Aptitude - Number System

Numbers

In Decimal number system, there are ten symbols namely 0,1,2,3,4,5,6,7,8 and 9 called digits. A number is denoted by group of these digits called as numerals.

Face Value

Face value of a digit in a numeral is value of the digit itself. For example in 321, face value of 1 is 1, face value of 2 is 2 and face value of 3 is 3.

Place Value

Place value of a digit in a numeral is value of the digit multiplied by 10^n where n starts from 0. For example in 321:

- Place value of 1 = 1 x 10^0 = 1 x 1 = 1
- Place value of 2 = 2 x 10^1 = 2 x 10 = 20
- Place value of 3 = 3 x 10^2 = 3 x 100 = 300

> *0^{th} position digit is called unit digit and is the most commonly used topic in aptitude tests.*

Types of Numbers

Natural Numbers - n > 0 where n is counting number; [1,2,3...]
Whole Numbers - n ≥ 0 where n is counting number; [0,1,2,3...].

> *0 is the only whole number which is not a natural number.*
> *Every natural number is a whole number.*

Integers - n ≥ 0 or n ≤ 0 where n is counting number;...,-3,-2,-1,0,1,2,3... are integers.

- **Positive Integers** - n > 0; [1,2,3...]
- **Negative Integers** - n < 0; [-1,-2,-3...]
- **Non-Positive Integers** - n ≤ 0; [0,-1,-2,-3...]
- **Non-Negative Integers** - n ≥ 0; [0,1,2,3...]

> *0 is neither positive nor negative integer.*

Even Numbers - n / 2 = 0 where n is counting number; [0,2,4,...]
Odd Numbers - n / 2 ≠ 0 where n is counting number; [1,3,5,...]
Prime Numbers - Numbers which is divisible by themselves only apart from 1.

> *1 is not a prime number.*
> *To test a number p to be prime, find a whole number k such that k > √p. Get all prime numbers less than or equal to k and divide p with each of these prime numbers. If no number divides p exactly then p is a prime number otherwise it is not a prime number.*

```
Example: 191 is prime number or not?
Solution:
```

```
Step 1 - 14 > √191
Step 2 - Prime numbers less than 14 are 2,3,5,7,11 and 13.
Step 3 - 191 is not divisible by any above prime number.
Result - 191 is a prime number.

Example: 187 is prime number or not?
Solution:
Step 1 - 14 > √187
Step 2 - Prime numbers less than 14 are 2,3,5,7,11 and 13.
Step 3 - 187 is divisible by 11.
Result - 187 is not a prime number.
```

Composite Numbers - Non-prime numbers > 1. For example, 4,6,8,9 etc.

> *1 is neither a prime number nor a composite number.*
> *2 is the only even prime number.*

Co-Primes Numbers - Two natural numbers are co-primes if their H.C.F. is 1. For example, (2,3), (4,5) are co-primes.

Divisibility

Following are tips to check divisibility of numbers.

Divisibility by 2 - A number is divisible by 2 if its unit digit is 0,2,4,6 or 8.

```
Example: 64578 is divisible by 2 or not?
Solution:
Step 1 - Unit digit is 8.
Result - 64578 is divisible by 2.

Example: 64575 is divisible by 2 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64575 is not divisible by 2.
```

Divisibility by 3 - A number is divisible by 3 if sum of its digits is completely divisible by 3.

```
Example: 64578 is divisible by 3 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 8 = 30
which is divisible by 3.
```

```
Result - 64578 is divisible by 3.

Example: 64576 is divisible by 3 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 6 = 28
which is not divisible by 3.
Result - 64576 is not divisible by 3.
```

Divisibility by 4 - A number is divisible by 4 if number formed using its last two digits is completely divisible by 4.

```
Example: 64578 is divisible by 4 or not?
Solution:
Step 1 - number formed using its last two digits is 78
which is not divisible by 4.
Result - 64578 is not divisible by 4.

Example: 64580 is divisible by 4 or not?
Solution:
Step 1 - number formed using its last two digits is 80
which is divisible by 4.
Result - 64580 is divisible by 4.
```

Divisibility by 5 - A number is divisible by 5 if its unit digit is 0 or 5.

```
Example: 64578 is divisible by 5 or not?
Solution:
Step 1 - Unit digit is 8.
Result - 64578 is not divisible by 5.

Example: 64575 is divisible by 5 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64575 is divisible by 5.
```

Divisibility by 6 - A number is divisible by 6 if the number is divisible by both 2 and 3.

```
Example: 64578 is divisible by 6 or not?
Solution:
Step 1 - Unit digit is 8. Number is divisible by 2.
Step 2 - Sum of its digits is 6 + 4 + 5 + 7 + 8 = 30
which is divisible by 3.
Result - 64578 is divisible by 6.
```

```
Example: 64576 is divisible by 6 or not?
Solution:
Step 1 - Unit digit is 8. Number is divisible by 2.
Step 2 - Sum of its digits is 6 + 4 + 5 + 7 + 6 = 28
which is not divisible by 3.
Result - 64576 is not divisible by 6.
```

Divisibility by 8 - A number is divisible by 8 if number formed using its last three digits is completely divisible by 8.

```
Example: 64578 is divisible by 8 or not?
Solution:
Step 1 - number formed using its last three digits is 578
which is not divisible by 8.
Result - 64578 is not divisible by 8.

Example: 64576 is divisible by 8 or not?
Solution:
Step 1 - number formed using its last three digits is 576
which is divisible by 8.
Result - 64576 is divisible by 8.
```

Divisibility by 9 - A number is divisible by 9 if sum of its digits is completely divisible by 9.

```
Example: 64579 is divisible by 9 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 9 = 31
which is not divisible by 9.
Result - 64579 is not divisible by 9.

Example: 64575 is divisible by 9 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 5 = 27
which is divisible by 9.
Result - 64575 is divisible by 9.
```

Divisibility by 10 - A number is divisible by 10 if its unit digit is 0.

```
Example: 64575 is divisible by 10 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64578 is not divisible by 10.

Example: 64570 is divisible by 10 or not?
```

```
Solution:
Step 1 - Unit digit is 0.
Result - 64570 is divisible by 10.
```

Divisibility by 11 - A number is divisible by 11 if difference between sum of digits at odd places and sum of digits at even places is either 0 or is divisible by 11.

```
Example: 64575 is divisible by 11 or not?
Solution:
Step 1 - difference between sum of digits at odd places
and sum of digits at even places = (6+5+5) - (4+7) = 5
which is not divisible by 11.
Result - 64575 is not divisible by 11.

Example: 64075 is divisible by 11 or not?
Solution:
Step 1 - difference between sum of digits at odd places
and sum of digits at even places = (6+0+5) - (4+7) = 0.
Result - 64075 is divisible by 11.
```

Tips on Division

If a number n is divisible by two co-primes numbers a, b then n is divisible by ab.
(a-b) always divides (a^n - b^n) if n is a natural number.
(a+b) always divides (a^n - b^n) if n is an even number.
(a+b) always divides (a^n + b^n) if n is an odd number.

Division Algorithm

When a number is divided by another number then

Dividend = (Divisor x Quotient) + Reminder

Series

Following are formulaes for basic number series:

$(1+2+3+...+n) = (1/2)n(n+1)$
$(1^2+2^2+3^2+...+n^2) = (1/6)n(n+1)(2n+1)$
$(1^3+2^3+3^3+...+n^3) = (1/4)n^2(n+1)^2$