

NUMBERS AND DIVISIBILITY RULES

Numbers are the fundamental units of mathematics.

In this section, we will discuss **numbers**, their **types**, **important facts**, **divisibility rules**, and other essential concepts for **aptitude preparation**.

Types of Numbers

Integers	All numbers whose fractional part is zero. Example: -3, -2, -1, 0, 1, 10, 100
Natural Numbers	Counting numbers starting from 1. Example: 1, 2, 3, 4, 5, 6, ...
Whole Numbers	All natural numbers including 0. Example: 0, 1, 2, 3, 4, 5, ...
Prime Numbers	Numbers that have only two distinct factors — 1 and the number itself. Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
Composite Numbers	Numbers greater than 1 that are not prime. Example: 4, 6, 8, 9, 10, 60, 91, 100

Important Points on Prime Numbers

- 0 and 1 are neither prime nor composite.
- 2 is the only even prime number.
- There are 25 prime numbers less than 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime Check Rule: To check if a number p is prime:

- Find the smallest natural number n such that $n^2 \geq p$.
- Check if p is divisible by any prime number $\leq n$.
- If p is not divisible by any such prime, then it is prime; otherwise, it is not.

Co-Primes: Two numbers a and b are co-prime if their HCF is 1. Example: (8, 15).

Number of Divisors: If $n = p_1^{e_1} \times p_2^{e_2} \times \dots \times p_k^{e_k}$, then divisors = $(e_1+1)(e_2+1)\dots(e_k+1)$
Example: $200 = 2^3 \times 5^2 \rightarrow$ Divisors = $(3+1)(2+1) = 12$.

Divisibility Rules

Divisible By	Rule	Example
2	The last digit should be even (0, 2, 4, 6, or 8).	$124 \rightarrow$ last digit is 4 (even), so divisible by 2.
3	The sum of all digits should be divisible by 3.	$12321 \rightarrow 1 + 2 + 3 + 2 + 1 = 9 \rightarrow$ divisible by 3.
4	The number formed by the last two digits should be divisible by 4.	$12324 \rightarrow$ last two digits 32 \rightarrow divisible by 4.
5	The last digit should be 0 or 5.	$85 \rightarrow$ last digit is 5 \rightarrow divisible by 5.
6	The number must be divisible by both 2 and 3.	$114 \rightarrow$ last digit 4 (even), and sum $1+1+4=6 \rightarrow$ divisible by 6.
7	Remove the last digit, double it, and subtract from the remaining number.	$196 \rightarrow$ Remove the last digit 6, double it to 12, and subtract from 19 \rightarrow 19 - 12 = 7, divisible by 7.
8	The number formed by the last three digits should be divisible by 8.	$1232 \rightarrow$ last three digits 232 \rightarrow divisible by 8.
9	The sum of all digits should be divisible by 9.	$12321 \rightarrow 1+2+3+2+1=9 \rightarrow$ divisible by 9.
11	The difference between the sum of digits at odd and even positions should be 0 or divisible by 11.	$1234567890 \rightarrow$ Sum of odd digits $(1+3+5+7+9) = 25$ and even digits $(2+4+6+8+0) = 20$. Difference $25 - 20 = 5$, not divisible by 11.