On the Robotic Uncertainty of Fully Autonomous Traffic

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CTS2022
August 15-16

Intelligent Transportation Thrust
Systems Hub, HKUST(GZ)

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1. Introduction

Human Driven Vehicles:

- **Accidents** caused by human errors *(main cause of car crashes)*
- **Congestion propagation** due to the heterogeneity of driver intentions

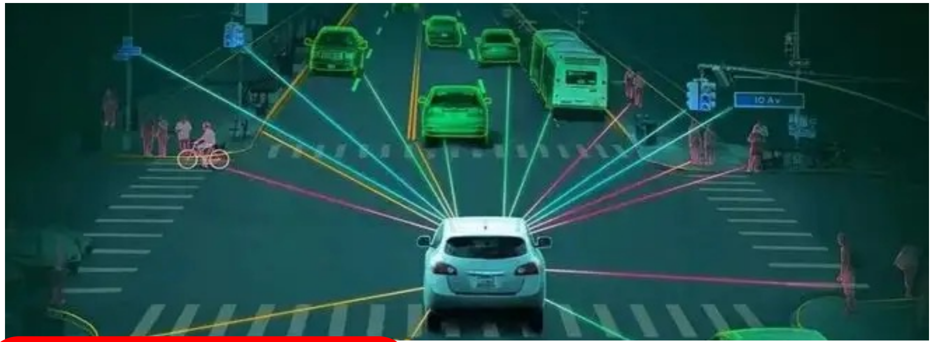
Autonomous Vehicles *(assumed)*:

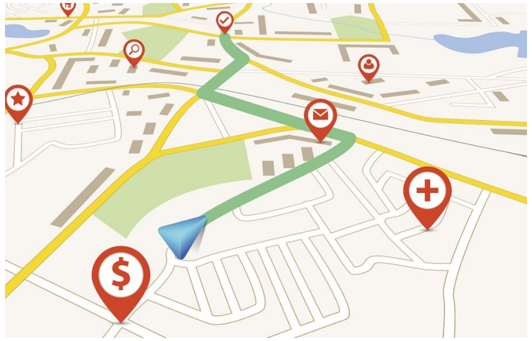
- **Intelligent** decision-making abilities
- **Accurate** machinery operations
- **Shorter** stable *headway*
- **Increased** roadway *capacity*

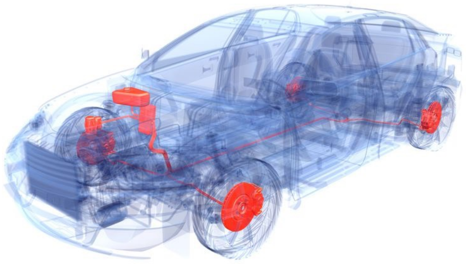
AVs offer the potential to improve both traffic **safety** and **efficiency.**

1. Introduction

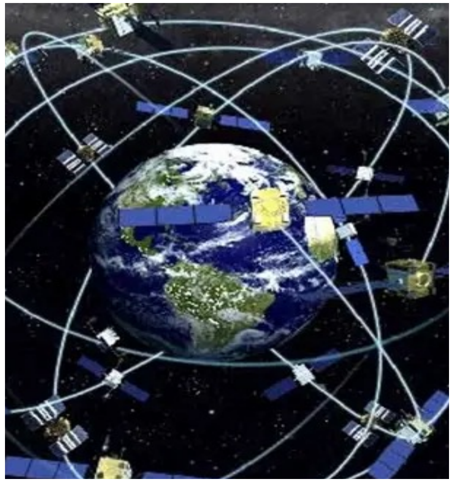
AVs
with
ERROR



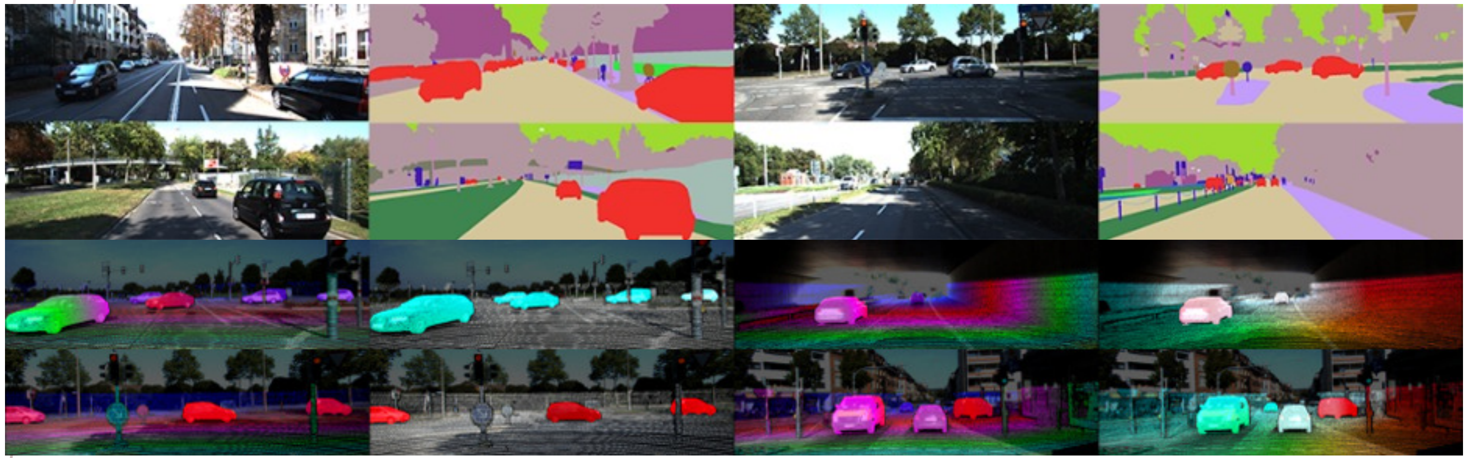




Perception → Localization → Planning → Control



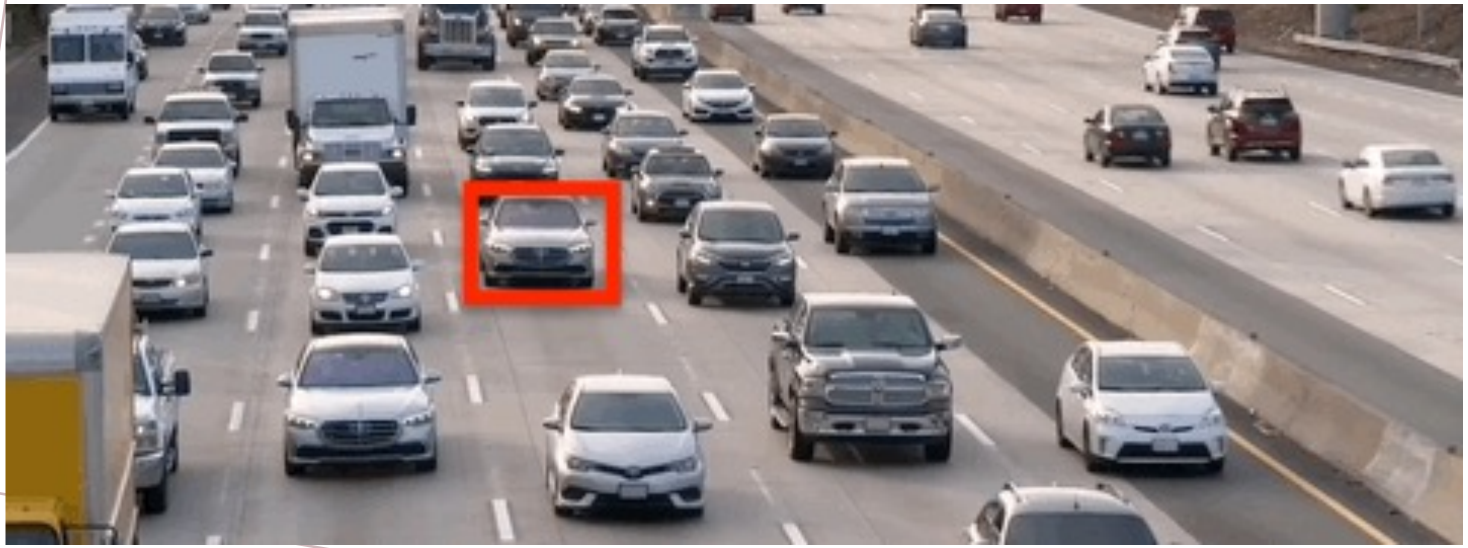
1. Introduction



- Perception algorithms
- Fusion localization
- Control strategies

Development

Ensure **stability** but neglect **traffic benefits**

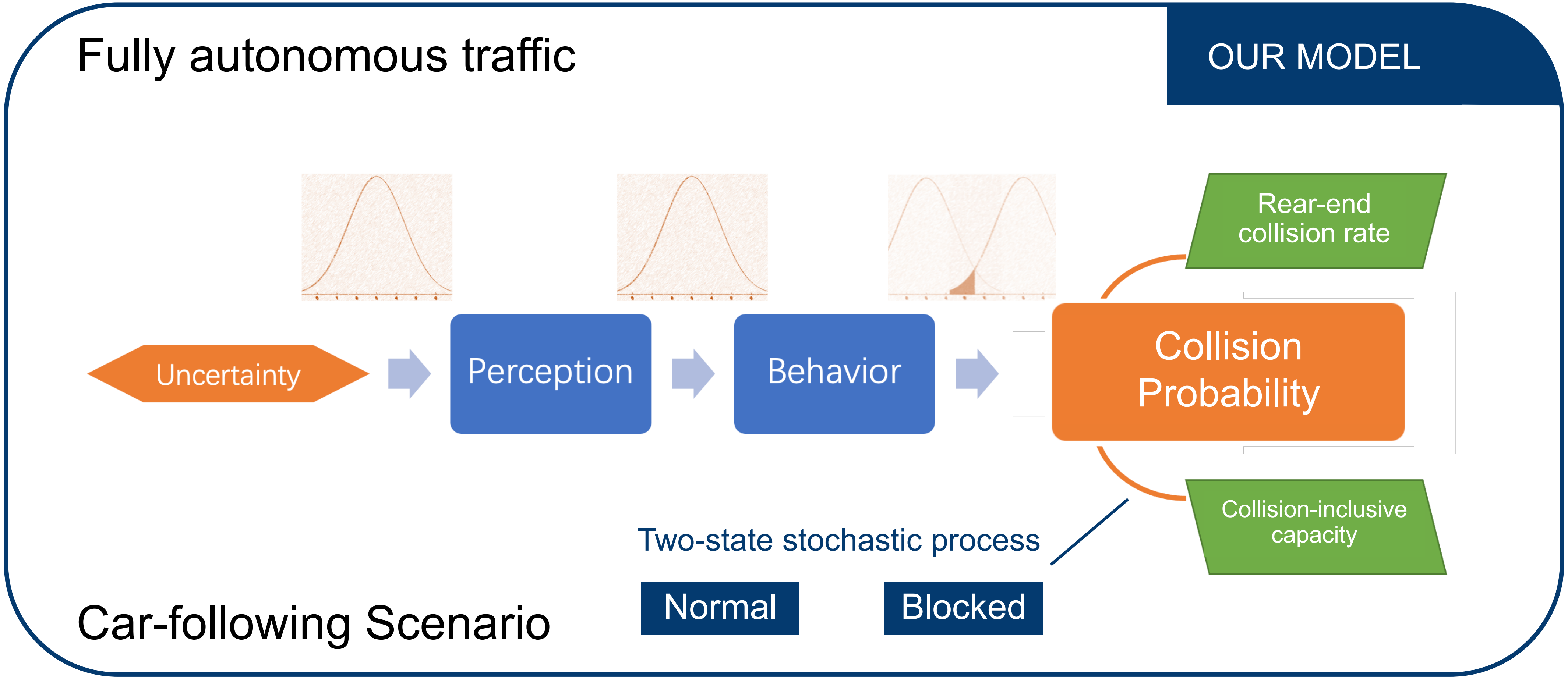


Focus on **safety** but sacrifice **efficiency**

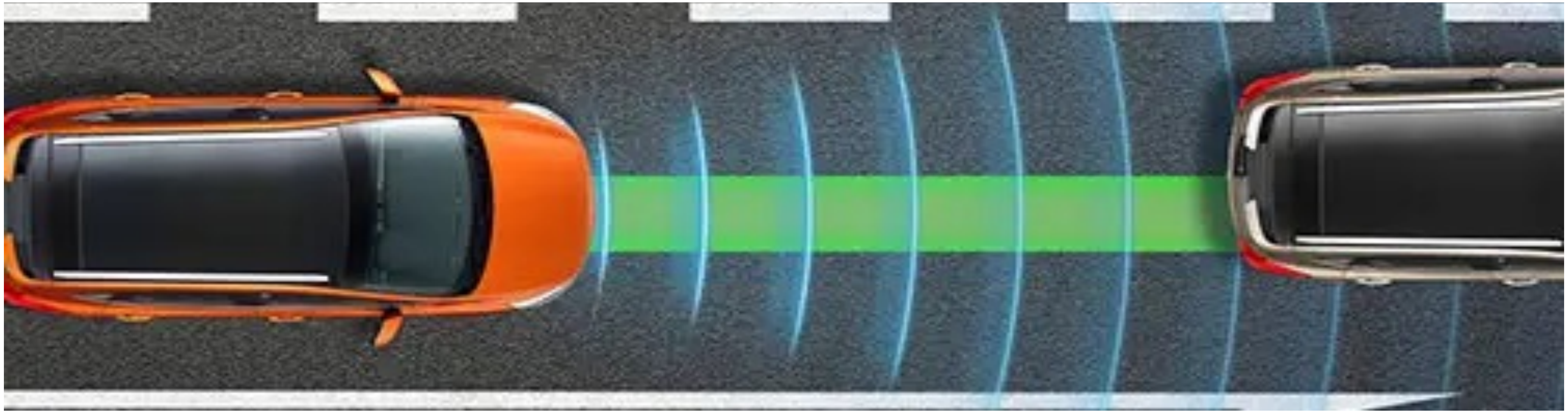
Validation

- Uncertainty
- Lack of laws and rules
- Public concerns

1. Introduction



2. Model - Scenario



- **Forward perception** – only follow the certain car in front
- **Longitudinal control** – only acceleration and braking are considered
- **Rear-end collision** – consider single-lane one-time rear-end collisions as the only source of capacity reduction
- **Predetermined speed** – view speed as an exogenous variable independent of traffic density
- **Ideal road section** – Independent section has no bottleneck at the downstream

2. Model – Car following with perceptual error

Newell's model:

$$v_1(t) = v$$

$$x_1(t) = x_1(0) + vt$$

$$x_2(t) = x_1(t - \boxed{\tau}) - \boxed{\delta}$$

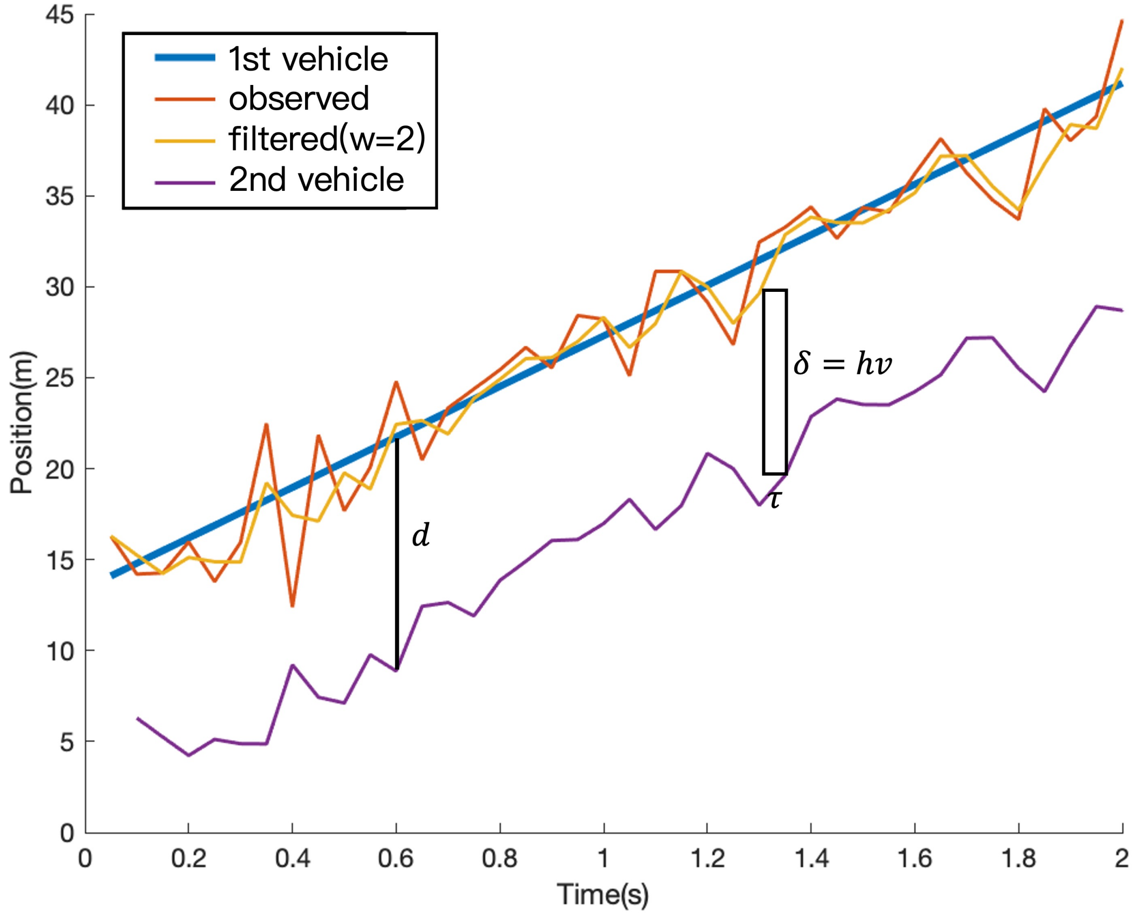
with perceptual error:

$$x_1^o(t) = x_1(0) + vt + \boxed{\varepsilon_o(t)}$$

$$\varepsilon_o \sim \mathcal{N}(0, \sigma_o^2)$$

$$x_2(t) = \boxed{\frac{1}{w}} \sum_{i=1}^{w} x_1^o(t - i\tau) - \delta$$

$$\varepsilon_{x_2} \sim \mathcal{N}(0, \frac{1}{w}\sigma_o^2)$$



2. Model – Error propagation

$$x_f(t) = x_f(0) + vt + \varepsilon_f(t)$$
$$\varepsilon_f \sim \mathcal{N}(0, \sigma_x^2)$$
$$x_e(t) = x_f(0) + vt - \frac{1+w}{2}v\tau - \delta + \varepsilon_e(t)$$
$$\varepsilon_e \sim \mathcal{N}(0, \frac{1}{w}\sigma_x^2 + \frac{1}{w}\sigma_o^2)$$

When ego vehicle (e) follows the front stochastic vehicle (f) with observation error (o)

$$\varepsilon_f = \varepsilon_e$$
$$\sigma_x^2 = \frac{1}{w}\sigma_x^2 + \frac{1}{w}\sigma_o^2$$
$$\sigma_x^2 = \frac{1}{w-1}\sigma_o^2$$

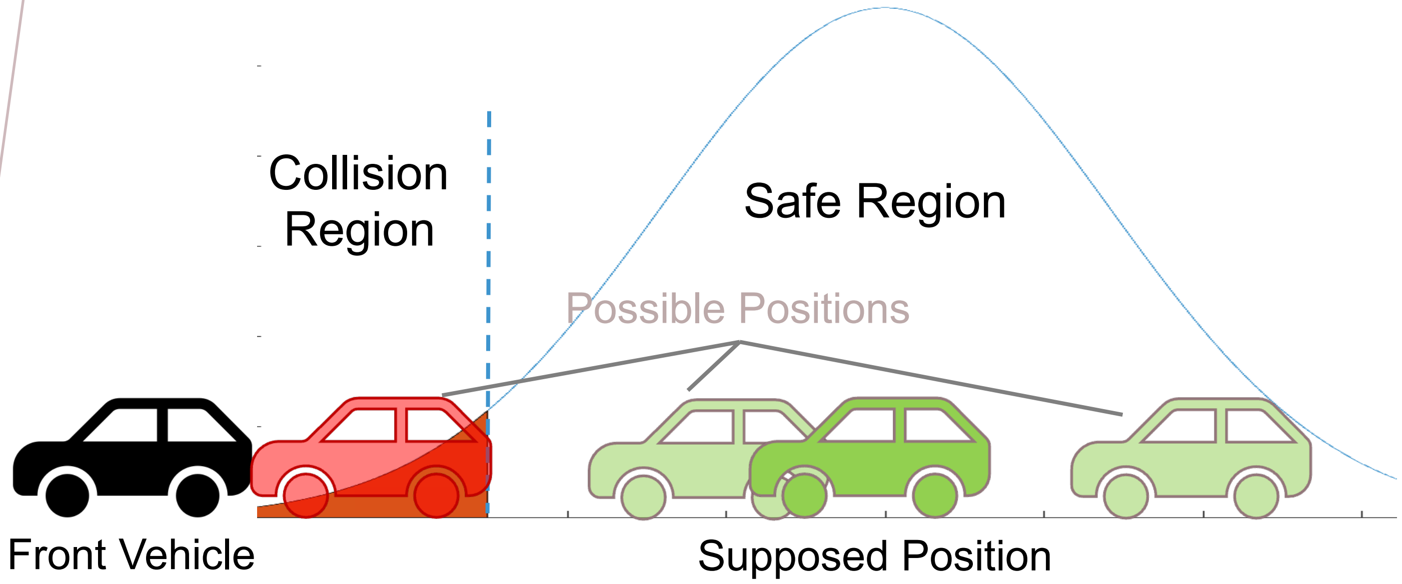
When randomness **stabilizes**,
stochasticity of each vehicle **converges**
and
distance (d) becomes **time-invariant**

$$d(t) = x_f(t) - x_e(t) = \delta + \frac{1+w}{2}v\tau + \varepsilon_x(t)$$
$$\varepsilon_x \sim \mathcal{N}(0, \frac{2}{w-1}\sigma_o^2)$$

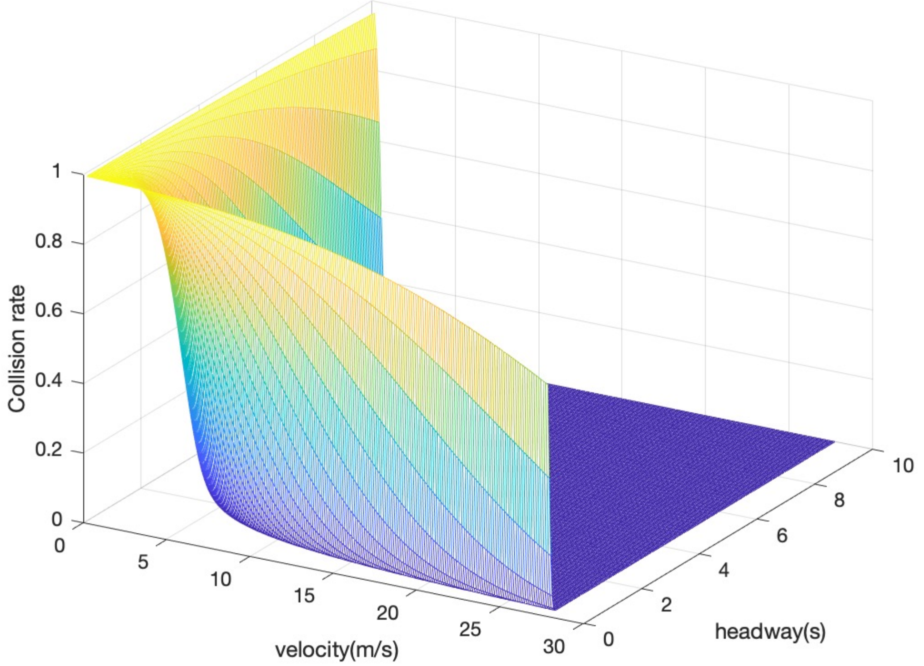
$$\Rightarrow$$

$$d(v,h) = d(t) = (h + \frac{1+w}{2}\tau)v + \varepsilon_x$$
$$\sim \mathscr{D} = \mathcal{N}((h + \frac{1+w}{2}\tau)v, \frac{2}{w-1}\sigma_o^2)$$

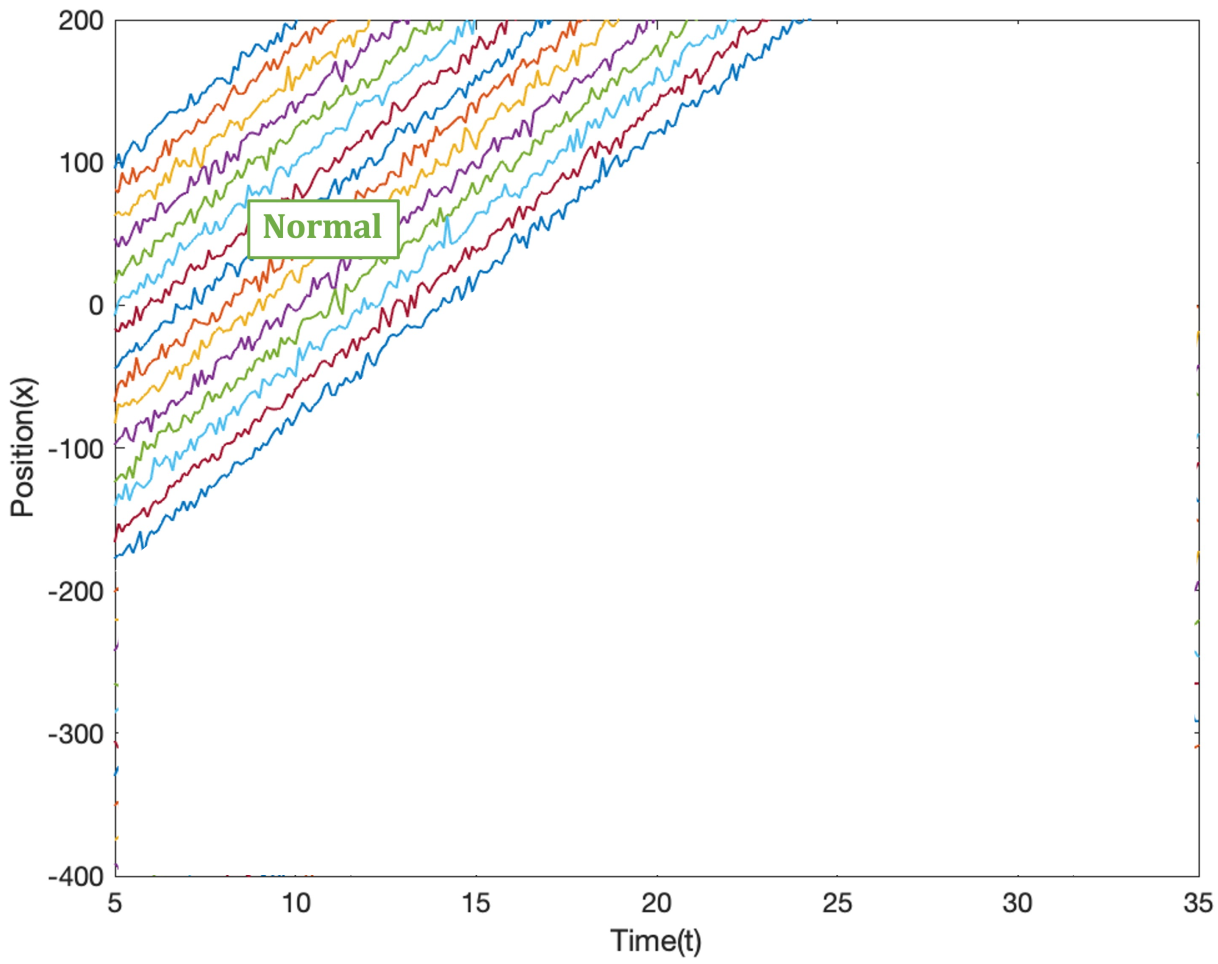
2. Model – Safety measurement



$$P_c(v,h) = \int_{-\infty}^{l} \frac{1}{\sqrt{\frac{4\pi}{w-1}\sigma_o^2}} exp\left(-\frac{(d-(h+\frac{1+w}{2}\tau)v)^2}{\frac{4}{w-1}\sigma_o^2}\right)\mathbf{d}d$$



2. Model – Collision-inclusive capacity



Normal

$$s^{+}(h) = \frac{3600}{h + \frac{1+w}{2}\tau}$$ (normal capacity)

Blocked (duration of TCT)

$$s^{-} = 0$$ (zero capacity)

$$\bar{n}(v,h) = \frac{T(v,h)}{h + \frac{1+w}{2}\tau}$$ (reduced flow per collision)

Transitions **Collision** **Clearance**

2. Model – Collision-inclusive capacity

Total Clearance Time (TCT)

$$T(v,h) = T(v) = 54v + 1800|_{v \leq 33.33m/s(120km/h)}$$

Collision-inclusive capacity

$$s(v,h) = s^+(h) - \boxed{\frac{s(v,h)}{s^+(h)}\frac{3600}{\tau}P_c}\boxed{\frac{T(v)}{h+\frac{1+w}{2}\tau}}$$

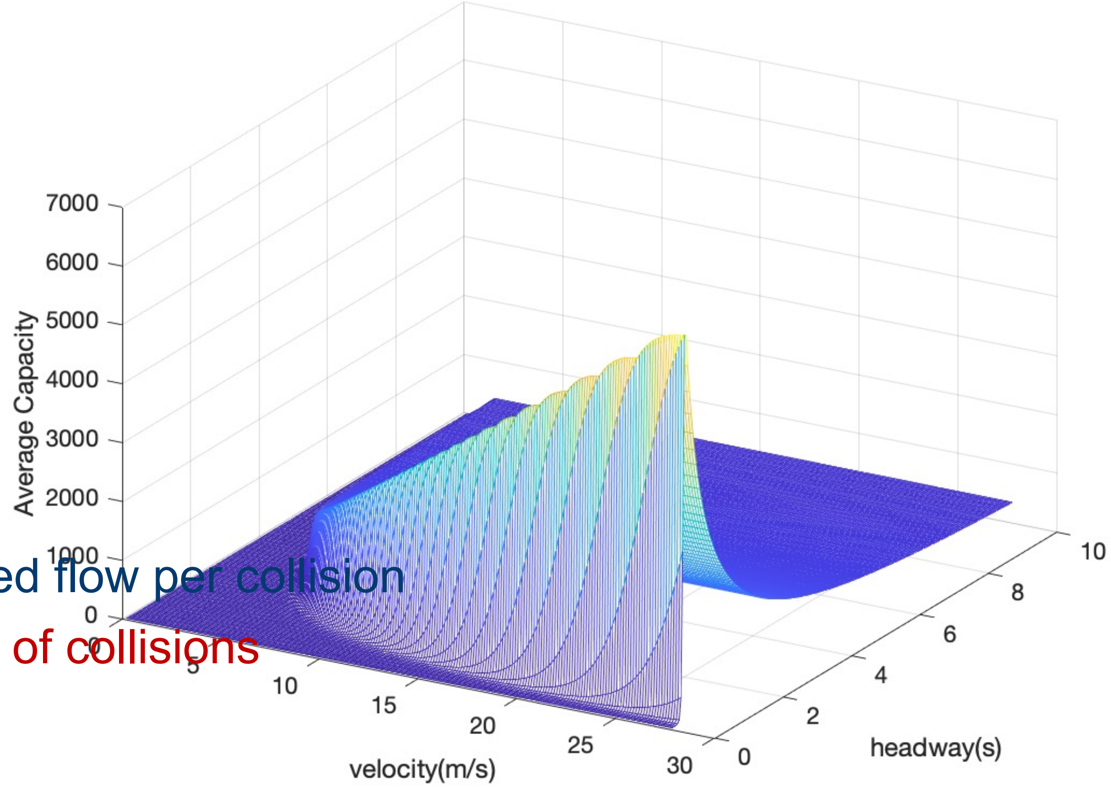
$$= s^+(h) - \frac{s(v,h)P_cT(v)}{\tau}$$

$$s(v,h) = \frac{s^+(h)}{1+\frac{T(v)}{\tau}P_c}$$

$$= \frac{3600}{(h+\frac{1+w}{2}\tau) + \frac{(h+\frac{1+w}{2})(54v+1800)}{\tau}\int_{-\infty}^{l}\frac{1}{\sqrt{\frac{4\pi}{w-1}\sigma_o^2}}exp(-\frac{(d-(h+\frac{1+w}{2}\tau)v)^2}{\frac{4}{w-1}\sigma_o^2})\mathbf{d}d}$$

reduced flow per collision

expected number of collisions



3. Discussion – Parameters

$$s(v,h)=\frac{s^+(h)}{1+\frac{T(v)}{\tau}P_c}=\frac{3600}{(h+\frac{1+w}{2}\tau)+\frac{(h+\frac{1+w}{2})(54v+1800)}{\tau}\int_{-\infty}^{l}\frac{1}{\sqrt{\frac{4\pi}{w-1}\sigma_o^2}}exp(-\frac{(d-(h+\frac{1+w}{2}\tau)v)^2}{\frac{4}{w-1}\sigma_o^2})\mathbf{d}d}$$

Length of a vehicle (l)
Safety (-) Capacity (-) → **Compact vehicles** may be more favored.

Precision of observation(σ_O)
Safety (+) Capacity (+) → **More precise sensors** lead to safer traffic and thereby contributing to greater traffic capacity and overall benefit.

Time step (τ)*
- Contribution to headway. *(Could be neglected for general values.)*
- Number of updates of a discrete system. *Negative to capacity sorely but allow larger w.*

Size of sliding window (w)*
- Contribution to headway. *(Could be neglected for general values.)*
- Error propagation from perception to motion. *Positive to both safety and capacity but reduces sensitivity in dynamic scenes.*

** See our latest paper for more details.*

3. Discussion – Suggested control on variables

Set capacity $s(v,h)$ as the goal of optimization:

$$\max_{v}\left[\max_{h} s(v,h)\right]$$

Two-stage: h controlled by single vehicle and v decided by the platoon.

$$\min_{h} S_v(h) = (h+\frac{1+w}{2}\tau) + \frac{(h+\frac{1+w}{2}\tau)(54v+1800)}{\tau}\int_{-\infty}^{l}\frac{1}{\sqrt{\frac{4\pi}{w-1}\sigma_o^2}} exp(-\frac{(d-(h+\frac{1+w}{2}\tau)v)^2}{\frac{4}{w-1}\sigma_o^2})\mathbf{d}d$$

Derivative:

$$S_v'(h) = 1 - \frac{T(v)d_m(v,h)}{\tau}\phi_{\sigma_x}(d_m(v,h)-l) + \frac{T(v)}{\tau}\Phi_{\sigma_x}(d_m(v,h)-l)$$

(*) For most of cases, $S_v(h)$ decreases first and then increases. The only solution to $S_v'(h) = 0$ exists and serves as the optimal h^*.

** See our latest paper for proof and more details.*

3. Discussion – Suggested control on variables

Set capacity $s(v, h)$ as the goal of optimization:

$$\max_v [\max_h s(v,h)]$$

Two-stage: h controlled by single vehicle and v decided by the platoon.

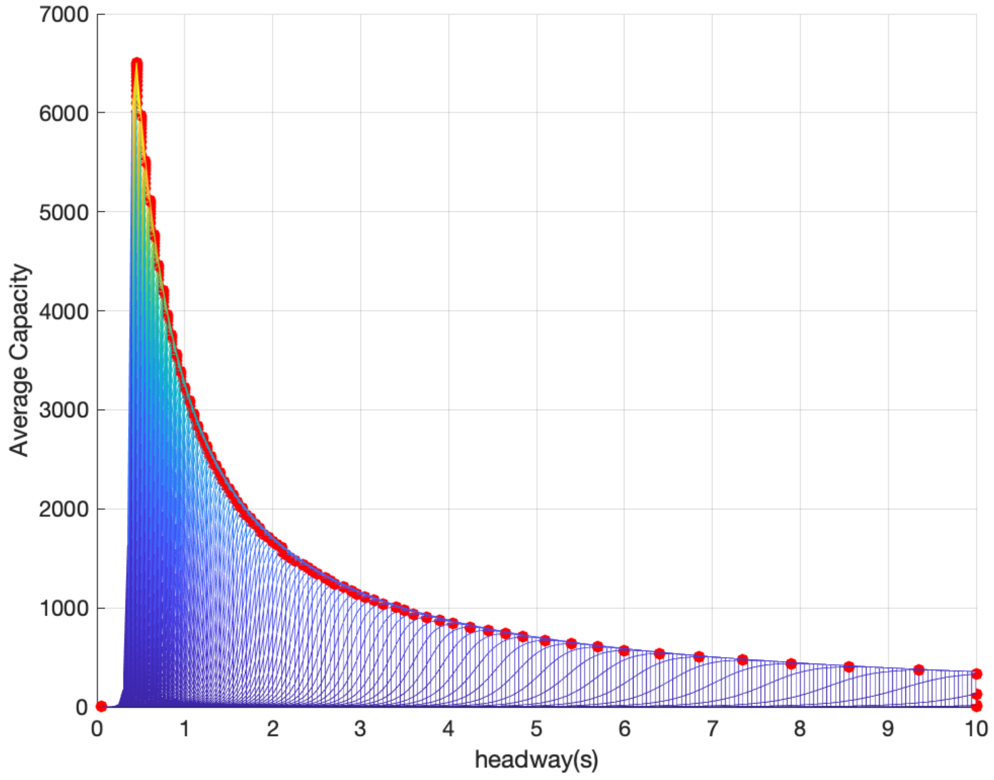
$$\max_v s(v, h^*(v)) \iff \min_v S(v, h^*(v))$$

$$S(v,h^*(v)) = h^*(v) + \frac{(h^*(v) + \frac{1+w}{2}\tau)(54v+1800)}{\tau} \int_{-\infty}^{l} \frac{1}{\sqrt{\frac{4\pi}{w-1}\sigma_o^2}} exp(-\frac{(d-(h^*(v)+\frac{1+w}{2}\tau)v)^2}{\frac{4}{w-1}\sigma_o^2})\mathbf{d}d$$

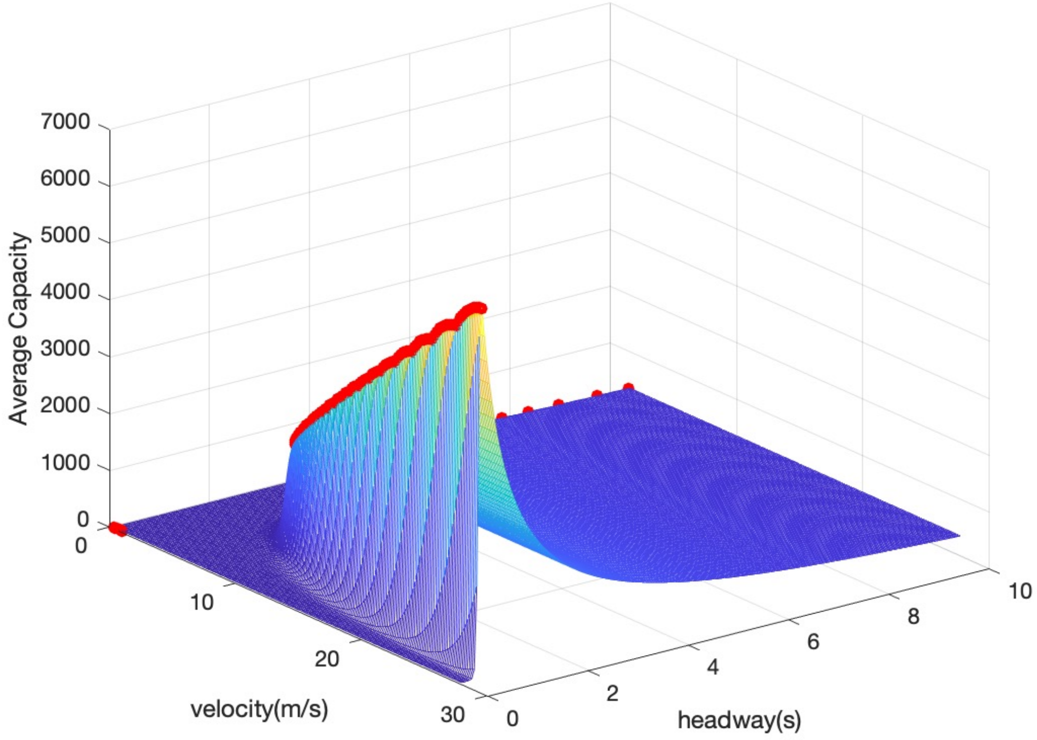
(*) As long as a h^* exists, $S(v, h^*(v))$ monotonically decreases. The original capacity function's monotonically increasing suggests that **driving as fast as possible** only when under reasonable **limits by road conditions**, such as the curvature, slope, unevenness, etc.

** See our latest paper for proof and more details.*

3. Discussion – Suggested control on variables



Optimal control on headway



General suggested control

3. Discussion – Generalization

From **Car-following** to **other scenarios**

- **error propagation**

$$p^{e}_{ego} = p_{ego} + \varepsilon_L \qquad \textit{(Localization)}$$

$$u_{ego} = u^{d}_{ego} + \varepsilon_D \qquad \textit{(Decision-making)}$$

$$p_{ego} = p^{*}_{ego} + \varepsilon_P(\varepsilon_L, \varepsilon_D) \sim \mathscr{X} \qquad \textit{(Position with uncertainty)}$$

- **safety measurement**

$$P_c(k) = \int_{p \in \mathbf{C}} f_{\mathscr{X}}(p)\mathbf{d}p$$

- **collision-inclusive capacity**

$$s^{+} = kv \qquad s(k,v) = \frac{kv}{1 + T(v)P_c(k)\frac{1}{\tau}}$$

As long as **AV systems** and the **traffic environment** are modeled,

safety performance and **collision-inclusive capacity** could be analytically derived.

4. Conclusion

Model

- **Robotic error:** from *perception* to *position* uncertainty, leading to possible **collisions**
- **Safety:** integrated **collision probability**
- **Capacity:** average **collision-inclusive** capacity reduced by collision blocks

Discussion

- **Parameters:** simple with *vehicle length* and *perception precision*, but complex with *time step* and *size of sliding window*
- **Optimal control:** locally optimal control on *headway*, and monotone relationship with *speed*
- **Generalization:** from the simple car-following to more **general scenarios**

Future plan

- Consider more realistic **high-order models** to include *random acceleration* and *speed*;
- Reveal the perceptual error in practice and validate the model by **experiments**;
- Conduct **optimization** with goals and constraints of realistic meanings based on our model.

Thank you
for your attention

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