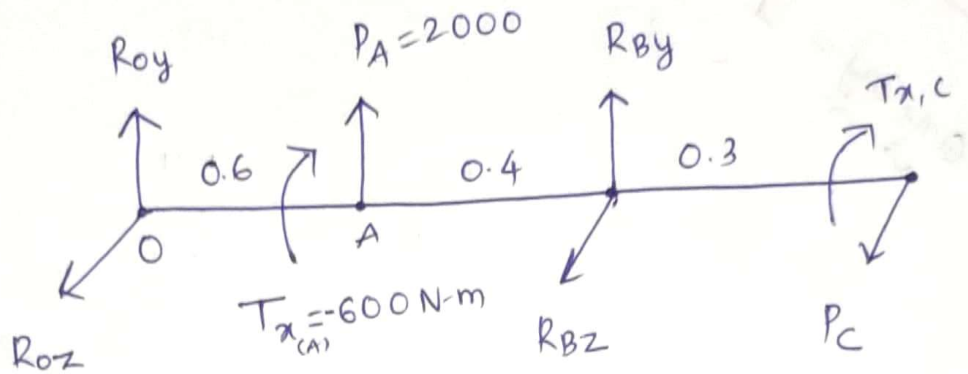
ME423 Mid Sem Q1 (Soln)

FBD of shaft:



Finding P_c:

As constant Torque is getting transmitted (1M)

$$\sum T_x = 0 \Rightarrow T_{xA} + T_{xB} = 0 \Rightarrow -600 + P_c \times 0.15 = 0$$

$$\Rightarrow P_c = 4000\,N$$

Finding All Reaction forces:

Force Balance (Y Axis) (1M)

$$\sum F_y = 0 \Rightarrow R_{oy} + 2000 + R_{By} = 0 \quad \text{— ①}$$

Moment Balance (Z axis at O)

from ① & ②

1200 N

Force Balance (z axis)

$$\sum F_z = 0 \Rightarrow R_{OZ} + R_{BZ} + 4000 = 0 \quad -③$$

Moment Balance (Y Axis w.r.t O):

$$\sum M_y\Big|_O = 0 \Rightarrow R_{BZ} \times 1 + P_C \times 1.2 = 0 \quad -④$$

from ③ & ④

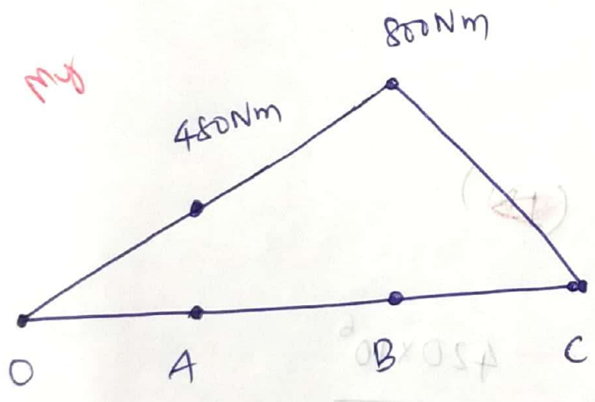
$R_{BZ} = -4800\,N$ (1M)

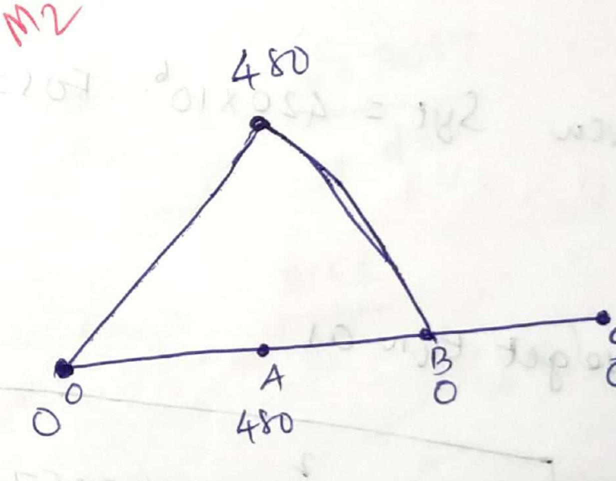
$R_{OZ} = 800\,N.$

(1M)

Finding critical section:

Drawing BMD's





Maximum Bending Moment,

$$M_A = \sqrt{480^2 + 480^2} = 678.82$$

$$M_B = \sqrt{800^2 + 0^2} = 800\,Nm \text{ (critical section)}.$$

(a)

Finding Equivalent σ & τ at critical section: (Static case)

(1M)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32 \times 800}{\pi d^3} = \frac{8152}{d^3}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 600}{\pi d^3} = \frac{3057}{d^3}$$

} if directly Using MFT formula check values if (MFT)

Using D.E.T for find Eq. stress (1M)

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{S_{yt}}{FoS}$$

given $S_{yt} = 420 \times 10^6$ $FoS = 2$

(~~1M~~)

So we get Eqn as

$$\sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{420 \times 10^6}{2}$$

Upon solving for d

we get $d = 35.9\,mm$ 1M

(b) Fatigue failure case:

we can get that Moment is Completely reversing and Torque

is constant at Critical section

so, $M_m = 0$ $M_a = M$ & $T_m = T$ and $T_a = 0$. (1M).

We get $\sigma_m = 0$ and $\sigma_a = \frac{8152}{d^3}$. (1M) 1M

and $\tau_m = \frac{3057}{d^3}$ and $\tau_a = 0$.

Using D.E.T we get (1M)

$$(\sigma_{eq})_m = \sqrt{(\sigma_m)^2 + 3(\tau_m)^2} = \sqrt{3}\,\tau_m = \frac{\sqrt{3}\,3057}{d^3}$$

$$(\sigma_{eq})_a = \sqrt{(\sigma_a)^2 + 3(\tau_a)^2} = \sigma_a = \frac{8152}{d^3}$$ (1M)

Using Goodman Eqn $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{FOS}$ ($S_e = 250 \times 10^6$ given)

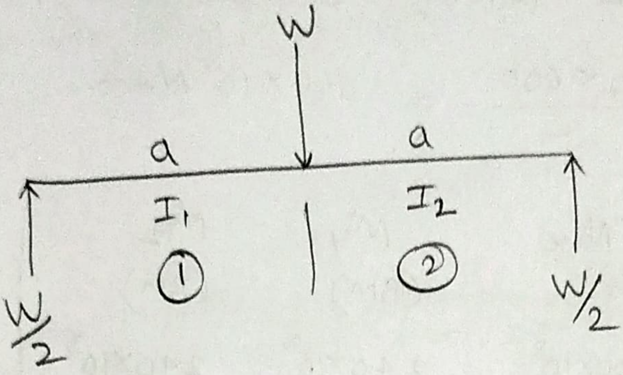
we get Expression as $\frac{8152}{250 \times 10^6 \times d^3} + \frac{\sqrt{3} \times 3057}{d^3 \times 560 \times 10^6} = \frac{1}{2}$.

$d = 43.8\ mm$ 1M

Marking Scheme

Finding Value of P_c	1M
Finding the Reaction Forces	1M
Identifying the Correct Critical Section	1M
Writing Proper Expressions for σ and τ using Correct Values of M and T and Writing Proper expression for Equivalent stress using D.E.T **(or)** Writing Direct Expression for Equivalent D.E.T Stress with Correct Values of T and M	1M+1M
Final Value of d (Diameter)	1M
Identifying that Bending Moment (BM) is Completely Reversed, Torsion is Constant, and Writing Values for M_m, M_a, T_m, T_a	1M
Writing Respective σ and τ, Using D.E.T to Calculate Equivalent Stresses, and Correct Expression for Goodman Equation (or Combined D.E.T and Goodman Equation) with Proper Substitution of M_m, M_a, T_m, T_a	1M + 1M
Final Value of d (Diameter)	1M
	Total Marks: 10

Q.2



$I_2 = 2I_1$

FBD + Reactions → 1

$U = U_1 + U_2$

$$U = \int_0^a \frac{M_{x_1}^2 \, dx}{2EI_1} + \int_a^{2a} \frac{M_{x_2}^2 \, dx}{2EI_2} \Bigg\} — 1$$

$$\frac{\partial U}{\partial W} = \int_0^a \frac{M_{x_1}}{EI_1} \frac{\partial M_{x_1}}{\partial W} dx + \int_a^{2a} \frac{M_{x_2}}{EI_2} \frac{\partial M_{x_2}}{\partial W} dx$$

$$M_{x_1} = \frac{W}{2} \times x \qquad \frac{\partial M_{x_1}}{\partial W} = \frac{x}{2} \quad — 1$$

$$M_{x_2} = \frac{W}{2}x - W(x-a) \qquad \frac{\partial M_{x_2}}{\partial W} = \frac{x}{2} - (x-a) \quad — 1$$

$$\delta = \frac{\partial U}{\partial W} = \int_0^a \frac{W\left(\frac{x}{2}\right)^2 dx}{EI_1} + \int_a^{2a} \frac{W\left(\frac{x}{2} - (x-a)\right)^2 dx}{EI_2} \quad — 1$$

$$\delta = \int_0^a \frac{Wx^2 \, dx}{4EI_1} + \int_a^{2a} \frac{W\left(-\frac{x}{2} + a\right)^2}{2EI_1} dx$$

$$\delta = \frac{W}{4EI_1}\left(\frac{x^3}{3}\right)_0^a + \frac{W}{2EI_1} \left.\frac{\left(-\frac{x}{2}+a\right)^3}{-3/2}\right|_a^{2a}$$

$$\delta = \frac{Wa^3}{12EI_1} + \frac{W}{EI_1}\left[0 + \frac{a^3}{8 \times 3}\right) = \frac{Wa^3}{12EI_1} + \frac{Wa^3}{24EI_1}$$

$$\boxed{\delta = \frac{Wa^3}{8EI_1}} \quad 2$$

$$W_c = \frac{2\pi \times 9000}{60} \text{ rad/s.} \longrightarrow 1$$

300π , 942.47

$$W = 50 \times 10 = 500 N$$

$$\left.\begin{aligned} W_c &= \sqrt{g/\delta} \\ \left(\frac{2\pi \times 9000}{60}\right)^2 &= \frac{10 \times 10^4}{\delta} \end{aligned}\right\} 1$$

$$\Rightarrow \delta = 0.0113 \text{ mm.}$$

$$\delta = \frac{Wa^3}{8EI_1} \qquad a = 300 \text{mm}, \quad E = 200 \text{ GPa},$$

$$\Rightarrow I_1 = 7.4947 \times 10^5 \text{ mm}^4$$

$$I_1 = \frac{\pi}{64} d_1^4$$

$$\Rightarrow \boxed{d_1 = 62.5 \text{ mm}} \text{ — } 1$$

3) a) Objective minimize $M = 2\pi r t L \rho$

s.t $T = \pi r^2 t \sigma_y$

$N_{cr} \geq N_{cr}^*$ +1

b) We have $f = c' \sqrt{\frac{k}{m}}$

For a shaft $N_{cr} = c'' \sqrt{\frac{k_{shaft}}{m_{shaft}}}$

We have $\delta = \tilde{c} \frac{PL^3}{EI}$ → deflection of a beam / or shaft

$\therefore k_{shaft} = \frac{P}{\delta} = \tilde{c} \frac{EI}{L^3}$. Mass of the shaft $m_{shaft} = M$.

$N_{cr} = c' \sqrt{\frac{\tilde{c} EI}{ML^3}} = c \sqrt{\frac{EI}{ML^3}}$ +3

c) $N \geq N_{cr}^*$

or $N^2 \geq N_{cr}^{*2}$

$\frac{c^2 EI}{ML^3} \geq N_{cr}^{*2}$

Now $I = \pi r^3 t = \frac{\pi r^4 t^2}{rt}$

$= \frac{2T^2 L \rho}{M \sigma_y^2}$

$\frac{c^2 E}{ML^3} \cdot \frac{2T^2 L \rho}{M \sigma_y^2} \geq N_{cr}^{*2}$ +3

$M^2 \leq 2c^2 \frac{T^2}{N_{cr}^{*2}} \frac{1}{L^2} \frac{E\rho}{\sigma_y^2}$

or $M \leq \sqrt{2}\, c \frac{T}{N_{cr}^*} \cdot \frac{1}{L} \underbrace{\frac{(E\rho)^{1/2}}{\sigma_y}}_{\text{material index}}$.

For a fixed T, N_{cr}^*, L to minimize M we need to choose a material with as low +1 of $\frac{(E\rho)^{1/2}}{\sigma_y}$ +1 as possible.

For the given materials

	MI.
Mild steel	0.18
High strength steel	0.10
Al alloy.	0.05 ✓ Lowest MI

chosen material Al alloy.

+1 if the material index is correct

4) $\sigma_{ult} = 1000$ MPa, $\sigma_y = 800$ MPa, $\sigma_e' = 500$ MPa. $k_a = 0.679$, $k_b = 1$.

$T_m = 45$ Nm, $M_a = 70$ Nm

$d_r = d - 2r = 0.65D$

$d = 0.8D$.

+7 - for iteration 1. I have shown one way to solve the problem. Another way is to assume dr and then check the FOS.

$$\frac{d_r}{d} = \frac{0.65}{0.8}$$

$$\therefore \frac{d_r}{d} = 1 - \frac{2r}{d}.$$

$$\therefore \frac{r}{d} = \frac{1}{2}\left(1 - \frac{0.65}{0.8}\right) = 0.094 \approx 0.1.$$

$K_t = 1.7$ $\quad K_{ts} = 1.5$.

$\therefore$ For the first iteration we will assume that $\underline{\underline{q = 1}}$ (To know q in addition to knowing σ_{ult} & r/d, we also need r.
$\therefore [k_f = K_t$ & $k_{fs} = K_{ts}]$.

The minimum diameter is d_r and that is the most critical regions as compared to the locations shown in the figure.

Now $\sigma_e = \underbrace{k_a k_b k_c k_d k_e k_f}_{1} \sigma_e' = 339.5$ MPa.

Also: $\sigma_a = k_f \dfrac{32 M_a}{\pi d_r^3}$, $\quad \tau_m = k_{fs} \dfrac{16 T_m}{\pi d_r^3}$.

in these formulae, the diameter to be chosen is the smallest diameter, i.e. dr

$$\sigma_{eq,a} = \frac{32 k_f M_a}{\pi d_r^3}, \quad \sigma_{eqm} = \frac{\sqrt{3} \cdot 16 \cdot k_{fs} T_m}{\pi d_r^3}.$$

We have $\dfrac{\sigma_{eqa}}{\sigma_e} + \dfrac{\sigma_{eqm}}{\sigma_{ult}} = \dfrac{1}{FOS}$.

Solving for d_r; $d_r = 20.27$ mm., $D = 31.19$ mm
$d = 24.95$ mm, $r = 2.34$ mm.

For the second iteration, we calculate k_b using d_r & q using d & $\frac{r}{d}$. We find FOS & if it is close to 2 we accept the values of d_r, D, d & r. Else using FOS, k_b, & q we calculate new d_r & repeat the above procedure. [In our case FOS = 1.9] +3

If we calculate d_r, then $d_r = 20.62$ mm. To be on the safe we can choose $d_r = 20.62$ mm $\approx$ 21.0 mm.