# Variable Bandwidth M-Path Filter with Fixed Coefficients Formed by M-Path Polyphase Filter Engines

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**ABSTRACT** - Software defined radios require selectable bandwidth digital filters to process the many different bandwidth signals pulled from or inserted into the ether. Implementation considerations favor fixed length digital filters with fixed coefficients that can be implemented with hardwired multipliers. This paper describes two filter architectures that use M-path polyphase partitions to implement variable bandwidth filters with fixed coefficients and with fixed computational resources.

*Keywords: Variable Bandwidth, M-Path Filter, Polyphase Filter*

## I. INTRODUCTION

Software defined radios require variable bandwidth digital filters to process signals defined by different standards as well as evolving standards. Conceptually, the variable bandwidth filter has the form suggested in Figure 1.

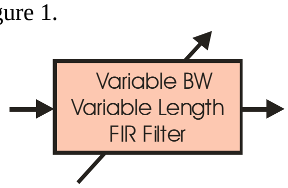


Figure 1. Desired Capability: Variable Bandwidth FIR Filter.

Designers of digital FIR filters are faced with two conflicting requirements. On one hand we desire digital filters that can switch rapidly between selectable filter bandwidths without the time delay required to compute and upload new sets of filter coefficients. On the other hand, resource allocation and power considerations favor the design of fixed length filters with fixed coefficients that allow the multipliers to be implemented by hardwired logic rather than hardware intensive Booth multipliers. Resolution of these conflicting requirements is found in the two filter structures we describe here. Both structures employ multirate signal processing techniques to convert the processing task to an alternate task that is better able to support the variable bandwidth option. One of the options offers continuously variable bandwidth changes while the other offers incremental bandwidth changes.

## II. CONTINUOUSLY VARIABLE OPTION

We first note that FIR filter length is inversely proportional to its transition bandwidth $\Delta f/f_S$ and that in many systems the transition bandwidth scales with the filter bandwidth. We describe this property as constant form factor. For such a filter the ratio of its stop band frequency to its pass band frequency is a constant, independent of bandwidth. When displayed on a log-frequency axis the spectral response of any bandwidth change is seen to be a simple frequency translation on the log-frequency axis. When the bandwidth of a filter is reduced by a factor of 2 the transition bandwidth is reduced by the same factor of 2 which in turn increases the number of taps in the filter by the factor 2. We then conclude that FIR filter with variable bandwidth and proportional transition bandwidth must also be of variable length.

The important concept here is that the impulse response of a reduced bandwidth filter spans a longer time interval. At a fixed sample rate, the filter would contain more samples to span the wider interval. An alternate method of having the filter span a longer time interval is to keep the number of samples fixed but increase the time interval between samples by reducing the sample rate. The relative bandwidth of a filter is fixed as a fraction of the sample rate, but the absolute bandwidth is proportional to the sample rate. Thus we can reduce the absolute bandwidth of a filter by operating it at a reduced sample rate. We accomplish this by interpolating the input time series to an intermediate, reduced sample rate, series. The interpolated values are processed by the filter operating at this reduced sample rate. The output of the filtered time series is then interpolated back to the original input sample rate. The implementation of this process is shown in the block diagram of Figure 2.

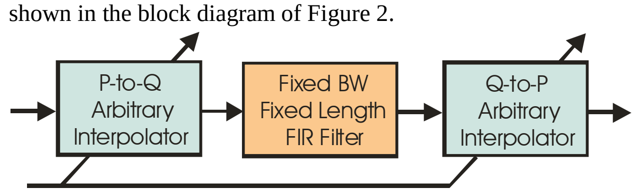


Figure 2. Variable Bandwidth, Fixed Length FIR Filter.

The system works as follow: The initially over sampled input signal is subjected to a sample rate reduction in a polyphase interpolator that preserves the dynamic range of the signal while forming samples at the reduced rate. This reduced rate still satisfies the Nyquist sampling criterion. The reduced sample rate signal is processed by the fixed number of weights low pass filter. The output of this filter is then up sampled by a second polyphase interpolator that returns the signal to the original input sample rate.

One efficient implementation of the arbitrary ratio interpolator is shown in Figure 3. This implementation uses two M-path polyphase filters that computes sample values of the interpolant and the sample derivative at the offset position k/M from the interpolator output phase center. The computed output is formed from a local Taylor series as shown in (1).

$$y(n+\frac{k}{M}+\frac{\delta}{M}) \cong y(n+\frac{k}{M})+\delta \cdot \dot{y}(n+\frac{k}{M}) \qquad (1)$$

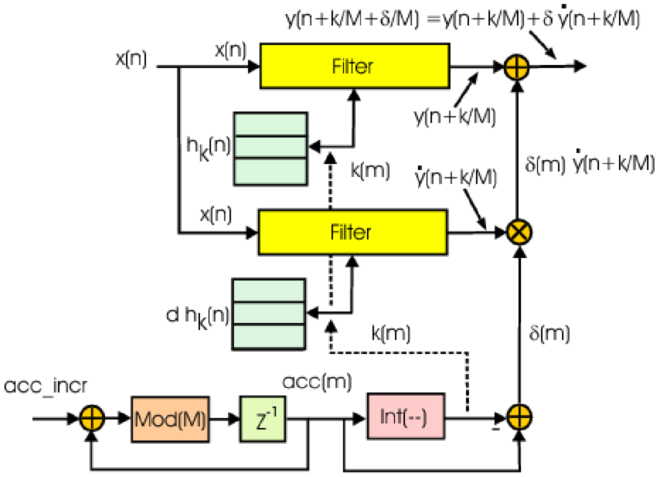


Figure 3. Block Diagram Low Complexity Arbitrary Interpolator.

Figure 4 shows the impulse response and frequency response for two interpolation ratios of the system shown in Figure 2. The input and output interpolators were implemented by 64 path versions of Figure 3. Note the 100 dB dynamic range of the variable bandwidth system formed by all three components of Figure 2 designed for the 100 dB range. The first and second subplots of Figure 5 show the spectra at the input and output of the input interpolator of Figure 2 at their respective sample rates. The third subplot of Figure 5 shows the output spectrum of the intermediate filter at the down sampled interpolated rate while the fourth subplot of Figure 5 shows spectrum at the output of the output interpolator at the up-sampled interpolated rate. Note the greater than 100 dB range of the composite process.

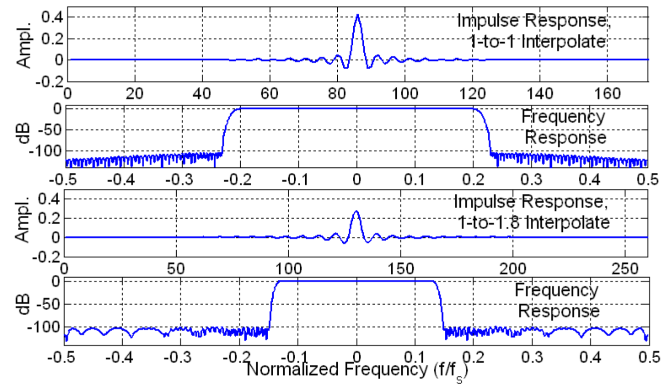


Figure 4. Impulse Response and Frequency Response of Variable Bandwidth Filter for Two Interpolating Ratios.

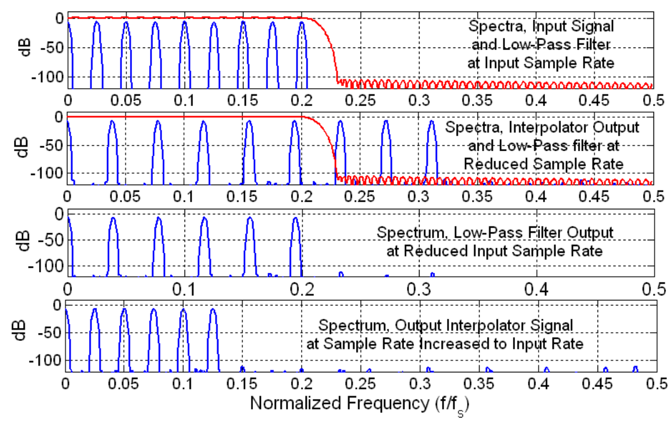


Figure 5. Spectra of Input, Intermediate, and Output Ports of Variable Bandwidth Filter.

## III. INCREMENTAL VARIABLE BANDWIDTH OPTION

The second variable bandwidth technique implements a pair of M-path perfect reconstruction filter banks one for analysis and one for synthesis. The input analysis filter bank partitions the input spectrum into M contiguous output channels with bandwidth and channel spacing of $f_S$/M. The non-maximally decimated filter bank avoids aliasing of the channel filter band edges by operating at an output sample rate of 2-samples per channel bandwidth. The corresponding synthesis bank accepts signals samples at 2-samples per channel bandwidth and up-samples 1-to-M/2 to obtain the original and desired output

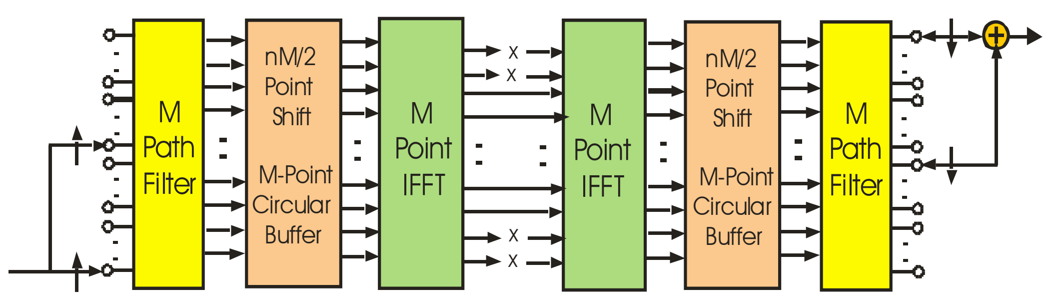


Figure 6. Cascade Polyphase Analysis and Synthesis Filter Banks Satisfying Nyquist Rate per Channel by Operating at Two Samples per Channel Bandwidth which Avoids Aliasing of Channel Transition Band Edges.

sample rate matched to the input sample rate $f_S/M$. Changes in composite system bandwidth are achieved by enabling or disabling the connection between the output ports of the analysis filter bank and the corresponding input port of the synthesis filter bank. A block diagram of the analysis-synthesis selectable bandwidth filter is shown in Figure 6. In this structure, the M-path polyphase filter arms are variants of the standard polyphase partition. The input commutator delivers M/2 input samples simultaneously to two halves of the M-path partition. Due to the M/2 resampling of the original M-path filter partition the filter path transfer functions $H_r(Z^M)$ originally polynomials in $Z^M$ become $H_r(Z^2)$ now polynomials in $Z^2$. The M/2 resampler also passes through the $Z^{(-M/2)}$ part of the $Z^{(-M/2-r)}$ delays in the lower half of the polyphase partition converting them into $Z^{-1}$ delays. The fractional delays at the input rate are absorbed by the dual input commutator ports while the delays in the polyphase arms define the path filters of the resampled partition. The filters in the r-th row of the upper half of the partition are the polynomials $H_r(Z^2)$ and those in the corresponding lower half are the polynomials $Z^{-1}H_{(r+M/2)}(Z^2)$. Block diagrams of the polyphase filter arms are shown in Figure 7.

The polynomials $H_r(Z^2)$ and the delayed $Z^{-1}H_{(r+M/2)}(Z^2)$ in the upper and lower halves respectively permit the current M/2 sample values from the top half of the filter to interact with the previous M/2 sample values from the lower half of the filter. The time offset between the upper and lower half of the filter is responsible for a frequency dependent phase shift between successive time shifts. This phase offset is removed by the successive shifts of the M/2 circular buffer between the polyphase filter and the IFFT. This process is described in the next paragraph.

When a standard polyphase filter performs an M-to-1 down sampling each multiple of the output sample rate is aliased to DC and the separation of the aliases is performed by the phase rotators of the IFFT. When the modified polyphase filter performs an M/2-to-1 down sampling, the even multiples of the output sample rate continue to alias DC but the odd multiples of the output sample rate alias to the half sample rate.

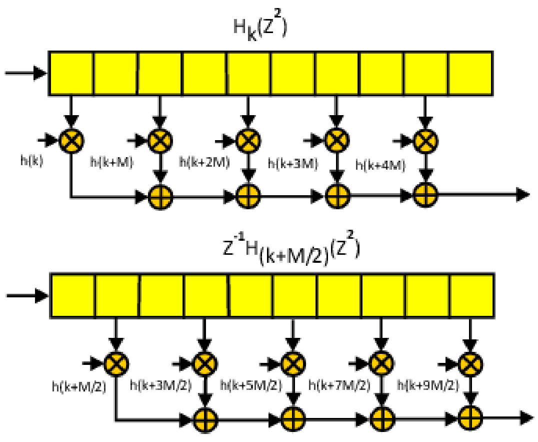


Figure 7. Path Filters in $Z^2$ with and without Input Delay.

There is now a mismatch between the alias positions and the phase rotators of the IFFT. We can respond to this change by alternating the signs of the samples taken from the odd-indexed IFFT bins or we can perform an equivalent compensation in the time domain by performing a circular shift of M/2 samples on alternate outputs from the polyphase filter arms. We can understand this fix by visualizing a single cycle of a scaled sine wave input to the IFFT in successive shifts of M/2 for a sequence of length M as is shown in Figure 8. Here we see that successive shifts of M/2 samples for a sinusoid with period M is observed by the IFFT as sign reversals to which the IFFT responds by folding the output signal component to the half sample rate.

We have described the general architecture of the input M/2-to-1 M-path analysis channelizer of Figure 6. Space limitations prevent our development of the output synthesis channelizer but we remind the reader that the output synthesis filter bank performs the inverse task of the input synthesis filter bank and the input and output processing blocks are seen to be each others' dual graphs.

Figure 9 shows the impulse response and frequency response for two enabled masks of the system shown in Figure 6. Note the 100 dB dynamic range of the variable bandwidth filter formed by the two components of Figure 6. Figure 10 shows the spectra at the input and output of the cascade channelizers. Note the greater than 100 dB range of the process. Here we see the artifacts of the channelizer process as the -110 dB spurious responses between the signal tone lines. We note that the granularity of this variable bandwidth filter is equal to the channel bandwidth of the analysis-synthesis bank. For this example, the 120 point IFFT partitions the 12.0 MHz input sample rate into 100 kHz channels so this variable bandwidth filter can only change bandwidth in 100 kHz increments.

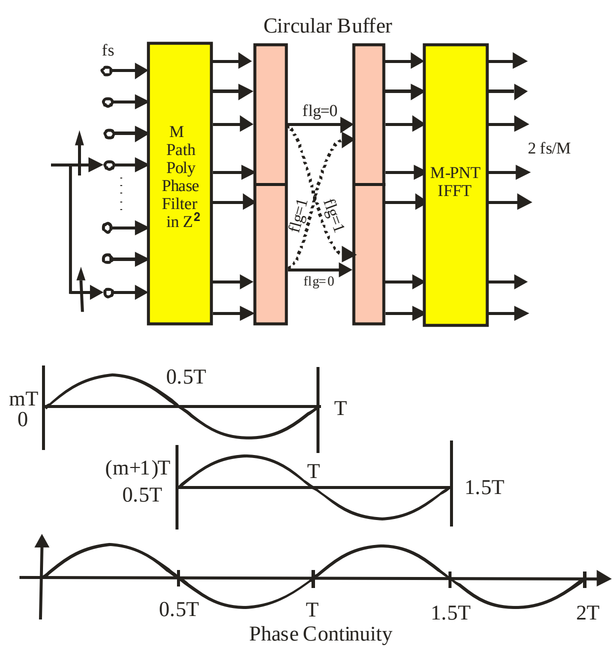


Figure 8. Sign Reversals of Successive M/2 Sample Shifts for a Single Cycle Length M Sinusoid Cancelled by M/2 Cyclic Shift of Alternate M length Sequences.

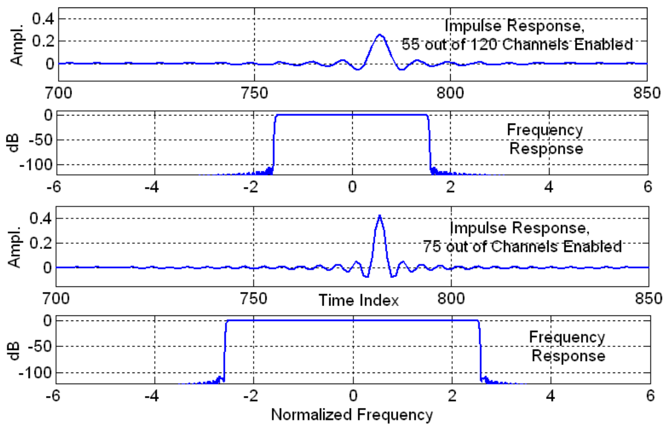


Figure 9. Impulse Response and Frequency Response of Variable Bandwidth Filter for Enabled Masks.

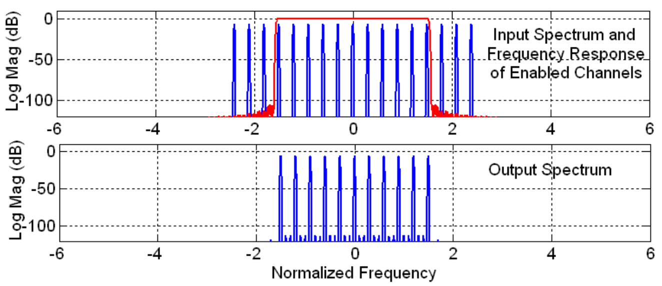


Figure 10. Spectra of Input Signal, Enabled Equivalent Filter, and Output Signal.

The 120 path channelizers used to implement the variable bandwidth filter shown here partitioned an 841 tap prototype low pass filter. Interestingly, while the group delay of the prototype linear phase filter is 420 samples, the group delay, corresponding to the peak position of the system impulse response, as seen in Figure 9, is 781 samples. This is 60 samples less than twice the group delay of the prototype filter. The doubling of the group delay is the result of passing through the filter twice, once at the input and once at the output. The 60 sample reduction is due to the input commutator that delivers data in 60 element vectors placing the probing impulse located at position zero of the input stream at position 59 of the polyphase partitioned filter.

While discussion the partition, we note that the 120 path partition of the 840 tap prototype filter results in 7-taps per path for both the input and output filter banks. In this system, the input and output commutators access two paths for each input-output sample pair. This means that 4-path computations, or 48 multiply-add operations are required for each input-output sample pair. If we use an estimate of 300 multiplies and adds for a Winograd based Good-Thomas version of the 120 point IFFT and amortize this computational workload over 60 input samples we arrive at 5 multiplies and adds for each IFFT per input-output sample pair. Thus the total workload for the filter is 58 multiply-adds to implement a single filter for real input-output pairs or 53 multiply-adds to implement each filter of a complex pair for complex baseband input-output pairs. These numbers are less than 7% of the workload required for the direct implementation the 841 tap prototype filter.

## IV. CLOSING COMMENTS

We have described two interesting applications of M-path polyphase filters to perform variable bandwidth filtering with fixed coefficients with essentially fixed computational resources. These are two nice examples of thinking outside the box. In this light we suggest that the IFFTs in the cascade channelizer should be implemented by the Good-Thomas prime-factor algorithms using the Winograd transform. Similarly, any high quality interpolator is a valid contender for the interpolator based filter. A final observation to keep in mind is that, while both filter structures are linear phase filters when their prototypes are linear phase, the two filter structures have very different group delay characteristics. The interpolator based system forms constant form factor filters with reduced transition bandwidths and hence longer duration impulse responses with commensurately longer group delay as its bandwidth is reduced. The channelizer based system forms constant transition bandwidth filters and hence exhibits a constant group delay as its bandwidth is varied.

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