Calculating Curvature & Getting Integrand for GHCV Clebsh Up

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Getting the Clebsch Map in terms of r & t.

cup - Clebsch Map from Circle to Surface; calculated by Dr. Betten in another Maple worksheet. We divide everything by the 4th equation to move everything to affine space. P_initial_check should be the same as P below.

$$cup := \left[-16\,y0^3 + \left(4\,y1^2 + 4\,y2^2\right) y0,\ -\left(16\,y0^2 - 4\,y1^2 + y2^2\right) y1,\ \left(4\,y0^2 + 4\,y1^2 - y2^2\right) y2,\ 10\,y0\,y1\,y2\right]:$$

$$cupx := \frac{cup[1]}{cup[4]}:$$

$$cupy := \frac{cup[2]}{cup[4]}:$$

$$cupz := \frac{cup[3]}{cup[4]}:$$

$$general\_cupaffine := [cupx, cupy, cupz]:$$

$$P\_initial\_check := subs(y0 = r\cdot\cos(t), y1 = r\cdot\sin(t), y2 = 1, general\_cupaffine)$$

$$P\_initial\_check := \left[\frac{-16\,r^3\cos(t)^3 + \left(4\,r^2\sin(t)^2 + 4\right) r\cos(t)}{10\,r^2\cos(t)\sin(t)},\ -\frac{16\,r^2\cos(t)^2 - 4\,r^2\sin(t)^2 + 1}{10\,r\cos(t)},\ \frac{4\,r^2\cos(t)^2 + 4\,r^2\sin(t)^2 - 1}{10\,r^2\cos(t)\sin(t)}\right] \tag{1}$$

Getting the integrand, or the curvature equation based on the Gaussian definition.

Let P(r,t) the parametrized equation of a circle projected onto the GHCV surface through a clebsh map with original equation as $\frac{5}{2}\cdot(x\cdot y\cdot z) - \left(x^2 + y^2 + z^2\right) + 1$. Let r be the circle's radius.

$$\frac{5}{2}\,x\,y\,z - x^2 - y^2 - z^2 + 1 \tag{2}$$

$$restart;$$

$$P(r,t) := simplify\left(\left[\frac{-16\cdot(r\cdot\cos(t))^3 + \left(4\cdot(r\cdot\sin(t))^2 + 4\right)\cdot(r\cdot\cos(t))}{10\cdot(r\cdot\cos(t))\cdot(r\cdot\sin(t))},\ -\frac{16\cdot(r\cdot\cos(t))^2 - 4\cdot(r\cdot\sin(t))^2 + 1}{10\cdot(r\cdot\cos(t))},\ \frac{4\cdot(r\cdot\cos(t))^2 + 4\cdot(r\cdot\sin(t))^2 - 1}{10\cdot(r\cdot\cos(t))\cdot(r\cdot\sin(t))}\right]\right):$$

Let P_r and P_t denote the partial derivatives of P with respect to r and t.

$$P\_r := simplify(diff(P(r,t), r))$$

$$P\_r := \left[-\frac{2\left(5r^2\cos(t)^2 - r^2 + 1\right)}{5r^2\sin(t)}, \frac{-20r^2\cos(t)^2 + 4r^2 + 1}{10\cos(t)\,r^2}, \frac{1}{5r^3\cos(t)\sin(t)} \right] \tag{3}$$

$P\_t := simplify(diff(P(r,t),t))$

$$P\_t := \left[\frac{2\cos(t)\left(5r^2\cos(t)^2 - 9r^2 + 1\right)}{5r\left(\cos(t)^2 - 1\right)}, \frac{\sin(t)\left(20r^2\cos(t)^2 + 4r^2 - 1\right)}{10r\cos(t)^2}, -\frac{\left(2\cos(t)^2 - 1\right)\left(4r^2 - 1\right)}{10r^2\cos(t)^2\sin(t)^2} \right] \tag{4}$$

Now lets' get the normal vector to the surface.

We firstly need to vectorize the equations as we had it as a list of equations before, then we find the cross product. After we find the cross product of the 2 vectors, we can then find the normalized vector by taking the cross product and dividing it by the magnitude. Take this normalized vector and then do the partial derivative in terms of r and t.

$with(Student\text{:-}MultivariateCalculus):$
$with(VectorCalculus):$
$P\_rVec := \langle P\_r \rangle :$
$P\_tVec := \langle P\_t \rangle :$
$P\_Cross := simplify(CrossProduct(P\_rVec, P\_tVec)):$
$P\_Magnitude := Norm(P\_Cross):$

$$P\_Normalized := (r,t) \mapsto \left[seq\left(simplify\left(\frac{P\_Cross[i]}{P\_Magnitude} \right), i = 1..3 \right) \right];$$

$P\_N\_r := \langle simplify(diff(P\_Normalized(r,t), r)) \rangle :$
$P\_N\_t := \langle simplify(diff(P\_Normalized(r,t), t)) \rangle :$

$$P\_Normalized := (r,t) \mapsto \left[seq\left(simplify\left(P\_Cross_i \, P\_Magnitude^{-1} \right), i = 1..3 \right) \right] \tag{5}$$

Now let's get the first and second fundamental forms of the surface. This is for the calculation of Gaussian curvature. Note, P_N_t and P_N_r are the partial (with respect to t & r) of P_N & are already vector datatypes here already.

$$first\_fundamental := \begin{bmatrix} \langle P\_r \rangle . \langle P\_r \rangle & \langle P\_r \rangle . \langle P\_t \rangle \\ \langle P\_t \rangle . \langle P\_r \rangle & \langle P\_t \rangle . \langle P\_t \rangle \end{bmatrix}:$$

$$second\_fundamental := -\begin{bmatrix} \langle P\_r \rangle \cdot P\_N\_r & \langle P\_r \rangle \cdot P\_N\_t \\ \langle P\_t \rangle \cdot P\_N\_r & \langle P\_t \rangle \cdot P\_N\_t \end{bmatrix}:$$

and the Gaussian Curvature:

$$K\_Gauss := simplify\left(\frac{|second\_fundamental|}{|first\_fundamental|} \right);$$

$$K\_Gauss := -\Bigg(\Bigg(\cos(t)^{12} r^{12} + \left(-\frac{3}{10} r^{10} - \frac{9}{5} r^{12}\right)\cos(t)^{10} + \frac{129\left(r^4 + \frac{1}{86} r^2 + \frac{67}{688}\right) r^8 \cos(t)^8}{100} - \frac{59\left(r^6 - \frac{285}{472} r^4 + \frac{183}{944} r^2 + \frac{231}{7552}\right) r^6 \cos(t)^6}{125} + \frac{117\left(r^8 - \frac{227}{156} r^6 + \frac{289}{1248} r^4 + \frac{17}{312} r^2 + \frac{115}{19968}\right) r^4 \cos(t)^4}{1250} - \frac{6\left(r^6 - \frac{29}{16} r^4 - \frac{25}{64} r^2 + \frac{5}{64}\right) r^2 \left(r + \frac{1}{2}\right)^2 \left(r - \frac{1}{2}\right)^2 \cos(t)^2}{625} + \frac{(r+1)^2 \left(r + \frac{1}{2}\right)^4 (r-1)^2 \left(r - \frac{1}{2}\right)^4}{2500}\Bigg) \sin(t)^4 r^4 \cos(t)^4\Bigg) \Bigg/ \Bigg(25\Bigg(\cos(t)^{12} r^{10} + \left(-\frac{3}{10} r^8 - \frac{9}{5} r^{10}\right)\cos(t)^{10} + \left(\frac{4}{25} r^8 + \frac{17}{400} r^6 + \frac{26}{25} r^{10}\right)\cos(t)^8 + \left(-\frac{9}{1000} r^4 + \frac{29}{125} r^8 + \frac{1}{80} r^6 - \frac{34}{125} r^{10}\right)\cos(t)^6 + \frac{21 r^2\left(r^8 - \frac{89}{21} r^6 - \frac{71}{112} r^4 + \frac{5}{112} r^2 + \frac{11}{672}\right)\cos(t)^4}{625} - \frac{\left(r + \frac{1}{2}\right)^2 \left(r^6 - 16 r^4 - 2 r^2 + \frac{15}{16}\right) \left(r - \frac{1}{2}\right)^2 \cos(t)^2}{625} - \frac{\left(r + \frac{1}{2}\right)^4 \left(r - \frac{1}{2}\right)^4}{625}\Bigg)^2\Bigg) \tag{6}$$

Lets integrate this value with respect to r & t (Symbolics)

$integrated\_P\_t := simplify(int(simplify(K\_Gauss), t)) :$
$integrated\_P\_t\_definite := simplify(subs(t = 2\,\mathrm{pi}, integrated\_P\_t));$

$$integrated\_P\_t\_definite := -\left(5\,r^2\left(\left(\frac{\left(r+\frac{1}{2}\right)^4\left(r-\frac{1}{2}\right)^4\tan(2\,\pi)^{12}}{1163283302187008}\right.\right.\right.$$

$$+\frac{\left(r+\frac{1}{2}\right)^2\left(r-\frac{1}{2}\right)^2\left(r^6-10\,r^4-5\,r^2+\frac{21}{16}\right)\tan(2\,\pi)^{10}}{1163283302187008}$$

$$+\left(\frac{43}{2326566604374016}\,r^8+\frac{229}{9306266417496064}\,r^6+\frac{75}{9306266417496064}\,r^4\right.$$

$$\left.-\frac{17}{4653133208748032}\,r^2-\frac{1}{72705206386688}\,r^{10}+\frac{45}{148900262679937024}\right)\tan(2\,\pi)^8$$

$$+\frac{3\left(r^8+\frac{3}{16}\,r^6+\frac{411}{512}\,r^4-\frac{211}{1024}\,r^2+\frac{85}{6144}\right)\left(r^2+\frac{1}{2}\right)\tan(2\,\pi)^6}{36352603193344}+\Bigg($$

$$-\frac{1}{4544075399168}\,r^{10}-\frac{151}{1163283302187008}\,r^8+\frac{151}{2326566604374016}\,r^6$$

$$+\frac{105}{4653133208748032}\,r^4-\frac{143}{18612532834992128}\,r^2+\frac{165}{297800525359874048}\Bigg)$$

$$\tan(2\pi)^4 + \frac{\left(r^6 - \frac{5}{32} r^4 - \frac{35}{256} r^2 + \frac{81}{4096}\right) \left(r + \frac{1}{2}\right)^2 \left(r - \frac{1}{2}\right)^2 \tan(2\pi)^2}{4544075399168}$$

$$+ \frac{\left(r + \frac{1}{2}\right)^4 \left(r - \frac{1}{2}\right)^4}{72705206386688}\Bigg) \Bigg($$

$$\_R = RootOf\big((256 r^8 - 256 r^6 + 96 r^4 - 16 r^2 + 1) \_Z^{12} + (256 r^{10} - 2688 r^8 + 16 r^6 + 816 r^4 - 248 r^2 + 21) \_Z^{10} + (-409$$

$$\Bigg(581641651093504 \Bigg((\_R^8 - 4 \_R^6 - 16 \_R^4 + 64 \_R^2) r^{38} + \Bigg(\_R^{10}$$

$$+ \frac{12657737}{270848} \_R^8 - \frac{13688919}{67712} \_R^6 - \frac{7805289}{16928} \_R^4 + \frac{10393847}{4232} \_R^2 + 4\Bigg) r^{36}$$

$$+ \Bigg(\frac{7917897}{270848} \_R^{10} + \frac{4868111677}{8667136} \_R^8 - \frac{605266431}{270848} \_R^6 - \frac{1725432513}{541696} \_R^4$$

$$+ \frac{7045056119}{270848} \_R^2 + \frac{10224567}{67712} \Bigg) r^{34} + \Bigg(\frac{2241200429}{8667136} \_R^{10} + \frac{36661604761}{69337088} \_R^8$$

$$- \frac{10835616379}{17334272} \_R^6 + \frac{178600803157}{8667136} \_R^4 + \frac{3820045673}{541696} \_R^2 + \frac{829570331}{541696} \Bigg) r^{32}$$

$$+ \Bigg(-\frac{8866199331}{69337088} \_R^{10} + \frac{2975758780715}{2218786816} \_R^8 + \frac{86978809021}{69337088} \_R^6$$

$$+ \frac{321595940237}{34668544} \_R^4 - \frac{554733965645}{69337088} \_R^2 - \frac{8795760081}{17334272} \Bigg) r^{30} + \Bigg($$

$$-\frac{283484472213}{2218786816} \_R^{10} - \frac{449173250447}{1109393408} \_R^8 - \frac{52899736017}{17334272} \_R^6$$

$$- \frac{17231762308659}{2218786816}\,\_R^{4} - \frac{371073569207}{277348352}\,\_R^{2} - \frac{88095522387}{138674176} \Big) r^{28}$$

$$+ \Big(\frac{246856424835}{4437573632}\,\_R^{10} - \frac{20535170537195}{35500589056}\,\_R^{8} - \frac{9075663510225}{17750294528}\,\_R^{6}$$

$$- \frac{210280148970075}{35500589056}\,\_R^{4} - \frac{2516303187295}{4437573632}\,\_R^{2} + \frac{288463410795}{1109393408} \Big) r^{26}$$

$$+ \Big(\frac{164207259865}{4437573632}\,\_R^{10} + \frac{13683168913307}{71001178112}\,\_R^{8} + \frac{22457981810053}{17750294528}\,\_R^{6}$$

$$+ \frac{81669719746483}{71001178112}\,\_R^{4} + \frac{4740594916339}{35500589056}\,\_R^{2} + \frac{15852110645}{2218786816} \Big) r^{24} + \Big($$

$$- \frac{644356988799}{35500589056}\,\_R^{10} + \frac{1258020058669}{142002356224}\,\_R^{8} + \frac{70948317832935}{284004712448}\,\_R^{6}$$

$$+ \frac{511106564016975}{284004712448}\,\_R^{4} + \frac{133773228720347}{284004712448}\,\_R^{2} + \frac{49206059793}{4437573632} \Big) r^{22} + \Big($$

$$- \frac{2502223417079}{568009424896}\,\_R^{10} - \frac{9627870844891}{1136018849792}\,\_R^{8} - \frac{304689702857943}{1136018849792}\,\_R^{6}$$

$$- \frac{334923059414157}{1136018849792}\,\_R^{4} - \frac{22792861713947}{568009424896}\,\_R^{2} - \frac{77876894621}{35500589056} \Big) r^{20}$$

$$+ \Big(\frac{2013553392829}{568009424896}\,\_R^{10} + \frac{838436917979}{71001178112}\,\_R^{8} - \frac{193887488051001}{4544075399168}\,\_R^{6}$$

$$- \frac{1083811710026889}{4544075399168}\,\_R^{4} - \frac{322517153823571}{4544075399168}\,\_R^{2} - \frac{1312892609651}{142002356224} \Big) r^{18} + \Big($$

$$- \frac{79479804399}{284004712448}\,\_R^{10} - \frac{49734247892555}{9088150798336}\,\_R^{8} + \frac{707788132457601}{18176301596672}\,\_R^{6}$$

$$+ \frac{287738699382501}{4544075399168}\,\_R^{4} + \frac{70223258081695}{9088150798336}\,\_R^{2} + \frac{1270448880399}{284004712448} \Big) r^{16} + \Big($$

$$- \frac{3549575647289}{18176301596672}\,\_R^{10} + \frac{11258235558629}{145410412773376}\,\_R^{8} - \frac{37956636032233}{18176301596672}\,\_R^{6}$$

$$+ \frac{532688610540527}{72705206386688}\,\_R^{4} + \frac{427641884089513}{72705206386688}\,\_R^{2} - \frac{781189105811}{1136018849792}\Bigg)\,r^{14}$$

$$+ \Bigg(\frac{146563069815}{9088150798336} + \frac{6694481739045}{145410412773376}\,\_R^{10} + \frac{94444884643801}{290820825546752}\,\_R^{8}$$

$$- \frac{55637931289603}{36352603193344}\,\_R^{6} - \frac{1131748985085371}{290820825546752}\,\_R^{4} - \frac{5956334413229}{4544075399168}\,\_R^{2}\Bigg)\,r^{12}$$

$$+ \Bigg(\frac{223706535591}{72705206386688} - \frac{34727403831}{72705206386688}\,\_R^{10} - \frac{64971947671933}{2326566604374016}\,\_R^{8}$$

$$+ \frac{305502963211257}{1163283302187008}\,\_R^{6} + \frac{593272895004921}{2326566604374016}\,\_R^{4} - \frac{74848130574461}{1163283302187008}\,\_R^{2}\Bigg)\,r^{10}$$

$$+ \Bigg(-\frac{353596530687}{145410412773376} - \frac{429215530857}{581641651093504}\,\_R^{10} - \frac{41467093078711}{4653133208748032}\,\_R^{8}$$

$$- \frac{12892925621925}{2326566604374016}\,\_R^{6} + \frac{236410074876795}{4653133208748032}\,\_R^{4} + \frac{25135332924007}{581641651093504}\,\_R^{2}\Bigg)\,r^{8}$$

$$+ \Bigg(\frac{411868290829}{290820825546752} + \frac{10339729308601}{9306266417496064}\,\_R^{8} - \frac{1129485934997}{404620279021568}\,\_R^{6}$$

$$- \frac{39687753120317}{4653133208748032}\,\_R^{4} - \frac{14178664384393}{4653133208748032}\,\_R^{2} + \frac{59926289731}{4653133208748032}\,\_R^{10}\Bigg)\,r^{6}$$

$$+ \frac{1}{37225065669984256}\Bigg(596959167567\,(\_R^{2}+1)\,\Bigg(\_R^{8} + \frac{576624739669}{85279881081}\,\_R^{6}$$

$$+ \frac{15213205734245}{596959167567}\,\_R^{4} - \frac{675189967943}{596959167567}\,\_R^{2} - \frac{3818616291024}{198986389189}\Bigg)\,r^{4}\Bigg)$$

$$- \frac{1}{74450131339968512}\Bigg(138741730369\,\Bigg(\_R^{6} + \frac{99015746642}{8161278257}\,\_R^{4}$$

$$+ \frac{266988951121}{138741730369}\,\_R^{2} - \frac{2177989506224}{138741730369}\Bigg)\,(\_R^{2}+1)^{2}\,r^{2}\Bigg)$$

$$+ \frac{9743984625\,(\_R-1)\,(\_R+1)\,(\_R^{2}+16)\,(\_R^{2}+1)^{3}}{148900262679937024}\Bigg)\,\ln(\tan(2\,\pi) - \_R)\Bigg) \Bigg/$$

$$\left(5\,\_R\left(\left(\_R+2\right)^{3}\left(\_R^{2}-\frac{4}{5}\right)\left(\_R-2\right)^{3}r^{10}+\left(-\frac{168}{5}+\frac{6}{5}\,\_R^{10}-\frac{21}{2}\,\_R^{8}\right.\right.\right.$$

$$\left.+\frac{86}{5}\,\_R^{6}+\frac{198}{5}\,\_R^{4}-\frac{302}{5}\,\_R^{2}\right)r^{8}$$

$$-\frac{6\left(\_R^{8}-\frac{101}{96}\,\_R^{6}-\frac{577}{32}\,\_R^{4}-25\,\_R^{2}-\frac{1}{6}\right)\left(\_R^{2}+1\right)r^{6}}{5}$$

$$+\frac{9\left(\_R^{2}+1\right)^{2}\left(\_R^{6}+\frac{61}{12}\,\_R^{4}+\frac{11}{2}\,\_R^{2}+\frac{107}{12}\right)r^{4}}{20}$$

$$\left.\left.\left.-\frac{3\left(\_R^{2}+1\right)^{3}\left(\_R^{4}+\frac{119}{12}\,\_R^{2}+\frac{151}{12}\right)r^{2}}{40}+\frac{3\left(\_R^{2}+\frac{27}{2}\right)\left(\_R^{2}+1\right)^{4}}{640}\right)\right)\right)$$

$$+\left(\left(r+\frac{1}{2}\right)^{4}\left(r^{28}+\frac{8188745}{270848}\,r^{26}+\frac{2499990093}{8667136}\,r^{24}+\frac{10351935461}{69337088}\,r^{22}\right.\right.$$

$$-\frac{188037616085}{2218786816}\,r^{20}-\frac{298189655103}{4437573632}\,r^{18}+\frac{173814313935}{17750294528}\,r^{16}+\frac{389099117373}{35500589056}\,r^{14}$$

$$-\frac{279944465535}{284004712448}\,r^{12}-\frac{768413574115}{1136018849792}\,r^{10}+\frac{273314251415}{4544075399168}\,r^{8}$$

$$+\frac{256186706175}{18176301596672}\,r^{6}-\frac{123215290385}{145410412773376}\,r^{4}-\frac{2643037103}{12644383719424}\,r^{2}$$

$$\left.+\frac{9743984625}{581641651093504}\right)\left(r-\frac{1}{2}\right)^{4}\tan(2\,\pi)^{10}+\left(r+\frac{1}{2}\right)^{2}\left(r^{34}+\frac{7917897}{270848}\,r^{32}\right.$$

$$+\frac{1581590861}{8667136}\,r^{30}-\frac{114443254963}{69337088}\,r^{28}-\frac{3140886626133}{2218786816}\,r^{26}+\frac{1812911470039}{4437573632}\,r^{24}$$

$$+\frac{12530392966947}{17750294528}\,r^{22}+\frac{2374746690441}{35500589056}\,r^{20}-\frac{46743546794409}{284004712448}\,r^{18}$$

$$-\frac{25926526020787}{1136018849792}\,r^{16}+\frac{94687348271295}{4544075399168}\,r^{14}+\frac{17149819806355}{18176301596672}\,r^{12}$$

$$-\frac{160368838799285}{145410412773376}\,r^{10}+\frac{4356473952381}{290820825546752}\,r^{8}+\frac{13182510562433}{581641651093504}\,r^{6}$$

$$\left.-\frac{22878586323}{72705206386688}\,r^{4}-\frac{765312812025}{2326566604374016}\,r^{2}+\frac{48719923125}{2326566604374016}\right)\left(r\right.$$

$$\left.-\frac{1}{2}\right)^{2}\tan(2\,\pi)^{8}+\left(-\frac{181222611668523}{1163283302187008}\,r^{10}-\frac{81846209380117}{2326566604374016}\,r^{8}\right.$$

$$+\frac{1456044471585}{404620279021568}\,r^{6}+\frac{12175330857381}{18612532834992128}\,r^{4}-\frac{517248269135}{4653133208748032}\,r^{2}$$

$$- \frac{6317344721405}{17750294528} r^{26} - \frac{14320565718771}{17750294528} r^{24} - \frac{31373246666043}{284004712448} r^{22}$$

$$+ \frac{275956215845299}{1136018849792} r^{20} + \frac{79179540863905}{4544075399168} r^{18} - \frac{607778611064661}{18176301596672} r^{16}$$

$$+ \frac{3322173520575}{2272037699584} r^{14} + \frac{243511507134419}{145410412773376} r^{12} - 12 r^{38} - \frac{29536949}{67712} r^{36}$$

$$- \frac{1223761215}{270848} r^{34} - \frac{6970603155}{17334272} r^{32} + \frac{189675894589}{69337088} r^{30} + \frac{2628711250785}{2218786816} r^{28}$$

$$+ \frac{341039461875}{7445013133996851 2} \Big) \tan(2\pi)^6 + \Big(- \frac{198899948044791}{2326566604374016} r^{10}$$

$$- \frac{219765863458789}{4653133208748032} r^8 + \frac{10870131732045}{4653133208748032} r^6 + \frac{239322225333}{202310139510784} r^4$$

$$- \frac{6177233928935}{37225065669984256} r^2 - \frac{29651269069}{17334272} r^{30} - \frac{9850766972445}{2218786816} r^{28}$$

$$- \frac{6582010272155}{35500589056} r^{26} - \frac{5083528660929}{71001178112} r^{24} - \frac{67901019884553}{284004712448} r^{22}$$

$$+ \frac{250472336205299}{1136018849792} r^{20} + \frac{8695487012305}{197568495616} r^{18} - \frac{83522947631217}{2272037699584} r^{16}$$

$$+ \frac{19660726437615}{72705206386688} r^{14} + \frac{522838966243493}{290820825546752} r^{12} + 48 r^{38} + \frac{31018379}{16928} r^{36}$$

$$+ \frac{10837682715}{541696} r^{34} + \frac{133268464695}{8667136} r^{32} + \frac{243599615625}{37225065669984256} \Big) \tan(2\pi)^4$$

$$- 64 \left(r + \frac{1}{2}\right)^2 \left(r - \frac{1}{2}\right)^2 \Big(r^{34} + \frac{10478487}{270848} r^{32} + \frac{7256599657}{17334272} r^{30}$$

$$+ \frac{8531745361}{34668544} r^{28} - \frac{386538528891}{4437573632} r^{26} - \frac{1077915230579}{17750294528} r^{24} - \frac{2603528959023}{284004712448} r^{22}$$

$$+ \frac{254847368193}{98784247808} r^{20} + \frac{82213686714819}{18176301596672} r^{18} + \frac{4175876943721}{36352603193344} r^{16}$$

$$- \frac{61541744278665}{145410412773376} r^{14} - \frac{1356989369485}{145410412773376} r^{12} + \frac{67421402387915}{4653133208748032} r^{10}$$

$$+ \frac{18288422906727}{9306266417496064} r^8 - \frac{9765178581359}{18612532834992128} r^6 - \frac{10602181076919}{148900262679937024} r^4$$

$$+ \frac{5894451719115}{297800525359874048} r^2 - \frac{633359000625}{595601050719748096} \Big) \tan(2\pi)^2 - 4 \Big(r^{28}$$

$$+ \frac{10495415}{270848} r^{26} + \frac{912721107}{2166784} r^{24} + \frac{19408089071}{69337088} r^{22} - \frac{19110789755}{554696704} r^{20}$$

$$- \frac{214060449813}{4437573632} r^{18} - \frac{156942875595}{8875147264} r^{16} - \frac{2437370517}{35500589056} r^{14} + \frac{222510383505}{71001178112} r^{12}$$

$$
-\frac{77853191965}{1136018849792}r^{10}-\frac{137739831665}{2272037699584}r^{8}-\frac{48628761915}{4544075399168}r^{6}
$$

$$
-\frac{16755186455}{36352603193344}r^{4}+\frac{58172467139}{72705206386688}r^{2}-\frac{9743984625}{145410412773376}\Big)\Big(r
$$

$$
+\frac{1}{2}\Big)^{4}\left(r-\frac{1}{2}\right)^{4}\Big)\tan(2\,\pi)\Big)\Big)\Bigg/\Bigg(8\Big(r^{32}+\frac{5047215}{135424}r^{30}+\frac{3204739203}{8667136}r^{28}
$$

$$
+\frac{12509264099}{34668544}r^{26}+\frac{85473155445}{2218786816}r^{24}-\frac{8903694471}{96468992}r^{22}-\frac{1491449715143}{35500589056}r^{20}
$$

$$
+\frac{673465543713}{142002356224}r^{18}+\frac{1096706338029}{142002356224}r^{16}+\frac{100313925341}{1136018849792}r^{14}
$$

$$
-\frac{2893107792903}{4544075399168}r^{12}+\frac{1945936262463}{36352603193344}r^{10}+\frac{1501635123059}{145410412773376}r^{8}
$$

$$
-\frac{121750112265}{72705206386688}r^{6}+\frac{365059595913}{2326566604374016}r^{4}-\frac{101311927699}{4653133208748032}r^{2}
$$

$$
+\frac{6250781925}{4653133208748032}\Big)\Big(\left(r+\frac{1}{2}\right)^{4}\left(r-\frac{1}{2}\right)^{4}\tan(2\,\pi)^{12}+\left(r+\frac{1}{2}\right)^{2}\left(r-\frac{1}{2}\right)^{2}\Big(r^{6}
$$

$$
-10\,r^{4}-5\,r^{2}+\frac{21}{16}\Big)\tan(2\,\pi)^{10}+\Big(-16\,r^{10}+\frac{43}{2}r^{8}+\frac{229}{8}r^{6}+\frac{75}{8}r^{4}-\frac{17}{4}r^{2}
$$

$$
+\frac{45}{128}\Big)\tan(2\,\pi)^{8}+\Big(-\frac{137}{16}r^{2}+96\,r^{10}+66\,r^{8}+\frac{1377}{16}r^{6}+\frac{75}{4}r^{4}
$$

$$
+\frac{85}{128}\Big)\tan(2\,\pi)^{6}+\Big(\frac{105}{4}r^{4}-\frac{143}{16}r^{2}-256\,r^{10}-151\,r^{8}+\frac{151}{2}r^{6}
$$

$$
+\frac{165}{256}\Big)\tan(2\,\pi)^{4}+\Big(r^{6}+\frac{321}{16}r^{4}-\frac{151}{32}r^{2}+256\,r^{10}-168\,r^{8}+\frac{81}{256}\Big)\tan(2\,\pi)^{2}
$$

$$
+16\left(r+\frac{1}{2}\right)^{4}\left(r-\frac{1}{2}\right)^{4}\Big)\Big)
$$

So, at this point I have symbolically integrated it in terms of t. Which is all fine and good and the next part would be to integrate it in terms of r. I tried this but I think Maple either could not calculate it or, because the P t definite is such a big intergrand, it's integration package is slower. I would eventually like to evatuate this in terms of r, maybe with Mathematica, but for now numerical integration done by our Matlab program will be good enough.