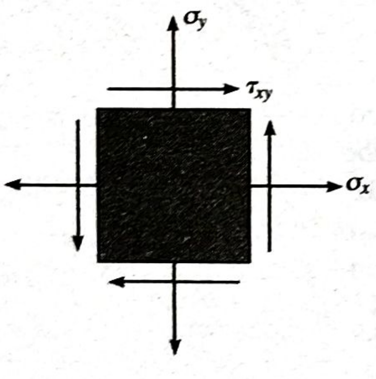
IMPORTANT POINTS

- A *force* does work when it moves through a *displacement*. If the force is increased gradually in magnitude from zero to F, the work is $U = (F/2)\Delta$, whereas if the force is constant when the displacement occurs then $U = F\Delta$.
- A *couple moment* does work when it moves through a *rotation*.
- *Strain energy* is caused by the internal work of the normal and shear stresses. It is always a *positive* quantity.
- The strain energy can be related to the resultant internal loadings N, V, M, and T.
- As the beam becomes longer, the strain energy due to bending becomes much larger than the strain energy due to shear. For this reason, the *shear strain energy* in beams can generally be *neglected*.

PROBLEMS

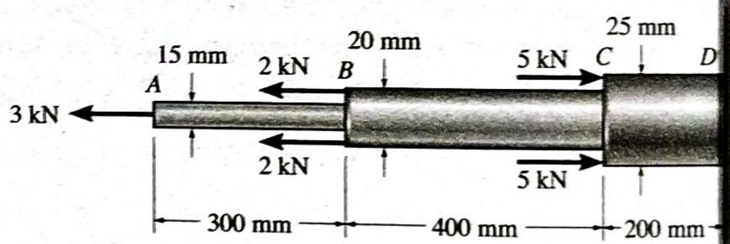
14-1. A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



Prob. 14–1

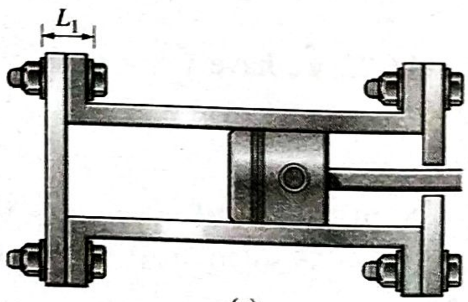
14-2. The strain-energy density must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain-energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

14-3. Determine the strain energy in the rod assembly. Portion AB is steel, BC is brass, and CD is aluminum. $E_{st} = 200$ GPa, $E_{br} = 101$ GPa, $E_{al} = 73.1$ GPa.



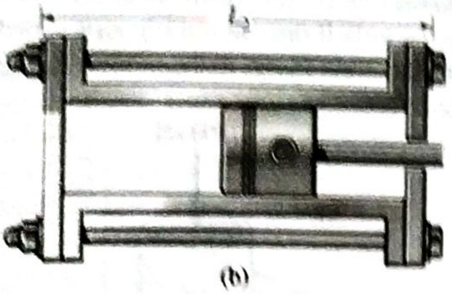
Prob. 14–3

***14-4.** Using bolts of the same material and cross-sectional area, two possible attachments for a cylinder head are shown. Compare the strain energy developed in each case, and then explain which design is better for resisting an axial shock or impact load.



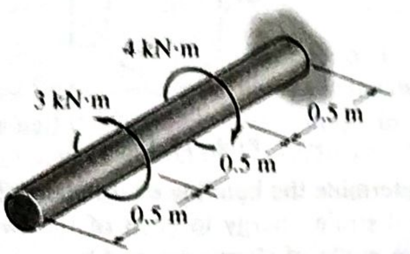
(a)

Prob. 14–4



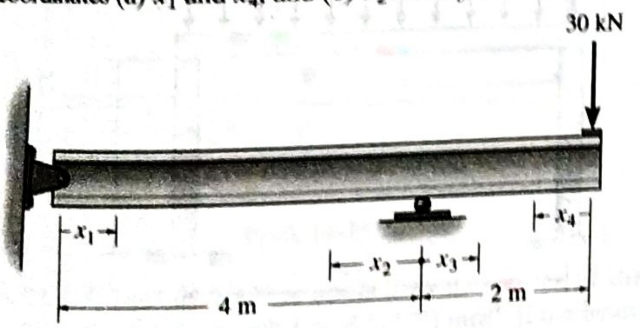
Prob. 14–4 (cont.)

14–5. Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 30 mm. $G = 75$ GPa.



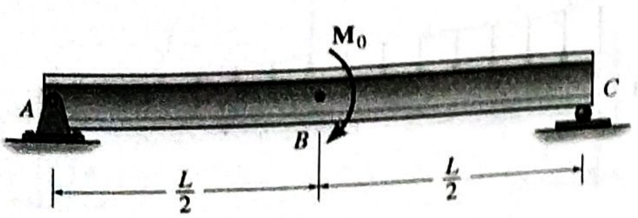
Prob. 14–5

14–6. Determine the bending strain energy in the A-36 structural steel W250 × 18 beam. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3. $E = 200$ GPa.



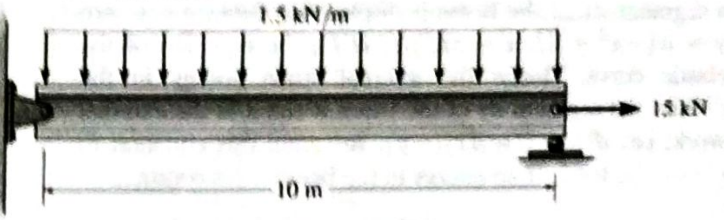
Prob. 14–6

14–7. Determine the bending strain energy in the beam due to the loading shown. EI is constant.



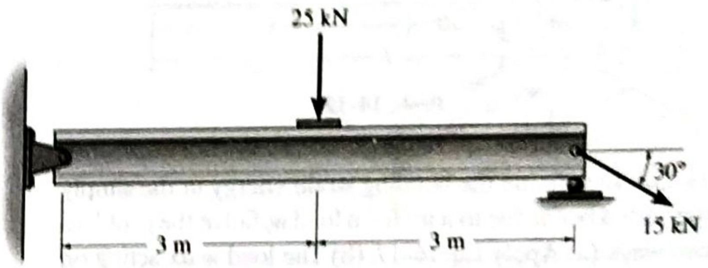
Prob. 14–7

***14–8.** Determine the total axial and bending strain energy in the A-36 steel beam. $A = 2300\ \text{mm}^2$, $I = 9.5(10^6)\ \text{mm}^4$, $E = 200$ GPa.



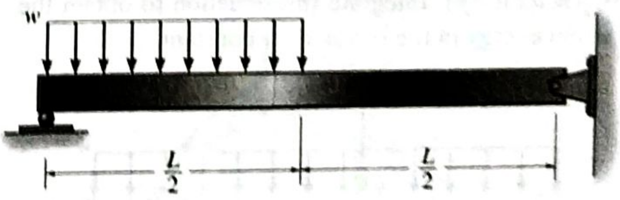
Prob. 14–8

14–9. Determine the total axial and bending strain energy in the A-36 structural steel W200 × 86 beam. $E = 200$ GPa.



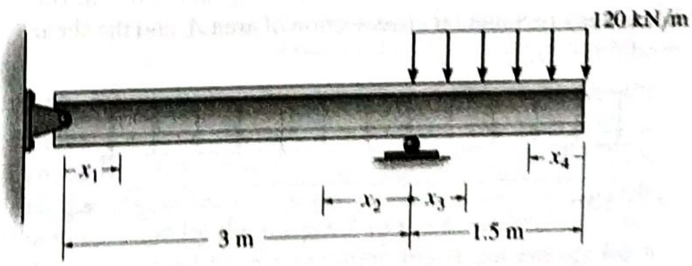
Prob. 14–9

14–10. The simply supported beam is subjected to the loading shown. Determine the bending strain energy in the beam.



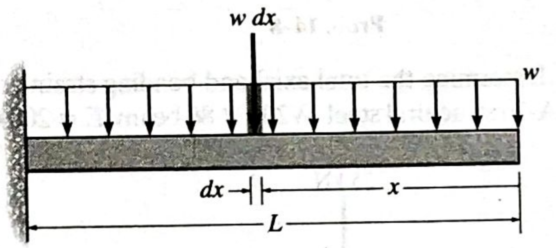
Prob. 14–10

14–11. Determine the bending strain energy in the A-36 steel beam due to the loading shown. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3. $I = 21(10^6)\ \text{mm}^4$. $E = 200$ GPa.



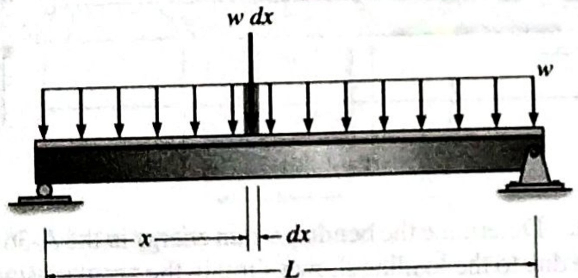
Prob. 14–11

***14-12.** Determine the bending strain energy in the cantilevered beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



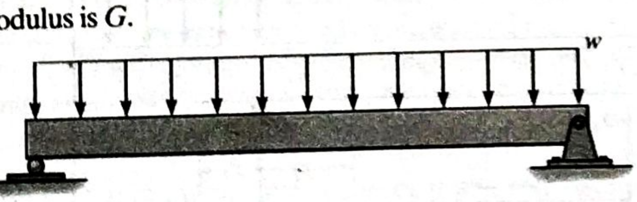
Prob. 14–12

14-13. Determine the bending strain energy in the simply supported beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on the segment dx of the beam is displaced a distance y, where $y = \frac{w}{24EI}(-x^4 + 4L^3x - 3L^4)$ the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



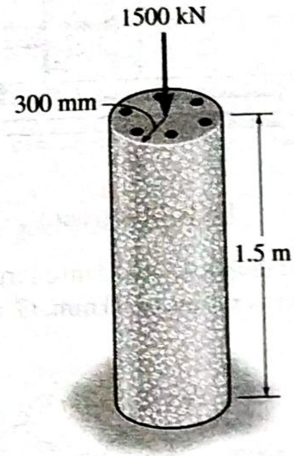
Prob. 14–13

14-14. Determine the shear strain energy in the beam. The beam has a rectangular cross section of area A, and the shear modulus is G.



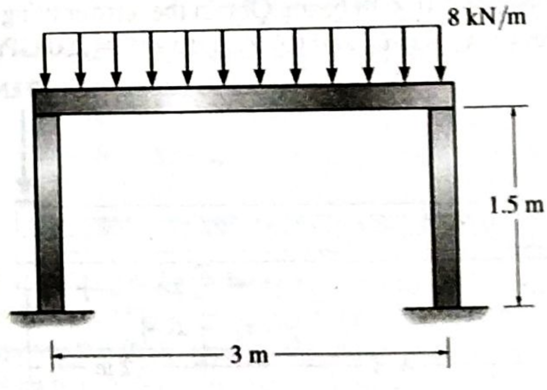
Prob. 14–14

14-15. The concrete column contains six 25-mm-diameter steel reinforcing rods. If the column supports a load of 1500 kN, determine the strain energy in the column. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



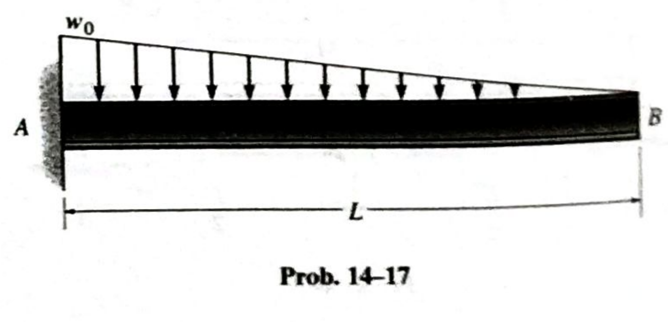
Prob. 14–15

***14-16.** Determine the bending strain energy in the beam and the axial strain energy in each of the two posts. All members are made of aluminum and have a square cross section 50 mm by 50 mm. Assume the posts only support an axial load. $E_{al} = 70$ GPa.



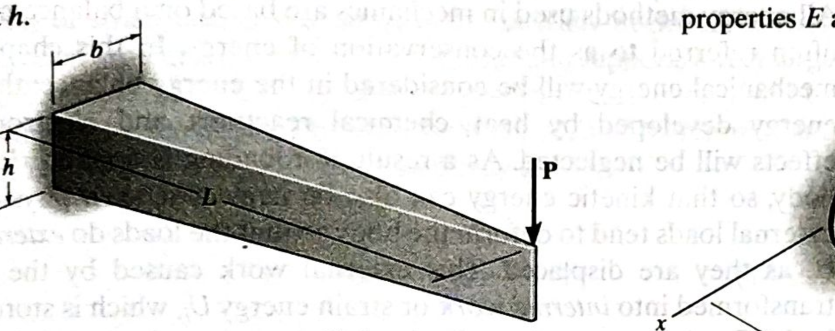
Prob. 14–16

14-17. Determine the bending strain energy in the beam due to the distributed load. EI is constant.



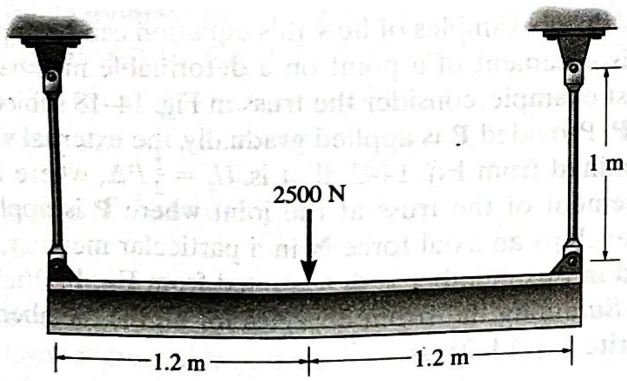
Prob. 14–17

14-18. The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.



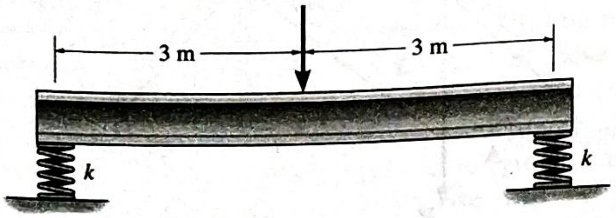
Prob. 14–18

14-19. Determine the total strain energy in the steel assembly. Consider the axial strain energy in the two 12-mm-diameter rods and the bending strain energy in the beam, which has a moment of inertia of $I = 17(10^6)\ \text{mm}^4$ about its neutral axis. $E_{st} = 200$ GPa.



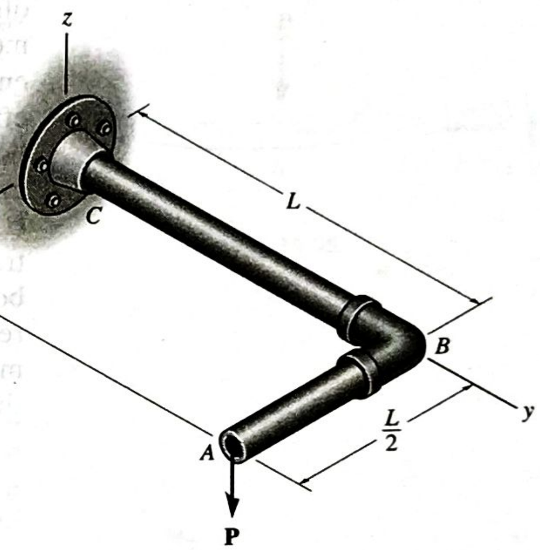
Prob. 14–19

***14-20.** A load of 5 kN is applied to the center of the A-36 steel beam, for which $I = 4.5(10^6)\ \text{mm}^4$. If the beam is supported on two springs, each having a stiffness of $k = 8$ MN/m, determine the strain energy in each of the springs and the bending strain energy in the beam. $E = 200$ GPa.



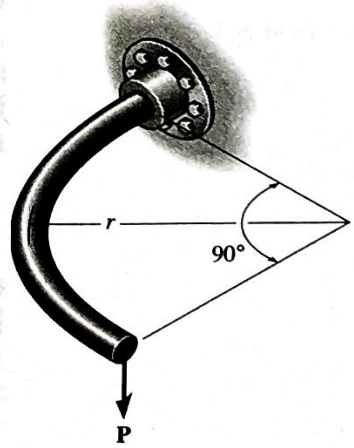
Prob. 14–20

14-21. The pipe lies in the horizontal plane. If it is subjected to a vertical force **P** at its end, determine the strain energy due to bending and torsion. Express the results in terms of the cross-sectional properties I and J, and the material properties E and G.



Prob. 14–21

14-22. Determine the strain energy in the *horizontal* curved bar due to torsion. There is a *vertical* force **P** acting at its end. JG is constant.



Prob. 14–22

14-23. Consider the thin-walled tube of Fig. 5–30. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20. *Hint:* Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.