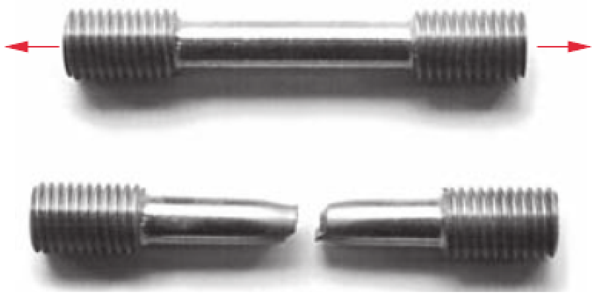
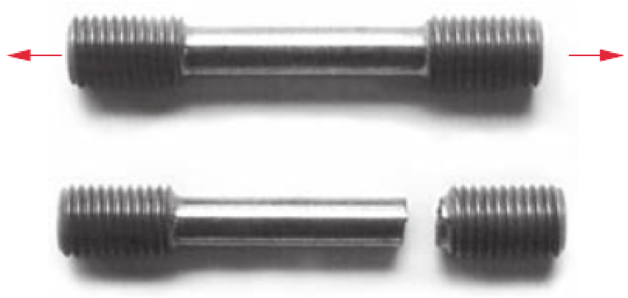
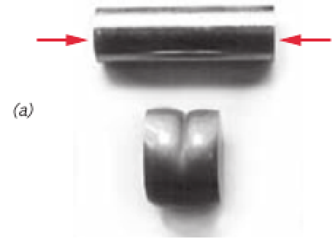
Types of Failure for Ductile and Brittle Material



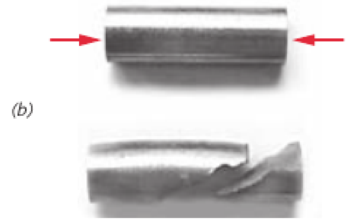
Ductile Specimen
Tensile Load



Brittle Specimen
Tensile Load

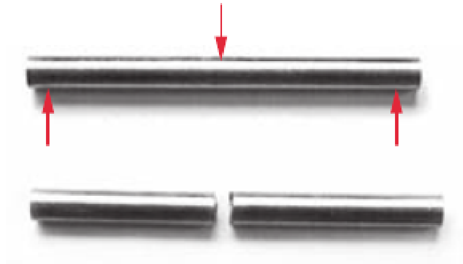


Ductile Specimen
Compressive Load



Brittle Specimen
Compressive Load

Ductile Specimen
Bending Load



Brittle Specimen
Bending Load

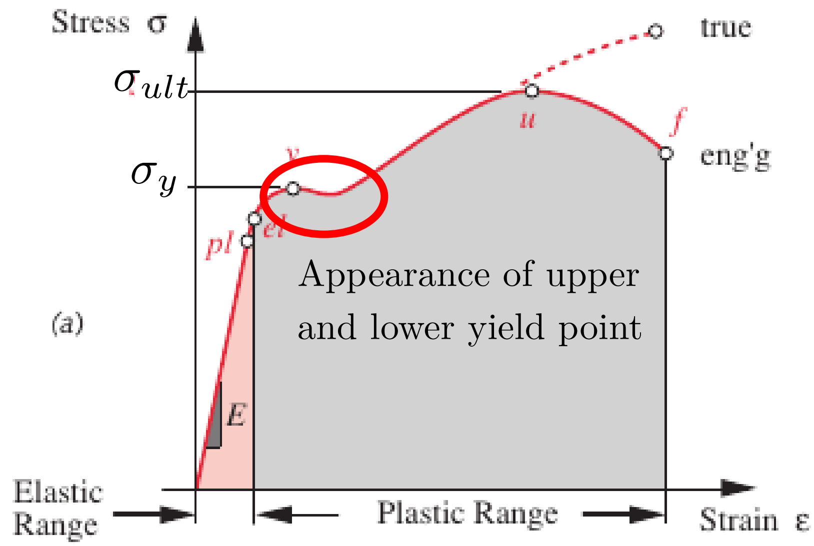
Ductile Specimen
Torsional Load

Brittle Specimen
Torsional Load

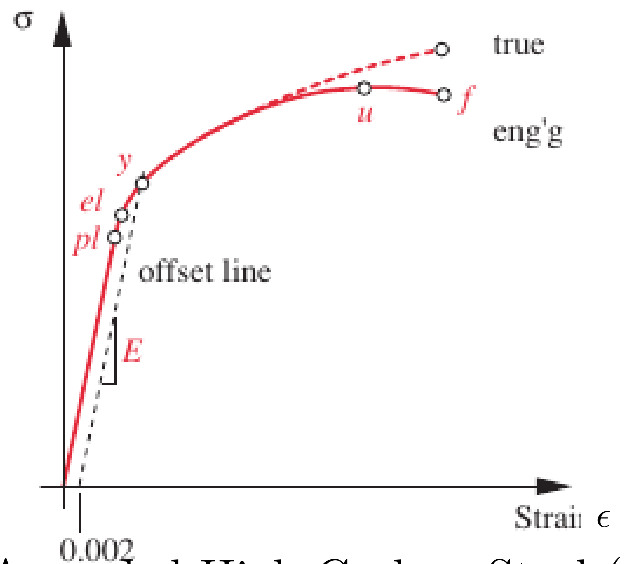
Yielding and **distortion** is associated with **ductile** material

Fracture and **separation** is associated with **brittle** material

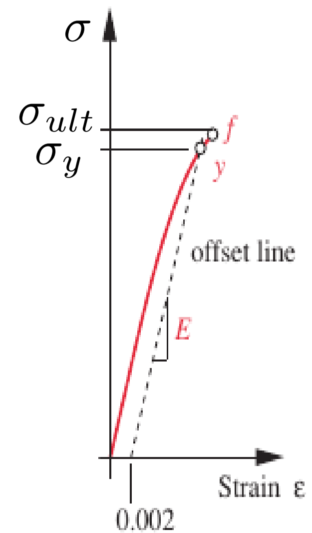
Stress-Strain Curves for Ductile and Brittle Materials



Low Carbon Steel (very ductile)



Annealed High Carbon Steel (less ductile)



Cast Iron (brittle)

Engineering stress $\sigma = \dfrac{P}{A_o}$

Engineering strain $\epsilon = \dfrac{L - L_o}{L_o}$

A_o Original c/s of the specimen

L_o Original gauge length

L Current gauge length

pl – proportional limit

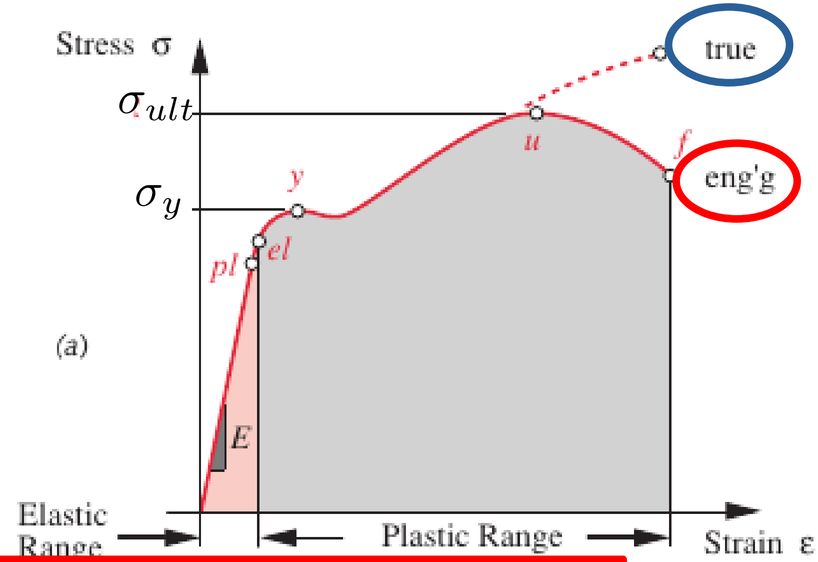
el – elastic limit

y – yield point

u – ultimate tensile strength point

f – fracture point

Engineering Stress and Strain Vs True Stress and Strain



$$\boxed{\text{Engineering stress} \quad \sigma = \frac{P}{A_o} \qquad \text{Engineering strain} \quad \epsilon = \frac{L - L_o}{L_o}}$$

A_o Original c/s of the specimen
L_o Original gauge length
L Current gauge length
A Current c/s of the specimen

- Engineering stress and strain do not give a true indication of the deformation characteristics of the material as they are based on the original dimensions of the specimens and these dimensions change continuously during the test.
- True stress and strain are based on instantaneous dimensions

$$\boxed{\text{True stress} \quad \sigma_{true} = \frac{P}{A} \qquad \text{True strain} \ \epsilon_{true} = \ln \frac{L}{L_o}}$$

Relation between different strain and stress measures

$$\epsilon_{true} = \ln(\epsilon + 1) \qquad \sigma_{true} = \sigma(\epsilon_{true} + 1)$$

Comparison of True strain and Engineering strain

True strain ϵ_{true}	0.002	0.010	0.100	0.500	1.000	4.000
Engineering strain ϵ	0.002	0.010	0.105	0.649	1.718	53.598

The two measures give nearly identical measures up to strain of 1%

Norton, Machine Design: An Integrated Approach

Problem

A tensile specimen with a 12 mm initial diameter and 50 mm gauge length reaches maximum load at 90 kN and fractures at 70 kN. The minimum diameter at fracture is 10 mm. Determine the engineering stress at the maximum load (UTS), the engineering stress at the fracture load and the true stress at the fracture load.

Given data:

$do := 12\ \mathrm{mm} \quad Lo := 50\ \mathrm{mm} \quad Pm := 90\ \mathrm{kN} \quad Pf := 70\ \mathrm{kN} \quad df := 10\ \mathrm{mm}$

$$\sigma u := \frac{Pm}{\frac{\pi}{4}\cdot(do)^2} \qquad \sigma u = 7.9577\cdot 10^{8}\ \mathrm{Pa}$$

Ultimate tensile stress

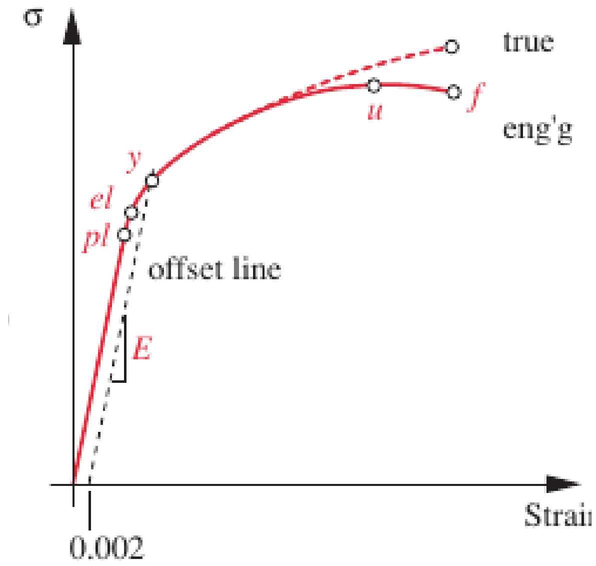
$$\sigma f := \frac{Pf}{\frac{\pi}{4}\cdot do^2} \qquad \sigma f = 6.1894\cdot 10^{8}\ \mathrm{Pa}$$

Engineering fracture stress

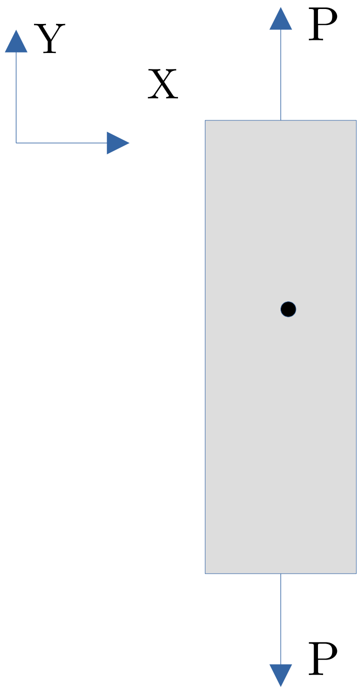
$$\sigma fT := \frac{Pf}{\frac{\pi}{4}\cdot df^2} \qquad \sigma fT = 8.9127\cdot 10^{8}\ \mathrm{Pa}$$

True fracture stress

Failure Theories

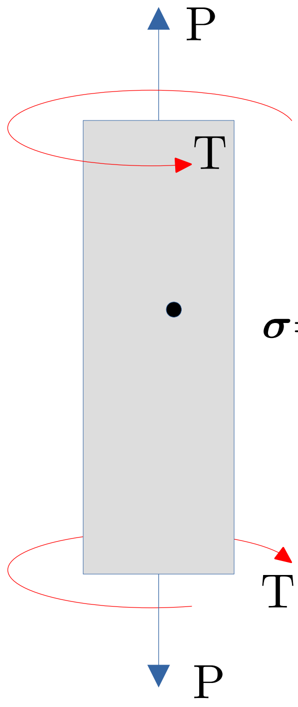


Test data from a uniaxial tension test



$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Circular rod subjected to uniaxial load



$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Circular rod subjected to multiaxial/combined load

When does the component fail ? $\sigma_{yy} = \sigma_y$

$\boxed{???}$

Failure Theories

- For the case of uniaxial loading when only a single non-zero component of stress exists, the conditions at which yielding/fracture occurs can be found out using the results of a uniaxial tension test.
- For the case of multiaxial loading when a three-dimensional state of stress exists, there is no theoretical technique to correlate the conditions at which yielding/fracture occurs can be to the results of a uniaxial tension test.
- Failure theories are essentially **empirical** theories which try to correlate the three dimensional state of stress with experimental results.
- In practice, the three dimensional state of stress are usually calculated at only a few critical points in the components
- The location of these points either found by applying principles of mechanics of materials or using theory of elasticity approach or by experimental techniques.

Static Failure Theories

- **Static Loads** – Loads that are slowly applied and remain essentially constant with time

- **Part Failure** – Part yields and distorts to not to function properly OR it fractures and separates

- Commonly used failure theories for ductile materials (true strain at fracture greater than 5%)
  - Maximum shear stress theory
  - Distortion Energy OR von Mises-Hencky theory

- Commonly used failure theories for brittle material (true strain at fracture less than 5%)
  - Maximum normal stress theory

Maximum Shear Stress Theory or Tresca Yield Criterion

Applicable to Ductile Materials

Yielding occurs when the the maximum shear stress at a point in the part reaches the shear stress that causes the same material to yield when it is subjected to uniaxial tension.

$$\boxed{\tau_{max} \geq \tau_y}$$

- If $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the principal stresses, then the maximum shear stress is given by

$$\tau_{max} = \frac{|\sigma_1 - \sigma_3|}{2}$$

- For the case of uniaxial tension at yielding $\sigma_1 = \sigma_y,\ \sigma_2 = 0,\ \sigma_3 = 0$. Therefore shear stress at yielding is

$$\tau_y = \frac{\sigma_y}{2}$$

- Therefore yielding occurs when

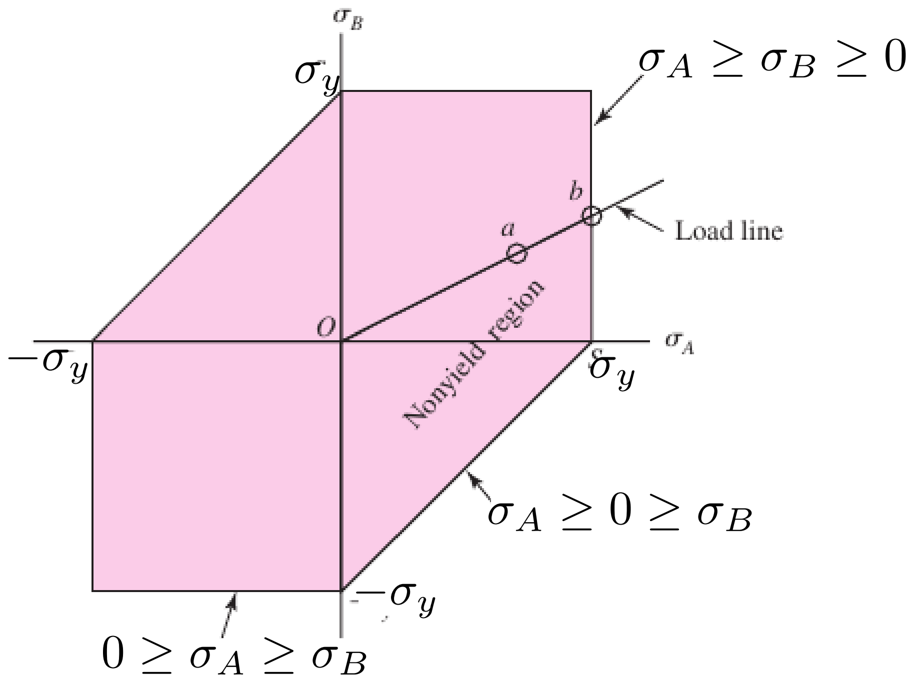
$$\boxed{|\sigma_1 - \sigma_3| \geq \sigma_y}$$

- For design purposes one introduces a factor of safety N and uses the following equation

$$\boxed{|\sigma_1 - \sigma_3| = \frac{\sigma_y}{N}}$$

Maximum Shear Stress Theory for Plane Stress Problems

- Let σ_A and σ_B represent the two in-plane principal stresses and let $\sigma_C = 0$ (out of plane stress component).
- Then the yield locus is given by



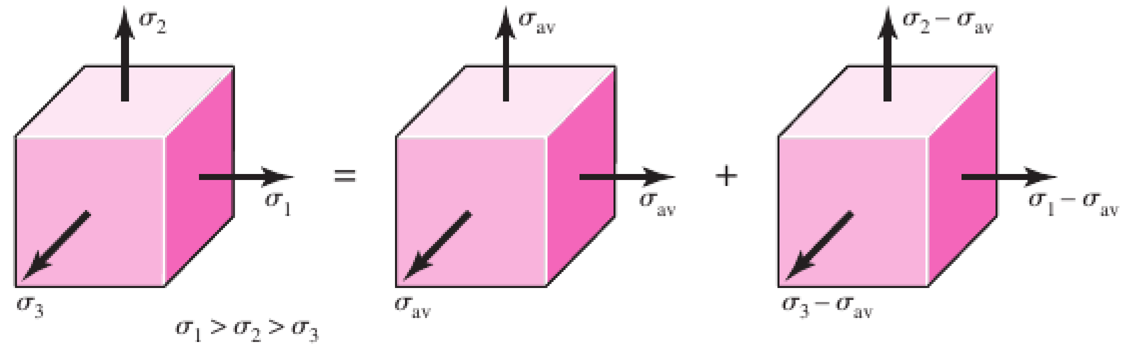
Factor of safety available at point a against yielding

$$n = \frac{l(Ob)}{l(Oa)}$$

Distortion Energy Theory or von Mises-Hencky Theory

Applicable to Ductile Materials

Yielding occurs when the the distortion strain energy per unit volume at a point in a part reaches or exceeds the distortion strain energy per unit volume at yield in the same material when it is subjected to uniaxial tension.



Triaxial state of stress	Hydrostatic component (responsible for volume change)	Deviatoric/Distortional Component (responsible for shape change)

Distortion Energy Theory or von Mises-Hencky Theory

- The distortional strain energy density per unit volume for a general state of stress is

$$U_d = \frac{1}{6G}\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]$$

- For the case of uniaxial tension at yielding $\sigma_1 = \sigma_y,\ \sigma_2 = 0,\ \sigma_3 = 0$

$$U_d^{uniaxial} = \frac{1}{6G}\sigma_y^2$$

- Yielding occurs when

$$U_d \geq U_d^{uniaxial}$$

or

$$\boxed{\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq \sigma_y}$$

- von Mises or equivalent stress is defined as

$$\sigma_{vm} = \sigma_{eq} = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

- Then yielding occurs when

$$\boxed{\sigma_{vm} \geq \sigma_y}$$

- For design purposes one introduces a factor of safety N and uses the following equation

$$\boxed{\sigma_{vm} = \frac{\sigma_y}{N}}$$

Distortion Energy Theory or von Mises-Hencky Theory

- The distortion energy theory is also called:
  - The shear-energy theory
  - The octahedral-shear-stress theory

- The von Mises stress terms of the components of the stress tensor referred to a Cartesian coordinate system is given by

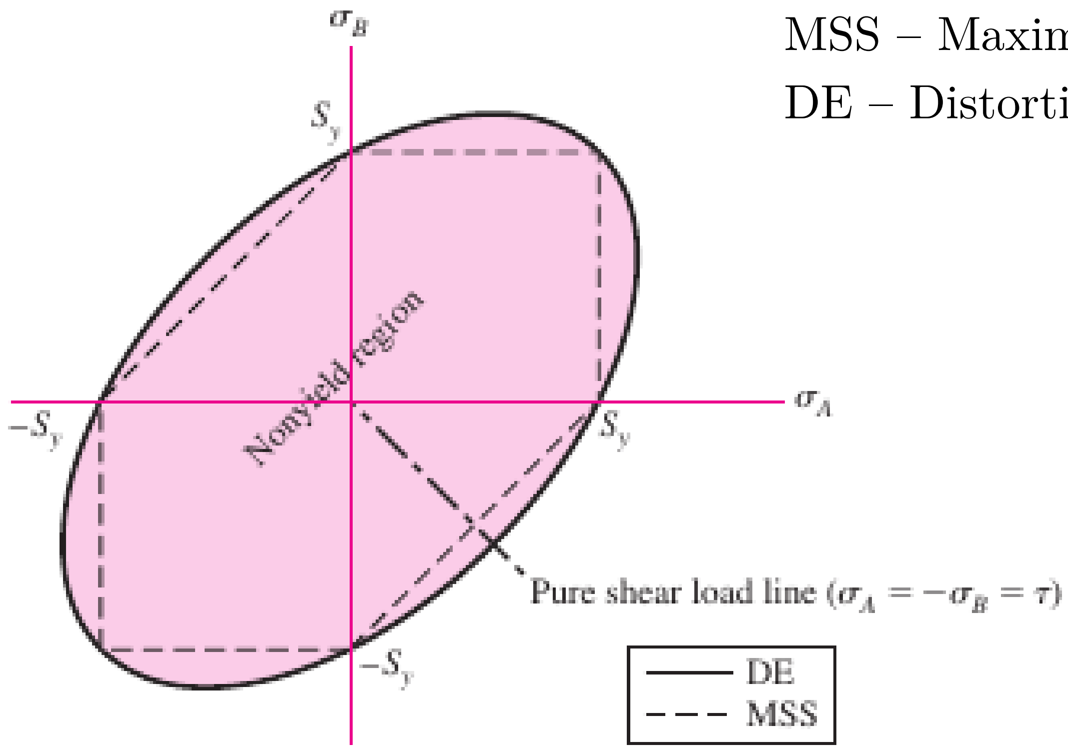
$$\sigma_{vm} = \frac{1}{\sqrt{2}}\left[(\sigma_{xx}-\sigma_{yy})^2+(\sigma_{yy}-\sigma_{zz})^2+(\sigma_{zz}-\sigma_{xx})^2+6(\sigma_{xy}^2+\sigma_{yz}^2+\sigma_{zx}^2)\right]^{1/2}$$

- The von Mises stress terms of the components of the stress tensor referred to a Cartesian for case of plane stress is

$$\sigma_{vm} = (\sigma_{xx}^2-\sigma_{xx}\sigma_{yy}+\sigma_{yy}^2+3\sigma_{xy})^{1/2}$$

Distortion Energy Theory for Plane Stress Problems

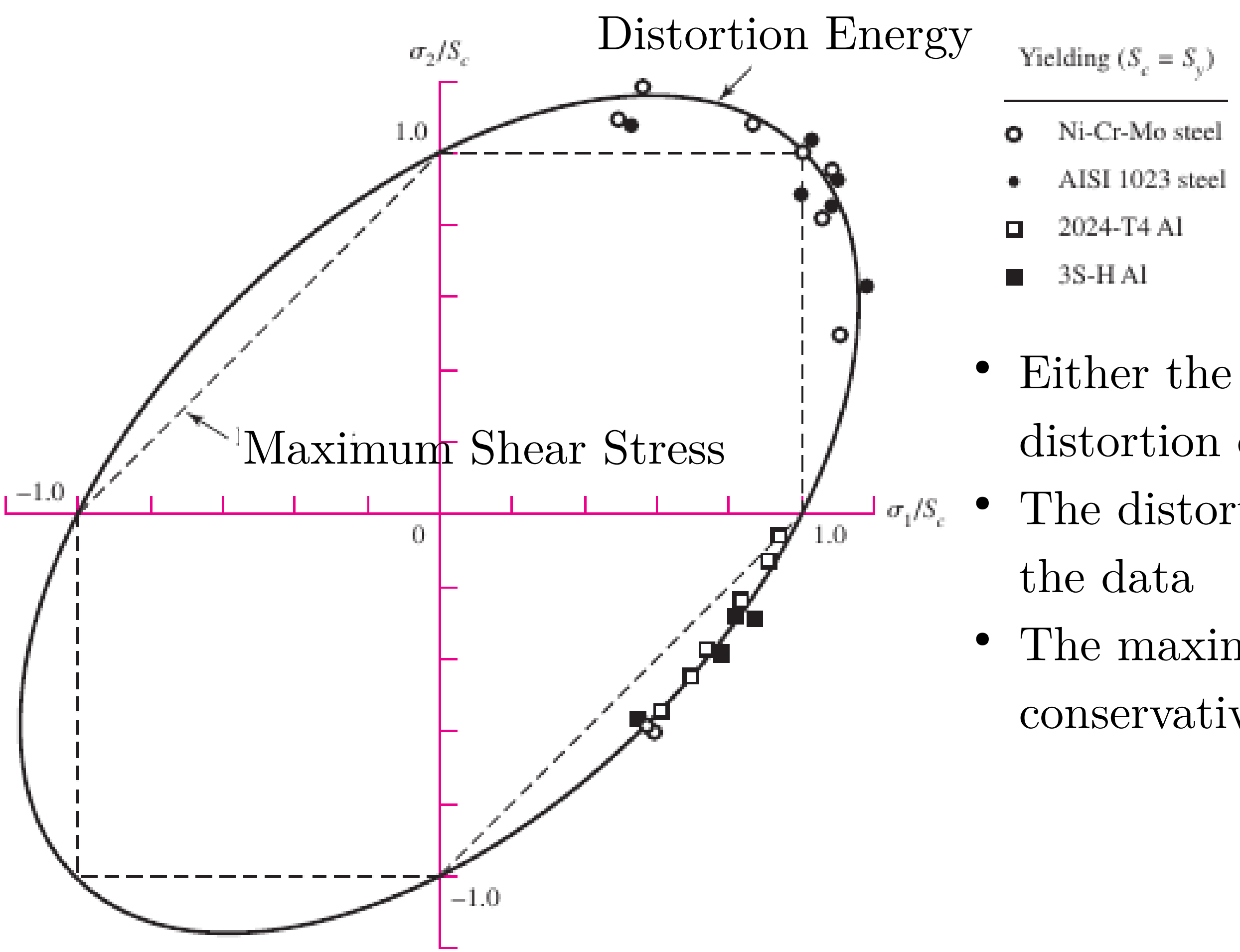
- Let σ_A and σ_B represent the two in-plane principal stresses and let $\sigma_C = 0$ (out of plane stress component).
- Then the yield locus is given by $\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B = \sigma_y^2$



MSS – Maximum shear stress theory

DE – Distortion Energy Theory

Comparison of Failure Theories for Ductile Materials



- Either the maximum shear stress theory or the distortion energy theory is acceptable
- The distortion energy theory gives better fit to the data
- The maximum shear stress theory is more conservative

Problem – Ductile Material

A stationary shaft, 50 mm in diameter and made of AISI 1060 hot-rolled steel, is subjected to a maximum bending moment of 3000 Nm and a maximum torque of 2000 Nm. Find the factor of safety corresponding to failure based on the distortion energy theory. The yield stress is 372 MPa.

$$d := 50\ \mathrm{mm} \qquad M := 3000\ \mathrm{N}\cdot 1\ \mathrm{m} \qquad T := 2000\ \mathrm{N}\cdot 1\ \mathrm{m} \qquad \sigma y := 372\ \mathrm{MPa}$$

$$\sigma xx := \frac{\left(M\cdot\frac{d}{2}\right)}{\frac{\pi}{64}\cdot d^4} \qquad \sigma xx = 2.4446\cdot 10^8\ \mathrm{Pa}$$

$$\sigma xy := \frac{\left(T\cdot\frac{d}{2}\right)}{\frac{\pi}{32}\cdot d^4} \qquad \sigma xy = 8.1487\cdot 10^7\ \mathrm{Pa}$$

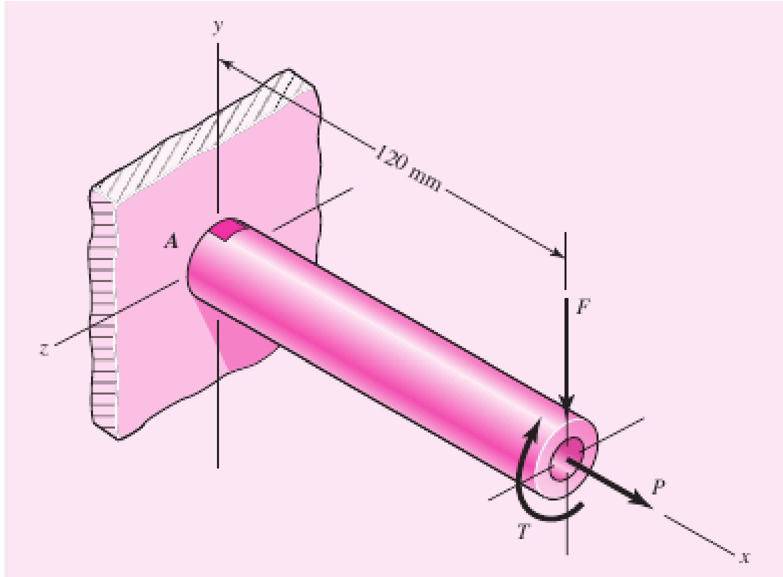
$$\sigma yy := 0\ \mathrm{Pa}$$

$$\sigma vm := \left(\sigma xx^2 + \sigma yy^2 - \sigma xx\cdot\sigma yy + 3\cdot\sigma xy^2\right)^{\frac{1}{2}} \qquad \sigma vm = 2.8228\cdot 10^8\ \mathrm{Pa}$$

$$FOS := \frac{\sigma y}{\sigma vm} \qquad FOS = 1.3178$$

Problem – Ductile Material

The cantilevered tube shown in Figure is to be made of 2014 Aluminum alloy treated to obtain a specified minimum yield strength of 276 MPa. We wish to select a stock-size tube from Table given below using a design factor $n_d = 4$. The bending load is $F = 1.75$ kN, the axial tension is $P = 9.0$ kN, and the torsion is $T = 72$ Nm. What is the realized factor of safety? Use both Maximum Shear Stress Theory and Distortion Energy Theory.



Size, mm	*m*	*A*	*I*	*k*	*Z*	*J*
12 × 2	0.490	0.628	0.082	0.361	0.136	0.163
16 × 2	0.687	0.879	0.220	0.500	0.275	0.440
16 × 3	0.956	1.225	0.273	0.472	0.341	0.545
20 × 4	1.569	2.010	0.684	0.583	0.684	1.367
25 × 4	2.060	2.638	1.508	0.756	1.206	3.015
25 × 5	2.452	3.140	1.669	0.729	1.336	3.338
30 × 4	2.550	3.266	2.827	0.930	1.885	5.652
30 × 5	3.065	3.925	3.192	0.901	2.128	6.381
42 × 4	3.727	4.773	8.717	1.351	4.151	17.430
42 × 5	4.536	5.809	10.130	1.320	4.825	20.255
50 × 4	4.512	5.778	15.409	1.632	6.164	30.810
50 × 5	5.517	7.065	18.118	1.601	7.247	36.226

w_a = unit weight of aluminum tubing, lbf/ft
w_s = unit weight of steel tubing, lbf/ft
m = unit mass, kg/m
A = area, in^2 (cm^2)
I = second moment of area, in^4 (cm^4)
J = second polar moment of area, in^4 (cm^4)
k = radius of gyration, in (cm)
Z = section modulus, in^3 (cm^3)
d, t = size (OD) and thickness, in (mm)

Maximum Normal Stress Theory

Applicable to Brittle Materials (primarily applied to isotropic material)

Failure occurs whenever one of the three principal stresses at a point in a part reaches or exceeds the strength of the material either in tension or compression

- Let $\sigma_1 \geq \sigma_2 \geq \sigma_3$ be the principal stresses at a point
- The material fails whenever

$$\boxed{\sigma_1 \geq \sigma_{ultT},\ \sigma_3 \leq -\sigma_{ultC}}$$

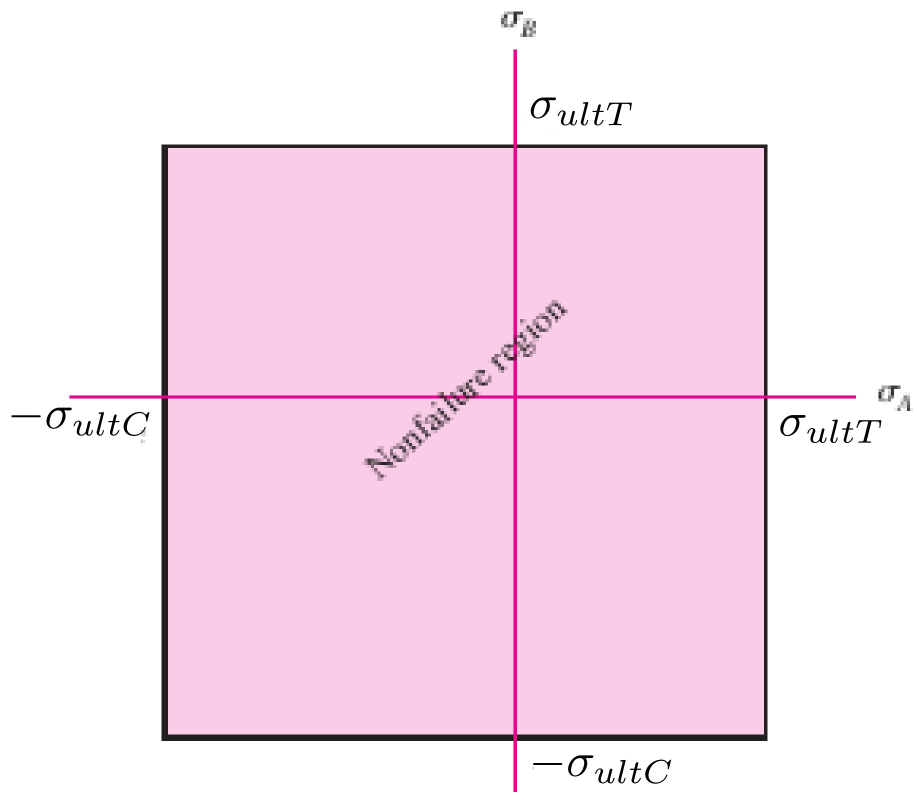
Here σ_{ultT} and σ_{ultC} are the ultimate strength in tension and compression, respectively

- For design purposes one introduces a factor of safety N and uses the following equation

$$\boxed{\sigma_1 = \frac{\sigma_{ultT}}{N},\ \text{or}\ \sigma_3 = \frac{-\sigma_{ultC}}{N}}$$

Maximum Normal Stress Theory for Plane Stress Problems

- Let σ_A and σ_B represent the two in-plane principal stresses and let $\sigma_C = 0$ (out of plane stress component). Let $\sigma_A \geq \sigma_B$
- Then the yield locus is



Choice of Failure Theory

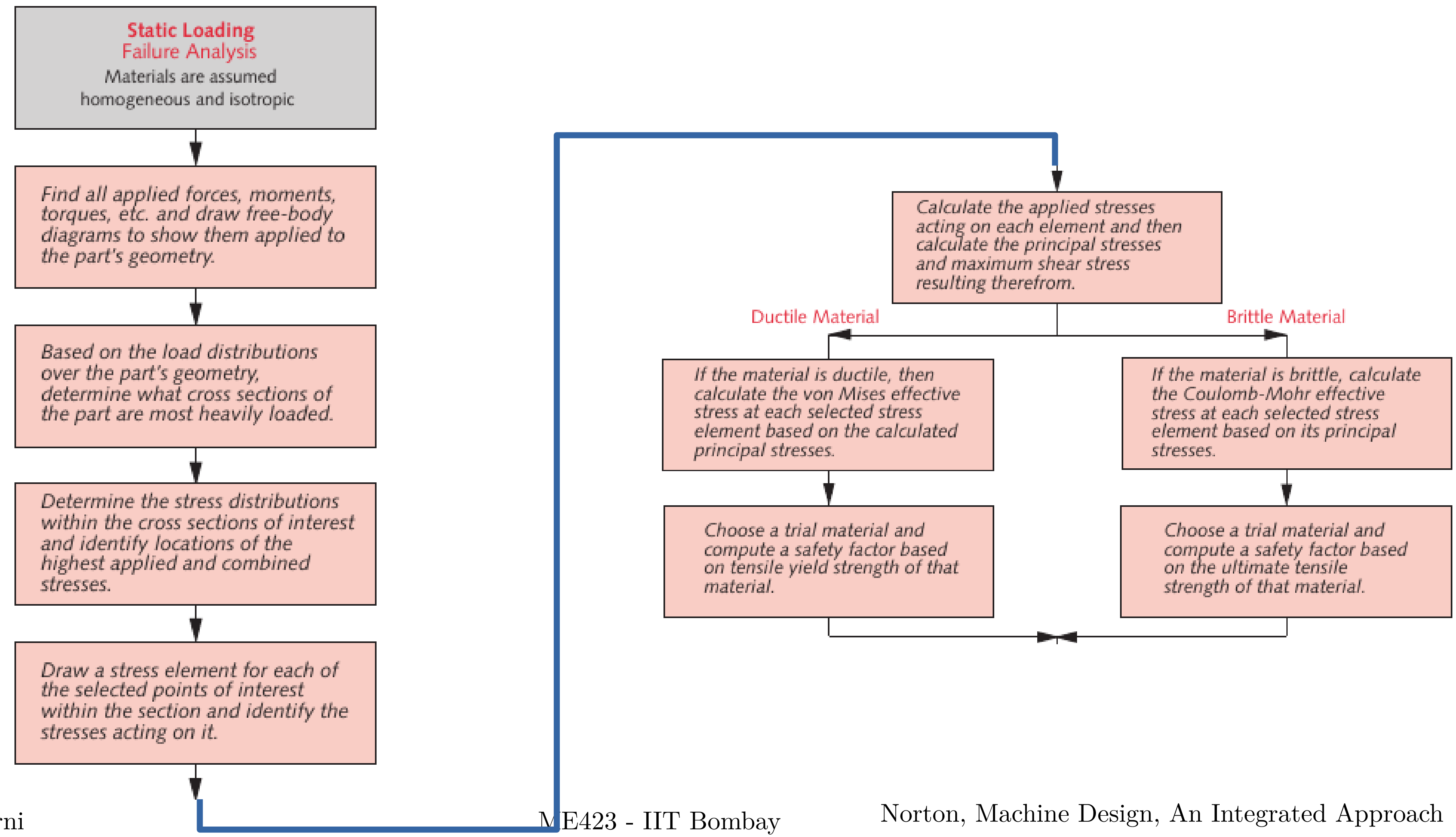
- For **isotropic** material that fail by yielding, use the distortion energy theory

- For **isotropic** material that fail by yielding, use maximum shear stress is more conservative than the distortion energy theory

- For **isotropic** material that fail by brittle fracture, use the maximum normal stress theory.

- For materials that fail by brittle fracture but whose compressive ultimate compressive strength is significantly different from the tensile ultimate strength, use the modified Mohr's theory (not covered in class)

Flow Chart for Static Analysis

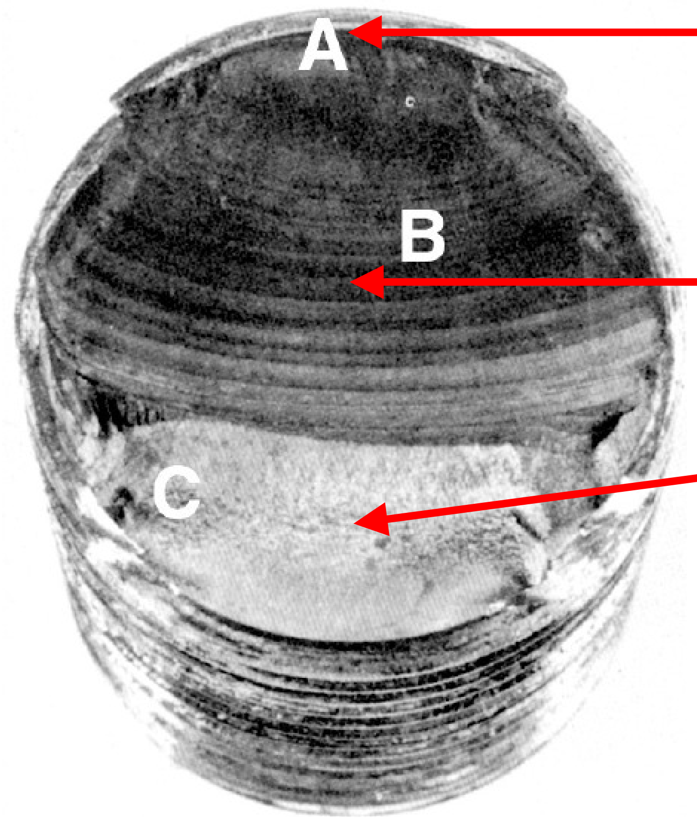


End

Fatigue Failure in Metals

- Metallic machine parts subjected to repetitive/fluctuating/alternate/variables stresses fail at a stress value much lower than the stress required to cause failure on a single application of load. This is referred to as fatigue failure.
- Fatigue failure accounts for about 90% of all service failures due to mechanical causes.
- Fatigue failure usually initiates at a point of stress concentration such as a sharp corner or a notch.
- While failures due to static loads give a visible warning (yielding, large deformation), fatigue failure gives no warning. It is sudden and total, and hence dangerous.
- A fatigue failure has an appearance similar to a brittle fracture, as the fracture surfaces are flat and perpendicular to the stress axis with the absence of necking.

Stages in Fatigue Failure in Metals



Stage I: Crack initiation
(normally not visible to naked eye)

Stage II: Crack propagation
(appearance of beach marks/clamshell marks)

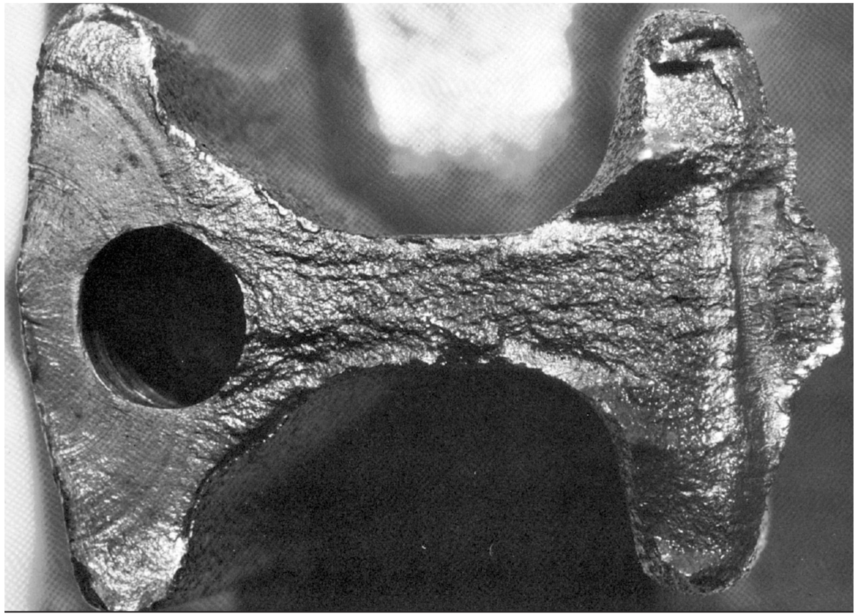
Stage III: Final fracture
(The remaining material cannot support the loads, leading to fracture)

clamshell

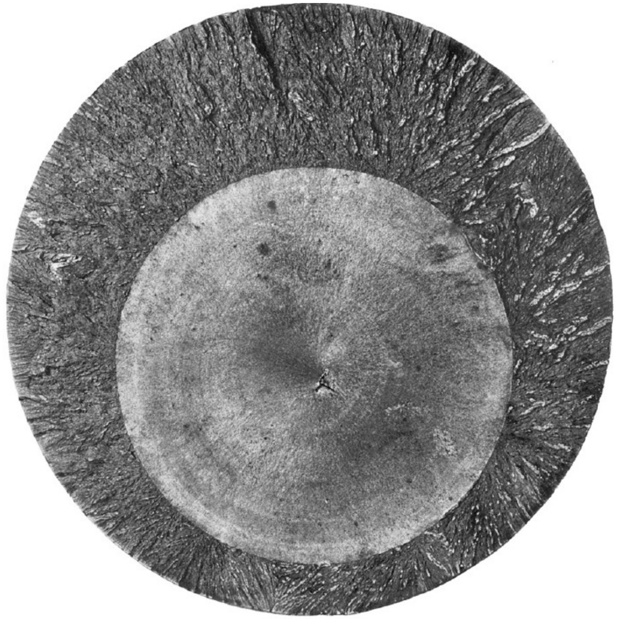
Shigley's Mechanical Engineering Design

https://en.wikipedia.org/wiki/Atlantic_surf_clam#/media/File:Spisula_solidissima_shell.jpg

Examples of Fatigue Failures



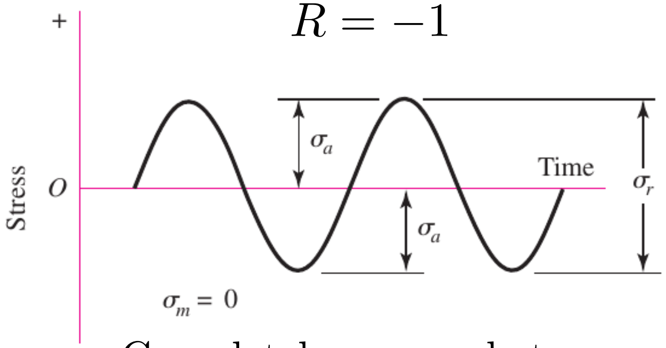
Fatigue fracture surface of a forged connecting rod of AISI 8640 steel. The fatigue crack origin is at the left edge,



Fatigue fracture surface of a 200 mm diameter piston rod of an alloy steel steam hammer used for forging. Fatigue fracture is caused by pure tension where surface stress concentrations are absent and a crack may initiate anywhere in the cross section.

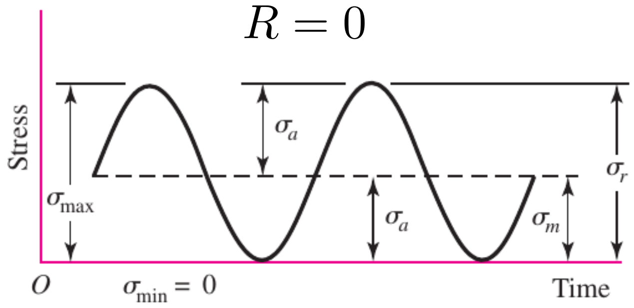
Representative Types of Loading and Terminology

$R = -1$



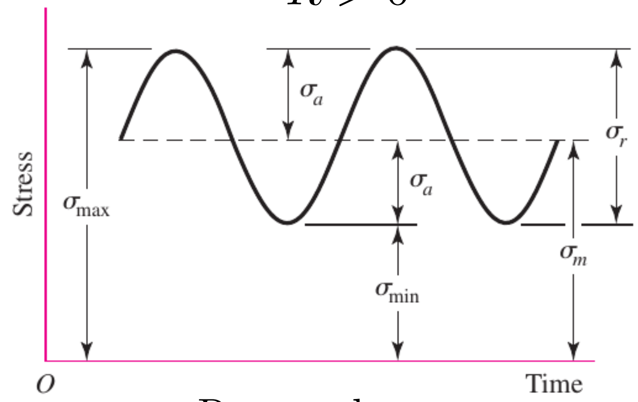
Completely reversed stress

$R = 0$



Fluctuating stress

$R > 0$



Reversed stress

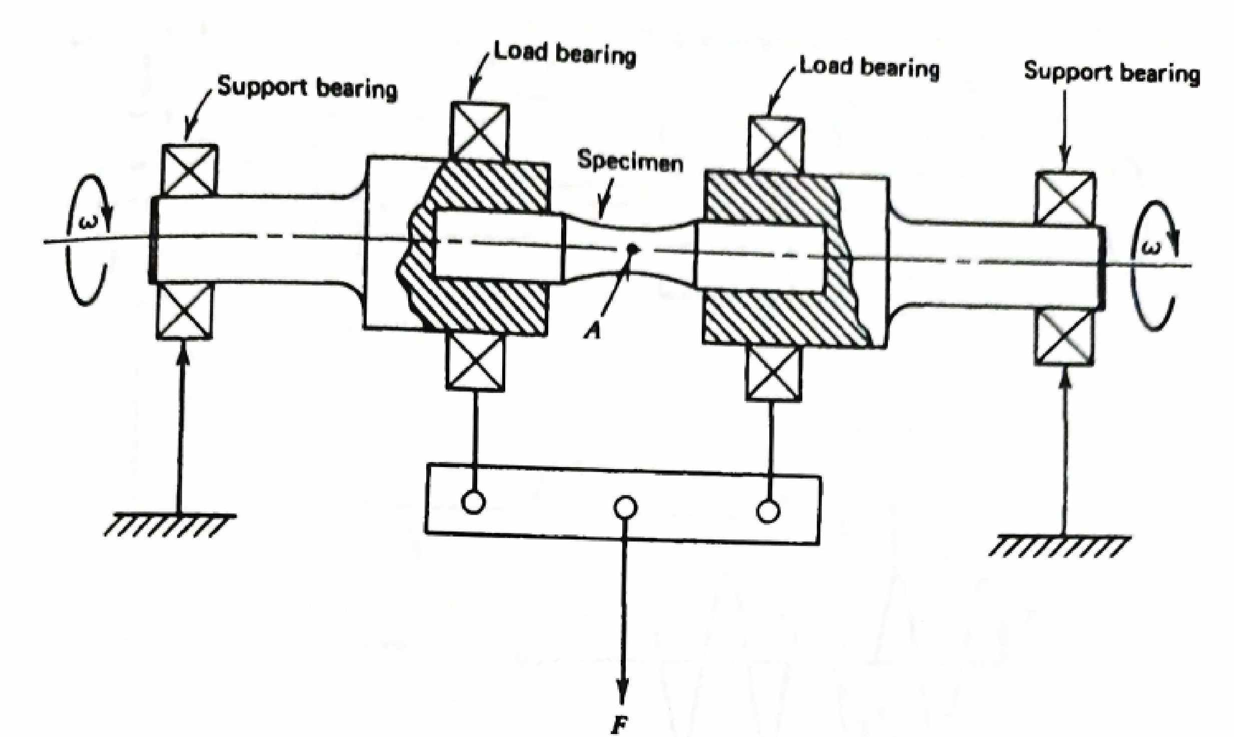
σ_{max}: max stress	σ_{min}: min stress
σ_a: stress amplitude	σ_m: mean stress
σ_r: stress range	

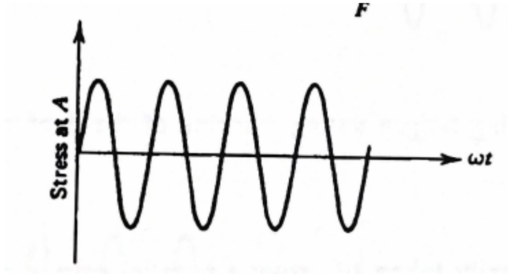
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad R = \frac{\sigma_{min}}{\sigma_{max}} \quad \text{Stress ratio}$$

$$\sigma_a = \frac{|\sigma_{max} - \sigma_{min}|}{2} \qquad A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R} \quad \text{Amplitude ratio}$$

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

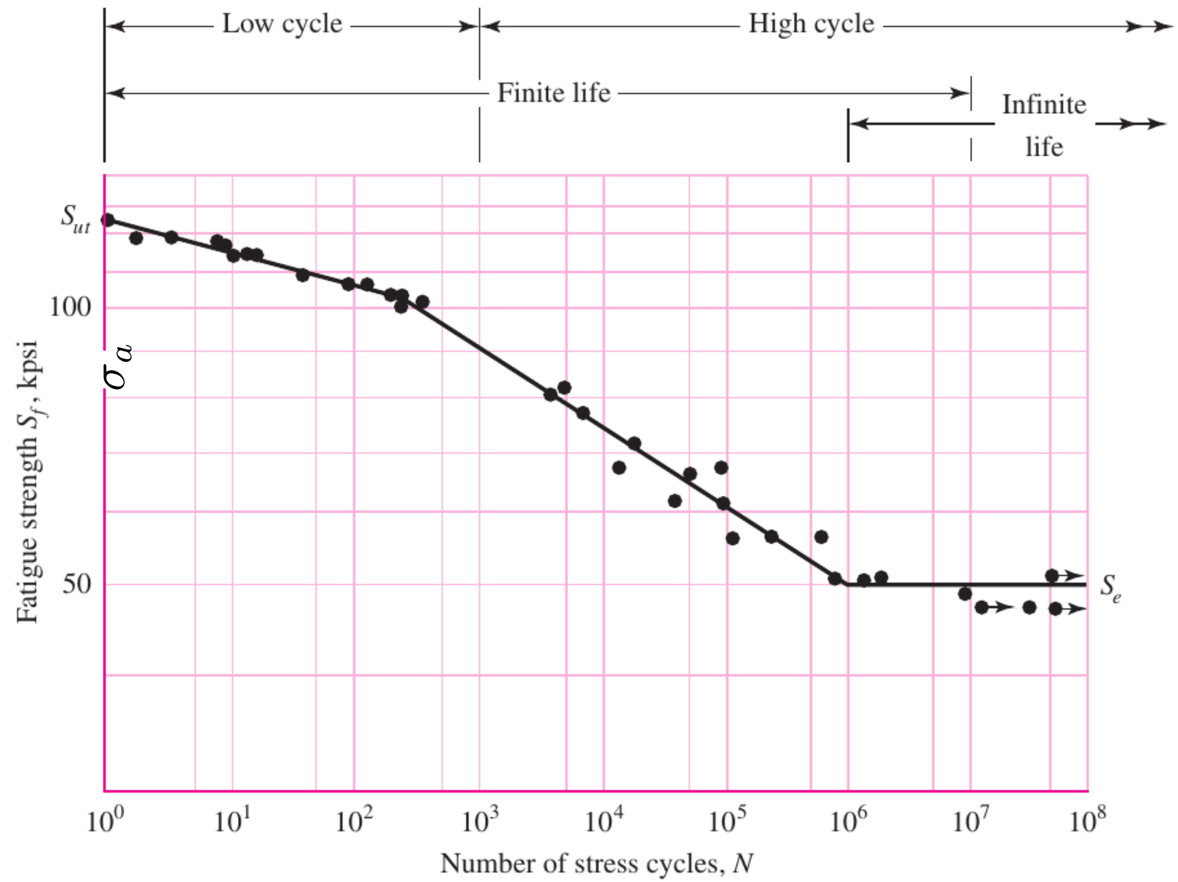
Rotating Bending Fatigue Testing Machine





Collins, Failure of Materials in Mechanical Design

S-N Diagram for a Steel Alloy



$1 \text{ kpsi} = 6.895 \text{ MPa}$

σ'_e

Shigley's Mechanical Engineering Design

Fatigue Life Methods

Attempt to predict the life in number of cycles to failure, N, for a specific level of loading.

- **Stress life method** – used for high cycle fatigue ($N \geq 10^3$)
  - Based on stress levels only, assumes stresses are within elastic limit
  - Easiest to implement for a wide range of design applications
  - Has ample supporting data (experimental results)
  - Works best when the load amplitudes are predictable and consistent over the life of the part
  - Is the least accurate approach, especially for low-cycle applications.
- **Strain life method** – used for low cycle fatigue ($N \leq 10^3$)
  - Involves detailed analysis of the plastic deformation at localized regions.
  - Gives a good picture of the crack initiation stage
  - In applying this method, several idealizations must be made and hence uncertainties exist in the results. Hence not very widely used.
- **Linear elastic fracture mechanics method**
  - Assumes a crack is already present and detected.
  - It is employed to predict crack growth with respect to stress intensity.
  - Used in conjunction with computer codes and a periodic inspection program.