraint for central bank lending to banks. Our findings indicate that increased demand for CBDC leads to a drop in banks' profit.

Second, our study contributes to the literature on the equivalence of payment systems.

Existing works by Brunnermeier and Niepelt (2019) and Niepelt (2022) propose a compensation mechanism where households shifting from deposits to CBDC can be offset by central bank lending to banks.

However, these models abstract from modeling the collateral constraint for central bank lending to banks. Notably, Piazzesi and Schneider (2022) show that when banks are required to hold liquid assets to back their deposits and face asset management costs, the equivalence between alternative payment instruments breaks down, even if banks can be refinanced directly by the central bank.

In light of this, we review the equivalence result established in Niepelt (2022) and extend the analysis by incorporating a collateral constraint for banks. We derive a new central bank lending rate that depends on the restrictiveness of the collateral requirement. Our findings reveal that the more binding the collateral constraint, the lower the central bank's loan rate must post.

Finally, we contribute to the literature on the relationship between CBDC and bank deposits by examining the effects of the introduction of CBDC considering different degrees of substitutability across means of payments [see, e.g., Bacchetta and Perazzi (2022), Barrdear and Kumhof (2022) and Kumhof and Noone (2021)].

The paper is organized as follows. Section 2 describes the model. Section 3 presents and discusses the analysis of the equivalence between operating payment systems. Section 4 characterizes the general equilibrium in which the household holds CBDC and deposits and discusses the dynamic effects of households' preferences shocks. Section 5 concludes.

2 Model

The model is based on Niepelt (2022) and is an extended Sidrauski (1967) model that embeds banks, deposits, government bonds, reserves, and CBDC into the baseline Real Business Cycle model. There is a continuum of mass one of homogeneous infinitely-lived households who own a succession of two-period-lived banks and of one-period-lived firms. The consolidated government determines monetary and fiscal policy.

2.1 Households

The representative household wants to maximize the discounted felicity function $\mathscr{U}$, which is increasing, strictly concave and satisfies Inada conditions. It takes prices, w_t and r_t; returns on asset i, R_t^i; profits, π_t; and taxes, τ_t as given and solves

$$\begin{aligned}
&\max_{\{c_t, x_t, k_{t+1}^h, m_{t+1}, n_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathscr{U}(c_t, x_t, z_{t+1}) \\
\text{s.t.} \quad & c_t + k_{t+1}^h + m_{t+1} + n_{t+1} + \tau_t = w_t(1 - x_t) + \pi_t + k_t^h R_t^k + m_t R_t^m + n_t R_t^n, \qquad (1) \\
& k_{t+1}^h, m_{t+1}, n_{t+1} \geq 0,
\end{aligned}$$

where $\beta \in (0, 1)$ is the positive discount factor, c_t and x_t denote household consumption of the good and leisure at date t, respectively; k_{t+1}^h is capital at date $t + 1$; and $z_{t+1} = z(m_{t+1}, n_{t+1})$ are effective real balances carried from date t to $t + 1$. Effective real balances are a function of both CBDC, m_{t+1}, and bank deposits, n_{t+1}.

Households value liquidity, as suggested by the *money in the utility function* specification. In this setting, it only matters that households demand liquidity services, not why they do. Equation (1) represents the household's budget constraint. The household finances consumption, taxes, investments in capital, and real balances, out of wage income, which equals the sum of the wage times the labor supply, distributed profits, and the gross return on its portfolio.

We assume interior solutions for capital, CBDC, and deposits, and we define the stochastic discount factor as $\Lambda_{t+s} \equiv \beta \mathscr{U}_c(c_{t+s}, x_{t+s}, z_{t+1+s}) / \mathscr{U}_c(c_t, x_t, z_{t+1})$. To express the Euler equations for CBDC and deposits in a more compact form, we define the risk-free rate and the liquidity premium on asset i, respectively:

$$R_{t+1}^f \equiv \frac{1}{\mathbb{E}_t \Lambda_{t+1}}, \qquad \mathscr{X}_{t+1}^i \equiv 1 - \frac{R_{t+1}^i}{R_{t+1}^f} \text{ for } i \in \{m, n\}.$$

Assuming that the interest rates on CBDC and deposits are risk-free, we can summarize the Euler equations as

$$\begin{aligned}
x_t: &\quad \mathscr{U}_x(c_t, x_t, z_{t+1}) = \mathscr{U}_c(c_t, x_t, z_{t+1})w_t, && (2)\\
k_{t+1}^h: &\quad 1 = \mathbb{E}_t R_{t+1}^k \Lambda_{t+1}, && (3)\\
m_{t+1}: &\quad \mathscr{U}_c(c_t, x_t, z_{t+1})\mathscr{X}_{t+1}^m = \mathscr{U}_z(c_t, x_t, z_{t+1})z_m(m_{t+1}, n_{t+1}), && (4)\\
n_{t+1}: &\quad \mathscr{U}_c(c_t, x_t, z_{t+1})\mathscr{X}_{t+1}^n = \mathscr{U}_z(c_t, x_t, z_{t+1})z_n(m_{t+1}, n_{t+1}). && (5)
\end{aligned}$$

Combining equations (4) and (5) we get that

$$z_m(m_{t+1}, n_{t+1})\mathscr{X}_{t+1}^n = z_n(m_{t+1}, n_{t+1})\mathscr{X}_{t+1}^m.$$

2.2 Banks

Within the literature, a motivation for introducing CBDC is when banks have some market power [e.g., Andolfatto (2021)]. The main implication is that banks can reduce deposit rates to extract rents, and households accept this markdown as they value the liquidity service provided by deposits. In particular, we follow Niepelt (2022) and assume that each bank is a monopsonist in its regional deposit market, such that households in a region can only access the regional bank.

A bank at date t issues deposits, n_{t+1}, borrows from the central bank, l_{t+1}, and collects government subsidies on deposit at rate θ_t. It invests in reserves, r_{t+1}, government bonds, b_{t+1}, and capital.[4] Without loss of generality, we abstract from bank equity. We follow Burlon et al. (2022) and assume that banks are subjective to a collateral requirement such that the loans they get from the central bank must be lower than a fraction θ_b of government bond holdings. In this setting, government bonds are the only assets that can be pledged as collateral. For simplification, we abstract away from interbank loans with collateral. Holding government bonds gives liquidity benefits to banks since they can use their holdings to obtain funding from the central bank. In other words, banks are willing to forego a spread on the risk-free rate because of the collateral benefits of holding government bonds (i.e., convenience yield). The convenience yield of government bonds reflects the additional benefits banks derive from holding these bonds beyond their financial yield. Therefore, government bonds are remunerated at a slightly lower rate than the risk-free rate.

The operating costs in the retail payment system, ν_t, are a negative function of the reserve-to-deposit ratio, ζ_{t+1}. This is analogous to a binding minimum reserves requirement,

[4] Bank's capital is defined as $k_{t+1}^b = n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}$. Alternatively, the bank can invest in loans to firms that eventually fund physical capital accumulation.

as larger reserves holdings relative to deposits lower the bank's operating costs. We also allow ν_t to vary with the stock of reserves and deposits of other banks, $\bar{\zeta}_{t+1}$, so as to capture positive externalities of reserve holdings. Differently from Niepelt (2022), we use a unit resource cost of managing deposit-based payments in the form $\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1})$. To simplify the analysis, we make some assumptions which imply that in equilibrium $\zeta_{t+1} = \bar{\zeta}_{t+1}$, and reserves are strictly positive if and only if deposits are strictly positive: when a bank holds no deposits, its operating costs are null, and when all other banks have no deposits, the bank's operating costs are large but bounded. In this way, we rule out asymmetric equilibrium in the bank's deposits and other banks' deposits. Otherwise, the operating cost function, $\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1})$, is strictly decreasing in both arguments, strictly convex, and satisfies $\nu_{\zeta\bar{\zeta},t} = 0$ or $\nu_{\zeta\zeta,t} \geq \nu_{\bar{\zeta}\bar{\zeta},t}$, as well as $\lim_{\zeta_{t+1}\to 0} \nu_{\zeta,t} = \infty$.

The bank takes the risk-free rate, as well as the rates of return on capital, reserves, and government bonds, the households' stochastic discount factor, and the subsidy rate as given. In contrast, the bank chooses the quantity of deposits and central bank loans subject to the deposit funding schedule of households.[5] Since the bank acts as a monopsonist in its regional deposit market, it takes the deposit funding schedule (rather than the deposit and the central bank loan rates) as given.

The program of the bank at date t reads

$$
\begin{aligned}
&\max_{n_{t+1}, l_{t+1}, r_{t+1}, b_{t+1}} \left\{ \pi^b_{1,t} + \mathbb{E}_t \left[\Lambda_{t+1} \pi^b_{2,t+1} \right] \right\} \\
\text{s.t.} \quad & \pi^b_{1,t} = -n_{t+1} \left(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t \right), \\
& \pi^b_{2,t+1} = (n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}) R^k_{t+1} + r_{t+1} R^r_{t+1} + b_{t+1} R^b_{t+1} - n_{t+1} R^n_{t+1} - l_{t+1} R^l_{t+1}, \\
& l_{t+1} \leq \theta_b \frac{b_{t+1}}{R^l_{t+1}}, \\
& n_{t+1}, l_{t+1}, b_{t+1} \geq 0.
\end{aligned}
$$

where

$$
\zeta_{t+1} \equiv \frac{r_{t+1}}{n_{t+1}}, \text{ and } \bar{\zeta}_{t+1} \equiv \frac{\bar{r}_{t+1}}{\bar{n}_{t+1}},
$$

and $\pi^b_{1,t}$, $\pi^b_{2,t+1}$ denote the cash flow generated in the first and second period of the bank's operations, respectively.

We assume interior solutions for deposits, loans, and government bonds, and we make use of the household's Euler equation for capital and the definition of the risk-free rate. Also, we

[5] In the model, the central bank's loan funding schedule mirrors the deposit funding schedule. This assumption plays a crucial role in the context of the equivalence analysis.

define the elasticity of the asset i with respect to the rate of returns on i as

$$\eta_{i,t+1} = \frac{\partial i_{t+1}}{\partial R^i_{t+1}} \frac{R^i_{t+1}}{i_{t+1}}, \qquad \text{for } i \in \{n, l\},$$

and the liquidity premium as

$$\mathcal{X}^i_{t+1} = 1 - \frac{R^i_{t+1}}{R^f_{t+1}}, \qquad \text{for } i \in \{n, l, r, b\}.$$

We follow Burlon et al. (2022) and assume that the collateral constraint is binding in equilibrium:[6]

$$\mu_t > 0, \qquad l_{t+1} = \theta_b \frac{b_{t+1}}{R^l_{t+1}}.$$

We can rewrite the bank's optimality conditions as

$$n_{t+1}: \qquad \mathcal{X}^n_{t+1} - \left(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t - \nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1})\zeta_{t+1}\right) = \frac{1}{\eta_{n,t+1}} \frac{R^n_{t+1}}{R^f_{t+1}}, \tag{6}$$

$$r_{t+1}: \qquad -\nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \mathcal{X}^r_{t+1}, \tag{7}$$

$$l_{t+1}: \qquad \mathcal{X}^l_{t+1} - \mu_t \left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}} \frac{R^l_{t+1}}{R^f_{t+1}}, \tag{8}$$

$$b_{t+1}: \qquad \mu_t \frac{\theta_b}{R^l_{t+1}} = \mathcal{X}^b_{t+1}. \tag{9}$$

The optimal conditions for deposits and reserves are analogous to the ones derived in Niepelt (2022). We first comment on the liability side of the bank's balance sheet, starting with deposits. The left-hand side (LHS) of equation (6) represents the marginal profit from issuing deposits, which is given by the difference between the bank's gain from the positive deposit liquidity premium and the marginal cost associated with increased deposit issuance. The right-hand side (RHS) equals the profit loss of inframarginal deposits, as higher deposit issuance is associated with increased interest rates on deposits. Similarly, the condition for central bank's loans, equation (8), states that the sum of the bank's marginal benefits of taking on more central bank's loans and the gain coming from the positive loan liquidity premium should be equal to the profit loss from the marginal cost associated with central bank's loans. In fact, higher loan holdings are associated with an increase in the interest rate on the central bank's loans.

Turning to the asset side of the bank's balance sheet, equation (7) equalizes the bank's

[6] See Appendix A for checking under which condition the collateral constraint binds.

gain from lower operating costs with the loss due to the bank's lower return coming from a positive spread on reserves. Looking at equation (9), the optimal choice of government bonds is when the bank's marginal costs of bond holdings are equal to the loss coming from the bank's lower return with a positive spread on government bonds.

We can combine the optimality conditions and comment on the implications. First, the combination of equations (6) and (7) gives:

$$\mathscr{X}^n_{t+1} - \Big[\Big(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t\Big) + \mathscr{X}^r_{t+1}\zeta_{t+1}\Big] = \frac{1}{\eta_{n,t+1}} \frac{R^n_{t+1}}{R^f_{t+1}}. \tag{6a}$$

This result implies that the bank's net benefit of issuing more deposits must equal the inframarginal cost of deposits.

Combining equations (8) and (9) results in the following relation:

$$\mathscr{X}^l_{t+1} - \mathscr{X}^b_{t+1} \frac{R^l_{t+1}}{\theta_b} \left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}} \frac{R^l_{t+1}}{R^f_{t+1}}. \tag{8a}$$

The marginal cost of taking on more central bank loans must equal the bank's net benefit of taking on more loans. This is given by the difference between the liquidity benefits given by the central bank loans and the marginal cost associated with the collateral constraint.

2.3 Firms

Neoclassical firms rent capital, k_t, and labor, ℓ_t, to produce the output good to maximize the profit π^f_t. The representative firm takes wages, w_t; the rental rate of capital, $R^k_t + \delta - 1$; and the good price as given and solves

$$\begin{aligned} & \max_{k_t, \ell_t} \pi^f_t \\ \text{s.t.} \quad & \pi^f_t = f_t(k_t, \ell_t) - k_t(R^k_t + \delta - 1) - w_t \ell_t, \end{aligned}$$

where f_t is the neoclassical production function.

The first-order conditions read

$$k_t : \qquad f_k(k_t, \ell_t) = R^k_t + \delta - 1, \tag{10}$$

$$\ell_t : \qquad f_l(k_t, \ell_t) = w_t. \tag{11}$$

2.4 Consolidated government

The consolidated government collects taxes and subsidies deposits, lends to banks against collateral, l_{t+1} invest in capital, k^g_{t+1}, and issues CBDC and reserves. The government budget constraint reads

$$\begin{aligned} k^g_{t+1} + l_{t+1} - b_{t+1} - m_{t+1} - r_{t+1} =& k^g_t R^k_t + l_t R^l_t - b_t R^b_t - m_t R^m_t - r_t R^r_t + \tau_t + \\ & - n_{t+1}\theta_t - m_{t+1}\mu^m - r_{t+1}\rho, \end{aligned} \tag{12}$$

where μ^m and ρ are the unit resource costs of managing CBDC and reserves payments, respectively.

2.5 Market clearing

Market clearing in the labor market requires that firms' labor demand equals the household's labor supply:

$$\ell_t = 1 - x_t. \tag{13}$$

Market clearing for capital requires that firms' demand for capital equals capital holdings of the household, banks, and the government:

$$k_t = k^h_t + (n_t + l_t - r_t - b_t) + k^g_t. \tag{14}$$

Profits distributed to the household must equal the sum of the banks' and firms' profits:

$$\pi_t = \pi^b_{1,t} + \pi^b_{2,t} + \pi^f_t. \tag{15}$$

By Walras' law, market clearing on labor and capital markets and the budget constraints of households, banks, firms, and the consolidated government imply market clearing on the goods market.

To derive the aggregate resource constraint for the economy, we plug equation (15) into the household's budget constraint, equation (1), and we impose market clearing conditions (13) and (14). Then, in combination with the government's budget constraint, equation (12), the resulting expression is the aggregate resource constraint:

$$k_{t+1} = f_t(k_t, 1 - x_t) + k_t(1 - \delta) - c_t \Omega^{rc}_t, \tag{16}$$

where we defined

$$\Omega_t^{rc} = 1 + \frac{m_{t+1}}{c_t}\mu^m + \frac{n_{t+1}}{c_t}(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) + \zeta_{t+1}\rho). \tag{17}$$

2.6 Functional Form Assumptions

In our analysis of the general equilibrium, we will impose the following functional form assumptions for households' preferences and real balances, operating costs for deposit-based payments, and firms' production function, respectively:

$$\mathscr{U}(c_t, x_t, z_{t+1}) = \frac{\left((1-\upsilon)c_t^{1-\psi} + \upsilon z_{t+1}^{1-\psi}\right)^{\frac{1-\sigma}{1-\psi}}}{1-\sigma} x_t^{\upsilon}, \tag{18}$$

$$z_{t+1}(m_{t+1}, n_{t+1}) = \left(\lambda_t m_{t+1}^{1-\epsilon_t} + n_{t+1}^{1-\epsilon_t}\right)^{\frac{1}{1-\epsilon_t}}, \tag{19}$$

$$\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \phi_1 \zeta_{t+1}^{1-\varphi} + \phi_2 \bar{\zeta}_{t+1}^{1-\varphi}, \tag{20}$$

$$f_t(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha}, \tag{21}$$

where $\upsilon, \psi \in (0,1); \sigma > 0, \neq 1; \epsilon_t > 0; \phi_1 > 0, \phi_2 \geq 0; \varphi > 1$. The parameter $\lambda_t > 0$ represents the liquidity benefits of holding CBDC relative to the liquidity benefits of holding deposits.

3 Equivalence between operating payment systems

In this section, we study the equivalence between operating payment systems. We study the problem under different degrees of substitutability between CBDC and deposits. First, we consider the case of perfect substitutes, as in Niepelt (2022), then the more general case of imperfect substitutability between different means of payment, assuming constant elasticity of substitution (CES) for real balances.

3.1 Perfect substitutability between payment instruments

We start by analyzing the equivalence of operating payment systems considering the case of perfect substitutability between payment instruments, as in Niepelt (2022). We do so by letting ϵ be zero in the household's real balances function, equation (29).

Consider a policy that implements an equilibrium with deposits, n_{t+1}; reserves, r_{t+1}; no central bank loans, $l_{t+1} = 0$; and no government bonds, $b_{t+1} = 0$. We want to analyze whether there exists another policy, indicated by circumflexes, that in equilibrium guarantees fewer deposits, $\hat{n}_{t+1}$, and reserves, $\hat{r}_{t+1}$; more CBDC, $\hat{m}_{t+1}$; central bank loans, $\hat{l}_{t+1}$; government bonds, $\hat{b}_{t+1}$; a different ownership structure of capital; possibly households taxes, $\hat{T}_{1,t}$ and $\hat{T}_{2,t+1}$; but the same equilibrium allocation and price system.

Denoting by Δ the intervention, the two policies and equilibrium described above coincide, except that

$$\hat{n}_{t+1} - n_{t+1} = -\Delta, \qquad \hat{r}_{t+1} - r_{t+1} = -\zeta_{t+1}\Delta,$$

$$\hat{m}_{t+1} - m_{t+1} = \frac{1}{\lambda_t}\Delta, \qquad \hat{l}_{t+1} - l_{t+1} = (1-\zeta_{t+1})\Delta, \qquad \hat{b}_{t+1} - b_{t+1} = \frac{\hat{l}_{t+1}R^l_{t+1}}{\theta_b},$$

$$\hat{k}^g_{t+1} - k^g_{t+1} = -\left(1 - \frac{1}{\lambda_t}\right)\Delta + \hat{b}_{t+1}, \qquad \hat{k}^h_{t+1} - k^h_{t+1} = \left(1 - \frac{1}{\lambda_t}\right)\Delta,$$

where l_{t+1} and b_{t+1} are normalized to zero.[7]

The proposed household portfolio change does not alter effective real balances, the aggregate capital stock, and the reserves-to-deposits ratio, i.e.,

$$\hat{z}_{t+1} = z_{t+1}, \qquad \hat{k}_{t+1} = k_{t+1}, \qquad \hat{\zeta}_{t+1} = \zeta_{t+1}.$$

The cash flows generated in the first and second period of the bank's operations are, respectively:

$$\pi^b_{1,t} = -n_{t+1}\left(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t\right),$$

$$\pi^b_{2,t+1} = (n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1})R^k_{t+1} + r_{t+1}R^r_{t+1} + b_{t+1}R^b_{t+1} - n_{t+1}R^n_{t+1} - l_{t+1}R^l_{t+1}.$$

Dropping the arguments of the ν_t function, the changes in bank profits at date t and at date $t+1$ are, respectively:

$$\hat{\pi}^b_{1,t} - \pi^b_{1,t} = \Delta\left(\nu_t(\dots) - \theta_t\right),$$

$$\hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1} = \Delta\left(R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)R^l_{t+1}\right).$$

[7] To guarantee the non-negativity of deposits, capital holdings, and reserves, Δ must not be too large. Specifically, we impose

$$\Delta \leq n_{t+1}, \qquad \zeta_{t+1}\Delta \leq r_{t+1},$$

$$\left(1 - \frac{1}{\lambda_t}\right)\Delta \leq k^g_{t+1}, \qquad \left(1 - \frac{1}{\lambda_t}\right)\Delta \geq -k^h_{t+1}.$$

Let $\hat{T}_{1,t}$ be a tax at date t that compensates for the reduced bank losses borne by households. Also, let $\hat{T}_{2,t+1}$ be a tax at date $t+1$ that compensates for the change in the households portfolio's return as well as for the change in bank profits that households collect at date $t+1$.

We define these quantities as

$$\begin{aligned}
\hat{T}_{1,t} &= \hat{\pi}^b_{1,t} - \pi^b_{1,t} \\
&= \Delta\left(\nu_t(\dots) - \theta_t\right), \\
\hat{T}_{2,t+1} &= (\hat{k}^h_{t+1} - k^h_{t+1})R^k_{t+1} + (\hat{n}_{t+1} - n_{t+1})R^n_{t+1} + (\hat{m}_{t+1} - m_{t+1})R^m_{t+1} + \hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1} \\
&= \left(\Delta - \left[\frac{1}{\lambda_t}\left(n^{1-\epsilon_t}_{t+1} - \hat{n}^{1-\epsilon_t}_{t+1}\right) + m^{1-\epsilon_t}_{t+1}\right]^{\frac{1}{1-\epsilon_t}} + m_{t+1}\right) R^k_{t+1} - \Delta R^n_{t+1} \\
&+ \left(\left[\frac{1}{\lambda_t}\left(n^{1-\epsilon_t}_{t+1} - \hat{n}^{1-\epsilon_t}_{t+1}\right) + m^{1-\epsilon_t}_{t+1}\right]^{\frac{1}{1-\epsilon_t}} - m_{t+1}\right) R^m_{t+1} \\
&+ \Delta\left(R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1}\right).
\end{aligned}$$

Using conditions from the household's optimization problem, we can write the market value of the two taxes as time t as

$$\begin{aligned}
&\hat{T}_{1,t} + \mathbb{E}_t\Lambda_{t+1}\hat{T}_{2,t+1} \\
&= \Delta\left\{\left(\nu_t(\dots) - \theta_t\right) + \frac{R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1}}{R^f_{t+1}}\right\}.
\end{aligned}$$

For the intervention to be neutral to the economy, the market value of taxes must be zero. We can derive the central bank's loan rate that ensures this is true:

$$R^l_{t+1} = \frac{R^n_{t+1} + \left(\nu_t(\dots) - \theta_t\right) R^f_{t+1} - \zeta_{t+1}R^r_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}. \tag{22}$$

The market value of the changes in bank profits is

$$\begin{aligned}
&(\hat{\pi}^b_{1,t} - \pi^b_{1,t}) + \mathbb{E}_t\Lambda_{t+1}(\hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1}) \\
&= \Delta\left(\nu_t(\dots) - \theta_t\right) + \mathbb{E}_t\Lambda_{t+1}\Delta\left(R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1}\right).
\end{aligned}$$

Substituting in equation (22) and the definition of the risk-free rate, we get:

$$
\begin{aligned}
&(\hat{\pi}^b_{1,t} - \pi^b_{1,t}) + \mathbb{E}_t \Lambda_{t+1} (\hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1}) \\
&= \Delta \left(\nu_t(\dots) - \theta_t \right) + \frac{1}{R^f_{t+1}} \Delta \Bigg(R^n_{t+1} - \zeta_{t+1} R^r_{t+1} + \\
&- (1 - \zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b} \right) \frac{R^n_{t+1} + (\nu_t(\dots) - \theta_t)\, R^f_{t+1} - \zeta_{t+1} R^r_{t+1}}{(1 - \zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b} \right)} \Bigg) \\
&= 0.
\end{aligned}
$$

It follows that if the central bank offers the lending rate derived in equation (22), the market values of the taxes and of the changes in bank profits are zero.

As for households, we show that also the government's dynamic and intertemporal budget constraints continue to be satisfied with the modified portfolios and policy. The consolidated government budget constraint at date t reads:

$$
k^g_{t+1} - m_{t+1} - r_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t - n_{t+1} \theta_t - m_{t+1} \mu^m - r_{t+1} \rho.
$$

The government budget constraint at date t, after the portfolio's change, is

$$
\hat{k}^g_{t+1} + \hat{l}_{t+1} - \hat{m}_{t+1} - \hat{r}_{t+1} - \hat{b}_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t - \hat{n}_{t+1} \theta_t - \hat{m}_{t+1} \mu^m - \hat{r}_{t+1} \rho + \hat{T}_{1,t}.
$$

Rearranging, simplifying, and collecting terms:

$$
k^g_{t+1} - m_{t+1} - r_{t+1} + \Delta \left\{ \frac{\mu^m}{\lambda_t} - (\nu_t(\dots) + \rho \zeta_{t+1}) \right\} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t - n_{t+1} \theta_t - m_{t+1} \mu^m - r_{t+1} \rho.
$$

Assume that the resource cost per unit of effective real balances is the same for CBDC and deposits:

$$
\frac{\mu^m}{\lambda_t} = \nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) + \zeta_{t+1} \rho. \tag{23}
$$

Under condition (23), the constraint after the intervention is identical to the constraint before the portfolio's change.

The government budget constraint at date $t+1$, after the portfolio's change, is

$$
\begin{aligned}
k^g_{t+2} - m_{t+2} - r_{t+2} = \hat{k}^g_{t+1} R^k_{t+1} + \hat{l}_{t+1} R^l_{t+1} - \hat{m}_{t+1} R^m_{t+1} - \hat{r}_{t+1} R^r_{t+1} - \hat{b}_{t+1} R^b_{t+1} + \tau_{t+1} + \\
- n_{t+2} \theta_{t+1} - m_{t+2} \mu^m_{t+1} - r_{t+2} \rho_{t+1} + \hat{T}_{2,t+1}.
\end{aligned}
$$

Rearranging, simplifying, and collecting terms:

$$k^g_{t+2} - m_{t+2} - r_{t+2} = k^g_{t+1} R^k_{t+1} - m_{t+1} R^m_{t+1} - r_{t+1} R^r_{t+1} + \tau_{t+1} + \\ - n_{t+2}\theta_{t+1} - m_{t+2}\mu^m_{t+1} - r_{t+2}\rho_{t+1}.$$

Equation (22) implies that the constraint at date $t+1$ is equivalent to the constraint before the portfolio's change.

We claimed initially that the proposed intervention does not change the price system. In this case, firms' optimal production decisions and profits are unchanged. Lastly, we must show that the modified bank's portfolio is still optimal.

Before the intervention, the bank's choice set is determined by the cost function, the subsidy rate, the household's stochastic discount factor, rates on returns on capital and reserves, and the deposit funding schedule. The intervention leaves unchanged the cost function, the subsidy rate, the stochastic discount factor, and the rates on returns on capital and reserves. After the intervention, as households hold more CBDC, there is a modified deposit funding schedule, together with a central bank's loan funding schedule. To induce non-competitive banks to go along with the equivalent balance sheet positions as before the intervention, the central bank needs to post an appropriate loan funding schedule. Subject to this schedule, the bank chooses loans that make up for the reduction in funding from households, net of reserves, at the same effective price. Previously, we have derived the central bank's lending rate that makes the market values of the changes in bank profit equal to zero:

$$R^l_{t+1} = \frac{R^n_{t+1} + (\nu_t(\dots) - \theta_t)\, R^f_{t+1} - \zeta_{t+1} R^r_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}.$$

We can demonstrate that the term in parenthesis at the denominator on the RHS is positive. From the households' problem, we know that $R^k_{t+1} \leq R^f_{t+1}$, assuming that the rate of return on capital is not risky, we can approximate $R^k_{t+1} \simeq R^f_{t+1}$. We also know that, due to the liquidity benefits banks have from holding government bonds, $R^b_{t+1} < R^f_{t+1}$. Recalling that $\theta_b \in [0,1]$, it follows that the additional term is positive.

The rate we derived differs from the one derived in Niepelt (2022) precisely from the additional terms in parenthesis at the denominator on the RHS. It follows that our equivalent loan rate is lower than the one in Niepelt (2022). The intuition is that when banks are not collateral constrained, they can borrow as much as they want from the central bank. With a collateral requirement, the central bank needs to offer a lower lending rate to incentivize banks to borrow the same quantity as in the absence of the constraint, such that their balance sheet remains unaffected. The equivalent loan rate we derived depends on how restrictive is the collateral constraint: the more binding it is, the lower the lending rate the central bank needs to offer. The equivalent loan rate is at its highest when all bonds held by the banks can be pledged, i.e., $\theta_b = 1$.

3.2 Relationship between the equivalent loan rate and the collateral constraint level

In Section 3.1, we derived the central bank's lending rate that makes the modified bank portfolio optimal. Next, we investigate the relationship between this equivalent loan rate and the level of the collateral constraint, which is proxied by the fraction of government bonds that can be pledged as collateral.

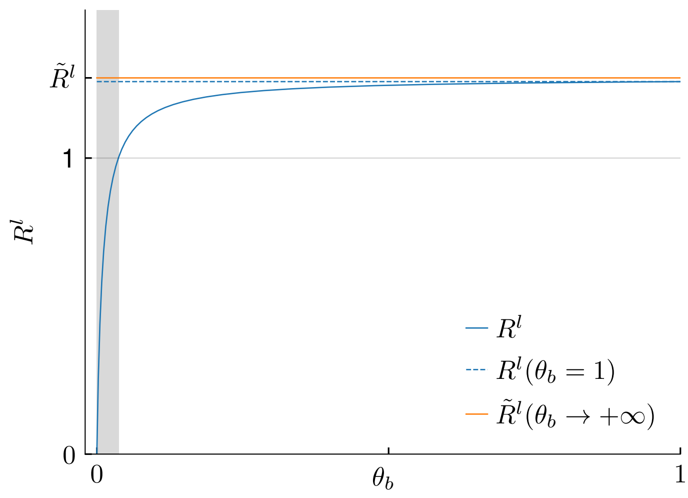

Figure 1: Equivalent loan rate, R^l_{t+1}, as function of the collateralized government bonds, θ_b

Figure 1 shows the equivalent loan rate we derive above, equation (22), as a function of the fraction of government bonds pledged as collateral. The maximum equivalent loan rate, the blue dashed line, is reported next to the orange line, representing the maximum value of the equivalent loan rate obtained in Niepelt (2022), which we call $\tilde{R}^l_{t+1}$.[8] It was previously noted that the equivalent rate we obtain is below Niepelt's and is logarithmically increasing in the fraction of government bonds pledged as collateral. Values of R^l_{t+1} equal or lower than unity imply that the central bank needs to pay banks to be able to lend to them, as the

[8]The author does not account for the central bank's collateral requirement; therefore it is like considering $\theta_b \to +\infty$.

collateral constraint is so restrictive that banks do not want to borrow from the central bank. This scenario is unlikely, as the corresponding values of θ_b in the shaded grey area are too low to be seen. In fact, empirically the value of θ_b is around 0.995 [see, e.g., Burlon et al. (2022)].

We can ask what would happen if the central bank offered a lending rate slightly above or below the equivalent loan rate. Nevertheless, the equivalence analysis has to be seen as a comparison of two steady states: one in which there is no CBDC (or a very little quantity in circulation) and the other in which both CBDC and bank deposits are used as means of payment. We can conjecture that, with a central bank's lending rate around the equivalent loan rate, we would still be in a situation very close to the optimum, and our conclusions would not change.

3.3 Imperfect substitutability between payment instruments

Imperfect substitutability between CBDC and bank deposits has been widely assumed in the existing literature.[9] However, there is little guidance on the degree of substitutability between the two payment methods. Barrdear and Kumhof (2022) offer a possible approach by calibrating the elasticity of substitution between CBDCs and bank deposits based on the elasticity of substitution across retail deposit accounts at different banks. They argue that the level of substitutability is low, given that even with high variability of interest rates offered on instant-access accounts by different banks, most households tend to remain with their current providers. They suggest that people tend to stick with what they know and are familiar with.

Although the lack of empirical evidence on the relationship between CBDC and bank deposits, CBDC would most likely not be a perfect substitute for bank deposits [see, e.g., Bacchetta and Perazzi (2022)]. In this section, we study the equivalence of operating payment systems assuming that the household values CBDC and deposits according to equation (29), such that CBDC and deposits are imperfect substitutes.

As in Section 3.1, consider a policy that implements an equilibrium with deposits, n_{t+1}; reserves, r_{t+1}; no central bank loans, $l_{t+1} = 0$; and no government bonds, $b_{t+1} = 0$. We want to analyze whether there exists another policy, indicated by circumflexes, that in equilibrium guarantees fewer deposits, $\hat{n}_{t+1}$, and reserves, $\hat{r}_{t+1}$; more CBDC, $\hat{m}_{t+1}$; central bank loans, $\hat{l}_{t+1}$; government bonds, $\hat{b}_{t+1}$; a different ownership structure of capital; possibly households taxes, $\hat{T}_{1,t}$ and $\hat{T}_{2,t+1}$; but same allocation and price system.

[9] See, e.g., Bacchetta and Perazzi (2022), Barrdear and Kumhof (2022), Burlon et al. (2022) and Kumhof and Noone (2021).

The two policies and equilibrium coincide, except that

$$\hat{n}_{t+1} - n_{t+1} = -\Delta, \qquad \hat{r}_{t+1} - r_{t+1} = -\zeta_{t+1}\Delta,$$

$$\hat{m}_{t+1} - m_{t+1} = \left[\frac{1}{\lambda_t}\left(n_{t+1}^{1-\epsilon_t} - \hat{n}_{t+1}^{1-\epsilon_t}\right) + m_{t+1}^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}} - m_{t+1},$$

$$\hat{l}_{t+1} - l_{t+1} = (1-\zeta_{t+1})\Delta, \qquad \hat{b}_{t+1} - b_{t+1} = \frac{\hat{l}_{t+1}R^l_{t+1}}{\theta_b},$$

$$\hat{k}^g_{t+1} - k^g_{t+1} = -\left(\hat{k}^h_{t+1} - k^h_{t+1}\right) + \hat{b}_{t+1}, \qquad \hat{k}^h_{t+1} - k^h_{t+1} = \Delta - \left(\hat{m}_{t+1} - m_{t+1}\right),$$

where l_{t+1} and b_{t+1} are normalized to zero.

We still observe that the proposed household portfolio change does not alter effective real balances, the aggregate capital stock, and the reserves-to-deposits ratio. We define the two taxes $\hat{T}_{1,t}$ and $\hat{T}_{2,t+1}$ as in Section 3.1. Using the household's Euler equation for capital and from the definition of the risk-free rate, we can express the market value of taxes at time t as $\hat{T}_{1,t} + \mathbb{E}_t\Lambda_{t+1}\hat{T}_{2,t+1}$. For the intervention to be neutral to the economy, the market value of taxes must be zero. This is true when the central bank's lending rate is equal to:[10]

$$R^l_{t+1} = \frac{\lambda_t\left(\frac{\hat{m}_{t+1}-m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon_t} R^n_{t+1} - \zeta_{t+1}R^r_{t+1} + \left(\nu_t(\dots) - \theta_t + 1 - \lambda_t\left(\frac{\hat{m}_{t+1}-m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon_t}\right)R^f_{t+1}}{(1-\zeta_{t+1})\left(1+\frac{R^k_{t+1}-R^b_{t+1}}{\theta_b}\right)}.$$

The market value of the changes in bank profits is

$$\begin{aligned}&(\hat{\pi}^b_{1,t} - \pi^b_{1,t}) + \mathbb{E}_t\Lambda_{t+1}(\hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1})\\ &= \Delta\left(\nu_t(\dots) - \theta_t\right) + \mathbb{E}_t\Lambda_{t+1}\Delta\left(R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1+\frac{R^k_{t+1}-R^b_{t+1}}{\theta_b}\right)R^l_{t+1}\right).\end{aligned}$$

Substituting in the expression we derived for R^l_{t+1}, the last expression becomes

$$\begin{aligned}&(\hat{\pi}^b_{1,t} - \pi^b_{1,t}) + \mathbb{E}_t\Lambda_{t+1}(\hat{\pi}^b_{2,t+1} - \pi^b_{2,t+1})\\ &= \mathbb{E}_t\Lambda_{t+1}\Delta\left(R^n_{t+1} - \lambda_t\left(\frac{\hat{m}_{t+1}-m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon_t}R^n_{t+1} - \left(1-\lambda_t\left(\frac{\hat{m}_{t+1}-m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon_t}\right)R^f_{t+1}\right),\end{aligned}$$

which does not reduce to zero for any central bank's lending rate. In other words, the central

[10] In deriving the central bank's loan rate, we use the condition $\frac{m_{t+1}}{n_{t+1}} = \left(\lambda_t\frac{\chi^n_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1}{\epsilon_t}}$ from the household's optimization problem.

bank cannot make banks indifferent to the competition from CBDC. In fact, a change in banks' profitability implies that the new policy does not guarantee the same allocations as before, implying that the introduction of CBDC has real effects on the economy.

4 General Equilibrium

In this section, we analyze an equilibrium in which the household holds both CBDC and deposits. Note that to pin down the demands for CBDC and deposits, we assume the functional form for real balances as in equation (29).

4.1 Equilibrium conditions

The set of equilibrium conditions resembles a Real Business Cycle model augmented with some "pseudo wedges".

The Euler equation, leisure choice, and resource constraint are, respectively:

$$c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} x_{t+1}^{\upsilon} \Omega_{t+1}^c R_{t+1}^k \right], \tag{24}$$

$$\frac{c_t^{1-\sigma}}{1-\sigma} \upsilon x_t^{\upsilon-1} \Omega_t^x = w_t c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c, \tag{25}$$

$$k_{t+1} = k_t^{\alpha}(1-x_t)^{1-\alpha} + k_t(1-\delta) - c_t \Omega_t^{rc}, \tag{26}$$

where the return on capital and the real wage are

$$R_{t+1}^k = 1 - \delta + \alpha \left(\frac{k_{t+1}}{1-x_{t+1}} \right)^{\alpha-1}, \tag{27}$$

$$w_t = (1-\alpha) \left(\frac{k_t}{1-x_t} \right)^{\alpha}. \tag{28}$$

The demand for effective real balances, CBDC, and bank deposits are, respectively:

$$z_{t+1} = c_t \left(\frac{v}{1-v} \frac{1}{\chi_{t+1}} \right)^{\frac{1}{\psi}}, \tag{29}$$

$$m_{t+1} = z_{t+1} \left(\lambda_t \frac{\chi_{t+1}}{\chi_{t+1}^m} \right)^{\frac{1}{\epsilon_t}}, \tag{30}$$

$$n_{t+1} = z_{t+1} \left(\frac{\chi_{t+1}}{\chi_{t+1}^n} \right)^{\frac{1}{\epsilon_t}}. \tag{31}$$

The cost of liquidity reads

$$\chi_{t+1} = \frac{\chi^m_{t+1}\chi^n_{t+1}}{\left(\lambda_t^{\frac{1}{\epsilon_t}}\left(\chi^n_{t+1}\right)^{\frac{1-\epsilon_t}{\epsilon_t}} + \left(\chi^m_{t+1}\right)^{\frac{1-\epsilon_t}{\epsilon_t}}\right)^{\frac{\epsilon_t}{1-\epsilon_t}}}. \tag{32}$$

From the bank's optimality condition:

$$\chi^n_{t+1} - \chi^n_{t+1}\left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon_t}\right)^{-1} = (\phi_1\varphi + \phi_2)\left(\frac{\chi^r_{t+1}}{\phi_1(\varphi-1)}\right)^{\frac{\varphi-1}{\varphi}} - \theta_t, \tag{33}$$

where

$$s_t = \frac{\lambda_t^{\frac{1}{\epsilon_t}}\left(\chi^n_{t+1}\right)^{\frac{1-\epsilon_t}{\epsilon_t}}}{\lambda_t^{\frac{1}{\epsilon_t}}\left(\chi^n_{t+1}\right)^{\frac{1-\epsilon_t}{\epsilon_t}} + \left(\chi^m_{t+1}\right)^{\frac{1-\epsilon_t}{\epsilon_t}}}. \tag{34}$$

The CBDC and reserve spreads are, respectively:

$$\chi^m_{t+1} = 1 - \frac{R^m_{t+1}}{R^f_{t+1}}, \tag{35}$$

$$\chi^r_{t+1} = 1 - \frac{R^r_{t+1}}{R^f_{t+1}}. \tag{36}$$

Recall the definition of the risk-free rate:

$$R^f_{t+1} = \frac{1}{\mathbb{E}_t[\Lambda_{t+1}]}, \tag{37}$$

where

$$\Lambda_{t+1} = \beta\frac{c_{t+1}^{-\sigma}x^{\upsilon}_{t+1}\Omega^c_{t+1}}{c_t^{-\sigma}x^{\upsilon}_t\Omega^c_t}. \tag{38}$$

Finally, the auxiliary variables are:

$$\Omega^c_t = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}}\left(1+\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}}\chi_{t+1}^{1-\frac{1}{\psi}}\right)^{\frac{\psi-\sigma}{1-\psi}}, \tag{39}$$

$$\Omega^x_t = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}}\left(1+\left(\frac{\upsilon}{1-\upsilon}\right)^{\frac{1}{\psi}}\chi_{t+1}^{1-\frac{1}{\psi}}\right)^{\frac{1-\sigma}{1-\psi}}, \tag{40}$$

$$\Omega^{rc}_t = 1 + \frac{m_{t+1}}{c_t}\mu^m + \frac{n_{t+1}}{c_t}\left((\phi_1+\phi_2)\left(\frac{\chi^r_{t+1}}{\phi_1(\varphi-1)}\right)^{\frac{\varphi-1}{\varphi}} + \left(\frac{\chi^r_{t+1}}{\phi_1(\varphi-1)}\right)^{-\frac{1}{\varphi}}\rho\right). \tag{41}$$

4.2 Dynamic effects of households' preferences shocks

After characterizing the general equilibrium, we aim to investigate the potential threat to financial stability should CBDC crowds out deposits. We address this concern by studying the economy's responses to changes in the households' relative preferences for CBDC over bank deposits.

Specifically, we first assess the effects of a positive shock to the liquidity share of CBDC, λ_t, and assume it follows a log AR(1) process of the form

$$\log(\lambda_t) = (1-\rho^{\lambda})\log(\lambda) + \rho^{\lambda}\log(\lambda_{t-1}) + e_t^{\lambda}, \tag{42}$$

where where ρ^{λ} is the persistence parameter, λ is the steady-state value, and e_t^{λ} is the exogenous one-time shock.

Secondly, we evaluate the impact of a positive shock to the inverse of the elasticity of substitution, ϵ_t, assuming it also follows a log AR(1) process of the form

$$\log(\epsilon_t) = (1-\rho^{\epsilon})\log(\epsilon) + \rho^{\epsilon}\log(\epsilon_{t-1}) + e_t^{\epsilon}, \tag{43}$$

where ρ^{ϵ} is the persistence parameter, ϵ is the steady-state value, and e_t^{ϵ} is the exogenous one-time shock. This shock corresponds to a negative shock to the substitutability between CBDC and deposits. Was a CBDC to be issued, the substitutability parameter would represent the willingness of households to substitute away one payment method for the other.

Bacchetta and Perazzi (2022) study the effect of the elasticity of substitution between CBDC and bank deposits on the demand for CBDC, conditioning on the relative level of interest paid by CBDC with respect to the interest paid by bank deposits. They find that the relationship between the interest rate on CBDC and demand for it is affected by the elasticity of substitution between CBDC and deposits. When the interest rate on CBDC is below that of deposits, demand for CBDC decreases in the elasticity of substitution. On the other hand, when the CBDC interest rate is higher but not significantly above that of deposits, the same effect is seen but in the opposite direction, meaning as the substitutability increases, demand for CBDC increases. However, when the CBDC interest rate is significantly above that of deposits, the holdings of CBDC decrease as the substitutability between the two instruments increases. The intuition is that when the two instruments are easily substitutable, people are less willing to use the more expensive option, but when they are less substitutable, it takes more of one to replace the other.

4.3 Parameters

The model is quarterly. We assume that in the steady state, the household perceives CBDC and deposits as equally useful for liquidity purposes, i.e., $\lambda = 1$. The household's discount factor β is set to the standard value of 0.99. Additionally, we assume that in the steady state the inverse substitutability between the two liquid assets, ϵ, is 1/6. This corresponds to a medium degree of substitutability following Bacchetta and Perazzi (2022). We follow Niepelt (2022) and set the inverse intertemporal elasticity of substitution, σ, to 0.5. The leisure function coefficient, υ, is set to 0.85 to match a steady-state labor supply of approximately 1/3. We assume that consumption and liquidity services are complements. Therefore, the inverse intratemporal elasticity of substitution between the two, ψ, is set higher than σ and equal to 0.6. The capital share of output, α, and the rate of capital depreciation, δ, are set to the standard values of 1/3 and 0.025, respectively.

Throughout this section, we assume that the government does not extend subsidies to banks, i.e., $\theta_t = 0$. We follow Niepelt (2022) and set the government's marginal cost of providing reserves, ρ, to 0.01. We follow the standard and set the persistence parameters in the log AR(1) processes, ρ^ϵ and ρ^λ, to 0.9.

Appendix C describes the model calibration of the remaining parameters. For simplicity, we assume that $\phi_1 = \phi_2 = \phi$. We calibrate the banks' cost function coefficients, ϕ and φ, as well as the utility weight of liquidity, $\boldsymbol{\upsilon}$, to match three steady-state quantities: CBDC-to-deposits ratio, m/n, reserve-to-deposits ratio, ζ, and consumption velocity, c/z. To minimize the compositional effect on the resource cost of liquidity provision, we set the unit resource costs of managing CBDC, μ, equal to the total resource cost of providing deposits.

Table 1 summarizes the parameter values.

Parameter	Value	Source
λ	1	Assumption
β	0.99	Standard
ϵ	1/6	Bacchetta and Perazzi (2022)
σ	0.5	Niepelt (2022)
υ	0.85	Assumption (Match steady-state labor supply $\approx 1/3$)
ψ	0.6	Assumption (Ensure $\psi > \sigma$)
α	1/3	Standard
δ	0.025	Standard
θ_t	0	Assumption
ρ	0.01	Niepelt (2022)
$\rho^\epsilon, \rho^\lambda$	0.9	Standard
ϕ	0.00061	Model
φ	2.00924	Model
v	0.01200	Model
μ	0.00745	Model

Table 1: Model Parameters

4.4 Impulse responses

Figure 2 shows the impulse responses as deviations from the steady state to a 10% increase in the liquidity share of CBDC, λ_t.

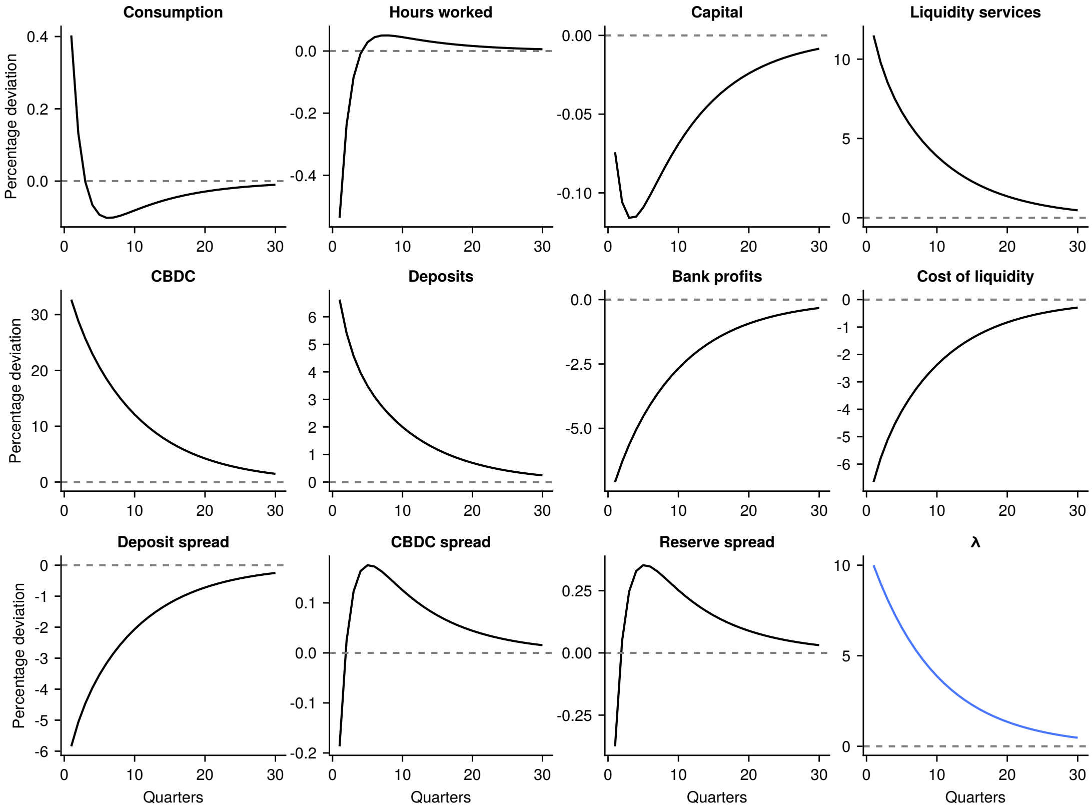

Figure 2: Impulse responses to 10% increase in the liquidity share of CBDC, λ_t

As the liquidity share of CBDC increases, the spreads on both CBDC and deposits decrease. Since the CBDC rate is kept constant, the marginal decrease in CBDC spread is due to a slight reduction in the risk-free rate.[11] On the other hand, the deposit spread decreases by a much larger magnitude. An increase in λ_t means CBDC is becoming a more attractive source of liquidity for households; thus, CBDC demand increases significantly. Banks, facing tougher competition from CBDC, decrease the price of deposits by a large margin to stem the potential deposit outflows. Intuitively, a more attractive CBDC diminishes the market power of banks. Thus, the markup on deposit spread (over the marginal cost of deposit issuance) that banks impose on households is reduced, and bank profits drop.

With both spreads decreasing, households' (average) cost of liquidity becomes lower, which induces them to hold more liquidity services and increase consumption. This is because a lower cost of liquidity increases households' current marginal utility of consumption. In other words, the opportunity cost of savings has increased, incentivizing households to save less and consume more. At the same time, households' marginal benefit of leisure is now higher

[11]The risk-free rate response is not plotted in the figure, but it is analogous to that of the reserve spread.

than the marginal cost, inducing them to decrease labor supply. Lastly, with the increased liquidity holdings by households, the societal resource costs associated with liquidity provision are higher and thus reduce capital accumulation.

Figure 3 shows the impulse responses, as deviations from steady state, to a 10% increase in inverse elasticity of substitution between CBDC and deposits, ϵ_t. The impulse responses are essentially the same as with an increase in λ_t.

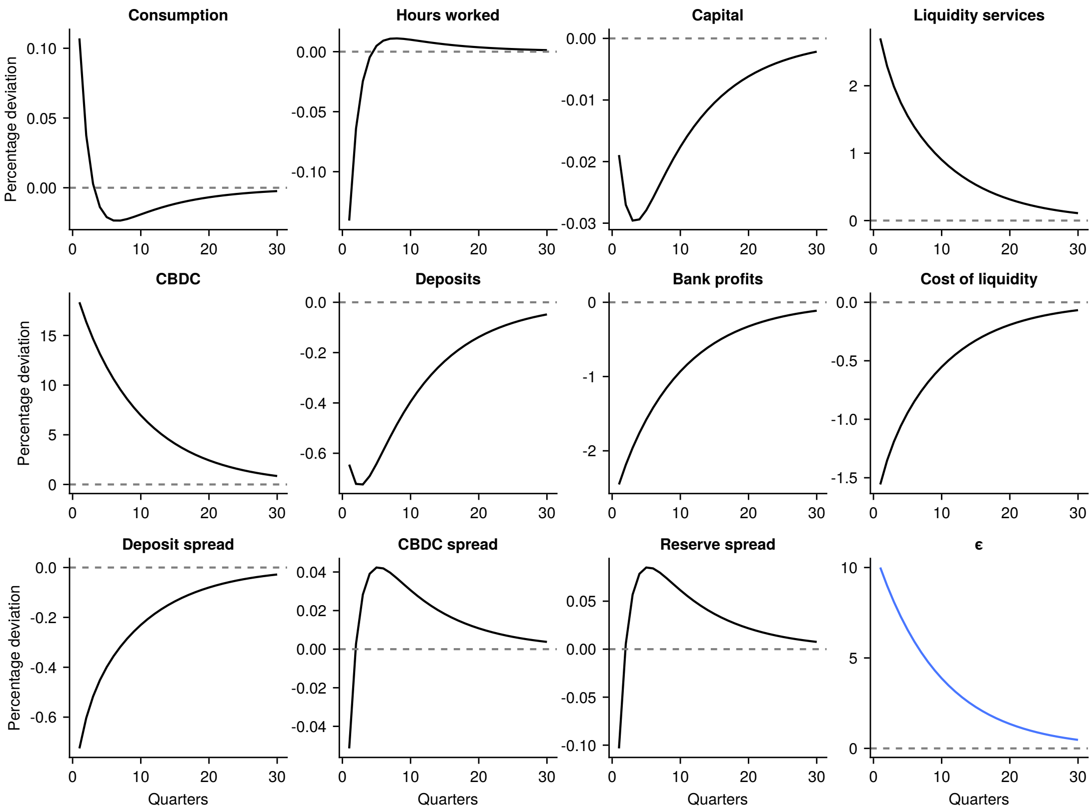

Figure 3: Impulse responses to 10% increase in inverse elasticity of substitution between CBDC and deposits, ϵ_t

4.5 Robustness checks

An aspect of uncertainty regarding any practical implementation of CBDC would be the households' perception of its usefulness relative to bank deposits. Therefore, we test the robustness of our results by changing the liquidity weight of CBDC and the substitutability between CBDC and deposits in the steady state.

Firstly, we change the steady-state liquidity weight of CBDC, λ, to 0.5 and 1.5. Figures 4 and 5 in the Appendix D show the impulse responses to a 10% increase in λ_t when its steady-state value is 0.5 and 1.5, respectively. Comparing these responses to the main specification in Figure 2, we see that the results remain the same. The shapes of the impulse responses are identical, while there are some minor differences in magnitudes. Figures 8 and 9 in the Appendix D show the impulse responses to a 10% increase in ϵ_t when steady-state λ is 0.5 and 1.5, respectively. Comparing these to Figure 3, we see that the main takeaways from the previous section stand. The only major difference is in Figure 8 when the steady-state CBDC liquidity weight is 0.5, the response of deposits is of the opposite sign. All other responses remain identical in shape and similar in magnitude.

Secondly, we change the elasticity of substitution between CBDC and deposits to 1/9 and 1/3. Figures 6 and 7 show the impulse responses to a 10% increase in λ_t when steady-state ϵ is 1/9 and 1/3, respectively. Figures 10 and 11 show the impulse responses to a 10% increase in ϵ_t when its steady-state value is 1/9 and 1/3, respectively. Comparing these impulse responses to their main counterparts in Figures 2 and 3, we see that they are all identical in shape and similar in magnitude.

5 Conclusion

Our work highlights the importance of considering the degree of substitutability between CBDC and bank deposits when evaluating the potential risk to financial stability resulting from the introduction of a retail CBDC. We find that, when CBDC and deposits are perfect substitutes, as long as they have the same resource cost per unit of effective real balances, the central bank can offer loans to banks such that they maintain the same balance sheet positions as before the introduction of CBDC, making it neutral to the real economy. Additionally, it is crucial to account for the collateral requirement that banks must respect when borrowing from the central bank, as the central bank's lending rate depends on how restrictive the collateral constraint is. The tighter the constraint is (the lower the fraction of banks' bond holdings that can be pledged as collateral), the lower the central bank's loan rate should be to keep the allocations unchanged when introducing a CBDC.

However, when CBDC and deposits are imperfect substitutes, issuing a CBDC changes banks' profitability irrespective of the central bank's lending rate, meaning that the central bank cannot make banks indifferent to the competition from CBDC. In fact, a change in banks' profitability implies that the new policy does not guarantee the same allocations as before, implying that the introduction of CBDC has real effects on the economy. Nevertheless, based on our dynamic effects analysis, there seems to be no risk of CBDC crowding out deposits when considering changes in households' preferences for CBDC over deposits.

Overall, our findings can help policymakers and central bankers design and implement CBDCs to minimize the risk of financial instability. The natural next step in the analysis of the equivalence result is to investigate the transition from the steady state with no CBDC to the steady state after CBDC has been introduced and identify the driving forces governing the transition between the two.

A Check under which condition the collateral constraint binds

Assuming interior solutions, the bank's optimality conditions for loans and bonds are, respectively:

$$\mathbb{E}_t \left[\Lambda_{t+1}(R^k_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \mu_t \left(1 + \theta_b \frac{b_{t+1}}{R^{l2}_{t+1}} \frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right),$$

$$\mathbb{E}_t \left[\Lambda_{t+1}(R^k_{t+1} - R^b_{t+1})\right] = \mu_t \frac{\theta_b}{R^l_{t+1}}.$$

Subtracting the condition for bonds from the one for loans:

$$\mathbb{E}_t \left[\Lambda_{t+1}(R^b_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \mu_t \left(1 - \frac{\theta_b}{R^l_{t+1}} + \theta_b \frac{b_{t+1}}{R^{l2}_{t+1}} \frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right). \tag{44}$$

To define the sign of the RHS, recall that $\theta_b \in [0, 1]$, and since the rate of return on reserves is positive, and we assumed interior solutions, all the terms are positive.

We define the elasticity of central bank's loans with respect to their rate of returns as

$$\eta_{l,t+1} = \frac{\partial l_{t+1}}{\partial R^l_{t+1}} \frac{R^l_{t+1}}{l_{t+1}},$$

such that we can rewrite the last term on the LHS as

$$\frac{1}{\eta_{l,t+1}} R^l_{t+1}.$$

Expression (44) says that the collateral constraint is binding if:

$$R^b_{t+1} - R^l_{t+1} > \frac{1}{\eta_{l,t+1}} R^l_{t+1}.$$

Rearranging

$$R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}} R^l_{t+1}.$$

We can conclude that the collateral constraint is binding if the sum of the cost of borrowing from the central bank and the bank's cost of taking on more loans is cheaper than the return banks get from government bonds:[12]

$$\mu_t > 0, \qquad l_{t+1} = \theta_b \frac{b_{t+1}}{R^l_{t+1}} \qquad \text{iff } R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}} R^l_{t+1}.$$

B Steady State

Variables without time subscripts denote the steady-state values. The discount factor determines the return on capital and the risk-free rate:

$$R^k = R^f = \frac{1}{\beta}.$$

Conditional on the policy rates, R^m and R^r, we know CBDC and reserve spreads, χ^m and χ^r, respectively. We find the deposit spread, χ^n, using equilibrium condition (33). Knowing the above-mentioned spreads, we derive χ^z, Ω^c, Ω^x. We find the capital-labor ratio dividing the return on capital, expression (27), by the labor supply:

$$\frac{k}{1-x} = \left(\frac{1}{\alpha}\left(R^k - 1 + \delta\right)\right)^{\frac{1}{\alpha-1}}.$$

Next, we divide the resource constraint (26) by the labor supply to get the consumption-labor ratio:

$$\frac{c}{1-x} = \left(\left(\frac{k}{1-x}\right)^{\alpha} - \delta\left(\frac{k}{1-x}\right)\right)\frac{1}{\Omega^{rc}}.$$

From the household's leisure choice, condition (25), we get the consumption-leisure ratio:

$$\frac{c}{x} = \frac{(1-\sigma)w}{\upsilon}\frac{\Omega_c}{\Omega_x},$$

where the wage rate is given by condition (28):

$$w = (1-\alpha)\left(\frac{k}{1-x}\right)^{\alpha}.$$

[12]We replicated the same analysis in the setting by Burlon et al. (2022), and we got an analogous result.

Lastly, we combine the consumption-leisure and consumption-labor ratios to derive the steady-state leisure:

$$x = \frac{c}{1-x}\left(\frac{c}{1-x} + \frac{c}{x}\right)^{-1}.$$

Having derived the above-mentioned steady-state values, it is straightforward to find all other quantities.

C Calibration

The household's first-order conditions for CBDC and deposits imply the following relation:

$$\chi^n = \frac{1}{\gamma}\left(\frac{m}{n}\right)^{\epsilon}\chi^m,$$

where m/n is the desired steady-state ratio of CBDC to deposits. To simulate a situation with limited CBDC adaptation, we set this value equal to $1/10$.[13]

We use the last relation to derive χ^z and, consequently, the left-hand side of the bank's optimality condition (33), denoted by LHS. Having derived LHS, we again use expression (33) to find φ as

$$\varphi = \frac{LHS + \chi^r\zeta}{LHS - \chi^r\zeta},$$

where ζ is the desired steady-state reserves-to-deposits ratio. This ratio is set to 0.25 to align with U.S. data.[14]

We use the right-hand side of expression (33) to find ϕ:

$$\phi = \frac{\chi^r}{\zeta^{-\varphi}(\varphi - 1)}.$$

Rearranging the household's demand for effective real balances, expression (29), we derive $\boldsymbol{\upsilon}$

[13]To find the steady-state level of the CBDC spread, χ^m, we set the steady-state CBDC rate, R^m, equal to 0.99, to simulate a negative net return in real terms. We make this conjecture since most of the central banks' research projects indicate that CBDCs may offer low or no nominal interests.

[14]To find the steady-state level of the reserve spread, χ^r, we set the steady-state reserve rate, R^r, equal to 1.0.

as

$$\upsilon = \frac{\left(\frac{z}{c}\right)^{\psi} \chi^{z}}{1 + \left(\frac{z}{c}\right)^{\psi} \chi^{z}}.$$

where c/z is the consumption velocity, targeted at 1.147 to align with U.S. data.

Lastly, to minimize the compositional effect on the resource cost of liquidity provision, we set the unit resource costs of managing CBDC, μ, to be equal to the total resource cost of providing deposits:

$$\mu = 2\phi\zeta^{1-\varphi} + \zeta\rho.$$

D Robustness checks

D.1 Impulse responses to 10% increase in λ_t with alternative specifications

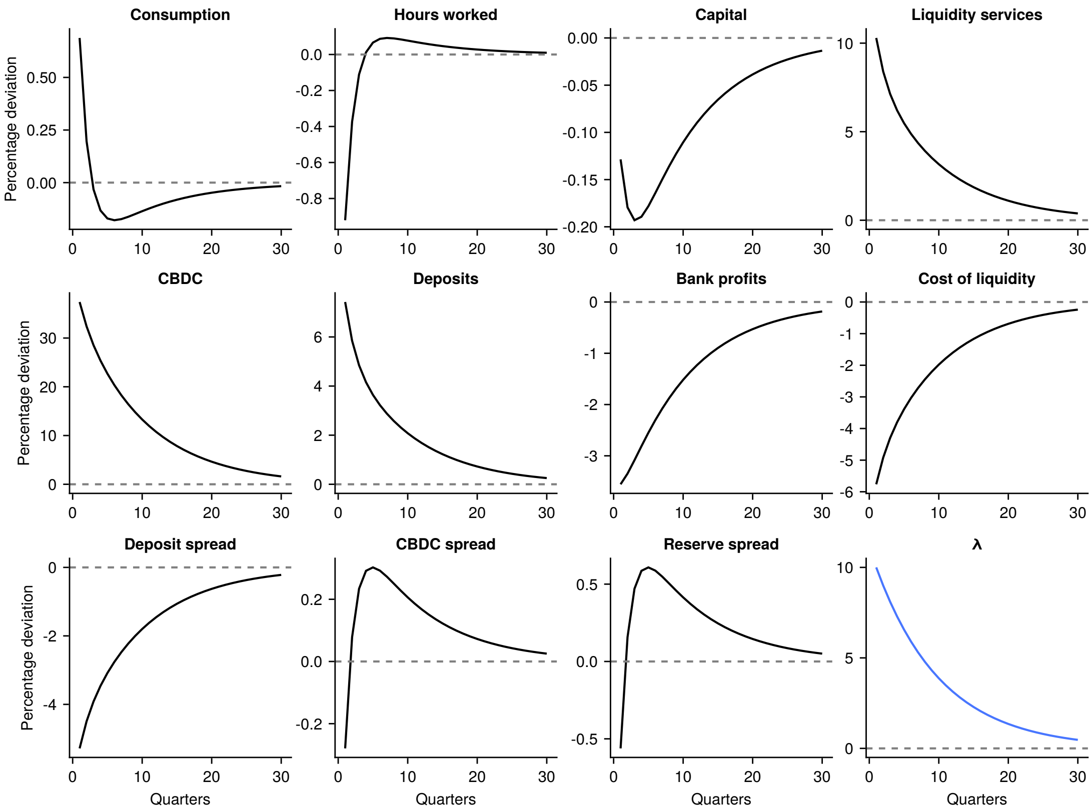

Figure 4: Lower steady-state $\lambda = 0.5$

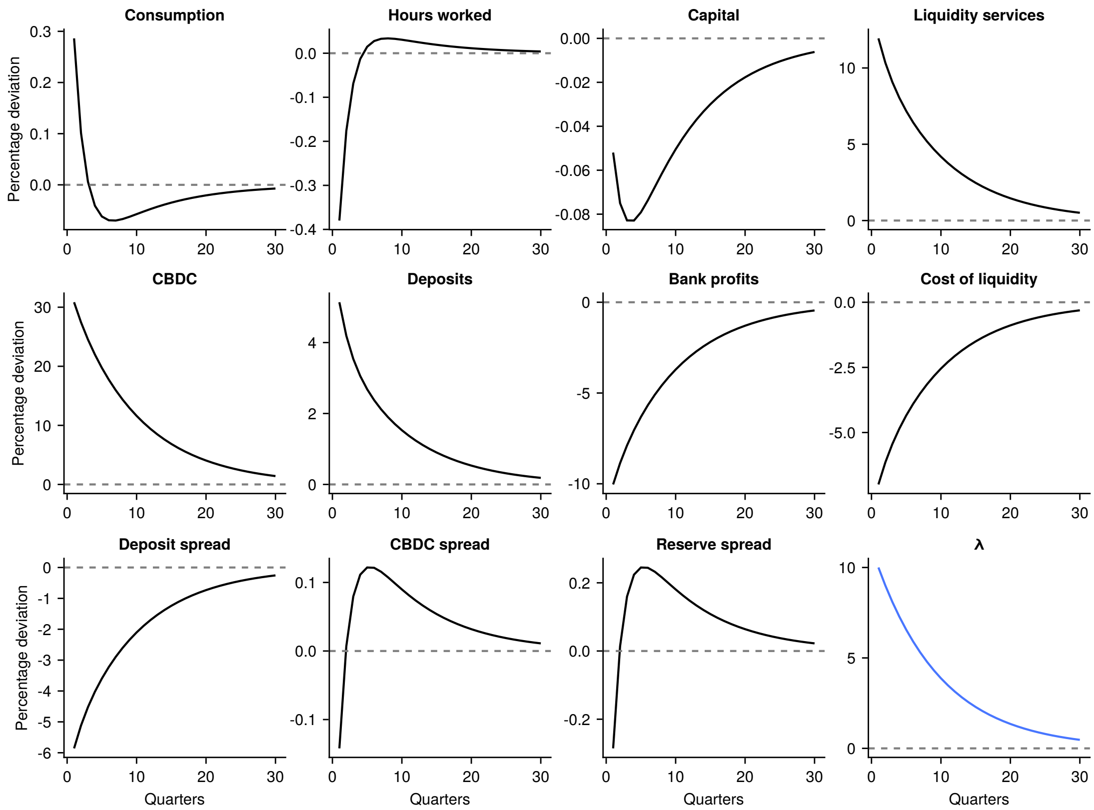

Figure 5: Higher steady-state $\lambda = 1.5$

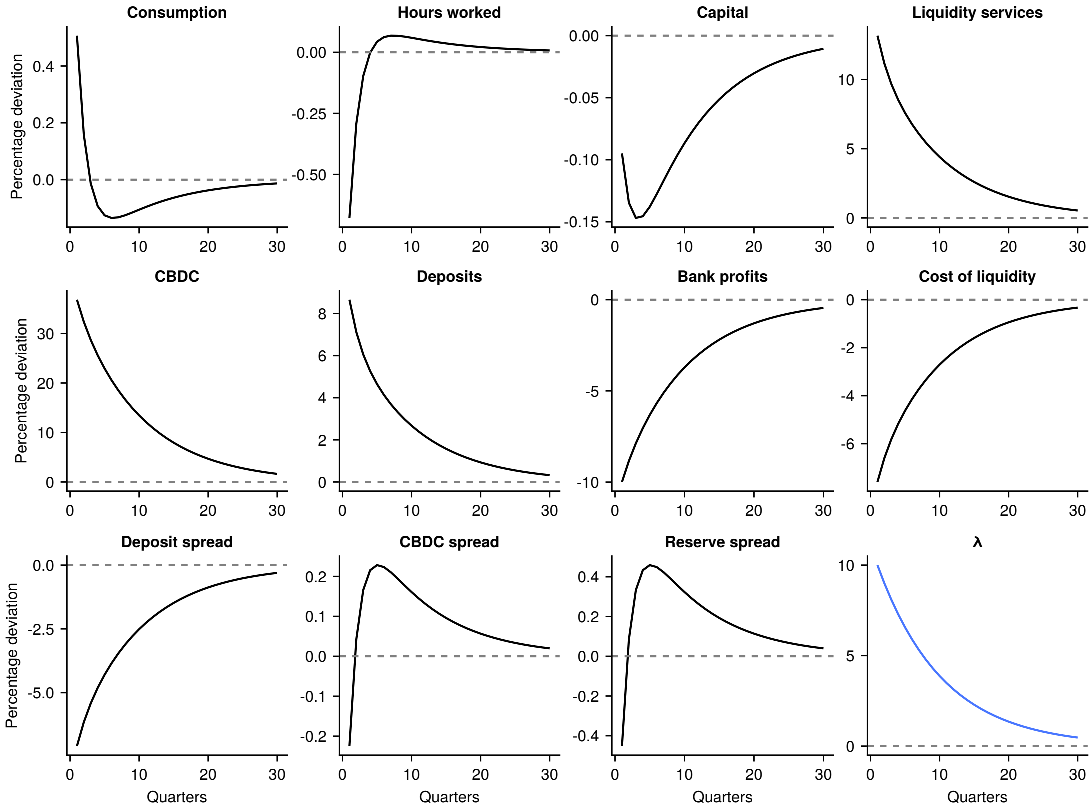

Figure 6: Lower steady-state $\epsilon = 1/9$

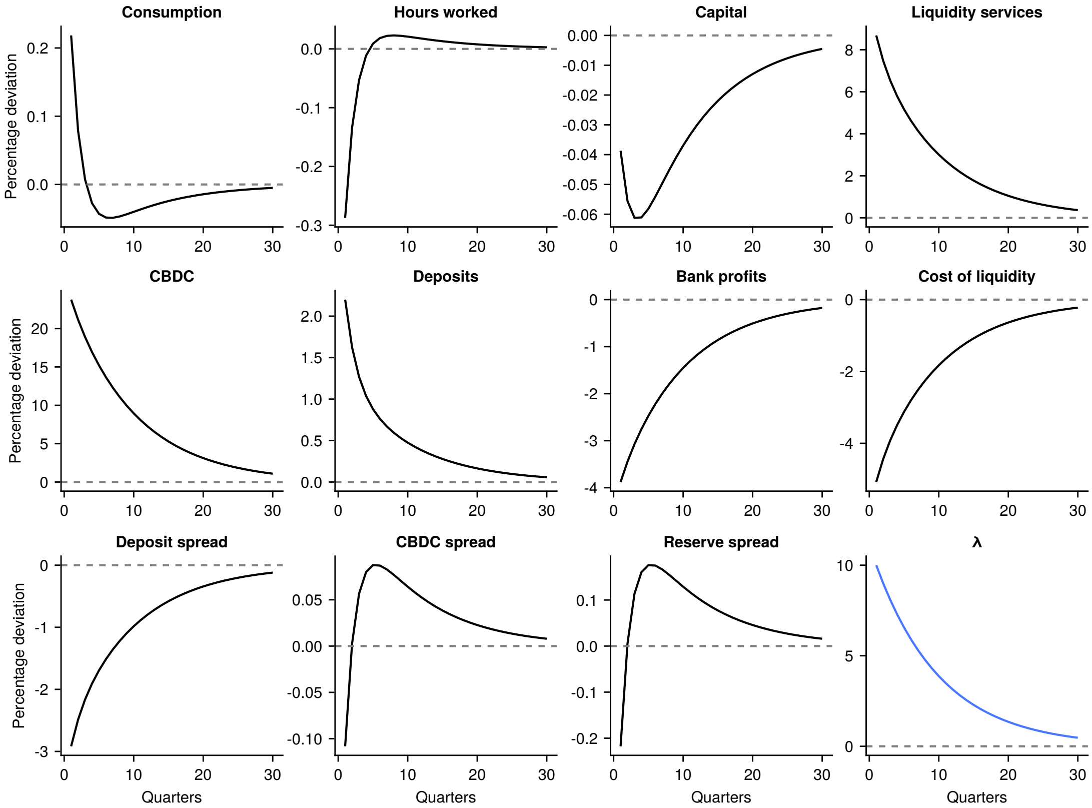

Figure 7: Higher steady-state $\epsilon = 1/3$

D.2 Impulse responses to 10% increase in ϵ_t with alternative specifications

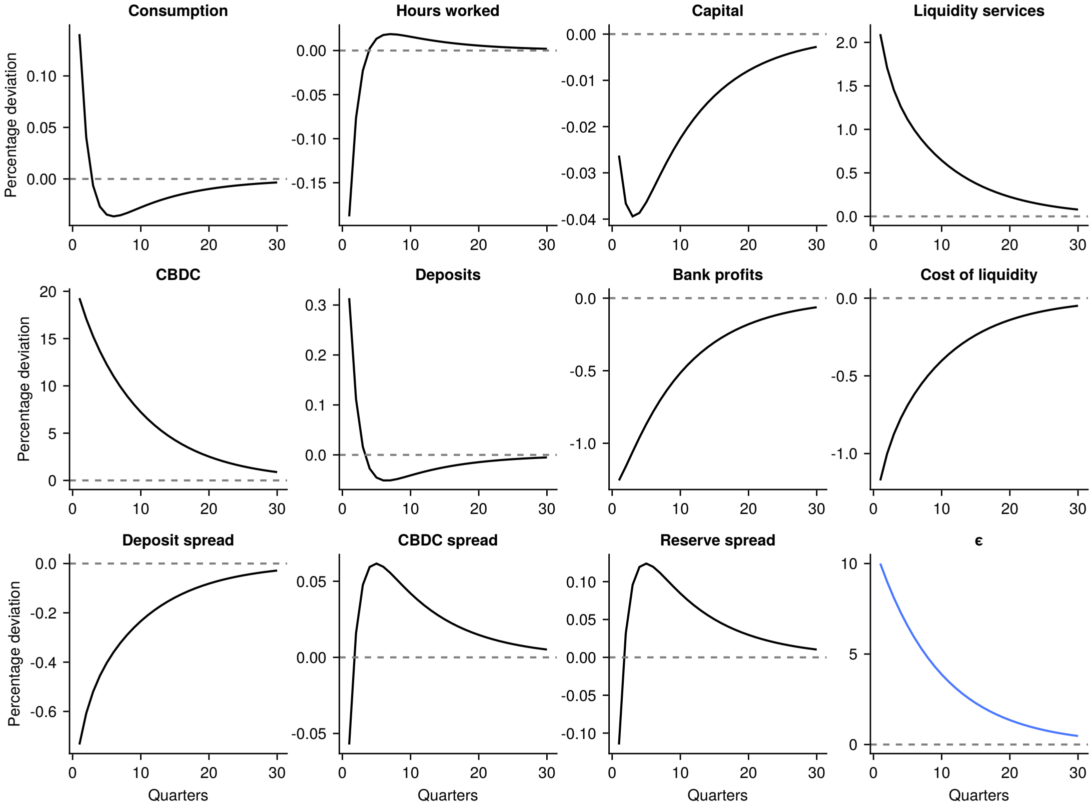

Figure 8: Lower steady-state $\lambda = 0.5$

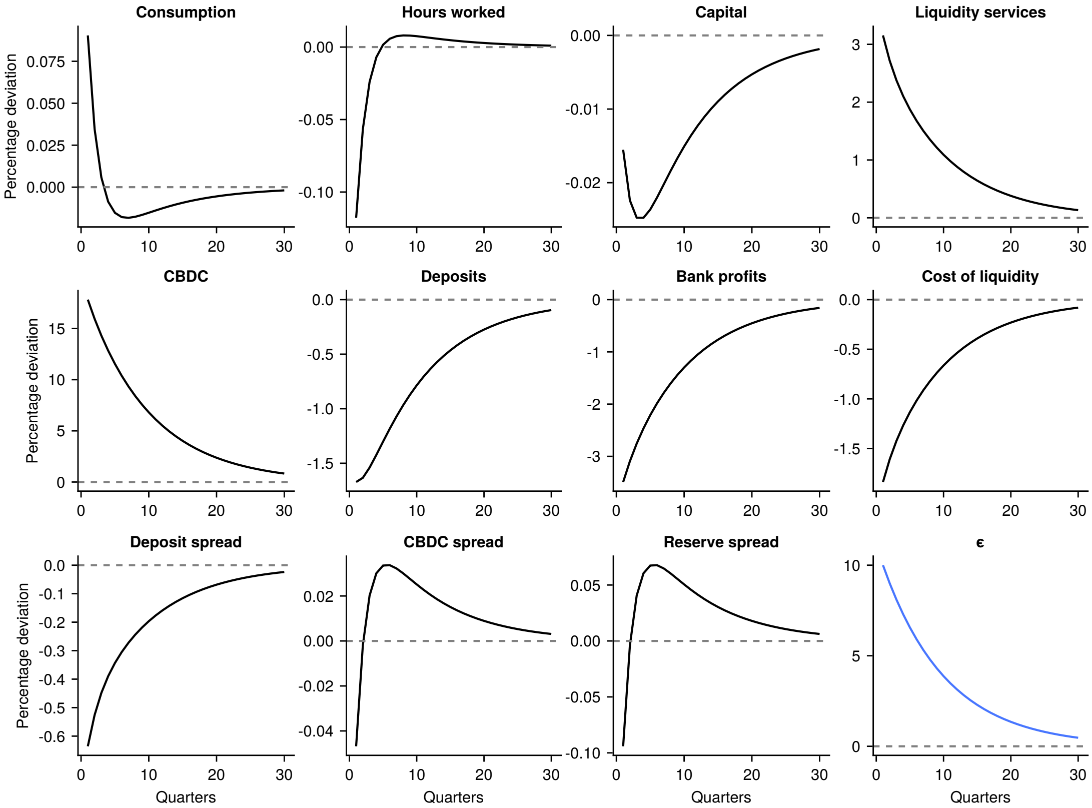

Figure 9: Higher steady-state $\lambda = 1.5$

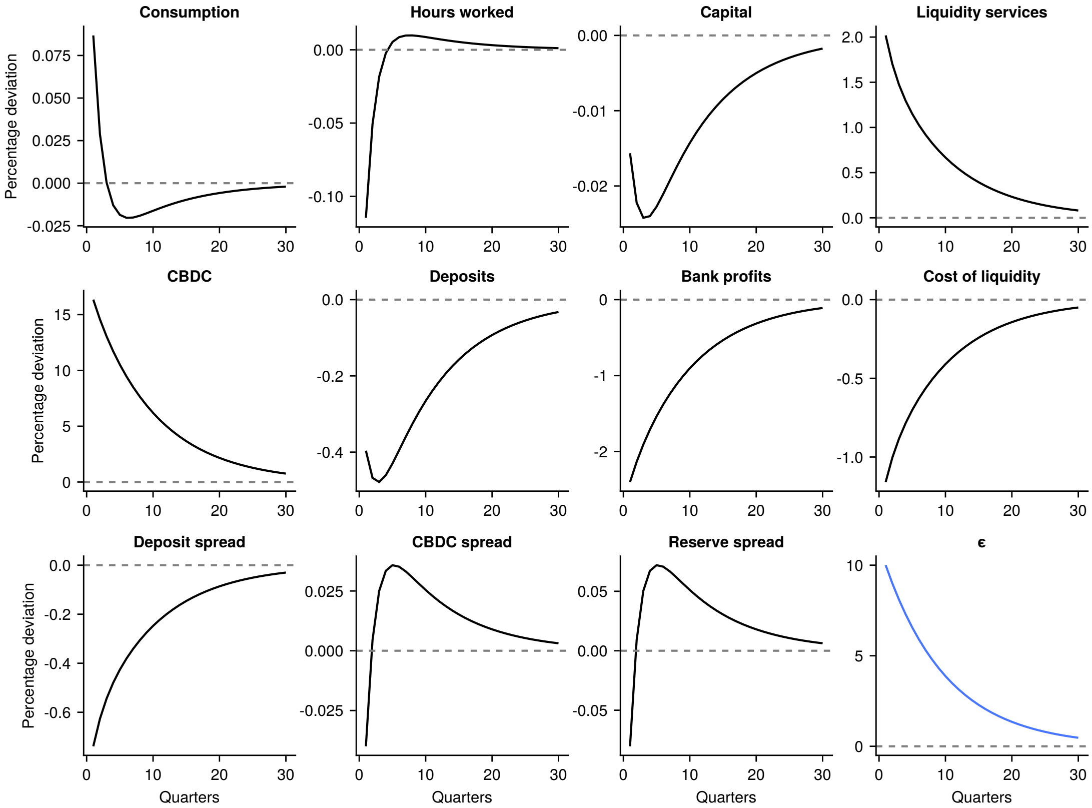

Figure 10: Lower steady-state $\epsilon = 1/9$

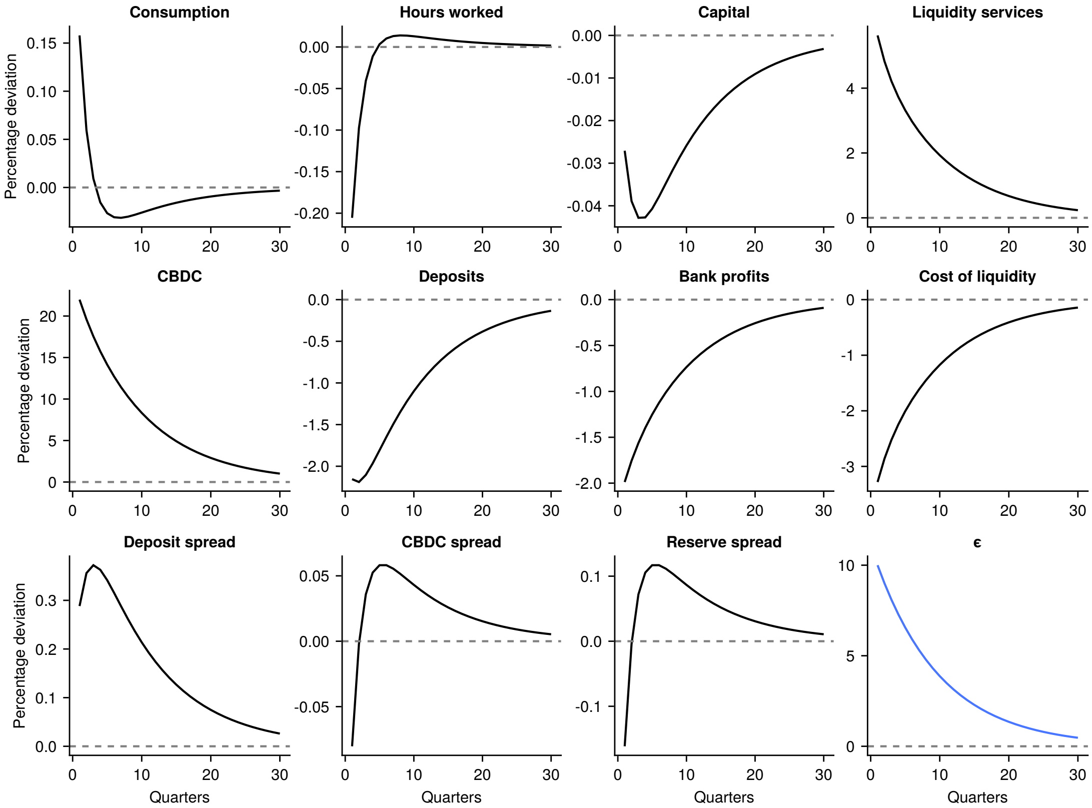

Figure 11: Higher steady-state $\epsilon = 1/3$

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