midterm 2

1) I: set of items

w_i: weight of item i

s_i: importance of item i

x_i: $\begin{cases} 1 & \text{if we choose item } i \\ 0 & \text{otherwise} \end{cases}$

a) $\max \sum_{i \in I} s_i \cdot x_i$

$\sum_{i \in I} w_i \cdot x_i \leq 15$

b) z: $\begin{cases} 1 & \text{if 1,6,4 carried together} \\ 0 & \text{otherwise} \end{cases}$

$x_1 \geq 1 \cdot z$

$x_4 \geq 1 \cdot z$

$x_6 \geq 1 \cdot z$

c) $x_2 + x_5 + x_7 + x_9 = 3$

2) D.V: x_{kt}: number of product type k, produced in period t

y_{kt}: $\begin{cases} 1 & \text{if we produced type k in period t} \\ 0 & \end{cases}$

s_{kt}: the number of product type k we stock.

$\min \left(\sum_{t \in T} \sum_{k \in K} p_{kt} \cdot x_{kt}\right) + \left(\sum_{t \in T} \sum_{k \in K} q_{kt} \cdot y_{kt}\right) + \left(\sum_{t \in T} \sum_{k \in K} h_{kt} \cdot s_{kt}\right)$

$s_{kt-1} + x_{kt} = d_{kt} + s_{kt}$

$s_0 = 0$

$x_{kt} \leq M y_{kt}$

$x_{kt}, s_{kt} \geq 0 \quad y_{kt} \in \{0, 1\}$

b) $z_t = \begin{cases} 1 & \text{if we produce any product in period } t \\ 0 \end{cases}$

$z_6 = 0$

c) $\sum_{k \in K} y_{kt} \leq 5 \qquad \forall t \in T$

d) $y_{73} \geq y_{43}$

3) I: set of items

J: set of assembled produc

a_i: number of available item i

s_i: sales price to other for item i

S_j: sales price to other for product j

d_{ij}: demand of item i to produce product j

x_i: the number of item i directly sell in the market

y_j: number of product j that made by ~~item~~

$$\max \sum_{j \in J} S_j \cdot y_j + \sum_{i \in I} s_i \cdot x_i$$

$$\cancel{\sum_{j \in J} d_{ij} \cdot y_j \leq a_i \quad i \in I}$$

$$\sum_{j \in J} d_{ij} \cdot y_j + x_i \leq a_i \quad i \in I$$

$$x_i, y_j \geq 0, \text{ integer}$$

4) I: set of districts (A - H)
J: set of potential location (1-7)

$a_{ij}: \begin{cases} 1 & \text{if location } j \text{ can cover district } j \\ 0 \end{cases}$

f_j: fixed cost of open location j

$x_j: \begin{cases} 1 & \text{if a police station set up in location } j \\ 0 \end{cases}$

a)

$\min \sum_{j \in J} f_j \cdot x_j$

$- \sum a_{ij} \cdot x_j \geq 1$

b) $\sum_{j \in J} a_{3j} \cdot x_j \geq 2$

$a_{c1} + a_{c2} + a_{c3}$

c) $x_4 + x_5 \leq 1$

d)

$\cancel{\max \sum_{i \in I} \sum_{j \in J} a_{ij} \cdot x_j}$

$y_i: \begin{cases} 1 & \text{if district } i \text{ is covered} \\ 0 \end{cases}$

$\max \sum_{i \in I} y_i$

$y_i \leq \sum a_{ij} \cdot x_j \quad i \in I$

$\sum_{j \in J} f_j \cdot x_j \leq 1700000$

5) I: set of machine

f_i: fixed cost of machine i

c_i: variable cost of machine i

u_i: capacity of machine i

$y_i: \begin{cases} 1 & \text{if we use machine } i \\ 0 \end{cases}$

x_i: number of product that produced by machin i

$\min \sum_{i \in I} c_i \cdot x_i + \sum_{i \in I} f_i \cdot y_i$

$x_i < u_i \, y_i \quad i \in I$

$\sum x_i \geq 2000$

6) I: set of store type
J: set of number of stores

u_i: square footage need of store type i

P_{ij}: profit of number j store of store type i

x_{ij}: $\begin{cases} 1 & \text{if we open } j \text{ set of store type } i \\ 0 \end{cases}$

$$\max \left(\sum_{i \in I} \sum_{j \in J} j \cdot P_{ij} \cdot x_{ij} \right) \cdot 0.05$$

$$\sum_{j \in J} v_i \cdot x_{ij} \leq 10000 \qquad i \in I$$

$x_{10} = 0$, $x_{20} = 0$, $x_{30} = 0$, $x_{40} = 1$, $x_{50} = 0$

$0 \leq \sum x_{1j} \leq 3$

midterm 3

? ☆ 1) I: type of closet

U_i: capacity of producing type i Assembling
V_i: " " " " Painting
P_i: selling price type i closet

x_i: number of type i produced.

$$\max \sum_{i \in I} P_i x_i$$

? $\sum_{i \in I} x_i \leq \sum_{i \in I} U_i$, $\sum_{i \in I} x_i \leq \sum_{i \in I} P_i$

$x_i \geq 0$, integer

2) I: set of door ~~J: set of workers~~

P_i: Profit of door i

x_i: number of door i that produce in one day

a) $$\max \sum_{i \in I} P_i x_i$$

$x_1 \leq 4$ $\quad x_2 \leq 6$

$8x_1 + 6x_2 \leq 40$, $x_i \geq 0$, integer

b) $x_1 P_1 \geq 3000$ the $x_2 P_2 \geq 5000$

$$\Rightarrow \begin{cases} g(x) = x_1 P_1 - 3000 \geq 0 \\ f(x) = x_2 P_2 - 5000 \geq 0 \end{cases} \Rightarrow \begin{cases} g(x) \leq Mz \\ f(x) \geq -M(1-z) \end{cases}$$

3) I: set of items

w_i: weight of item i

s_i: importance of item i

x_i: $\begin{cases} 1 & \text{if we choose item } i \\ 0 & \text{otherwise} \end{cases}$

a) $\max \sum_{i \in I} s_i \cdot x_i$

$\sum_{i \in I} w_i \cdot x_i \leq 15$

b)

$x_5 = x_6 = x_8$

c) $x_2 + x_5 + x_7 + x_8 = 2$

d) $\begin{cases} x_3 \geq x_2 + x_5 - 1 \\ x_4 \geq x_2 + x_5 - 1 \end{cases}$

e) $\begin{cases} x_3 \leq 1 - x_7 \\ x_4 \leq 1 - x_7 \\ x_5 \leq 1 - x_7 \end{cases}$

$x_i \in \{0,1\}$

4) I: set of month

d_i: demand of month i

c_i: purchase price month i

s_i: Stock price

x_i: amount of product that ~~produce~~ purchase in month i

y_i: amount of product in the stock in month i

$x_1 = 43 + y_1$

$y_1 + x_2 = 73 + y_2$

$y_2 + x_3 = 23 + y_3$

$y_3 + x_4 = 78 + y_4$

$y_4 + x_5 = 99 + y_5$

$\min \sum_{i \in I} c_i x_i - \underbrace{y_5 \cdot 1600} + \sum s_i y_i$

$x_i \geq 0$, integer

$y_i \geq 0$, integer

5) I: set of tasks
J: set of employees

t_{ij}: time of employee j to do task i

a_{ij}: preference of employee j to do task i

x_{ij}: $\begin{cases} 1 & \text{if task } i \text{ is assign to employee } j \\ 0 \end{cases}$

a) $\max \sum_{i \in I} \sum_{j \in J} a_{ij} \cdot x_{ij}$

$\sum_{i \in I} t_{ij} \cdot x_{ij} \leq 350 \quad j \in J$

$\sum_{j \in J} x_{ij} = 1 \quad i \in I \qquad x_{ij} \in \{0,1\}$

b) $\sum_{i \in I} x_{ij} \leq 3 \quad j \in J$

c) $\sum_{i \in I} a_{ij} \cdot x_{ij} \geq 40 \quad j \in J$

d) if $x_{13} = 1$ then $x_{36} = 1 \Rightarrow x_{36} \geq x_{13}$

e) $x_{44} + x_{47} + x_{49} + x_{4\,11} + x_{4\,13} \leq 3$

change i, j

f) if $x_{32} = 1$ and $x_{82} = 1$ then $x_{54} = 1$ and $x_{64} = 1$

$\Rightarrow \begin{cases} x_{54} \geq x_{32} + x_{82} - 1 \\ x_{64} \geq x_{32} + x_{82} - 1 \end{cases}$

g) $\sum_{i \in I} a_{i1} \cdot x_{i1} \geq 30$ then $\sum a_{i3} \cdot x_{i3} \geq 40$

$\begin{cases} g(x) = \sum a_{i1} \cdot x_{i1} - 30 \geq 0 \\ f(x) = \sum a_{i3} \cdot x_{i3} - 40 \geq 0 \end{cases} \Rightarrow \begin{cases} g(x) \leq Mz \\ f(x) \geq -M(1-z) \end{cases}$

$\frac{13}{100} \times x = \frac{40}{100}$ $\quad w_{12}$ $\quad \frac{10}{10} \times 80 = fk$

$\frac{13}{100}$ $\quad \cancel{40} = \frac{100}{100}$

6) I: set of metals (1, 2, 3, 4) $\quad$ J: set of type of ~~metals~~

w_{ij}: weight of metal i typj

P_{ij}: cost of metal i type j

N_{ij}: Nickel % in metal i type j

C_{ij}: carbon % in metal i type j

$x_{ij}: \begin{cases} 1 & \text{if we use type j of metal i} \\ 0 & \text{otherwise} \end{cases}$

y_{ij}: the amount of metal i type j in final product

a)

$\min \sum P_{ij} \cdot y_{ij}$

$\sum_{i \in I} \sum_{j \in J} w_{ij} \cdot \cancel{x_{ij}}\, y_{ij} \geq 40$ $\qquad \sum_{i \in I} \sum_{j \in J} N_{ij} \cdot \cancel{x_{ij}}\, y_{ij} = \frac{7}{100} \sum \sum y_{ij}$

$x_{ij} \in \{0,1\}$ $\qquad \sum_{i \in I} \sum_{j \in J} C_{ij} \cdot \cancel{x_{ij}}\, y_{ij} = \frac{13}{100} \sum y_{ij}$

b) ~~$\min \sum w_{ij} \cdot x_{ij} \cdot P_{ij}$~~ $\quad y_{1j} = w_{1j}$

c) $y_{21} \geq 5 \Rightarrow y_{22} \leq 6 \Rightarrow$

$\begin{cases} g(x) = y_{21} - 5 \geq 0 \\ f(x) = 6 - y_{22} \geq 0 \end{cases} \Rightarrow \begin{cases} g(x) \leq Mz \\ f(x) \geq -M(1-z) \end{cases}$

d) if $y_{31} > 0$ then $y_{32} \leq 0 \Rightarrow \{$

☆ 7) I: set of printers   K: set of customers
J: set of items   S: set of suppliers

a_{ij}: number of item j need to produce printer i

d_{ik}: demand of printer i from customer k

u_{js}: number of item j in each batch of supplier s

~~min~~ x_s: number of batches ~~of~~ ~~the~~ buy from supplier s

a) $\boxed{\min \sum_{s \in S} x_s}$ $\sum\sum u_{js} \cdot x_s \geq \sum a_{ij}$

$\sum_{i \in I}\sum_{j \in J}\sum_{s \in S} a_{ij} \cdot u_{js} \cdot x_s \geq \sum_{i \in I} d_{ik}$ $k \in K$

? b) $\left(\min \sum d_{ik} - \sum a_{ij} \cdot u_{js} \cdot x_s\right) \times P_i$

x_i: amount of printer i that produced

$\sum \left(d_i - \sum x_i\right) \cdot P_i$

8

I: set of factories

J: set of ceteres

p_i: number of shippment from factory i

d_j: demand of center j

v_{ij}: distance between factory i to center j

x_{ij}: shipment from factory i to centre j

$$\min \sum_{i \in J} \sum_{j \in J} (246 + 20\, v_{ij}) \cdot x_{ij}$$

$$\sum_{j \in J} x_{ij} \leq p_i \qquad i \in I$$

$$\sum_{i \in I} x_{ij} \geq d_j \qquad j \in J$$

midterm 4

1) I: set of items

w_i: weight of item i

s_i: importance of item i

$x_i: \begin{cases} 1 & \text{if item } i \text{ choosen} \\ 0 & \text{otherwise} \end{cases}$

a) $\max \sum_{i \in I} s_i \cdot x_i$

$\sum_{i \in I} w_i \cdot x_i \leq 20$

$x_i \in \{0,1\}$

b) $x_2 = x_4 = x_6$

c) $x_1 + x_2 + x_5 + x_8 = 3$

d) $\begin{cases} x_2 \geq x_1 + x_4 - 1 \\ x_3 \geq x_1 + x_4 - 1 \end{cases}$

e) $1 - x_8 \geq x_1$

$1 - x_8 \geq x_2$

$1 - x_8 \geq x_6$

2) I: set of factories

J: set of centers

P_i: shipments of factory i

d_j: demand of center j

a_{ij}: distance between factory i and center j

x_{ij}: ~~distance~~ number of shipments from factory i to center j

$\min \sum_{i \in I} \sum_{j \in J} (246 + 20\, a_{ij}) \cdot x_{ij}$

$\sum_{j \in J} x_{ij} \leq P_i \quad i \in I$

$x_{ij} \geq 0$, integer

$\sum_{i \in I} x_{ij} \geq d_j \quad j \in J$

3) I: set of items (TV, radio)

d_i: space that item i need to stock

P_i: profit of item i

x_i: the number of item i are in stock.

$$\max \sum_{i \in I} P_i \cdot x_i$$

$$\sum_{i \in I} d_i \cdot x_i \leq 350$$

$$340 x_1 + 6 x_2 \leq 4500$$

$$x_2 \geq \frac{70}{100}(x_2 + x_1) \qquad x_i \geq 0, \text{ integer} \quad i \in I$$

4) I: set of boxes (B_1, B_2, B_3)

J: set of machines ( )

U_{ij}: capacity of machine j to produce box i

P_i: profit of box i

d_i: demand of box i

x_i: number of box i that produced

a) $\max \sum_{i \in I} P_i \cdot x_i \qquad \sum_{i \in I} x_i \geq \sum_{i \in I} d_i \qquad \sum_{i \in I} x_i \leq \sum_{i \in I} u_{ij} \quad j \in$

b) $x_1 - x_3 \leq 100$

c) $x_1 \geq 500 \Rightarrow x_3 \leq 200 \quad \Rightarrow g(x) = x_1 - 500 \geq 0$

$f(x) = 200 - x_3 \geq 0$

$$\begin{cases} g(x) \leq Mz \\ f(x) \geq -M(1-z) \end{cases}$$

d) $|x_2 - x_3| \leq 200$

e) $x_2 \geq 400$ or $x_1 - x_3 \geq 300$

$$\Rightarrow \begin{cases} g(x) = x_2 - 400 \geq 0 \\ f(x) = x_1 - x_3 - 300 \geq 0 \end{cases} \Rightarrow \begin{cases} g(x) \leq Mz \\ f(x) \geq -M(1-z) \end{cases}$$

5) I: set of windows J: set of workers

P_i: profit of each window i

x_i: number of door i that produced in one day

$$\max \sum_{i \in I} P_i \cdot x_i$$

$x_1 \leq 6$, $x_2 \leq 5$

$6.5 x_1 + 7.25 x_2 \leq 55$

$x_i \geq 0$, integer

3 kg 1

5 deg

☆

(6) I: set of supplier

J: set of bars in each batch (L, M, S)

c_i: cost per batch from supplier i

a_{ij}: number of bar j in batch of supplier i

~~x_i: number of batches we buy from supplier i~~

d_j: demand of number of bars

$\min \sum_{i \in I} c_i x_i$, $\sum_{j \in J} a_{ij} \cdot x_i \leq 1000 \quad i \in I$

$\sum_{i \in I} a_{ij} \cdot x_i \geq \sum_{i \in I} d_j$

b) $a_{45} \cdot x_4 + a_{55} \cdot x_5 \geq \frac{60}{100} \sum_{i \in I} a_{i5} \cdot x_i$

7) I: set of chocolate

J: set of characteristic

c_i: cost of chocolati per kg

a_{ij}: percentage of characteristic j in chocolate i

x_i: amount of chocolate i used for blending

y_i: if we use chclate i 1

$\min \sum_{i \in I} c_i \cdot x_i$

$\sum_{i \in I} x_i \geq 5000$, $\sum_{i \in I} a_{i1} \cdot x_i \leq \frac{40}{100} \cdot \sum_{i \in I} x_i \cancel{a_{i1}}$

$\sum a_{i1} \cdot x_i \leq 40$

b) $\begin{cases} x_2 + x_5 \geq 3000 \\ x_1 + x_4 \leq 3500 \end{cases} \Rightarrow \begin{matrix} g(x) = 3000 - x_2 - x_5 \leq 0 \\ f(x) = x_1 + x_4 - 3500 \leq 0 \end{matrix} \Rightarrow \begin{cases} g(x) \leq M(1-z) \\ f(x) \leq Mz \end{cases}$

r

c) $y_i \begin{cases} 1 & \text{if we use choclate } i \\ 0 & \end{cases}$ $y_i \in \{0,1\}$

$$\sum_{i \in I} y_i \leq 3$$

8) I: set of desks
J: set of items
K: set of machines

c_j: cost of buying item j

a_{ij}: number of item j need to produce desk i

t_{jk}: time of machin k to produce desk i

P_i: price sale of desk i

x_i: number of desk i that produce

y_i: $\begin{cases} 1 & \text{if we produce desk } i \\ 0 & \text{otherwise} \end{cases}$ $\sum x_i \leq a_{ij} y_i$

$$\max \sum x_i \cdot P_i - \sum c_j \cdot a_{ij} \cdot x_j = \sum (P_i - c_j \cdot a_{ij}) x_i$$

a) $\sum_{i \in I} \sum_{j \in J} t_{ik} x_i < 65$

$x_i \geq 0$

$y_i \{0,1\}$

b) $\sum_{i \in I} y_i \geq 4$

c) $1 - y_3 \geq y_1$

$1 - y_5 \geq y_1$

d) $y_1 + y_3 \geq 2y_4$

e) $\begin{cases} y_1 \geq y_3 \\ y_4 \geq y_3 \end{cases}$

f) $1 - y_5 \geq -1 + y_2 + y_4$

midterm 1

4) I: set of friends $\{1,2\}$ P_i: profit per friend i

x_i: fraction for friend i

$$\max \sum_{i \in I} P_i \cdot x_i$$

$$18700\, x_1 + 19600\, x_2 \leq 34000$$

$$850\, x_1 + 650\, x_2 \leq 800$$

$$0 \leq x_i \leq 1 \quad i \in I$$

5) I: set of supplier

C_i: cost of a batch to buy from supplier i

L_i: number of larg valves in each batch of supplier i

M_i: " " medium " " " " " " "

S_i: " " small " " " " " " "

x_i: number of batches we buy from supplier i

a)

$$\min \sum_{i \in I} C_i \cdot x_i$$

$$\sum_{i \in I} L_i \cdot x_i \geq 300 \qquad x_i(L_i + M_i + S_i) \leq 800 \quad i \in I$$

$$\sum_{i \in I} M_i \cdot x_i \geq 700$$

$$\sum_{i \in I} S_i \cdot x_i \geq 500$$

b) $S_3 x_3 \geq \frac{70}{100} \sum_{i \in I} S_i \cdot x_i$

6) I: set of Alloy

J: set of characteristic

a_{ij}: percentage of characteristic j in alloy i

c_i: cost of per kg of alloy i

b_j: percentage of characteristic j in final alloy

x_i: amount of alloy i in final alloy

a) $\min \sum_{i \in I} c_i \cdot x_i$ $\qquad \sum_{i \in I} x_i = 40$

$$\sum_{i \in I} a_{ij} \cdot x_i = b_j \cdot \sum_{i \in I} x_i \qquad j \in J$$

$$x_i \geq 0$$

b) $x_1 + x_2 \leq 1000$ or $x_3 + x_4 \geq 4000$

$$\begin{cases} g(x) = x_1 + x_2 - 1000 \leq 0 \\ f(x) = 4000 - x_3 - x_4 \leq 0 \end{cases} \Rightarrow \begin{cases} g(x) \leq M(1-z) \\ f(x) \leq Mz \end{cases} \qquad z \in \{0, 1\}$$

c) $y_i : \begin{cases} 1 & \text{if we use alloy } i \\ 0 & \text{other wise} \end{cases}$

$$\sum_{i \in I} y_i \leq 3 \qquad y_i \in \{0,1\}$$

$$x_i \leq M y_i \quad i \in I$$

7) I: set of food

J: set of nutrition

c_i: cost of food i per unit

a_{ij}: amount of nutrition j in food i

x_i: amount of food i

$$\min \sum_{i \in I} c_i \cdot x_i$$

$$300 \leq \sum_{i \in I} a_{i2} \cdot x_i \leq 700 \quad , \quad \sum a_{i1} \cdot x_i \leq \frac{60}{100} \sum_{i \in I} a_{i2} \cdot x_i$$

$$\sum_{i \in I} a_{i3} \cdot x_i \geq 50 \quad , \quad \sum_{i \in I} a_{i4} \cdot x_i \geq 10$$

$$x_1 = 3 \quad , \quad x_5 + x_6 \leq 2 \quad , \quad x_i \geq 0, \text{ integer } i \in I$$

8) I: set of parts

J: set of machines

t_{ij}: time required part i in machine j

p_i: profit of part i

x_i: amount of part i that produced

$$\max \sum_{i \in I} p_i \, x_i$$

$$\sum_{i \in I} t_{ij} \cdot x_i \leq 50 \quad j \in J \quad , \quad x_i \geq 0, \text{ integer}$$

b) $y_i : \begin{cases} 1 & \text{if we produce part } i \\ 0 \end{cases}$

$$\sum_{i \in I} y_i \leq 3$$

c) if $y_1 = 1$ or $y_2 = 1 \Rightarrow y_4 = 0$

$$\begin{cases} 1 - y_4 \geq y_1 \\ 1 - y_4 \geq y_2 \end{cases}$$

d) if $y_1 = 1$ and $y_2 = 1 \Rightarrow y_4 = 0$

$$1 - y_4 \geq y_1 + y_2 - 1$$

e) if $y_1 = 1 \Rightarrow y_4 = 1$ and $y_5 = 1$

$$\begin{cases} y_5 \geq y_1 \\ y_4 \geq y_1 \end{cases}$$

f) if $y_1 = 1 \Rightarrow y_4 = 1$ or $y_5 = 1$

$y_4 + y_5 \cancel{\leq} \geq y_1$

$y_4 \geq y_1$

$y_5 \geq y_1$

if $x_6 = 1$ or $x_7 = 1 \xrightarrow{\text{the}} x_3 = 1$

$$x_3 \geq x_6$$

$$x_3 \geq x_7$$

$$2x_3 \geq x_6 + x_7$$

~~if $x_3 = 1 \rightarrow x_6$ 2) x_7 2)~~

~~$x_3 \leq x_6$~~

~~$x_3 \leq x_7$~~

~~$2x_3 \leq x_6 + x_7$~~

1) I: set of vegetables

w_i : amount of veg i in an acre

t_i : time of working for veg i in an acre

P_i : sale price of veg i per kg

x_i : amount of acre of veg i

a) $\max \sum_{i \in I} w_i \cdot P_i \cdot x_i$

$\sum_{i \in I} x_i \leq 60$ $\qquad$ $\sum_{i \in I} t_i x_i \leq 40$

$w_2 \cdot x_2 \geq 1000$ $\qquad$ $x_i \geq 0$

b)

$0 \leq | w_1 x_1 - w_2 x_2 | \leq 2000$