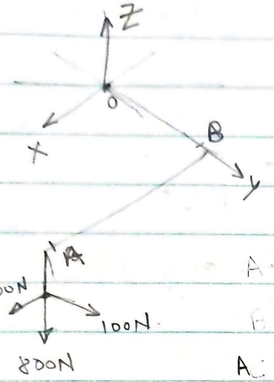
Problem 1

ME 423 - Quiz 1

1a. The internal resisting forces and moments at different locations are calculated using:



$$\underline{F}_R + \underline{F}_A = \underline{0}$$

$\underline{F}_R$ & $\underline{M}_R$ are internal resisting forces & moments

$$\& \quad \underline{M}_R + \underline{r} \times \underline{F}_A = \underline{0}$$

$$\underline{F}_A = 1500\hat{i} + 100\hat{j} - 800\hat{k}$$

$$\underline{F}_R = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\underline{M}_R = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

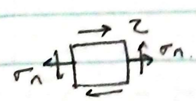
	F_x (N)	F_y (N)	F_z (N)	M_x (Nm)	M_y (Nm)	M_z (Nm)
A	−1500	−100	800	−5 (T)	75	0
B	−1500	−100	800	−5	−525 (T)	−75
O	−1500	−100	800	235	−525 (T)	375

From the table it is seen that the most critical section is at pt O.

We will ignore the effect of direct shear due to F_x & F_z. M_y represent the torque @ Y-axis. M_x represents bending moment @ x-axis & M_z represents bending @ z axis. The resultant bending moment is $\tilde{M} = \sqrt{M_y^2 + M_x^2}$

$$\sigma_{b|max} = \frac{32\tilde{M}}{\pi d^3} \qquad \sigma_n|_{axial} = \frac{4F_y}{\pi d^2}$$

$$\sigma_n = \sigma_{b|max} + \sigma_n|_{axial} = \frac{32\tilde{M}}{\pi d^3} + \frac{4F_y}{\pi d^2}.$$



$$\tau_{max} = \frac{16M_y}{\pi d^3}$$

$$\sigma_{vm} = (\sigma_n^2 + 3\tau_{max}^2)^{1/2}.$$

To find the minimum required diameter

$$\sigma_{vm} = \frac{\sigma_y}{FOS} \quad \text{or} \quad \sigma_{vm}^2 = \frac{\sigma_y^2}{FOS^2}$$

$$\therefore \sigma_n^2 + 3\tau_{max}^2 = \frac{\sigma_y^2}{FOS^2}$$

$$\therefore \left(\frac{32\tilde{M}}{\pi d^3} + \frac{4P}{\pi d^2}\right)^2 + 3\left(\frac{16M_y}{\pi d^3}\right)^2 = \frac{\sigma_y^2}{FOS^2} \qquad (A)$$

Solve the above equation (A) for d. It is not easy to solve unless you have the "right" calculator. To simplify, we assume that $\frac{4P}{\pi d^2} << \frac{32\tilde{M}}{\pi d^3}$. We will verify the assumption at the end. Then we get.

(can be solved using any calculator)

$$\left(\frac{32\tilde{M}}{\pi d^3}\right)^2 + 3\left(\frac{16M_y}{\pi d^3}\right)^2 = \frac{\sigma_y^2}{FOS^2} \qquad (B).$$

Solving B (approx): $d = 33.3012$ mm ≈ 33.3 mm

Solving A (exact): $d_2 = 33.3063$ mm ≈ 33.3 mm.

We see that the difference in ansr is about 0.02%.

$d = 33.3063$ mm.

For $\sigma_{axial} = \pm 0.11$ MPa.

$\sigma_b = 122.00$ MPa.

$\tau = 72.37$ MPa

$\therefore \sigma_{axial} << \sigma_b$.

$\sigma_{axial} << \tau$.

Hence our assumption of neglecting σ_a is justified. This is true in almost all problem of combined loading.

Problem 2

2) ① $(\sigma_{max})_1 = 340$ MPa, $(\sigma_{min})_1 = 160$ MPa, $n_1 = 8 \times 10^4$.
② $(\sigma_{max})_2 = 320$ MPa, $(\sigma_{min})_2 = -200$ MPa $n_2 = ?$
$\sigma_y = 350$ MPa, $\sigma_{ult} = 420$ MPa, $f = 0.9$ $\sigma_e = 175$ MPa

Miners rule $\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1$

$N_i \rightarrow$ number of cycles for failure at fully reversed (ie $\sigma_m = 0$) cycle with amplitude σ_{a_i}

Equivalent fully reversed stress amplitude for ①.

$$(\sigma_{a_1}' = \frac{\sigma_{a_1}}{1 - \frac{\sigma_{m_1}}{\sigma_{ult}}} \qquad \sigma_{a_1} = 90 \text{ MPa} \quad \sigma_{m_1} = 250 \text{ MPa}$$

$$= 222.35 \text{ MPa}$$

$$\sigma_{a_2}' = \frac{\sigma_{a_2}}{1 - \frac{\sigma_{m_2}}{\sigma_{ult}}} \qquad \sigma_{a_2} = 260 \text{ MPa} \quad \sigma_{m_2} = 60 \text{ MPa}$$

$$= 303.33 \text{ MPa}$$

Need to find N_1 & N_2 corresponding to σ_{a_1}' & σ_{a_2}' from the Basquin eqn.

$$\sigma_a = \sigma_f' (2N)^b$$

$$b = \frac{\ln(\sigma_a' / \sigma_e)}{\ln(10^3 / 10^6)} = \frac{\ln(f\sigma_{ult} / \sigma_e)}{\ln(10^3 / 10^6)} = -0.11148$$

$$\sigma_f = \frac{\sigma_e}{(2 \times 10^6)^b} = 882.07 \text{ MPa}$$

$$\therefore N_1 = \frac{1}{2}\left(\frac{\sigma_{a_1}'}{\sigma_f}\right)^{\frac{1}{b}} = 116705.75$$

$$N_2 = \frac{1}{2}\left(\frac{\sigma_{a_2}'}{\sigma_f}\right)^{\frac{1}{b}} = 7198.8558$$

Now $n_2 = N_2\left(1 - \frac{n_1}{N_1}\right) \approx 2264$ cycles.

Note that if you had used $(\sigma_{max})_1 = 360$ MPa, then $\frac{n_1}{N_1} > 1$

Problem 3

3. For a cantilever beam: $\delta = \frac{PL^3}{3EI}$. For a rectangular c/s $I = \frac{bh^3}{12}$

$$\therefore \quad \delta = \frac{4PL^3}{Ebh^3}$$

or $$P = \frac{Ebh^3\delta}{4L^3}$$

$$P_{max} = \frac{Ebh^3\delta_{max}}{4L^3}, \quad P_{min} = \frac{Ebh^3\delta_{min}}{4L^3}$$

$$\sigma_{max} = \frac{M_{max}\, h/2}{I} = \frac{(P_{max}L)\, h/2}{I} = \frac{3}{2}\frac{Eh\delta_{max}}{L^2}$$

Similarly $$\sigma_{min} = \frac{3}{2}\frac{Eh\delta_{min}}{L^2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max} - \delta_{min})$$

$$\sigma_m = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max} + \delta_{min})$$

Substituting $E = 200$ GPa, $h = 4$ mm, $L = 300$ mm, $\delta_{max} = 20$ mm, $\delta_{min} = 10$ mm

$\sigma_a = 66.66$ MPa, $\sigma_m = 200$ MPa.

Goodman criterion for infinite life

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n}$$

Substituting the above values with $\sigma_e = 0.5\,\sigma_{ult}$, $n = 1.4 > 1$

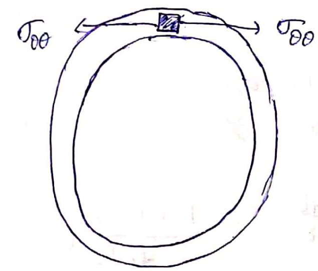
$\therefore$ Hence the factor of safety based on infinite life as per Goodman's criterion is 1.4.

Note: No correction factor is assumed for the endurance limit but not was mentioned/given. In practice, one does need to use the corrections factors to modify the uncorrected endurance strength

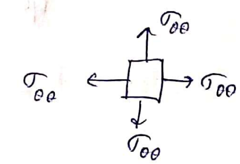
QUIZ-1

PRATMIK REDDY
17D100022

(1) Cut the sphere diametrically using a plane

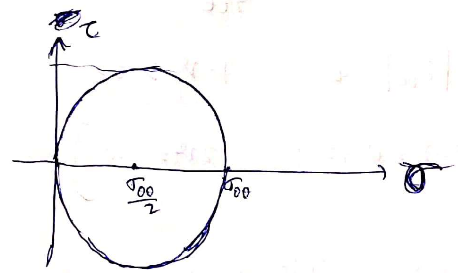


Top view:-



There is no r-r component of stress

$$\sigma_{\theta\theta} \times 2\pi r t = \pi r^2 \times p \Rightarrow \sigma_{\theta\theta} = \frac{Pr}{2t}$$



Max Tensile stress $= \sigma_{\theta\theta}$

Max shear stress $= \frac{\sigma_{\theta\theta}}{2}$

↳ In 45° plane.

Given Yield values :- In tension = 140,000 psi
In shear = 65,000 psi

~~If it yields first~~

a) Shear yield stress is less than half of Tensile yield stress. But ~~Ten~~ Max. Tensile stress is twice the Max Shear stress.

⇒ Shear yield will happen first if the failure happens by yield.

If it happens by yield, :-

$$\frac{Pr}{4t} = \left(\frac{\text{Shear yield stress}}{\text{FOS}}\right)$$

$$\Rightarrow P_1 = \frac{65000}{2.75} \times \frac{4t}{r} = \frac{130000}{11} t$$

If the normal Strain limit reaches first :- strain in rr :- $\frac{-Pr}{2tE} \times 2\nu$

Strain in θθ :- $\frac{Pr}{2tE}(1-\nu)$

$$\Rightarrow |\varepsilon_{rr}| < |\varepsilon_{\theta\theta}| \quad \text{as} \quad 1-\nu > 2\nu$$

$$\left[\because \; 1-\nu = 0.72 \;;\quad 2\nu = 0.56\right]$$

⇒ First Brittle failure if it happens, happens through θθ - direction

$$\Rightarrow \frac{Pr}{2tE}(1-\nu) = 1000\times10^{-6}$$

$$\Rightarrow P = \frac{1000\times10^{-6} \times 2\times E \times t}{(1-\nu)\times r}$$

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$$\Rightarrow P_2 = \frac{10^{-3} \times 2\times 30\times10^{6}}{0.72\times8} t \neq \cancel{\frac{250000}{3} t}$$

$$= \frac{31250}{3} t$$

$$P_2 < P_1$$

⇒ Failure will happen because of normal strain limit i.e; Brittle failure.

b) Min. permissible thickness :-

$$P_2 = \frac{31250}{3} t \Rightarrow \cancel{t = 3P_2 \times 31250}$$

$$t = \frac{3P_2}{31250} \Rightarrow t = \frac{3\times3000}{31250} \text{ in}$$

$$\Rightarrow t = 0.288 \text{ in}$$

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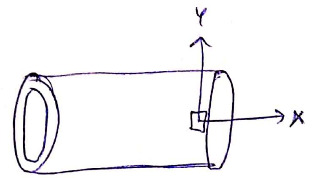
(2)

i) Failure will occur likely at either A or B.



Y → tangential direction

X → Cylinder's Axis direction.

~~Given P = 3.5 MPa (But required 1.5 times the operating pressure~~

$$\boxed{I = \pi r^3 t}$$

POINT A :-

$d_z = 250\,mm$; $P = 3.5\,MPa$; $r_{out} = 125\,mm$.

$r_{in} = 125\,mm - 6\,mm = 119\,mm$.

Due to pressure and Bending,

$$\sigma_x = \frac{M r_{out}}{I} + \frac{P r_1}{2t}$$

$$= \frac{(45\times1000)(750\times10^{-3})\times125\,mm}{(6.846\times10^{7}\,mm^4)} + \frac{(3.5\,MPa\times119\,mm)}{2\times6\,mm}$$

$$= 158\,MPa.$$

$$\sigma_y = \frac{P r_1}{t} = \frac{(3.5\,MPa)\times(119\,mm)}{6\,mm}$$

$$= 69.4\,MPa.$$

$$\tau_{xy} = \frac{T r_2}{J} = \frac{(45000\,N \times 125\,mm)\times(125\,mm)}{6.846\times10^{7}\,mm^4} = 10.27\,MPa.$$

At B there is no Bending as it is on neutral axis.

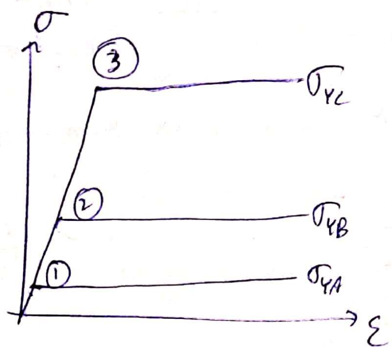
At A, there will be Maximum Bending load.

Every other stress is same between A and B

⇒ Stress at A > Stress at B.

⇒ Failure is likely to occur at A.

(3) All the tie-rods have same E [Young's Modulus]



Order of yielding:-

A, B, C

Deflection will be same in all the rods.

$\delta_A = \delta_B = \delta_C \Rightarrow \varepsilon_A = \varepsilon_B = \varepsilon_C$ [As L is same].

$F_A + F_B + F_C = F$

$\sigma_A \times A_A + \sigma_B \times A_B + \sigma_C \times A_C = F$

a) when A yields, $\sigma_A = \sigma_{YA}$

until yielding, $\sigma_A = E\varepsilon_A$, $\sigma_B = E\varepsilon_B$, $\sigma_C = E\varepsilon_C$

But, $\varepsilon_A = \varepsilon_B = \varepsilon_C$

$\Rightarrow \sigma_A = \sigma_B = \sigma_C = E\varepsilon$

$\Rightarrow \varepsilon = \dfrac{\sigma_{YA}}{E} = \dfrac{50\times10^6 \text{ Pa}}{100\times10^9 \text{ Pa}} = 5\times10^{-4}$

$\Rightarrow F = 50\times10^6 \times [5+3+1] \times 10^{-4}$

$= 50\times9\times10^2 = 45\times10^3 N.$

Deflection is $\delta = \varepsilon \times l$

$= 5\times10^{-4} \times \frac{1}{10} m = 0.05 mm$

b) when B yields,

Here, $\sigma_A = \sigma_{YA}$, $\sigma_B = \sigma_{YB}$, $\sigma_C = \sigma_{YB}$ [Hasn't yielded yet]

$\Rightarrow \varepsilon_B = \varepsilon_C = \dfrac{\sigma_{YB}}{E} = \varepsilon_A$

$F = \sigma_A A_A + \sigma_B A_B + \sigma_C A_C$

$\Rightarrow F = \sigma_{YA} \times A_A + (\sigma_{YB})(A_B + A_C)$

$\Rightarrow \varepsilon = \dfrac{\sigma_{YB}}{E} = \dfrac{100\times10^6}{100\times10^9} = 10^{-3}$ $\Rightarrow \delta = 10^{-3} \times \frac{1}{10} = 0.1 mm$

$\Rightarrow F = 50\times10^6 \times 5\times10^{-4} + 100\times10^6 (3+1) \times 10^{-4}$

$\Rightarrow F = 65000 N = 65\times10^3 N.$

c) when C yields,

$\sigma_A = \sigma_{YA}$, $\sigma_B = \sigma_{YB}$, $\sigma_C = \sigma_{YC}$

$\Rightarrow \varepsilon_C = \dfrac{\sigma_{YC}}{E} = \dfrac{400\times10^6}{100\times10^9} = 4\times10^{-3}$

$\Rightarrow \delta = 4\times10^{-3} \times \frac{1}{10} = 0.4 mm$

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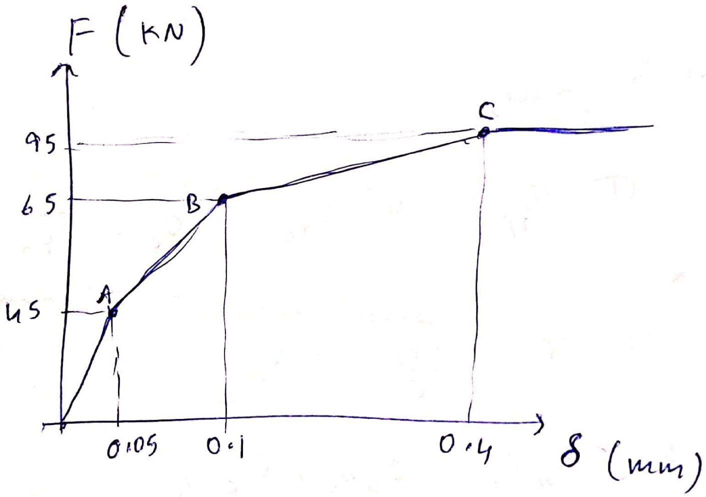
$$\Rightarrow F = \sigma_A A_A + \sigma_B A_B + \sigma_C A_C$$

$$= (50\times10^6 \times 5\times10^{-4}) + (100\times10^6 \times 3\times10^{-4}) + (400\times10^6 \times 1\times10^{-4})$$

$$= 95\times10^3\ N$$

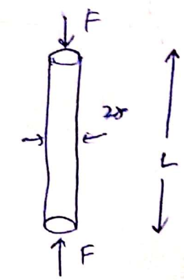


(4)

Given is the slender rod [legs of table]

Function:-	Table leg [should bear compressive load].
Objective:-	* Minimizing mass * Improving slenderness (minimize $\frac{r}{L}$)
Constraints:-	* L is given. * Avoid Buckling
Free variables:-	Radius of leg, Material.

$$m = \pi r^2 L \times \rho$$



* For Buckling, critical load will be :-

$$F_{Critical} = \frac{\pi^2 E I}{L^2}$$

where $I = \frac{\pi r^4}{4}$

Given load F must be below $F_{critical}$

$$\Rightarrow F < \frac{\pi^2 E}{L^2} \times \frac{\pi r^4}{4} = \frac{\pi^3 E r^4}{4L^2}$$

But, $r^2 = \frac{m}{\pi L \rho} \Rightarrow r^4 = \frac{m^2}{\pi^2 L^2 \rho^2}$

$$\Rightarrow F < \frac{\pi^3 E}{4L^2} \times \frac{m^2}{\pi^2 L^2 \rho^2}$$

$$\Rightarrow F < \frac{\pi}{4L^4} \times \frac{E}{\rho^2} m^2$$

$$\Rightarrow m > \sqrt{\frac{4F}{\pi}} L^2 \times \left(\frac{\rho}{E^{1/2}}\right)$$

$$\Rightarrow m > \sqrt{\frac{4F}{\pi}} \times L^2 \times \frac{1}{M_1} \quad \text{where} \quad \boxed{M_1 = \frac{E^{1/2}}{\rho}}$$

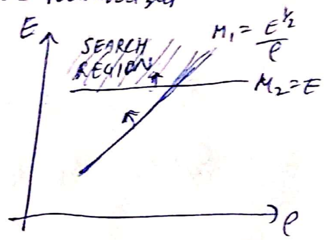
⇒ To minimize mass, M_1 needs to be maximized

Also, $F < \frac{\pi^3 E r^4}{4L^2} \Rightarrow r > \left(\frac{4FL^2}{\pi^3}\right)^{1/4} \times \left(\frac{1}{E}\right)^{1/4}$

$$\Rightarrow r > \left[\frac{4FL^2}{\pi^3}\right]^{1/4} \times \left[\frac{1}{M_2}\right]^{1/4} \quad \text{where} \quad \boxed{M_2 = E}$$

⇒ To minimize radius, M_2 needs to be maximized

From the Material selection chart, plotting two Material indices,



Choices of materials can be :-

	Material:	$M_1 = \frac{E^{1/2}}{\rho}$ $\left[\frac{(GPa)^{1/2}}{kg/m^3}\right]$	$M_2 = E$ [GPa]	Comments:
Best choice without cost	Mg Alloys	0.115	44.8	Very low M_1
	Al Alloys	0.327	70	Very low M_1
	CFRP	6.201	96	Best acc. to Indices but very costly.
Best choices with cost	GFRP	2.211	21	Very low M_2
	Wood.	4.234	9.3	Consider cost, wood is the best material after CFRP for M_1