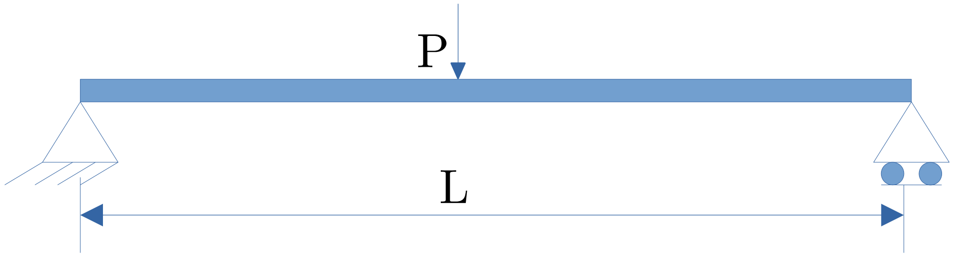
Influence of c/s Shape on the Performance

Problem: Want to design a light stiff simply supported beam



$$\delta = \frac{PL^3}{48EI} \qquad S = \frac{P}{\delta} = \frac{48EI}{L^3}$$

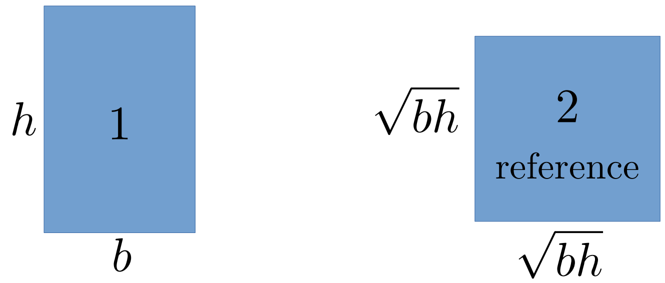
$$S \propto EI$$

Material property   Material distribution

For two beams with the same material

$$S \propto I$$

Consider two c/ having the same area



$$I_1 = \frac{bh^3}{12} \qquad I_2 = \frac{(\sqrt{bh})(\sqrt{bh})^3}{12} = \frac{b^2h^2}{12}$$

$$\frac{I_1}{I_2} = \frac{h}{b}$$

For $h > b$, $\quad I_1 > I_2$

For the same material, we get a better performance with a rectangular c/s as compared with the square c/s

Influence of c/s Shape on the Performance

- Shape is taken into account using **shape factors**
- Shape factor measures the efficiency of the material usage and is independent of the material
- Shape factor is a dimensionless quantity and is independent of scale
- The same c/s can have different shape factors depending on the type of loading conditions
- **Shape factor for stiffness (bending or torsional)**

$$\phi^e = \frac{\text{stiffness of the shaped c/s}}{\text{stiffness of the neutral or reference c/s}}$$

- **Shape factor for strength (bending or torsional)**

$$\phi^f = \frac{\text{strength of the shaped c/s}}{\text{strength of the neutral or reference c/s}}$$

- Neutral or reference c/s: solid c/s section with the same c/s area as that of the shaped section
- For bending, the reference c/s is a square c/s with the same area
- For torsion, the reference c/s is a circular c/s with the same area

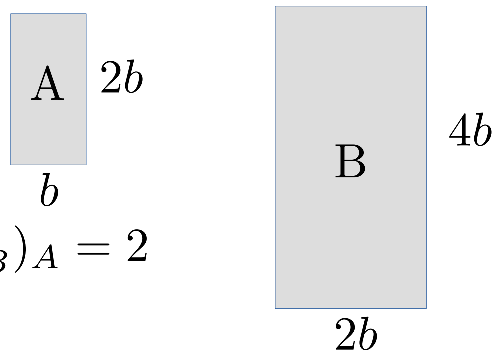
Influence of c/s Shape on the Performance

Shape factor for Bending Stiffness

$$\phi_B^e = \frac{I}{I_{NS}}, \; I_{NS} = \frac{b^4}{12} = \frac{A^2}{12}$$

$$\boxed{\phi_B^e = \frac{12I}{A^2}}$$

Scale Independence



$$(\phi_B^e)_A = 2$$

$$(\phi_B^e)_B = 2$$

Shape factor for Bending Strength

- Bending Strength $\sigma = \dfrac{M}{I/y} = \dfrac{M}{Z}$

Section modulus

- For a given moment M, to minimize σ we need to maximize Z

$$\phi_B^f = \frac{Z}{Z_{NS}}, \; Z_{NS} = \frac{b^3}{6} = \frac{A^{3/2}}{6}$$

$$\boxed{\phi_B^f = \frac{6Z}{A^{3/2}}}$$

Simultaneous Selection of Material and Shape of c/s to Maximize Performance

- Have looked at material selection and shape selection independently up to this point
- In practice, they are not independent – Aluminum is available as thin walled tubes while wood is not.

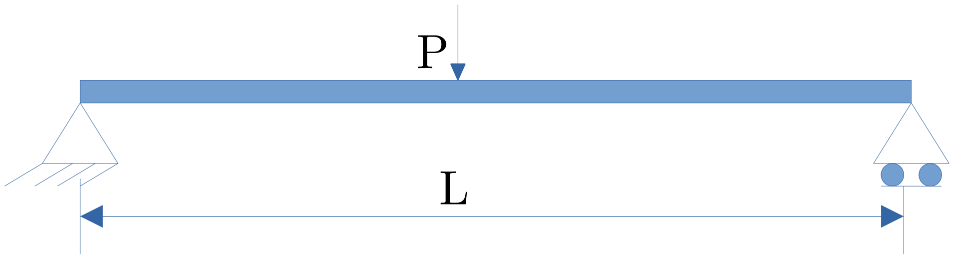
- Will now develop a **performance index** which account for both the material and c/s shape

https://alorwood.com/en/what-is-natural-wood

https://www.chaluminium.com/extruded-aluminum-tubes-manufacturing-applications-and-advantages

Example

Choose a material and c/s for a simply supported light stiff beam. The stiffness of the beam should be at least S^*



Function: The beam supports a load P

Constraint: stiffness should be at least S^*

Objective: minimize the mass of the beam

Free variable: material, shape of c/s, A

Material	(kg/m³)	E(GPa)	ϕ_B^e
1020 Steel	7850	205	20
6061 Al	2700	70	15
Wood (oak)	900	13.5	2
GFRP	1750	28	8

$$\delta = \frac{PL^3}{48EI} \qquad \phi_B^e = \frac{12I}{A^2}$$

$$\text{Stiffness} \quad S = \frac{P}{\delta} = \frac{48EI}{L^3} = \frac{4EA^2\phi_B^e}{L^3}$$

Example

Mass $m = \rho A L$

Stiffness $S = \dfrac{4EA^2\phi_B^e}{L^3}$

$S \geq S^*$ constraint

$$\frac{4EA^2\phi_B^e}{L^3} \geq S^*$$

or

$$\frac{4Em^2\phi_B^e}{L^5\rho^2} \geq S^*$$

$$m \geq \left(\frac{1}{2}\sqrt{S^*}\right)\left(L^{5/2}\right)\left(\frac{\rho}{(E\phi_B^e)^1/2}\right)$$

Functional parameters — **G**eometric parameters — **M**aterial and c/s properties

$m \geq f_1(F) f_2(G) f_3(MS)$ separable form

To minimize the mass we therefore need to maximize the performance index $(E\phi_B^e)^{1/2}/\rho$

Material	ρ (kg/m^3)	E(GPa)	ϕ_B^e	$E^{1/2}/\rho$	$(E\phi_B^e)^{1/2}/\rho$
1020 Steel	7850	205	20	58	258
6061 Al	2700	70	15	98	**380**
Wood(oak)	900	13.5	2	**129**	183
GFRP	1750	28	8	96	270

Select this

End