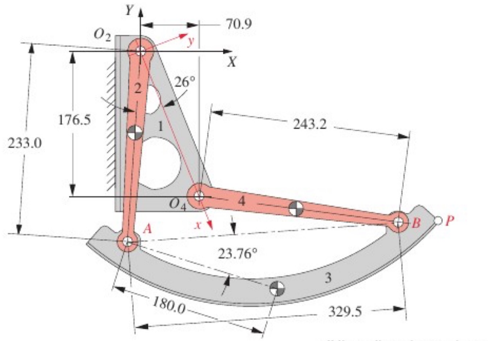
Ganesh Iyer

210100059

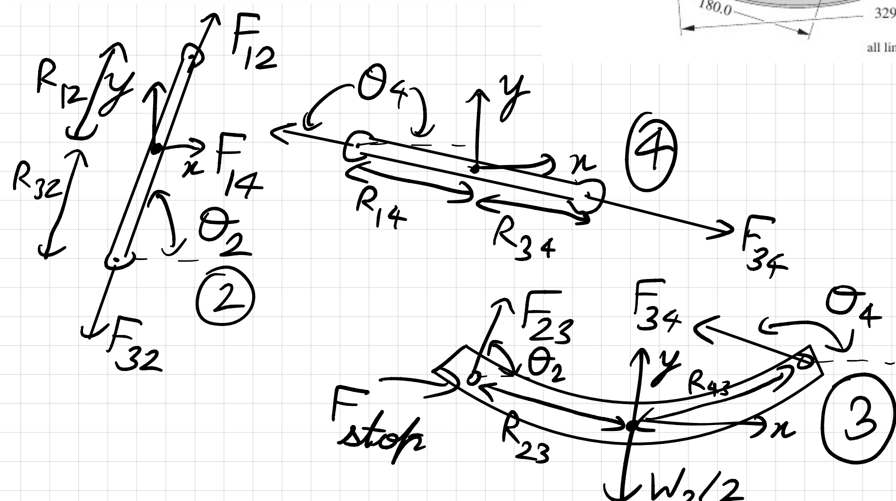
ME-423: Tut-1

2. [Norton, Chapter 3] Figure shows an aircraft overhead bin mechanism in end view. For the position shown, draw free-body diagrams of links 2 and 4 and the door (3). There are stops that prevent further clockwise motion of link 2 (and the identical link behind it at the other end of the door) resulting in horizontal forces being applied to the door at points A. Assume that the mechanism is symmetrical so that each set of links 2 and 4 carry one half of the door weight. Ignore the weight of links 2 and 4 as they are negligible.
Also determine the pin forces on the door (3), and links 2 & 4 and the reaction force on each of the two stops. Available data:

R_{23}	180 mm @ 160.345°
R_{43}	180 mm @ 27.862°
W_3	45 N
θ_2	85.879°
θ_4	172.352°



all linear dimensions are in mm



Eqbm for (3):

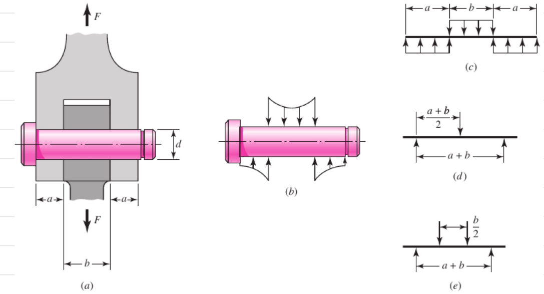
$$\Sigma F_x = 0 \Rightarrow F_{stop} + F_{23,x} + F_{43,x} = 0$$

$$\Sigma F_y = 0 \Rightarrow F_{23,y} + F_{43,y} - W_3/2 = 0$$

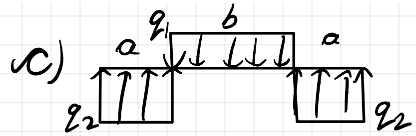
$$\Sigma M = 0 \Rightarrow -F_{stop} \cdot R_{23,y} + F_{23,y} \cdot R_{23,x} - F_{23,x} R_{23,y} + F_{43y} \cdot R_{43x} - R_{43,y} \cdot F_{43x} = 0$$

We can solve for the variables, and get pin force F_{23}, F_{43}, F_{stop}, and reaction on (2), (4) $-F_{23}$, $-F_{43}$.

3. [Shigley, Chapter 3] A pin in a knuckle joint carrying a tensile load F deflects somewhat on account of this loading, making the distribution of reaction and load as shown in part (b) of the figure. A common simplification is to assume uniform load distributions, as shown in part (c). To further simplify, designers may consider replacing the distributed loads with point loads, such as in the two models shown in parts d and e. If a = 1.20 cm, b = 1.8 cm, d = 1.20 cm, and F = 4500 N, estimate the maximum bending stress and the maximum shear stress due to V for the three simplified models. Compare the three models from a designer's perspective in terms of accuracy, safety, and modeling time.

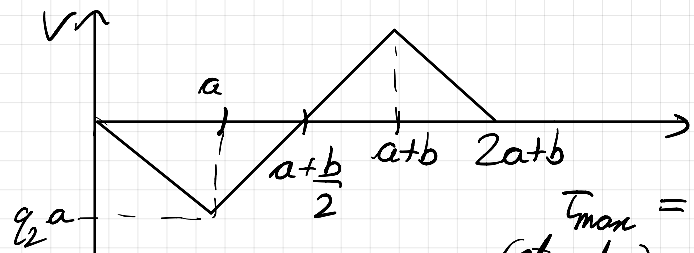


c)



$$q_1 b = F \Rightarrow q_1 = \frac{\cancel{4500} \times 10^3}{\cancel{18}} = 250\ kN/m$$

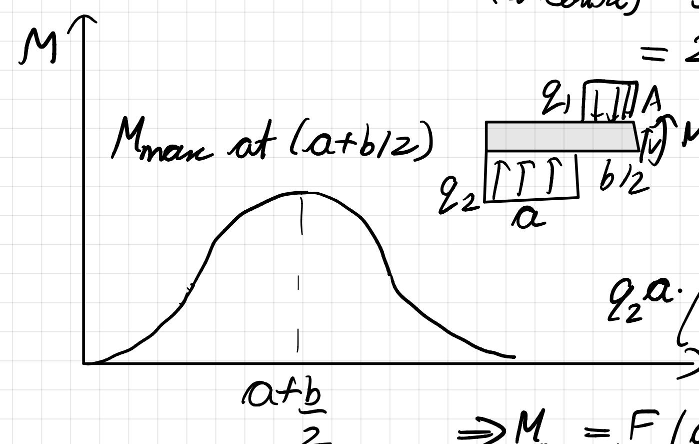
$$2 q_2 a = F \Rightarrow q_2 = \frac{\cancel{4500} \times 10^3}{2 \times \cancel{12}} = 187.5\ kN/m$$



$$V_{max} = q_2 a = F/2$$

For circular beam,

$$\tau_{max} \text{ (at centre)} = \frac{4 V_{max}}{3A} = \frac{4 \times F/2}{3 \times \pi d^2/4} = 26.52\ MPa$$



Balance about A,

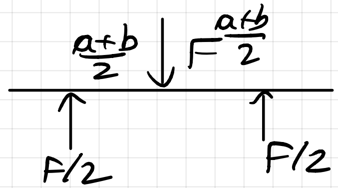
$$q_2 a \cdot \left(\frac{a+b}{2}\right) - \frac{q_1 b}{2} \cdot \frac{b}{4} = M_{max}$$

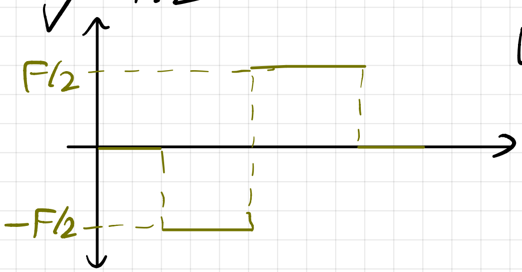
$$\Rightarrow M_{max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = 23.625\ N\text{-}m$$

$$I = \frac{\pi d^4}{64} = 1.018 \times 10^{-9}$$

$$\sigma_{max} = \frac{M(d/2)}{I} = \frac{23.625 \times 0.6 \times 10^{-2}}{1.018 \times 10^{-9}} = 139.26 \text{ MPa}$$

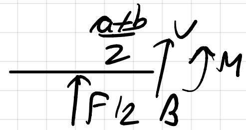
d)



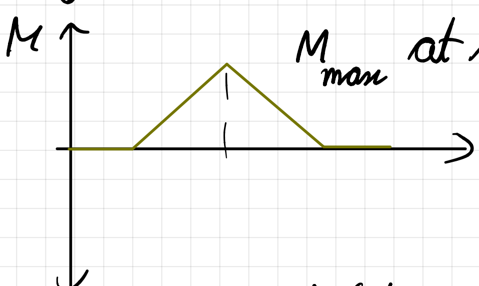


$$V_{max} = F/2$$

$$\tau_{max} = \frac{4 \times F/2}{3A} = 26.52 \text{ MPa}$$



M_{max} at centre

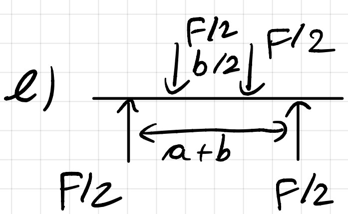


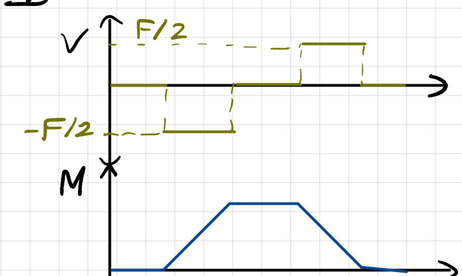
About B,

$$M = \frac{F}{2}\left(\frac{a+b}{2}\right) = 33.75 \text{ N-m}$$

$$\sigma_{max} = \frac{M(d/2)}{I} = 198.91 \text{ MPa}$$

e)





$V_{max} = F/2$

$$M_{max} \text{ (around centre)} = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{b}{4}\right) = \frac{F}{4}\left(a + \frac{b}{2}\right) = 23.625 \text{ N-m}$$

$$\tau_{max} = \frac{4 \times F/2}{3A} = 26.52 \text{ MPa}$$

$$\sigma_{max} = \frac{M(d/2)}{I} = 139.26 \text{ MPa}$$

Distributed Load most accurate, since actual loads are not point loads.

τ_{max} same in all, but σ_{max} highest in 3-point loading, most conservative, good from safety point of view.

From modelling perspective, 3-point loading has min no. of loads so easiest to analyze.

5. [Shigley, Chapter 3]. A gear reduction unit uses the countershaft shown in the figure. Gear A receives power from another gear with the transmitted force F_A applied at the 20° pressure angle as shown. The power is transmitted through the shaft and delivered through gear B through a transmitted force F_B at the pressure angle shown.

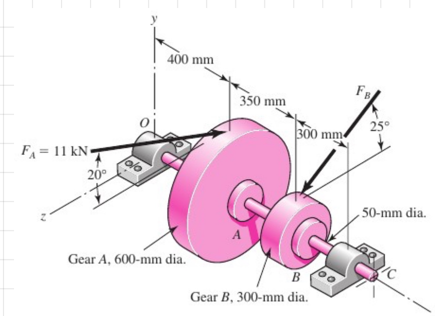
(a) Determine the force F_B, assuming the shaft is running at a constant speed.

(b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

(c) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.

(d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.

(e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

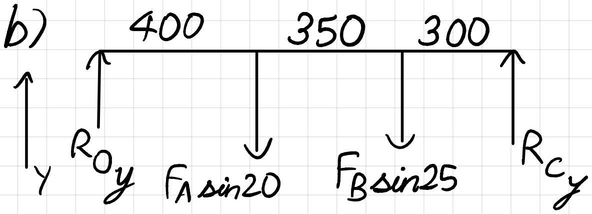


a) Torque Balance about axis,

$$F_A \cos 20 \times r_A = F_B \cos 25 \, r_B$$

$$\Rightarrow F_B = 22.81 \text{ KN}$$

b)

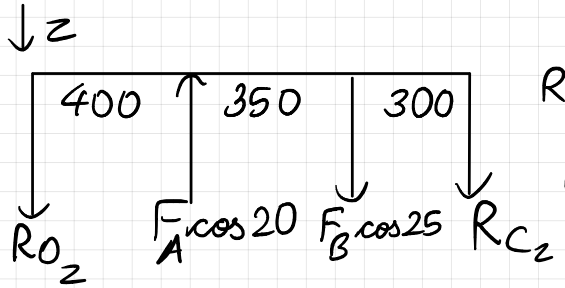


$R_{O_y} + R_{C_y} = 13.4$ KN

$F_A \sin 20 \times 400 + F_B \sin 25 \times 750 = R_{C_y} \times 1050$

$R_{C_y} = 8.32$ KN

$R_{O_y} = 5.08$ KN



$R_{O_z} + R_{C_z} = -10.33$

$F_A \cos 20 \times 400 = F_B \cos 25 \times 750 + R_{C_z} \times 1050$

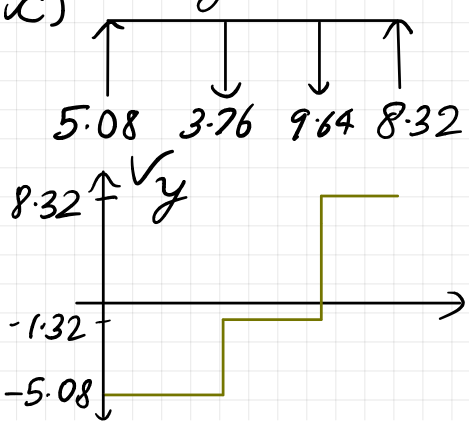
$R_{C_z} = -10.83$ KN

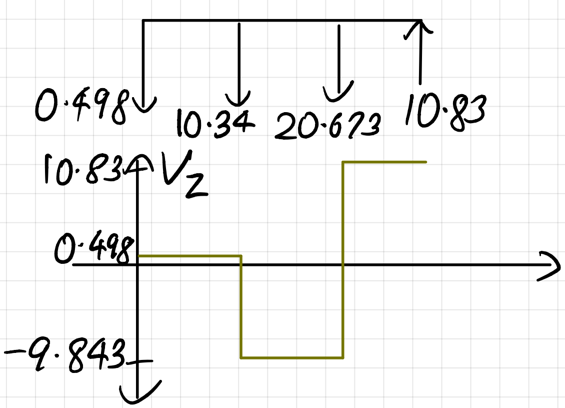
$R_{O_z} = 0.498$ KN

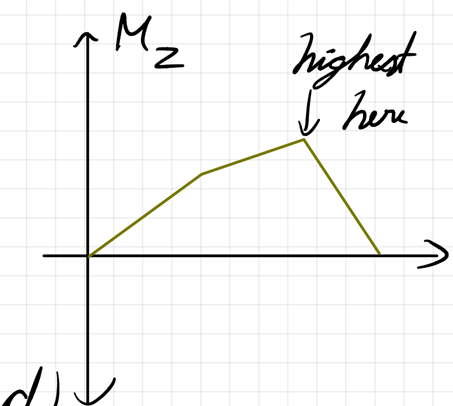
$R_C = \sqrt{R_{C_y}^2 + R_{C_z}^2} = 13.657$ KN

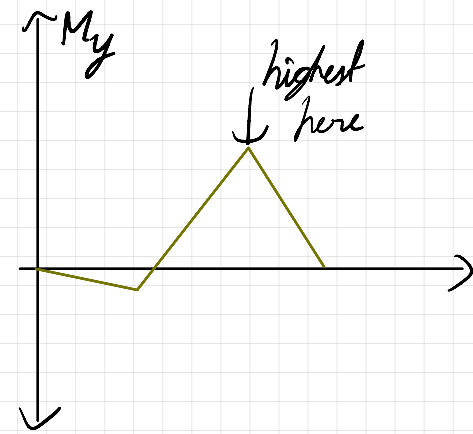
$R_O = \sqrt{R_{O_y}^2 + R_{O_z}^2} = 5.1$ KN

c)









d)

Critical location at B, max bending moment.

$M_z = 300 \times R_{C_y} = 2.5$ KN-m

$M_y = 300 \times R_{C_z} = 3.25$ KN-m

$M = \sqrt{M_y^2 + M_z^2} = 4.1$ KN-m

$T = 11 \cos 20 \times r_A = 3.1$ KN-m

$$\sigma = \frac{Mr}{I} = \frac{Md}{2 \times \frac{\pi d^{\cancel{4}3}}{\cancel{64}\,32}} = \frac{32M}{\pi d^3} = \frac{32 \times 4.1}{\pi (0.05)^3} = 334 \text{ MPa}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = 126.3 \text{ MPa}$$

e) $\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$= 376, -42.4$ MPa

$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 209$ MPa

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ME-423: Tut - 2

1. Estimate σ_e' in MPa for the following materials:
(a) AISI 1035 CD steel.
(b) AISI 1050 HR steel.
(c) 2024 T4 aluminum.
(d) AISI 4130 steel heat-treated to a tensile strength of 1620 MPa.

a) $\sigma_{ult} = 552 \text{ MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 276 \text{ MPa}$

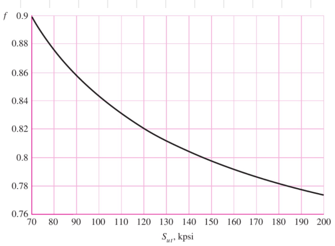
b) $\sigma_{ult} = 621 \text{ MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 310.5 \text{ MPa}$

c) $\sigma_{ult} = 441 \text{ MPa (Al)} \Rightarrow \sigma_{f@5\times10^8} = 130 \text{ MPa} \; (\sigma_{ult} > 300)$

d) $\sigma_{ult} = 1620 \text{ MPa} (> 1400 \text{ MPa}) \Rightarrow \sigma_e' = 700 \text{ MPa}$

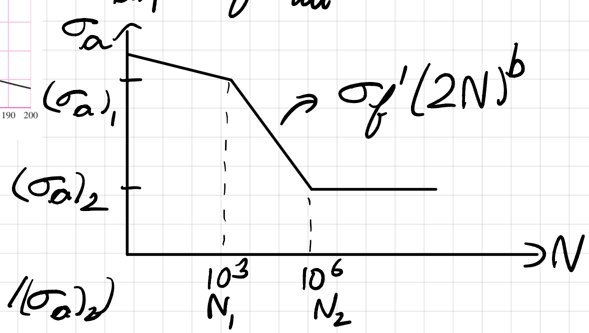
2. A steel rotating-beam test specimen has an ultimate strength of 830 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 480 MPa.

2. $\sigma_{ult} = 830 \text{ MPa} = \frac{830}{6.894} \text{ Kpsi} = 120.38 \text{ Kpsi}$ $\quad \sigma_e' \approx 0.5\sigma_{ult} = 415 \text{ MPa} \rightarrow (\sigma_a)_2$



$f \approx 0.82$

$(\sigma_a)_1 \approx f\sigma_{ult} = 680.6 \text{ MPa}$



$$\Rightarrow b = \frac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.0716, \quad \sigma_f' = \frac{(\sigma_a)_2}{(2N_2)^b} = 1172.99 \approx 1173 \text{ MPa}$$

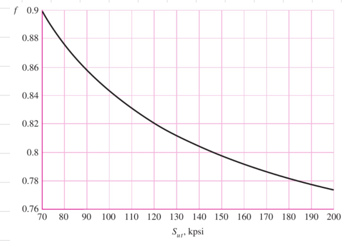
N for $\sigma_a = 480$ MPa

$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 131431.81$$

$\Rightarrow N \approx 131400$ cycles

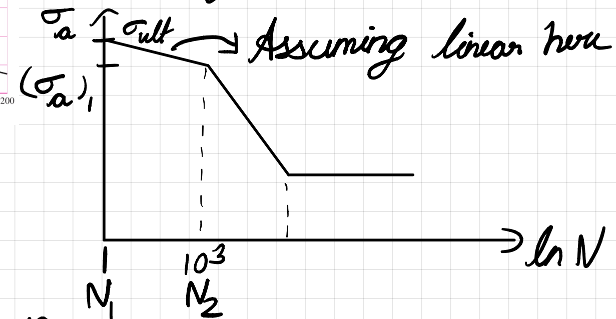
3. A steel rotating-beam test specimen has an ultimate strength of 1030 MPa and a yield strength of 930 MPa. It is desired to test low-cycle fatigue at approximately 500 cycles. Check if this is possible without yielding by determining the necessary reversed stress amplitude.

3. $\sigma_{ult} = 1030 \text{ MPa} = \frac{1030}{6.894} \text{ Kpsi} = 149.4 \text{ Kpsi}$ $\quad \sigma_e' \approx 0.5\sigma_{ult} = 515 \text{ MPa} \rightarrow (\sigma_a)_2$



$f \approx 0.8$

$(\sigma_a)_1 \approx f\sigma_{ult} = 824 \text{ MPa}$



$(\sigma_a)_1 \approx f\sigma_{ult}$, $f \sim 0.8$

$\Rightarrow (\sigma_a)_1 = 824$ MPa

Taking relation as $\sigma_f = aN^b$,

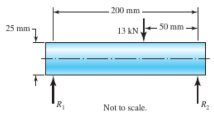
$\sigma_{ult} = aN_1^b \Rightarrow 1030 = a(1)^b \Rightarrow a = 1030$

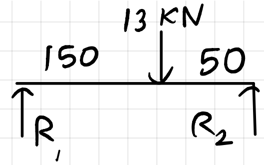
$824 = 1030 \times (10^3)^b \Rightarrow b = -0.032$

At $N = 500$, $\sigma_a = 1030 \times 500^{-0.032} = 844.25 \text{ MPa}$

$\sigma_a < \sigma_y = 930 \text{ MPa} \Rightarrow$ Does not yield

4. A rotating shaft of 25-mm diameter is simply supported by bearing reaction forces R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine
(a) the minimum static factor of safety based on yielding.
(b) the endurance limit, adjusted as necessary with correction (Marin) factors.
(c) the minimum fatigue factor of safety based on achieving infinite life.
(d) If the fatigue factor of safety is less than 1, then estimate the life of the part in number of rotations of rotations.



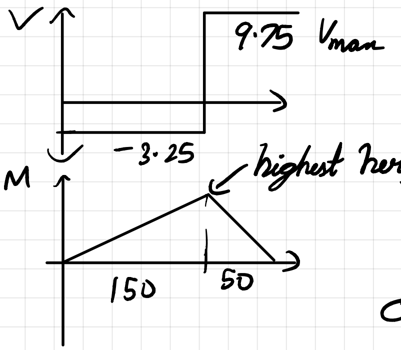


$3R_1 = R_2$ (moment balance)

$R_1 + R_2 = 13$

$\Rightarrow R_1 = \frac{13}{4},\ R_2 = \frac{39}{4}$ KN

a)



highest here $\rightarrow M = \frac{50 \times R_2}{1000} = 487.5$ N-m

$\sigma = \frac{M \times d/2}{\pi d^4/64} = \frac{32M}{\pi d^3}$

$\sigma = 317.8$ MPa

Top, bottom

$\tau = \frac{4V_{max}}{3A} = \frac{4V_{max}}{3(\pi d^2/4)} = 26.48$ MPa

At center

$\|\tau\| << \|\sigma\|$. Hence critical points top/bottom.

σ_y (1045 HR Steel) $= 310$ MPa

σ_{ult} " $= 565$ MPa

$N = \sigma_y / \sigma = 0.975$

$\Rightarrow$ It will fail by yielding

b) $\sigma_e' = 0.5\sigma_{ult} = 0.5 \times 565 = 282.5$ MPa

K_a(Surface) $= a\sigma_{ult}^b$, For machining,
$= 4.51(565)^{-0.265} = 0.8411$

K_b(Size) $= 1.24\, d^{-0.107} = 0.8787$

K_c(Load) $= 1$, $K_d, K_e, K_f = 1$ (No info given)

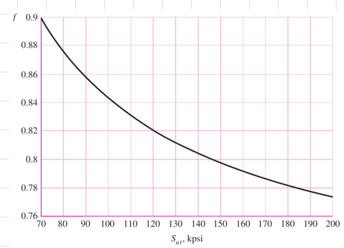
$\Rightarrow \sigma_e = 0.8411 \times 0.8787\, \sigma_e' = 208.8$ MPa

c) $N_f = \frac{\sigma_e}{\sigma} = \frac{208.8}{317.8}$

$\Rightarrow N_f = 0.657$ Fatigue factor of safety

d) As $N_f < 1$, finite life

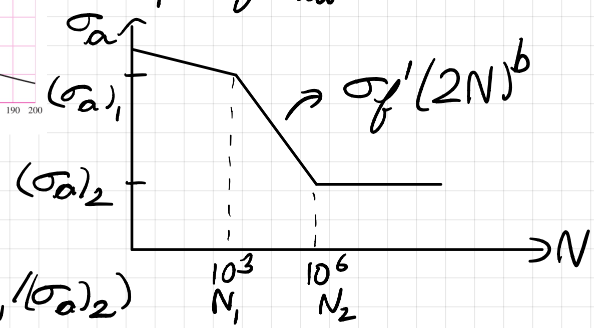
$\sigma_{ult} = 565$ MPa
$= \frac{565}{6.894}$ kpsi $= 81.97$ kpsi



$f \approx 0.875$

$(\sigma_a)_1 \approx f\sigma_{ult} = 494.375$ MPa

$(\sigma_a)_2 = \sigma_e = 208.8$ MPa



$\Rightarrow b = \frac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.12477$, $\sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 1276.27$ MPa

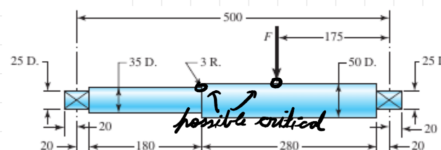
For $\sigma_a = 317.8$ MPa,

$$N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 34528.96$$

$\Rightarrow N \approx 34500$ cycles

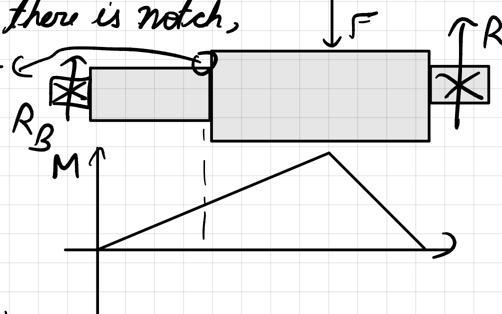
But as $\sigma_a > \sigma_y$, It will yield before fatigue.

5. The rotating shaft shown in the figure is machined from AISI 1020 CD (cold rolled) steel. It is subjected to a force of F = 6 kN. Find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding. All the dimensions are in mm.



$$\sigma_0 = \frac{32M}{\pi d^3}$$

As $\sigma_0 \propto 1/d^3$, and there is notch, it would be higher at



$R_A + R_B = F = 6$

$\circlearrowright$ Balance,

$R_B \times 500 = F \times 175$

$\Rightarrow R_B = \frac{7}{20}F = 2.1$ kN $\Rightarrow R_A = 3.9$ kN

M at critical point : $M = R_A \times 300 - F(125)$
$= 420$ N-m

$$\sigma_0 = \frac{32 \times 420}{\pi(35\times10^{-3})^3} = 99.78 \text{ MPa}$$

Notch (3mm radius) $\Rightarrow \frac{r}{d} = \frac{3}{35} = 0.085$, $\frac{D}{d} = \frac{50}{35} \approx 1.5$

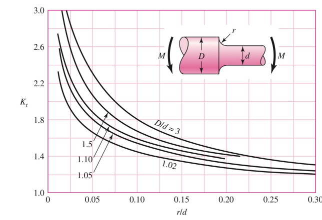


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

$\Rightarrow k_t \approx 1.7$

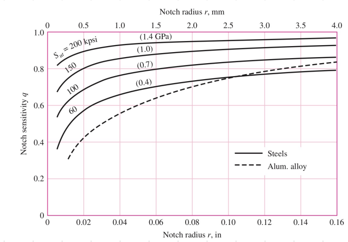


Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.), Metal Fatigue, McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

$\sigma_{ult} = 60.9$ kpsi

$\Rightarrow q \approx 0.78$

$K_f = 1 + q(K_t - 1) = 1.546$

$\sigma_{max} = K_f \sigma_0 = 154.26$ MPa

$\sigma_e' = 0.5\sigma_{ult} = 0.5 \times 420 = 210$ MPa

Taking highest D for lowest size factor K_b,

$K_b = 1.24 \times 50^{-0.107} = 0.816$

$K_a = 4.51 \times 420^{-0.265} = 0.91$ (Machined) $\Rightarrow \sigma_e = K_a \cdot K_b \sigma_e' = 155.93$ MPa

As $\sigma_{max} < \sigma_e$, infinite life

$\sigma_y = 350$ MPa, $\sigma_{max} < \sigma_y$, No yielding

6. A steel part is loaded with a combination of bending, axial, and torsion such that the following stresses are created at a particular location:
Bending: Completely reversed, with a maximum stress of 60 MPa
Axial: Constant stress of 20 MPa
Torsion: Repeated load, varying from 0 MPa to 70 MPa
Assume the varying stresses are in phase with each other. The part contains a notch such that $(K_f)_{\text{bending}} = 1.4$, $(K_f)_{\text{axial}} = 1.1$, and $(K_f)_{\text{torsion}} = 2.0$. The material properties are $\sigma_y = 300$ MPa and $\sigma_{ult} = 400$ MPa. The completely adjusted (corrected) endurance limit is found to be $\sigma_e = 160$ MPa. Find the factor of safety for fatigue based on infinite life, using the Goodman criterion (assume proportional loading). If the life is not infinite, estimate the number of cycles, using the SWT criterion to find the equivalent completely reversed stress. Be sure to check for yielding.

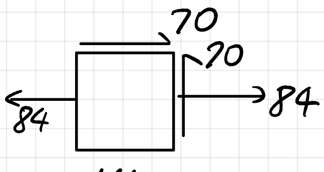
(Complete Reverse)

$$\sigma_{bending} = (K_f)_{bending} \times \sigma_0 = 1.4 \times 60 = 84 \text{ MPa}$$

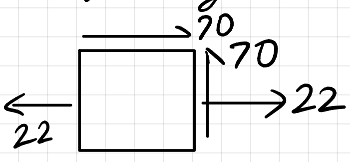
$$\sigma_{axial} = 1.1 \times 20 = 22 \text{ MPa}$$

$$\sigma_{torsion} = 0 \text{ to } \underbrace{70 \times 2}_{140 \text{ MPa}}$$

i.e mean 70 MPa, alternating 70 MPa



Alternating



Mean

Von-Mises:

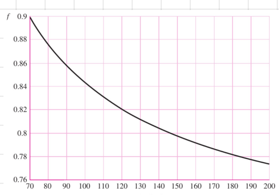
$$\sigma_{vm,a} = (84^2 + 3 \times 70^2)^{1/2} = 147.5 \text{ MPa}$$

$$\sigma_{vm,m} = (22^2 + 3 \times 70^2)^{1/2} = 123.22 \text{ MPa}$$

$$\frac{\sigma_{vm,a}}{\sigma_e} + \frac{\sigma_{vm,m}}{\sigma_{ult}} = \frac{147.5}{160} + \frac{123.22}{400} = 1.23 > 1$$

$\Rightarrow$ Point lies above Goodman line, finite life.

$$\text{SWT: } \sigma_a' = \sqrt{(\sigma_m + \sigma_a)\sigma_a} = \sqrt{(123.22 + 147.5)(147.5)} = 199.83 \text{ MPa}$$



$$\sigma_{ult} = 53 \text{ Kpsi} \Rightarrow f \approx 1$$

$$(\sigma_a)_1 = 400 \text{ MPa}, \ (\sigma_a)_2 = \sigma_e = 160 \text{ MPa}$$

$$b = \ln(400/160) / \ln(10^3/10^6) = -0.1326$$

$$\sigma_f' = (\sigma_a)_1 / (2000)^b = 1096.3 \text{ MPa}$$

For equivalent σ_a',

$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{199.83}{1096.3}\right)^{1/b} \approx 188000 \text{ cycles}$$

$$\sigma_{max} = \sigma_m + \sigma_a = 270.72 < \sigma_y = 300 \text{ MPa}$$

(Not yielded)

7. A machine part will be cycled at ±350 MPa for 5×10^3 cycles. Then the loading will be changed to ±260 MPa for 5×10^4 cycles. Finally, the load will be changed to ±225 MPa. Using the Miner's rule, estimate the of cycles of operation that can be expected at this stress level before the part fails? For the part, $\sigma_{ult} = 530$ MPa, $f = 0.9$, and has a fully corrected endurance strength of $\sigma_e = 210$ MPa.

$$(\sigma_a)_1 = f\sigma_{ult} = 477 \text{ MPa}, \ (\sigma_a)_2 = \sigma_e = 210 \text{ MPa}$$

$$N_1 = 10^3, \ N_2 = 10^6 \text{ cycles}$$

$$\sigma_a = \sigma_f'(2N)^b \Rightarrow b = \frac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.1187$$

$$\Rightarrow \sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 1176.44 \text{ MPa}$$

Now, $(\sigma_a)_A = 350$ MPa, $n_A = 5 \times 10^3$

$$N_A = \frac{1}{2}\left(\frac{(\sigma_a)_A}{\sigma_f'}\right)^{1/b} = 13553.7$$

$$\Rightarrow N_A \approx 13500 \text{ cycles}$$

$(\sigma_a)_B = 260$ MPa, $n_A = 5 \times 10^4$

$$N_B = \frac{1}{2}\left(\frac{(\sigma_a)_B}{\sigma_f'}\right)^{1/b} = 165585.2$$

$$\Rightarrow N_A \approx 165500 \text{ cycles}$$

So that estimate for n_c is low, conservative

$(\sigma_a)_C = 225$ MPa

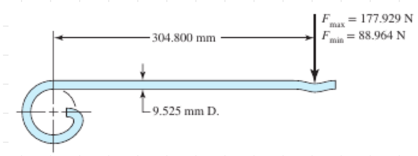
$$N_C = \frac{1}{2}\left(\frac{(\sigma_a)_C}{\sigma_f'}\right)^{1/b} = 559388.5$$

$$\approx 559300 \text{ cycles}$$

$$n_C = N_C\left(1 - \frac{n_A}{N_A} - \frac{n_B}{N_B}\right)$$

$$n_C = 183179 \approx 183100 \text{ cycles}$$

8. The figure shows a formed round-wire cantilever spring subjected to a varying force. The inner radius of the bend is 20 mm. The hardness tests made on 50 springs gave a minimum hardness of 400 Brinell. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. Estimate the number of cycles to likely to cause failure using the Goodman criterion.
1. If the curvature effects on the bending stress are ignored.
2. If the curvature effects on the bending stress are not ignored



$\sigma_{ult} \approx 3.1\ HB = 1240$ MPa , $F_m = 133.4465$ N, $F_a = 44.825$ N

1) 

$M = 0.3048 \times F$, $\sigma = \frac{32M}{\pi d^3} = 3592693 \times F$ Pa

$\sigma_a = 159.811$ MPa , $\sigma_a' = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_{ult}}\right)}$

$\sigma_m = 475.77$ MPa

$\sigma_a' = 259.3$ MPa

$\sigma_{ult} = 1240$ MPa $= 179.86$ Kpsi

$\sigma_e' = 0.5\,\sigma_{ult} = 620$ MPa

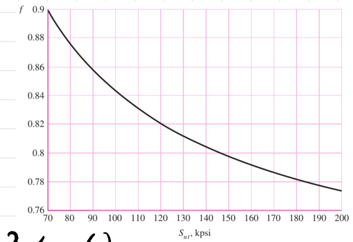
$k_a = 57.7(1240)^{-0.718}$ Hot rolled
$= 0.3468$

$k_b = 1.24(9.525)^{-0.107} = 0.974$

$\sigma_e = k_a \cdot k_b \cdot \sigma_e' = 209.485$ MPa

$\sigma_a' > \sigma_e \Rightarrow$ Finite Life

$f \approx 0.78$



$(\sigma_a)_1 = f\sigma_{ult} = 967.2$ MPa

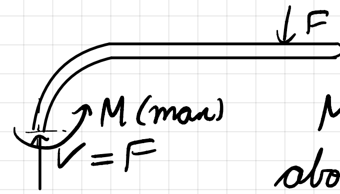
$(\sigma_a)_2 = \sigma_e = 209.485$ MPa

$b = \ln\left((\sigma_a)_1 / (\sigma_a)_2\right) / \ln(10^3/10^6)$
$= -0.2214$

$\sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 5204.32$ MPa

At $\sigma_a' = 259.3$ MPa,

$N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 382907$
≈ 382900 cycles

2) 

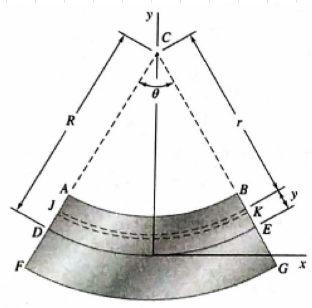
$M = (304.8 + 20 + \frac{1}{2} \times 9.525) \times F$ about neutral axis
$= 0.3295625\ F$ N-m

$\bar{r} = 20 + \frac{1}{2} \times 9.525$
$= 24.7625$, $c = 0.5 \times 9.525 = 4.7625$ mm

$R = \frac{A}{2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)} = 24.53$ mm

$\sigma_{\theta\theta} = \frac{-My}{A(R-y)e}$, $e = \bar{r} - R = 0.2325$ mm , $y = R - r$

$$\Rightarrow \sigma_{\theta\theta}(r) = \frac{-M(R-r)}{\pi c^2 r e} = \frac{-M}{\pi c^2 e}\left(\frac{R}{r}-1\right)$$



Highest at $r = r_i = 20\,mm$

$$\sigma_{\theta\theta,max} = 4.5057\,F \text{ MPa}$$

$$\sigma_m = 4.5057\,F_m = 601.27 \text{ MPa}$$

$$\sigma_a = 201.97 \text{ MPa}$$

$$\sigma_a' = \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_{ult}}} = 392.091 \text{ MPa}$$

At $\sigma_a' = 392.091$ MPa, (Basquin found in (1))

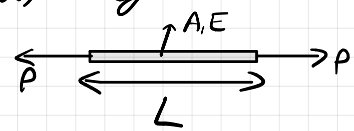
$$N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 60591.62$$

$$\approx 60500 \text{ cycles}$$

Ganesh Iyer, 210100059 ME-423: Tutorial-3

1. Derive the material indices for the following cases:
   a. A light truss with stiffness greater than S^*
   b. A light shaft with stiffness greater than S^*

a) Objective: min m Constraint: $S \geq S^*$



$$\delta = \frac{PL}{EA}$$ Truss → Axial

$$S = \frac{P}{\delta} = \frac{EA}{L} \geq S^*$$

$$m = \rho AL \Rightarrow A = \frac{m}{\rho L}$$

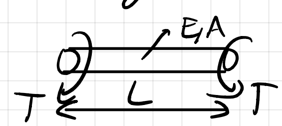
$$\Rightarrow \frac{E}{L} \times \frac{m}{\rho L} \geq S^* \Rightarrow m \geq S^* \times \left(\frac{\rho}{E}\right) \times L^2$$

Material

$$\min(m) \Rightarrow \max\left(\frac{E}{\rho}\right)$$

$\frac{E}{\rho}$: material index

b) Objective: min m Constraint: $S \geq S^*$



$$\theta = \frac{TL}{GJ}$$ Shaft → Torsion

$$S = \frac{T}{\theta} = \frac{GJ}{L} \geq S^*$$

$$m = \rho AL \Rightarrow A = \frac{m}{\rho L}$$ Let us take circular c/s

$$\Rightarrow A = \pi d^2, \quad J = \frac{\pi d^4}{32} \Rightarrow J = \frac{A^2}{32\pi}$$

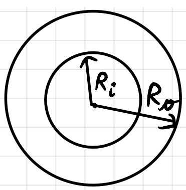
$$\Rightarrow J = \frac{m^2}{32\pi\rho^2 L^2}$$

Now, $$\frac{G}{L}\left(\frac{m^2}{32\pi\rho^2 L^2}\right) \geq S^*$$

$$\Rightarrow m \geq S^{*1/2} \times \left(\frac{\rho}{\sqrt{G}}\right) \times (32\pi L^3)^{1/2}$$

min(m) $\Rightarrow$ max$\left(\frac{G^{1/2}}{\rho}\right)$ material index

2. Derive the shape factor for annular cross-section with inner radius R_i and outer radius R_o for torsional stiffness.



$$J = \frac{\pi(R_o^4 - R_i^4)}{2}$$

Reference c/s : $\cancel{\pi} R^2 = \cancel{\pi}(R_o^2 - R_i^2)$

$$J_{ref} = \frac{\pi R^4}{2} = \frac{\pi(R_o^2 - R_i^2)^2}{2}$$

$$S_{tors.} = \frac{GJ}{L} \propto J$$

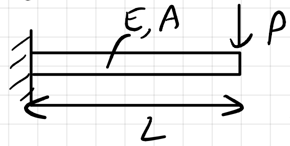
$$\Rightarrow \varphi = \frac{J}{J_{ref}} = \frac{R_o^4 - R_i^4}{(R_o^2 - R_i^2)^2} = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}$$

3. Derive the performance indices for the following cases:

a. A light beam with maximum stress less than or equal to σ_f (material property)

b. A light hollow shaft with stiffness greater than S^*

a) Objective : min m , Constraint: $\sigma_{max} \leq \sigma_f$



$$M_{max} = PL$$

$$\sigma_{max} = \frac{M_{max}\, y}{I} = \frac{M_{max}}{Z}$$

For bending strength, shape factor

$$\varphi_B^f = 6Z/A^{3/2} \Rightarrow Z = \varphi_B^f \cdot A^{3/2}/6$$

$$\Rightarrow \sigma_{max} = \frac{6 M_{max}}{\varphi_B^f A^{3/2}}, \quad A = \frac{m}{\rho L} \Rightarrow \sigma_{max} = \frac{6(PL)(\rho L)^{3/2}}{\varphi_B^f m^{3/2}}$$

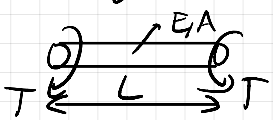
$$\sigma_{max} \leq \sigma_f$$

$$\Rightarrow \frac{6P\rho^{3/2}L^{5/2}}{\varphi_B^f m^{3/2}} \leq \sigma_f$$

$$\Rightarrow m \geq (6P)^{2/3} \cdot (L)^{5/3} \cdot \left(\frac{\rho}{\varphi_B^{2/3}\sigma_f^{2/3}}\right)$$

min(m) $\Rightarrow$ max$\left(\frac{\sigma_f^{2/3} \cdot \varphi_B^{2/3}}{\rho}\right)$: Material Index

b) Objective: min m Constraint: $S \geq S^*$



$$\theta = \frac{TL}{GJ}$$ Shaft → Torsion

$$S = \frac{T}{\theta} = \frac{GJ}{L} \geq S^* \qquad m = \rho A L$$

For shaft, $\varphi_T^e = \frac{2\pi J}{A^2}$ from Q.(2).

$$J = \frac{\varphi_T^e A^2}{2\pi} = \frac{\varphi_T^e (m/\rho L)^2}{2\pi} = \frac{\varphi_e^T m^2}{2\pi \rho^2 L^2}$$

$$\Rightarrow \frac{G}{L} \times \left(\frac{\varphi_e^T m^2}{2\pi \rho^2 L^2}\right) \geq S^*$$

$$\Rightarrow m \geq (2\pi S^*)^{1/2} \times (L)^{3/2} \times \left(\frac{\rho}{\sqrt{G \varphi_e^T}}\right)$$

Performance Index: $\frac{\sqrt{G\varphi_e^T}}{\rho}$

4. Derive the material index to maximize the slenderness ratio (L/r) of a column with circular c/s subject to the constraint that it must not buckle under a given load F.

$$P_{cr} = C \pi^2 \frac{EI}{L^2}$$ C depends on B.Cs, some constant

Objective: $\max\left(\frac{L}{r}\right)$ Constraint: $P_{cr} \geq F$

$$C\pi^2 \frac{EI}{L^2} \geq F \Rightarrow \frac{C\pi^3 E r^4}{4L^2} \geq F$$

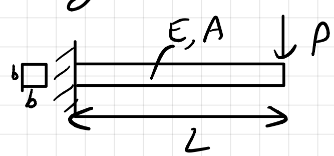
$$\Rightarrow \frac{C\pi^3 E L^2}{4} \left(\frac{r}{L}\right)^4 \geq F$$

$$\Rightarrow \frac{L}{r} \leq \left(\frac{C\pi^3}{4}\right)^{1/4} E^{1/4} \times \frac{1}{F^{1/4}} \times L^{1/2}$$

$\max\left(\frac{L}{r}\right) \Rightarrow \max\left(E^{1/4}\right)$ material index

5. Derive the material index to minimize the cost of a beam with stiffness greater than S^*. Note that the cost of the beam, C, can be assumed to be directly proportional to the mass of the beam, i.e. $C = C_m \boldsymbol{m}$ where C_m is the cost per unit mass and is a material property. Your material index will now include E, ρ and C_m.

Objective: min C, Constraint: $S \geq S^*$



$$\delta = \frac{PL^3}{3EI}, \quad S = \frac{P}{\delta} = \frac{3EI}{L^3}$$

$$S = \frac{3Eb^4}{12L^3} = \frac{EA^2}{4L^3} \quad (A = b^2)$$

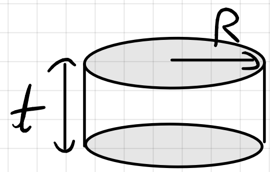
$$C = C_m \times m = C_m \rho A L \Rightarrow A = C/(C_m \rho L)$$

$$S = \frac{Ec^2}{4L^5 C_m^2 \rho^2} \geq S^*$$

$$\Rightarrow C \geq (4L^5)^{1/2} \times \left(\frac{C_m S}{E^{1/2}} \right)$$

$$\min(C) \Rightarrow \max\left(\frac{E^{1/2}}{C_m S} \right) \leftarrow \text{index}$$

6. Derive the material index to maximize the energy stored per unit mass in a flywheel of fixed outer radius R, radius t and rotating with angular speed ω. Note that the maximum stress induced in the flywheel should be less than or equal to the failure stress σ_f, a material property. Note that at this stress the flywheel bursts. The maximum principal stress in a spinning disk of radius R with uniform thickness is $\sigma_{max} = \frac{3+\nu}{8}\rho R^2\omega^2$.



$$E = \frac{1}{2} I \omega^2 = \frac{1}{4} m R^2 \omega^2$$

$$E/m = \frac{1}{4} R^2 \omega^2$$

$$\sigma_{max} = \frac{3+\nu}{8} \rho R^2 \omega^2 \leq \sigma_f$$

$$\Rightarrow \frac{3+\nu}{2} \rho \left(\frac{E}{m} \right) \leq \sigma_f$$

$$\Rightarrow \frac{E}{m} \leq \frac{2\sigma_f}{\rho(3+\nu)} \leftarrow \text{material index}$$

ME423 HW 4 21/09/2024

1. A shaft is loaded in bending and torsion such that $M_a = 70$ Nm, $T_a = 45$ Nm, $M_m = 55$ Nm, and $T_m = 35$ Nm. For the shaft, ultimate strength = 700 MPa, yield strength = 560 MPa, and a fully corrected endurance limit of = 210 MPa is assumed. Let $K_f = 2.2$ and $Kfs = 1.8$. For a factor of safety of 2.0 determine the minimum acceptable diameter of the shaft. Use the Goodman criterion. Clearly mention any assumptions that you make. Assuming $q = q_s = 1 \Rightarrow K_t = 2.2$ $K_{ts} = 1.8$

$M_a = 70\,Nm$ $T_a = 45\,Nm$

$M_m = 55\,Nm$ $T_m = 35\,Nm$

$\sigma_{ult} = 700\,MPa$

$\sigma_y = 560\,MPa$

$\sigma_e = 210\,MPa$

$K_f = 2.2$ $K_{fs} = 1.8$ $n = 2$

$$\sigma_{xx} = \frac{My}{I} = \frac{32M}{\pi d^3} \qquad \tau_{xy} = Tr = \frac{16T}{\pi d^3}$$

$$\sigma_m = \left(4K_f^2 \sigma_{xxm}^2 + 3K_{fs}^2 \tau_{xym}^2\right)^{1/2} = \frac{16}{\pi d^3}\left(4K_f^2 M_m^2 + 3K_{fs}^2 T_m^2\right)^{1/2}$$

$$= \frac{16}{\pi d^3}\left(242^2 + 3(63)^2\right)^{1/2} = \frac{16}{\pi d^3}(265.46)$$

$$\sigma_a = \frac{16}{\pi d^3}\left(4K_f^2 M_a^2 + 3K_{fs}^2 T_a^2\right)^{1/2} = \frac{16}{\pi d^3}\left(308^2 + 3(81)^2\right)^{1/2} = \frac{16}{\pi d^3}(338.45)$$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \qquad \frac{16}{\pi d^3}\left(\frac{338.45}{210} + \frac{265.46}{700}\right) = \frac{1}{2} \qquad d_{min} = 27.3\,mm$$

$$d \geq d_{min}$$

$$\sigma_{x\,max} = \frac{32}{\pi d^3}(K_t M_m + K_f M_a) = 137.74\,MPa$$

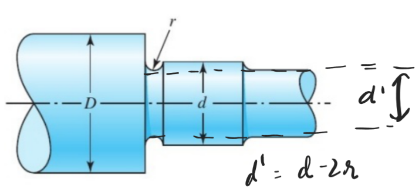
$$\tau_{xy\,max} = \frac{16}{\pi d^3}(K_{ts} T_m + K_{fs} T_a) = 36\,kPa$$

$$(\sigma_{x\,max}^2 + 3\tau_{xy\,max}^2)^{1/2} \approx \sigma_{x\,max} < \sigma_y \text{ (no yielding)}$$

2. The section of shaft shown in the figure is to be designed to approximate relative sizes of $d = 0.75D$ and $r = D/20$ with diameter d conforming to that of standard rolling-bearing bore sizes. The shaft is to be made of SAE 2340 steel, heat-treated to obtain minimum strengths in the shoulder area of 1200 MPa ultimate tensile strength and 1100 MPa yield strength with a Brinell hardness not less than 370. At the shoulder the shaft is subjected to a completely reversed bending moment of 67.800 Nmm, accompanied by a steady torsion of 45.200 Nmm. Use a facor of safety of 2.5 and size the shaft for an infinite life using the DE-Goodman criterion.

Problem 7–3

Section of a shaft containing a grinding-relief groove. Unless otherwise specified, the diameter at the root of the groove $d_r = d - 2r$, and though the section of diameter d is ground, the root of the groove is still a machined surface.



$d' = d - 2r$

$\sigma_{ult} = 1200\,MPa$ $\quad M_a = 67.8\,Nmm$ $\quad T_a = 0$

$\sigma_y = 1100\,MPa$ $\quad M_m = 0$ $\quad T_m = 45.2\,Nmm$

$\sigma_e = 600\,MPa$ $\quad \frac{d'}{D} = \frac{d}{D} - \frac{2r}{D} = 0.65$ $\quad \frac{r}{d'} = \frac{r}{D}\frac{D}{d'} = \frac{1}{20 \times 0.65} = 0.07$

For given d'/D, r/D, $K_t = 1.4$ $\quad K_{ts} = 1.2$

Let $q = q_{shear} = 1 \Rightarrow K_f = 1.4$ $\quad K_{fs} = 1.2$

$\sigma_{xa} = \frac{32 K_f M_a}{\pi d^3} = \frac{16}{\pi d^3}(0.189)$ $\quad \tau_{xya} = 0$

$\sigma_{xm} = 0$ $\quad \tau_{xym} = \frac{16 K_{fs} T_m}{\pi d^3} = \frac{16}{\pi d^3}(0.054)$

$$\sigma_a = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = \frac{16}{\pi d^3}(0.189)$$

$$\sigma_m = (\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2} = \frac{16}{\pi d^3}(0.094)$$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \Rightarrow \frac{16}{\pi d^3}\left(\frac{189}{600} + \frac{94}{1200}\right) = \frac{1}{2.5}$$

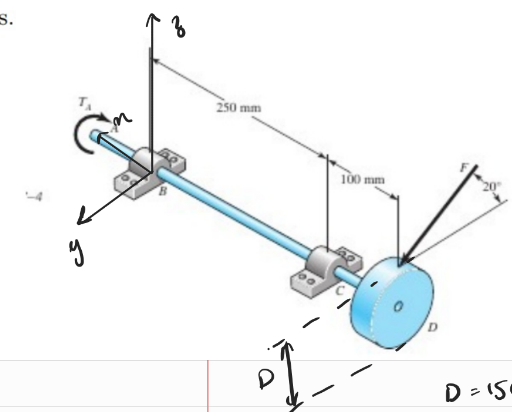
$\Rightarrow d'_{min} = 1.71\,mm$

$D_{min} = 1.12\,mm$

$\sigma_{x\,max} = \sigma_{xa}$ $\quad \tau_{xy\,max} = \tau_{xym}$

$(\sigma_{xa}^2 + 3\tau_{xym}^2)^{1/2} = 215\,MPa < \sigma_y$ no yielding

3. The rotating solid steel shaft is simply supported by bearings at points B and C and is driven by a gear (not shown) which meshes with the spur gear at D, which has a 150-mm pitch diameter. The force F from the drive gear acts at a pressure angle of 20°. The shaft transmits a torque to point A of TA = 340 Nm. The shaft is machined from steel with yield strength = 420 MPa and ultimate tensile strength = 560 MPa. Using a factor of safety of 2.5, determine the minimum allowable diameter of the 250-mm section of the shaft based on (a) a static yield analysis using the distortion energy theory and (b) a fatigue-failure analysis. Assume sharp fillet radii at the bearing shoulders for estimating stress-concentration factors.



$\sigma_y = 420\,MPa$

$\sigma_{ult} = 560\,MPa$ $\quad D = 150\,mm$

$\sigma_e = 280\,MPa$

At B $\exists$ R_{By} R_{Bz}

At C $\exists$ R_{Cy} R_{Cz}.

$\Sigma T_x = 0 \Rightarrow T_A = F\cos 20^\circ \frac{D}{2}$

$\boxed{F} = \frac{340 \times 2}{\cos 20^\circ \times 150 \times 10^{-3}} = \boxed{4.82\,kN}$

$\Sigma M_{By} = 0 \Rightarrow R_{Cz}(250) = F\sin 20^\circ (350)$

$\boxed{R_{Cz} = 2.31\,kN}$

$\Sigma M_{Bz} = 0 \Rightarrow R_{Cy}(250) + F\cos 20^\circ (350) = 0$

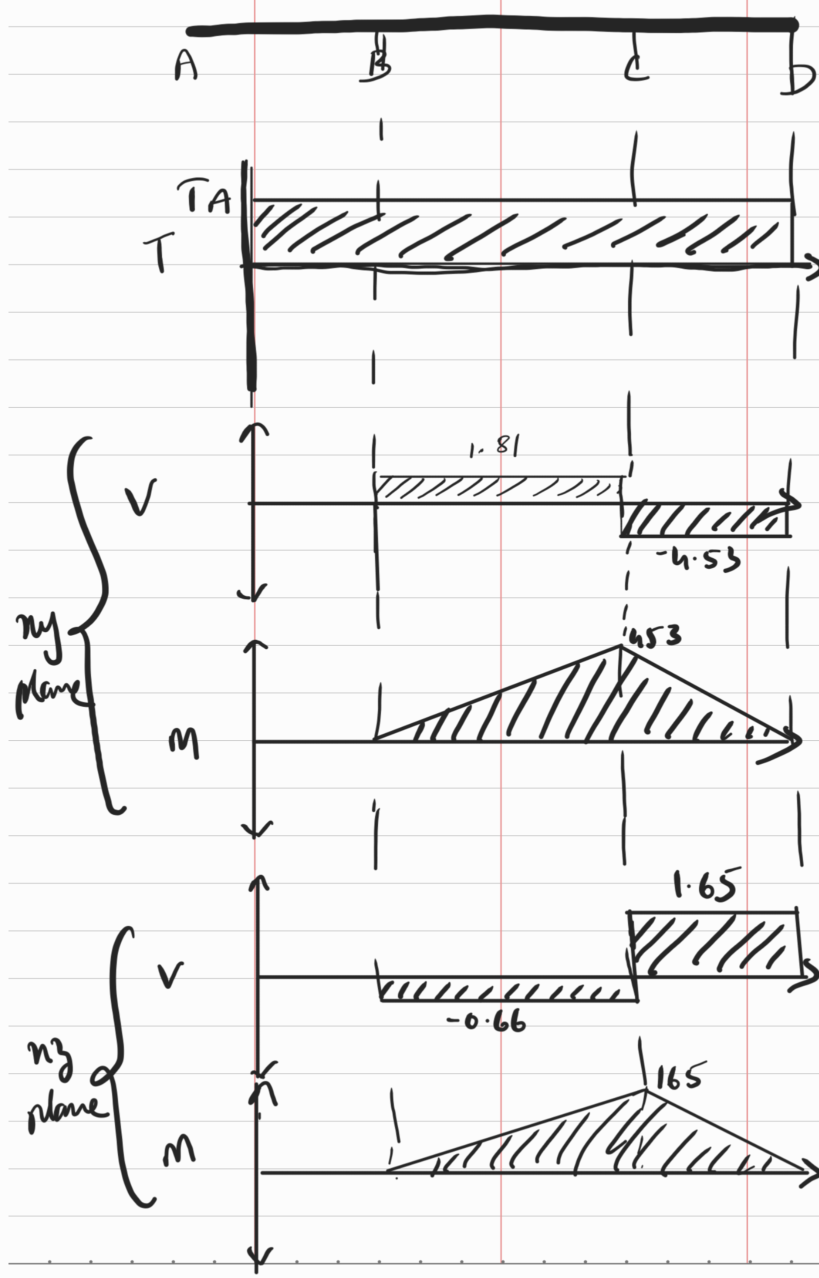
$\boxed{R_{Cy} = -6.34\,kN}$

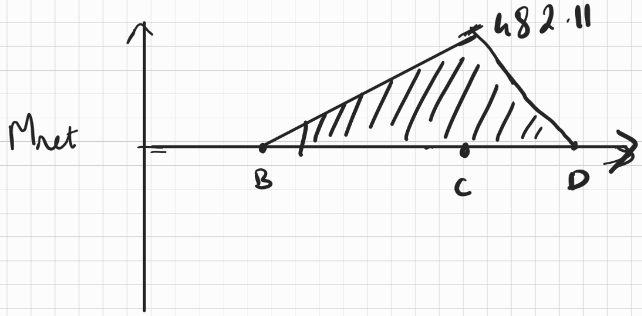
$\Sigma F_y = 0 \quad R_{By} = -R_{Cy} - F\cos 20^\circ$

$\Sigma F_z = 0 \quad R_{Bz} = -R_{Cz} + F\sin 20^\circ$

$\boxed{R_{By} = 1.81\,kN}$
$\boxed{R_{Bz} = -0.66\,kN}$

$R_{By} = 1.81\,kN$
$R_{Bz} = -0.66\,kN$
$R_{Cy} = 6.34\,kN$
$R_{Cz} = 2.31\,kN$





At C, $M_a = 482.11$ N m $\qquad T_a = 0$

$M_m = 0 \qquad T_m = 340$ Nm

For sharp fillet $\qquad$ Bending $\qquad$ Torsion

$K_t = 2.7 \qquad K_{ts} = 2.2$

Let $\quad K_f = 2.7 \qquad K_{fs} = 2.2$

$$\sigma_{nna} = \frac{16}{\pi d^3}(2K_f M_a) = \frac{16}{\pi d^3}(2603) \qquad \sigma_{nym} = \frac{16}{\pi d^3}(K_{ts} T_m) = \frac{16}{\pi d^3}(748)$$

(a) $\sigma_{nmax} = \frac{16}{\pi d^3}(2603) \qquad \sigma_{nymax} = \frac{16}{\pi d^3}(748)$

$$\left(\sigma_{nmax}^2 + 3\sigma_{nymax}^2\right)^{1/2} = \sigma_{eq} = \frac{16}{\pi d^3}(2908) < \frac{\sigma_y}{N}$$

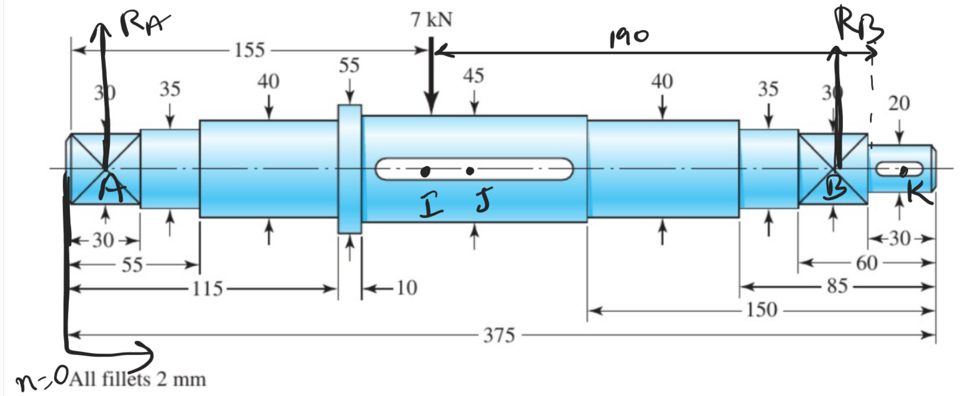
$$\Rightarrow d_{min} = 44.51\text{mm}$$

(b) $\sigma_a = \sigma_{nna} = \frac{16}{\pi d^3}(2603) \qquad \sigma_m = \sqrt{3}\sigma_{nym} = \frac{16}{\pi d^3}(1296)$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \Rightarrow \frac{16}{\pi d^3}\left(\frac{2603}{280} + \frac{1296}{560}\right) = \frac{1}{2.5}$$

$$d_{min} = 52.88 \text{ mm}$$

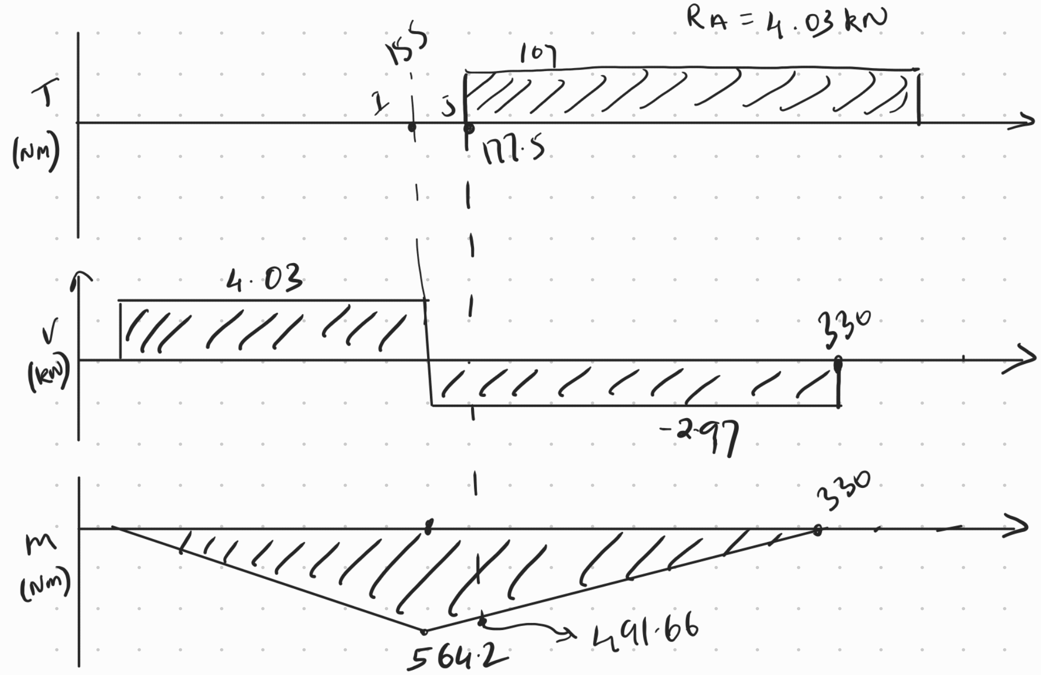
4. An AISI 1020 cold-drawn steel shaft with the geometry shown in the figure carries a transverse load of 7 kN and a torque of 107 Nm. Examine the shaft for strength and deflection. If the largest allowable slope at the bearings is 0.001 rad and at the gear mesh is 0.0005 rad, what is the factor of safety guarding against damaging distortion? What is the factor of safety guarding against a fatigue failure assuming the Goodman criterion? If the shaft turns out to be unsatisfactory, what would you recommend to correct the problem?



All fillets 2 mm

$\Sigma M = 0 \qquad 7(155-15) = R_B(375-45) \qquad R_B = 2.97$ kN

$R_A = 4.03$ kN



J seems to be a critical pt.

$M_a = 491.66$ Nm $\quad T_a = 0$

$M_m = 0 \quad T_m = 107$ Nm

For keyway, $K_f = 2.14$ bending $\quad K_t = 3$ torsion.

$$\sigma_{xxa} = \frac{32 K_f M_a}{\pi d^3} = 117.67 \text{ MPa}$$

$$\tau_{xym} = \frac{16 K_t T_m}{\pi d^3} = 17.95 \text{ MPa}$$

$\sigma_a = 117.67$ MPa

$\sigma_m = \sqrt{3}\, 17.95 = 31.09$ MPa

$\sigma_{ult} = 420$ MPa $\Rightarrow \sigma_e = 210$ MPa

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{117.67}{210} + \frac{31.09}{420} = 0.632 = \frac{1}{n}$$

$n = 1.58$ (FoS)

$\sigma_{xxmax} = 117.67$ MPa $\quad \tau_{xymax} = 17.95$ MPa $\quad \sigma_{eq} = 121.7$ MPa

$\sigma_{eq} < \sigma_y \Rightarrow$ no yielding.

Assuming pin joint at both bearing and roller at right (simply supported beam).

$$u = \frac{-Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

since I, J are almost in the middle they will have lesser slope than K.

slope @K $\leq 5\times10^{-4}$ rad.

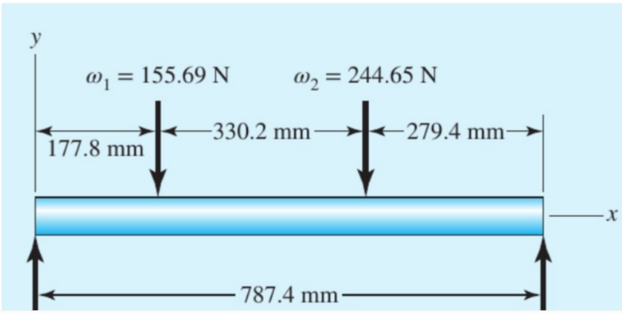
slope @ A $\leq 10^{-3}$ rad.

since shaft is almost symmetric, slope @ A ≈ B

slope @ K = slope @ B.

$\Rightarrow \boxed{\theta(0) \leq 5\times10^{-4}}$

5. Consider a simply supported steel shaft as depicted in with 25.4 mm diameter and a 787.4 mm span between bearings, carrying two gears weighing 155.69 and 244.65 N. Estimate the first critical speed in rpm using Rayleigh's method.



$$u = -\frac{Pun}{6EIL}(L^2 - u^2 - n^2)$$

$$u_{11} = -\frac{w_1(609.6)(177.8)}{6EIL}(787.4^2 - 609.6^2 - 177.8^2)$$

$$u_{12} = -\frac{w_1(609.6)(508)}{6EIL}(787.4^2 - 609.6^2 - 508^2)$$

$$u_{21} = -\frac{w_2(279.4)(177.8)}{6EIL}(787.4^2 - 279.4^2 - 177.8^2)$$

$$u_{22} = -\frac{w_2(279.4)(508)}{6EIL}(787.4^2 - 279.4^2 - 508^2)$$

$u_{11} = \frac{-3.66}{6EIL}$ $\quad u_{12} = \frac{0.47}{6EIL}$ $\quad u_{21} = \frac{-6.2}{6EIL}$ $\quad u_{22} = \frac{9.86}{6EIL}$

$u_1 = u_{11} + u_{21} = \frac{-9.86}{6EIL}$ $\quad u_2 = u_{12} + u_{22} = \frac{-9.39}{6EIL}$

$$\omega_1^2 = \frac{g\sum m_i \delta_i}{\sum m_i \delta_i^2} = \frac{g\sum w_i \delta_i}{\sum w_i \delta_i^2}$$

$$= \frac{6EILg\,(155.69(-9.86) + (244.65)(-9.39))}{155(9.86)^2 + 244.65(9.39)^2}$$

$$= \frac{6EILg\,(-3832.37)}{(36707)}$$

$$= 0.62\,EIgL$$

$E = 210\,GPa$ $\quad I = \frac{\pi d^3}{64}$ $\quad g = 10$ $\quad L = 787.4\,mm$

$$= \frac{\pi(25.4)^3}{64} = 804\,mm^3$$

$$\omega_1^2 = 210 \times 0.62 \times 804 \times 10 \times 787.4 \times 10^9$$

$$\boxed{\omega_1 = 9\,GHz}$$