COMPSCI 762 Tutorial 9

Tutorial on Reinforcement Learning and Association Rule Mining

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Topics

Reinforcement Learning

Association Rule Mining

Reinforcement Learning

Terminology

- **Agent:** A hypothetical entity which performs actions in an environment to gain some reward.
- **Environment:** A scenario the agent has to face.
- **Action $a_t \in A$:** All the possible moves that the agent can take.
- **State $s_t \in S$:** Current situation returned by the environment.
- **Reward $R(s_t, a_t)$:** An immediate return sent back from the environment to evaluate the last action by the agent.
- **Policy $\pi : S \to A$:** The strategy that the agent employs to determine next action based on the current state.
- **Value $V^\pi(s)$:** The expected long-term return with discount γ, as opposed to the short term reward R. $V^\pi(s)$ is defined as the expected long term return of the current state s under policy π.
- **Q-value, action-value $Q^\pi(s, a)$:** is similar to **Value**, except it takes the current action a. $Q^\pi(s, a)$ refers to the long term return of the current state s, taking action a under policy π.

Markov Decision Process (MDP)

MDP is defined by $(S, A, R, \mathbb{P}, \gamma)$

- S: Set of possible states $s_t \in S$
- A: Set of possible actions $a_t \in A$
- R: Immediate reward given by the state and action pair $R(s_t, a_t)$
- $\mathbb{P}$: Transition probability at state s if an action a is taken
- γ: Discount factor with $0 \leq \gamma < 1$; The weight of future rewards

Markov Property:

$$P(s_{t+1} | s_t, a_t, s_{t-1}, a_{t-1}, s_{t-2}, a_{t-2}, \ldots) = P(s_{t+1} | s_t, a_t)$$

A *stochastic process*, where the future is solely determined by the current state and action. The past and the future are independent.

Markov Decision Process

- Assume the reward r_t the Markov property:

$$P(r_t| < s_t, a_t >, < s_{t-1}, a_{t-1} >, < s_{t-2}, a_{t-2} >, \ldots) = P(r_t|s_t, a_t)$$

  Immediate reward r_t is solely based on the current state and action pair $R(s_t, a_t)$.

- **The task:** Learn a policy $\pi : S \rightarrow A$ to maximizes the expected current and future rewards

$$\mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

  for every possible starting state s_0.

- **Intuition:** How we got here doesn't matter, what is the current best move?

Escape the grid-world: Solving an MDP - Value Iteration

Your task is to design the AI to help a robot to escape the room. The door is at right top corner. The actions are $\{\text{Left}, \text{Up}, \text{Right}, \text{Down}\}$.

$$+10$$

0	0	0
0	0	0

Initially, no action is taken, we set the value to 0 for all states.

Value Iteration - 1 Action

If we can only take 1 action in the game.

+10

0	0	10
0	0	0

Value Iteration - 2 Actions

When we take more than one action, we have to balance immediate reward and future reward. γ controls the importance of future rewards.

Let $\gamma = 0.5$, state values with 2 actions:

$$
\begin{aligned}
V^{\pi}(s) \quad &= \mathbb{E}[\textstyle\sum_{t\geq 0} \gamma^t r_t] = r_0 + \gamma r_1 + \gamma^2 r_2 + \dots \\
&= 0 + 0.5 \times 10 = 5
\end{aligned}
$$

+10

0	5	10
0	0	5

Value Iteration - 3 Actions

Let $\gamma = 0.5$, state values with 3 actions:

$$V^{\pi}(s) = 0 + 0 + 0.5^2 \times 10 = 2.5$$

+10

2.5	5	10
0	2.5	5

Value Iteration

Let $\gamma = 0.5$, state values with 4 actions:

$$V^{\pi}(s) = 0 + 0 + 0 + 0.5^3 \times 10 = 1.25$$

+10

2.5	5	10
1.25	2.5	5

Caveat: Do not mix value $V^{\pi}(s)$ and reward $R(s_t, a_t)$.

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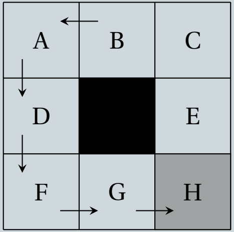
Example

- The black cell cannot be entered.
- The actions are $\{\text{Left}, \text{Up}, \text{Right}, \text{Down}\}$.
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- Discount factor, $\gamma = 0.5$



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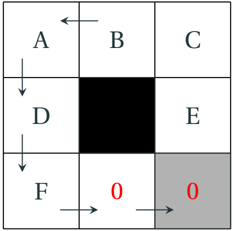
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- $\gamma = 0.5$

Give for each state the value of the value function, V, for the given policy. You can ignore states for which no policy is defined.



- The target state, H, requires 0 action, $V^{\pi}(s = H) = 0$. Note: There is no action called "stay".
- State G requires 1 action, $V^{\pi}(s = G) = 0$

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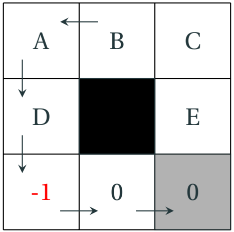
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- $\gamma = 0.5$



- State F requires 2 action, $V^{\pi}(s = F) = -1 + 0.5 \times 0 = -1$

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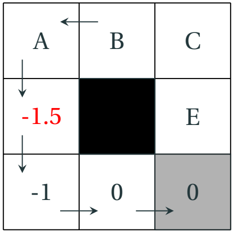
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- $\gamma = 0.5$



- State D requires 3 action, $V^{\pi}(s = D) = -1 + 0.5 \times (-1) + 0.5^2 \times 0 = -1.5$

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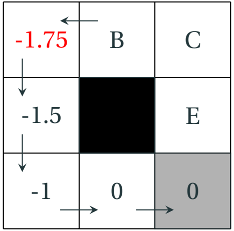
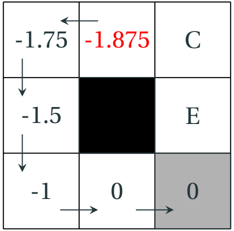
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- $\gamma = 0.5$



- State A requires 4 action, $V^{\pi}(s=A) = -1 + 0.5 \times (-1) + 0.5^2 \times (-1) + 0.5^3 \times 0 = -1.75$

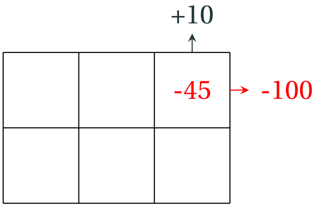
- The reward for actions that bring it into the target is 0. Other actions get a reward of -1.
- The partial policy, π, is given by the arrows in the grid.
- $\gamma = 0.5$



- State B requires 5 action,
  $V^{\pi}(s = A) = -1 + 0.5 \times (-1) + 0.5^2 \times (-1) + 0.5^3 \times (-1) + 0.5^4 \times 0 = -1.875$

Limitations on MDP - The Cliff Sernario

- The value of a state is the expected reward from taking the best action in the state.
- Accumulating rewards from random actions would calculate the expected reward from random actions in the state.
- E.g.: We would learn that any state near a cliff is bad, because you get a negative score if you jump off, even though you don't have to jump off.



Q-Learning

Intuition: Instead of computing value $V^{\pi}(s)$, we learn **Q-value**, $Q^{\pi}(s,a)$, which considers the state s and the action a as a pair.

- Q-learning can identify an **optimal action-selection policy** for any given a finite Markov decision process (FMDP).
- $Q : S \times A \to \mathbb{R}$, calculating the quality of a state–action combination
- Iterative method, Q is initialized to an arbitrary fixed value. At each time t, the agent selects an action a_t, observes a reword r_t, enters a new state s_{t+1}, and Q is updated.

The Q-value is updated by:

$$Q^{\text{new}}(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \cdot [R(s_t, a_t) + \gamma \cdot \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t)]$$

where α is the learning rate. (In a fully deterministic environment, a learning rate of $\alpha = 1$ is optimal.)

Q-Learning: 361 Exam Question 8, 2020

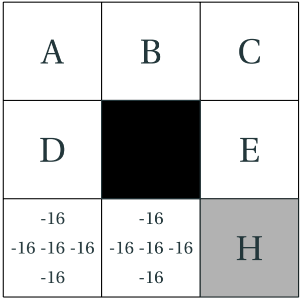
Example

- For the transitions into non-existing cell the next state, s', is equal to the current state, s.
- For each action, the agent gets a reward of -1. The reward for actions that bring it into the target is 0, and the black state is -2.
- Discount factor, $\gamma = 0.5$
- Let the initial value of Q be -16.

A	B	C
D		E
F	G	H

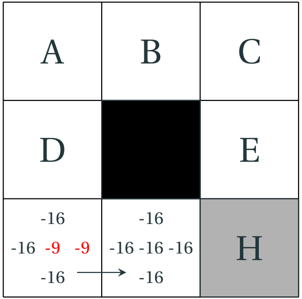
Q-Learning: 361 Exam Question 8, 2020

Let the agent walks the path (F, right) $\rightarrow$ (G, up) $\rightarrow$ (G, right). Calculate values for the Q-table after every step of the path. What is the value of $V(G)$ after the last step?



Q-Learning: 361 Exam Question 8, 2020

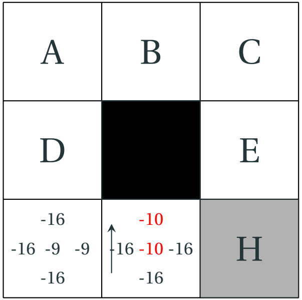
Let the agent walks the path (F, right) $\rightarrow$ (G, up) $\rightarrow$ (G, right). Calculate values for the Q-table after every step of the path. What is the value of $V(G)$ after the last step?



Given $\gamma = 0.5$, $Q(s = F, a = \text{right}) = -1 + 0.5 \times (-16) = -9$

Q-Learning: 361 Exam Question 8, 2020

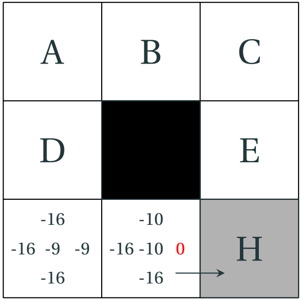
Let the agent walks the path (F, right) $\rightarrow$ (G, up) $\rightarrow$ (G, right). Calculate values for the Q-table after every step of the path. What is the value of $V(G)$ after the last step?



$$Q(G, \text{up}) = -2 + 0.5 \times (-16) = -10$$

Q-Learning: 361 Exam Question 8, 2020

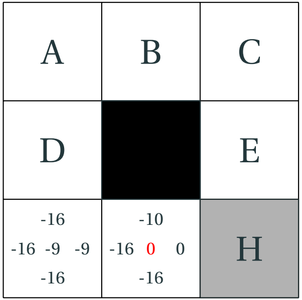
Let the agent walks the path (F, right) $\rightarrow$ (G, up) $\rightarrow$ (G, right). Calculate values for the Q-table after every step of the path. What is the value of $V(G)$ after the last step?



$$Q(G, \text{right}) = 0 + 0.5 \times 0 = 0$$

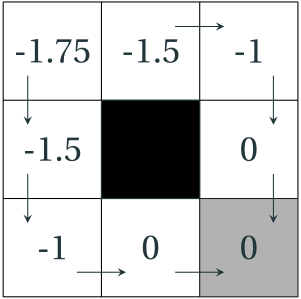
Q-Learning: 361 Exam Question 8, 2020

Let the agent walks the path (F, right) $\rightarrow$ (G, up) $\rightarrow$ (G, right). Calculate values for the Q-table after every step of the path. What is the value of $V(G)$ after the last step?



$$V(G) = 0$$

Create an optimal policy π^*, and draw it on the given diagram. What are the corresponding values of V^*? What is the most desirable state for the agent?



Note: The initial Q-value does not affect $V(s)$.

Association Rule Mining

Association Rule Mining

Apriori Algorithm

FP-Tree