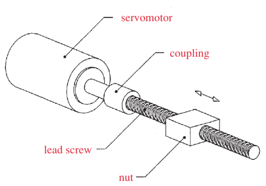
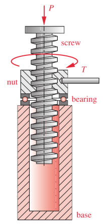
Power or Lead Screws

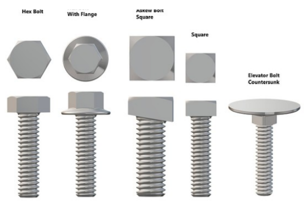
Helical-thread screw is the basis of:
power or lead screws which change angular motion to linear motion to transmit power or to develop large forces (presses, jacks, etc.), and
threaded fasteners, an important element in nonpermanent joints





Power-Screw Jack


Screw Press


Types of Fasteners

https://www.design-engineering.com/what-is-a-lead-screw-1004040732/
https://www.mtixtl.com/EQ-YLJ-SP.aspx
https://forum.digikey.com/t/types-of-threaded-fasteners-screws-and-bolts/7954

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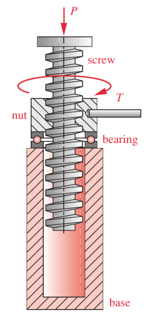
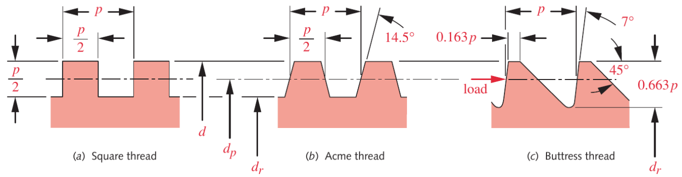
Power or Lead Screws

Power screws, also called lead screws, are used to convert rotation to linear motion.
They are capable of very large mechanical advantages and so can lift or move large loads.





Typical Threads used in Power Screws: Square, Acme, and Buttress Threads

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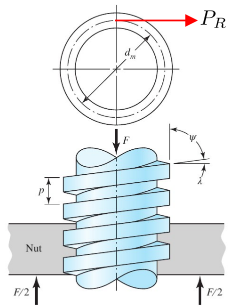
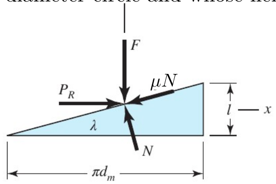
Force and Torque Analysis: Square Threads

A square-threaded power screw with single thread having a mean diameter d_m, a pitch p and a lead angle is loaded by the axial compressive force F. Want to find an expression for the torque required to raise this load, and another expression for the torque required to lower the load



- Imagine that a single thread of the screw is unrolled or developed for exactly a single turn.
- Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean thread diameter circle and whose height is the lead.



P_R - horizontal load
F – axial force
N – normal force
- coefficient of friction between nut and screw

$$\tan\lambda = \frac{l}{\pi d_m} \qquad \mu = 0.15 \pm 0.05$$

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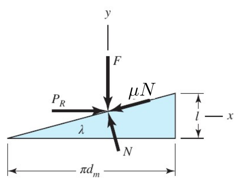
Force and Torque Analysis: Square Threads



Load Raising Case

$$\sum F_x = 0, \text{ or } P_R - \mu N\cos\lambda - N\sin\lambda = 0$$
$$P_R = N(\mu\cos\lambda + \sin\lambda)$$
$$\sum F_y = 0, \text{ or } -F - \mu N\sin\lambda + N\cos\lambda = 0$$
$$N = \frac{F}{\cos\lambda - \mu\sin\lambda} \qquad P_R = F\left(\frac{\mu\cos\lambda + \sin\lambda}{\cos\lambda - \mu\sin\lambda}\right)$$

$$T_R = P_R\frac{d_m}{2} = \frac{Fd_m}{2}\left(\frac{\mu\cos\lambda + \sin\lambda}{\cos\lambda - \mu\sin\lambda}\right) = \frac{Fd_m}{2}\left(\frac{\mu\pi d_m + l}{\pi d_m - \mu l}\right)$$

The required torque depends on the load, coefficient of friction and the geometry of the screw

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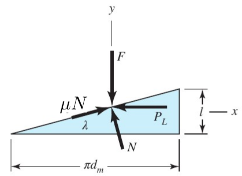
Force and Torque Analysis: Square Threads



Load Lowering Case

$$\sum F_x = 0, \text{ or } -P_L + \mu N\cos\lambda - N\sin\lambda = 0$$

$$P_L = N(\mu\cos\lambda - \sin\lambda)$$

$$\sum F_y = 0, \text{ or } -F + \mu N\sin\lambda + N\cos\lambda = 0$$

$$N = \frac{F}{\cos\lambda + \mu\sin\lambda} \qquad P_L = F\left(\frac{\mu\cos\lambda - \sin\lambda}{\cos\lambda + \mu\sin\lambda}\right)$$

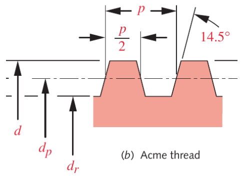
$$T_L = P_L\frac{d_m}{2} = \frac{Fd_m}{2}\left(\frac{\mu\cos\lambda - \sin\lambda}{\cos\lambda + \mu\sin\lambda}\right) = \frac{Fd_m}{2}\left(\frac{\mu\pi d_m - l}{\pi d_m + \mu l}\right)$$

If $\mu < \frac{l}{\pi d_m} = \tan\lambda$ then the load will lower itself without the application of any torque

Condition for self-locking (positive torque required to lower the load)

$$\mu > \frac{l}{\pi d_m} = \tan\lambda$$

Force and Torque Analysis: ACME Threads

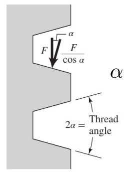


Torque required to raise the load

$$T_R = \frac{Fd_m}{2}\left(\frac{\mu\pi d_m + l\cos\alpha}{\pi d_m\cos\alpha - \mu l}\right)$$

Torque required to lower the load

$$T_L = \frac{Fd_m}{2}\left(\frac{\mu\pi d_m - l\cos\alpha}{\pi d_m\cos\alpha + \mu l}\right)$$



$\alpha = 14.5^o$

Change in the Normal Force due to α

Condition for self-locking

$$\mu > \tan\lambda\cos\alpha$$

- ACME thread is not as efficient as the square thread because of the additional friction
- It is preferred because it is easier to machine

Efficiency of Power Screws

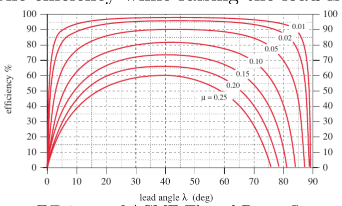
The efficiency of the screw is defined as $e = \frac{W_{out}}{W_{in}} = \frac{FL}{2\pi T}$

Square Thread

The efficiency while raising the load is given by $e = \frac{FL}{2\pi T_R} = \frac{1 - \mu\tan\lambda}{1 + \mu\cot\lambda}$

ACME Thread

The efficiency while raising the load is given by $e = \frac{FL}{2\pi T_R} = \frac{\cos\alpha - \mu\tan\lambda}{\cos\alpha + \mu\cot\lambda}$



Efficiency of ACME Thread Power Screw

Typical lead angle

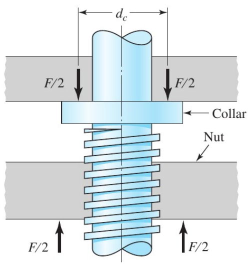
$2^o \leq \lambda \leq 5^o$

Efficiency is quite low

$18\% \leq e \leq 36\%$ $(\mu = 0.15)$

Thrust Collars and Power Requirement

When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component.



- Friction torque due to thrust collar

$$T_c = \mu_c F\frac{d_c}{2}$$

- Total Torque required to raise the load

$$T_{Rt} = T_R + T_c$$

- Total Torque required to lower the load

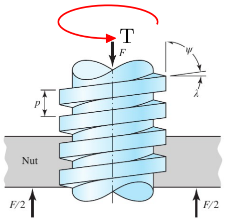
$$T_{Lt} = T_L + T_c$$

- Power required to drive the lead screw at a constant angular velocity

$$P = T_{total}\omega$$

Design of Screw and Nut

- Screws are typically made up of plain carbon steel and nut is made up of bronze
- The body of the screw is subject to a torsion T and axial force F



	Boundary conditions	c	k
1.	P-P	1.00	1.00
2.	P-C	2.05	0.70
3.	C-F	0.25	2.00
4.	C-C	4.00	0.50

P-Pinned, C-Clamped, F-Free

- Maximum nominal shear stress in the screw body

$$\tau = \frac{16T}{\pi d_r^3}$$

- Compressive axial stress in the screw body

$$\sigma = -\frac{4F}{\pi d_r^2}$$

- For a lead screw of length L, the critical buckling load is estimated using the Euler formula (assuming the lead screw is slender)

$$P_{cr} = c\pi^2 \frac{EI}{L^2} \text{ or } P_{cr} = \pi^2 \frac{EI}{(kL)^2}, \; I = \frac{\pi d_r^4}{64}$$

- Critical Speed – refer to catalogues

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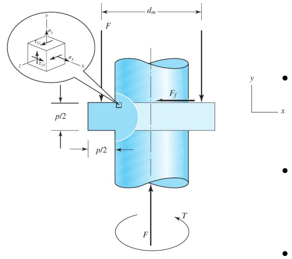
Design of Screw and Nut - Thread Stresses



- Bearing stress due to the force F acting on the surface of the thread

$$\sigma_B = \sigma_{yy} = -\frac{F}{(\pi d_m p/2) n_t} = \frac{2F}{\pi d_m p n_t} \quad n_t - \text{number of engaged threads}$$

- Bending stress at the root of the thread

$$\sigma_{xx} = \frac{M}{Z} = \frac{(Fp/4)}{(\pi d_r n_t)(p/2)^2/6} = \frac{6F}{\pi d_r n_t p}$$

- Shear stress τ_{xz} at the root of the thread

$$\tau_{xz} = \frac{T}{A_r r_r} = \frac{T}{\pi d_r n_t p / 2 r_r} = \frac{4T}{\pi d_r^2 n_t p}$$

- Shear stress τ_{yz} at the root of the thread

$$\tau_{yz} = \frac{16T}{\pi d_r^3}$$

Using these stress components, the equivalent stress (von Mises stress) at the root of the thread can be obtained.

The stresses at the root of the thread of the nut can be estimated similarly

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Design of Screw and Nut - Thread Stresses

- The equations derived earlier assume all engaged threads are equally sharing the load (non conservative assumption)
- A power screw lifting a load is in compression and its thread pitch is shortened by elastic deformation. Its engaging nut is in tension and its thread pitch is lengthened.
- **The engaged threads do not share the load equally**.
- Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load.
- In estimating thread stresses by the equations derived earlier, substituting $0.38F$ for F and setting n_t to 1 gives the largest level of stresses in the thread-nut combination.

Useful Information

Table 8–4 Screw Bearing Pressure p_b

Screw Material	Nut Material	Safe p_b, MPa	Notes
Steel	Bronze	17.2–24.1	Low speed
Steel	Bronze	11.0–17.2	≤50 mm/s
	Cast iron	6.9–17.2	≤40 mm/s
Steel	Bronze	5.5–9.7	100–200 mm/s
	Cast iron	4.1–6.9	100–200 mm/s
Steel	Bronze	1.0–1.7	≥250 mm/s

Table 8–6 Thrust-Collar Friction Coefficients

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

Table 8–5 Coefficients of Friction f for Threaded Pairs

	Nut Material			
Screw Material	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

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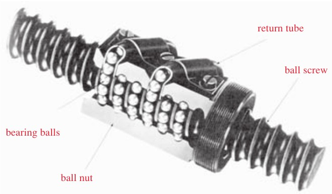
Ball Screws



- The efficiency of lead screws is poor
- Ball Screw is an adaptation of the lead screw wherein the friction between the screw and the nut is minimized by using bearing balls
- Load path: Screw thread to bearing balls to the nut to the driven device
- Efficiency is as high as 90%

Working of a ball screw
https://www.youtube.com/watch?v=kl6qNn9-nkk

Design Considerations
- Axial load exerted by the screw
- Rotational speed (critical speed)
- Supporting end conditions of the screw (critical speed, buckling)
- Length of the screw
- Expected life – this calculation is similar to the rolling bearing calculation

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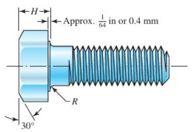
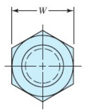
END

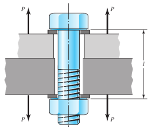
Introduction - Screw Fasteners

The purpose of a fastener is to clamp two or more parts together.

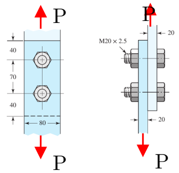




Screw fasteners are classified in different ways
- By their intended use
- By their thread type
- By their head style
- By their strength



Bolted Joint in Tension



Bolted Joint in Shear

Fasteners are available in wide variety of materials – steel, stainless steel, aluminum, brass, bronze and plastics

Manufacturing – Thread: cutting, rolling, Head: forming

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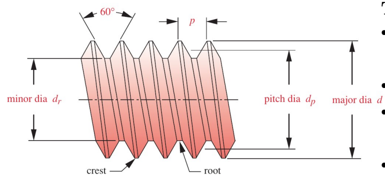
Screws Fasteners



Unified National and ISO Standard Thread Form

Metric thread specification

M8×1.25 – 8mm diameter by 1.25 mm pitch thread in ISO coarse series

$d_p = d - 0.649519p$ Pitch diameter
$d_r = d - 1.226869p$ Root diameter
p is the pitch in mm

Tensile strength area $A_t = \frac{\pi}{4}\left(\frac{d_p + d_r}{2}\right)^2$

Terminology
- **Pitch** p is the distance between adjacent thread forms measured parallel to the thread axis.
- **Major diameter** d is the largest diameter of a screw thread.
- **Minor (or root) diameter** d_r is the smallest diameter of a screw thread.
- **Pitch diameter** d_p is a theoretical diameter between the major and minor diameters.

	Coarse Threads			Fine Threads		
Major Diameter d (mm)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm^2)	Pitch p mm	Minor Diameter d_r (mm)	Tensile Stress Area A_t (mm^2)
3.0	0.50	2.39	5.03			
3.5	0.60	2.76	6.78			
4.0	0.70	3.14	8.78			
5.0	0.80	4.02	14.18			
6.0	1.00	4.77	20.12			
7.0	1.00	5.77	28.86			
8.0	1.25	6.47	36.61	1.00	6.77	39.17
10.0	1.50	8.16	57.99	1.25	8.47	61.20
12.0	1.75	9.85	84.27	1.25	10.47	92.07
14.0	2.00	11.55	115.44	1.50	12.16	124.55
16.0	2.00	13.55	156.67	1.50	14.16	167.25
18.0	2.50	14.93	192.47	1.50	16.16	216.23
20.0	2.50	16.93	244.79	1.50	18.16	271.50
22.0	2.50	18.93	303.40	1.50	20.16	333.06
24.0	3.00	20.32	352.50	2.00	21.55	384.42
27.0	3.00	23.32	459.41	2.00	24.55	495.74
30.0	3.50	25.71	560.59	2.00	27.55	621.20
33.0	3.50	28.71	693.55	2.00	30.55	760.80
36.0	4.00	31.09	816.72	3.00	32.32	864.94
39.0	4.00	34.09	975.75	3.00	35.32	1028.39

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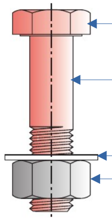
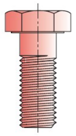
Screw Fasteners - Classification

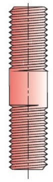
By intended use



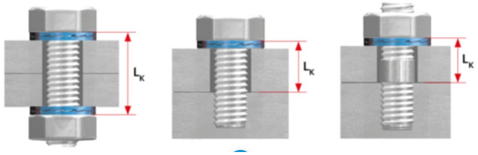
Bolt, washer, nut



Machine Screw



Stud

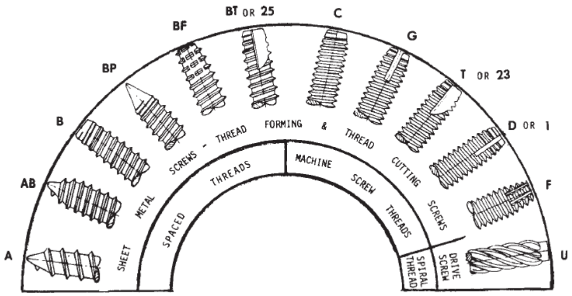


https://www.nord-lock.com/learnings/bolting-tips/2017/optimize-bolted-joint-through-clamped-length/

Screw Fasteners - Classification

By thread use

Tapping screws – Fasteners that own hole or make their own thread are called as tapping screws



Styles of Threads on tapping screws

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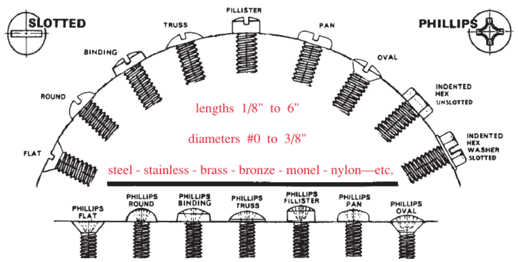
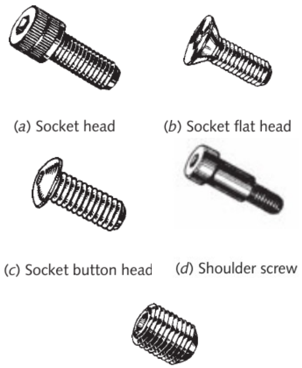
Screw Fasteners - Classification

By Head Type



Heads Used on Small Machine Screws
(small torque requirement)



(*a*) Socket head (*b*) Socket flat head

(*c*) Socket button head (*d*) Shoulder screw

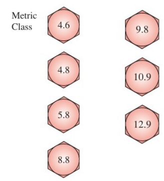
(*e*) Socket set screw

Styles of Threads on tapping screws

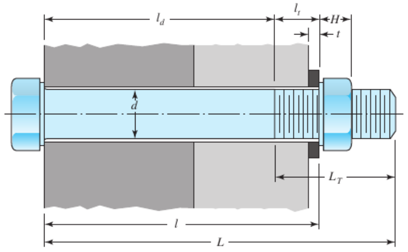
Strength of Steel Bolts -Metric Specifications

Proof Strength: The stress at which the bolt begins to to take a permanent deformation/set. It is slightly lower than the yield strength

Class Number	Size Range Outside Diameter (mm)	Minimum Proof Strength (MPa)	Minimum Yield Strength (MPa)	Minimum Tensile Strength (MPa)	Material
4.6	M5–M36	225	240	400	low or medium carbon
4.8	M1.6–M16	310	340	420	low or medium carbon
5.8	M5–M24	380	420	520	low or medium carbon
8.8	M3–M36	600	660	830	medium carbon, Q&T
9.8	M1.6–M16	650	720	900	medium carbon, Q&T
10.9	M5–M36	830	940	1 040	low-carbon martensite, Q&T
12.9	M1.6–M36	970	1 100	1 220	alloy, quenched & tempered



Metric Bolts - Dimensions

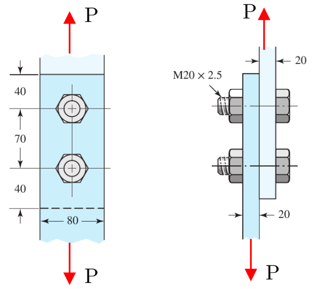


D – bolt diameter (mm)
P – pitch (mm)
H – nut thickness (table)
t – washer thickness (table)
l – thickness of all material squeezed between the face of the bolt and face of the nut.
L – Bolt length. $L > l + H + t$

$$L_T = \begin{cases} 2d + 6 \text{ mm}, & L \le 125 \text{ mm}, d \le 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

l_d – length of unthreaded portion in the grip. $l_d = L - L_T$
l_t – length of threaded portion in the grip. $l_t = l - l_d$

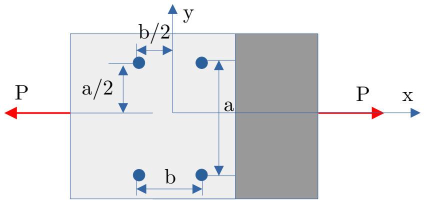
Fasteners in Shear



Typical Failure Modes

- Shear failure of fasteners
- Crushing failure of fasteners
- Crushing failure of plates
- Tensile failure of plates

Fasteners in Shear – Symmetric Load



Consider two plates joined together using fasteners of equal size and subjected to loading as shown

Shear stress in a fastener $\tau = \dfrac{P/n}{A_s}$

n – number of fasteners

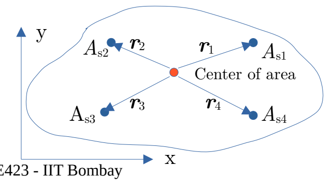
A_s – area of fastener resisting shear

The load distributes equally to all the fasteners as it Acted along the line of symmetry of the fasteners

Center of area

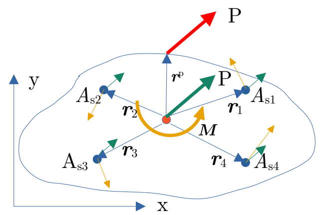
$$x_c = \frac{\sum_{i=1}^{n} A_{si} x_i}{\sum_{i=1}^{n} A_{si}}, \; y_c = \frac{\sum_{i=1}^{n} A_{si} y_i}{\sum_{i=1}^{n} A_{si}}$$

A_{si} – area of shear in the fastener
x_i, y_i – coordinates of the center of the bolt



Fasteners in Shear – Eccentric Load

Line of action of force doe not pass through the center of area



The parallel force $\boldsymbol{P}$ will produce a force in each fastener given by

$$\boldsymbol{F}_i^p = \frac{|\boldsymbol{P}|}{n}\hat{\boldsymbol{P}} \qquad \hat{\boldsymbol{P}} \text{ is a unit vector along } \boldsymbol{P}$$

The moment $\boldsymbol{M}$ will produce a forces $\boldsymbol{F}_i^m, i = 1, \ldots, n$ in the fasteners.

The magnitude of these forces is assumed to be proportional to the distance of the fastener center from the center of area

$$|\boldsymbol{F}_i^m| \propto |\boldsymbol{r}_i| = c|\boldsymbol{r}_i| \qquad c \text{ is an unknown}$$

The direction of these forces is assumed to be perpendicular to the position vector of the fastener center from the center of area

$$\hat{\boldsymbol{F}}_i^m = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{r}}_i$$

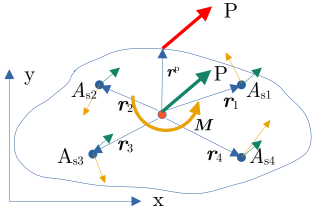
Therefore

$$\boldsymbol{F}_i^m = |\boldsymbol{F}_i^m|\hat{\boldsymbol{F}}_i^m = c|\boldsymbol{r}_i|(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{r}}_i) = c(\hat{\boldsymbol{k}} \times \boldsymbol{r}_i)$$

(c is an unknown)

The eccentric load is replaced by:
A parallel force passing through the center of the area
A moment about about the center of the area

Fasteners in Shear – Eccentric Load



Now $\boldsymbol{r}^p \times \boldsymbol{P} = \sum_{i=1}^{n} \boldsymbol{r}_i \times \boldsymbol{F}_i^m$

$$= c \sum_{i=1}^{n} \boldsymbol{r}_i \times (\hat{\boldsymbol{k}} \times \boldsymbol{r}_i) = c\hat{\boldsymbol{k}} \sum_{i=1}^{n} |\boldsymbol{r}_i|^2$$

Now $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$

Therefore $c = \dfrac{(\boldsymbol{r}^p \times \boldsymbol{P}) \cdot \hat{\boldsymbol{k}}}{\sum_{i=1}^{n} |\boldsymbol{r}_i|^2}$

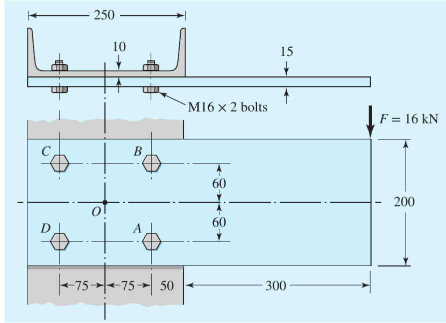
The resultant force acting on the fastener is

$$\boldsymbol{F}_i = \boldsymbol{F}_i^p + \boldsymbol{F}_i^m,\ i = 1, \ldots, n$$

The shear stress in the fastener is

$$\tau_i = \frac{|\boldsymbol{F}_i|}{A_{si}},\ i = 1, \ldots, n$$

Fasteners in Shear – Eccentric Load



All dimensions in mm

Assuming that all the load is carried by the bolts, find the loads acting on each bolt

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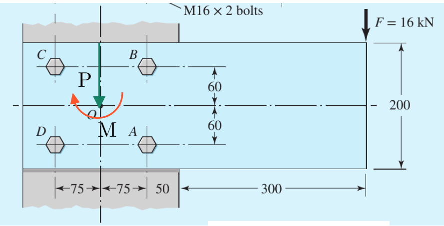
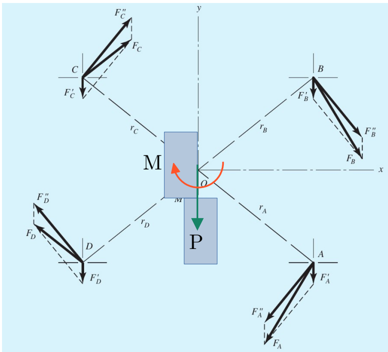
Fasteners in Shear – Eccentric Load



P = 16 kN, M = 6800 Nm



$\boldsymbol{r}^p = 425\hat{\boldsymbol{\imath}} + 100\hat{\boldsymbol{\jmath}}$ $\quad \boldsymbol{r}_C = -75\hat{\boldsymbol{\imath}} + 100\hat{\boldsymbol{\jmath}}$

$\boldsymbol{r}_A = 75\hat{\boldsymbol{\imath}} - 100\hat{\boldsymbol{\jmath}}$ $\quad \boldsymbol{r}_D = -75\hat{\boldsymbol{\imath}} - 100\hat{\boldsymbol{\jmath}}$

$\boldsymbol{r}_B = 75\hat{\boldsymbol{\imath}} + 100\hat{\boldsymbol{\jmath}}$

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```
(%i20) d:16*mm
(%i21) rp:vec(425*mm,100*mm,0)
(%i22) P:vec(0,(-16)*kN,0)
(%i23) ra:vec(75*mm,(-100)*mm,0)
(%i24) rb:vec(75*mm,100*mm,0)
(%i25) rc:vec((-75)*mm,100*mm,0)
(%i26) rd:vec((-75)*mm,(-100)*mm,0)
(%i27) ramag:vecmag(ra)
(%i32) M:crossprod(rp,P)
                           [     0     ]
                           [           ]
(%o32)                     [     0     ]
                           [           ]
                           [ - 6800.0  ]
(%i33) k:vec(0,0,1)
(%i34) c:dotprod(M,k)/(4*ramag^2)
(%o34)                       - 108800.0
(%i35) Fd:P/4
                           [     0     ]
                           [           ]
(%o35)                     [ - 4000.0  ]
                           [           ]
                           [     0     ]
(%i36) Fma:c*crossprod(k,ra)
                          [ - 10880.0 ]
                          [           ]
(%o36)                    [ - 8160.0  ]
```
- 1 -

```
                           [           ]
                           [     0     ]
(%i37) magFma:vecmag(Fma)
(%o37)                        13600.0
(%i38) uvFma:unitvec(Fma)
                          [ - 0.79999999 ]
                          [              ]
(%o38)                    [    - 0.6     ]
                          [              ]
                          [      0       ]
(%i39) Fta:Fd+Fma
                           [ - 10880.0 ]
                           [           ]
(%o39)                     [ - 12160.0 ]
                           [           ]
                           [     0     ]
(%i40) magFta:vecmag(Fta)
(%o40)                       16316.862
(%i41) uvFta:unitvec(Fta)
                          [ - 0.66679485 ]
                          [              ]
(%o41)                    [ - 0.74524131 ]
                          [              ]
                          [      0       ]
(%i42) Fmb:c*crossprod(k,rb)
                            [ 10880.0   ]
                            [           ]
```
- 2 -

```
(%o42)                  [ - 8160.0 ]
                        [          ]
                        [    0     ]
(%i43) magFmb:vecmag(Fmb)
(%o43)                    13600.0
(%i44) uvFmb:unitvec(Fmb)
                       [ 0.79999999 ]
                       [            ]
(%o44)                 [   - 0.6    ]
                       [            ]
                       [     0      ]
(%i45) Ftb:Fd+Fmb
                       [  10880.0  ]
                       [           ]
(%o45)                 [ - 12160.0 ]
                       [           ]
                       [     0     ]
(%i46) magFtb:vecmag(Ftb)
(%o46)                   16316.862
(%i47) uvFtb:unitvec(Ftb)
                      [  0.66679485  ]
                      [              ]
(%o47)                [ - 0.74524131 ]
                      [              ]
                      [      0       ]
(%i48) Fmc:c*crossprod(k,rc)
                        [ 10880.0 ]
```

- 3 -

```
                        [         ]
(%o48)                  [ 8160.0  ]
                        [         ]
                        [    0    ]
(%i49) magFmc:vecmag(Fmc)
(%o49)                    13600.0
(%i50) uvFmc:unitvec(Fmc)
                       [ 0.79999999 ]
                       [            ]
(%o50)                 [    0.6     ]
                       [            ]
                       [     0      ]
(%i51) Ftc:Fd+Fmc
                        [ 10880.0 ]
                        [         ]
(%o51)                  [ 4160.0  ]
                        [         ]
                        [    0    ]
(%i52) magFtc:vecmag(Ftc)
(%o52)                   11648.175
(%i53) uvFtc:unitvec(Ftc)
                       [ 0.93405183 ]
                       [            ]
(%o53)                 [ 0.35713746 ]
                       [            ]
                       [     0      ]
(%i54) Fmd:c*crossprod(k,rd)
```

- 4 -


```
                        [ - 10880.0 ]
                        [           ]
(%o54)                  [  8160.0   ]
                        [           ]
                        [     0     ]
(%i55) magFmd:vecmag(Fmd)
(%o55)                    13600.0
(%i56) uvFmd:unitvec(Fmd)
                       [ - 0.79999999 ]
                       [              ]
(%o56)                 [     0.6      ]
                       [              ]
                       [      0       ]
(%i57) Ftd:Fd+Fmd
                        [ - 10880.0 ]
                        [           ]
(%o57)                  [  4160.0   ]
                        [           ]
                        [     0     ]
(%i58) magFtd:vecmag(Ftd)
(%o58)                   11648.175
(%i59) uvFtd:unitvec(Ftd)
                       [ - 0.93405183 ]
                       [              ]
(%o59)                 [  0.35713746  ]
                       [              ]
                       [      0       ]
```

- 5 -

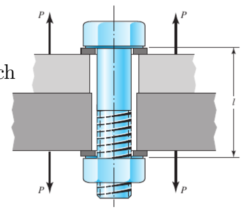
Design of Bolted Joints

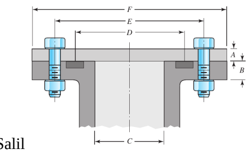
Designing of bolted joints include:

- Determining the number of bolts
- Bolt sizes
- Bolt placements/pattern
- Appropriate preload for the bolt and the torque that must be applied to achieve the desired preload.

Bolts in Tension

- Primary application of bolts and nuts is clamping parts together in situations which the applied loads put the bolts in tension
- Common practice is to preload the the joint by the tightening the bolt with sufficient torque to create tensile stress in it which approaches its proof strength
- Need to understand how the elasticities of the bolt and the clamped members interact when the bolt is tightened followed by the application of the external load





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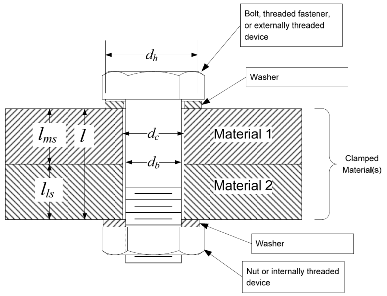
Requirements of a Preloaded Joint



A preloaded joint must meet the following requirement:

- The bolt must have adequate strength.
- The joint must not separate at the maximum load to be applied to the joint.
- The bolt must have adequate fracture and fatigue life.

- Bolt strength is checked at maximum external load and maximum preload.
- Joint separation is checked at maximum external load and minimum preload.

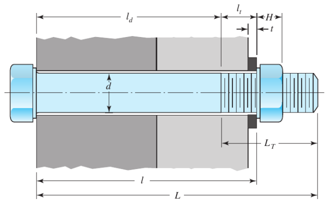
Analytical Approach for Modeling of Bolted Joints

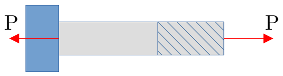
- Implicitly assumes an axisymmetic stress field due to a single preloaded bolt.

- Any geometric or material effects that significantly violate this assumption make the approach invalid.
  - Bolts very close together
  - Bolts near a physical boundary
  - Non axisymmetric geometries

If the bolted joint of interest does not meet these assumptions (and the additional assumptions of the approach) then it is recommended that a finite element analysis be used for the joint.

Bolt Stiffness





$$\delta = \delta_d + \delta_t$$

$$\frac{Pl}{AE_b} = \frac{Pl_d}{A_d E_b} + \frac{Pl_t}{A_t E_b}$$

$$\boxed{\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}}$$

A_t – tensile stress area (table)
l_t – length of portion of the grip
A_d – major diameter
L_d – length of unthreaded portion in the grip

$$k_b = \frac{A_d A_t E_b}{A_d l_t + A_t l_d}$$

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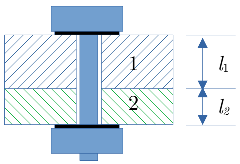
Stiffness of Clamped Members



The clamped members are viewed as springs in series

$$\delta = \delta_1 + \delta_2$$

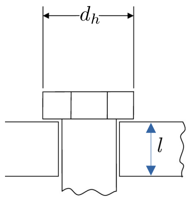
$$\frac{P}{k_m} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}$$

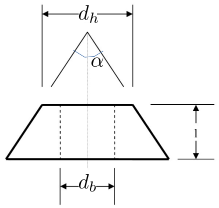
$\mathrm{k}_1 = \frac{A_{m1}E_1}{l_1}$, $k_2 = \frac{A_{m2}E_2}{l_2}$

A_{m1}, A_{m2} – effective clamped areas of members 1 and 2

Shigley's Approach – the stress field is approximated as a hollow frustum of a cone



A Bolt Through a Plate



The Assumed Stress Field

$$k_i = \frac{\pi E d_b \tan(\alpha)}{\ln\left(\frac{(2l\tan\alpha + d_h - d_b)(d_h + d_b)}{(2l\tan\alpha + d_h + d_b)(d_h - d_b)}\right)}$$

Stiffness of Clamped Members

Morrow's approach – based on the Finite Element Analysis valid for two material joint

$$k_m = E_{eff} d_b (0.9991 x_G + 0.2189 n + 0.5234)$$

$$E_{eff} = \left(\frac{1}{\frac{1}{E_{ms}} + n\left(\frac{1}{E_{ls}} - \frac{1}{E_{ms}}\right)} \right),\ n = \frac{l_{ls}}{l},\ x_G = \frac{d_b}{l}\left(\frac{d_h^2 - d_c^2}{1.25 d_b^2}\right)$$

d_b – diameter of the bolt

d_c – diameter of the clearance hole

d_h – Diameter of the load bearing area between the bolt head and the clamping material

E_{ms} – Young's modulus of the more stiff material

E_{ls} – Young's modulus of the less stiff material

l_{ls} – thickness of the less stiff plate

l – total thickness

Stiffness of Clamped Members

Wileman's approach – based on the Finite Element Analysis valid for a joint made up of one material only

$$\frac{k_m}{Ed} = A \exp(Bd/l)$$

d – bolt diameter

l - thickness of all material squeezed between face of bolt and face of nut

Material Used	Poisson Ratio	Elastic Modulus GPa	Elastic Modulus Mpsi	*A*	*B*
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

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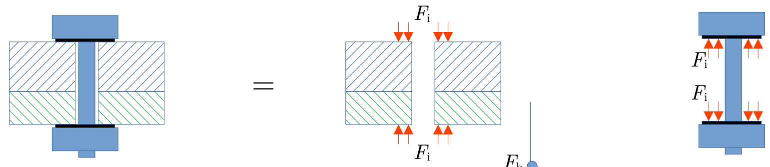
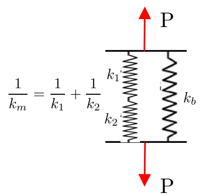
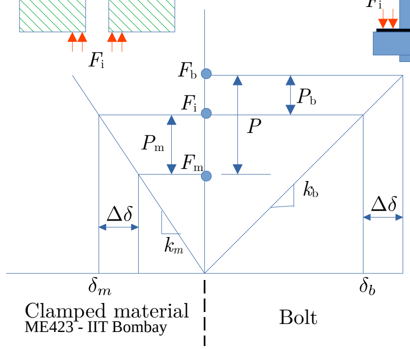
Partitioning the Applied Tensile Load

Bolt Preload F_i



Applied Load P





Partitioning the Applied Tensile Load

Bolt Preload: F_i

Applied load: $P = P_b + P_m$

Load in the bolt (tensile): $F_b = P_b + F_i$

Load in the members (compressive): $F_m = F_i - P_m$

Compatibility

$$\Delta\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m} \quad \text{or} \quad P_b = \frac{k_b}{k_m} P_m$$

Part of the applied load carried by the bolt (tensile)

$$P_b = \frac{k_b}{k_b + k_m} P = CP, \text{ where } C = \frac{k_b}{k_b + k_c}$$

C - stiffness constant of the joint

C is typically less than 0.2, i.e. clamped members take 80% of the applied load

Part of the applied load carried by the clamped members (compressive)

$$P_m = \frac{k_m}{k_m + k_b} P = (1 - C)P$$

Total load carried by the bolt (tensile)

$$F_b = F_i + P_b$$
$$= F_i + CP$$

Total load carried by the clamped members (compressive)

$$F_m = F_i - P_m$$
$$= F_i - (1 - C)P$$

Partitioning the Applied Tensile Load

Total load carried by the bolt (tensile)

$$F_b = F_i + P_b = F_i + CP$$

Total load carried by the clamped members (compressive)

$$F_m = F_i - P_m = F_i - (1-C)P$$

Joint separates when $F_m = 0$

$$P_o = \frac{F_i}{1-C}$$

Factor of safety against joint separation

$$(FOS)_{sep} = \frac{P_o}{P} = \frac{F_i}{P(1-C)}$$

Bolt Pretension and Bolt Torque

- High preload increases the force required to separate the joint
- Bolt pretension

$$\mathrm{F}_i = \begin{cases} 0.75 A_t S_p & \text{nonpermanant connections} \\ 0.90 A_t S_p & \text{permanant connections} \end{cases}$$

A_t – tensile strength area (catalog)
S_p – proof strength (catalog)

- The torque required to generate the required pretension can be estimated using

$$T = K d_b F_i$$

d_b – bolt diamter (catalog)
K – nut factor

Bolt Condition	*K*
Nonplated, black finish	0.30
Zinc-plated	0.20
Lubricated	0.18
Cadmium-plated	0.16
With Bowman Anti-Seize	0.12
With Bowman-Grip nuts	0.09

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Factors of Safety – Preloaded Joint

- yielding factor of safety

$$(FOS)_{yield} = \frac{S_p}{F_b/A_t} = \frac{S_p A_t}{F_i + CP}$$

- Load factor n_L– related to overloading. It applies only to P. Indicates the factor by which one can increase P without exceeding the proof strength

$$F_i + C n_L P = S_p A_t \quad \text{or} \quad n_L = \frac{S_p A_t - F_i}{CP}$$

- Factor of safety against joint separation

$$(FOS)_{sep} = \frac{P_o}{P} = \frac{F_i}{P(1-C)}$$

Fatigue Loading in Tension Joint

The joint is subjected to a cyclic load which varied between $P_{\min}$ and $P_{\max}$

The maximum and the minimum load carried by the bolt (initial pretension F_i) is

$$F_{b\min} = CP_{\min} + F_i$$
$$F_{b\max} = CP_{\max} + F_i$$

The alternating stress experiences by the bolt is given by

$$\sigma_a = \frac{(F_{bmax} - F_{bmin})/2}{A_t} = \frac{C(P_{max} - P_{min})}{2A_t}$$

The mean stress experiences by the bolt is given by

$$\sigma_m = \frac{(F_{bmax} + F_{bmin})/2}{A_t} = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t}$$

Fatigue Loading in Tension Joint

Fully corrected endurance limits including stress concentration effects (K_f)

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi
SAE 8	$\frac{1}{4}$–$1\frac{1}{2}$ in	23.2 kpsi
ISO 8.8	M16–M36	129 MPa
ISO 9.8	M1.6–M16	140 MPa
ISO 10.9	M5–M36	162 MPa
ISO 12.9	M1.6–M36	190 MPa

*Repeatedly applied, axial loading, fully corrected, including K_f as a strength reducer.

Goodman Criterion

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{(FOS)_{fatigue}}$$

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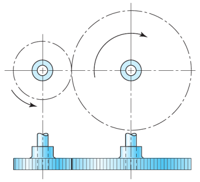
END

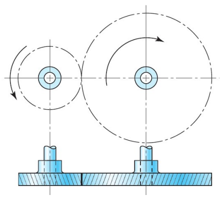
Introduction

Gears are mechanical components with teeth that interlock with other gears to transmit motion and force.

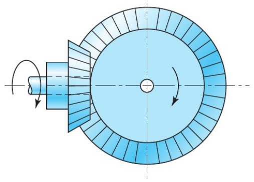
They are used to change the direction of motion, increase or decrease speed, and adjust torque in various machines



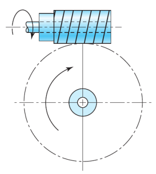
Spur



Helical



Bevel



Worm and Worm Wheel

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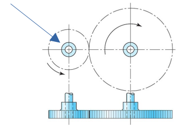
Gears – Common Types

Convention
Smaller of the two gears is called pinion

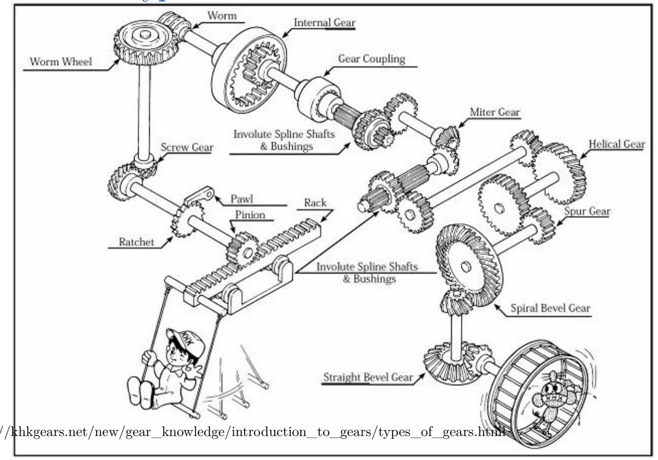


https://www.theengineeringchoice.com/what-is-gear/

Gears – Common Types

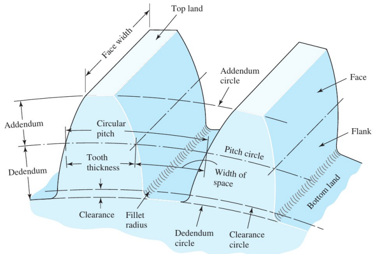
- **Spur gears** – have teeth parallel to the axis of rotation and are used to transmit motion/torque between parallel shafts
- **Helical gears** – have teeth inclined to the axis of rotation and are mostly used to transmit motion/torque between parallel shafts
- **Bevel gears** – have teeth formed on conical surfaces and are used mostly for transmitting motio between intersecting shafts, either perpendicular or at an angle
- **Worm and worm gear** – used to transmit motion between non-intersecting shafts and are used when the speed ratios are high, i.e. > 3
- Hypoid gears – similar to bevel gears except that shafts are offset and non-intersecting
- Mitre gears - bevel gears with ratio 1:1
- Herringbone gear - similar to double helical gears but with no gap

Gears – Common Types



https://khkgears.net/new/gear_knowledge/introduction_to_gears/types_of_gears.html

Nomenclature of the Spur-Gear Tooth



Terminology

- **Pitch circle**: theoretical circle on which all the calculations are based. The pitch circles of the mating gears are tangent to each other.
- **Pitch diameter** d is the diameter of the pitch circle.
- **Module** m is the ratio of the pitch diameter to the number of teeth, N. $m = \frac{d}{N}$
- **Addendum** is the radial distance between the top land and the pitch circle.
- **Dedendum** is the radial distance from the bottom land to the pitch circle.
- **Whole depth** h_t is the sum of the addendum and the dedendum
- **Backlash** – amount by which the width of the tooth space exceeds the thickness of the engaging tooth measured at the pitch circle
- **Circular pitch** p distance measure along the pitch circle from a point on one tooth to a corresponding point on adjacent tooth

https://khkgears.net/new/gear_knowledge/gear_technical_reference/calculation_gear_dimensions.html

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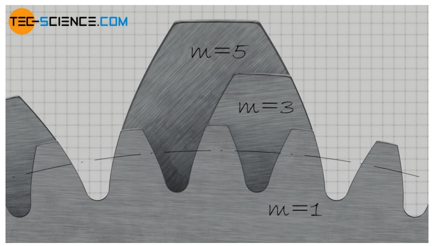
Module

Examples of modules in different applications (as the load increases, the module increases

Wrist watches ~ 0.05 – 0.2

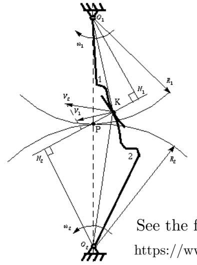
Printers, office copiers ~ 0.6 – 1.0

Transmissions for cars ~ 1.5 – 3.0



Conjugate Action and Involute Profile

- When tooth profiles or cams are designed to produce a constant angular velocity ratio during meshing, they are said to have conjugate action
- Fundamental law of gear tooth action – to obtain a constant angular velocity ratio during meshing, the common normal at the point of contact between a pair of gear teeth must always pass through a fixed point on the line of centers (pitch point).



$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{O_2P}{O_1P} = \frac{R_2}{R_1} \quad \text{(law of gearing)}$$

Involute profile satisfies the above requirement – most commonly used gear profile

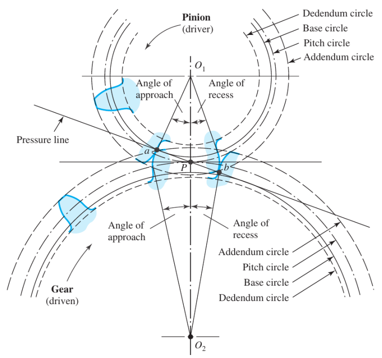
See video below for gears with cycloidal profile

See the following videos

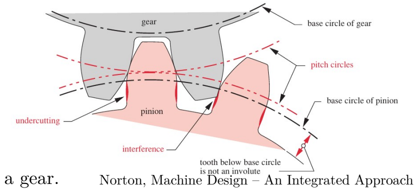
https://www.tec-science.com/mechanical-power-transmission/involute-gear/meshing-line-action-contact-pitch-circle-law/

https://www.tec-science.com/mechanical-power-transmission/cycloidal-gear/geometry-of-cycloidal-gears/

Tooth Action



- Pressure line = line of action = generating line
- Pressure angle = 20° (most common) – angle between the line of action and the direction of velocity at the pitch point
- Contact ratio – average number of teeth in contact at any given instant ~ 1.2
- Interference – contact of tooth profiles which are not conjugate is called interference (tooth profiles are not involute below the base circle)



Base circle is used to generate the involute profile – basic to a gear.

$$r_b = r_p \cos\phi$$

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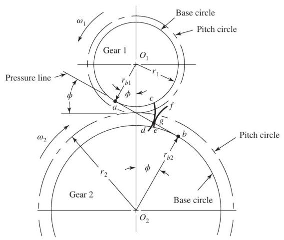
Problem

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The module is 12 mm, and the addendum and dedendum are 1 m and 1.25 m, respectively. The gears are cut using a pressure angle of 20°.

(a) Compute the circular pitch, the center distance, and the radii of the base circles.

(b) In mounting these gears, the center distance was incorrectly made 6.35 mm larger. Compute the new values of the pressure angle and the pitch-circle diameters.



Tooth Systems - American Gear Manufacturers Association (AGMA)

A tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle.

Table 13–1 Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	$22\frac{1}{2}$	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
	25	$1/P$ or m	$1.25/P$ or $1.25m$
			$1.35/P$ or $1.35m$
Stub	20	$0.8/P$ or $0.8m$	$1/P$ or m

Module m (mm/tooth)	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

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Calculation for Standard Spur Gears – Type 1

Item	Symbol/Formula
Center distance	a (**given**)
Speed ratio	i ($i = N_g/N_p$) (**given**)
module	m (**choose**)
Pressure angle	ϕ (**choose**)
Number of teeth	N_p, N_g (calculate)
Pitch diameter	dp = mNp,dg = mNg (calculate)
Base diameter	dbp = dp cosϕ dbg = dg cosϕ (calculate)
Addendum	m
Tooth depth	1.25 m
Tip diameter	dp+2m, dg+2m
Root diameter	dp-2.5m,dg-2.5m

$$N_p = \frac{2a}{m(i+1)}$$

$$N_g = \frac{2ai}{m(i+1)}$$

https://khkgears.net/new/gear_knowledge/gear_technical_reference/calculation_gear_dimensions.html

Calculation of Standard Spur Gears – Type 2 Problems

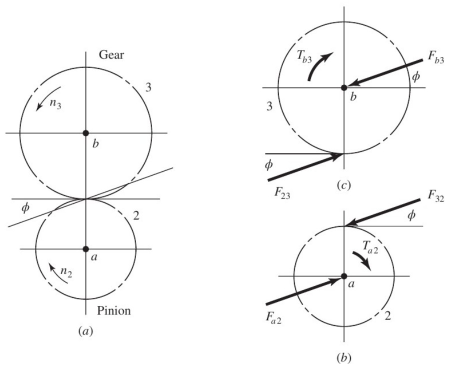
Item	Symbol/Formula
Module	m (**choose**)
Pressure angle	ϕ (**choose**)
Number of teeth	N_p, N_g (**given**)
Center distance	a=(dp+dg)/2 (calculate)
Pitch diameter	dp = mNp,dg = mNg (calculate)
Base diameter	dbp = dp cosϕ dbg = dg cosϕ (calculate)
Addendum	m
Tooth depth	1.25 m
Tip diameter	dp+2m, dg+2m
Root diameter	dp-2.5m,dg-2.5m

Force Analysis – Spur Gears

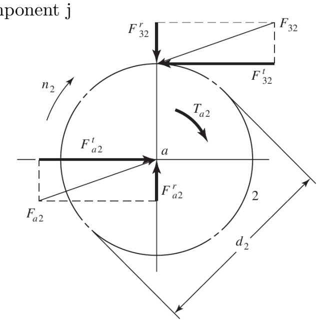
Input gear: 2 followed by 3,4,5, ...

Shaft: a, b, c, ...

Nomenclature: F_{ij} : Force exerted by component i on component j

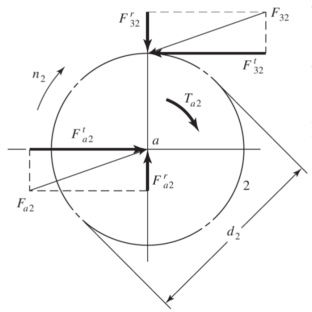


FBD for a gear pair



Force Resolution - Pinion

Force Analysis – Spur Gears - Continued



Transmitted load $W_t = F^t_{32}$ (this is the useful component)

Torque transmitted: $T = W_t d_2/2$

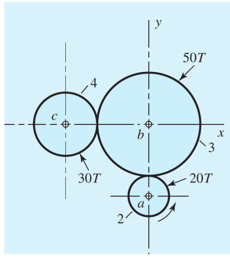
Power transmitted $P = T$

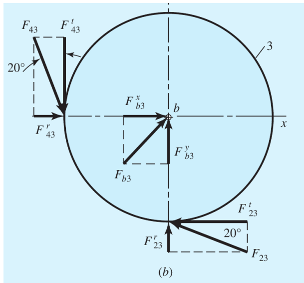
(efficiency of spur gears is ~ 98%)

Slip line velocity V = $(d/2)$

Force Analysis – Spur Gears - Problem

Pinion 2 in Figure runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of m = 2.5 mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.





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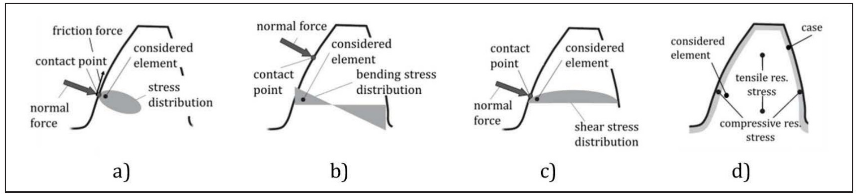
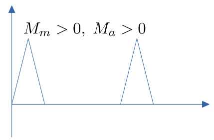
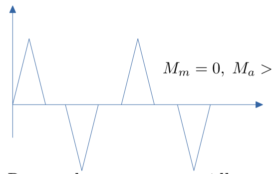
Gear Tooth: Types of Stresses



Contact Stress Bending Stress Shear Stress Residual Stress

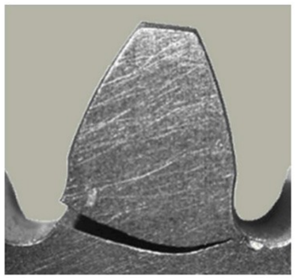


Repeated moment on a nonidler tooth

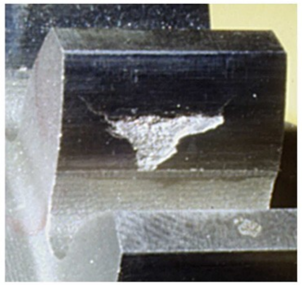


Reversed moment on an idler tooth

Gear Tooth: Common Failure Modes



Tooth Root Breakage
(Bending Fatigue Fracture)



Pitting
(Surface Fatigue Fracture)

Pitting – the fracture and separation of small pieces of materials from the surface – form of surface fatigue

https://www.geartechnology.com/articles/22175-tooth-flank-fracture-basic-principles-and-calculation-model-for-a-sub-surface-initiated-fatigue-failure-mode-of-case-hardened-gears

Gear Material

Steel: The most widely used material for gears.

Low Carbon Steel: Offers good machinability and is often used for gears that do not require high strength.

Medium Carbon Steel: Provides a balance of strength and toughness, suitable for many gear applications.

High Carbon Steel: Known for its high strength and wear resistance, used in heavy-duty gears.

Alloy Steel: Enhanced with elements like chromium, nickel, and molybdenum to improve strength, toughness, and wear resistance.

Cast Iron: Good wear resistance and damping properties, used for gears that operate under heavy loads and low speeds.

Non-Ferrous Alloys: Includes materials like bronze and brass, which offer good corrosion resistance and are often used in worm gears and other applications where lubrication is critical.

Powder Metallurgy Materials: Used for producing gears with complex shapes and fine details, offering good strength and wear resistance.

Plastics: Commonly used in applications where noise reduction and lightweight are important. Thermoplastics like nylon and acetal.

Each material has its own set of properties that make it suitable for different gear applications.
The choice of material depends on factors like load, speed, operating environment, and cost considerations.

AGMA Bending Stress Equation

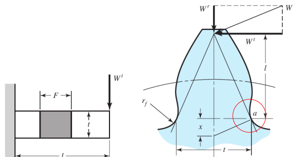
Assumptions

- The contact ratio is between 1 and 2.
- There is no interference between the tips and the root fillets of the mating teeth and no undercutting of the
- teeth above the theoretical start of the active profile.
- No teeth are pointed.
- There is nonzero backlash.
- The root fillets are standard, assumed smooth, and produced by a generating process.
- The friction force effects are neglected.
- Valid for external tooth only

$$\sigma_b = \frac{F_t}{FmJ}\frac{K_a K_m}{K_v}K_s K_B K_I$$

F_t - tangential load, F – face width, m - module

Based on Lewis's formula – Lewis developed his formula in 1892 and is based on considering the loaded tooth as a cantilever beam
Lewis's formula neglects the stress concentration effect at the root



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AGMA Bending Stress Equation

$$\sigma_b = \frac{F_t}{FmJ}\frac{K_a K_m}{K_v}K_s K_B K_I$$

- Bending strength geometry factor J – vary with number of teeth on the pinion and the gear (AGMA standard) (different for pinion and gear)
- Dynamic factor K_v – attempts to account for the internally generated vibration loads from tooth-tooth impacts induced by nonconjugate meshing of the gear teeth. Depends on the accuracy to which the gears are machined (estimate it from the standards)
- Load distribution factor K_m – takes into account the fact that the transmitted load will be distributed unevenly across the face of the tooth
- Application factor K_a – takes into account the non-uniform nature of the transmitted load (shock loading)
- Size factor K_S – similar to the size factor used in fatigue calculations (larger components have lower strength). As per AGMA recommendations normally $K_S = 1$.
- Rim thickness factor K_B – applicable in situations in which the gear is made with rim and spokes rather than a solid disk. For solid gears $K_B = 1$
- Idler factor K_I - takes into account the fact that an idler gear is subject to more load cycles per unit time as compared to a non-idler gear. $K_I = 1.42$ for idler and $K_I = 1.0$ for non-idler

AGMA Bending Stress Equation

Table 12-8 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with Tip Loading

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.24	0.24								
26	U	U	U	U	U	U	0.24	0.25	0.25	0.25						
35	U	U	U	U	U	U	0.24	0.26	0.25	0.26	0.26	0.26				
55	U	U	U	U	U	U	0.24	0.28	0.25	0.28	0.26	0.28	0.28	0.28		
135	U	U	U	U	U	U	0.24	0.29	0.25	0.29	0.26	0.29	0.28	0.29	0.29	0.29

Table 12-9 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with HPSTC Loading

Gear teeth	Pinion teeth 12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.33	0.33								
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

HPSTC - highest point of single-tooth contact
For contact ratio > 1 and gear with good Accuracy, it lies somewhere below the tip

U - undercutting

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AGMA Bending Stress Equation

$$\sigma_b = \frac{F_t}{FmJ}\frac{K_a K_m}{K_v}K_s K_B K_I$$

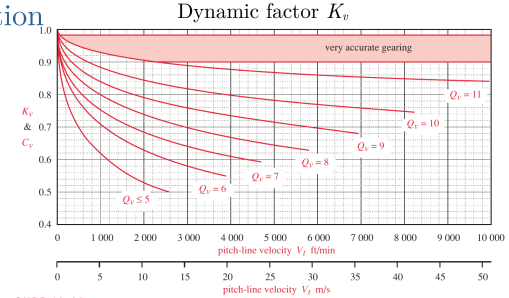
Table 12-16 Load Distribution Factors K_m

Face Width in (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Table 12-17 Application Factors K_a

Driving Machine	Driven Machine: Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

Dynamic factor K_v



The higher the quality number Q_v, the closer the actual geometry to what is specified
Depends on the gear manufacturing process and the specified tolerances
Tighter the tolerances, higher is the quality number

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Gear Quality Number

The quality number indicates the the accuracy of tooth shape and placement.
The four main parameters accounted for are:

The **tooth lead or tooth alignment** criterion applies to spur and helical-type gearing, and measures the variation between the specified lead (or helix angle) and the lead of the produced gear.

Involute profile variation is the difference between the specified profile and the measured profile of the tooth.

Pitch variation or spacing variation is the difference between the specified tooth location and the actual tooth location around the circumference of the gear.

Radial runout refers to the disparity in radial position of teeth on a gear – the variation in tooth distances from the center of rotation.

https://www.machinedesign.com/automation-iiot/article/21834626/gear-quality-what-its-all-about

AGMA Pitting Resistance Equation

$$\sigma_c = C_p \sqrt{\frac{F_t}{FId}\frac{C_a C_m}{C_v} C_s C_f}$$

Feature of contact stresses
$\sigma \propto \sqrt{F}$

F_t – tangential force
d – pitch circle diameter of the smaller of the two gears in mesh
F – face width
I – surface geometry factor accounts for the radii of curvature of the gear teeth and the pressure angle. Calculated for a gear pair in a mesh
C_p – elastic coefficient that accounts for the differences in the gear and pinion material constants
C_f - surface finish factor that accounts for the unusually rough finishes on the gear teeth.
$C_f = 1$ for gears made by conventional methods.
$C_a = K_a$ - Application factor
$C_m = K_m$ - Load distribution factor
$C_v = K_v$ - Dynamic factor
$C_s = K_s$ – Size factor

AGMA Pitting Resistance Equation

Table 12-18 AGMA Elastic Coefficient C_p in Units of $[psi]^{0.5}$ ($[MPa]^{0.5}$)*†

Pinion Material	E_p psi (MPa)	Gear Material: Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30E6 (2E5)	2 300 (191)	2 180 (181)	2 160 (179)	2 100 (174)	1 950 (162)	1 900 (158)
Malleable Iron	25E6 (1.7E5)	2 180 (181)	2 090 (174)	2 070 (172)	2 020 (168)	1 900 (158)	1 850 (154)
Nodular Iron	24E6 (1.7E5)	2 160 (179)	2 070 (172)	2 050 (170)	2 000 (166)	1 880 (156)	1 830 (152)
Cast Iron	22E6 (1.5E5)	2 100 (174)	2 020 (168)	2 000 (166)	1 960 (163)	1 850 (154)	1 800 (149)
Aluminum Bronze	17.5E6 (1.2E5)	1 950 (162)	1 900 (158)	1 880 (156)	1 850 (154)	1 750 (145)	1 700 (141)
Tin Bronze	16E6 (1.1E5)	1 900 (158)	1 850 (154)	1 830 (152)	1 800 (149)	1 700 (141)	1 650 (137)

$$C_p = \sqrt{\frac{1}{\pi\left(\left(\frac{1-\nu_p^2}{E_p}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right)}}$$

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AGMA Bending-Fatigue Strength Calculation for Gear Materials

σ_b – Tooth bending stress (pinion/gear)
S'_{fb}– Uncorrected (published) bending-fatigue strength for pinion/gear material
(Strength data is stated for 10^7 cycles of repeated stress and 99% reliability)
S_{fb}– Corrected bending-fatigue strength for pinion/gear material

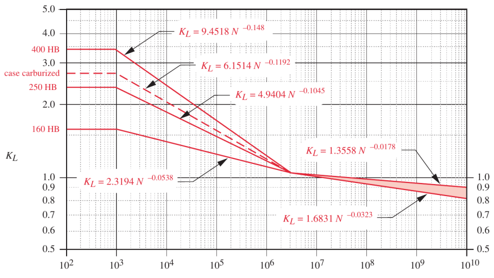
$$S_{fb} = \frac{K_L}{K_T K_R} S'_{fb}$$

K_L – Life factor. As test data is for 10^7 cycles, a shorter or longer life cycle requires modification of the bending-fatigue strength
K_T – Temperature factor, K_T =1 if the gear temperature is less than 120°
K_R – Takes into account reliability other than 99%

Calculate the the following FOS

$$N_{b_{pinion}} = \frac{S_{fb_{pinion}}}{\sigma_{b_{pinion}}} \qquad N_{b_{gear}} = \frac{S_{fb_{gear}}}{\sigma_{b_{gear}}}$$

AGMA Bending-Fatigue Strength Calculation for Gear Materials



The number of load cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed

Table 12-20 AGMA Bending-Fatigue Strengths S_{fb}' for a Selection of Gear Materials*

Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Bending-Fatigue Strength psi x 10^3	Bending-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	25–33	170–230
			Through hardened	240 HB	31–41	210–280
			Through hardened	300 HB	36–47	250–325
			Through hardened	360 HB	40–52	280–360
			Through hardened	400 HB	42–56	290–390
			Flame or induction hardened	Type A pattern 50–54 HRC	45–55	310–380
			Flame or induction hardened	Type B pattern	22	150
			Carburized and case hardened	55–64 HRC	55–75	380–520
		AISI 4140	Nitrided	84.6 HR15N†	34–45	230–310
		AISI 4340	Nitrided	83.5 HR15N	36–47	250–325
		Nitralloy 135M	Nitrided	90.0 HR15N	38–48	260–330
		Nitralloy N	Nitrided	90.0 HR15N	40–50	280–345
		2.5% Chrome	Nitrided	87.5–90.0 15N	55–65	380–450
Cast iron	20	Class 20	As cast		5	35
	30	Class 30	As cast	175 HB	8	69
	40	Class 40	As cast	200 HB	13	90
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	22–33	150–230
	A-7-c	80-55-06	Quenched and tempered	180 HB	22–33	150–230
	A-7-d	100-70-03	Quenched and tempered	230 HB	27–40	180–280
	A-7-e	120-90-02	Quenched and tempered	230 HB	27–40	180–280
Malleable iron (pearlitic)	A-8-c	45007		165 HB	10	70
	A-8-e	50005		180 HB	13	90
	A-8-f	53007		195 HB	16	110
	A-8-i	80002		240 HB	21	145
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	5.7	40
	Al/Br 3	ASTM B-148 78 alloy 954	Heat treated	90 ksi min tensile strength	23.6	160

Table 12-19
AGMA Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

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AGMA Surface-Fatigue Strength Calculation for Gear Materials

σ_c – Tooth surface stress (pinion)

S'_{fc}– Uncorrected surface-fatigue strength for pinion material

(Strength data is stated for 10^7 cycles of repeated stress and 99% reliability)

S_{fc}– Corrected surface-fatigue strength for pinion material

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S'_{fc}$$

C_L – surface life factor. As test data is for 10^7 cycles, a shorter or longer life cycle requires modification of the surface-fatigue strength

C_T – Temperature factor, $C_T = K_T$

C_R – Takes into account reliability other than 99%, $C_T = K_T$

C_H – function of the gear ratio and the relative hardness of pinion and gears, $C_H = 1$ if made from the same material

Calculate the the following FOS $N_{c_{pinion}} = \left(\frac{S_{fc_{pinion}}}{\sigma_{c_{pinion}}}\right)^2$ $\sigma \propto \sqrt{F}$

AGMA Surface-Fatigue Strength Calculation for Gear Materials

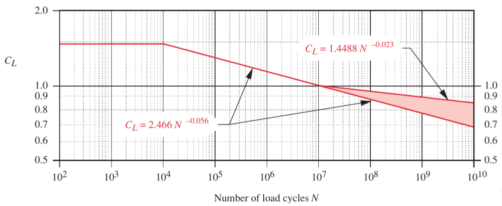


Table 12-21 AGMA Surface-Fatigue Strengths S_{fc}' for a Selection of Gear Materials*

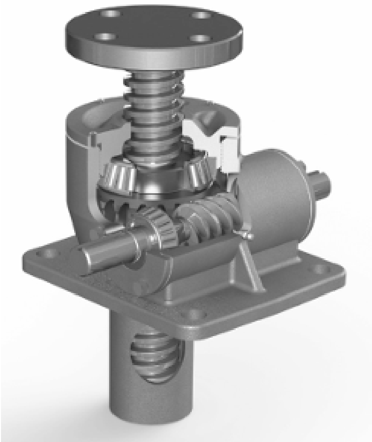
Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Surface-Fatigue Strength psi x 10^3	Surface-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	85–95	590–660
			Through hardened	240 HB	105–115	720–790
			Through hardened	300 HB	120–135	830–930
			Through hardened	360 HB	145–160	1000–1100
			Through hardened	400 HB	155–170	1100–1200
			Flame or induction hardened	50 HRC	170–190	1200–1300
			Flame or induction hardened	54 HRC	175–195	1200–1300
			Carburized and case hardened	55–64 HRC	180–225	1250–1300
		AISI 4140	Nitrided	84.6 HR15N†	155–180	1100–1250
		AISI 4340	Nitrided	83.5 HR15N	150–175	1050–1200
		Nitralloy 135M	Nitrided	90.0 HR15N	170–195	1170–1350
		Nitralloy N	Nitrided	90.0 HR15N	195–205	1340–1410
		2.5% Chrome	Nitrided	87.5 HR15N	155–172	1100–1200
		2.5% Chrome	Nitrided	90.0 HR15N	192–216	1300–1500
Cast iron	20	Class 20	As cast		50–60	340–410
	30	Class 30	As cast	175 HB	65–70	450–520
	40	Class 40	As cast	200 HB	75–85	520–590
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	77–92	530–630
	A-7-c	80-55-06	Quenched and tempered	180 HB	77–92	530–630
	A-7-d	100-70-03	Quenched and tempered	230 HB	92–112	630–770
	A-7-e	120-90-02	Quenched and tempered	230 HB	103–126	710–870
Malleable iron (pearlitic)	A-8-c	45007		165 HB	72	500
	A-8-e	50005		180 HB	78	540
	A-8-f	53007		195 HB	83	570
	A-8-i	80002		240 HB	94	650
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	30	450
	Al/Br 3	ASTM B-148 78 alloy 954	Heat–treated	90 ksi min tensile strength	65	450

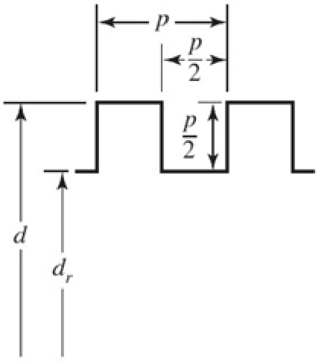
Norton, Machine Design – An Integrated Approach

END

TUT 7

1. A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in the following figure.





The given data include $\mu = \mu_c = 0.08$, d_c = 40 mm, and F = 6.4 kN per screw.

(a) Find the thread depth, thread width, mean diameter, minor diameter, and lead.

(b) Find the torque required to raise and lower the load.

(c) Find the efficiency during lifting the load.

(d) Find the body stresses, torsional and compressive.

(e) Find the bearing stress on the first thread.

(f) Find the thread bending stress at the root of the first thread.

(g) Determine the von Mises stress at the critical stress element where the root of the first thread interfaces with the screw body.

EXAMPLE 8–1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. 8–4. The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

(*a*) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
(*b*) Find the torque required to raise and lower the load.
(*c*) Find the efficiency during lifting the load.
(*d*) Find the body stresses, torsional and compressive.
(*e*) Find the bearing stress.
(*f*) Find the thread bending stress at the root of the thread.
(*g*) Determine the von Mises stress at the root of the thread.
(*h*) Determine the maximum shear stress at the root of the thread.

Solution

(*a*) From Fig. 8–3*a* the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

Answer

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

(*b*) Using Eqs. (8–1) and (8–6), the torque required to turn the screw against the load is

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)}\right] + \frac{6.4(0.08)40}{2}$$

Answer

$$= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m}$$

Using Eqs. (8–2) and (8–6), we find the load-lowering torque is

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right) + \frac{Ff_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)}\right] + \frac{6.4(0.08)(40)}{2}$$

Answer $$= -0.466 + 10.24 = 9.77 \text{ N} \cdot \text{m}$$

The minus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(c) The overall efficiency in raising the load is

Answer $$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

(d) The body shear stress τ due to torsional moment T_R at the outside of the screw body is

Answer $$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

Answer $$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(e) The bearing stress σ_B is, with one thread carrying $0.38F$,

Answer $$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(f) The thread-root bending stress σ_b with one thread carrying $0.38F$ is

Answer $$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$

(g) The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (d) of 6.07 MPa. The three-dimensional stresses, after Fig. 8–8, noting the y coordinate is into the page, are

$$\begin{aligned} \sigma_x &= 41.5 \text{ MPa} & \tau_{xy} &= 0 \\ \sigma_y &= -10.39 \text{ MPa} & \tau_{yz} &= 6.07 \text{ MPa} \\ \sigma_z &= 0 & \tau_{zx} &= 0 \end{aligned}$$

For the von Mises stress, Eq. (5–14) of Sec. 5–5 can be written as

Answer $$\sigma' = \frac{1}{\sqrt{2}}\{(41.5 - 0)^2 + [0 - (-10.39)]^2 + (-10.39 - 41.5)^2 + 6(6.07)^2\}^{1/2}$$

$$= 48.7 \text{ MPa}$$

Alternatively, you can determine the principal stresses and then use Eq. (5–12) to find the von Mises stress. This would prove helpful in evaluating $\tau_{\max}$ as well. The principal stresses can be found from Eq. (3–15); however, sketch the stress element and note that there are no shear stresses on the x face. This means that σ_x is a principal stress. The remaining stresses can be transformed by using the plane stress equation, Eq. (3–13). Thus, the remaining principal stresses are

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79,\ -13.18\ \text{MPa}$$

Ordering the principal stresses gives $\sigma_1, \sigma_2, \sigma_3 = 41.5, 2.79, -13.18$ MPa. Substituting these into Eq. (5–12) yields

Answer

$$\sigma' = \left\{\frac{[41.5 - 2.79]^2 + [2.79 - (-13.18)]^2 + [-13.18 - 41.5]^2}{2}\right\}^{1/2}$$
$$= 48.7\ \text{MPa}$$

(h) The maximum shear stress is given by Eq. (3–16), where $\tau_{\max} = \tau_{1/3}$, giving

Answer

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{41.5 - (-13.18)}{2} = 27.3\ \text{MPa}$$

2. Find the bolt spring rate of $M12 \times 1.25 \times 38.1$ mm (property class 8.8) bolt.

Table 8–11

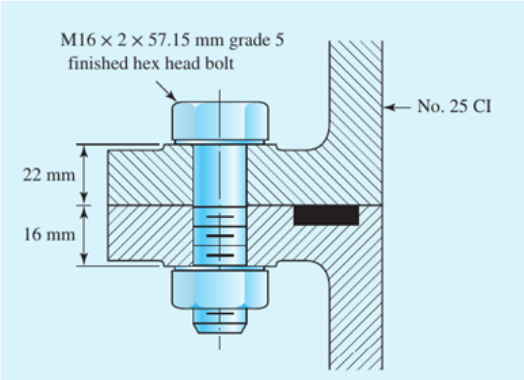
Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs

Property Class	Size Range, Inclusive	Minimum Proof Strength,* MPa	Minimum Tensile Strength,* MPa	Minimum Yield Strength,* MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6–M16	310	420	340	Low or medium carbon	4.8
5.8	M5–M24	380	520	420	Low or medium carbon	5.8
8.8	M16–M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	12.9

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Nominal Major Diameter d mm	Coarse-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²	Fine-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

3. The figure shown below is a cross section of a grade 25 cast-iron pressure vessel.



A total of N bolts are to be used to resist a separating force of 160 kN.

(a) Determine k_b, k_m, and C. You can estimate k_m using the following equation presented by Wileman et al.

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

This equation is valid only if the entire joint is made of the same material. Here E is the Young's modulus, d is the mean bolt diameter and l is the grip length. For the given joint: E = 100 GPa, A = 0.77871, B = 0.61616.

(b) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

(c) With the number of bolts obtained in part (b), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

Figure 8–19 is a cross section of a grade 25 cast-iron pressure vessel. A total of N bolts are to be used to resist a separating force of 36 kip.

(*a*) Determine k_b, k_m, and C.

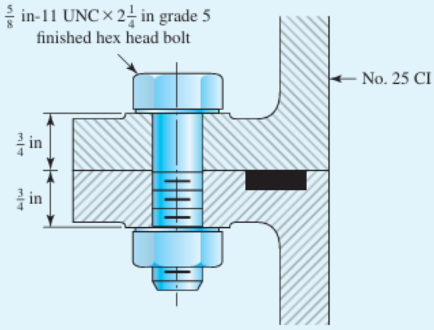
(*b*) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

(*c*) With the number of bolts obtained in part (*b*), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

(*a*) The grip is l = 1.50 in. From Table A–31, the nut thickness is $\frac{35}{64}$ in. Adding two threads beyond the nut of $\frac{2}{11}$ in gives a bolt length of

$$L = \frac{35}{64} + 1.50 + \frac{2}{11} = 2.229 \text{ in}$$

From Table A–17 the next fraction size bolt is $L = 2\frac{1}{4}$ in. From Eq. (8–13), the thread length is $L_T = 2(0.625) + 0.25 = 1.50$ in. Thus, the length of the unthreaded portion



in the grip is $l_d = 2.25 - 1.50 = 0.75$ in. The threaded length in the grip is $l_t = l - l_d = 0.75$ in. From Table 8–2, $A_t = 0.226$ in^2. The major-diameter area is $A_d = \pi(0.625)^2/4 = 0.3068$ in^2. The bolt stiffness is then

Answer

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068(0.226)(30)}{0.3068(0.75) + 0.226(0.75)} = 5.21 \text{ Mlbf/in}$$

From Table A–24, for no. 25 cast iron we will use $E = 14$ Mpsi. The stiffness of the members, from Eq. (8–22), is

Answer

$$k_m = \frac{0.5774\pi E d}{2 \ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(14)(0.625)}{2 \ln\left[5\dfrac{0.5774(1.5) + 0.5(0.625)}{0.5774(1.5) + 2.5(0.625)}\right]} = 8.95 \text{ Mlbf/in}$$

If you are using Eq. (8–23), from Table 8–8, $A = 0.778\,71$ and $B = 0.616\,16$, and

$$\begin{aligned} k_m &= EdA \exp(Bd/l) \\ &= 14(0.625)(0.778\,71) \exp[0.616\,16(0.625)/1.5] \\ &= 8.81 \text{ Mlbf/in} \end{aligned}$$

which is only 1.6 percent lower than the previous result.

From the first calculation for k_m, the stiffness constant C is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368$$

(*b*) From Table 8–9, $S_p = 85$ kpsi. Then, using Eqs. (8–31) and (8–32), we find the recommended preload to be

$$F_i = 0.75 A_t S_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

For N bolts, Eq. (8–29) can be written

$$n_L = \frac{S_p A_t - F_i}{C(P_{\text{total}}/N)} \tag{1}$$

or

$$N = \frac{C n_L P_{\text{total}}}{S_p A_t - F_i} = \frac{0.368(2)(36)}{85(0.226) - 14.4} = 5.52$$

Answer Six bolts should be used to provide the specified load factor.

(*c*) With six bolts, the load factor actually realized is

Answer

$$n_L = \frac{85(0.226) - 14.4}{0.368(36/6)} = 2.18$$

From Eq. (8–28), the yielding factor of safety is

Answer

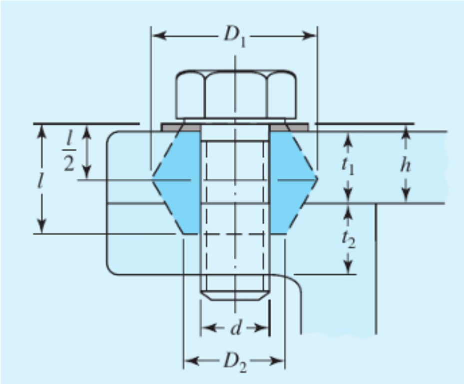
$$n_p = \frac{S_p A_t}{C(P_{\text{total}}/N) + F_i} = \frac{85(0.226)}{0.368(36/6) + 14.4} = 1.16$$

From Eq. (8–30), the load factor guarding against joint separation is

Answer

$$n_0 = \frac{F_i}{(P_{\text{total}}/N)(1 - C)} = \frac{14.4}{(36/6)(1 - 0.368)} = 3.80$$

4. Figure shows a connection using cap screws.



The joint is subjected to a fluctuating force whose maximum value is 22.24 kN per screw. The data for the cap screw is M16 × 2, (property class 8.8). The bolt stiffness k_b is 1.28 MN/mm while the member stiffness, k_m, is 3.07 MN/mm.

(a) Find all factors of safety – yield, overload, separation, fatigue.

EXAMPLE 8–5

Figure 8–21 shows a connection using cap screws. The joint is subjected to a fluctuating force whose maximum value is 5 kip per screw. The required data are: cap screw, 5/8 in-11 UNC, SAE 5; hardened-steel washer, $t_w = \frac{1}{16}$ in thick; steel cover plate, $t_1 = \frac{5}{8}$ in, $E_s = 30$ Mpsi; and cast-iron base, $t_2 = \frac{5}{8}$ in, $E_{ci} = 16$ Mpsi.

(*a*) Find k_b, k_m, and C using the assumptions given in the caption of Fig. 8–21.

(*b*) Find all factors of safety and explain what they mean.

Solution

(*a*) For the symbols of Figs. 8–15 and 8–21, $h = t_1 + t_w = 0.6875$ in, $l = h + d/2 = 1$ in, and $D_2 = 1.5d = 0.9375$ in. The joint is composed of three frusta; the upper two frusta are steel and the lower one is cast iron.

For the upper frustum: $t = l/2 = 0.5$ in, $D = 0.9375$ in, and $E = 30$ Mpsi. Using these values in Eq. (8–20) gives $k_1 = 46.46$ Mlbf/in.

For the middle frustum: $t = h - l/2 = 0.1875$ in and $D = 0.9375 + 2(l - h)$ tan 30° = 1.298 in. With these and $E_s = 30$ Mpsi, Eq. (8–20) gives $k_2 = 197.43$ Mlbf/in.

The lower frustum has $D = 0.9375$ in, $t = l - h = 0.3125$ in, and $E_{ci} = 16$ Mpsi. The same equation yields $k_3 = 32.39$ Mlbf/in.

Substituting these three stiffnesses into Eq. (8–18) gives $k_m = 17.40$ Mlbf/in. The cap screw is short and threaded all the way. Using $l = 1$ in for the grip and $A_t = 0.226\ \text{in}^2$ from Table 8–2, we find the stiffness to be $k_b = A_t E/l = 6.78$ Mlbf/in. Thus the joint constant is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{6.78}{6.78 + 17.40} = 0.280$$

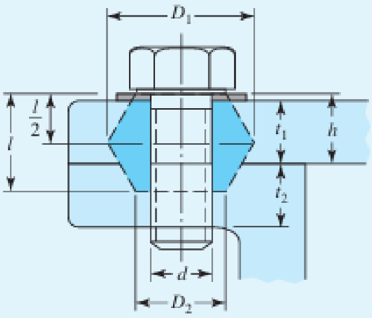


Figure 8–21

Pressure-cone frustum member model for a cap screw. For this model the significant sizes are

$$l = \begin{cases} h + t_2/2 & t_2 < d \\ h + d/2 & t_2 \geq d \end{cases}$$

$D_1 = d_w + l \tan \alpha = 1.5d + 0.577l$

$D_2 = d_w = 1.5d$

where l = effective grip. The solutions are for $\alpha = 30°$ and $d_w = 1.5d$.

(b) Equation (8–30) gives the preload as

$$F_i = 0.75F_p = 0.75A_tS_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

where from Table 8–9, $S_p = 85$ kpsi for an SAE grade 5 cap screw. Using Eq. (8–28), we obtain the load factor as the yielding factor of safety is

Answer
$$n_p = \frac{S_pA_t}{CP + F_i} = \frac{85(0.226)}{0.280(5) + 14.4} = 1.22$$

This is the traditional factor of safety, which compares the maximum bolt stress to the proof strength.

Using Eq. (8–29),

Answer
$$n_L = \frac{S_pA_t - F_i}{CP} = \frac{85(0.226) - 14.4}{0.280(5)} = 3.44$$

This factor is an indication of the overload on P that can be applied without exceeding the proof strength.

Next, using Eq. (8–30), we have

Answer
$$n_0 = \frac{F_i}{P(1 - C)} = \frac{14.4}{5(1 - 0.280)} = 4.00$$

If the force P gets too large, the joint will separate and the bolt will take the entire load. This factor guards against that event.

For the remaining factors, refer to Fig. 8–22. This diagram contains the modified Goodman line, the Gerber line, the proof-strength line, and the load line. The intersection

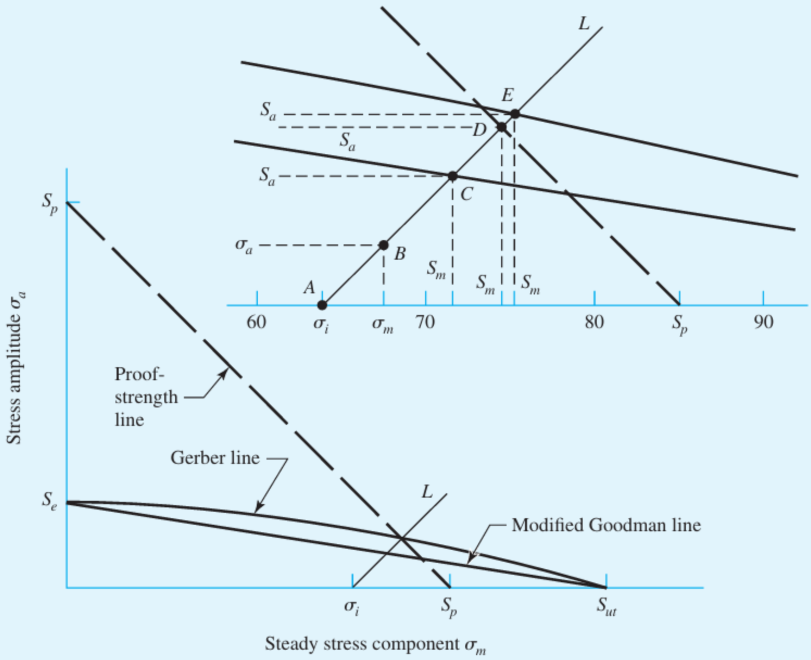


Figure 8–22

Designer's fatigue diagram for preloaded bolts, drawn to scale, showing the modified Goodman line, the Gerber line, and the Langer proof-strength line, with an exploded view of the area of interest. The strengths used are $S_p = 85$ kpsi, $S_e = 18.6$ kpsi, and $S_{ut} = 120$ kpsi. The coordinates are A, $\sigma_i =$ 63.72 kpsi; B, $\sigma_a = 3.10$ kpsi, $\sigma_m = 66.82$ kpsi; C, $S_a =$ 7.55 kpsi, $S_m = 71.29$ kpsi; D, $S_a = 10.64$ kpsi, $S_m =$ 74.36 kpsi; E, $S_a = 11.32$ kpsi, $S_m = 75.04$ kpsi.

of the load line L with the respective failure lines at points C, D, and E defines a set of strengths S_a and S_m at each intersection. Point B represents the stress state σ_a, σ_m. Point A is the preload stress σ_i. Therefore the load line begins at A and makes an angle having a unit slope. This angle is 45° only when both stress axes have the same scale.

The factors of safety are found by dividing the distances AC, AD, and AE by the distance AB. Note that this is the same as dividing S_a for each theory by σ_a.

The quantities shown in the caption of Fig. 8–22 are obtained as follows:

Point A

$$\sigma_i = \frac{F_i}{A_t} = \frac{14.4}{0.226} = 63.72 \text{ kpsi}$$

Point B

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.280(5)}{2(0.226)} = 3.10 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \sigma_i = 3.10 + 63.72 = 66.82 \text{ kpsi}$$

Point C

This is the modified Goodman criteria. From Table 8–17, we find $S_e = 18.6$ kpsi. Then, using Eq. (8–45), the factor of safety is found to be

Answer

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{18.6(120 - 63.72)}{3.10(120 + 18.6)} = 2.44$$

Point D

This is on the proof-strength line where

$$S_m + S_a = S_p \tag{1}$$

In addition, the horizontal projection of the load line AD is

$$S_m = \sigma_i + S_a \tag{2}$$

Solving Eqs. (1) and (2) simultaneously results in

$$S_a = \frac{S_p - \sigma_i}{2} = \frac{85 - 63.72}{2} = 10.64 \text{ kpsi}$$

The factor of safety resulting from this is

Answer

$$n_p = \frac{S_a}{\sigma_a} = \frac{10.64}{3.10} = 3.43$$

which, of course, is identical to the result previously obtained by using Eq. (8–29).

A similar analysis of a fatigue diagram could have been done using yield strength instead of proof strength. Though the two strengths are somewhat related, proof strength is a much better and more positive indicator of a fully loaded bolt than is the yield strength. It is also worth remembering that proof-strength values are specified in design codes; yield strengths are not.

We found $n_f = 2.44$ on the basis of fatigue and the modified Goodman line, and $n_p = 3.43$ on the basis of proof strength. Thus the danger of failure is by fatigue, not by overproof loading. These two factors should always be compared to determine where the greatest danger lies.

Point E

For the Gerber criterion, from Eq. (8–46), the safety factor is

Answer

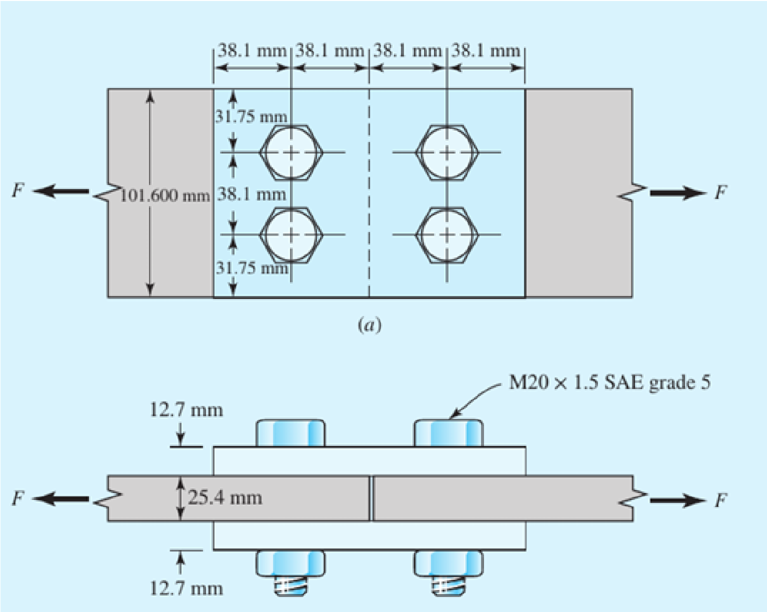
$$n_f = \frac{1}{2\sigma_a S_e}[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e]$$

$$= \frac{1}{2(3.10)(18.6)}[120\sqrt{120^2 + 4(18.6)(18.6 + 63.72)} - 120^2 - 2(63.72)(18.6)]$$

$$= 3.65$$

which is greater than $n_p = 3.43$ and contradicts the conclusion earlier that the danger of failure is fatigue. Figure 8–22 clearly shows the conflict where point *D* lies between points *C* and *E*. Again, the conservative nature of the Goodman criterion explains the discrepancy and the designer must form his or her own conclusion.

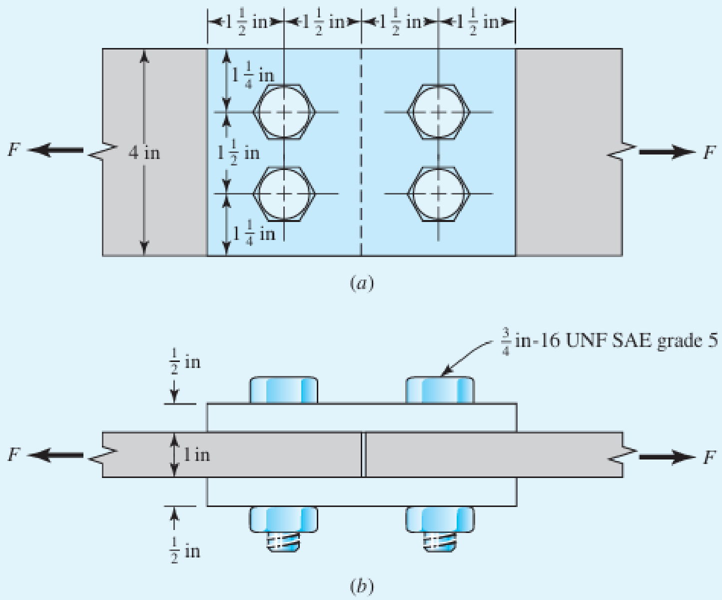
5. Two 25.4 by 101.6 mm 1018 cold-rolled steel bars are butt-spliced with two 12.7 by 101.6 mm 1018 cold-rolled splice plates using four M20 × 1.5 mm (property class 8.8) bolts as depicted in Figure 8–26. For a design factor of nd = 1.5 estimate the static load F that can be carried if the bolts lose preload.



EXAMPLE 8–6 Two 1- by 4-in 1018 cold-rolled steel bars are butt-spliced with two $\frac{1}{2}$- by 4-in 1018 cold-rolled splice plates using four $\frac{3}{4}$ in-16 UNF grade 5 bolts as depicted in Fig. 8–24. For a design factor of $n_d = 1.5$ estimate the static load F that can be carried if the bolts lose preload.

Solution From Table A–20, minimum strengths of $S_y = 54$ kpsi and $S_{ut} = 64$ kpsi are found for the members, and from Table 8–9 minimum strengths of $S_p = 85$ kpsi, $S_y = 92$ kpsi, and $S_{ut} = 120$ kpsi for the bolts are found.

Figure 8–24



$F/2$ is transmitted by each of the splice plates, but since the areas of the splice plates are half those of the center bars, the stresses associated with the plates are the same. So for stresses associated with the plates, the force and areas used will be those of the center plates.

Bearing in bolts, all bolts loaded:

$$\sigma = \frac{F}{2td} = \frac{S_y}{n_d}$$

$$F = \frac{2td\,S_y}{n_d} = \frac{2(1)(\frac{3}{4})92}{1.5} = 92 \text{ kip}$$

Bearing in members, all bolts active:

$$\sigma = \frac{F}{2td} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{2td(S_y)_{\text{mem}}}{n_d} = \frac{2(1)(\frac{3}{4})54}{1.5} = 54 \text{ kip}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$\tau = \frac{F}{4\pi d^2/4} = 0.577\frac{S_y}{n_d}$$

$$F = 0.577\pi d^2\frac{S_y}{n_d} = 0.577\pi(0.75)^2\frac{92}{1.5} = 62.5 \text{ kip}$$

If the bolt threads extend into a shear plane:

$$\tau = \frac{F}{4A_r} = 0.577\frac{S_y}{n_d}$$

$$F = \frac{0.577(4)A_r S_y}{n_d} = \frac{0.577(4)0.351(92)}{1.5} = 49.7 \text{ kip}$$

Edge shearing of member at two margin bolts: From Fig. 8–25,

$$\tau = \frac{F}{4at} = \frac{0.577(S_y)_{\text{mem}}}{n_d}$$

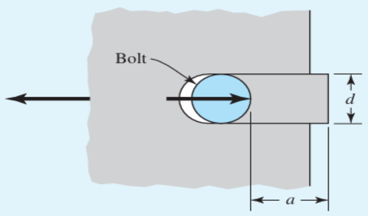
$$F = \frac{4at0.577(S_y)_{\text{mem}}}{n_d} = \frac{4(1.125)(1)0.577(54)}{1.5} = 93.5 \text{ kip}$$

Tensile yielding of members across bolt holes:

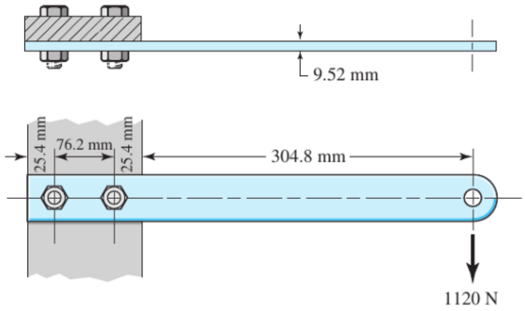
$$\sigma = \frac{F}{[4 - 2(\frac{3}{4})]t} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{[4 - 2(\frac{3}{4})]t(S_y)_{\text{mem}}}{n_d} = \frac{[4 - 2(\frac{3}{4})](1)54}{1.5} = 90 \text{ kip}$$

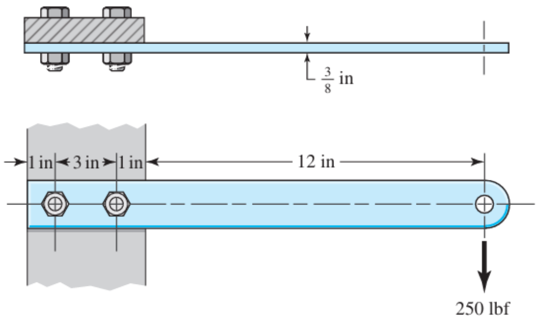
On the basis of bolt shear, the limiting value of the force is 49.7 kip, assuming the threads extend into a shear plane. However, it would be poor design to allow the threads to extend into a shear plane. So, assuming a *good* design based on bolt shear, the limiting value of the force is 62.5 kip. For the members, the bearing stress limits the load to 54 kip.



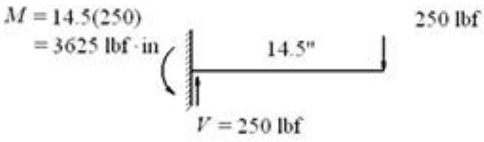
7. A 9.52 × 50.8 mm AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 1120 N as illustrated. The bar is secured to the support using two M10x1.5 (property class 5.8). Assume the bolt threads do not extend into the joint. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.

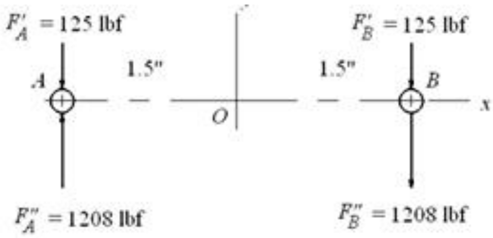


8–77 A $\frac{3}{8}$- × 2-in AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 250 lbf as illustrated. The bar is secured to the support using two $\frac{3}{8}$ in-16 UNC SAE grade 4 bolts. Assume the bolt threads do not extend into the joint. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.



Problem 8–77





$$F''_A = F''_B = \left(\frac{3625}{3}\right) = 1208 \text{ lbf}$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}, \qquad F_B = 1208 + 125 = 1333 \text{ lbf}$$

Bolt shear:

$$A_s = (\pi/4)(0.375^2) = 0.1104 \text{ in}^2$$

$$\tau_{\max} = \frac{F_{\max}}{A_s} = \frac{1333}{0.1104} = 12\ 070 \text{ psi}$$

From Table 8-10, $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{57.7}{12.07} = 4.78 \qquad \textit{Ans.}$$

Bearing on bolt: Bearing area is $A_b = td = 0.375\ (0.375) = 0.1406 \text{ in}^2$.

$$\sigma_b = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\ 481 \text{ psi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{9.481} = 10.55 \qquad \textit{Ans.}$$

Bearing on member: From Table A-20, $S_y = 54$ kpsi. Bearing stress same as bolt

$$n = \frac{S_y}{|\sigma_b|} = \frac{54}{9.481} = 5.70 \qquad \textit{Ans.}$$

Bending of member: At B, $M = 250(13) = 3250$ lbf·in

$$I = \frac{1}{12}\left(\frac{3}{8}\right)\left[2^3 - \left(\frac{3}{8}\right)^3\right] = 0.2484 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\ 080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13 \qquad \textit{Ans.}$$

EXAMPLE 8–3 A $\frac{3}{4}$ in-16 UNF × $2\frac{1}{2}$ in SAE grade 5 bolt is subjected to a load P of 6 kip in a tension joint. The initial bolt tension is F_i = 25 kip. The bolt and joint stiffnesses are k_b = 6.50 and k_m = 13.8 Mlbf/in, respectively.

(*a*) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(*b*) Specify the torque necessary to develop the preload, using Eq. (8–27).

(*c*) Specify the torque necessary to develop the preload, using Eq. (8–26) with $f = f_c = 0.15$.

6. A M20 × 1.5 × 63.5 mm (property class 5.8) bolt is subjected to a load P of 26.69 kN in a tension joint. The initial bolt tension is F_i = 111.205 kN. The bolt and joint stiffnesses are k_b = 1.14 and k_m = 2.42 MN/mm, respectively.

(a) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(b) Specify the torque necessary to develop the preload. Specify the torque necessary to develop the preload.

Solution From Table 8–2, A_t = 0.373 in^2.

(*a*) The preload stress is

Answer

$$\sigma_i = \frac{F_i}{A_t} = \frac{25}{0.373} = 67.02 \text{ kpsi}$$

The stiffness constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.320$$

From Eq. (8–24), the stress under the service load is

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = C\frac{P}{A_t} + \sigma_i$$

Answer

$$= 0.320\frac{6}{0.373} + 67.02 = 72.17 \text{ kpsi}$$

From Table 8–9, the SAE minimum proof strength of the bolt is S_p = 85 kpsi. The preload and service load stresses are respectively 21 and 15 percent less than the proof strength.

(*b*) From Eq. (8–27), the torque necessary to achieve the preload is

$$T = KF_i d = 0.2(25)(10^3)(0.75) = 3750 \text{ lbf} \cdot \text{in}$$

(*c*) The minor diameter can be determined from the minor area in Table 8–2. Thus $d_r = \sqrt{4A_r/\pi} = \sqrt{4(0.351)/\pi} = 0.6685$ in. Thus, the mean diameter is $d_m = (0.75 + 0.6685)/2 = 0.7093$ in. The lead angle is

$$\lambda = \tan^{-1}\frac{l}{\pi d_m} = \tan^{-1}\frac{1}{\pi d_m N} = \tan^{-1}\frac{1}{\pi(0.7093)(16)} = 1.6066°$$

For α = 30°, Eq. (8–26) gives

$$T = \left\{\left[\frac{0.7093}{2(0.75)}\right]\left[\frac{\tan 1.6066° + 0.15(\sec 30°)}{1 - 0.15(\tan 1.6066°)(\sec 30°)}\right] + 0.625(0.15)\right\}25(10^3)(0.75)$$

$$= 3551 \text{ lbf} \cdot \text{in}$$

which is 5.3 percent less than the value found in part (*b*).

TUT 8

4. A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 2980 watts to a 52-tooth disk gear. The module is 2.5 mm, the face width 38 mm, and the quality standard is No. 6. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Assume $J_P = 0.30$, $J_G = 0.40$, $I = 0.121$, $(K_S)_P = (K_S)_G = 1, C_H = 1$, $(C_f)_P = (C_g)_G = 1$. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90.

(a) Find the factor of safety of the gears in bending.

(b) Find the factor of safety of the gears in wear.

EXAMPLE 14–4 A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\,H}{V} = \frac{33\,000(4)}{801.1} = 164.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$\begin{aligned} A' &= 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3}) \\ &= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49 \end{aligned}$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\frac{10}{1.5}\frac{1.22(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_P\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

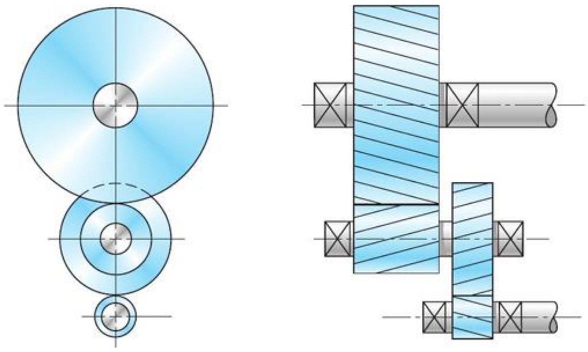
Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

1. As a rough guideline a reduction ratio (speed ratio, gear ratio) of pair of gears should not exceed 10:1. Note reduction ratio = number of teeth on the driven gear/number of teeth of the driver gear. To obtain higher reduction ratios, one uses a compounded gear train. A two-stage compounded gear train is shown below:



a. Select the appropriate number of teeth (20° pressure angle) of the a two-stage compounded gear train to obtain a reduction ratio of 30:1 (±1%). Assume that in each stage a reduction of approximately $\sqrt{30}$ is desired. Also, assume that the number of teeth on the pinion is 16. Note that the minimum number of teeth on a pinion to avoid interference for given speed ratio and pressure angle is

$$N_p = \frac{2}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right),\ m_G = N_G/N_P$$

b. Select the appropriate number of teeth of the a two-stage compounded gear train to obtain an **exact** reduction ratio of 30:1.

EXAMPLE 13–3 A gearbox is needed to provide a 30:1 (±1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate tooth numbers.

Solution Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

Answer

$$16\sqrt{30} = 87.64 \approx 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

EXAMPLE 13–4 A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \qquad \text{and} \qquad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Then

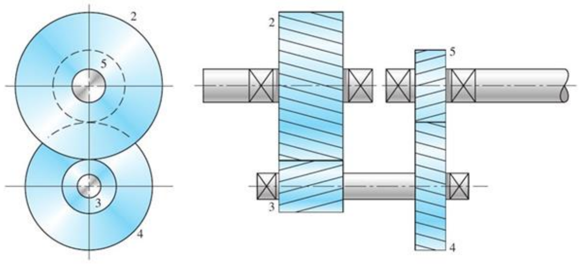
$$N_2 = 6N_3 = 6(16) = 96$$

$$N_4 = 5N_5 = 5(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

2. It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Figure. This configuration is called a **compound reverted gear train**. This requires the distances between the shafts to be the same for both stages of the train.



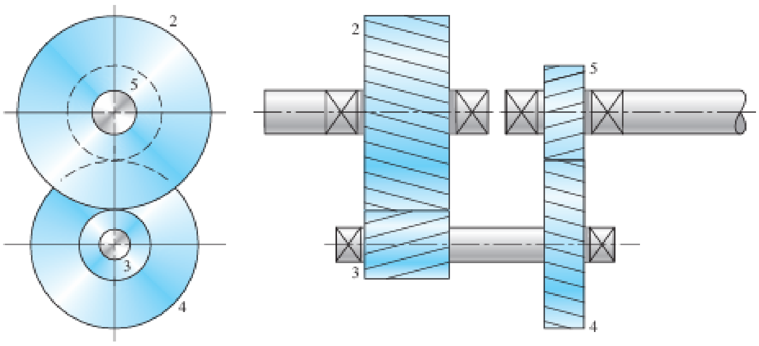
Select the appropriate number of teeth of a compound reverted gear train to obtain an **exact** reduction ratio of 30:1.

It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Fig. 13–29. This configuration is called a *compound reverted gear train*. This requires the distances between the shafts to be the same for both stages of the train, which adds to the complexity of the design task. The distance constraint is

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

29

erted gear



The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

EXAMPLE 13–5 A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$.
Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$

$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$

$$126 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 126/6 = 21$$

$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

$$N_2 = 108$$

$$N_3 = 18$$

$$N_4 = 105$$

$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

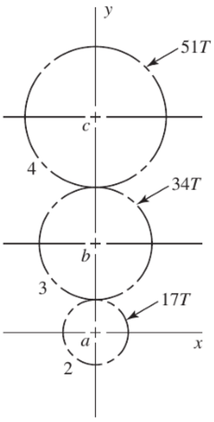
And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$

$$108 + 18 = 105 + 21$$

$$126 = 126$$

3. Shaft a in the figure has a power input of 75 kW at a speed of 1000 rev/min in the counterclockwise direction. The gears have a module of 5 mm and a 20° pressure angle. Gear 3 is an idler.



(a) Find the force F_{3b} that gear 3 exerts against shaft b.
(b) Find the torque T_{4c} that gear 4 exerts on shaft c.

13-31 (a)

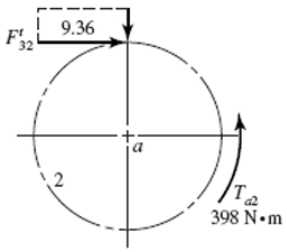
$$\omega = 2\pi n/60$$
$$H = T\omega = 2\pi T n/60 \qquad (T \text{ in N}\cdot\text{m},\ H \text{ in W})$$

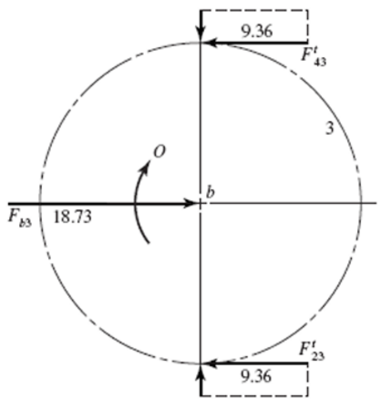
So

$$T = \frac{60H(10^3)}{2\pi n}$$
$$= 9550\ H/n \qquad (H \text{ in kW},\ n \text{ in rev/min})$$
$$T_a = \frac{9550(75)}{1800} = 398\ \text{N}\cdot\text{m}$$
$$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5\ \text{mm}$$

So

$$F_{32}^t = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36\ \text{kN}$$

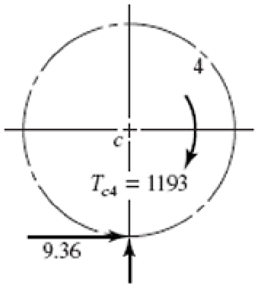




$$F_{3b} = -F_{b3} = 2(9.36) = 18.73\ \text{kN in the positive } x\text{-direction.} \qquad \textit{Ans.}$$

(b)

$$r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5\ \text{mm}$$
$$T_{c4} = 9.36(127.5) = 1193\ \text{N}\cdot\text{m ccw}$$
$$\therefore T_{4c} = 1193\ \text{N}\cdot\text{m cw} \qquad \textit{Ans.}$$



Note: The solution is independent of the pressure angle.

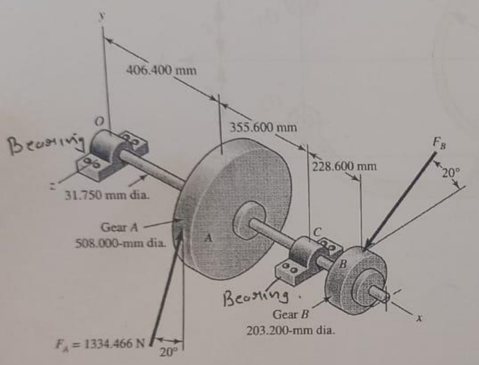
ME423 – Quiz 2

Wednesday
23rd October 2024

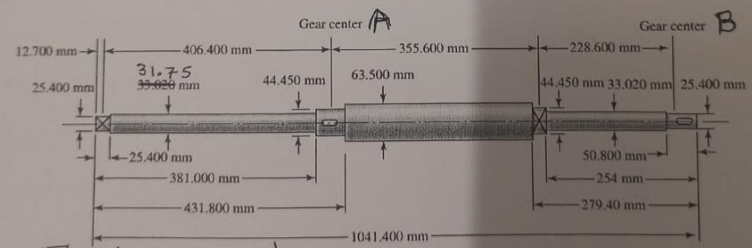
5.30 pm to 7.00 pm
Max Marks: 30

Instructions:
a. There are FOUR questions b. This is an open notes exam

1. Consider a shaft on which gears, A and B, are mounted as shown. The detailed drawing of the shaft is also shown. Select an appropriate parallel key for the gear A and gear B for a factor of safety 1.2. (you need to specify all its dimensions). The standard key sizes available are given in the table. The key material has a yield strength of 372 MPa in tension and compression **(10 marks)**



Shaft Diameter (mm)	Key Width x Height (mm)
$8 < d \le 10$	3 x 3
$10 < d \le 12$	4 x 4
$12 < d \le 17$	5 x 5
$17 < d \le 22$	6 x 6
$22 < d \le 30$	8 x 7
$30 < d \le 38$	10 x 8
$38 < d \le 44$	12 x 8
$44 < d \le 50$	14 x 9
$50 < d \le 58$	16 x 10
$58 < d \le 65$	18 x 11
$65 < d \le 75$	20 x 12
$75 < d \le 85$	22 x 14
$85 < d \le 95$	25 x 14



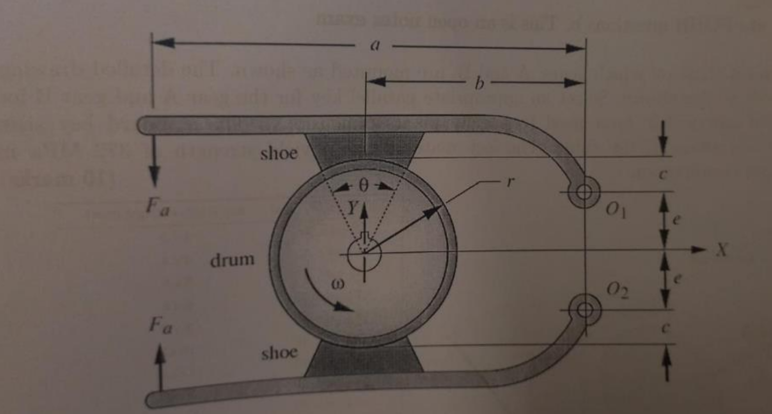
☒ – bearing location.

2. Consider problem 1. One would like to select appropriate ball bearings at the two locations from SKF's catalog with a desired life of 5000 hours for the shaft running at 1500 rpm. Note that SKF rates its bearings for 1 million cycles. Which catalog ratings (C_{10}) would one search in an SKF catalog? Assume a load application factor of 1.2. **(6 marks)**

3. Two ball bearings from different manufacturers are being considered for a certain application. Bearing A has a catalog rating of 2.0 kN based on a catalog rating system of 3000 hours at 500 rev/min. Bearing B has a catalog rating of 7.0 kN based on a catalog that rates at 10^6 cycles. For a given application, determine which bearing can carry the larger load. You need to give proper justification for your answer. **(4 marks)**

Please Turn Over

4. The figure shows a double short-shoe drum brake. Find its torque capacity and required actuating force for a = 90, b = 80, e = 30, r = 40, w = 60 mm (width), and θ = 25°. What value of c will make it self-locking? Assume p_{max} = 1.5 MPa and μ = 0.25. **(10 marks)**



Quiz 2

Soln - 1 For gear A

Shaft diameter $d_A = 44.45$ mm or $r_A = 22.225$ mm

from the given table we get $W \times H = 14 \text{ mm} \times 9 \text{ mm}$ (+1)

~~Torque acting on gear A = $r_A \times F_A \cos 20°$~~

~~$= 0.022225 \times 1384.466 \times 0.9397$~~

Torque acting on gear A $= R_A \times F_A \cos 20°$

$T_A = 0.254 \times 1334.466 \times 0.9397$

$= 318.52$ Nm

(+1) T_A or F_{key}

Force acting on the key $F_{key} = T_A / r_A$

$= 318.52 / 0.022225$

$= 14331.61$ N

Average direct shear $= F_{key} / A_{shear} = F_{key} / L_s W$

$= 14331.61 / L_s \times 14 = 1023.69 / L_s$ N/mm²

for shear failure $\tau_{max} = \dfrac{\sigma_{ys}}{FoS}$ $\left(\begin{array}{l} \sigma_{ys} = \sigma_y/2 \\ \sigma_{ys} = \sigma_y/\sqrt{3} \end{array} \right.$

Using maximum shear stress theory $\sigma_y/2$	Using distortion energy theory $\sigma_y/\sqrt{3}$
$\dfrac{1023.69}{L_s} = \dfrac{372}{1.2 \times 2}$	$\dfrac{1023.69}{L_s} = \dfrac{372}{1.2 \times \sqrt{3}}$
(+1) L_s	
$\Rightarrow L_s = 6.6$ mm	$L_s = 5.72$ mm

Average crushing stress $\sigma_c = F_{key} / A_{crush} = F_{key} / L_c H/2$

$$\Rightarrow \quad \sigma_c = \frac{2 \times 14331.61}{L_c \times 9} = \frac{3184.8}{L_c} \text{ N/mm}^2$$

& for crushing failure $\sigma_{c\,max} = \sigma_y / FOS$

$$\Rightarrow \quad \cancel{318.4} \quad \frac{3184.8}{L_c} = \frac{372}{1.2}$$

+1

or $L_c = 10.27$ mm

∴ length of key for gear A = max (L_s, L_c)

= 10.27mm ~~= 10.27mm~~

+1

for gear B

Shaft diameter = 25.4 mm

key's $W \times H$ = 8×7 mm^2. (+1)

Torque acting would be same & = 318.5 Nm

Forces acting on the key = T / r_{shaft}

F_B = 25078.7 N (+1), [2]

Average direct shear for B : τ_B = F_B / LW

= 3134.84 / L N/mm^2

∴ for shear failure $\tau_B = \sigma_y / \sqrt{3} \times FOS$

$$\Rightarrow \frac{3134.84}{L_s} = \frac{372}{\sqrt{3} \times 1.2}$$

(+1), [3]

L_s = 17.51 mm

using $\left(\frac{\sigma_y}{2}\right)$ L_s = 20.22 mm

Average crushing stress = $\sigma_{B\,crush}$ = $2F_B / LH$

= 7165.34 / L N/mm^2

∴ for crushing failure $\sigma_{B\,crush} = \sigma_y /$ FOS

$$\Rightarrow \frac{7165.34}{L_c} = \frac{372}{1.2}$$

or $L_c = 23.11$ mm +1

$\therefore$ length of key for gear B = $\max(L_s, L_c)$

$= 23.11$ mm. +1

ME 423 Quiz 2

2) Given :- Desired life $= L_{10h} = 5000$ hou

Speed $= N = 1500$ rpm

Load Application factor $= K_a = 1.2$

$F_A = 1334.466$ N

<u>To find</u>:- C_{10} for bearing at 'O' and 'C'

<u>Solution</u>:- For given shaft system.

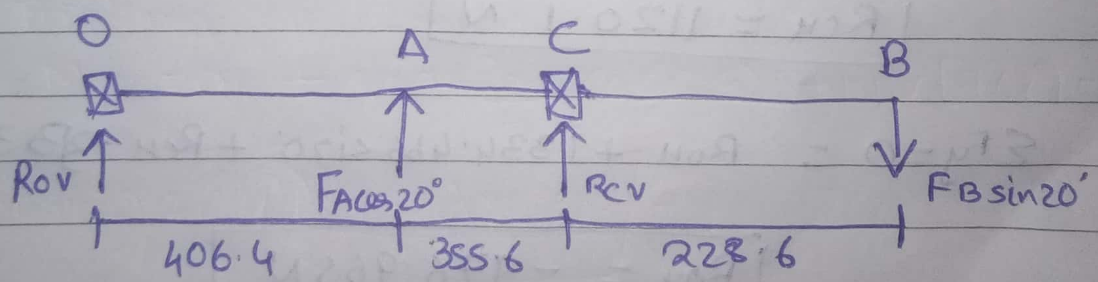
$$\text{Torque A} + \text{Torque B} = 0$$

$$F_A \cos 20^\circ \times \frac{508}{2} - F_B \times \cos 20^\circ \times \frac{203.2}{2} = 0$$

$$F_B = \frac{F_A \times 508}{203.2} = \frac{1334.466 \times 508}{203.2}$$

$$\boxed{F_B = 3336.165\ N}$$

Now vertical loading diagram.



$$\Sigma M_O = F_A \cos 20^\circ \times 406.4 + R_{cv}(762) - F_B \sin 20 (990.6) = 0$$

$$R_{cv} = \frac{3336.165 \times \sin 20 \times 990.6 - 13334.466 \times \cos 20^\circ \times 406.4}{762}$$

$$\boxed{R_{cv} = 3669.018\ N}$$

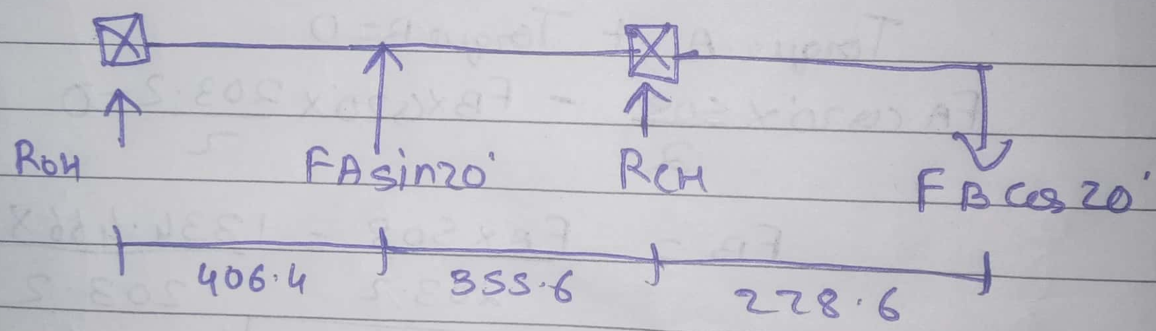
Date

$\Sigma F_y = 0 \quad R_{ov} + F_A \cos 20^\circ + R_{cv} - F_B \sin 20^\circ = 0$

$R_{ov} = 3336.165 \sin 20^\circ - 1334.466 \cos 20^\circ$
$- 3669.08$

$$\boxed{R_{ov} = -1167.85\ N}$$

Horizontal loading diagram :-



$\Sigma M_o = 0 = 1334.466 \times \sin 20^\circ \times 406.4 + R_{CH} \times 762$
$- 3336.165 \times 990.6 = 0$
$\cos 20^\circ$

$$\boxed{R_{CH} = 1120.1\ N}$$

$\Sigma F_y = 0 = R_{OH} + 1334.466 \sin 20^\circ + R_{CH} - 3336.165 \cos 20^\circ = 0$

$$\boxed{R_{OH} = -976.965\ N}$$

$\therefore$ Radial load at 'O' $= \sqrt{R_{OH}^2 + R_{OV}^2}$

$= \sqrt{(-976.965)^2 + (-1167.85)^2}$

$$\boxed{R_O = 1522.61 \text{ N}}$$

Radial load at 'C' $= \sqrt{R_{CH}^2 + R_{CV}^2}$

$= \sqrt{(1120.1)^2 + (3669.018)^2}$

$$\boxed{R_C = 3836.185 \text{ N}}$$

$\therefore$ Equivalent ~~eq.~~ dynamic load on Bearing 'O', 'C'

$P_{eO} = K_a R_O = 1.2 \times 1522.61$

$= 1827.132$ N

$P_{eC} = K_a R_C = 1.2 \times 3836.185$

$= 4603.422$ N

Now for bearing $L_{10} = \dfrac{L_{10h} \times N \times 60}{10^6}$ millia rev

$= \dfrac{5000 \times 1500 \times 60}{10^6}$

$= 450$

$L_{10} = \left(\dfrac{C}{P_e}\right)^3$ for ball bearing.

for Bearing 'O' $\quad 450 = \left[\dfrac{C}{1827.132}\right]^3 \quad C = 14.001$ kN

for Bearing 'C' $\quad 450 = \left[\dfrac{C}{4603.422}\right]^3 \quad C = 35.276$ kN

$a=3$ (roller Bearing)

Q3 Bearing A →

$F_A = 2\,kN$

$L_A = 3000 \times 500 \times 60$ cycles

$= 9 \times 10^7$ cycles

Bearing B

$C_B = 7\,kN$

$L_B = 10^6$ cycles

To compare the both, they need to be rated in terms of the same Catalog rating system. (10^6 cycles)

$$C_A = F_A \times \left(\frac{L_A}{L_B}\right)^{1/a} = 2 \times \left(\frac{9 \times 10^7}{10^6}\right)^{1/3} = 8.96\,kN$$

Catalogue rating of Bearing A for 10^6 cycles is $8.96\,kN$

Catalogue rating of Bearing B for 10^6 cycles is $7.0\,kN$

$\boxed{C_A > C_B}$

∴ Bearing A can carry the larger load.

→ Calculation of number of cycles → 1 mark

→ using $L_{10} C_{10}^3 = $ Constant → 1 mark

→ Finding $(C_{10})_A$ or $L_{10} C_{10}^3 = \boxed{\text{constant}}$ → 1 mark

(calculation of constant)

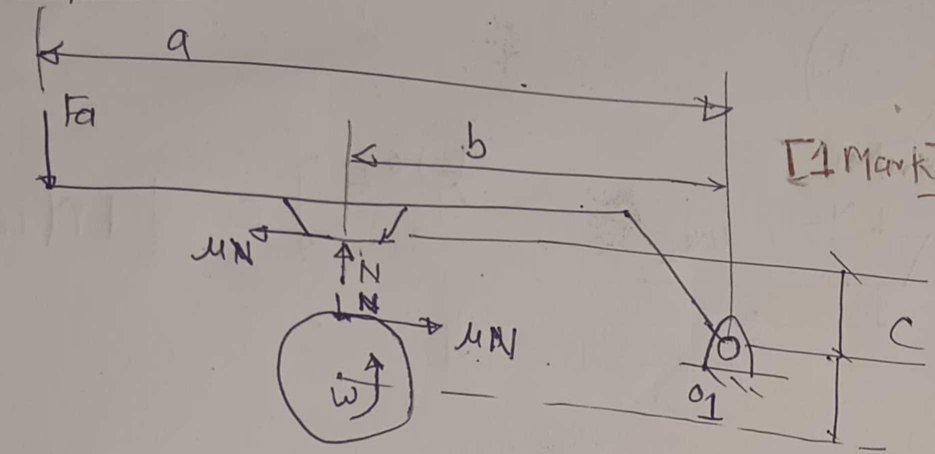
→ Final Comparision of Bearing A & B → 1 mark

[Q.4] Quiz2

Partial FBD

Top Shoe

FBD



[1 Mark]

given

$a = 90$

$b = 80$

$e = 30$

$r = 40$

$w = 60$

$\theta = 25°$

$P_{max} = 1.5\ MPa$

$\mu = 0.25$

① Normal reaction $N = PA = (r\theta w) = 1570.796\,(N)$ → [1 Mark]

② $T_1 = \mu N r = 15.708\,(Nm)$ [1 Mark]

③ $C = r - e = 40 - 30 = 10$

$+\circlearrowleft \Sigma M_{O1} = 0$

$+(Fa \cdot a) - (Nb) + \mu N C = 0\ ;\quad Fa = \dfrac{N(b - \mu c)}{a} = 1352.63\,(N)$

[1 Mark]

④ Self lock

If $\mu c \geq b$

here

$\mu c = 0.25 \times 10 = 2.5$

$b = 80 \Rightarrow \because \mu c \ngeq b \Rightarrow$ no self lock.

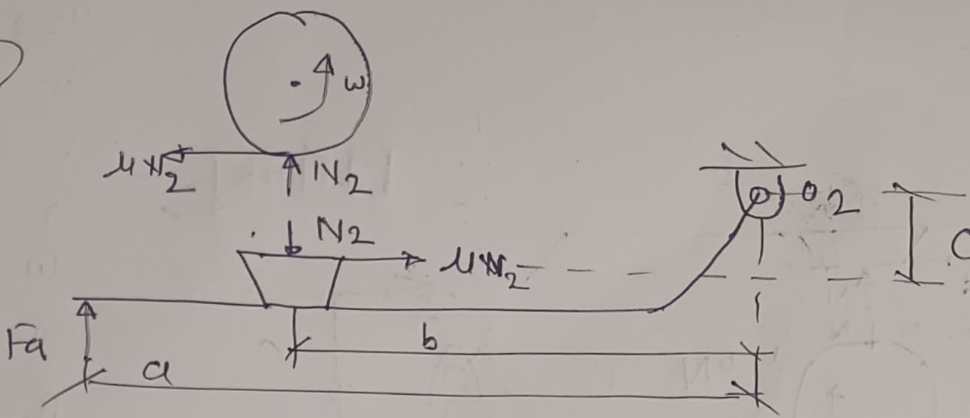
⑤ Value of c for self lock $= b/\mu = 80/0.25 = 320\,(mm)$

[1 Mark]

⑥ bottom Shoe :-

Partial FBD

FBD



[1 Mark]

$+\circlearrowleft \sum M_{O_2} = 0$

$-[Fa \cdot a] + [N_2 \cdot b] + [\mu N_2 c] = 0$;

$Fa = \frac{N_2(b+\mu c)}{a} \Rightarrow N_2 = Fa \cdot \left[\frac{a}{b+\mu c}\right]$

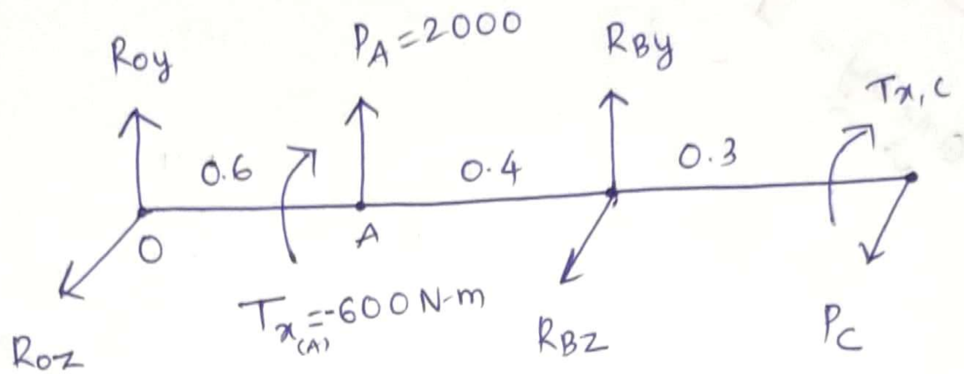
$N_2 = 1475.596$ (N) [1 Mark]

⑦ $T_2 = (\mu N_2 r) = 14.7559$ (Nm) $\simeq 14.756$ (Nm) [1 Mark]

⑧ total torque Capacity $= T_1 + T_2 = 30.464$ (N m) [2 Mark]

ME423 Mid Sem Q1 (Sol'n)

FBD of shaft:



Finding P_c:

As constant Torque is getting transmitted (1M)

$$\sum T_x = 0 \Rightarrow T_{xA} + T_{xB} = 0 \Rightarrow -600 + P_c \times 0.15 = 0$$

$$\Rightarrow P_c = 4000\,N$$

Finding All Reaction forces:

Force Balance (Y Axis) (1M)

$$\sum F_y = 0 \Rightarrow R_{oy} + 2000 + R_{By} = 0 \quad \text{—— ①}$$

Moment Balance (Z axis at O)

from ① & ②

1200 N

Force Balance (z axis)

$$\sum F_z = 0 \Rightarrow R_{oz} + R_{Bz} + 4000 = 0 \quad -③$$

Moment Balance (Y Axis w.r.t O):

$$\sum M_y \Big|_O = 0 \Rightarrow R_{Bz} \times 1 + P_c \times 1.2 = 0 \quad -④$$

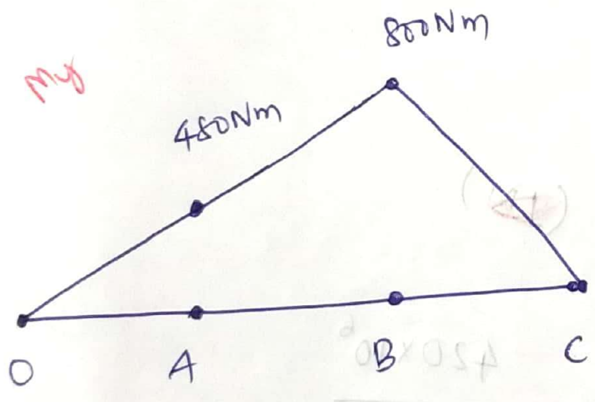
from ③ & ④

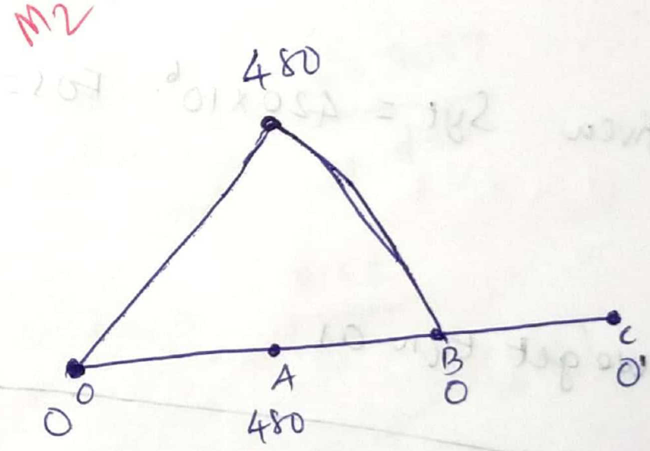
$$\Rightarrow R_{Bz} = -4800 N$$
(1M)
$$R_{oz} = 800 N.$$

(1M)

Finding critical section:

Drawing BMD's





Maximum Bending Moment,

$$M_A = \sqrt{480^2 + 480^2} = 678.82$$

$$M_B = \sqrt{800^2 + 0^2} = 800 Nm \text{ (critical section)}.$$

(a)

Finding Equivalent σ & τ at critical section: (static case)

(1M)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32 \times 800}{\pi d^3} = \frac{8152}{d^3}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 600}{\pi d^3} = \frac{3057}{d^3}$$

if directly Using M & T formula check values of (M & T)

Using D.E.T for find Eq. stress (1M)

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{S_{yt}}{F_{os}}$$

given $S_{yt} = 420 \times 10^6$ $F_{os} = 2$

(~~1M~~)

So we get Eqn as

$$\sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{420 \times 10^6}{2}$$

Upon solving for d

we get $d = 35.9$ mm

1M

(b) Fatigue failure case:

we can get that Moment is completely reversing and Torque is constant at critical section

so, $M_m = 0$ $M_a = M$ & $T_m = T$ and $T_a = 0$. (1M).

we get $\sigma_m = 0$ and $\sigma_a = \frac{8152}{d^3}$. (1M) 1M

and $\tau_m = \frac{3057}{d^3}$ and $\tau_a = 0$.

Using D.E.T we get (1M)

$$(\sigma_{eq})_m = \sqrt{(\sigma_m)^2 + 3(\tau_m)^2} = \sqrt{3}\,\tau_m = \frac{\sqrt{3}\, 3057}{d^3}$$

$$(\sigma_{eq})_a = \sqrt{(\sigma_a)^2 + 3(\tau_a)^2} = \sigma_a = \frac{8152}{d^3}$$ (1M)

Using Goodman Eqn $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{FOS}$ ($S_e = 250\times10^6$ given)

we get Expression as $\frac{8152}{250\times10^6\times d^3} + \frac{\sqrt{3}\times3057}{d^3\times560\times10^6} = \frac{1}{2}$.

$d = 43.8\ mm$ (1M)

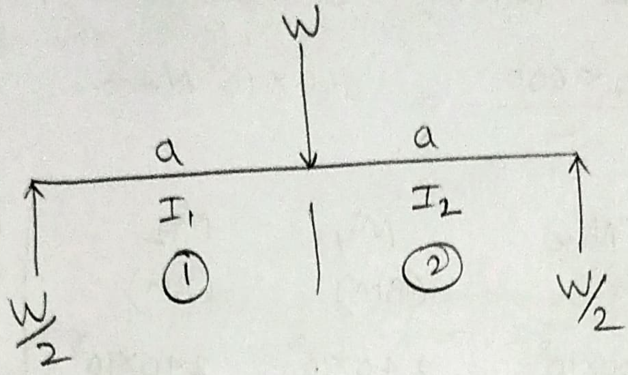
Marking Scheme

Finding Value of P_c	1M
Finding the Reaction Forces	1M
Identifying the Correct Critical Section	1M
Writing Proper Expressions for σ and τ using Correct Values of M and T and Writing Proper expression for Equivalent stress using D.E.T **(or)** Writing Direct Expression for Equivalent D.E.T Stress with Correct Values of T and M	1M+1M
Final Value of d (Diameter)	1M
Identifying that Bending Moment (BM) is Completely Reversed, Torsion is Constant, and Writing Values for M_m, M_a, T_m, T_a	1M
Writing Respective σ and τ, Using D.E.T to Calculate Equivalent Stresses, and Correct Expression for Goodman Equation (or Combined D.E.T and Goodman Equation) with Proper Substitution of M_m, M_a, T_m, T_a	1M + 1M
Final Value of d (Diameter)	1M

Total Marks: 10

Q.2

$I_2 = 2I_1$



FBD + Reactions (1)

$U = U_1 + U_2$

$$U = \int_0^a \frac{M_{x_1}^2\,dx}{2EI_1} + \int_a^{2a} \frac{M_{x_2}^2\,dx}{2EI_2} \quad \text{(1)}$$

$$\frac{\partial U}{\partial W} = \int_0^a \frac{M_{x_1}}{EI_1}\frac{\partial M_{x_1}}{\partial W}\,dx + \int_a^{2a} \frac{M_{x_2}}{EI_2}\frac{\partial M_{x_2}}{\partial W}\,dx$$

$$M_{x_1} = \frac{W}{2} \times x \qquad \frac{\partial M_{x_1}}{\partial W} = \frac{x}{2} \quad \text{(1)}$$

$$M_{x_2} = \frac{W}{2}x - W(x-a) \qquad \frac{\partial M_{x_2}}{\partial W} = \frac{x}{2} - (x-a) \quad \text{(1)}$$

$$\delta = \frac{\partial U}{\partial W} = \int_0^a \frac{W\left(\frac{x}{2}\right)^2 dx}{EI_1} + \int_a^{2a} \frac{W\left(\frac{x}{2} - (x-a)\right)^2 dx}{EI_2} \quad \text{(1)}$$

$$\delta = \int_0^a \frac{Wx^2\,dx}{4EI_1} + \int_a^{2a} \frac{W\left(-\frac{x}{2} + a\right)^2}{2EI_1}\,dx$$

$$\delta = \frac{W}{4EI_1}\left(\frac{x^3}{3}\right)_0^a + \frac{W}{2EI_1}\left.\frac{\left(-\frac{x}{2} + a\right)^3}{-3/2}\right|_a^{2a}$$

$$\delta = \frac{Wa^3}{12EI_1} + \frac{W}{EI_1}\left[0 + \frac{a^3}{8 \times 3}\right) = \frac{Wa^3}{12EI_1} + \frac{Wa^3}{24EI_1}$$

$$\boxed{\delta = \frac{Wa^3}{8EI_1}} \quad \text{(2)}$$

$$W_c = \frac{2\pi \times 9000}{60} \text{ rad/s.} \longrightarrow 1$$

300π , 942.47

$$W = 50 \times 10 = 500N$$

$$\left.\begin{aligned} W_c &= \sqrt{g/\delta} \\ \left(\frac{2\pi \times 9000}{60}\right)^2 &= \frac{10 \times 10^4}{\delta} \end{aligned}\right\} 1$$

$$\Rightarrow \delta = 0.0113 \text{ mm.}$$

$$\delta = \frac{Wa^3}{8EI_1} \qquad a = 300\text{ mm}, \quad E = 200 \text{ GPa},$$

$$\Rightarrow I_1 = 7.4947 \times 10^5 \text{ mm}^4$$

$$I_1 = \frac{\pi}{64} d_1^4$$

$$\Rightarrow \boxed{d_1 = 62.5 \text{ mm}} \text{ — } 1$$

3) a) Objective minimize $M = 2\pi r t L \rho$

s.t $T = \pi r^2 t \sigma_y$

$N_{cr} \geq N_{cr}^*$

+1

b) We have $f = c'\sqrt{\frac{k}{m}}$

For a shaft $N_{cr} = c''\sqrt{\frac{k_{shaft}}{m_{shaft}}}$

We have $\delta = \tilde{c}\frac{PL^3}{EI}$ → deflection of a beam / or shaft

$\therefore k_{shaft} = \frac{P}{\delta} = \tilde{c}\frac{EI}{L^3}$. Mass of the shaft $m_{shaft} = M$.

$N_{cr} = c'\sqrt{\frac{\tilde{c}\,EI}{ML^3}} = c\sqrt{\frac{EI}{ML^3}}$ +3

c) $N \geq N_{cr}^*$

or $N^2 \geq N_{cr}^{*2}$

$\frac{c^2 EI}{ML^3} \geq N_{cr}^{*2}$

Now $I = \pi r^3 t = \frac{\pi r^4 t^2}{rt}$

$= \frac{2T^2 L \rho}{M\sigma_y^2}$

$\frac{c^2 E}{ML^3} \cdot \frac{2T^2 L\rho}{M\sigma_y^2} \geq N_{cr}^{*2}$ +3

$M^2 \leq 2c^2 \frac{T^2}{N_{cr}^{*2}} \frac{1}{L^2} \frac{E\rho}{\sigma_y^2}$

or $M \leq \sqrt{2}\,c\,\frac{T}{N_{cr}^*} \cdot \frac{1}{L} \underbrace{\frac{(E\rho)^{1/2}}{\sigma_y}}_{\text{material index.}}$

For a fixed T, N_{cr}^*, L to minimize M we need to choose a material with as low of $\frac{(E\rho)^{1/2}}{\sigma_y}$ as possible. +1 +1

For the given materials:

	MI.
Mild steel	0.18
High strength steel	0.10
Al alloy.	0.05 ✓ Lowest MI

chosen material Al alloy.

+1 if the material index is correct

4) $\sigma_{ult} = 1000$ MPa, $\sigma_y = 800$ MPa, $\sigma_e' = 500$ MPa. $k_a = 0.679$, $k_b = 1$.

$T_m = 45$ Nm, $M_a = 70$ Nm

$d_r = d - 2r = 0.65D$

$d = 0.8D$.

+7 - for iteration 1. I have shown one way to solve the problem. Another way is to assume dr and then check the FOS.

$$\frac{d_r}{d} = \frac{0.65}{0.8}$$

$$\therefore \frac{d_r}{d} = 1 - \frac{2r}{d}.$$

$$\therefore \frac{r}{d} = \frac{1}{2}\left(1 - \frac{0.65}{0.8}\right) = 0.094 \approx 0.1.$$

$K_t = 1.7$ $K_t = 1.5$

$\therefore$ For the first iteration we will assume that $\underline{\underline{q = 1}}$ (To know q in addition to knowing σ_{ult} & r/d, we also need r. $\therefore [K_f = K_t$ & $K_{fs} = K_t]$.

The minimum diameter is d_r and that is the most critical regions as compared to the locations shown in the figure.

Now $\sigma_e = \underbrace{k_a k_b k_c k_d k_e k_f}_{1} \sigma_e' = 339.5$ MPa.

in these formulae, the diameter to be chosen is the smallest diameter, i.e. dr

Also: $\sigma_a = K_f \dfrac{32 M_a}{\pi d_r^3}$, $\tau_m = K_{fs} \dfrac{16 T_m}{\pi d_r^3}$.

$$\sigma_{eq,a} = \frac{32 K_f M_a}{\pi d_r^3}, \quad \sigma_{eq\,m} = \frac{\sqrt{3}\cdot 16 \cdot K_{fs} T_m}{\pi d_r^3}.$$

We have $\dfrac{\sigma_{eq\,a}}{\sigma_e} + \dfrac{\sigma_{eq\,m}}{\sigma_{ult}} = \dfrac{1}{FOS}$.

Solving for d_r; $d_r = 20.27$ mm., $D = 31.19$ mm

$d = 24.95$ mm, $r = 2.34$ mm.

For the second iteration, we calculate K_b using d_r & q using d & r. We find FOS & if it is close to 2 we accept the values of d_r, D, d & r. Else using FOS, K_b, & q we calculate new d_r & repeat the above procedure. [In our case FOS = 1.9] +3

If we calculate d_r, then $d_r = 20.62$ mm. To be on the safe we can choose $d_r = 20.62$ mm ≈ 21.0 mm.

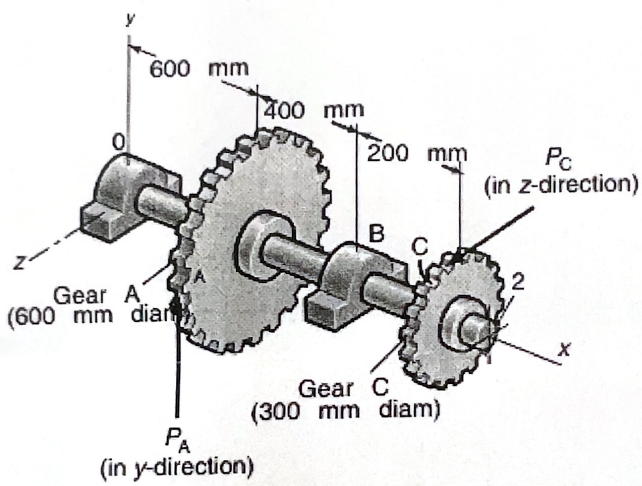
ME423 – Midsemester Exam

Saturday
21st September 2024

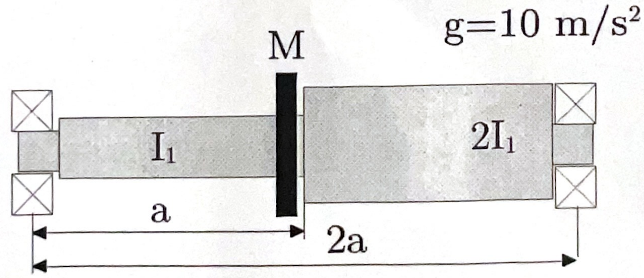
1.30 pm to 3.30 pm
Max Marks: 40

Instructions:
a. There are FOUR questions b. This is an open notes exam

1. The rotating solid steel shaft of constant diameter shown in the figure has two gears mounted on it and is supported by bearings at points O and B. The gears transmit a constant torque caused due to P_A = 2000 N acting vertically as shown. The shaft is machined from steel with yield strength = 420 MPa and ultimate tensile strength = 560 MPa. The corrected endurance limit is 250 MPa. Using a factor of safety of 2.0, determine the minimum allowable diameter of the shaft based on (a) a static yield analysis using the distortion energy theory and (b) a fatigue-failure analysis (Goodman + distortion energy theory). **(10 marks)**



2. The figure shows a stepped shaft which is simply supported and on which a pulley of mass $M = 50$ kg is mounted. The moment of inertia's of the two sections are $I_1 = \frac{\pi}{64}d^4$ and $I_2 = 2I_1$. Calculate d that ensures that the first critical speed is at least 9000 rpm.



The shaft is made of steel with E = 200 GPa. The distance a = 300 mm and the mass of the shaft is neglected. The effect of transverse shear is also neglected. You can assume $g = 10\ m/s^2$. **(10 marks)**

3. One would like to select the material for a **light** thin walled hollow circular transmission shaft (mean radius r and thickness t). The shaft has to be designed in such a way that its critical speed is at least N^*_{cr} rpm. It should also be able to transmit a torque T without yielding. To proceed: **(10 marks)**

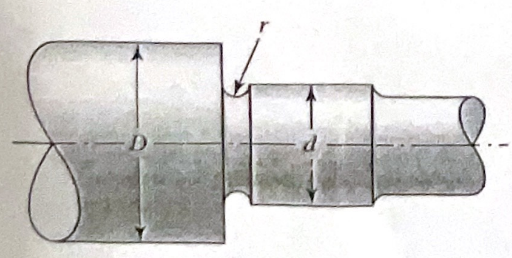
- Clearly identify the objective, the equality constraint and the inequality constraint.
- Show that the critical speed of the shaft is of the form

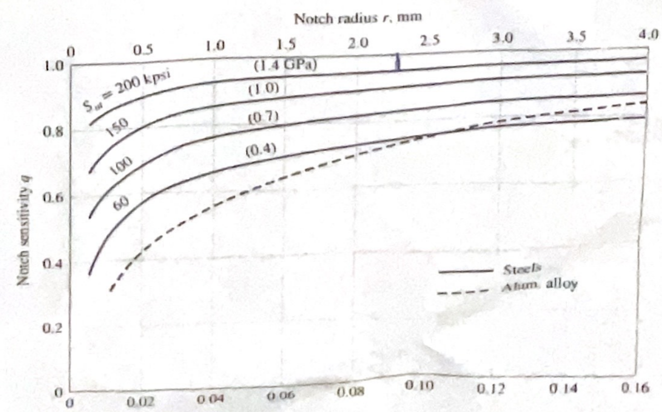
$$N_{cr} = c\sqrt{\frac{EI}{ML^3}}$$

  Here c is a constant, E is the Young's modulus, I is the moment of inertia, M mass of the shaft and L is the length of the shaft. Hint: The natural frequency of oscillation of an elastic system is of the form $f = c'\sqrt{k/m}$ where c' is a constant, k is its stiffness and m is its mass.
- Start with the inequality constraint. Express the moment of inertia of the cross-section, I, in terms of the torque carrying capacity T, yield strength σ_y, length L of the shaft, density ρ and mass M of the shaft. Note that for a shaft of length L with a thin circular cross-section with mean radius r and thickness t, $M = 2\pi rtL\rho$, moment of inertia $I = \pi r^3 t$ and torque carrying capacity is $T = \pi r^2 t\sigma_y$.
- Use of the expression of I derived above to find the material index. Hint: This expression will be in a form of inequality which includes M, L, ρ, E, σ_y, T , N^*_{cr}
- Based on the material index derived, choose the appropriate material from table given below.

Material	E (GPa)	ρ (kg/m3)	σy (MPa)
Mild steel	210	7800	220
High strength steel	210	7800	400
Aluminum alloy	70	2700	300

4. A section of shaft shown in the figure. The shaft is made steel with $\sigma_{ult} = 1000$ MPa and $\sigma_y = 800$ MPa and is subjected to completely reversed bending and steady torsion. The uncorrected endurance limit is 500 MPa and the surface finish factor for all surfaces is $k_a = 0.679$. At the location shown, $M_a = 70\ Nm$ and $T_m = 45\ Nm$. The diameter at the root of the groove is $d_r = d - 2r$, where d is the shoulder diameter. The relative sizes of the diameters are as follows: $d_r = 0.65D$ and $d = 0.8D$. One needs to size the shaft for an infinite life using the DE-Goodman criterion with a factor of safety 2.0. **(10 marks)** Show **only the first iteration** of your calculations, i.e. find d, d_r, D: you can assume $k_b = 1$. Then clearly **describe the methodology in WORDS** to obtain the final design. You also need to highlight the changes that you anticipate that need to be made from the first iteration. In case you feel some information is missing, make appropriate assumptions and clearly state them.





Stress Concentration Factors for the first iteration

	Bending	Torsional
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5