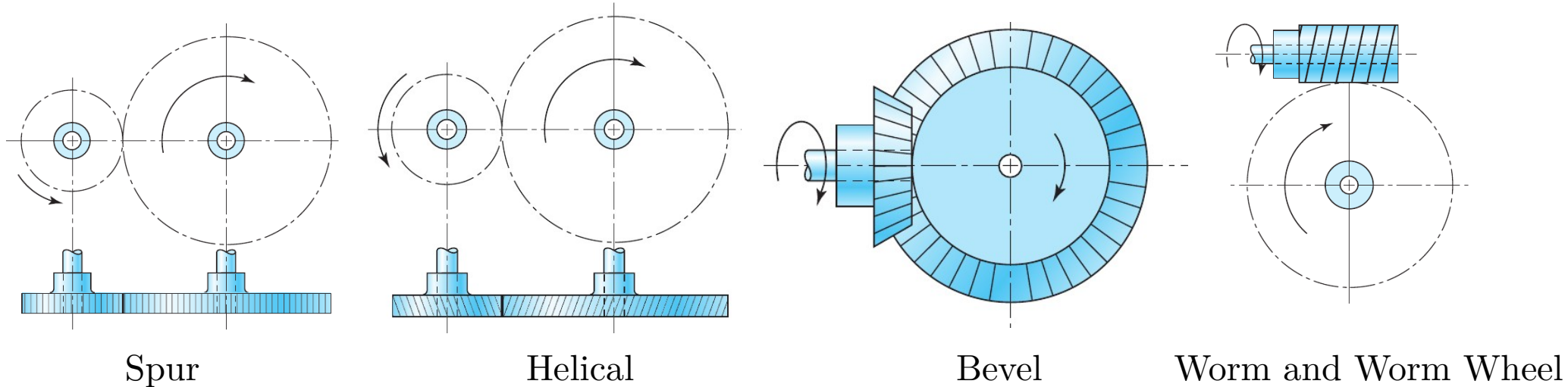


Introduction

Gears are mechanical components with teeth that interlock with other gears to transmit motion and force.

They are used to change the direction of motion, increase or decrease speed, and adjust torque in various machines



Gears – Common Types

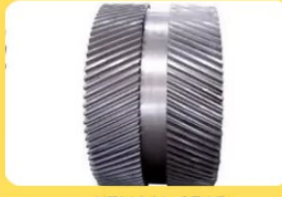
TYPES OF GEAR



SPUR GEAR



HELICAL GEAR



HELICAL GEAR



BEVEL GEAR



SCREW GEAR



SPIRAL BEVEL GEAR



WORM GEAR



MITRE GEAR



INTERNAL GEAR



RACK AND PINION

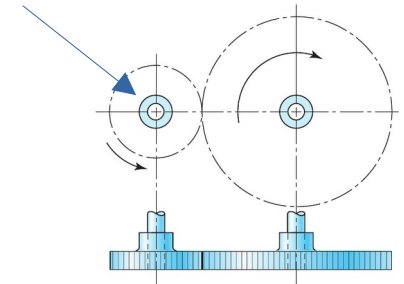


HERRINGBONE GEAR



HYPOID GEAR

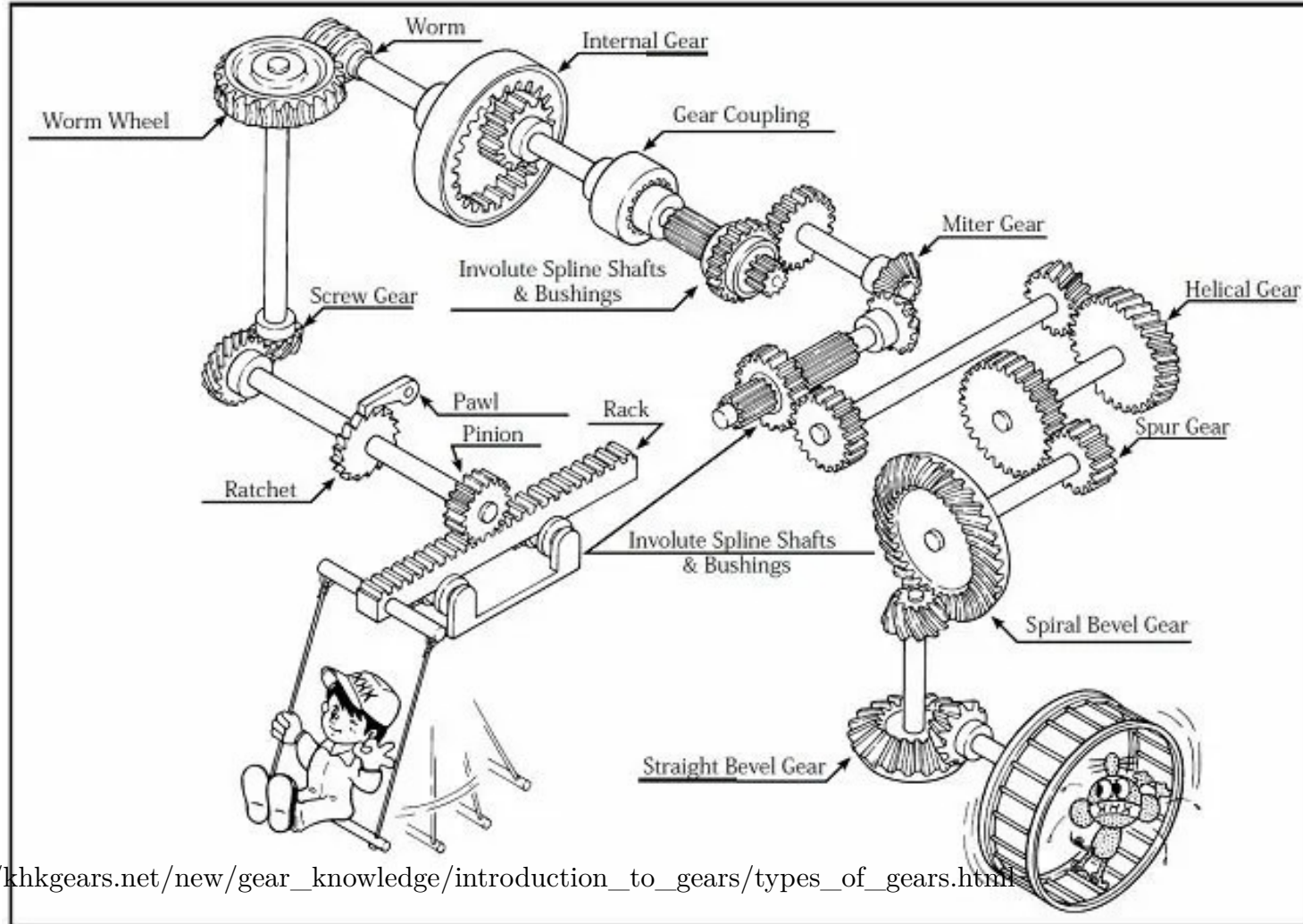
Convention
Smaller of the two gears is
called pinion



Gears – Common Types

- **Spur gears** – have teeth parallel to the axis of rotation and are used to transmit motion/torque between parallel shafts
- **Helical gears** – have teeth inclined to the axis of rotation and are mostly used to transmit motion/torque between parallel shafts
- **Bevel gears** – have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts, either perpendicular or at an angle
- **Worm and worm gear** – used to transmit motion between non-intersecting shafts and are used when the speed ratios are high, i.e. > 3
- Hypoid gears – similar to bevel gears except that shafts are offset and non-intersecting
- Mitre gears - bevel gears with ratio 1:1
- Herringbone gear - similar to double helical gears but with no gap

Gears – Common Types



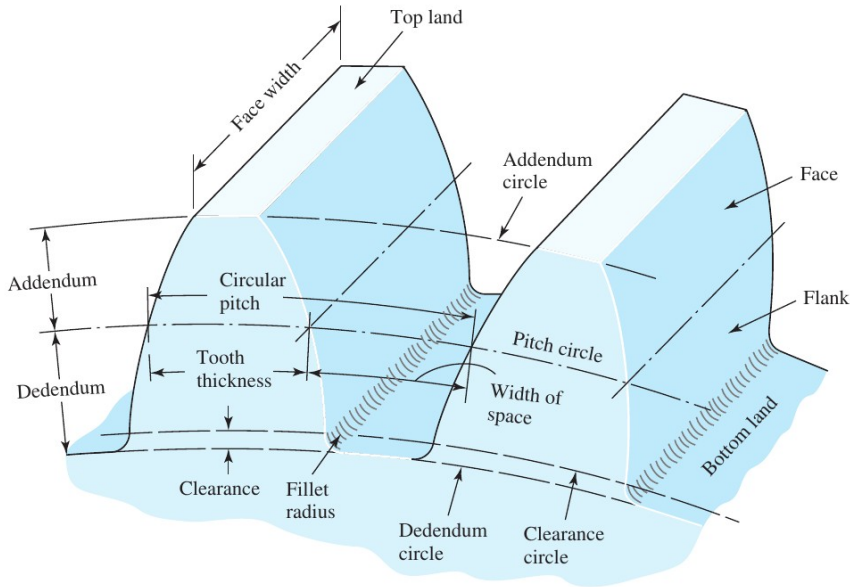
https://khkgears.net/new/gear_knowledge/introduction_to_gears/types_of_gears.html

Nomenclature of the Spur-Gear Tooth



Terminology

- **Pitch circle:** theoretical circle on which all the calculations are based. The pitch circles of the mating gears are tangent to each other.
- **Pitch diameter** d is the diameter of the pitch circle.
- **Module** m is the ratio of the pitch diameter to the number of teeth, N .
$$m = \frac{d}{N}$$
- **Addendum** is the radial distance between the top land and the pitch circle.
- **Dedendum** is the radial distance from the bottom land to the pitch circle.



- **Whole depth** h_t is the sum of the addendum and the dedendum
- **Backlash** – amount by which the width of the tooth space exceeds the thickness of the engaging tooth measured at the pitch circle
- **Circular pitch** p distance measure along the pitch circle from a point on one tooth to a corresponding point on adjacent tooth

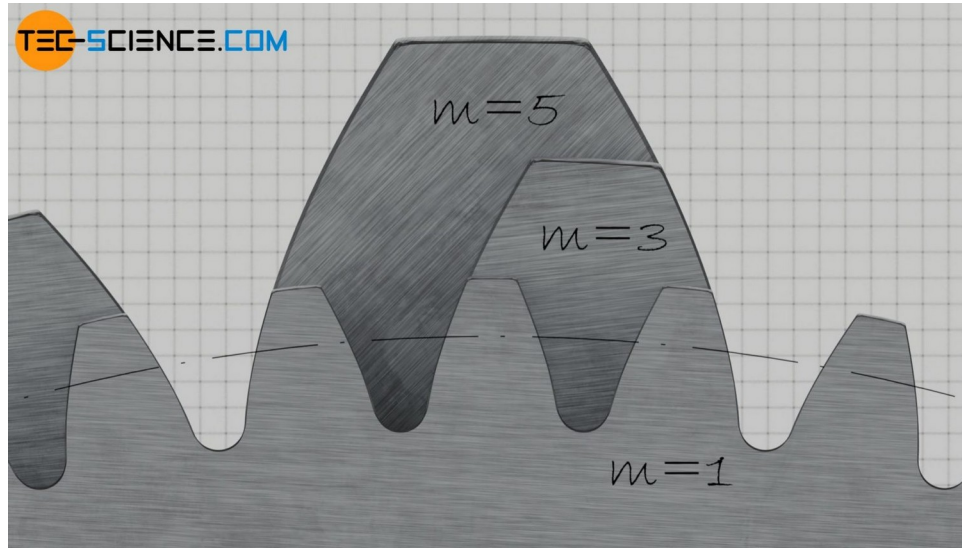
Module

Examples of modules in different applications (as the load increases, the module increases)

Wrist watches $\sim 0.05 - 0.2$

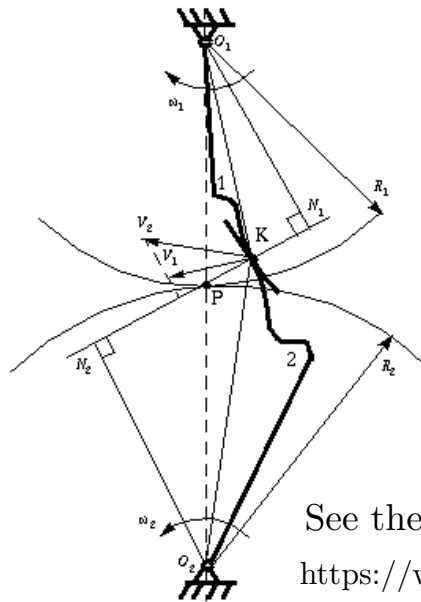
Printers, office copiers $\sim 0.6 - 1.0$

Transmissions for cars $\sim 1.5 - 3.0$



Conjugate Action and Involute Profile

- When tooth profiles or cams are designed to produce a constant angular velocity ratio during meshing, they are said to have conjugate action
- Fundamental law of gear tooth action – to obtain a constant angular velocity ratio during meshing, the common normal at the point of contact between a pair of gear teeth must always pass through a fixed point on the line of centers (pitch point).



$$\left| \frac{\omega_1}{\omega_2} \right| = \frac{O_2P}{O_1P} = \frac{R_2}{R_1} \quad (\text{law of gearing})$$

Involute profile satisfies the above requirement – most commonly used gear profile

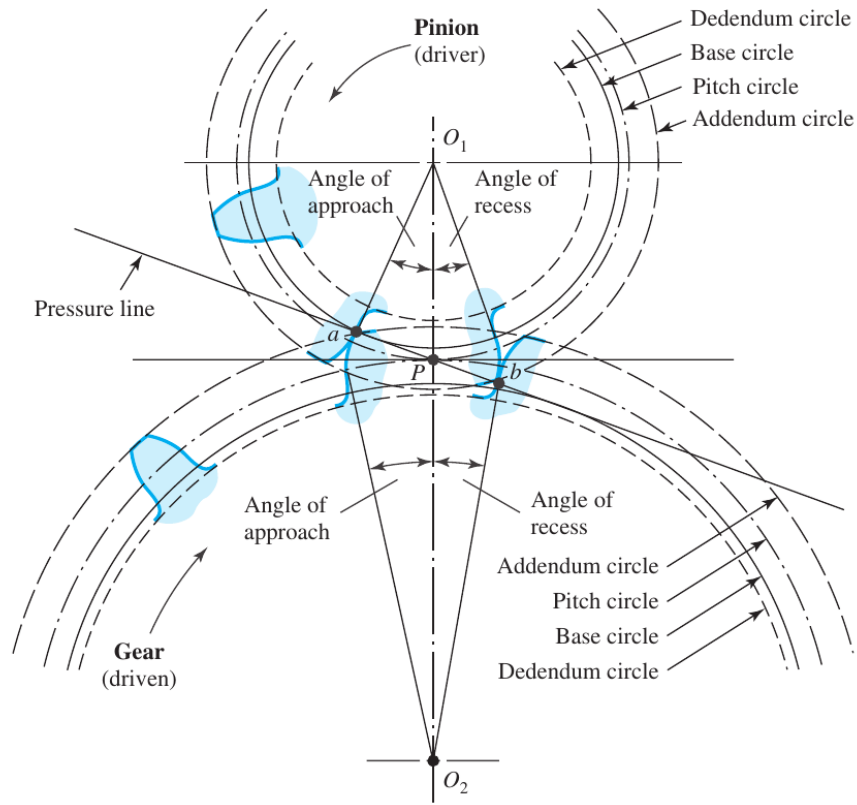
See video below for gears with cycloidal profile

See the following videos

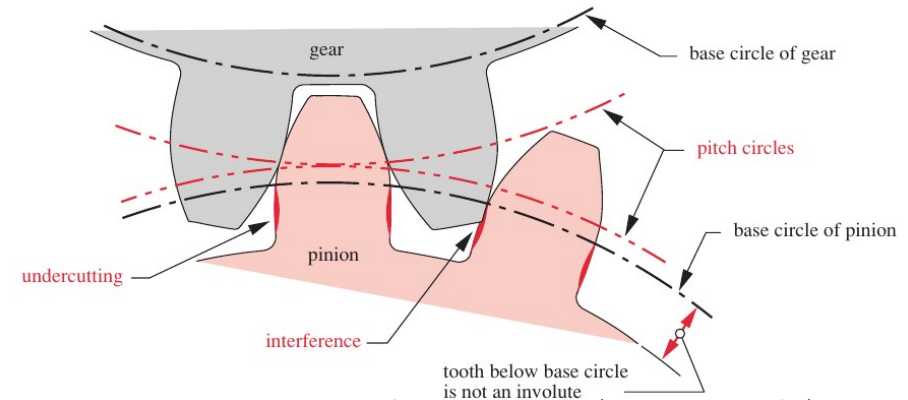
<https://www.tec-science.com/mechanical-power-transmission/involute-gear/meshing-line-action-contact-pitch-circle-law/>

<https://www.tec-science.com/mechanical-power-transmission/cycloidal-gear/geometry-of-cycloidal-gears/>

Tooth Action



- Pressure line = line of action = generating line
- Pressure angle = 20° (most common) – angle between the line of action and the direction of velocity at the pitch point
- Contact ratio – average number of teeth in contact at any given instant ~ 1.2
- Interference – contact of tooth profiles which are not conjugate is called interference (tooth profiles are not involute below the base circle)



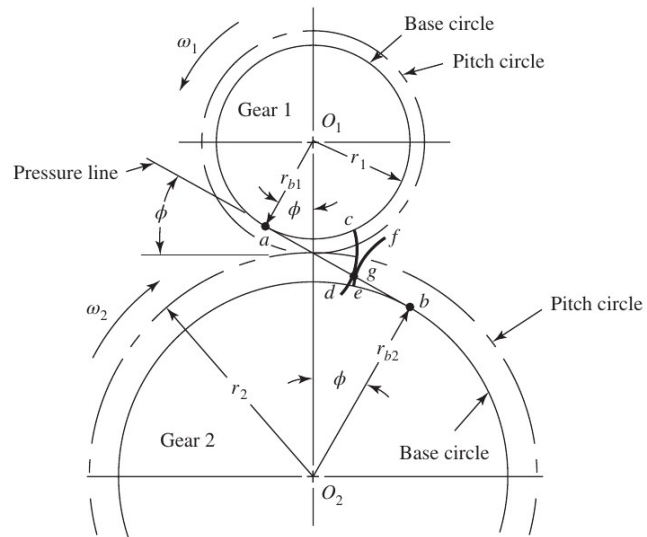
Base circle is used to generate the involute profile – basic to a gear.

$$r_b = r_p \cos \phi$$

Problem

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The module is 12 mm, and the addendum and dedendum are 1 m and 1.25 m, respectively. The gears are cut using a pressure angle of 20° .

- (a) Compute the circular pitch, the center distance, and the radii of the base circles.
- (b) In mounting these gears, the center distance was incorrectly made 6.35 mm larger. Compute the new values of the pressure angle and the pitch-circle diameters.



Tooth Systems - American Gear Manufacturers Association (AGMA)

A tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle.

Table 13–1 Standard and Commonly Used Tooth Systems for Spur Gears

Tooth System	Pressure Angle ϕ , deg	Addendum a	Dedendum b
Full depth	20	$1/P$ or m	$1.25/P$ or $1.25m$ $1.35/P$ or $1.35m$
	$22\frac{1}{2}$	$1/P$ or m	$1.25/P$ or $1.25m$ $1.35/P$ or $1.35m$
	25	$1/P$ or m	$1.25/P$ or $1.25m$ $1.35/P$ or $1.35m$
Stub	20	$0.8/P$ or $0.8m$	$1/P$ or m

Module m (mm/tooth)	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45



Calculation for Standard Spur Gears – Type 1

Item	Symbol/Formula
Center distance	a (given)
Speed ratio	i ($i = N_g/N_p$) (given)
module	m (choose)
Pressure angle	ϕ (choose)
Number of teeth	N_p, N_g (calculate)
Pitch diameter	$d_p = mN_p, d_g = mN_g$ (calculate)
Base diameter	$d_{bp} = d_p \cos\phi, d_{bg} = d_g \cos\phi$ (calculate)
Addendum	m
Tooth depth	$1.25\ m$
Tip diameter	$d_p + 2m, d_g + 2m$
Root diameter	$d_p - 2.5m, d_g - 2.5m$

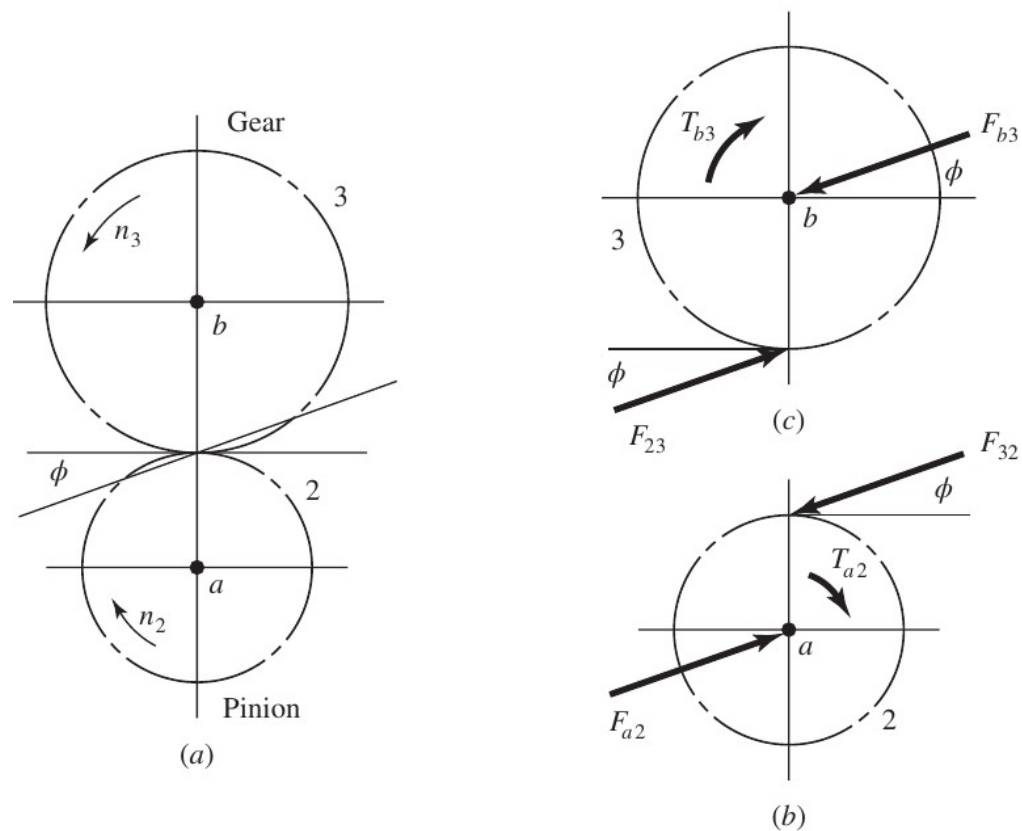
$$N_p = \frac{2a}{m(i + 1)}$$

$$N_g = \frac{2ai}{m(i + 1)}$$

Calculation of Standard Spur Gears – Type 2 Problems

Item	Symbol/Formula
Module	m (choose)
Pressure angle	ϕ (choose)
Number of teeth	N_p, N_g (given)
Center distance	$a = (d_p + d_g)/2$ (calculate)
Pitch diameter	$d_p = mN_p, d_g = mN_g$ (calculate)
Base diameter	$d_{bp} = d_p \cos \phi, d_{bg} = d_g \cos \phi$ (calculate)
Addendum	m
Tooth depth	$1.25 m$
Tip diameter	$d_p + 2m, d_g + 2m$
Root diameter	$d_p - 2.5m, d_g - 2.5m$

Force Analysis – Spur Gears

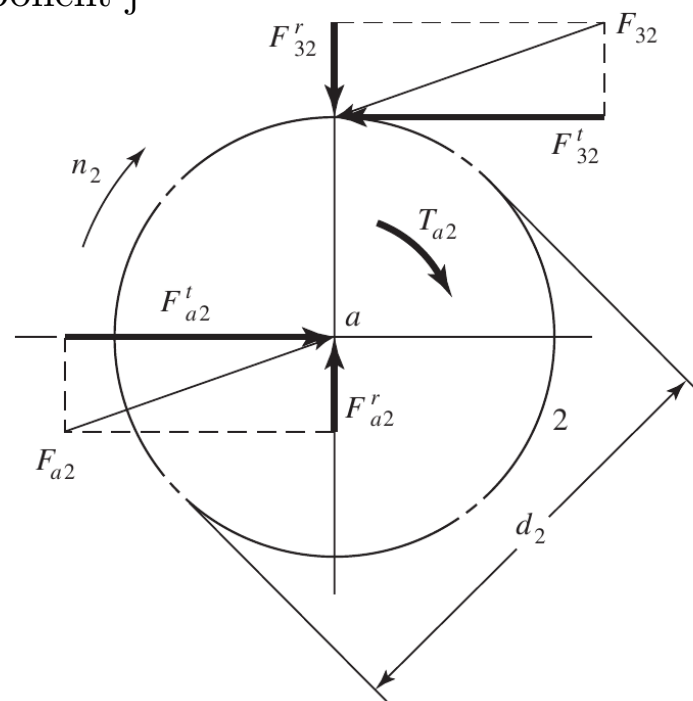


FBD for a gear pair

Input gear: 2 followed by 3,4,5, ...

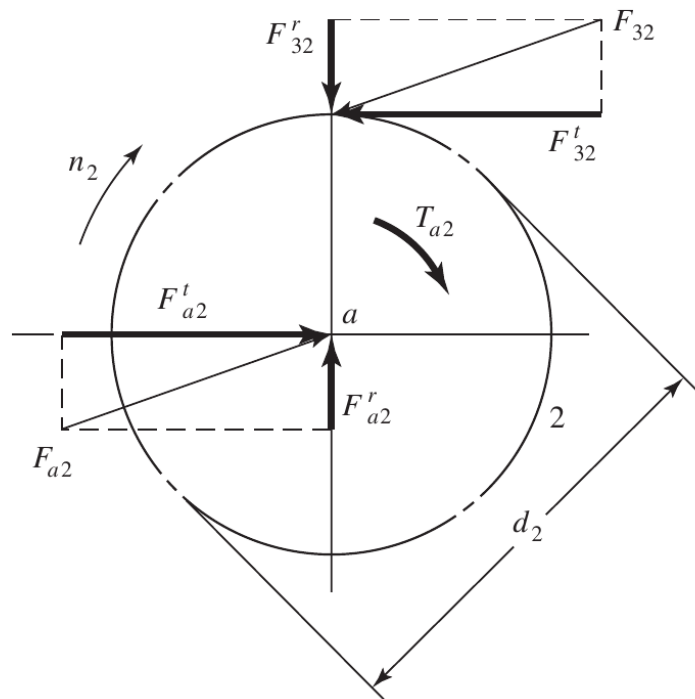
Shaft: a, b, c, ...

Nomenclature: F_{ij} : Force exerted by component i on component j



Force Resolution - Pinion

Force Analysis – Spur Gears - Continued



Transmitted load $W_t = F_{32}^t$ (this is the useful component)

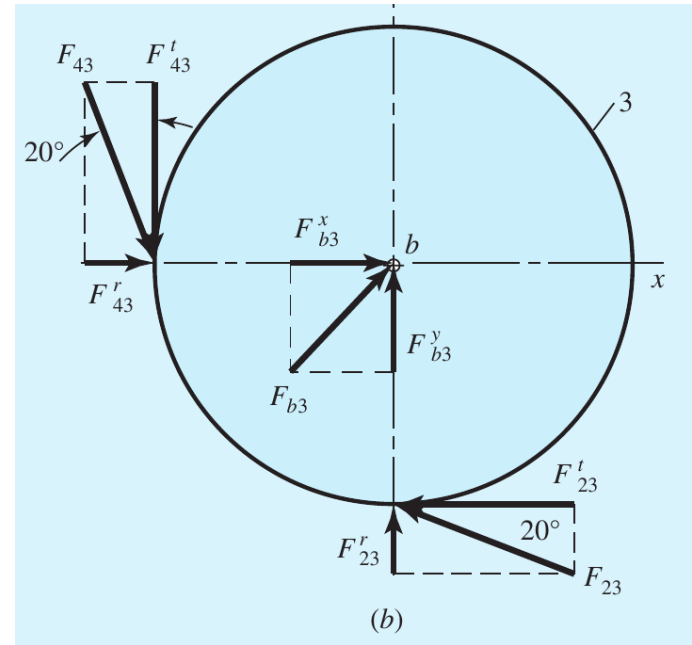
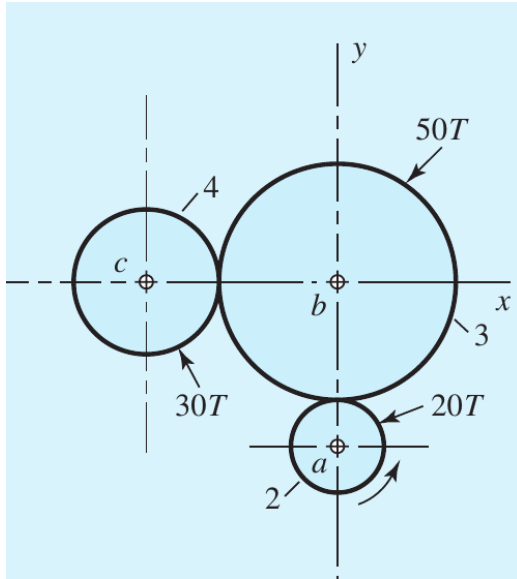
Torque transmitted: $T = W_t d_2 / 2$

Power transmitted $P = T$
(efficiency of spur gears is $\sim 98\%$)

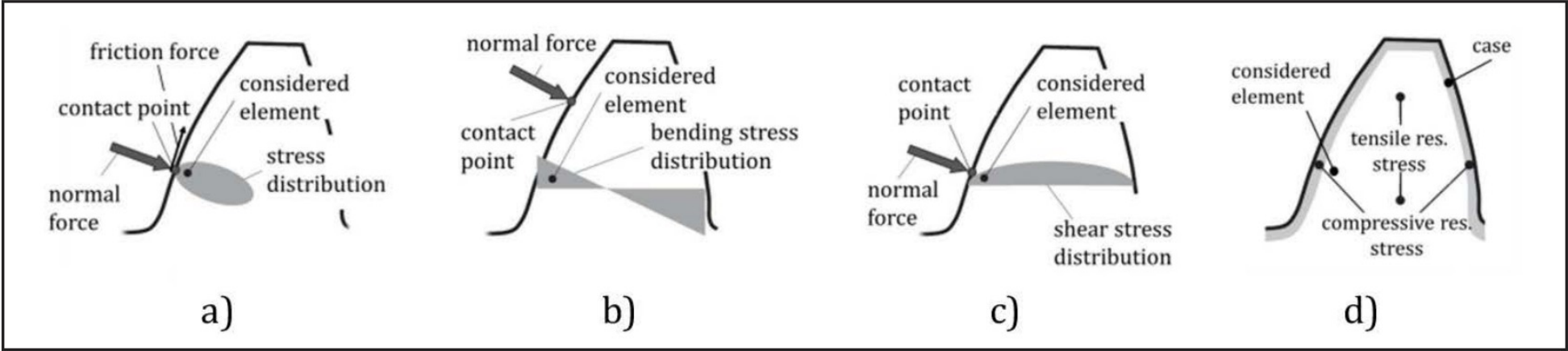
Slip line velocity $V = (d_2/2) n_2$

Force Analysis – Spur Gears - Problem

Pinion 2 in Figure runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.



Gear Tooth: Types of Stresses

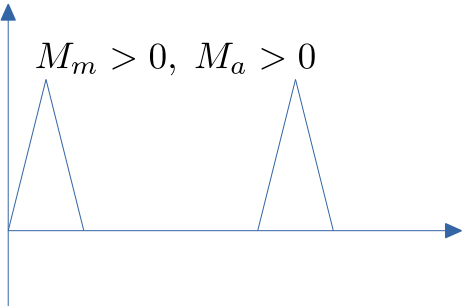


Contact Stress

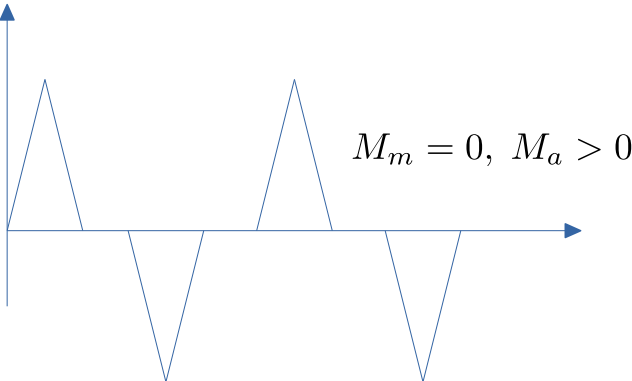
Bending Stress

Shear Stress

Residual Stress

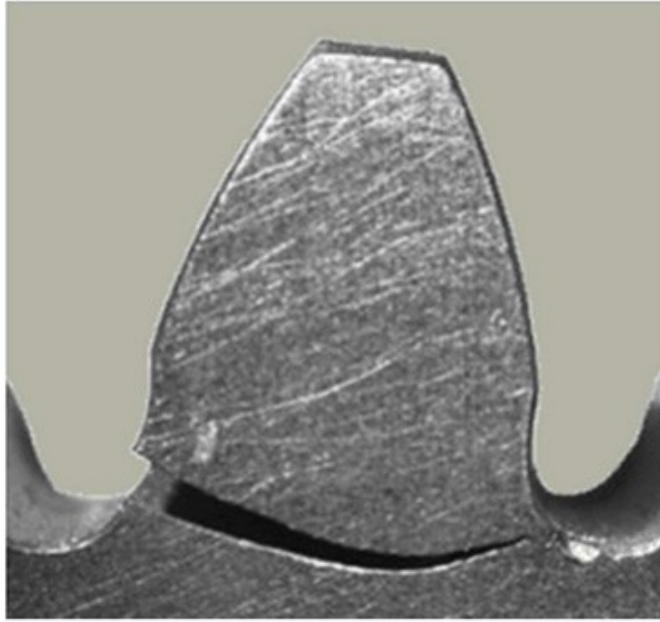


Repeated moment on a nonidler tooth

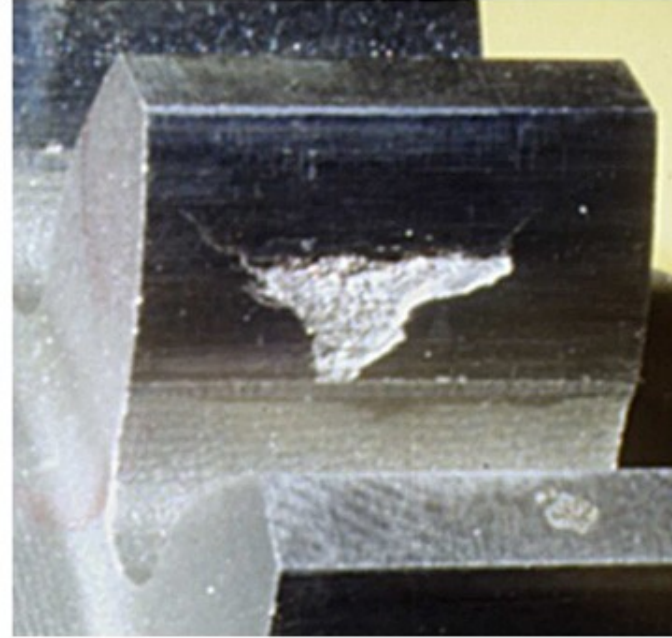


Reversed moment on an idler tooth

Gear Tooth: Common Failure Modes



Tooth Root Breakage
(Bending Fatigue Fracture)



Pitting
(Surface Fatigue Fracture)

Pitting – the fracture and separation of small pieces of materials from the surface – form of surface fatigue

<https://www.geartechology.com/articles/22175-tooth-flank-fracture-basic-principles-and-calculation-model-for-a-sub-surface-initiated-fatigue-failure-mode-of-case-hardened-gears>

Gear Material

Steel: The most widely used material for gears.

Low Carbon Steel: Offers good machinability and is often used for gears that do not require high strength.

Medium Carbon Steel: Provides a balance of strength and toughness, suitable for many gear applications.

High Carbon Steel: Known for its high strength and wear resistance, used in heavy-duty gears.

Alloy Steel: Enhanced with elements like chromium, nickel, and molybdenum to improve strength, toughness, and wear resistance.

Cast Iron: Good wear resistance and damping properties, used for gears that operate under heavy loads and low speeds.

Non-Ferrous Alloys: Includes materials like bronze and brass, which offer good corrosion resistance and are often used in worm gears and other applications where lubrication is critical.

Powder Metallurgy Materials: Used for producing gears with complex shapes and fine details, offering good strength and wear resistance.

Plastics: Commonly used in applications where noise reduction and lightweight are important. Thermoplastics like nylon and acetal.

Each material has its own set of properties that make it suitable for different gear applications.

The choice of material depends on factors like load, speed, operating environment, and cost considerations.



AGMA Bending Stress Equation

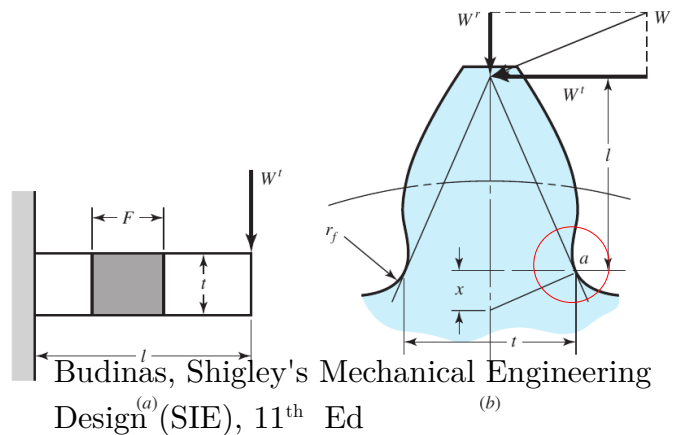
Assumptions

- The contact ratio is between 1 and 2.
- There is no interference between the tips and the root fillets of the mating teeth and no undercutting of the teeth above the theoretical start of the active profile.
- No teeth are pointed.
- There is nonzero backlash.
- The root fillets are standard, assumed smooth, and produced by a generating process.
- The friction force effects are neglected.
- Valid for external tooth only

$$\sigma_b = \frac{F_t}{FmJ} \frac{K_a K_m}{K_v} K_s K_B K_I$$

F_t - tangential load, F – face width, m - module

Based on Lewis's formula – Lewis developed his formula in 1892 and is based on considering the loaded tooth as a cantilever beam
Lewis's formula neglects the stress concentration effect at the root



AGMA Bending Stress Equation

$$\sigma_b = \frac{F_t}{FmJ} \frac{K_a K_m}{K_v} K_s K_B K_I$$

- Bending strength geometry factor J – vary with number of teeth on the pinion and the gear (AGMA standard) (different for pinion and gear)
- Dynamic factor K_v – attempts to account for the internally generated vibration loads from tooth-tooth impacts induced by nonconjugate meshing of the gear teeth. Depends on the accuracy to which the gears are machined (estimate it from the standards)
- Load distribution factor K_m – takes into account the fact that the transmitted load will be distributed unevenly across the face of the tooth
- Application factor K_a – takes into account the non-uniform nature of the transmitted load (shock loading)
- Size factor K_s – similar to the size factor used in fatigue calculations (larger components have lower strength). As per AGMA recommendations normally $K_s = 1$.
- Rim thickness factor K_B – applicable in situations in which the gear is made with rim and spokes rather than a solid disk. For solid gears $K_B = 1$
- Idler factor K_I – takes into account the fact that an idler gear is subject to more load cycles per unit time as compared to a non-idler gear. $K_I = 1.42$ for idler and $K_I = 1.0$ for non-idler



AGMA Bending Stress Equation

Table 12-8 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with Tip Loading

Gear teeth	Pinion teeth															
	12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.24	0.24								
26	U	U	U	U	U	U	0.24	0.25	0.25	0.25						
35	U	U	U	U	U	U	0.24	0.26	0.25	0.26	0.26	0.26				
55	U	U	U	U	U	U	0.24	0.28	0.25	0.28	0.26	0.28	0.28	0.28		
135	U	U	U	U	U	U	0.24	0.29	0.25	0.29	0.26	0.29	0.28	0.29	0.29	0.29

HPSTC - highest point of single-tooth contact
For contact ratio > 1 and gear with good Accuracy, it lies somewhere below the tip
U - undercutting

Table 12-9 AGMA Bending Geometry Factor J for 20°, Full-Depth Teeth with HPSTC Loading

Gear teeth	Pinion teeth															
	12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	U	U												
17	U	U	U	U	U	U										
21	U	U	U	U	U	U	0.33	0.33								
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35						
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.39	0.39				
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.40	0.42	0.43	0.43		
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.41	0.45	0.45	0.47	0.49	0.49

AGMA Bending Stress Equation

$$\sigma_b = \frac{F_t}{FmJ} \frac{K_a K_m}{K_v} K_s K_B K_I$$

Table 12-16

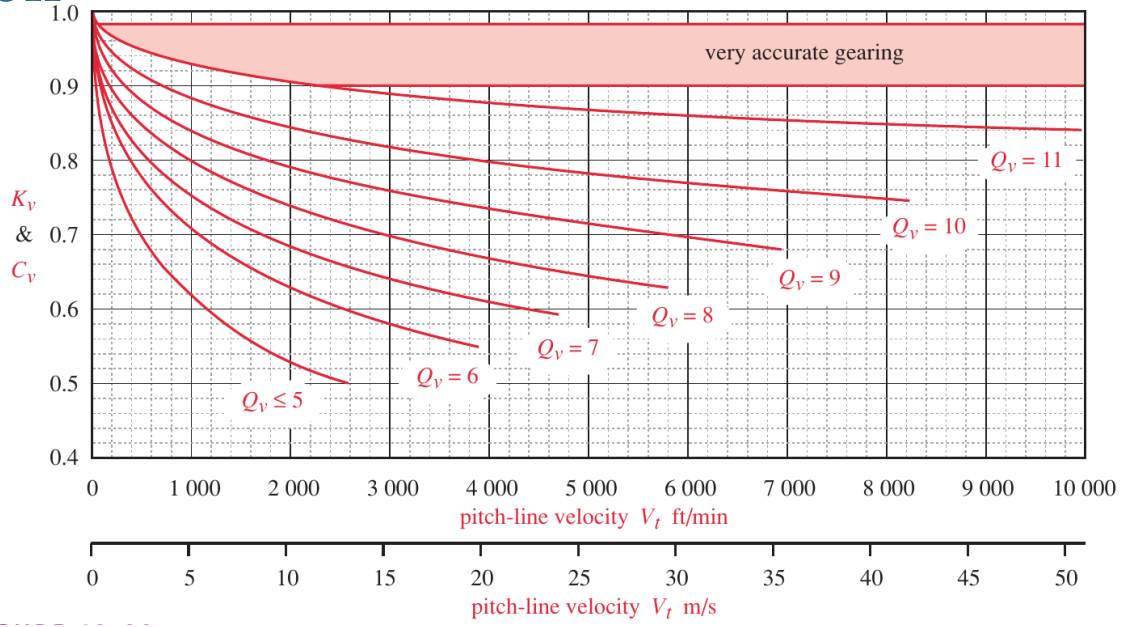
Load Distribution Factors K_m

Face Width in (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Table 12-17 Application Factors K_a

Driving Machine	Driven Machine		
	Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

Dynamic factor K_v



The higher the quality number Q_v , the closer the actual geometry to what is specified

Depends on the gear manufacturing process and the specified tolerances

Tighter the tolerances, higher is the quality number

Gear Quality Number

The quality number indicates the the accuracy of tooth shape and placement.

The four main parameters accounted for are:

The **tooth lead or tooth alignment** criterion applies to spur and helical-type gearing, and measures the variation between the specified lead (or helix angle) and the lead of the produced gear.

Involute profile variation is the difference between the specified profile and the measured profile of the tooth.

Pitch variation or spacing variation is the difference between the specified tooth location and the actual tooth location around the circumference of the gear.

Radial runout refers to the disparity in radial position of teeth on a gear – the variation in tooth distances from the center of rotation.

<https://www.machinedesign.com/automation-iiot/article/21834626/gear-quality-what-its-all-about>

AGMA Pitting Resistance Equation

$$\sigma_c = C_p \sqrt{\frac{F_t}{F I d} \frac{C_a C_m}{C_v} C_s C_f}$$

Feature of contact stresses

$$\sigma \propto \sqrt{F}$$

F_t – tangential force

d – pitch circle diameter of the smaller of the two gears in mesh

F – face width

I – surface geometry factor accounts for the radii of curvature of the gear teeth and the pressure angle. Calculated for a gear pair in a mesh

C_p – elastic coefficient that accounts for the differences in the gear and pinion material constants

C_f - surface finish factor that accounts for the unusually rough finishes on the gear teeth.

$C_f = 1$ for gears made by conventional methods.

$C_a = K_a$ - Application factor

$C_m = K_m$ - Load distribution factor

$C_v = K_v$ - Dynamic factor

$C_s = K_s$ - Size factor

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ME423 - IIT Bombay

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AGMA Pitting Resistance Equation

Table 12-18 AGMA Elastic Coefficient C_p in Units of $[\text{psi}]^{0.5}$ ($[\text{MPa}]^{0.5}$)^{*†}

Pinion Material	E_p psi (MPa)	Gear Material					
		Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30E6 (2E5)	2 300 (191)	2 180 (181)	2 160 (179)	2 100 (174)	1 950 (162)	1 900 (158)
Malleable Iron	25E6 (1.7E5)	2 180 (181)	2 090 (174)	2 070 (172)	2 020 (168)	1 900 (158)	1 850 (154)
Nodular Iron	24E6 (1.7E5)	2 160 (179)	2 070 (172)	2 050 (170)	2 000 (166)	1 880 (156)	1 830 (152)
Cast Iron	22E6 (1.5E5)	2 100 (174)	2 020 (168)	2 000 (166)	1 960 (163)	1 850 (154)	1 800 (149)
Aluminum Bronze	17.5E6 (1.2E5)	1 950 (162)	1 900 (158)	1 880 (156)	1 850 (154)	1 750 (145)	1 700 (141)
Tin Bronze	16E6 (1.1E5)	1 900 (158)	1 850 (154)	1 830 (152)	1 800 (149)	1 700 (141)	1 650 (137)

$$C_p = \sqrt{\frac{1}{\pi \left(\left(\frac{1-\nu_p^2}{E_p} \right) + \left(\frac{1-\nu_g^2}{E_g} \right) \right)}}$$

AGMA Bending-Fatigue Strength Calculation for Gear Materials

σ_b – Tooth bending stress (pinion/gear)

S'_{fb} – Uncorrected (published) bending-fatigue strength for pinion/gear material
(Strength data is stated for 10^7 cycles of repeated stress and 99% reliability)

S_{fb} – Corrected bending-fatigue strength for pinion/gear material

$$S_{fb} = \frac{K_L}{K_T K_R} S'_{fb}$$

K_L – Life factor. As test data is for 10^7 cycles, a shorter or longer life cycle requires modification of the bending-fatigue strength

K_T – Temperature factor, $K_T = 1$ if the gear temperature is less than 120°

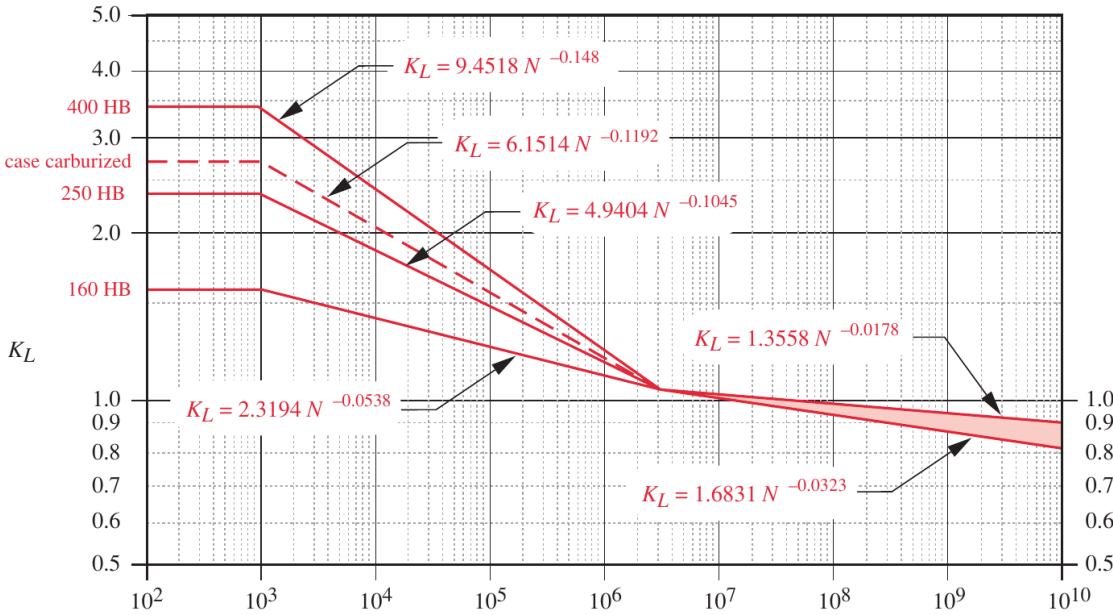
K_R – Takes into account reliability other than 99%

Calculate the the following FOS

$$N_{b_{pinion}} = \frac{S_{fb_{pinion}}}{\sigma_{b_{pinion}}}$$

$$N_{b_{gear}} = \frac{S_{fb_{gear}}}{\sigma_{b_{gear}}}$$

AGMA Bending-Fatigue Strength Calculation for Gear Materials



The number of load cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed

Table 12-19

AGMA Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Table 12-20 AGMA Bending-Fatigue Strengths S_{fb}' for a Selection of Gear Materials*

Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Bending-Fatigue Strength	
					psi x 10 ³	MPa
Steel	A1-A5		Through hardened	≤ 180 HB	25-33	170-230
			Through hardened	240 HB	31-41	210-280
			Through hardened	300 HB	36-47	250-325
			Through hardened	360 HB	40-52	280-360
			Through hardened	400 HB	42-56	290-390
			Flame or induction hardened	Type A pattern 50-54 HRC	45-55	310-380
		AISI 4140 AISI 4340 Nitralloy 135M Nitralloy N 2.5% Chrome	Flame or induction hardened	Type B pattern	22	150
			Carburized and case hardened	55-64 HRC	55-75	380-520
			Nitrided	84.6 HR15N [†]	34-45	230-310
			Nitrided	83.5 HR15N	36-47	250-325
			Nitrided	90.0 HR15N	38-48	260-330
			Nitrided	90.0 HR15N	40-50	280-345
Cast iron	20	Class 20	As cast		5	35
	30	Class 30	As cast	175 HB	8	69
	40	Class 40	As cast	200 HB	13	90
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	22-33	150-230
	A-7-c	80-55-06	Quenched and tempered	180 HB	22-33	150-230
	A-7-d	100-70-03	Quenched and tempered	230 HB	27-40	180-280
	A-7-e	120-90-02	Quenched and tempered	230 HB	27-40	180-280
Malleable iron (pearlitic)	A-8-c	45007		165 HB	10	70
	A-8-e	50005		180 HB	13	90
	A-8-f	53007		195 HB	16	110
	A-8-i	80002		240 HB	21	145
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	5.7	40
	Al/Br 3	ASTM B-148 78 alloy 954	Heat treated	90 ksi min tensile strength	23.6	160

AGMA Surface-Fatigue Strength Calculation for Gear Materials

σ_c – Tooth surface stress (pinion)

S'_{fc} – Uncorrected surface-fatigue strength for pinion material

(Strength data is stated for 10^7 cycles of repeated stress and 99% reliability)

S_{fc} – Corrected surface-fatigue strength for pinion material

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S'_{fc}$$

C_L – surface life factor. As test data is for 10^7 cycles, a shorter or longer life cycle requires modification of the surface-fatigue strength

C_T – Temperature factor, $C_T = K_T$

C_R – Takes into account reliability other than 99%, $C_T = K_T$

C_H – function of the gear ratio and the relative hardness of pinion and gears, $C_H = 1$ if made from the same material

Calculate the the following FOS $N_{c_{pinion}} = \left(\frac{S_{fc_{pinion}}}{\sigma_{c_{pinion}}} \right)^2$ $\sigma \propto \sqrt{F}$

AGMA Surface-Fatigue Strength Calculation for Gear Materials

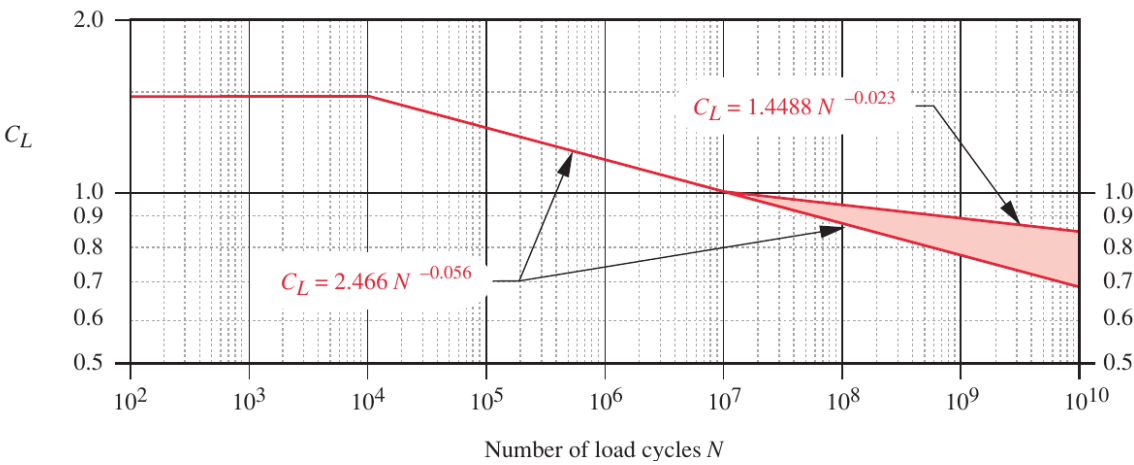


Table 12-21 AGMA Surface-Fatigue Strengths S_{fc}' for a Selection of Gear Materials*					
Material	AGMA Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Surface-Fatigue Strength psi x 10^3 MPa
Steel	A1–A5		Through hardened	≤ 180 HB	85–95 590–660
			Through hardened	240 HB	105–115 720–790
			Through hardened	300 HB	120–135 830–930
			Through hardened	360 HB	145–160 1000–1100
			Through hardened	400 HB	155–170 1100–1200
			Flame or induction hardened	50 HRC	170–190 1200–1300
			Flame or induction hardened	54 HRC	175–195 1200–1300
			Carburized and case hardened	55–64 HRC	180–225 1250–1300
		AISI 4140	Nitrided	84.6 HR15N [†]	155–180 1100–1250
		AISI 4340	Nitrided	83.5 HR15N	150–175 1050–1200
		Nitralloy 135M	Nitrided	90.0 HR15N	170–195 1170–1350
		Nitralloy N	Nitrided	90.0 HR15N	195–205 1340–1410
Cast iron	20 30 40	Class 20	As cast		50–60 340–410
		Class 30	As cast	175 HB	65–70 450–520
		Class 40	As cast	200 HB	75–85 520–590
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	77–92 530–630
	A-7-c	80-55-06	Quenched and tempered	180 HB	77–92 530–630
	A-7-d	100-70-03	Quenched and tempered	230 HB	92–112 630–770
	A-7-e	120-90-02	Quenched and tempered	230 HB	103–126 710–870
Malleable iron (pearlitic)	A-8-c	45007		165 HB	72 500
	A-8-e	50005		180 HB	78 540
	A-8-f	53007		195 HB	83 570
	A-8-i	80002		240 HB	94 650
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	30 450
	Al/Br 3	ASTM B-148 78 alloy 954	Heat-treated	90 ksi min tensile strength	65 450

END