ME-423: Tut - 2

Ganesh Iyer,
210100059

1. Estimate σ_e' in MPa for the following materials:
(a) AISI 1035 CD steel.
(b) AISI 1050 HR steel.
(c) 2024 T4 aluminum.
(d) AISI 4130 steel heat-treated to a tensile strength of 1620 MPa.

a) $\sigma_{ult} = 552 \text{ MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 276 \text{ MPa}$

b) $\sigma_{ult} = 621 \text{ MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 310.5 \text{ MPa}$

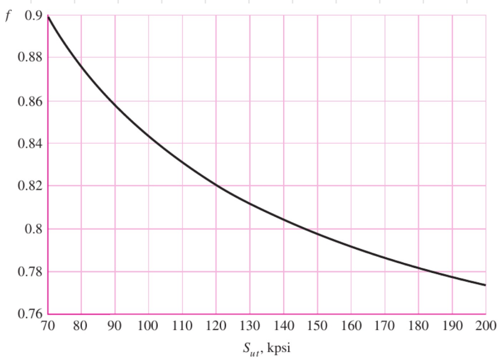
c) $\sigma_{ult} = 441 \text{ MPa (Al)} \Rightarrow \sigma_{f@5\times10^8} = 130 \text{ MPa} \left(\sigma_{ult} > 300\right)$

d) $\sigma_{ult} = 1620 \text{ MPa} (> 1400 \text{ MPa}) \Rightarrow \sigma_e' = 700 \text{ MPa}$

2. A steel rotating-beam test specimen has an ultimate strength of 830 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 480 MPa.

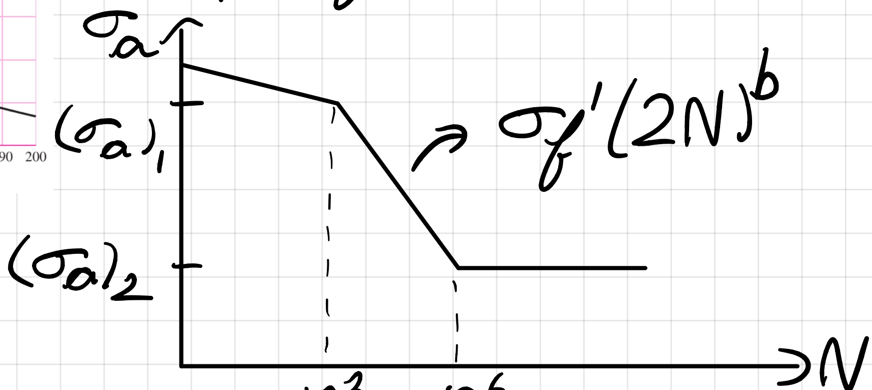
2. $\sigma_{ult} = 830 \text{ MPa}$ $\quad \sigma_e' \approx 0.5\sigma_{ult} = 415 \text{ MPa} \rightarrow (\sigma_a)_2$

$= \dfrac{830}{6.894} \text{ Kpsi} = 120.38 \text{ Kpsi}$



$f \approx 0.82$

$(\sigma_a)_1 \approx f\sigma_{ult} = 680.6 \text{ MPa}$



10^3 N_1 $\quad$ 10^6 N_2

$$\Rightarrow b = \frac{\ln\left((\sigma_a)_1/(\sigma_a)_2\right)}{\ln(N_1/N_2)} = -0.0716, \quad \sigma_f' = \frac{(\sigma_a)_2}{(2N_2)^b} = 1172.99 \approx 1173 \text{ MPa}$$

N for $\sigma_a = 480 \text{ MPa}$

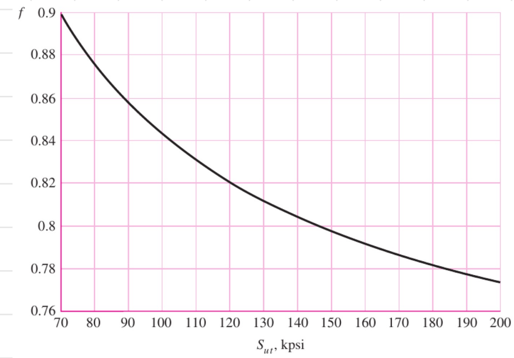
$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 131431.81$$

$$\Rightarrow N \approx 131400 \text{ cycles}$$

3. A steel rotating-beam test specimen has an ultimate strength of 1030 MPa and a yield strength of 930 MPa. It is desired to test low-cycle fatigue at approximately 500 cycles. Check if this is possible without yielding by determining the necessary reversed stress amplitude.

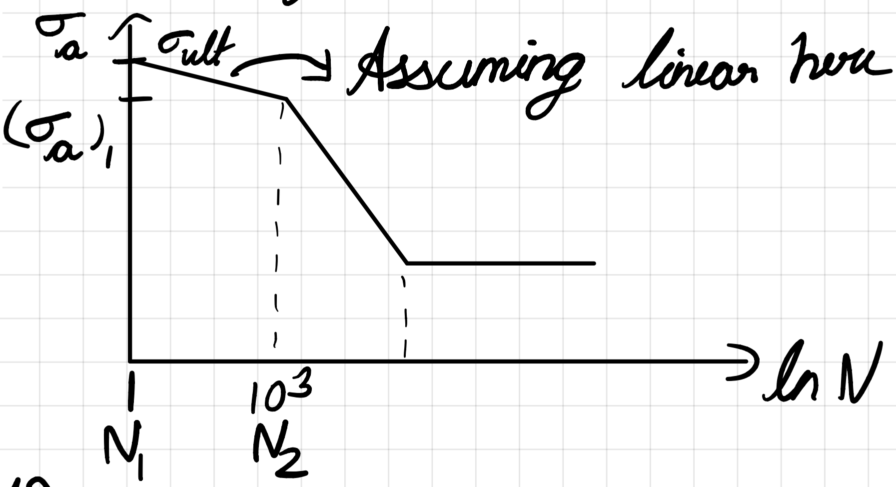
3. $\sigma_{ult} = 1030 \text{ MPa} = \frac{1030}{6.894} \text{ Kpsi} = 149.4 \text{ Kpsi}$

$\sigma_e' \approx 0.5\sigma_{ult} = 515 \text{ MPa} \rightarrow (\sigma_a)_2$



$f \approx 0.8$

$(\sigma_a)_1 \approx f\sigma_{ult} = 824 \text{ MPa}$



$$(\sigma_a)_1 \approx \underbrace{f}_{\sim 0.8}\sigma_{ult}$$

$$\Rightarrow (\sigma_a)_1 = 824 \text{ MPa}$$

Taking relation as $\sigma_f = aN^b$,

$$\sigma_{ult} = aN_1^b \Rightarrow 1030 = a(1)^b \Rightarrow a = 1030$$

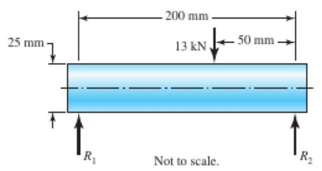
$$824 = 1030 \times (10^3)^b \Rightarrow b = -0.032$$

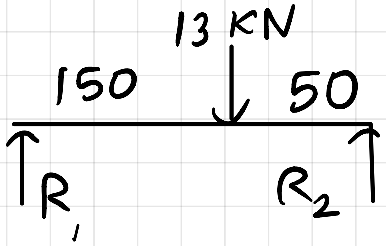
At $N = 500$, $\sigma_a = 1030 \times 500^{-0.032}$

$$= 844.25 \text{ MPa}$$

$\sigma_a < \sigma_Y = 930 \text{ MPa} \Rightarrow$ Does not yield

4. A rotating shaft of 25-mm diameter is simply supported by bearing reaction forces R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine
(a) the minimum static factor of safety based on yielding.
(b) the endurance limit, adjusted as necessary with correction (Marin) factors.
(c) the minimum fatigue factor of safety based on achieving infinite life.
(d) If the fatigue factor of safety is less than 1, then estimate the life of the part in number of rotations of rotations.



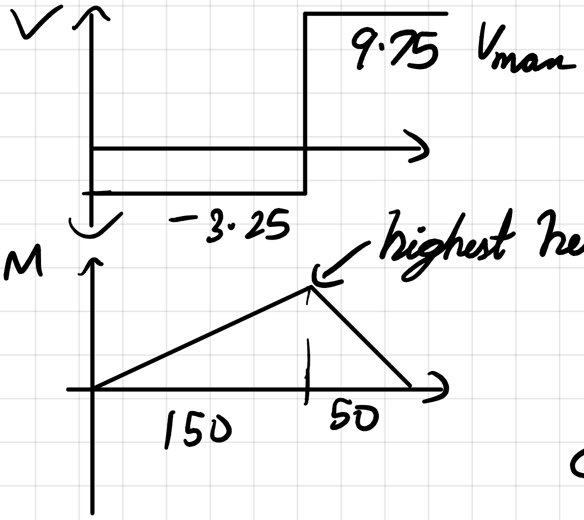


$3R_1 = R_2$ (moment balance)

$R_1 + R_2 = 13$

$\Rightarrow R_1 = \frac{13}{4}, R_2 = \frac{39}{4}$ KN

a)



highest here $\rightarrow$ $M = \frac{50}{1000} \times R_2 = 487.5$ N-m

$$\sigma = \frac{M \times d/2}{\pi d^4/64} = \frac{32M}{\pi d^3}$$

$\sigma = 317.8$ MPa

Top, bottom

$$\tau = \frac{4V_{max}}{3A} = \frac{4V_{max}}{3(\pi d^2/4)} = 26.48 \text{ MPa}$$

At center

$\|\tau\| << \|\sigma\|$. Hence critical points top/bottom.

σ_y (1045 HR Steel) = 310 MPa

σ_{ult} " = 565 MPa

$N = \sigma_y / \sigma = 0.975$

$\Rightarrow$ It will fail by yielding

b) $\sigma_e' = 0.5\sigma_{ult} = 0.5 \times 565 = 282.5$ MPa

K_a(Surface) $= a\sigma_{ult}^b$, For machining,

$= 4.51(565)^{-0.265} = 0.8411$

$K_b(\text{Size}) = 1.24\, d^{-0.107} = 0.8787$

$K_c(\text{Load}) = 1$, $K_d, K_e, K_f = 1$ (No info given)

$\Rightarrow \sigma_e = 0.8411 \times 0.8787\, \sigma_e' = 208.8$ MPa

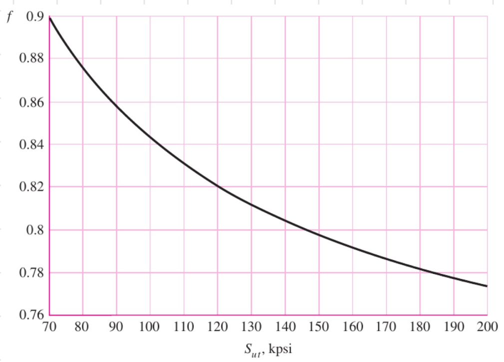
c) $N_f = \dfrac{\sigma_e}{\sigma} = \dfrac{208.8}{317.8}$

$\Rightarrow N_f = 0.657$ Fatigue factor of safety

d) As $N_f < 1$, finite life

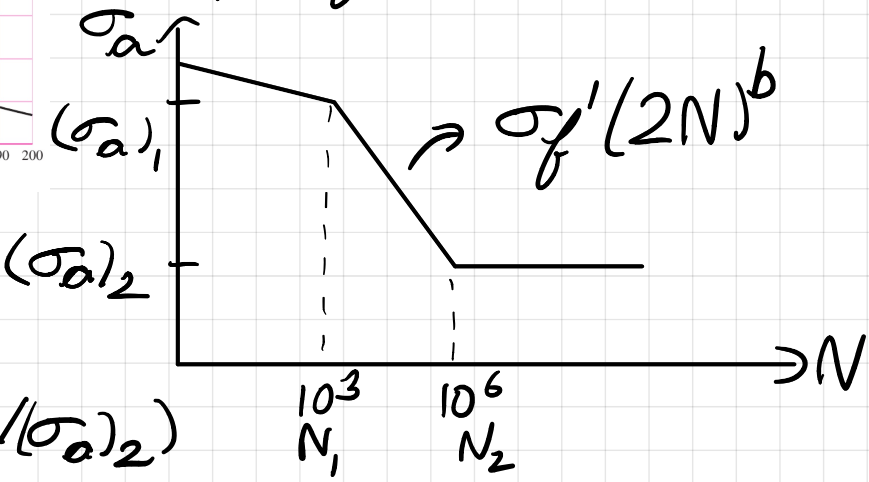
$\sigma_{ult} = 565$ MPa $= \dfrac{565}{6.894}$ kpsi $= 81.97$ kpsi

$(\sigma_a)_2 = \sigma_e = 208.8$ MPa



$f \approx 0.875$

$(\sigma_a)_1 \approx f \sigma_{ult} = 494.375$ MPa



$\Rightarrow b = \dfrac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.12477$, $\sigma_f' = \dfrac{(\sigma_a)_1}{(2N_1)^b} = 1276.27$ MPa

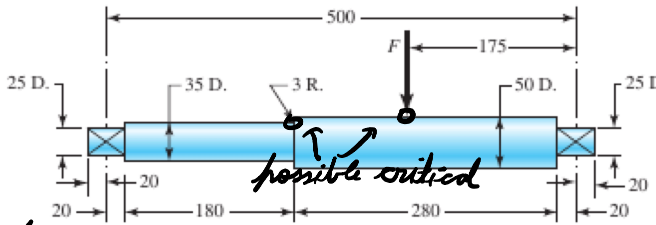
For $\sigma_a = 317.8$ MPa,

$$N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 34528.96$$

$\Rightarrow N \approx 34500$ cycles

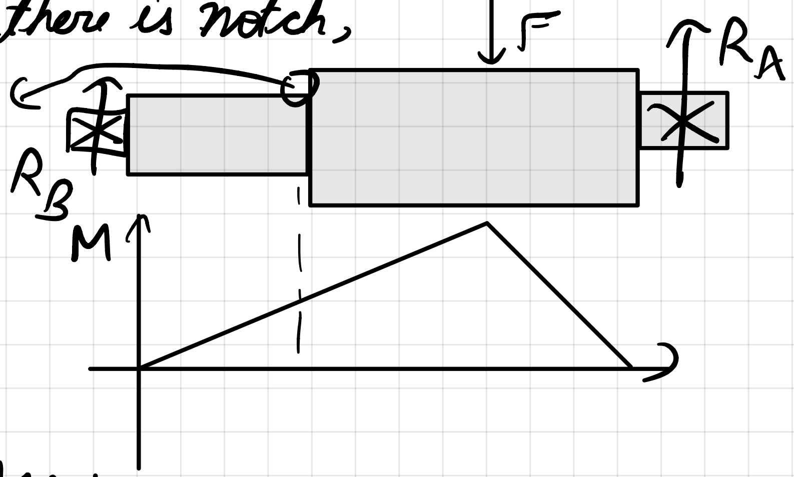
But as $\sigma_a > \sigma_y$, It will yield before fatigue.

5. The rotating shaft shown in the figure is machined from AISI 1020 CD (cold rolled) steel. It is subjected to a force of F = 6 kN. Find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding. All the dimensions are in mm.



$$\sigma_0 = \frac{32M}{\pi d^3}$$

As $\sigma_0 \propto 1/d^3$, and there is notch, it would be higher at



$R_A + R_B = F = 6$

Balance,

$R_B \times 500 = F \times 175$

$\Rightarrow R_B = \frac{7}{20} F = 2.1$ kN $\Rightarrow R_A = 3.9$ kN

M at critical point : $M = R_A \times 300 - F(125)$

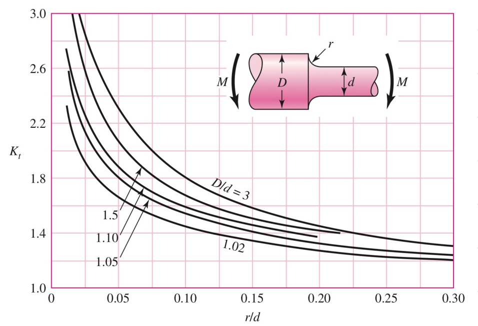
$= 420$ N-m

$$\sigma_0 = \frac{32 \times 420}{\pi (35\times10^{-3})^3} = 99.78 \text{ MPa}$$

Notch (3mm radius) $\Rightarrow$ $\frac{r}{d} = \frac{3}{35} = 0.085$, $\frac{D}{d} = \frac{50}{35} \approx 1.5$

Figure A–15–9

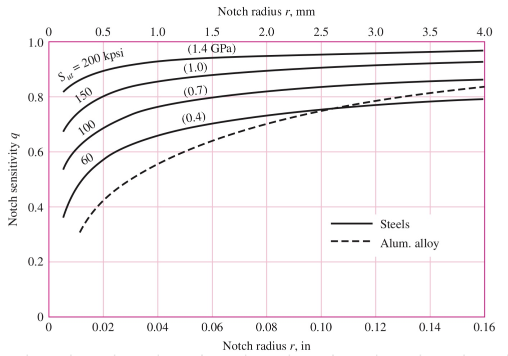
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$\Rightarrow k_t \approx 1.7$

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



$\sigma_{ult} = 60.9 \text{ kpsi}$

$\Rightarrow q \approx 0.78$

$$K_f = 1 + q(K_t - 1) = 1.546$$

$$\sigma_{max} = K_f \sigma_0 = 154.26 \text{ MPa}$$

$$\sigma_c' = 0.5\sigma_{ult} = 0.5 \times 420 = 210 \text{ MPa}$$

Taking highest D for lowest size factor K_b,

$$K_b = 1.24 \times 50^{-0.107} = 0.816$$

$$K_a = 4.51 \times 420^{-0.265} = 0.91 \quad \Rightarrow \sigma_c = K_a \cdot K_b \sigma_c'$$

(Machined) $\qquad = 155.93 \text{ MPa}$

As $\sigma_{max} < \sigma_c$, infinite life

$\sigma_y = 350 \text{ MPa}$, $\sigma_{max} < \sigma_y$, No yielding

6. A steel part is loaded with a combination of bending, axial, and torsion such that the following stresses are created at a particular location:

Bending: Completely reversed, with a maximum stress of 60 MPa

Axial: Constant stress of 20 MPa

Torsion: Repeated load, varying from 0 MPa to 70 MPa

Assume the varying stresses are in phase with each other. The part contains a notch such that $(K_f)_{\text{bending}} = 1.4$, $(K_f)_{\text{axial}} = 1.1$, and $(K_f)_{\text{torsion}} = 2.0$. The material properties are $\sigma_y = 300$ MPa and $\sigma_{ult} = 400$ MPa. The completely adjusted (corrected) endurance limit is found to be $\sigma_e = 160$ MPa. Find the factor of safety for fatigue based on infinite life, using the Goodman criterion (assume proportional loading). If the life is not infinite, estimate the number of cycles, using the SWT criterion to find the equivalent completely reversed stress. Be sure to check for yielding.

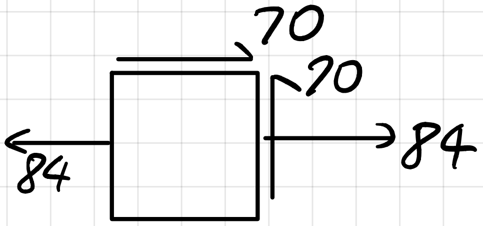
(Complete Reverse)

$$\sigma_{\text{bending}} = (K_f)_{\text{bending}} \times \sigma_0 = 1.4 \times 60 = 84 \text{ MPa}$$

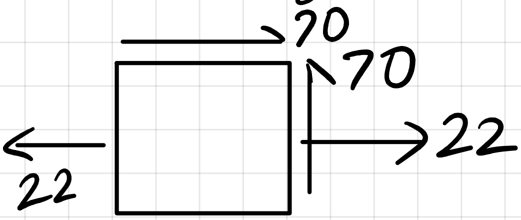
$$\sigma_{\text{axial}} = 1.1 \times 20 = 22 \text{ MPa}$$

$$\sigma_{\text{torsion}} = 0 \text{ to } \underbrace{70 \times 2}_{140 \text{ MPa}}$$

i.e. mean 70 MPa, alternating 70 MPa



Alternating



Mean

Von-Mises:

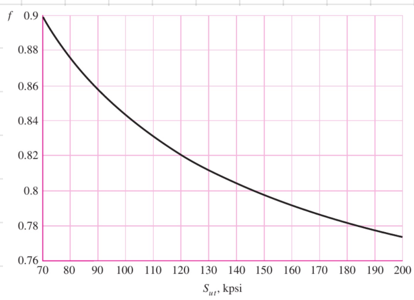
$$\sigma_{vm,a} = (84^2 + 3 \times 70^2)^{1/2} = 147.5 \text{ MPa}$$

$$\sigma_{vm,m} = (22^2 + 3 \times 70^2)^{1/2} = 123.22 \text{ MPa}$$

$$\frac{\sigma_{vm,a}}{\sigma_e} + \frac{\sigma_{vm,m}}{\sigma_{ult}} = \frac{147.5}{160} + \frac{123.22}{400} = 1.23 > 1$$

$\Rightarrow$ Point lies above Goodman line, finite life.

$$\text{SWT: } \sigma_a' = \sqrt{(\sigma_m + \sigma_a)\sigma_a} = \sqrt{(123.22 + 147.5)(147.5)} = 199.83 \text{ MPa}$$



$$\sigma_{ult} = 53 \text{ Kpsi} \Rightarrow f \approx 1$$

$$(\sigma_a)_1 = 400 \text{ MPa}, (\sigma_a)_2 = \sigma_e = 160 \text{ MPa}$$

$$b = \ln(400/160) / \ln(10^3/10^6) = -0.1326$$

$$\sigma_f' = (\sigma_a)_1 / (2000)^b = 1096.3 \text{ MPa}$$

For equivalent σ_a',

$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{199.83}{1096.3}\right)^{1/b} \approx 188000 \text{ cycles}$$

$$\sigma_{max} = \sigma_m + \sigma_a = 270.72 < \sigma_y = 300 \text{ MPa}$$

(Not yielded)

7. A machine part will be cycled at ±350 MPa for $5\text{x}10^3$ cycles. Then the loading will be changed to ±260 MPa for $5\text{x}10^4$ cycles. Finally, the load will be changed to ±225 MPa. Using the Miner's rule, estimate the of cycles of operation that can be expected at this stress level before the part fails? For the part, σ_{ult} = 530 MPa, f = 0.9, and has a fully corrected endurance strength of σ_e = 210 MPa.

$$(\sigma_a)_1 = f\sigma_{ult} = 477 \text{ MPa}, \quad (\sigma_a)_2 = \sigma_e = 210 \text{ MPa}$$

$$N_1 = 10^3, \quad N_2 = 10^6 \text{ cycles}$$

$$\sigma_a = \sigma_f'(2N)^b \Rightarrow b = \frac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.1187$$

$$\Rightarrow \sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 1176.44 \text{ MPa}$$

Now, $(\sigma_a)_A = 350$ MPa, $n_A = 5\times10^3$

$$N_A = \frac{1}{2}\left(\frac{(\sigma_a)_A}{\sigma_f'}\right)^{1/b} = 13553.7$$

$$\Rightarrow N_A \approx 13500 \text{ cycles}$$

So that estimate for n_c is low, conservative

$(\sigma_a)_B = 260$ MPa, $n_A = 5\times10^4$

$$N_B = \frac{1}{2}\left(\frac{(\sigma_a)_B}{\sigma_f'}\right)^{1/b} = 165585.2$$

$$\Rightarrow N_A \approx 165500 \text{ cycles}$$

$$(\sigma_a)_C = 225 \text{ MPa}$$

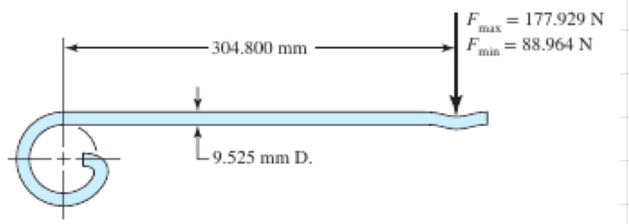
$$N_C = \frac{1}{2}\left(\frac{(\sigma_a)_C}{\sigma_f'}\right)^{1/b} = 559388.5$$

$$\approx 559300 \text{ cycles}$$

$$n_C = N_C\left(1 - \frac{n_A}{N_A} - \frac{n_B}{N_B}\right)$$

$$n_C = 183179 \approx 183100 \text{ cycles}$$

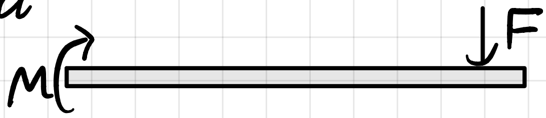
8. The figure shows a formed round-wire cantilever spring subjected to a varying force. The inner radius of the bend is 20 mm. The hardness tests made on 50 springs gave a minimum hardness of 400 Brinell. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. Estimate the number of cycles to likely to cause failure using the Goodman criterion.
1. If the curvature effects on the bending stress are ignored.
2. If the curvature effects on the bending stress are not ignored



$\sigma_{ult} \approx 3.1\ HB = 1240\ MPa$, $F_m = 133.4465\ N$

$F_a = 44.825\ N$

1)



$$M = 0.3048 \times F, \quad \sigma = \frac{32M}{\pi d^3} = 3592693 \times F \text{ Pa}$$

$\sigma_a = 159.811\ MPa$

$\sigma_m = 475.77\ MPa$

$\sigma_a' = 259.3\ MPa$

$$, \quad \sigma_a' = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_{ult}}\right)}$$

$\sigma_{ult} = 1240\ MPa = 179.86\ Kpsi$

$\sigma_e' = 0.5\,\sigma_{ult} = 620 MPa$

$K_a = 57.7(1240)^{-0.718}$ Hot rolled

$= 0.3468$

$K_b = 1.24(9.525)^{-0.107} = 0.974$

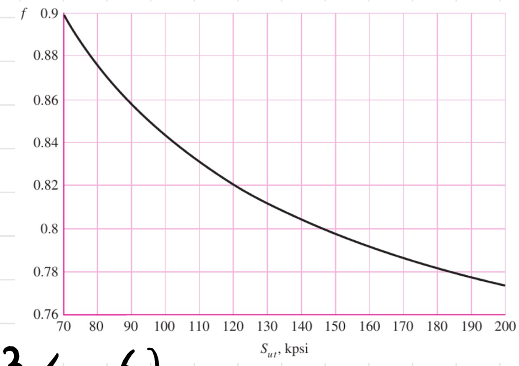
$\sigma_e = K_a \cdot K_b \cdot \sigma_e' = 209.485\ MPa$

$\sigma_a' > \sigma_e \Rightarrow$ Finite Life

$f \approx 0.78$

$(\sigma_a)_1 = f\sigma_{ult} = 967.2$ MPa

$(\sigma_a)_2 = \sigma_c = 209.485$ MPa



$b = \ln((\sigma_a)_1/(\sigma_a)_2)/\ln(10^3/10^6)$
$= -0.2214$

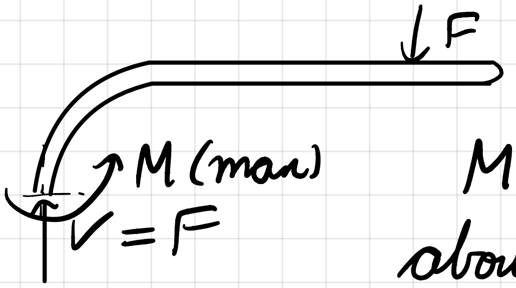
$\sigma_f' = \dfrac{(\sigma_a)_1}{(2N_1)^b} = 5204.32$ MPa

At $\sigma_a' = 259.3$ MPa,

$N = \dfrac{1}{2}\left(\dfrac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 382907$

≈ 382900 cycles

2)



$M = \left(304.8 + 20 + \dfrac{1}{2} \times 9.525\right) \times F$

about neutral axis

$= 0.3295625\, F$ Nm

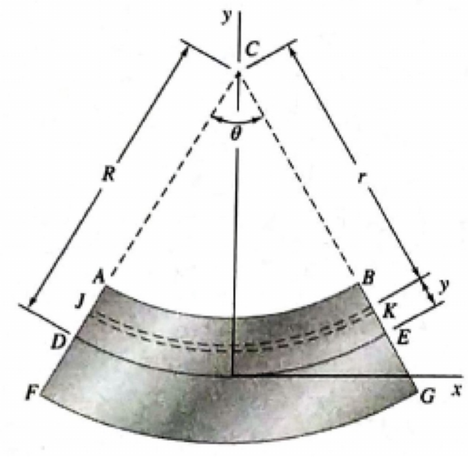
$\bar{r} = 20 + \dfrac{1}{2} \times 9.525$

$= 24.7625$, $c = 0.5 \times 9.525 = 4.7625$ mm

$R = \dfrac{A}{2\pi(\bar{r} - \sqrt{\bar{r}^2 - c^2})} = 24.53$ mm

$\sigma_{\theta\theta} = \dfrac{-My}{A(R-y)e}$, $e = \bar{r} - R = 0.2325$ mm, $y = R - r$

$$\Rightarrow \sigma_{\theta\theta}(r) = \frac{-M(R-r)}{\pi c^2 r e} = -\frac{M}{\pi c^2 e}\left(\frac{R}{r} - 1\right)$$



Highest at $r = r_i = 20\,mm$

$$\sigma_{\theta\theta,max} = 4.50577\,F\ MPa$$

$$\sigma_m = 4.50577\,F_m = 601.27\,MPa$$

$$\sigma_a = 201.97\,MPa$$

$$\sigma_a' = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_{ult}}} = 392.091\,MPa$$

At $\sigma_a' = 392.091\,MPa$, (Basquin found in (1))

$$N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 60591.62$$

$$\approx 60500 \text{ cycles}$$