ME423[S2] -Information

- **Lectures**: Wednesday and Fridays, 11.05 am to 12.30 pm in LC101
- **Credit Structure**: 2-1-2 (8 credits)
- **Instructor**: Salil S. Kulkarni
  - Office: F-38, ME Building
  - Email: salil.kulkarni@iitb.ac.in
  - Office Hours: Tuesday 2.30 pm to 3.30 pm or by appointment
- **Textbooks**:
  - Shigley's Mechanical Engineering Design, R.G. Budynas, J.K. Nisbett; Tata Mcgraw-Hill Publishing Co. Ltd., 2012.
  - Machine Design: An integrated approach, R.L. Norton; Pearson Education Inc. (India), 2nd edition, 2000.
- **Group Project**: Groups of 8 students will work on design project which will run for the entire duration of the semester.
- Attendance will be taken in every lecture.
- Assignments/tutorials will be assigned roughly once every 2 weeks.

ME423[S2] -Information

Course Objectives:

- Apply concepts from Solid Mechanics and Strength of Material courses to design machine parts.
- Use failure theories to analyse and evaluate the design of machine parts.
- Use appropriate materials in the design of machine parts

This course will involve significant amount of problem solving using hand calculations.

Grading Policy:

Assignments and Tutorials	5%
Quizzes (2)	10%
Midterm Exam	25%
Final Exam	25%
Project	35% (15% insem evaluation+20% final evaluation)

ME423[S2] – Course Contents

- Introduction to Machine Design
- Review of Mechanics of Materials - combined stresses, strains
- Static Failure Theories (steady loads) - ductile and brittle materials
- Fatigue Failure Theories (variable loads) - high cycle fatigue, low cycle fatigue
- Material selection
- Design of shafts, axles
- Design of welded joints
- Design of screws and fasteners
- Selection of bearings
- Design of spur gears
- Design of mechanical springs
- Selection of Sensors and Actuators
- Design of clutches and brakes

Introduction

Mechanical Design: It is a process of using scientific principles and tools of engineering to produce products or systems that are mechanical in nature (machines, structures, devices) and which satisfy a certain requirement.

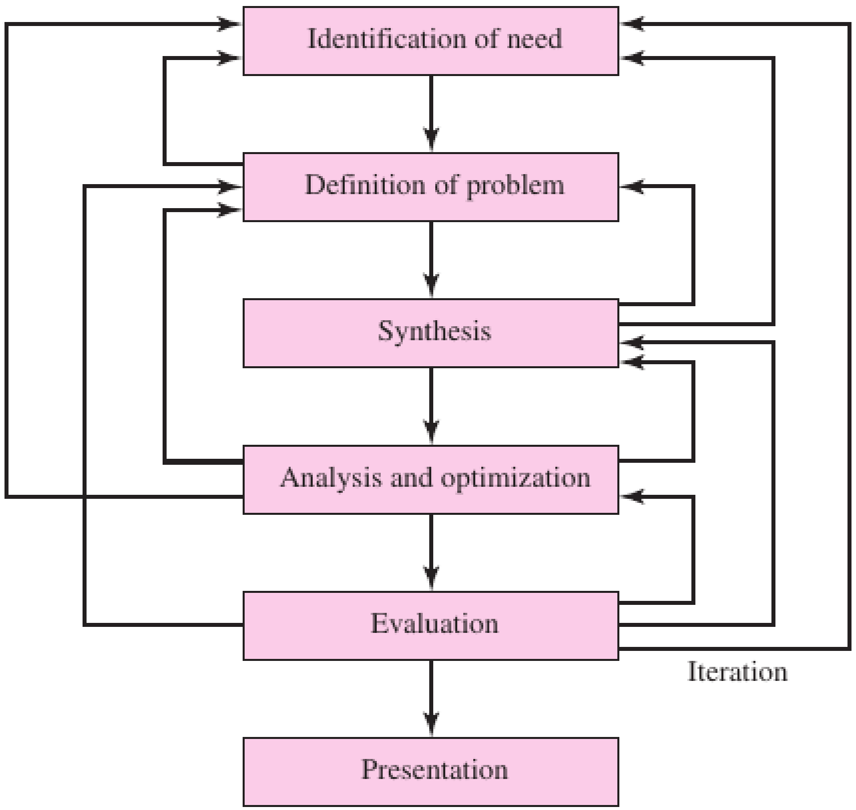
Mechanical design utilizes mathematics, material sciences and engineering mechanics.

The main goal in **machine design** is to **size and shape the parts** (machine elements) and **choose appropriate material** and **manufacturing processes** so that the resulting machine can be expected to **perform its intended function without failure**.

Design problems have no unique answers.

Design problems are always subject to some constraints.

Mechanical Design Process



Mechanical design process is iterative in nature

- Identification of need
- Define the problem
- Synthesis
  - Generate new ideas
- Analyse and optimize
  - Analyse the different ideas
  - Select the most promising solution
- Evaluate
  - Detailed design and analysis of the selected solution
  - Build and test a prototype
- Refine the design if required
- Documentation

Shigley's Mechanical Engineering Design

Design Considerations

- **Functionality**: All machine parts must be capable of transmitting the necessary forces and performing the necessary motion (strength, stiffness, flexibility, noise).

- **Safety**: Failure must not occur in any part before a predetermined span of operating life has elapsed.

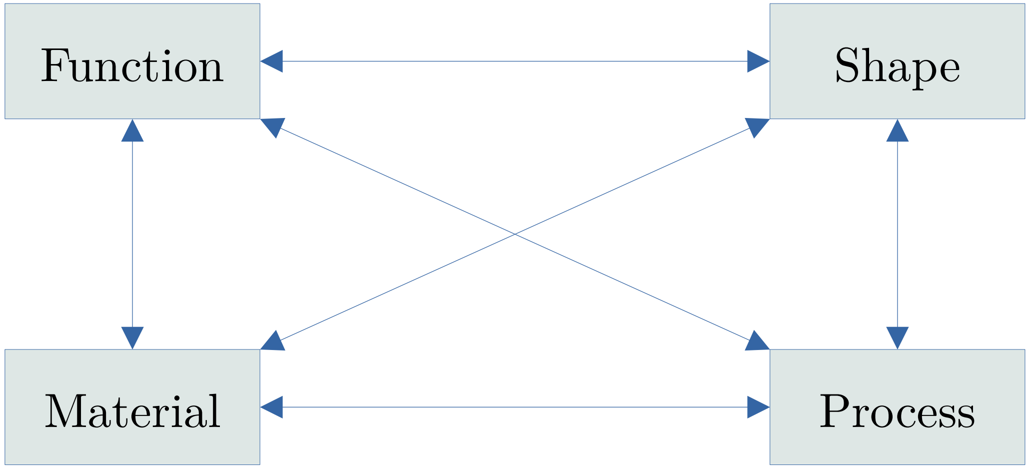
- **Manufacturability**: It must be possible to manufacture the part and assemble it in the machine.

- **Cost**: The cost of the finished part must be consistent with the application.

- **Material selection**: Choose material which is consistent with the safety, manufacturability

Design of Machine Elements

- Selecting a suitable type of machine element taking into account its function
- Estimating the size of the machine element that is likely to be safe
- Evaluating the machine element's performance against design requirements and constraints
- Modifying the design and dimensions until the performance is satisfactory



Interrelated factors

Design of Machine Elements

- **Physical/Actual Machine Element**
- **Analytical Model**: It is an imaginary model which resembles the actual system and captures its important features. It is obtained by making some simplifying assumptions.

  Important points to note while developing the analytical model:
    - Behaviour in many systems is a complex phenomenon.
    - Important variables may not be readily identified.
    - Cause and effect relations may not be readily apparent.
    - Interactions between variables may not be known.
- **Mathematical Model**: Obtained by applying physical laws to the analytical models, e.g. conservation of mass, energy, momentum balance, etc.
- Solution of the Mathematical Model
- Comparison with the behaviour of the actual problem
- If results do not match go to back to the development of the analytical model and repeat

Standards and Codes

- **Standards**: A standard can be defined as a set of technical definitions and guidelines that function as instructions for designers, manufacturers, operators, or users of equipment e.g. ASTM E8 | ASTM E8M (standard for performing tensile test). Standards are intended to achieve uniformity and reliability. Some organisations granting standards are:
  - ASME – American Society of Mechanical Engineering
  - ASTM – American Society of Testing and Materials
  - AMS – American Welding Society
  - ISO – International Standards Organization
  - SAE – Society of Automotive Engineers International

- **Codes**: A set of specifications for the analysis, design, manufacturing of something in order to achieve a specified degree of safety and performance, e.g. Pressure vessels in India must conform with IS 2825 published by the Bureau of Indian Standards (BIS). It specifies the design, fabrication, inspection, testing, and certification requirements for unfired pressure vessels.

Uncertainty in Machine Design

Typical uncertainties in machine design

- Composition of material and the effect of variation on properties.
- Variations in properties from place to place within a bar of stock.
- Effect of nearby assemblies such as weldments and shrink fit
- Intensity and distribution of loading.
- Validity of mathematical models used to represent reality.
- Intensity of stress concentrations.
- Influence of time on strength and geometry.

Uncertainties are accounted in either **deterministic** or **stochastic** manner.

Common Instruments

- Stapler
- Mechanical Pencil
- Ball pen

Assignment 1 – 6 page report maximum

- How do the **stapler** function ?
- Does it have/require any safety features ?
- How are the different parts of the stapler made and how do you think they are assembled ?
- What is the material that is used for different parts ? Can you justify ?

End

Review of Mechanics

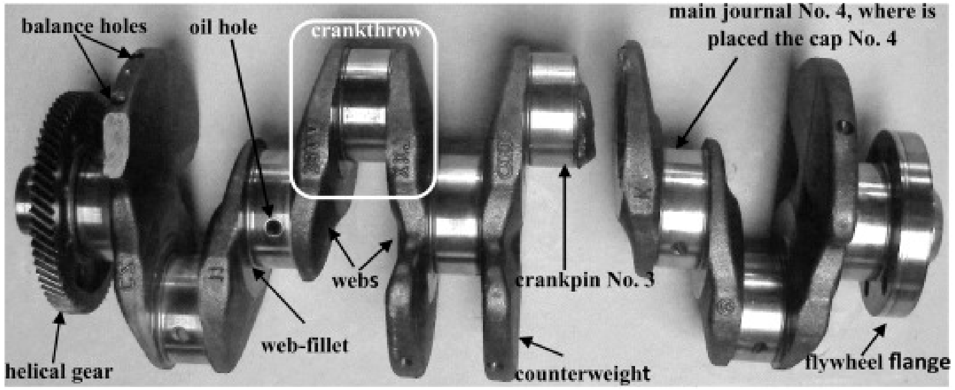
Mechanical Failures

Before After

Piston

Before After

Bearing



Crankshaft

M. Fonte, M. Freitas, L. Reis, Failure analysis of a damaged diesel motor crankshaft, Engineering Failure Analysis, Volume 102, 2019, Pages 1-6

Mechanical Failures

Cycle Fork

Shaft

Cycle Rim

Hip Prosthesis

Modes of Mechanical Failure

Mechanical Failure: Any change in the size, shape, or material properties of a structure, machine, or machine part that renders it incapable of satisfactorily performing its intended function.

- **Appearance of failure**
  - Elastic deformation
  - Plastic deformation
  - Fracture or rupture
  - Material change - metallurgical, chemical
- **Failure inducing agents**
  - Force - steady, transient, cyclic, random
  - Time - very short, short, long
  - Temperature - low, room, elevated, steady, transient, cyclic, random
  - Environment - reactive, nuclear
- **Failure locations**
  - Body type
  - Surface type

Common Failure Modes

- Force and/or temperature induced elastic deformation
- Yielding
- Fracture - ductile/brittle
- Fatigue - high cycle, low cycle, thermal
- Creep
- Buckling
- Corrosion
- Thermal shock
- Wear
- Fretting

All these involve solving problems in mechanics

Common Solution Procedure and Simplifying Assumptions

- Three ingredients essential for analysing mechanical systems in equilibrium are:
    - Equilibrium equations.
    - Strain displacement relations.
    - Constitutive behaviour (stress strain relations)

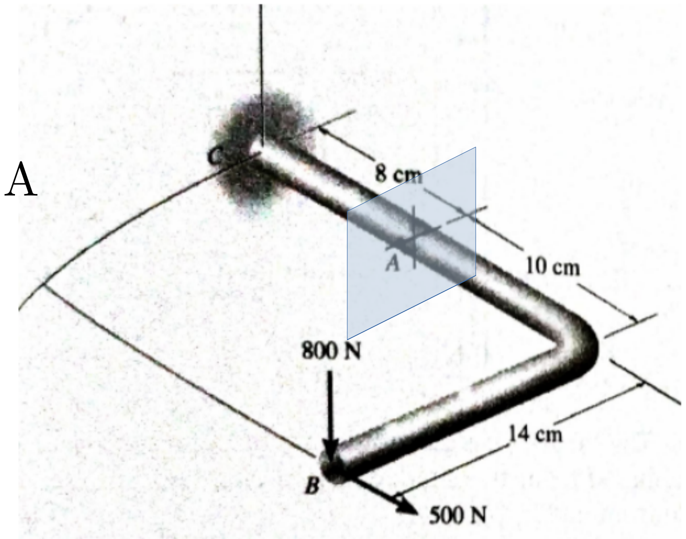
- Two most common assumptions that we make are:
  - displacements and their gradients are small (geometry)
  - stresses are linearly proportional to the strains (material)

Free Body Diagram and Internal Resisting Forces/Moments

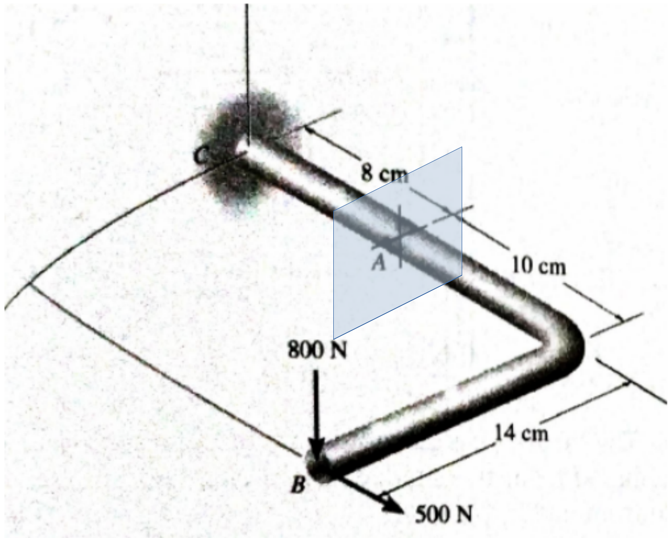
- **External forces** acting on a body are of two types – **surface forces**, e.g. contact forces and **body forces**, e.g. gravitational force.
- For a body in equilibrium, **free body diagrams** are used to identify reaction forces, moments and **internal forces, moments** transmitted across a section using

$$\sum \mathbf{F} = \mathbf{0}, \sum \mathbf{M} = \mathbf{0}$$

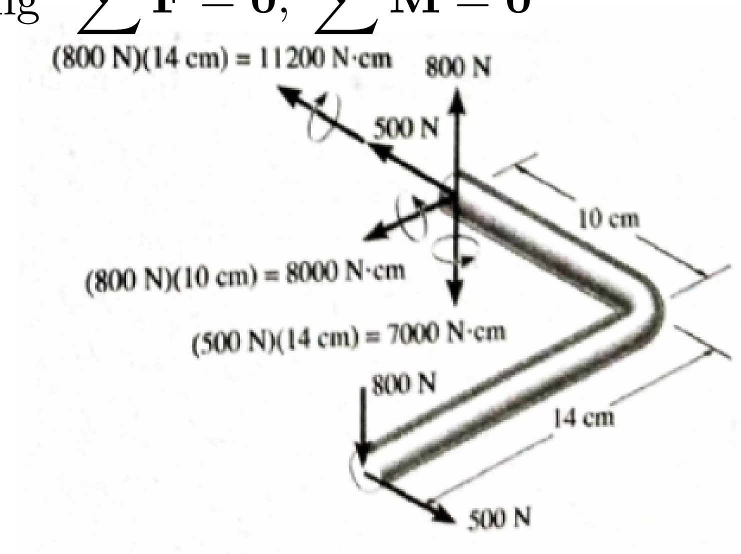
Determine the internal resisting forces and moments acting in the plane shown that passes through point A



Free Body Diagram and Internal Resisting Forces/Moments



Using $\sum \mathbf{F} = \mathbf{0}, \sum \mathbf{M} = \mathbf{0}$

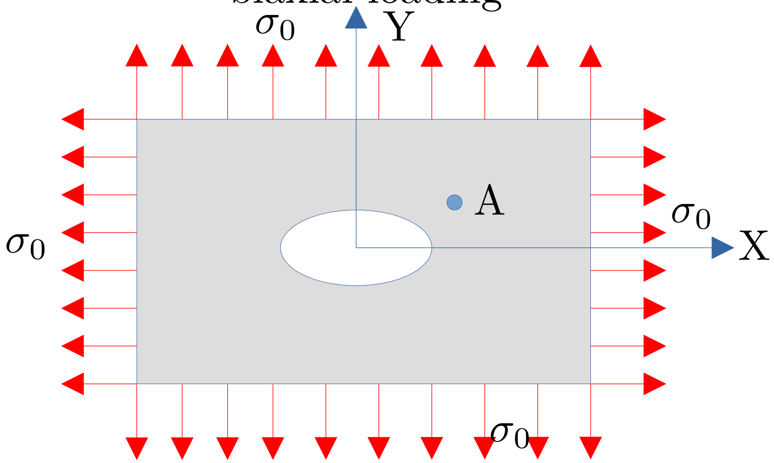


Free Body Diagram

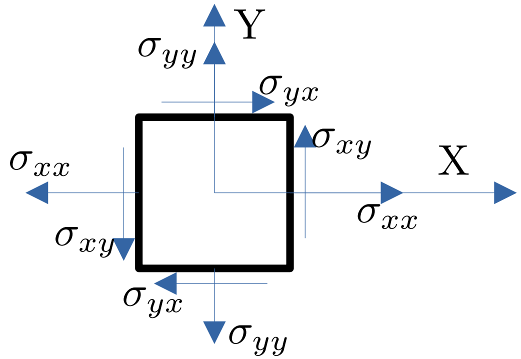
- The **internal forces acting a point** in the body can be characterized by a second order tensor called a **Cauchy stress tensor**. It is a symmetric tensor.
- The stress tensor has 6 independent components in 3D and 3 independent components in 2D.

Two-Dimensional State of Stress

Thin plate subjected to in-plane biaxial loading



State of stress at point A

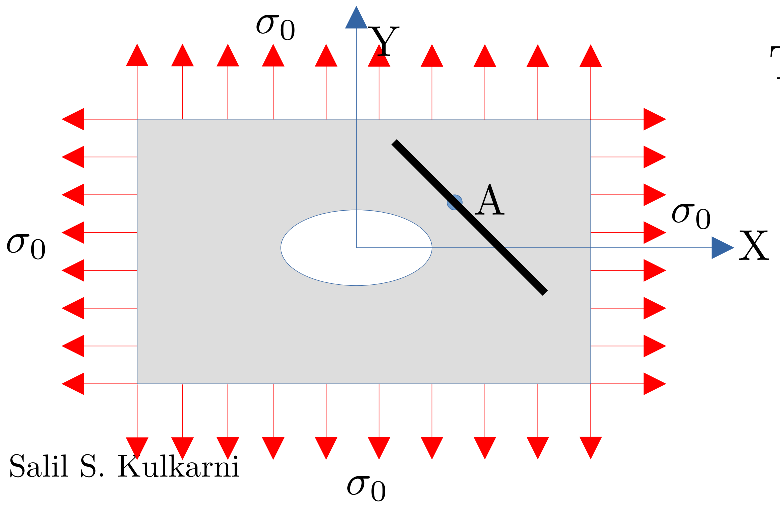


Cauchy stress tensor at point A referred to X-Y coordinate system

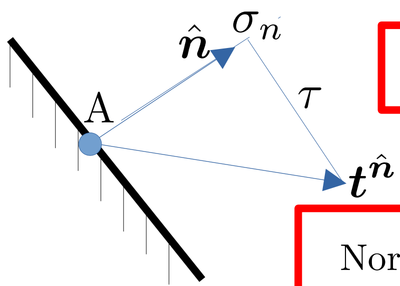
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}, \ \sigma_{xy} = \sigma_{yx}$$

$\sigma_{xx}, \ \sigma_{yy}$ normal stress components

σ_{xy} shear stress component



Traction vector acting on a plane with normal $\hat{\boldsymbol{n}}$



$$\boxed{\boldsymbol{t}^{\hat{\boldsymbol{n}}} = \boldsymbol{\sigma}\hat{\boldsymbol{n}}}$$

or

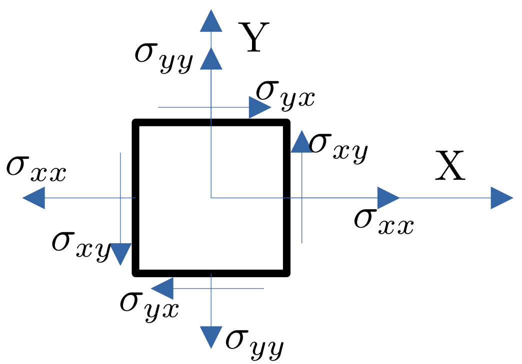
$$\begin{bmatrix} t_x^{\hat{\boldsymbol{n}}} \\ t_y^{\hat{\boldsymbol{n}}} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$

Normal component of the traction vector $\sigma_n = \boldsymbol{t}^{\hat{\boldsymbol{n}}} \cdot \hat{\boldsymbol{n}}$

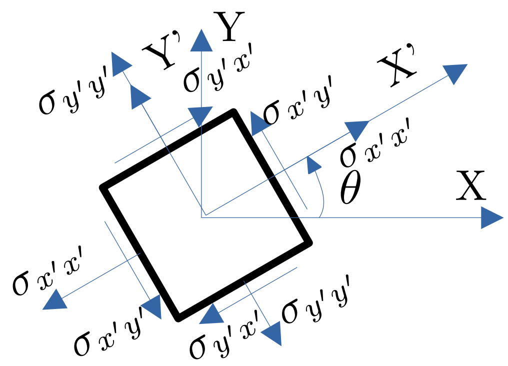
Shear component of the traction vector $\tau^2 = |\boldsymbol{t}^{\hat{\boldsymbol{n}}}|^2 - \sigma_n^2$

Stress Transformation in Two-dimensions

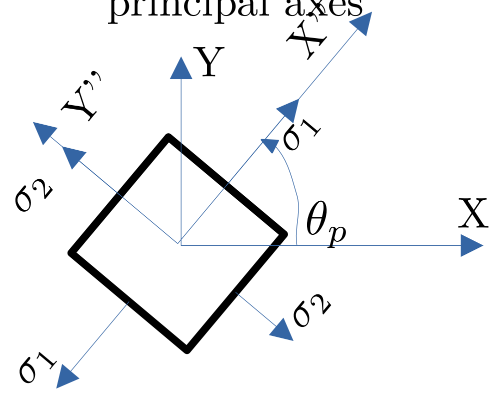
Stress components referred to XY system



Stress components referred to X'Y' system



Stress components referred to principal axes



Equations for stress transformation

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'y'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \tau_{xy}\cos 2\theta + \frac{\sigma_{yy}-\sigma_{xx}}{2}\sin 2\theta$$

Principal stresses and principal planes

$$\sigma_{1,2} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_{xx}-\sigma_{yy})/2}$$

Max. shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_n = \frac{\sigma_{xx}+\sigma_{yy}}{2}$$

$$\theta_s = \theta_p + \pi/4$$

Plane of max. shear

Principal Stresses and Principal Directions – Alternate Method

- Have

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}, \ \sigma_{xy} = \sigma_{yx}$$

- To find the principal stresses and the principal directions, solve

$$\boldsymbol{\sigma}\hat{\boldsymbol{n}} = \lambda\hat{\boldsymbol{n}}$$

- This is an **eigenvalue** problem, i.e. the eigenvalues and the corresponding eigenvectors of $\boldsymbol{\sigma}$

- Need to find λ and $\hat{\boldsymbol{n}}$ such that

$$(\boldsymbol{\sigma} - \lambda\boldsymbol{I})\hat{\boldsymbol{n}} = \boldsymbol{0}$$

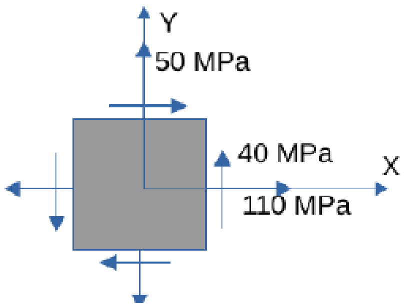
- As we are interested in a non-trivial solution, solve the following **quadratic** equation

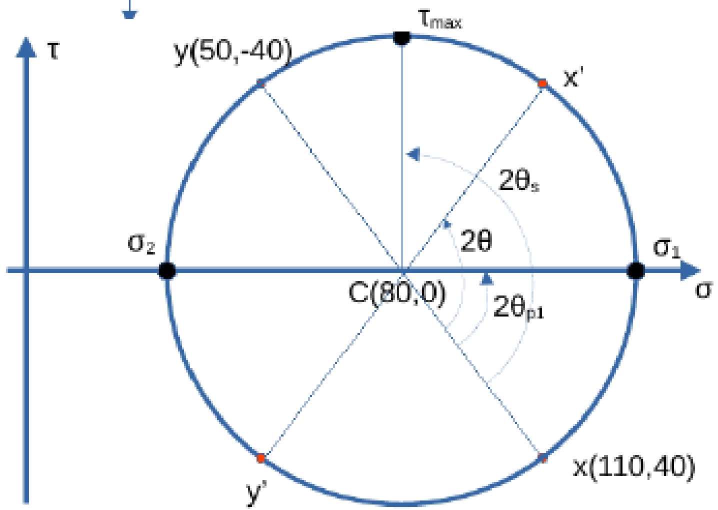
$$|\boldsymbol{\sigma} - \lambda\boldsymbol{I}| = 0$$

- The solution to the above equation, gives the eigenvalues principal stresses. Once the eigenvalues are obtained, then find the corresponding eigenvectors (principal directions)

Stress Transformation in Two-dimensions using Mohr's Circle

The state of stress at a point in a thin plate is shown below. Find the stress components w.r.t to x'y' axis inclined at 45^o with the xy axis. Also find the principal stresses, orientation of the principal planes, maximum shear stress and the orientation of the planes of maximum shear stress.





We have

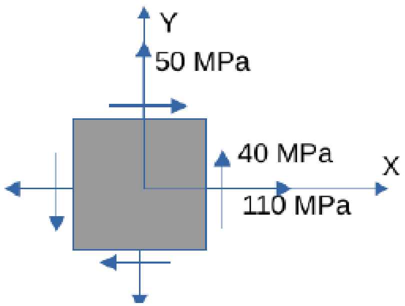
$$
\begin{aligned}
R &= \sqrt{30^2 + 40^2} = 50 \text{ MPa} \\
\tan 2\theta_{p1} &= \frac{40}{30}, \ \therefore \ 2\theta_{p1} = 53.1^o \\
\sigma_{x'x'} &= 80 + 50\cos(\pi/2 - \theta_{p1}) = 120 \text{ MPa} \\
\sigma_{y'y'} &= 80 - 50\cos(\pi/2 - \theta_{p1}) = 40 \text{ MPa} \\
\tau_{x'y'} &= -50\sin(\pi/2 - \theta_{p1}) = -30 \text{ MPa}
\end{aligned}
$$

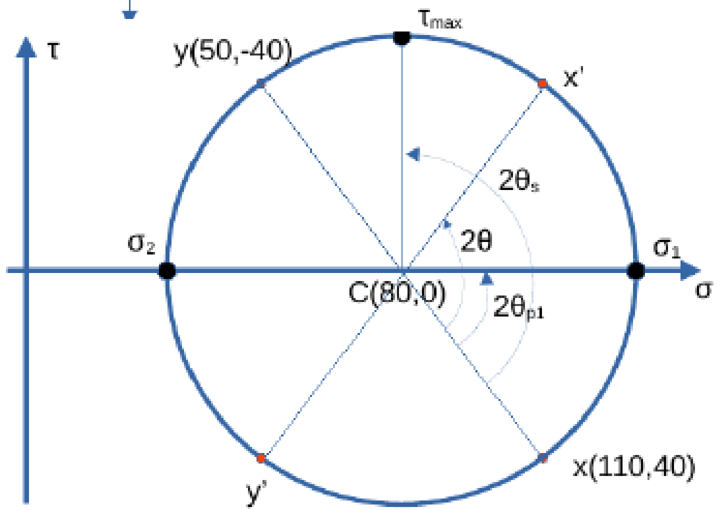
Principal stresses and principal planes

$$
\begin{aligned}
\sigma_1 &= 80 + 50 = 130 \text{ MPa} \\
\sigma_2 &= 80 - 50 = 30 \text{ MPa} \\
\theta_{p1} &= 26.6^o \\
\theta_{p2} &= 90^o + 26.6^o = 116.6^o
\end{aligned}
$$

Stress Transformation in Two-dimensions using Mohr's Circle

The state of stress at a point in a thin plate is shown below. Find the stress components w.r.t to x'y' axis inclined at 45° with the xy axis. Also find the principal stresses, orientation of the principal planes, maximum shear stress and the orientation of the planes of maximum shear stress.





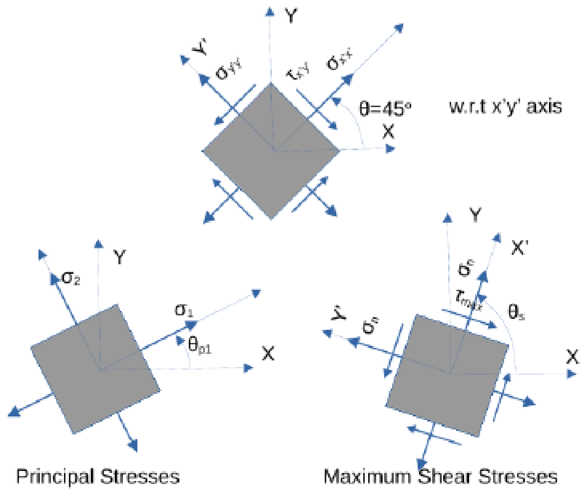
Maximum shear stress

$$\tau_{max} = R = 50 \text{ MPa}$$

$$\sigma_n = 80 \text{ MPa}$$

$$\theta_{s1} = 45^o + 26.6^o = 71.6^o$$

$$\theta_{s2} = 45^o + 71.6^o = 161.6^o$$



Construction of the Mohr's Circle

- **Mohr's Circle**: A graphical method for representing stress transformations. It helps in visualizing the normal and shear stress components acting on different planes through the point of interest.

- Sign Convention (Crandall):
  - Normal stress: Tension is positive and plotted to the right of the origin in the $\sigma - \tau$ plane. Compression is negative and is plotted to the left of the origin.

  - Shear stress: Positive shear stress is acted below the σ axis while a negative shear stress is plotted to above the σ axis. If the shear stress acting on the right face tends to rotate the element in an anticlockwise direction, then it is considered positive.

- Using the sign convention, locate point $x\,(\sigma_{xx}, \tau_{xy})$ and $y\,(\sigma_{yy}, -\tau_{xy})$. Note that points x and y lie on either side of the σ axis.

- Join points x and y with a straight line. The point of intersection of this line with the σ axis is $\mathrm{C}\left(\frac{\sigma_{xx}+\sigma_{yy}}{2}, 0\right)$, the center of the Mohr'scircle.

Construction of the Mohr's Circle - Continued

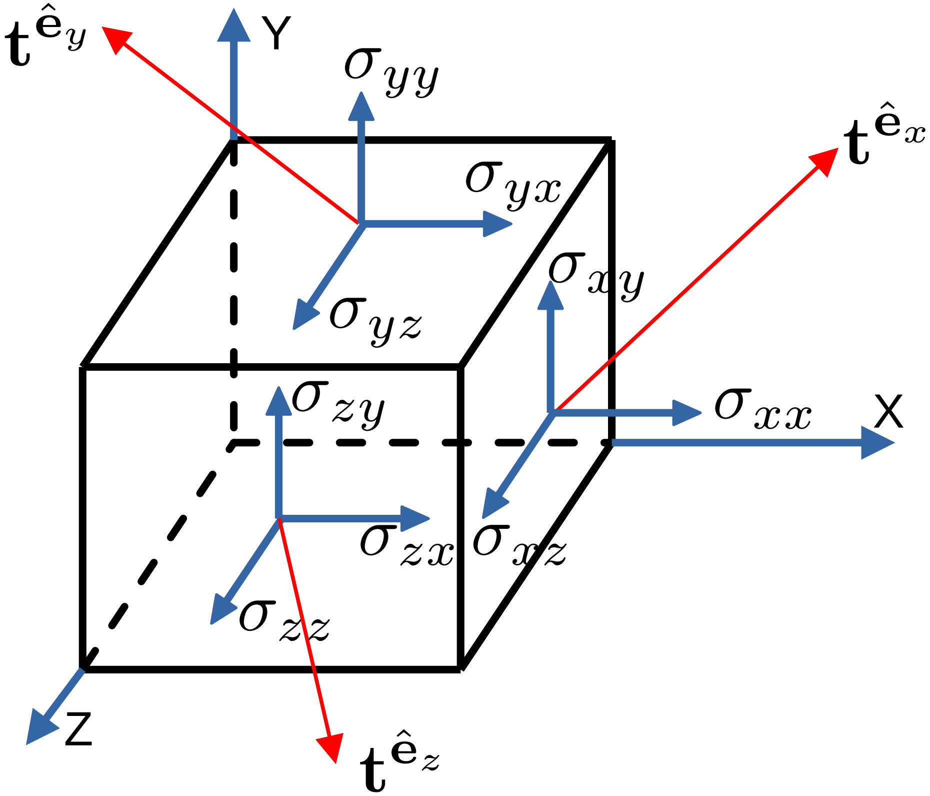
- With C as the center and diameter equal to $l(xy)$, draw a circle. This is the **Mohr's circle**.
- Locate diameter $x'y'$ with respect to diameter xy by rotating it by an angle 2θ measured in the **same sense** as the rotation θ.
- Using the sign convention introduced before, read off $\sigma_{x'x'}$ and $\tau_{x'y'}$ which are the coordinates of x' and $\sigma_{y'y'}$ and $\tau_{x'y'}$ which are the coordinates of y'.
- The intersection of the Mohr's circle with the σ axis gives the principal stresses σ_1 and σ_2. The plane on which σ_1 acts is located by reading off the angle between the xy line and the positive σ axis, say $2\theta_{p1}$. Then the orientation of the plane on which σ_1 acts is given by θ_{p1}, and is measured in the same direction as the rotation $2\theta_{p1}$. The location of the plane on which σ_2 acts is given by $\theta_{p1} + \frac{\pi}{2}$.

Construction of the Mohr's Circle - Continued

- The maximum shear stress, τ_{max} is given by the radius of the Mohr's circle. The plane on which τ_{max} acts is located by reading off the angle between the xy line and a line parallel to the τ axis, passing through point C and located above the σ axis, say $2\theta_s$. Then the orientation of one of the planes on which τ_s acts is given by θ_s, and is measured in the same direction as the rotation $2\theta_s$.

- The normal stress acting on the plane of maximum shear stress is the x coordinate of the point of intersection of the Mohr's circle and a line parallel to the τ axis, passing through point C and located above the σ axis.

Stress Tensor in Three-Dimensions



$\sigma_{xx},\ \sigma_{yy},\ \sigma_{zz}$ normal stress components

$\sigma_{yz},\ \sigma_{zx},\ \sigma_{xy}$ shear stress components

Cauchy Stress Tensor

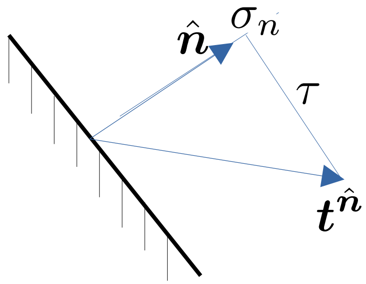
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\sigma_{xy} = \sigma_{yx},\ \sigma_{xz} = \sigma_{zx},\ \sigma_{yz} = \sigma_{zy}$$

Traction vector acting on a plane with normal $\hat{\boldsymbol{n}}$

$$\boldsymbol{t}^{\hat{\boldsymbol{n}}} = \boldsymbol{\sigma}\hat{\boldsymbol{n}} \quad \text{or}$$

$$\begin{bmatrix} t_x^{\hat{\boldsymbol{n}}} \\ t_y^{\hat{\boldsymbol{n}}} \\ t_z^{\hat{\boldsymbol{n}}} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$



Normal component of the traction vector

$$\sigma_n = \boldsymbol{t}^{\hat{\boldsymbol{n}}} \cdot \hat{\boldsymbol{n}}$$

Shear component of the traction vector

$$\tau^2 = |\boldsymbol{t}^{\hat{\boldsymbol{n}}}|^2 - \sigma_n^2$$

Stress Tensor in Three-Dimensions

- We have

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma_{xx} & \sigma_{xy} & \sigma_{xz}\\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz}\\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}\end{bmatrix}$$

- The principal stresses are obtained by solving for the following **cubic** equation

$$|\boldsymbol{\sigma}-\lambda\boldsymbol{I}| = \begin{vmatrix}\sigma_{xx}-\lambda & \sigma_{xy} & \sigma_{xz}\\ \sigma_{yx} & \sigma_{yy}-\lambda & \sigma_{yz}\\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}-\lambda\end{vmatrix} = 0$$

or

$$-\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$$

where

$I_1 = \text{trace}(\boldsymbol{\sigma}) = \sigma_{xx}+\sigma_{yy}+\sigma_{zz}$ First Invariant

$I_2 = \frac{1}{2}\left[(\text{trace}(\boldsymbol{\sigma}))^2 - \text{trace}(\boldsymbol{\sigma}^2)\right] = \sigma_{xx}\sigma_{yy}+\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx} - (\sigma_{xy}^2+\sigma_{yz}^2+\sigma_{zx}^2)$ Second Invariant

$I_3 = \det(\boldsymbol{\sigma})$ Third Invariant

Spherical and Deviatoric Stress Tensors

- We have

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

- The stress tensor can be written as:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_H + \boldsymbol{\sigma}_D$$

where

$$\boldsymbol{\sigma}_H = \sigma_h \boldsymbol{I}, = \sigma_h \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sigma_h = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Spherical stress tensor.
Responsible for volume change

and

$$\boldsymbol{\sigma}_D = \boldsymbol{\sigma} - \boldsymbol{\sigma}_H = \boldsymbol{\sigma} - \sigma_h \boldsymbol{I} = \begin{bmatrix} \sigma_{xx} - \sigma_h & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - \sigma_h & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - \sigma_h \end{bmatrix}$$

Deviatoric stress tensor.
Responsible for shape change

Equilibrium Equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + b_x = 0$$
$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + b_y = 0$$
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

Have to be satisfied at every point inside the body

End