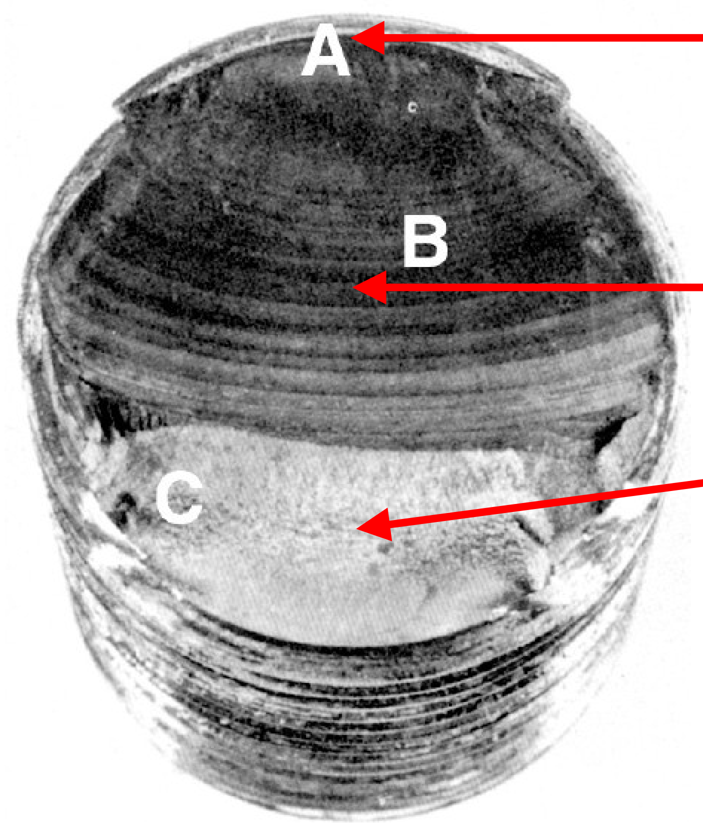
Fatigue Failure in Metals

- Metallic machine parts subjected to repetitive/fluctuating/alternate/variables stresses fail at a stress value much lower than the stress required to cause failure on a single application of load. This is referred to as fatigue failure.
- Fatigue failure accounts for about 90% of all service failures due to mechanical causes.
- Fatigue failure usually initiates at a point of stress concentration such as a sharp corner or a notch.
- While failures due to static loads give a visible warning (yielding, large deformation), fatigue failure gives no warning. It is sudden and total, and hence dangerous.
- A fatigue failure has an appearance similar to a brittle fracture, as the fracture surfaces are flat and perpendicular to the stress axis with the absence of necking.

Stages in Fatigue Failure in Metals



Stage I: Crack initiation
(normally not visible to naked eye)

Stage II: Crack propagation
(appearance of beach marks/clamshell marks)

Stage III: Final fracture
(The remaining material cannot support the loads, leading to fracture)

clamshell

https://en.wikipedia.org/wiki/Atlantic_surf_clam#/media/File:Spisula_solidissima_shell.jpg

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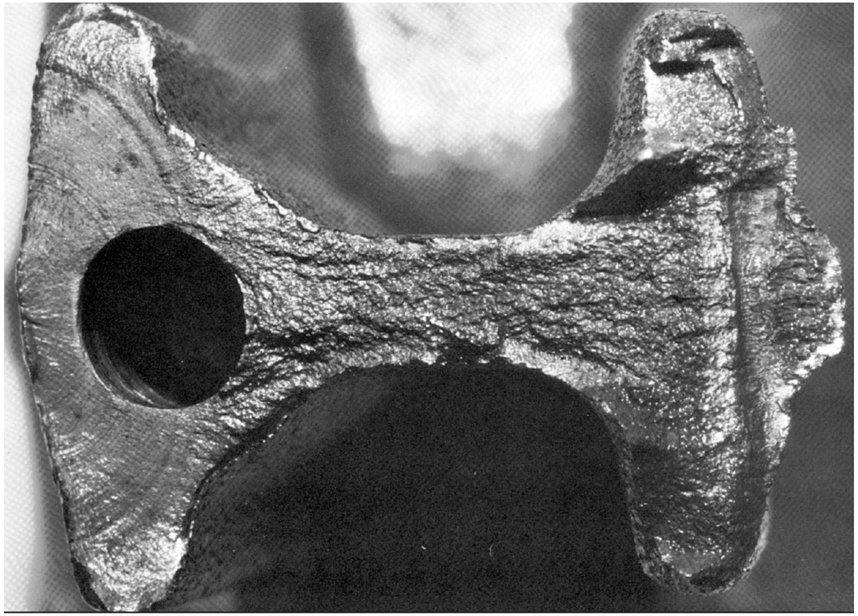
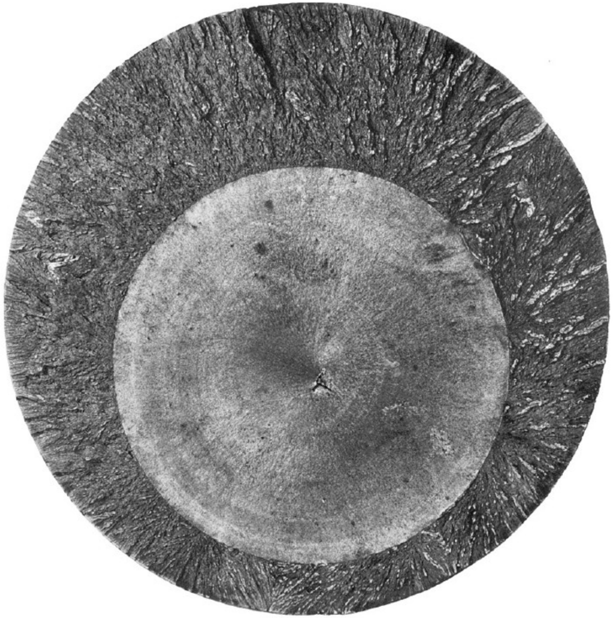
Examples of Fatigue Failures



Fatigue fracture surface of a forged connecting rod of AISI 8640 steel. The fatigue crack origin is at the left edge,



Fatigue fracture surface of a 200 mm diameter piston rod of an alloy steel steam hammer used for forging. Fatigue fracture is caused by pure tension where surface stress concentrations are absent and a crack may initiate anywhere in the cross section.

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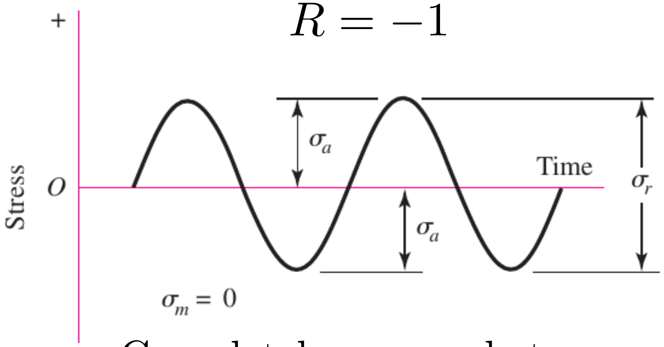
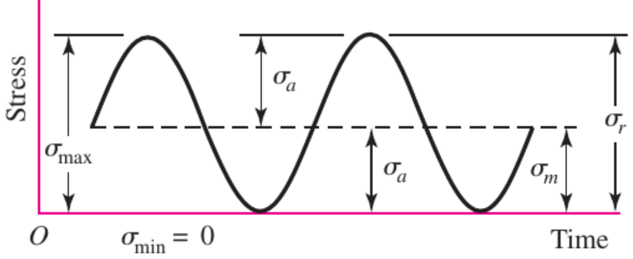
Representative Types of Loading and Terminology

$R = -1$



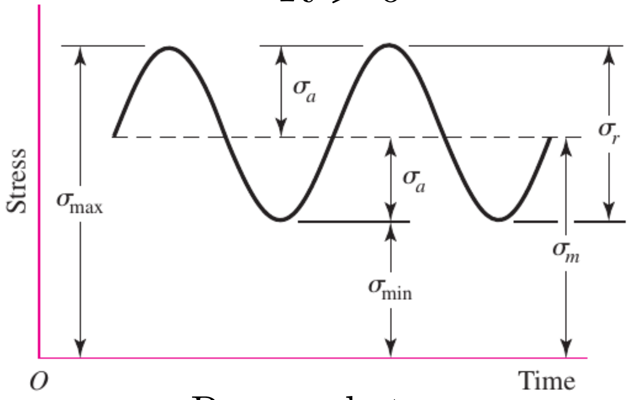
Completely reversed stress

$R = 0$



Fluctuating stress

$R > 0$



Reversed stress

σ_{max}: max stress	σ_{min}: min stress
σ_a: stress amplitude	σ_m: mean stress
σ_r: stress range	

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

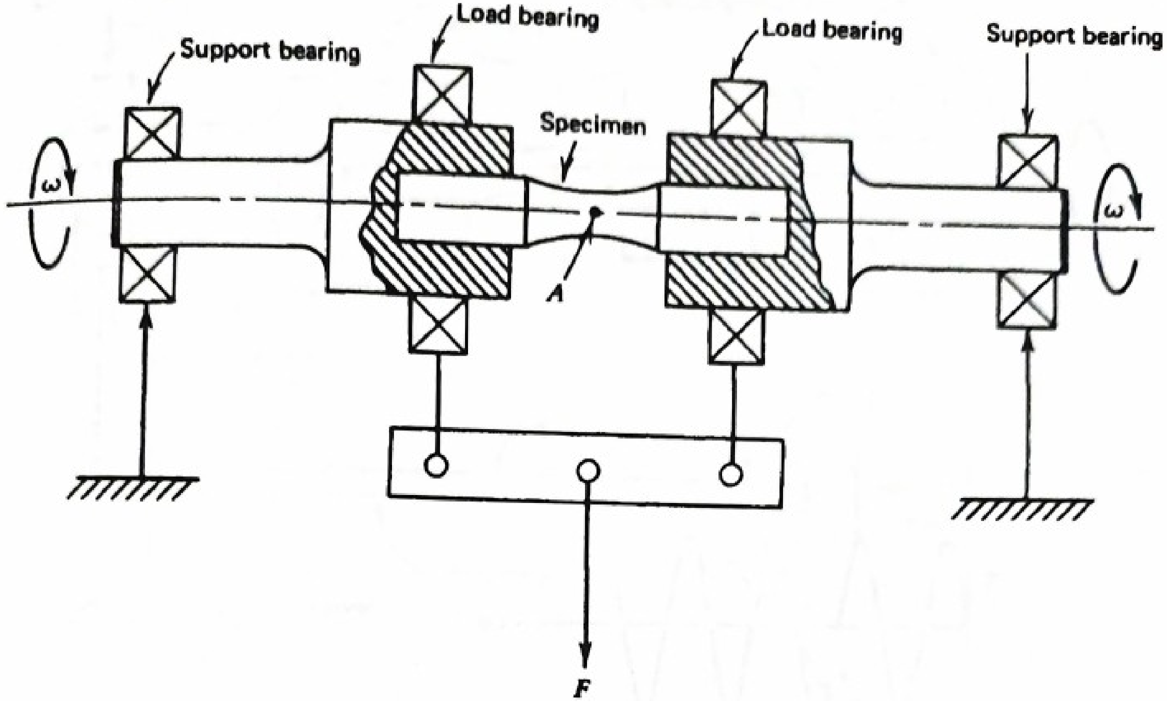
$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad \text{Stress ratio}$$

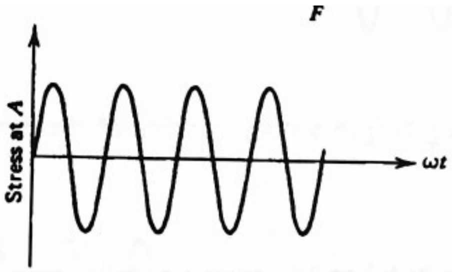
$$\sigma_a = \frac{|\sigma_{max} - \sigma_{min}|}{2}$$

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R} \quad \text{Amplitude ratio}$$

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Rotating Bending Fatigue Testing Machine





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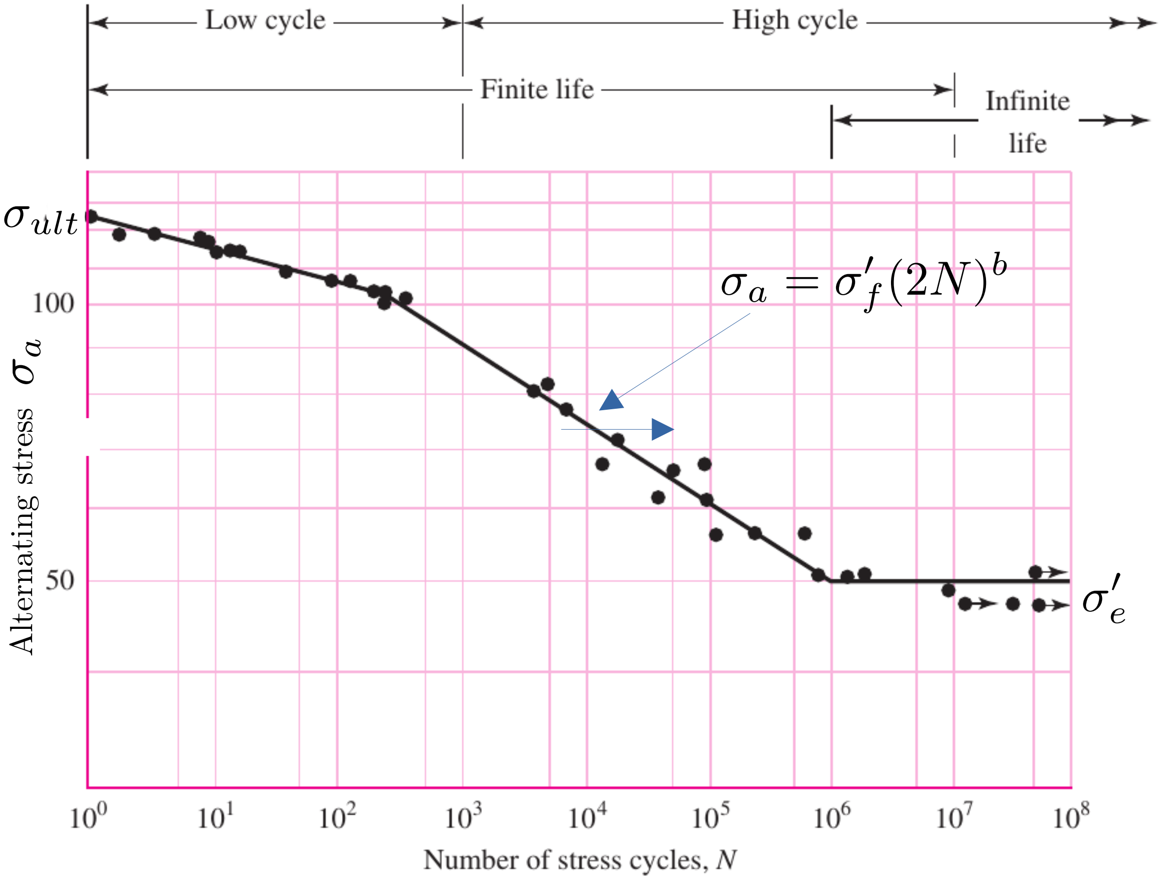
S-N Diagram for a Steel Alloy



1 kpsi = 6.895 MPa

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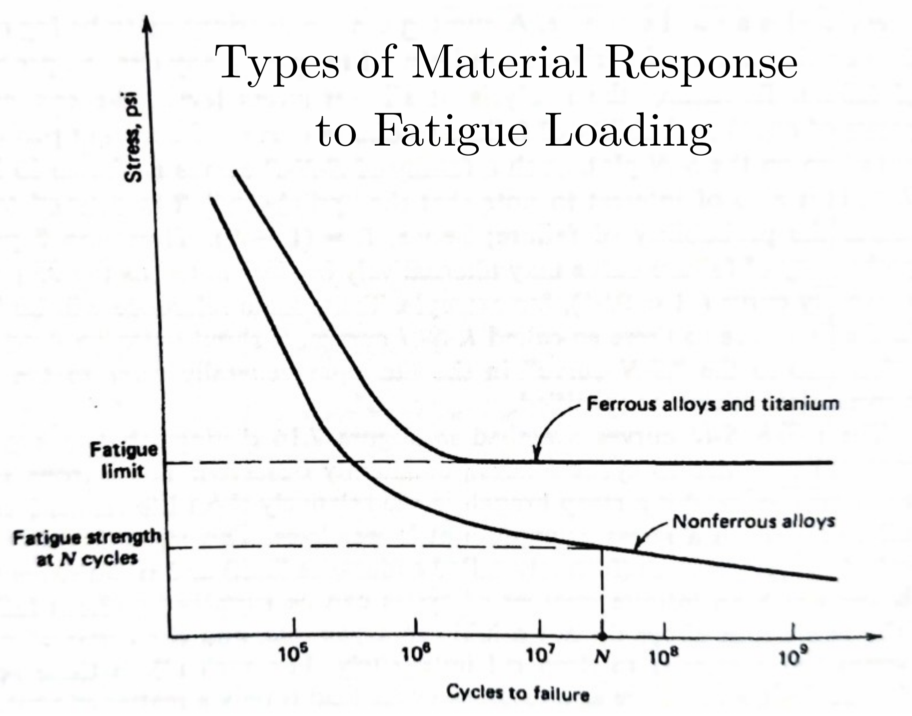
Fatigue Life Methods

Attempt to predict the life in number of cycles to failure, N, for a specific level of loading.

- **Stress life method** – used for high cycle fatigue ($N \geq 10^3$)
  - Based on stress levels only, assumes stresses are within elastic limit
  - Easiest to implement for a wide range of design applications
  - Has ample supporting data (experimental results)
  - Works best when the load amplitudes are predictable and consistent over the life of the part
  - Is the least accurate approach, especially for low-cycle applications.
- **Strain life method** – used for low cycle fatigue ($N \leq 10^3$)
  - Involves detailed analysis of the plastic deformation at localized regions.
  - Gives a good picture of the crack initiation stage
  - In applying this method, several idealizations must be made and hence uncertainties exist in the results. Hence not very widely used.
- **Linear elastic fracture mechanics method**
  - Assumes a crack is already present and detected.
  - It is employed to predict crack growth with respect to stress intensity.
  - Used in conjunction with computer codes and a periodic inspection program.

Stress Life Approach

- The part is designed based on the material's fatigue strength (or **fatigue/endurance limit**) and a safety factor.
- Attempts to keep the stress levels so low that the crack initiation stage never begins



- Presence of fatigue/endurance limit for Ferrous alloys and titanium
- Absence of well defined fatigue/endurance limit for nonferrous allows like aluminium alloys
- Presence of a well defined endurance limit indicates infinite-life if cycled below the endurance limit

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Stress Life Approach – Endurance Limits and Fatigue Strengths of Common Metals

Steels $$\sigma'_e \approx \begin{cases} 0.5\sigma_{ult} & \sigma_{ult} < 1400 \text{ MPa} \\ 700 \text{ MPa} & \sigma_{ult} \geq 1400 \text{ MPa} \end{cases}$$

Cast Iron $$\sigma'_e \approx \begin{cases} 0.4\sigma_{ult} & \sigma_{ult} < 400 \text{ MPa} \\ 160 \text{ MPa} & \sigma_{ult} \geq 400 \text{ MPa} \end{cases}$$

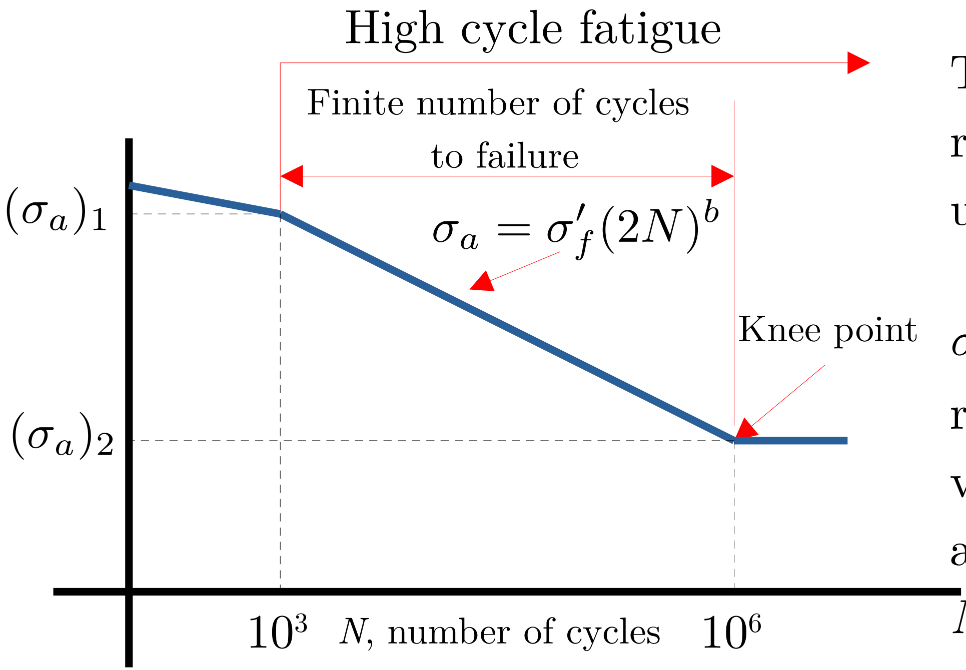
Aluminium $$\sigma'_{f@5e8} \approx \begin{cases} 0.4\sigma_{ult} & \sigma_{ult} < 330 \text{ MPa} \\ 130 \text{ MPa} & \sigma_{ult} \geq 300 \text{ MPa} \end{cases}$$

Copper $$\sigma'_{f@5e8} \approx \begin{cases} 0.4\sigma_{ult} & \sigma_{ult} < 280 \text{ MPa} \\ 100 \text{ MPa} & \sigma_{ult} \geq 280 \text{ MPa} \end{cases}$$

σ'_e, σ'_f are the endurance limit or fatigue strength obtained in lab conditions using polished specimens using bending fatigue tests

These are only estimates and are to be use only if the S-N data is unavailable and must be used with caution

Estimating Cycles to Failure in the High Cycle Regime



The number of cycles to failure in the high cycle regime up to 10^6 (endurance limit) can be estimated using the Basquin relation

$$\sigma_a = \sigma'_f(2N)^b$$

σ'_f and b are material constants. If they are not readily available, they can be estimated using the values of the alternate stress required for failure at two distinct points, typically at $N_1 = 10^3$ and $N_2 = 10^6$

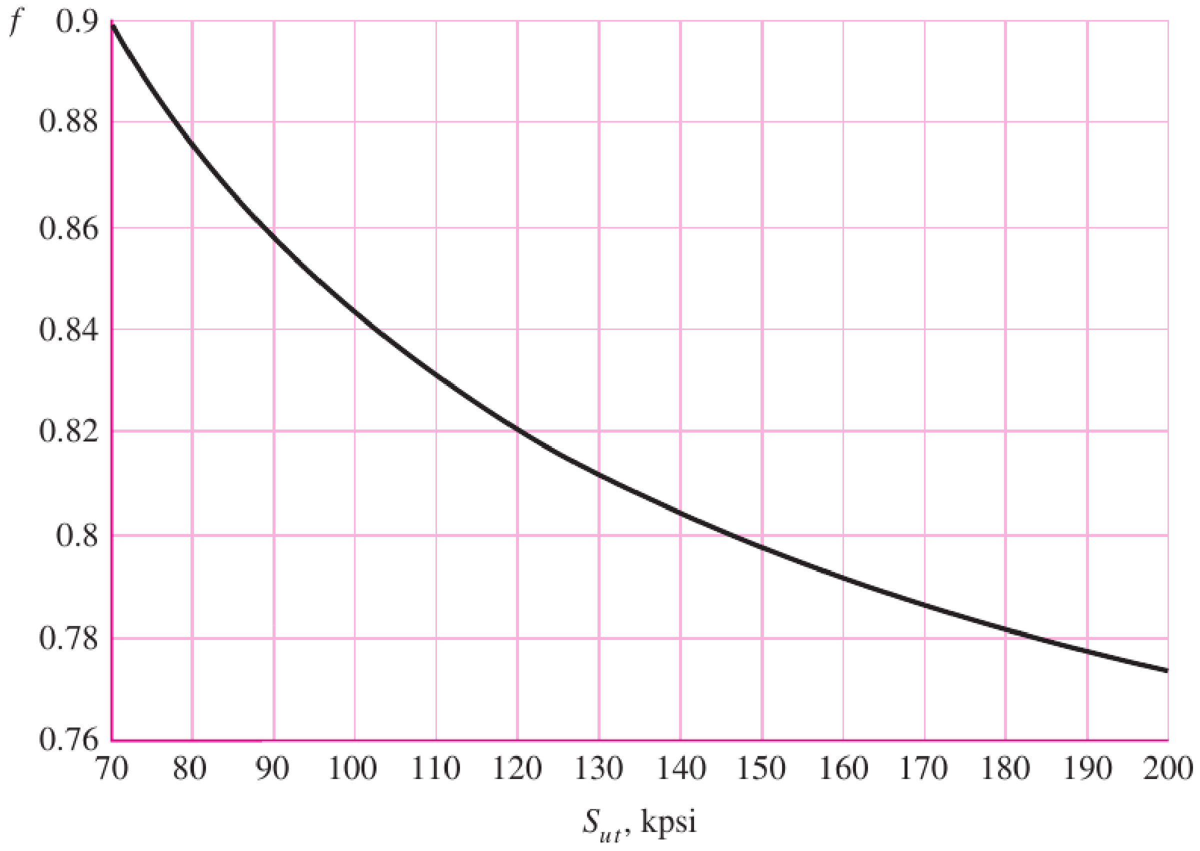
$$b = \frac{\log\left(\frac{(\sigma_a)_1}{(\sigma_a)_2}\right)}{\log\left(\frac{N_1}{N_2}\right)}, \quad \sigma'_f = \frac{(\sigma_a)_1}{(2N_1)^b} = \frac{(\sigma_a)_2}{(2N_2)^b}$$

$(\sigma_a)_2$ is the endurance limit

$(\sigma_a)_1$ is the stress amplitude that the specimen fails at $N = 10^3$

$(\sigma_a)_1 \approx f\sigma_{ult}$ if no data is available (See next slide for f)

Estimate of Fatigue Strength Fraction f of S_{ut} at 10^3 cycles for $S_e = 0.5\ S_{ut}$ at 10^6 cycles



Problem – No Endurance limit Correction & Zero Mean Stress

Given a 1050 HR (hot rolled) steel, estimate

(a) the rotating-beam endurance limit at 10^6 cycles.

(b) the fatigue strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure

(c) the expected life of a polished rotating-beam specimen under a completely reversed stress of 380 MPa.

UTS for HR 1050 steel

$sult := 620\ \text{MPa} \qquad f := 0.86$

Endurance strength

$s2 := 0.5 \cdot sult = 310\ \text{MPa} \qquad N2 := 10^6$

Stress amplitude corresponding to 1000 cycles

$s1 := f \cdot sult = 533.2\ \text{MPa} \qquad N1 := 10^3$

To find:

Fatigue strength at $N := 10^4$

Number of cycles to failure at stress amplitude $sp := 380\ \text{MPa}$

$$b := \frac{\log_e\left(\frac{s1}{s2}\right)}{\log_e\left(\frac{N1}{N2}\right)} = -0.0785 \qquad sf := \frac{s2}{(2 \cdot N2)^b} = 968.3945\ \text{MPa}$$

Answers

Fatigue strength at $N = 10000$

$$sN := sf \cdot (2 \cdot N)^b = 445.0212\ \text{MPa}$$

Number of cycles to failure at stress amplitude $sp = 380\ \text{MPa}$

$$Np := \frac{1}{2} \cdot \left(\frac{sp}{sf}\right)^{\frac{1}{b}} = 74772.5822$$

approximately 74700 cycles

Endurance Limit Modifying Factors

$$\sigma_e = k_a k_b k_c k_d k_e k_f \sigma_e'$$

σ_e = endurance limit of the machine part at the critical location

k_a = surface condition modification factor (highly polishes sample has a higher endurance limit as compared to an unpolished sample)

k_b = size modification factor (smaller sample has larger endurance limit as compared to a larger sample)

k_c = load modification factor (parts subjected to bending have higher endurance as compared to parts subjected to axial loads and torsion)

k_d = temperature modification factor (endurance limit for steel decreases for temperatures > 300° C)

k_e = reliability factor (higher reliability requirements leads to a lower value of k_e)

k_f = miscellaneous-effects modification factor (accounts for reduction in endurance limit due to all other factors – residual stress, corrosion, cyclic frequency)

σ_e' = rotary-beam test specimen endurance limit

Endurance Limit Modifying Factors

Surface Factor k_a

$$k_a = a\sigma_{ult}^{b}$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size Factor k_b

For bending and torsion

$$\mathrm{k}_b = \begin{cases} 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 \le d \le 254\,\text{mm} \end{cases}$$

For non rotating round bar in bending or noncircular c/s see Shigley

For axial loading $k_b = 1$

Load Factor k_c

$$\mathrm{k}_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

Temperature Factor k_d

Temperature, °C	S_T/S_{RT}
20	1.000
50	1.010
100	1.020
150	1.025
200	1.020
250	1.000
300	0.975
350	0.943
400	0.900
450	0.843
500	0.768
550	0.672
600	0.549

If σ'_e is known, then

$$k_d = \frac{S_T}{S_{RT}}$$

If σ'_e is unknown, then, calculate it using temperature compensated S_T

And use k_d =1

Endurance Limit Modifying Factors

Reliability Factor k_e

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

$$\boxed{\sigma_e = k_a k_b k_c k_d k_e k_f \sigma'_e}$$

Miscellaneous-Effects Factor k_f

Factors including residual stress, corrosion, cyclic frequency, electrolytic plating, metal spraying, frettage corrosion

Stress Concentration and Notch Sensitivity

- Have looked at the theoretical or geometrical stress concentration factors (K_t, K_{ts}) earlier - it depends on the geometry and loading
- Not all materials are equally sensitive to the presence of geometrically discontinuous
- For materials which are not fully sensitive to the presence of geometric discontinuous, a reduced value of K_t (Kt_s) are used.
- This reduced factor is referred to as fatigue stress concentration factor K_f or K_{fs}

$$\sigma_{max} = K_f \sigma_o \text{ or } \tau_{max} = K_{fs} \tau_o$$

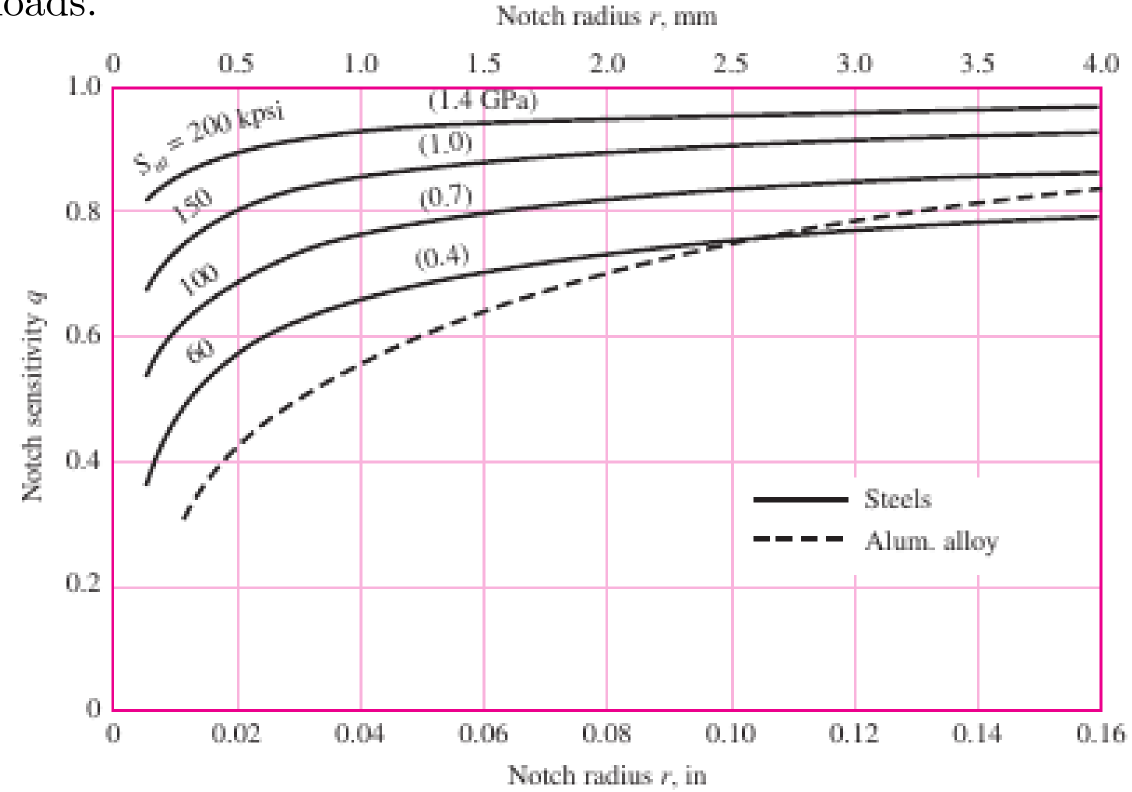
- Notch sensitivity factor is defined as

$$q = \frac{K_f - 1}{K_t - 1} \text{ or } q_{shear} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

- $q = 0$ indicates that the material is insensitive to a notch while $q = 1$ indicates that the material is fully sensitive to the notch

Stress Concentration and Notch Sensitivity

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads.



For a given material, geometric discontinuity and loading determine K_t and q. Using the equation for the notch sensitivity find K_f. i.e

$$K_f = 1 + q(K_t - 1)$$

The fatigue stress-concentration factor is then used as a multiplier of the nominal stress. We will use to modify both the mean stress component and the alternating stress component

Problem – Zero Mean Stress & Endurance Limit Correction

A 1050 hot-rolled steel bar has been **machined** to a **diameter of 2.5 cm**. It is to be placed in **reversed axial loading** for 10000 cycles to failure at room temperature.

Using ASTM minimum properties, and a **reliability of 99 percent**, estimate the endurance limit, fatigue strength at 10000 cycles and the number of cycles to fail at $\sigma_a = 380$ MPa.

Problem – Zero Mean Stress & Endurance Limit Correction

UTS for HR 1050 steel

$sult := 620\ \text{MPa}$ $\quad f := 0.86$

Endurance strength

$s2 := 0.5 \cdot sult = 310\ \text{MPa}$ $\quad N2 := 10^{6}$

Correction Factors

Calculation of surface factor

$a := 4.51$ $\quad bsurF := -0.265$

$$ka := a \cdot \left(\frac{sult}{1\ \text{MPa}}\right)^{bsurF} = 0.8207$$

$kb := 1$ No size effect for axial loading

$kc := 0.85$ Load factor

$kd := 1$ Temperature effect

$ke := 0.814$ reliability factor

$kf := 1$ Miscellaneous factor

$$s2C := ka \cdot kb \cdot kc \cdot kd \cdot ke \cdot kf \cdot s2 = 176.0345\ \text{MPa}$$

$s1 := f \cdot sult = 533.2\ \text{MPa}$ $\quad N1 := 10^{3}$

Problem: Find the fatigue strength at $N := 10000$

Number of cycles to fail at $sp := 380\ \text{MPa}$

$$b := \frac{\log_{e}\left(\frac{s1}{s2C}\right)}{\log_{e}\left(\frac{N1}{N2}\right)} = -0.1604 \qquad sf := \frac{s2C}{(2 \cdot N2)^{b}} = 1804.9991\ \text{MPa}$$

Answers

Fatigue strength at $N = 10000$

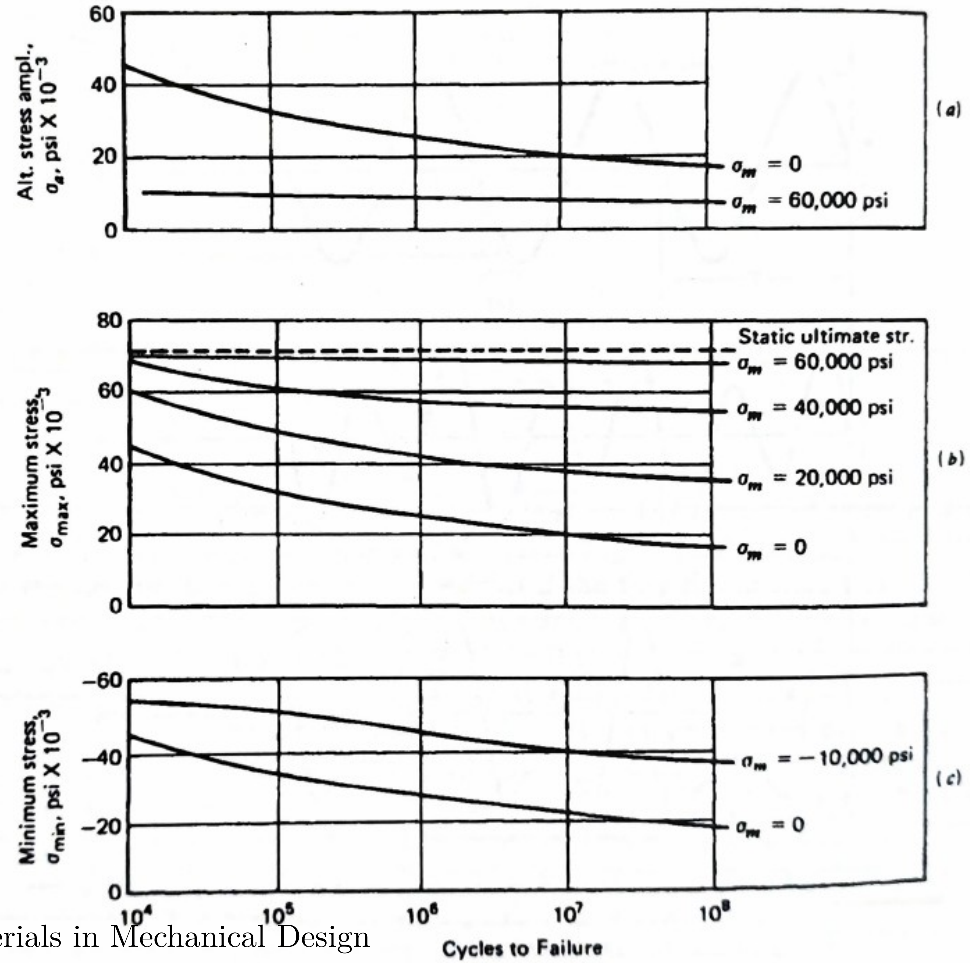
$$sN := sf \cdot (2 \cdot N)^{b} = 368.5185\ \text{MPa}$$

Number of cycles to fail at $sp = 380\ \text{MPa}$

$$Np := \frac{1}{2} \cdot \left(\frac{sp}{sf}\right)^{\frac{1}{b}} = 8259.3729$$

Approximately 8200 cycles

Effect of Mean Stress on Endurance Limit



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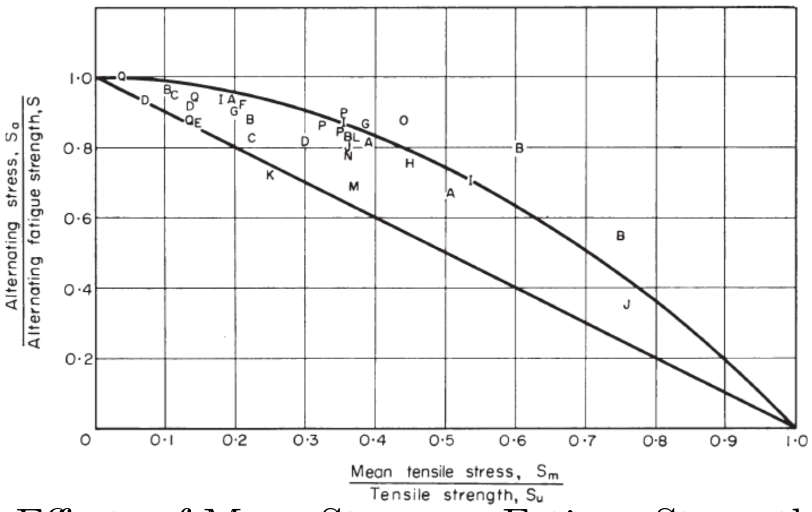
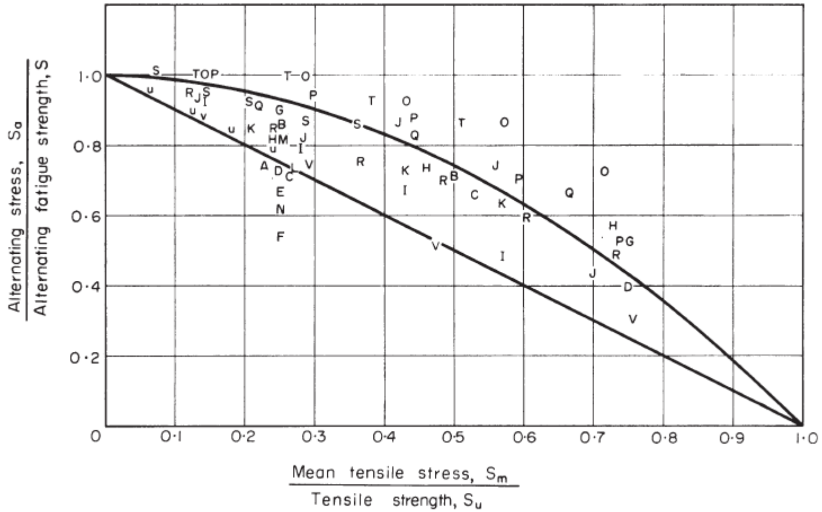
Tensile mean stress has a detrimental on the fatigue life while compressive mean stress has a beneficial effect

Effect of Tensile Mean Stress on the Fatigue Life



Effects of Mean Stress on Fatigue Strength of Steels based on 10^7 to 10^8 Cycles



Effects of Mean Stress on Fatigue Strength of Aluminum based on 5 x 10^8 Cycles

- A parabola called the Gerber line, can be fitted to the data with reasonable accuracy.
- A straight line connecting the fatigue strength with the ultimate strength, called the modified Goodman line, is a reasonable fit to the lower envelope of the data.

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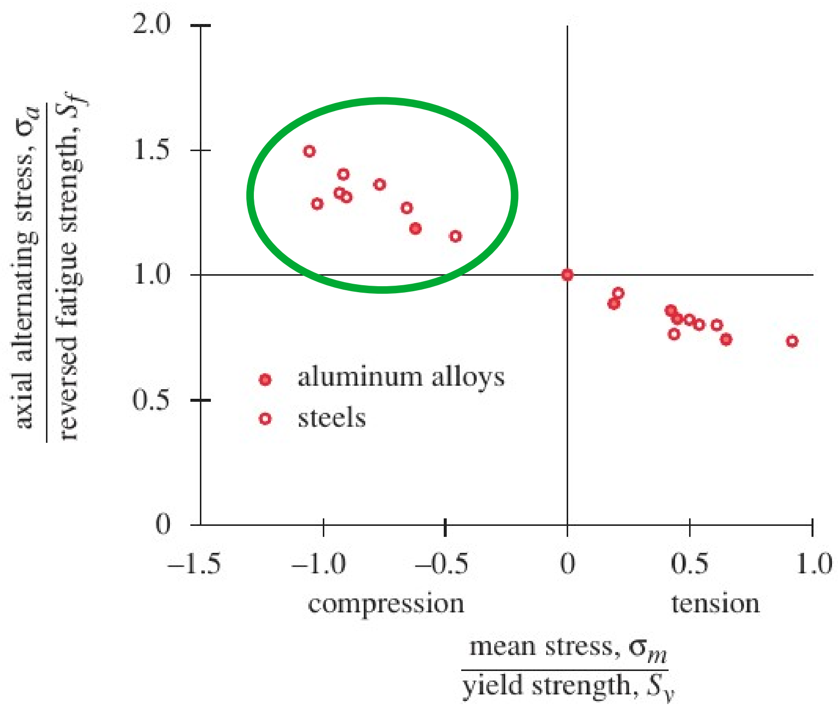
Effect of Mean Stress on the Fatigue Life



- Compressive mean stresses have a beneficial effect
- Tensile mean stresses have a detrimental effect
- To reduce the effects of alternating tensile stresses one deliberately introduce mean compressive stresses
- One way to do this is to create **residual compressive stress** in the material in regions where large alternating components are expected

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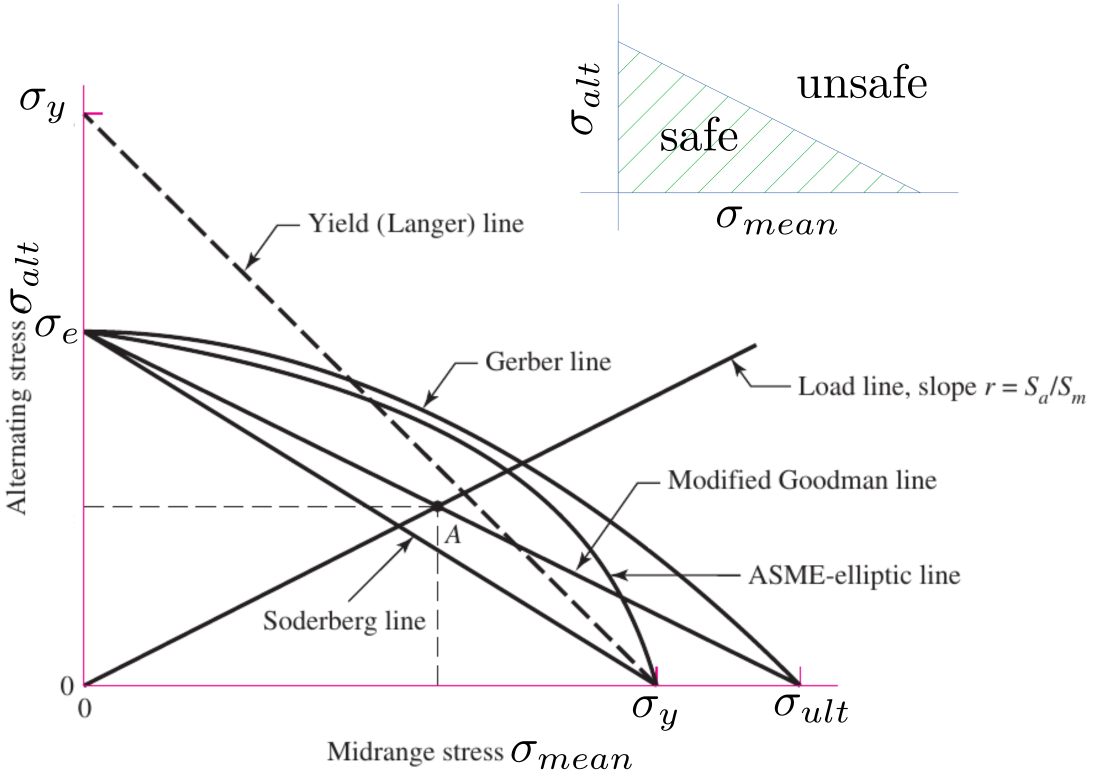
Fatigue Criteria in Presence of Mean Stress



Soderberg line $\dfrac{\sigma_a}{\sigma_e} + \dfrac{\sigma_m}{\sigma_y} = 1$

Modified Goodman line $\dfrac{\sigma_a}{\sigma_e} + \dfrac{\sigma_m}{\sigma_{ult}} = 1$

Gerber line $\dfrac{\sigma_a}{\sigma_e} + \left(\dfrac{\sigma_m}{\sigma_{ult}}\right)^2 = 1$

SME-elliptic line $\left(\dfrac{\sigma_a}{\sigma_e}\right)^2 + \left(\dfrac{\sigma_m}{\sigma_y}\right)^2 = 1$

Yield (Langer) line $\sigma_a + \sigma_m = \sigma_y$

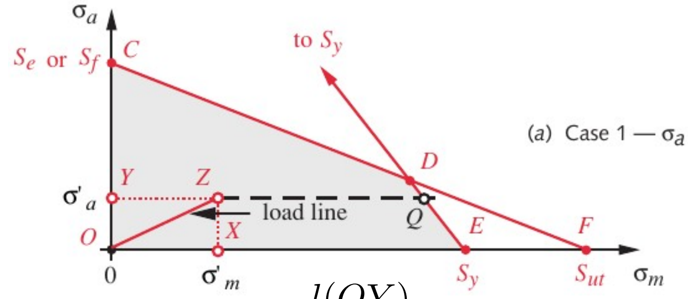
Smith-Watson-Topper Criterion

$$\sigma_e = \sqrt{\sigma_{max}\sigma_a} = \sqrt{(\sigma_m + \sigma_a)\sigma_a}$$

σ_e is the modified/corrected endurance limit

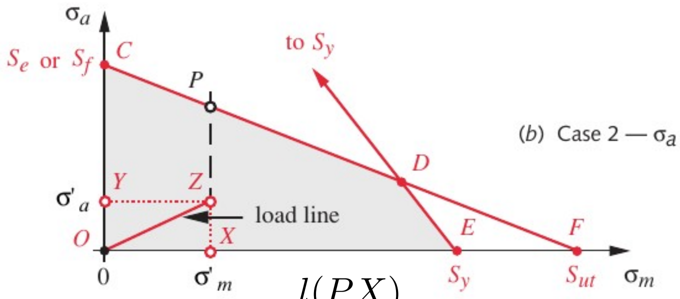
If N is the factor of safety then replace σ_m and σ_a with $N\sigma_m$ and $N\sigma_a$, respectively

Factor of Safety (FOS) in the Presence of Mean Stress using augmented modified Goodman diagram



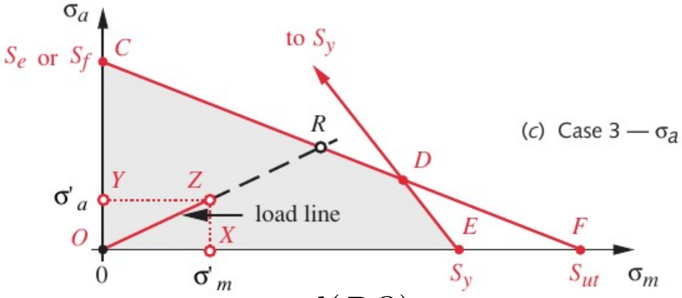
(a) Case 1 — σ_a constant and σ_m varies

$$FOS = \frac{l(QY)}{l(ZY)}$$



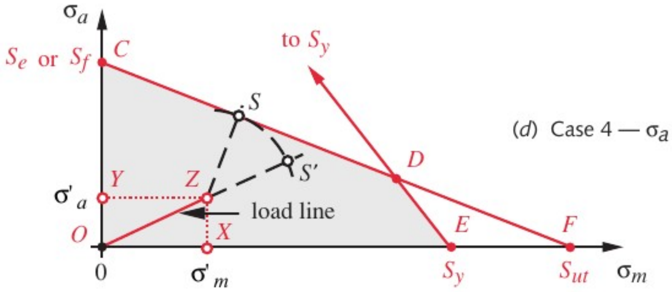
(b) Case 2 — σ_a varies and σ_m constant

$$FOS = \frac{l(PX)}{l(ZX)}$$



(c) Case 3 — σ_a / σ_m ratio constant

$$FOS = \frac{l(RO)}{l(ZO)}$$



(d) Case 4 — σ_a and σ_m vary independently

$$FOS = \frac{l(SZ) + l(ZO)}{l(ZO)}$$

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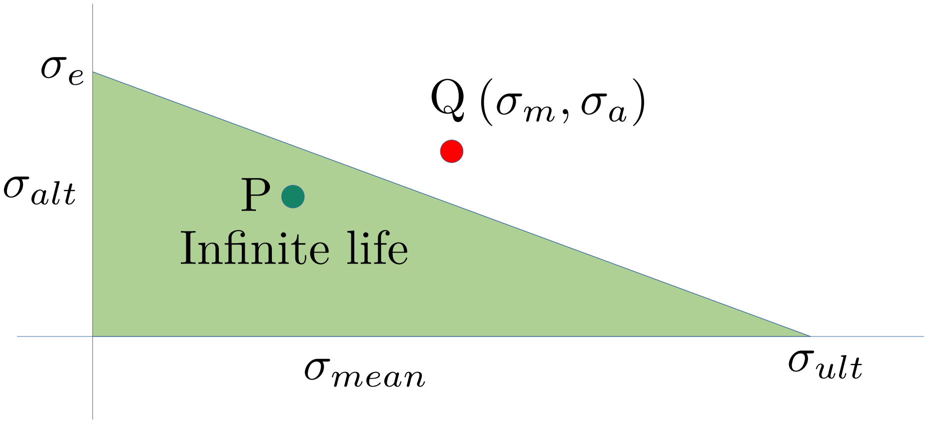
Finite Life in Presence of Mean Stress



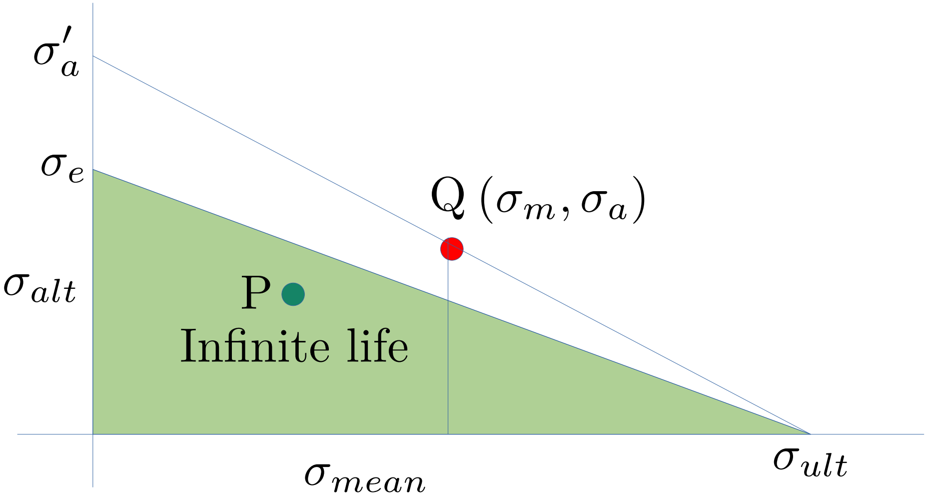
- At point P, the part has infinite life
- At point Q, the part will fail before 10^6 cycles (finite life)
- Want to estimate the number of cycles when it fails
- Data is generally available for completely reversed loading
- Procedure: Find the equivalent complete reversed stress amplitude σ'_a and then use the Basquin equation to estimate the number of cycles to failure

$$\sigma'_a = \sigma'_f (2N_f)^b$$

Finite Life in Presence of Mean Stress

Multiple approaches available to estimate the equivalent fully reversed stress amplitude, σ_a' corresponding to (σ_m, σ_a)

Approach based on Goodman Diagram



From similar triangles

$$\frac{\sigma_a'}{\sigma_{ult}} = \frac{\sigma_a}{(\sigma_{ult} - \sigma_m)}$$

Therefore

$$\boxed{\sigma_a' = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_{ult}}\right)}}$$

Approach based on Smith-Watson-Topper (SWT) criterion

$$\boxed{\sigma_a' = \sqrt{\sigma_{max}\sigma_a} = \sqrt{(\sigma_m + \sigma_a)\sigma_a}}$$

- Approach based on Goodman diagram is widely used but is overly conservative and inaccurate
- SWT is reasonably accurate and **does not require any material properties**

Problem – Non Zero Mean Stress

A 1050 hot-rolled steel bar has been **machined** to a **diameter of 2.5 cm**. It is to be placed in **reversed axial loading** for 10000 cycles to failure at room temperature.

Using ASTM minimum properties, and a **reliability of 99 percent**, estimate the endurance limit, and the number of cycles to fail at $\sigma_a = 160$ MPa, $\sigma_m = 100$ MPa using

1. approach based on the Goodman diagram
2. Approach based on the SWT criterion

Problem – Endurance Limit Correction & Non Zero Mean Stress

UTS for HR 1050 steel

$sult := 620\ \text{MPa} \qquad f := 0.86$

Endurance strength

$s2 := 0.5 \cdot sult = 310\ \text{MPa} \qquad N2 := 10^{6}$

Correction Factors

Calculation of surface factor

$a := 4.51 \quad bsurF := -0.265$

$$ka := a \cdot \left(\frac{sult}{1\ \text{MPa}}\right)^{bsurF} = 0.8207$$

$kb := 1$ No size effect for axial loading

$kc := 0.85$ Load factor

$kd := 1$ Temperature effect

$ke := 0.814$ reliability factor

$kf := 1$ Miscellaneous factor

$s2C := ka \cdot kb \cdot kc \cdot kd \cdot ke \cdot kf \cdot s2 = 176.0345\ \text{MPa}$

$s1 := f \cdot sult = 533.2\ \text{MPa} \qquad N1 := 10^{3}$

Endurance limit

Find Number of cycles to fail at $\quad sa := 160\ \text{MPa} \qquad sm := 100\ \text{MPa}$

Constants for the Basquin curve

$$b := \frac{\log_e\left(\frac{s1}{s2C}\right)}{\log_e\left(\frac{N1}{N2}\right)} = -0.1604 \qquad sf := \frac{s2C}{(2 \cdot N2)^{b}} = 1804.9991\ \text{MPa}$$

Answers

equivalent fully reversed stress amplitude

$$sap := \frac{sa}{\left(1 - \frac{sm}{sult}\right)} = 1.9077 \cdot 10^{8}\ \text{Pa}$$

$$Nf := \frac{1}{2} \cdot \left(\frac{sap}{sf}\right)^{\frac{1}{b}} = 6.0589 \cdot 10^{5}$$

Number of cycles to fail is approximately 605000

$$saswt := \sqrt{(sa + sm) \cdot sm} = 161.2452\ \text{MPa}$$

Equivalent stress amplitude based on SWT criteria 161.2 MPa. Less than the Endurance limit – does not fail

Combination of Loading Modes (Multiaxial Loading)

In general loading can be classified as

- Completely reversing simple loads (zero mean, but only one mode of loading)
- Fluctuating simple loads (nonzero mean, but only one mode of loading)
- Combinations of loading modes (axial, torsion, bending, e.g. rotating shaft subject to a static bending moment and carrying a torque)

Complications:

- The loading components can be at same/different frequencies, same/different phases
- As a result, in general, the direction of principal stresses change with time and damage accumulates in different planes

Classification of combined (multiaxial) loading:

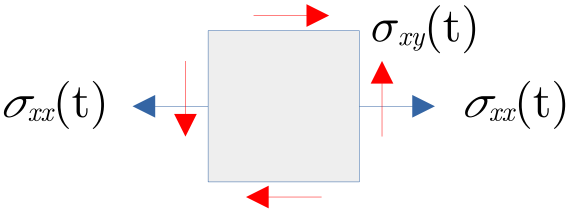
- **Simple multiaxial loading**

  loading where direction of principal stresses do not change with time

- **Complex multiaxial**

  loading where direction of principal stresses change with time

Combination of Loading Modes (Multiaxial Loading)



Simple Multiaxial Loading

(same frequency, same phase)

$$\sigma_{xx} = a_o \sin 2\pi t$$

$$\sigma_{xy} = b_o \sin 2\pi t$$

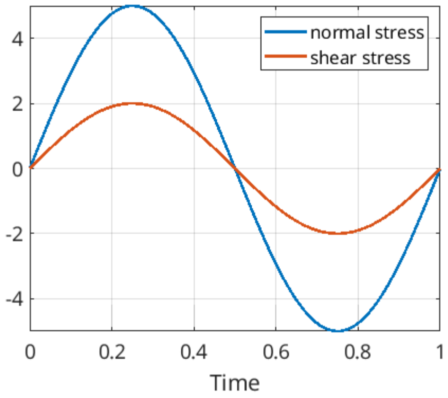
Complex Multiaxial Loading

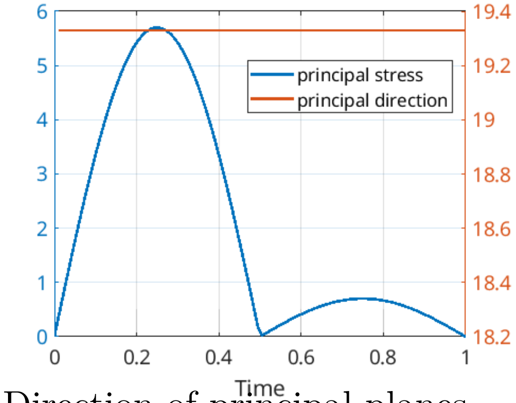
(same frequency, different phase)

$$\sigma_{xx} = a_o \sin 2\pi t$$

$$\sigma_{xy} = b_o \sin(2\pi t + \pi/3)$$

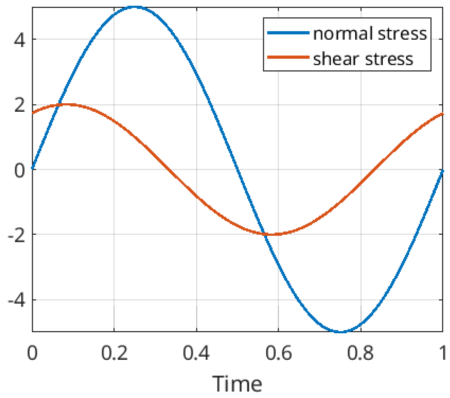
Simple Multiaxial Loading

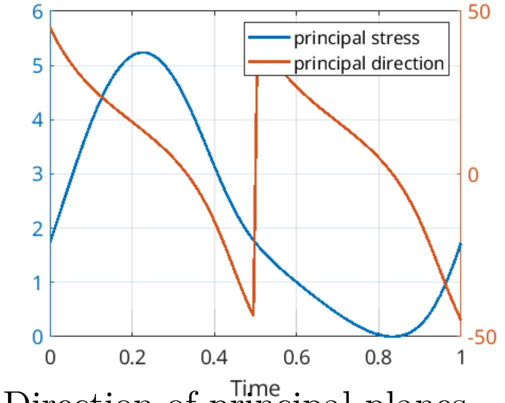




Direction of principal planes do not change

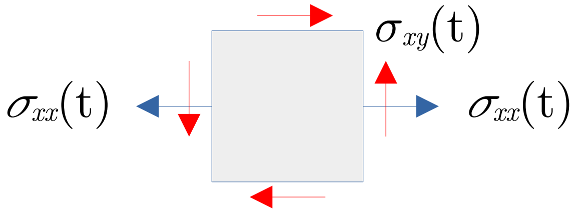
Complex Multiaxial Loading





Direction of principal planes changes

Combination of Loading Modes (Multiaxial Loading)

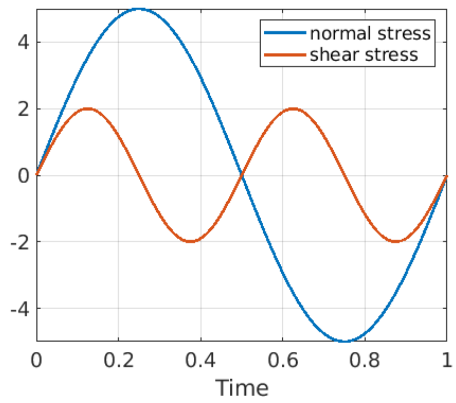


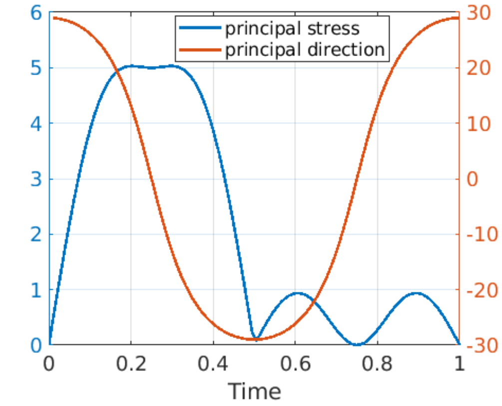
Complex Multiaxial Loading
(different frequencies)

$$\sigma_{xx} = a_o \sin 2\pi t$$

$$\sigma_{xy} = b_o \sin 4\pi t$$

Complex Multiaxial Loading



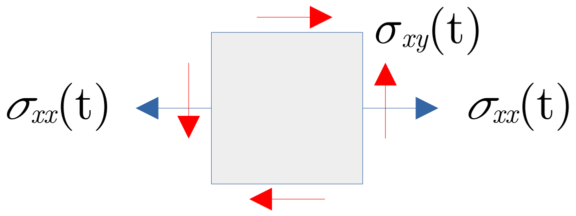


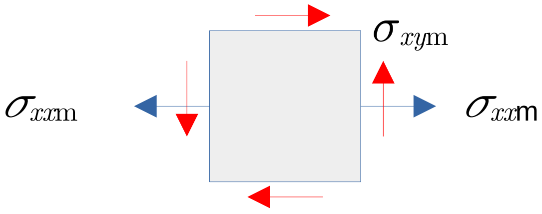
Direction of principal planes changes

Combination of Loading Modes (Multiaxial Loading)

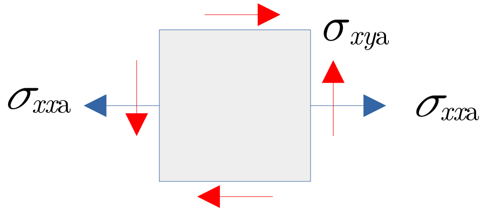
Approach:

- To analyse static failures we used the idea of von Mises stress to combine various stress components to come up with a single number characterizing the state of stress at a point.
- Here too we will use the von Mises stress to find the equivalent alternating component and the equivalent mean components





mean components



alternating components

Combination of Loading Modes (Multiaxial Loading)

Biaxial state of stress

$$\sigma_{vma} = (\sigma_{xxa}^2 - \sigma_{xxa}\sigma_{yya} + \sigma_{yya}^2 + 3\sigma_{xya})^{1/2}$$ Equivalent alternating stress

$$\sigma_{vmm} = (\sigma_{xxm}^2 - \sigma_{xxm}\sigma_{yym} + \sigma_{yym}^2 + 3\sigma_{xym})^{1/2}$$ Equivalent mean stress

Here

$$\sigma_{xxa} = \frac{(\sigma_{xx})_{max} - (\sigma_{xx})_{min}}{2}, \quad \sigma_{xxm} = \frac{(\sigma_{xx})_{max} + (\sigma_{xx})_{min}}{2} \qquad \sigma_{yya} = \frac{(\sigma_{yy})_{max} - (\sigma_{yy})_{min}}{2}, \quad \sigma_{yym} = \frac{(\sigma_{yy})_{max} + (\sigma_{yy})_{min}}{2}$$

$$\sigma_{xya} = \frac{(\sigma_{xy})_{max} - (\sigma_{xy})_{min}}{2}, \quad \sigma_{xym} = \frac{(\sigma_{xy})_{max} + (\sigma_{xy})_{min}}{2}$$

Triaxial state of stress

$$\sigma_{vma} = \frac{1}{\sqrt{2}}\left[(\sigma_{xxa} - \sigma_{yya})^2 + (\sigma_{yya} - \sigma_{zza})^2 + (\sigma_{zza} - \sigma_{xxa})^2 + 6(\sigma_{xya}^2 + \sigma_{yza}^2 + \sigma_{zxa}^2)\right]^{1/2}$$

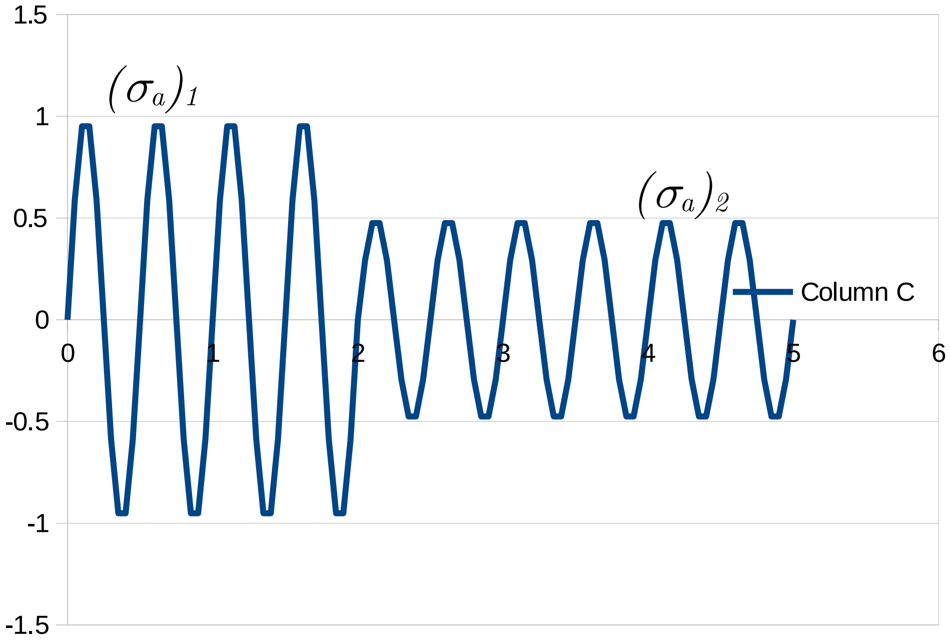
$$\sigma_{vmm} = \frac{1}{\sqrt{2}}\left[(\sigma_{xxm} - \sigma_{yym})^2 + (\sigma_{yym} - \sigma_{zzm})^2 + (\sigma_{zzm} - \sigma_{xxm})^2 + 6(\sigma_{xym}^2 + \sigma_{yzm}^2 + \sigma_{zxm}^2)\right]^{1/2}$$

Steps for Multiaxial Loading

- Theoretical/geometric stress concentration factors and notch sensitivity will be different for different loadings. Calculate the corresponding fatigue stress concentration factors
- Generate two stress elements—one for the alternating stresses and one for the mean stresses.
- Apply the appropriate fatigue stress concentration factors to each of the stresses; that is, apply $(K_f)_{\text{bending}}$ for the bending stresses, $(K_{fs})_{\text{torsion}}$ for the torsional stresses, and $(K_f)_{\text{axial}}$ for the axial stresses.
- Calculate an equivalent von Mises stress for each of these two stress elements, σ_a' and σ_m'
- Select a fatigue failure criterion.
- For the endurance limit, σ_e, the only correction factors that are affected by multiple load types are the size factor k_b and the load factor k_c.
- Find k_b, calculate it for each load and select the lowest one.
- The only load factor to be applied is corresponding to the axial load if it happens to be the dominant mode of loading. The load factor corresponding to the torsion load should not
- be applied as the that mode of loading is already considered while calculating the von Mises stress.

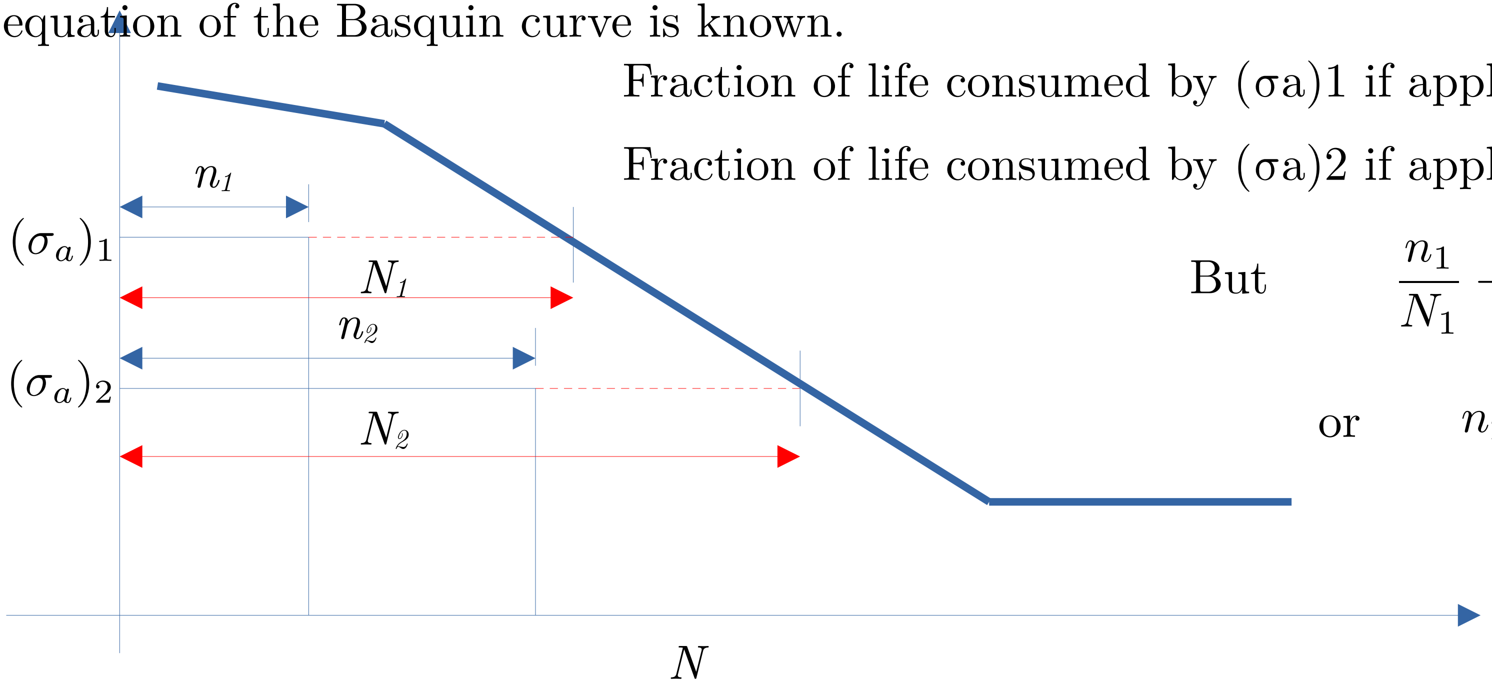
Cumulative Damage

A machine part is subject to n_1 (given) fully reversed stress cycles with amplitude $(\sigma_a)_1$ (given). It is then subject to n_2 (not known) fully reversed stress cycles of amplitude $(\sigma_a)_2$ (given) before it eventually fails. Find n_2. Assume $(\sigma_a)_1 > \sigma_e$, $(\sigma_a)_1 > \sigma_e$. Assume that the equation of the Basquin curve is known.



Cumulative Damage – Palmergren – Miner Theory

A machine part is subject to n_1 (given) fully reversed stress cycles with amplitude $(\sigma_a)_1$ (given). It is then subject to n_2 (not known) fully reversed stress cycles of amplitude $(\sigma_a)_2$ (given) before it eventually fails. Find n_2. Assume $(\sigma_a)_1 > \sigma_e$, $(\sigma_a)_1 > \sigma_e$. Assume that the equation of the Basquin curve is known.



Fraction of life consumed by (σa)1 if applied alone: n1/N1

Fraction of life consumed by (σa)2 if applied alone: n2/N2 (unknown)

But $$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1$$

or $$n_2 = N_2\left(1 - \frac{n_1}{N_1}\right)$$

Cumulative Damage – Palmergren-Miner Theory

In general, if fully reversed stress cycles with amplitude $(\sigma_a)_1,\ (\sigma_a)_2, \ldots, (\sigma_a)_k$ act for $n_1, n_2, \ldots, n_k$ cycles with

$$\sum_{i=1}^{k} n_i = n_{total},\ n_{total} \text{ total number of cycles to failure}$$

then

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = 1$$

and

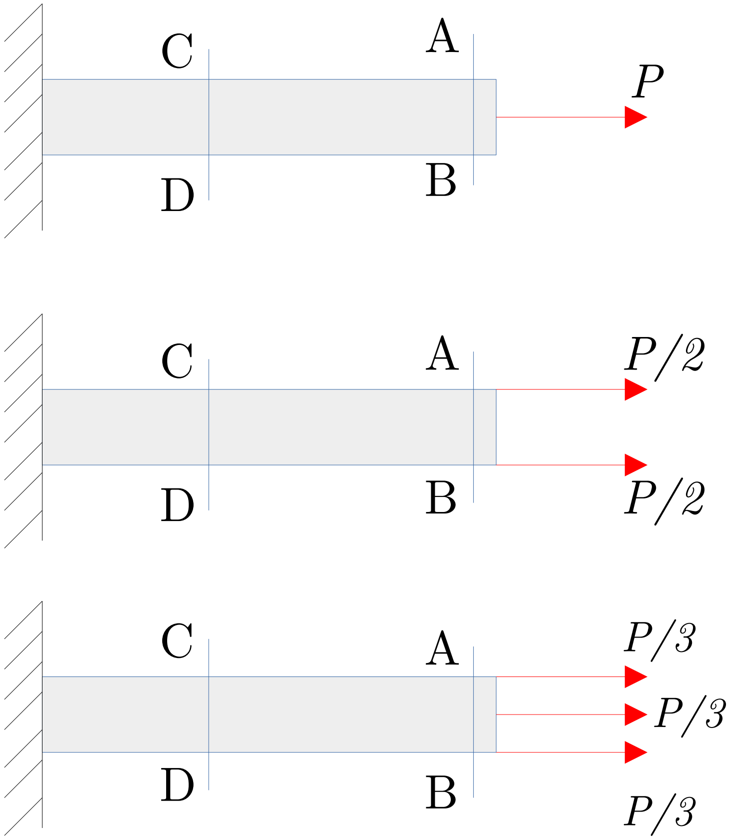
$$\sum_{i=1}^{k} \frac{\alpha_i}{N_i} = \frac{1}{n_{total}},\ \sum_{i=1}^{k} \alpha_i = 1,\ \alpha_i = \frac{n_i}{n_{total}}$$

$\alpha_1, \alpha_2, \ldots, \alpha_k$ are the fraction of the total life spent at $(\sigma_a)_1,\ (\sigma_a)_2, \ldots, (\sigma_a)_k$

Limitations of the Miner Rule:

Does not take into account the sequence in which the stress cycles are applied

Saint Venant's Principle



In each of the three cases the resultant load is the same

Saint Venant's principle states that the displacement, strain and stress distributions caused by <u>statically equivalent force distributions</u> in parts of the body which <u>are sufficiently far from the loading parts are approximately the same</u>

Accordingly the displacement, strain and stress distributions at section CD will be approximately the same in the three cases

The displacement, strain and stress distributions at section AB will be different in the three cases

Design Factor and Factor of Safety

A solid circular rod undergoes a bending moment $M = 100$ Nm. Assuming that the yield strength of the material is 170 MPa and a **design factor of 2.5**, determine the minimum diameter of the rod. From the available sizes, choose an appropriate sized rod and determine the **factor of safety**.

Given: $M = 100$ Nm, $\sigma_y = 170$ MPa, $n_d = 2.5$
To find: diameter of the circular rod and the factor of safety
For a rod of diameter d, the maximum bending stress is given by

$$\sigma_{max0} = \frac{Md/2}{I} = \frac{Md/2}{\frac{\pi}{64}d^4} = \frac{32M}{\pi d^3}$$

Using the design factor, the component is to be designed for σ_{max} given by

$$\sigma_{max} = n_d \sigma_{max0}$$

The minimum diameter, d_{min}, required to withstand σ_{max} is obtained by solving

$$\sigma_y = \sigma_{max}$$

From the catalog select $d_s > d_{min}$ which is closest to d_{min}. The factor of safety is then given by

$$\text{FOS} = \frac{\sigma_y}{32M/(\pi d_s^3)}$$

Design Factors to be used for Ductile Materials

1.25 to 2.0	Design of structures under static load for which there is a high level of confidence in all design data
2.0 to 2.5	Design of machine elements under dynamic loading with average confidence in all design data
2.5 to 4.0	Design of static structures or machine element under dynamic loading with uncertainty about loads, material properties, environment
4.0 and higher	Design of static structures or machine element under dynamic loading with uncertainty about some combinations of loads, material properties, environment

Design Factors to be used for Brittle Materials

3.0 to 4.0	Design of structures under static load for which there is a high level of confidence in all design data
4.0 to 8.0	Design of static structures or machine element under dynamic loading with uncertainty about loads, material properties, environment