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4^{th} assignment

8.1 a) $\bar{X} \sim N\left(12, \frac{(0.04)^2}{16}\right)$ mean = 12, standard deviation $= \sqrt{\frac{(0.04)^2}{16}} = 0.01$

b) $\bar{X} \sim N\left(12, \frac{(0.04)^2}{64}\right)$ mean = 12, standard deviation $= \sqrt{\frac{(0.04)^2}{64}} = 0.005$

c) $\bar{X} \sim N(12, (0.01)^2)$ $P(11.99 \le \bar{X} \le 12.01) = P\left(\frac{11.99-12}{0.01} \le Z \le \frac{12.01-12}{0.01}\right) = P(-1 \le Z \le 1)$

$= 0.8413 - 0.1587 = 0.6826$

d) $\bar{X} \sim N\left(12, \frac{(0.04)^2}{25}\right)$ $P(\bar{X} > 12.01) = P\left(Z > \frac{12.01-12}{\sqrt{\frac{(0.04)^2}{25}}}\right) = P(Z > 1.25) = 1 - 0.8944 = 0.1056$

8.2 a) $n = 64$ $\sigma_{64} = \sqrt{\frac{(5.6)^2}{64}} = 0.7$ $n = 196$ $\sigma_{196} = \sqrt{\frac{(5.6)^2}{196}} = 0.4$ $0.7 - 0.4 = 0.3$

b) $n = 784$ $\sigma_{784} = \sqrt{\frac{(5.6)^2}{784}} = 0.2$ $n = 49$, $\sigma_{49} = \sqrt{\frac{(5.6)^2}{49}} = 0.8$ $0.2 - 0.8 = -0.6$

8.24 $\bar{X} \sim N\left(40, \frac{2^2}{36}\right)$ $P\left(\bar{X} > \frac{1458}{36}\right) = P(\bar{X} > 40.5) = P\left(Z > \frac{40.5-40}{\sqrt{\frac{2^2}{36}}}\right) = P(Z > 1.5) = 1 - 0.9332 = 0.0668$

8.3 $\bar{X} \sim N\left(28, \frac{5^2}{40}\right)$ $P(\bar{X} > 30) = P\left(Z > \frac{30-28}{\sqrt{\frac{5^2}{40}}}\right) = P(Z > 2.53) = 1 - 0.9943 = 0.0057$

8.23 a) $E(X) = 4\times0.2 + 5\times0.4 + 6\times0.3 + 7\times0.1 = 5.3$ $E(X^2) = 4^2\times0.2 + 5^2\times0.4 + 6^2\times0.3 + 7^2\times0.1 = 28.9$

$\sigma^2 = E(X^2) - (E(X))^2 = 28.9 - 5.3^2 = 0.81$, $\mu = 5.3$

b) $\mu_{\bar{X}} = 5.3$ $\sigma^2_{\bar{X}} = \frac{0.81}{36} = 0.0225$

c) $\bar{X} \sim N(5.3, 0.0225)$ $P(\bar{X} < 5.5) = P\left(Z < \frac{5.5-5.3}{\sqrt{0.0225}}\right) = P(Z < 1.33) = 0.9082$

8.22 a) $\mu_{\bar{X}} = 174.5$, $\sigma_{\bar{X}} = \sqrt{\frac{6.9^2}{25}}$

b) $P(172.5 \le \bar{X} \le 175.8) = P\left(\frac{172.5-174.5}{\sqrt{\frac{6.9^2}{25}}} \le Z \le \frac{175.8-174.5}{\sqrt{\frac{6.9^2}{25}}}\right) = P(-1.45 \le Z \le 0.94) = 0.8264 - 0.0735$
$= 0.7529$

c) $P(\bar{X} < 172.0) = P\left(Z < \frac{172.0-174.5}{\sqrt{\frac{6.9^2}{25}}}\right) = P(Z < -1.81) = 0.0351$

8.26
$\bar{X} \sim N\left(3.2, \frac{1.6^2}{64}\right)$

a) $P(\bar{X} \le 2.7) = P\left(Z \le \frac{2.7-3.2}{\sqrt{\frac{1.6^2}{64}}}\right) = P(Z \le -2.5) = 0.0062$

b) $P(\bar{X} > 3.5) = P\left(Z > \frac{3.5-3.2}{\sqrt{\frac{1.6^2}{64}}}\right) = P(Z > 1.5) = 1 - 0.9332 = 0.0668$

c) $P(3.2 \le \bar{X} < 3.4) = P\left(\frac{3.2-3.2}{\sqrt{\frac{1.6^2}{64}}} \le Z < \frac{3.4-3.2}{\sqrt{\frac{1.6^2}{64}}}\right) = P(0 \le Z < 1) = 0.8413 - 0.5000 = 0.3413$

8.72 $\bar{X} \sim N\left(20, \frac{9}{n}\right)$ $P(19.9 \le \bar{X} \le 20.1) = P\left(\frac{19.9-20}{\frac{3}{\sqrt{n}}} \le Z \le \frac{20.1-20}{\frac{3}{\sqrt{n}}}\right) = P\left(-\frac{1}{30}\times\sqrt{n} \le Z \le \frac{1}{30}\times\sqrt{n}\right) = 0.95$

$P\left(0 \le Z \le \frac{1}{30}\times\sqrt{n}\right) = 0.475$ $P\left(Z \le \frac{1}{30}\times\sqrt{n}\right) = 0.975$ $\frac{1}{30}\sqrt{n} = 1.96$ $n = 3458$

8.4 $\bar{X}_A \sim N\left(6.5, \frac{0.9^2}{36}\right)$ $\bar{X}_B \sim N\left(6.0, \frac{0.8^2}{49}\right)$

$P(\bar{X}_A - \bar{X}_B \ge 1) = P\left(Z \ge \frac{1-(6.5-6.0)}{\sqrt{\frac{0.9^2}{36} + \frac{0.8^2}{49}}}\right) = P(Z \ge 2.65) = 1 - 0.9960 = 0.0040$

8.32 $\bar{X}_A \sim N\left(4.5, \frac{1}{36}\right)$ $\bar{X}_B \sim N\left(4.7, \frac{1}{36}\right)$

$P(\bar{X}_B - \bar{X}_A \ge 0.2) = P\left(Z \ge \frac{0.2-0}{\sqrt{\frac{1}{36} + \frac{1}{36}}}\right) = P(Z \ge 0.85) = 1 - 0.8023 = 0.1977$

8.28 $\bar{X}_1 \sim N(80, \frac{5^2}{25})$ $\bar{X}_2 \sim N(75, \frac{3^2}{36})$

$P(3.4 \le \bar{X}_1 - \bar{X}_2 < 5.9) = P\left(\frac{3.4-(80-75)}{\sqrt{\frac{5^2}{25}+\frac{3^2}{36}}} \le Z < \frac{5.9-(80-75)}{\sqrt{\frac{5^2}{25}+\frac{3^2}{36}}}\right) \approx P(-1.43 \le Z < 0.81) = 0.7910 - 0.0764$
$= 0.7146$

8.5 $\hat{p} \sim N\left(\frac{211}{528}, \frac{1}{120} \times \frac{211}{528} \times \frac{317}{528}\right)$

a) $\sigma = \sqrt{\frac{1}{120} \times \frac{211}{528} \times \frac{317}{528}} \approx 0.045$

b) $P\left(Z < \frac{0.33 - \frac{211}{528}}{\sqrt{\frac{1}{120} \times \frac{211}{528} \times \frac{317}{528}}}\right) = P(Z < -1.56) = 0.0594$

c) $P\left(\frac{0.4 - \frac{211}{528}}{\sqrt{\frac{1}{120} \times \frac{211}{528} \times \frac{317}{528}}} < Z < \frac{0.5 - \frac{211}{528}}{\sqrt{\frac{1}{120} \times \frac{211}{528} \times \frac{317}{528}}}\right) \approx P(0.01 < Z < 2.24) = 0.9875 - 0.5040 = 0.4835$

8.6 $\hat{P}_A \sim N(0.65, 0.65 \times 0.35 \times \frac{1}{200})$ $\hat{P}_B \sim N(0.65, 0.65 \times 0.35 \times \frac{1}{150})$

$P(\hat{P}_A - \hat{P}_B > 0.10) = P\left(Z > \frac{0.10}{\sqrt{\frac{0.65 \times 0.35}{200} + \frac{0.65 \times 0.35}{150}}}\right) \approx P(Z > 1.94) = 1 - 0.9738 = 0.0262$

8.7 $\hat{P}_f \sim N(0.2, \frac{0.2 \times 0.8}{50})$ $\hat{P}_m \sim N(0.23, \frac{0.23 \times 0.77}{55})$

$P(\hat{P}_f - \hat{P}_m < 0.1) = P\left(Z < \frac{0.1 - (0.2 - 0.23)}{\sqrt{\frac{0.2 \times 0.8}{50} + \frac{0.23 \times 0.77}{55}}}\right) \approx P(Z < 1.62) = 0.9474$

8.44 a) $t_{0.025,14} = 2.145$

b) $-t_{0.10,10} = -1.372$

c) $t_{0.995,7} = -t_{0.005,7} = -3.499$

8.45

a) $P(T < 2.365) = 1 - P(T \ge 2.365)^{v=7} = 1 - 0.025 = 0.975$

b) $P(T > -1.356)^{v=12} - P(T \ge 2.179)^{v=12} = 0.60 - 0.025 = 0.575$

c) $P(T > -2.567)^{v=17} = 0.51$

8.46

a) $1 - 0.005 - 0.01 = 0.985$

b) $1 - 0.025 = 0.975$

8.47 $v = 23$

a) $P(T > -2.069) = 0.975$ $P(T \ge K) = 0.975 - 0.965 = 0.01$ $K = 2.500$

b) $P(T < 2.807) = 1 - 0.005 = 0.995$ $P(T \le K) = 0.995 - 0.095 = 0.9$ $P(T > K) = 0.10$ $K = 1.319$

c) $P(T \ge K) = 0.05$ $K = 1.714$

8-8 $P\left(T > \frac{24-20}{\frac{4.1436}{\sqrt{9}}}\right) = P(T > 2.896)^{v=8} = 0.01$

8-50

$\bar{x} = 0.475 \quad s^2 = \frac{0.6^2+0.7^2+0.7^2+0.3^2+0.4^2+0.5^2+0.4^2+0.2^2}{8} - 0.475^2 \approx 0.03$

$P(\bar{x} < 0.475) = P\left(T < \frac{0.475-0.5}{\sqrt{\frac{0.03}{8}}}\right) = P(T < -0.408) \approx 0.35$

8-9 $P(\bar{x} \leq 65) = P\left(T \leq \frac{65-60}{\frac{10}{\sqrt{25}}}\right) = P(T \leq 2.500)^{v=24} \approx 0.99$

8-40

a) $\chi^2_{0.01,21} = 38.932$

b) $\chi^2_{0.05,6} = 12.592$

c) $\chi^2_{0.01,10} = 0.01 \quad \chi^2_{\alpha,10} = 0.025$

8-41

a) $P(s^2 > 9.1) = P\left(\chi^2 > \frac{24 \times 9.1}{6}\right) = P(\chi^2 > 36.4)^{v=24} \approx 0.05$

b) $P(3.462 < s^2 < 10.745) = P\left(\frac{3.462 \times 24}{6} < s^2 < \frac{10.745 \times 24}{6}\right) = P(13.840 < s^2 < 42.980)^{v=24} \approx 0.95 - 0.01 = 0.94$

8-10

$P(s^2 > 54.668 \text{ or } s^2 < 12.102) = P\left(\chi^2 > \frac{54.668 \times 15}{25}\right) + P\left(\chi^2 < \frac{12.102 \times 15}{25}\right) = P(\chi^2 > 32.801)^{v=15} + P(\chi^2 < 7.261)^{v=15}$

$= 0.005 + 1 - 0.95 = 0.055$

8-51 a) $f_{0.05}(7,15) = 2.71$

b) $f_{0.05}(15,7) = 3.51$

c) $f_{0.01}(24,19) = 2.92$

d) $f_{0.95}(19,24) = \frac{1}{f_{0.05}(24,19)} = \frac{1}{2.11} \approx 0.474$

e) $f_{0.99}(28,12) = \frac{1}{f_{0.01}(12,28)} = \frac{1}{2.90} \approx 0.345$

8.59 $P\left(\frac{s_1^2}{s_2^2} < 4.89\right) = P(F < 4.89)^{v_1=7, v_2=11} = 1 - 0.01 = 0.99$

8-11 $P\left(\frac{s_1^2}{s_2^2} < 4.03\right) = P(F < 4.03)^{v_1=9, v_2=14} = 1 - 0.01 = 0.99$

8-64 $P\left(\frac{s_1^2}{s_2^2} > 1.26\right) = P\left(F > 1.26 \times \frac{15}{10}\right) = P(F > 1.89)^{v_1=24, v_2=30} = 0.05$