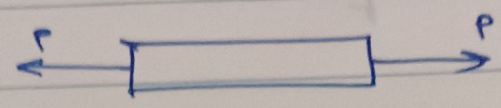
HW#3

① a) $S_{light} > S^*$

$S > S^*$

$\frac{AE}{L} > S^*$

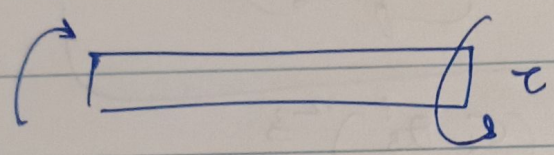


$\delta = \frac{PL}{AE} \Rightarrow S = \frac{AE}{L}$

$\frac{E}{L}\left(\frac{M}{\rho L}\right) > \delta^*$

$M > \delta^* L^2 \left(\frac{\rho}{E}\right)$ → material index

b)



$\frac{\tau}{\theta} = \frac{GJ}{L} = S_{beam} > \delta^*$

$\frac{GJ}{L} > \delta^* = \frac{G\pi d^4}{32L} > \delta^*$

$\delta^* < \frac{G}{2\pi L}\left(\frac{\pi d^4}{4}\right)^2 = \frac{G}{\pi L} A^2 = \frac{G}{2\pi L}\left(\frac{M}{\rho L}\right)^2$

$M > \sqrt{2\pi\delta^*}\, L^{3/2} \frac{\rho}{\sqrt{G}}$

② $K = \frac{GJ}{L} = \frac{G\pi d^4}{32L}$

$d_{NS}^4 = 4(R_o^2 - R_i^2)$

$K_{NS} = \frac{\pi G}{32L} \times 16(R_o^2 - R_i^2)$

$\Phi = \frac{K}{K_{NS}} = \frac{R_o^4 - R_i^4}{(R_o^2 - R_i^2)^2} = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}$

(3) a) $\sigma = \frac{M}{I_y} < \sigma_f$

$\theta_B^{\max} = \frac{\sigma z}{A^{3/2}} = z = \frac{\theta_B^{\max} A^{3/2}}{6}$

$\frac{M}{\sigma_f} < \frac{\theta_B^{\max}}{6} A^{3/2} = \frac{\theta_B^{\max}}{6} \left(\frac{M}{\rho L}\right)^{3/2}$

$$M > (6M)^{2/3} L \frac{\rho}{(\sigma_f \theta_B^{\max})^{2/3}}$$

maximize $\left(\frac{\sigma_f \theta_B^{\max}}{\rho}\right)^{2/3}$

b) $\phi_s^c = \frac{GJ/L}{(GJ_y)_{NS}} = \frac{2\pi J}{A^2}$

$\phi_s^c = \frac{2\pi J}{A^2}$

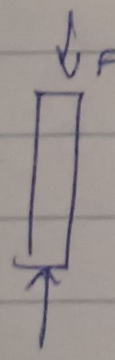
$S = \frac{G \phi_3^2 A^2}{2\pi} = \frac{G \phi_s^c}{2\pi} \left(\frac{M}{\rho L}\right)^2$

$\delta > \delta^{ss}$

$M > \sqrt{2\pi \delta^{ss}} L \frac{\rho}{\sqrt{G \phi_s^c}}$

maximize $\sqrt{\frac{\phi_s^c G}{\rho}}$

(4)

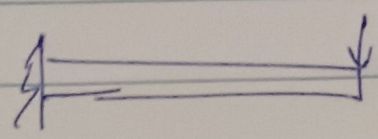


$F_{Buckling} = \frac{4\pi^2 EI}{L^2} \Rightarrow F = \frac{4\pi^2 E}{L^2} \times \frac{\pi r^4}{4}$

$F < \pi^3 E \left(\frac{r}{\ell}\right)^4 \ell^2$

$\frac{\ell}{r} < \left(\frac{\pi^3}{F}\right)^{3/4} \sqrt{2} E^{1/4}$

max $F^{1/4}$

(5) 

$$\delta \propto \frac{F}{EL} \qquad \delta > \delta^*$$

$$\delta^* < \frac{CF}{EI} = \frac{12CF}{Ea^2\phi_s^2} < \frac{12CF}{E\left(\frac{t}{\sqrt{12}}\right)^2\phi_s^2}$$

$$C < \sqrt{\frac{12CF}{E\delta^*}} < \frac{t\sqrt{\rho}}{\sqrt{E}\phi_s^2}$$

$$\phi_s^2 < 1$$

$$\text{material index} = \sqrt{\frac{E}{C\rho}}.$$

$$\text{maximize } \sqrt{E}/C\rho.$$

(6) $\sigma_{max} < \sigma_f$

$$\frac{3+\nu}{8}\rho R^2\omega^2 < \sigma_f$$

$$E = \frac{I\omega^2}{2} = \frac{mR^2\omega^2}{4} \Rightarrow \frac{E}{m} = \frac{R^2\omega^2}{4} = E_m$$

$$\frac{(3+\nu)}{8}\rho(GE_m)^2 < \sigma_f$$

$$E_m < \sqrt{\frac{\sigma_f}{2}}\frac{1}{\sqrt{\rho(3+\nu)}}$$

$$\text{material idex} = \frac{1}{\sqrt{\rho(3+\nu)}}$$

$$\text{minimize } \sqrt{\rho(3+\nu)}$$