ME423 HW 4 21/09/2024

1. A shaft is loaded in bending and torsion such that $M_a = 70$ Nm, $T_a = 45$ Nm, $M_m = 55$ Nm, and $T_m = 35$ Nm. For the shaft, ultimate strength = 700 MPa, yield strength = 560 MPa, and a fully corrected endurance limit of = 210 MPa is assumed. Let $K_f = 2.2$ and $Kfs = 1.8$. For a factor of safety of 2.0 determine the minimum acceptable diameter of the shaft. Use the Goodman criterion. Clearly mention any assumptions that you make. Assuming $q = q_s = 1 \Rightarrow K_t = 2.2 \quad K_{ts} = 1.8$

$M_a = 70\,Nm \qquad T_a = 45\,Nm$

$M_m = 55\,Nm \qquad T_m = 35\,Nm$

$\sigma_{ult} = 700\,MPa$

$\sigma_y = 560\,MPa$

$\sigma_e = 210\,MPa$

$K_f = 2.2 \qquad K_{fs} = 1.8 \qquad n = 2$

$\sigma_{xx} = \frac{My}{I} = \frac{32M}{\pi d^3} \qquad \sigma_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$

$$\sigma_m = \left(4K_f^2\sigma_{xxm}^2 + 3K_{fs}^2\sigma_{xym}^2\right)^{1/2} = \frac{16}{\pi d^3}\left(4K_f^2M_m^2 + 3K_{fs}^2T_m^2\right)^{1/2}$$

$$= \frac{16}{\pi d^3}\left(242^2 + 3(63)^2\right)^{1/2} = \frac{16}{\pi d^3}(265.46)$$

$$\sigma_a = \frac{16}{\pi d^3}\left(4K_f^2M_a^2 + 3K_{fs}^2T_a^2\right)^{1/2} = \frac{16}{\pi d^3}\left(308^2 + 3(81)^2\right)^{1/2} = \frac{16}{\pi d^3}(338.45)$$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \qquad \frac{16}{\pi d^3}\left(\frac{338.45}{210} + \frac{265.46}{700}\right) = \frac{1}{2} \qquad d_{min} = 27.3\,mm$$

$d \geq d_{min}$

$$\sigma_{xxmax} = \frac{32}{\pi d^3}(K_t M_m + K_f M_a) = 137.74\,\text{MPa}$$

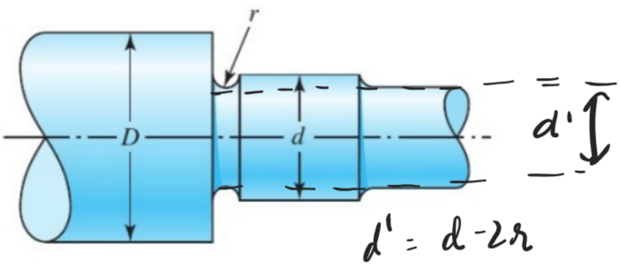
$$\sigma_{xymax} = \frac{16}{\pi d^3}(K_{ts} T_m + K_{fs} T_a) = 36\,\text{kPa}$$

$$(\sigma_{xxmax}^2 + 3\sigma_{xymax}^2)^{1/2} \approx \sigma_{xxmax} < \sigma_y \text{ (no yielding)}$$

2. The section of shaft shown in the figure is to be designed to approximate relative sizes of $d = 0.75D$ and $r = D/20$ with diameter d conforming to that of standard rolling-bearing bore sizes. The shaft is to be made of SAE 2340 steel, heat-treated to obtain minimum strengths in the shoulder area of 1200 MPa ultimate tensile strength and 1100 MPa yield strength with a Brinell hardness not less than 370. At the shoulder the shaft is subjected to a completely reversed bending moment of 67.800 Nmm, accompanied by a steady torsion of 45.200 Nmm. Use a facor of safety of 2.5 and size the shaft for an infinite life using the DE-Goodman criterion.

Problem 7–3

Section of a shaft containing a grinding-relief groove. Unless otherwise specified, the diameter at the root of the groove $d_r = d - 2r$, and though the section of diameter d is ground, the root of the groove is still a machined surface.



$d' = d - 2r$

$\sigma_{ult} = 1200\,\text{MPa}$ $\quad M_a = 67.8\,\text{Nmm}$ $\quad T_a = 0$

$\sigma_y = 1100\,\text{MPa}$ $\quad M_m = 0$ $\quad T_m = 45.2\,\text{Nmm}$

$\sigma_e = 600\,\text{MPa}$ $\quad \frac{d'}{D} = \frac{d}{D} - \frac{2r}{D} = 0.65$ $\quad \frac{r}{d'} = \frac{r}{D}\frac{D}{d'} = \frac{1}{20 \times 0.65} = 0.07$

For given d'/D, r/D, $\quad K_t = 1.4 \quad K_{ts} = 1.2$

Let $q = q_{shear} = 1 \Rightarrow K_f = 1.4 \quad K_{fs} = 1.2$

$$\sigma_{xxa} = \frac{32 K_f M_a}{\pi d^3} = \frac{16}{\pi d^3}(0.189) \qquad \sigma_{xya} = 0$$

$$\sigma_{xxm} = 0 \qquad \sigma_{xym} = \frac{16 K_{fs} T_m}{\pi d^3} = \frac{16}{\pi d^3}(0.054)$$

$$\sigma_a = \left(\sigma_{xma}^2 + 3\sigma_{xya}^2\right)^{1/2} = \frac{16}{\pi d^3}(0.189)$$

$$\sigma_m = \left(\sigma_{xmm}^2 + 3\sigma_{xym}^2\right)^{1/2} = \frac{16}{\pi d^3}(0.094)$$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \Rightarrow \frac{16}{\pi d^3}\left(\frac{189}{600} + \frac{94}{1200}\right) = \frac{1}{2.5}$$

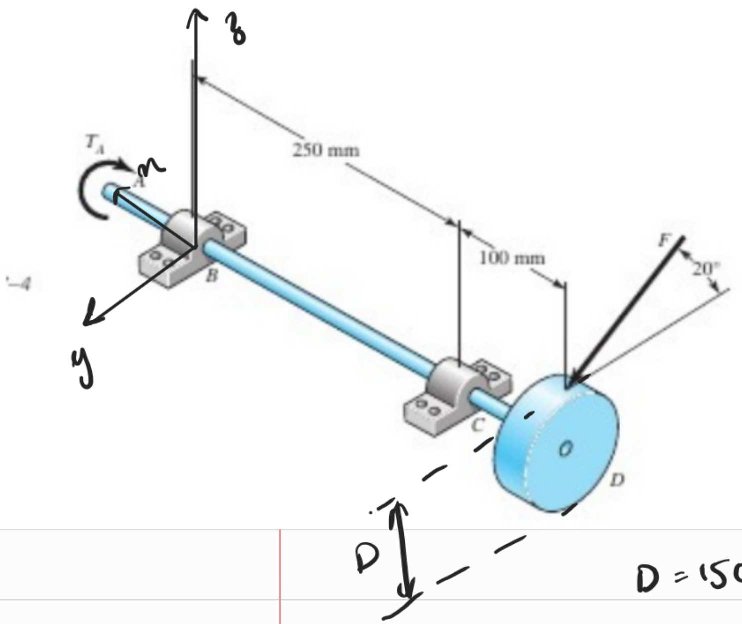
$$\Rightarrow d'_{min} = 1.71\,mm$$

$$D_{min} = 1.12mm$$

$$\sigma_{xmax} = \sigma_{xma} \qquad \sigma_{xymax} = \sigma_{xym}$$

$$\left(\sigma_{xma}^2 + 3\sigma_{xym}^2\right)^{1/2} = 215MPa < \sigma_y \text{ no yielding}$$

3. The rotating solid steel shaft is simply supported by bearings at points B and C and is driven by a gear (not shown) which meshes with the spur gear at D, which has a 150-mm pitch diameter. The force F from the drive gear acts at a pressure angle of 20°. The shaft transmits a torque to point A of TA = 340 Nm. The shaft is machined from steel with yield strength = 420 MPa and ultimate tensile strength = 560 MPa. Using a factor of safety of 2.5, determine the minimum allowable diameter of the 250-mm section of the shaft based on (a) a static yield analysis using the distortion energy theory and (b) a fatigue-failure analysis. Assume sharp fillet radii at the bearing shoulders for estimating stress-concentration factors.



$\sigma_y = 420\,MPa$

$\sigma_{ult} = 560\,MPa$

$D = 150mm$

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$\sigma_e = 280 MPa$

At B $\exists$ R_{By} R_{Bz}

At C $\exists$ R_{Cy} R_{Cz}.

$\Sigma T_x = 0 \Rightarrow T_A = F\cos 20° \frac{D}{2}$

$F = \frac{340 \times 2}{\cos 20° \times 150 \times 10^{-3}} = 4.82 kN$

$\Sigma M_{By} = 0 \Rightarrow R_{Cz}(250) = F \sin 20° (350)$

$R_{Cz} = 2.31 kN$

$\Sigma M_{Bz} = 0 \Rightarrow R_{Cy}(250) + F\cos 20° (350) = 0$

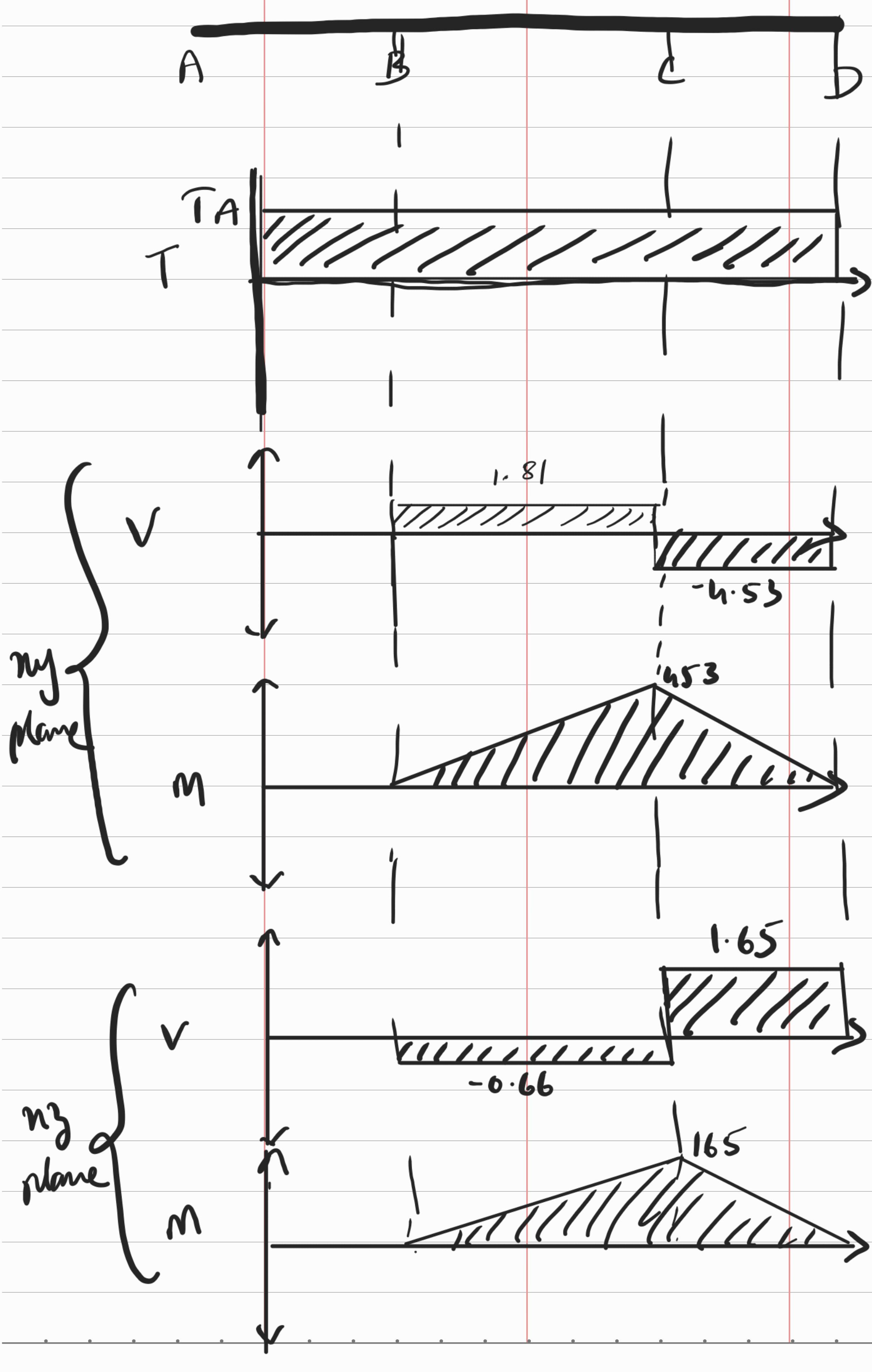
$R_{Cy} = -6.34 kN$

$\Sigma F_y = 0$ $R_{By} = -R_{Cy} - F\cos 20°$

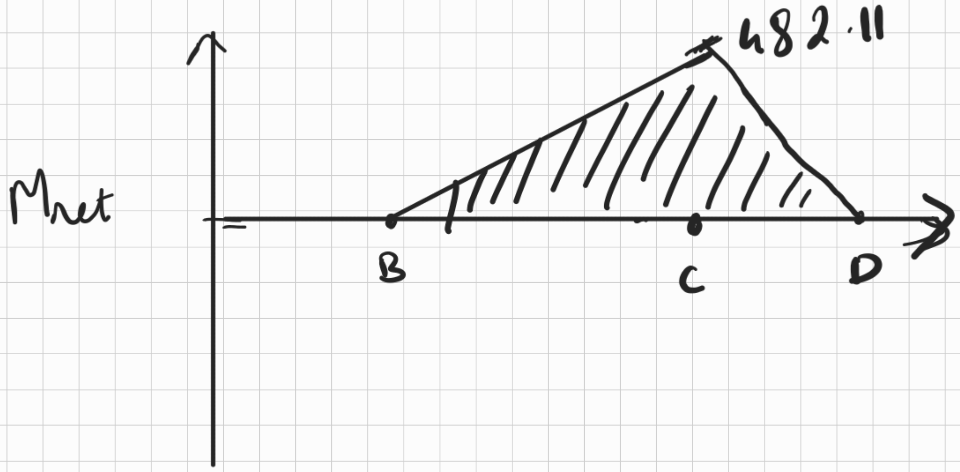
$\Sigma F_z = 0$ $R_{Bz} = -R_{Cz} + F\sin 20°$

$R_{By} = 1.81 kN$
$R_{Bz} = -0.66 kN$

$R_{By} = 1.81 kN$
$R_{Bz} = -0.66 kN$
$R_{Cy} = 6.34 kN$
$R_{Cz} = 2.31 kN$


A
B
C
D
TA
T
V
1.81
-4.53
xy
plane
453
M
1.65
V
-0.66
xz
plane
165
M

/ /



At C, $M_a = 482.11$ N m $\quad T_a = 0$

$M_m = 0 \quad T_m = 340$ Nm

For sharp fillet Bending Torsion

$K_t = 2.7 \quad K_{ts} = 2.2$

Let $K_f = 2.7 \quad K_{fs} = 2.2$

$$\sigma_{xxa} = \frac{16}{\pi d^3}(2K_f M_a) = \frac{16}{\pi d^3}(2603) \qquad \tau_{xym} = \frac{16}{\pi d^3}(K_{ts} T_m) = \frac{16}{\pi d^3}(748)$$

(a) $\sigma_{xxmax} = \frac{16}{\pi d^3}(2603) \qquad \tau_{xymax} = \frac{16}{\pi d^3}(748)$

$$\left(\sigma_{xxmax}^2 + 3\,\tau_{xymax}^2\right)^{1/2} = \sigma_{eq} = \frac{16}{\pi d^3}(2908) < \frac{\sigma_y}{N}$$

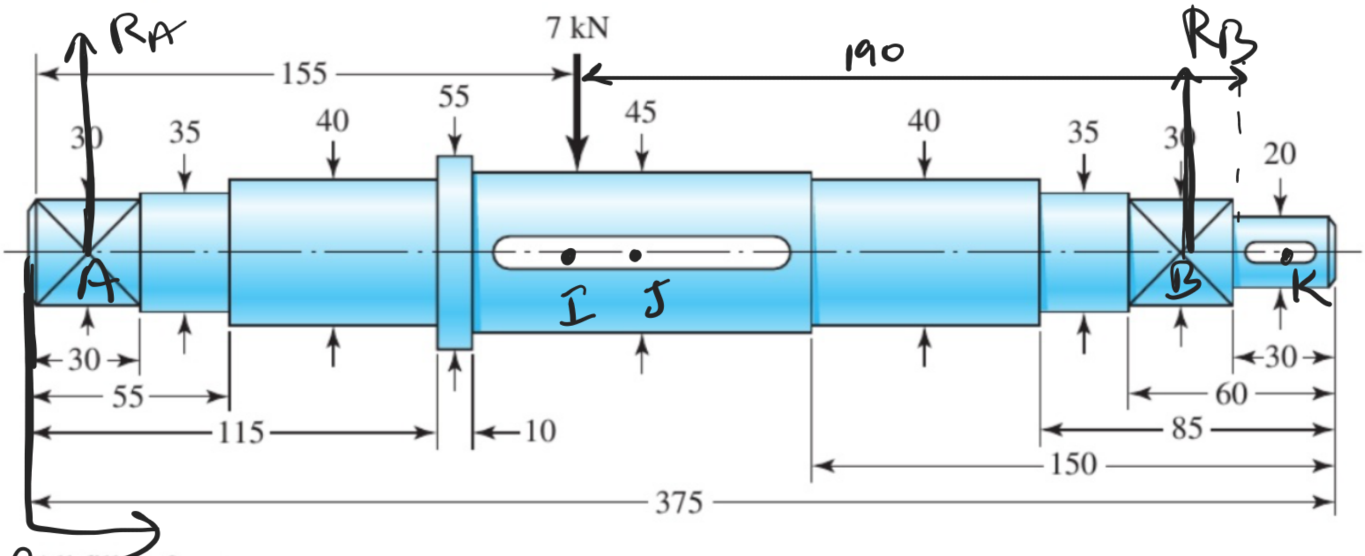
$\Rightarrow d_{min} = 44.51$ mm

(b) $\sigma_a = \sigma_{xxa} = \frac{16}{\pi d^3}(2603) \qquad \sigma_m = \sqrt{3}\,\tau_{xym} = \frac{16}{\pi d^3}(1296)$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n} \Rightarrow \frac{16}{\pi d^3}\left(\frac{2603}{280} + \frac{1296}{560}\right) = \frac{1}{2.5}$$

$d_{min} = 52.88$ mm

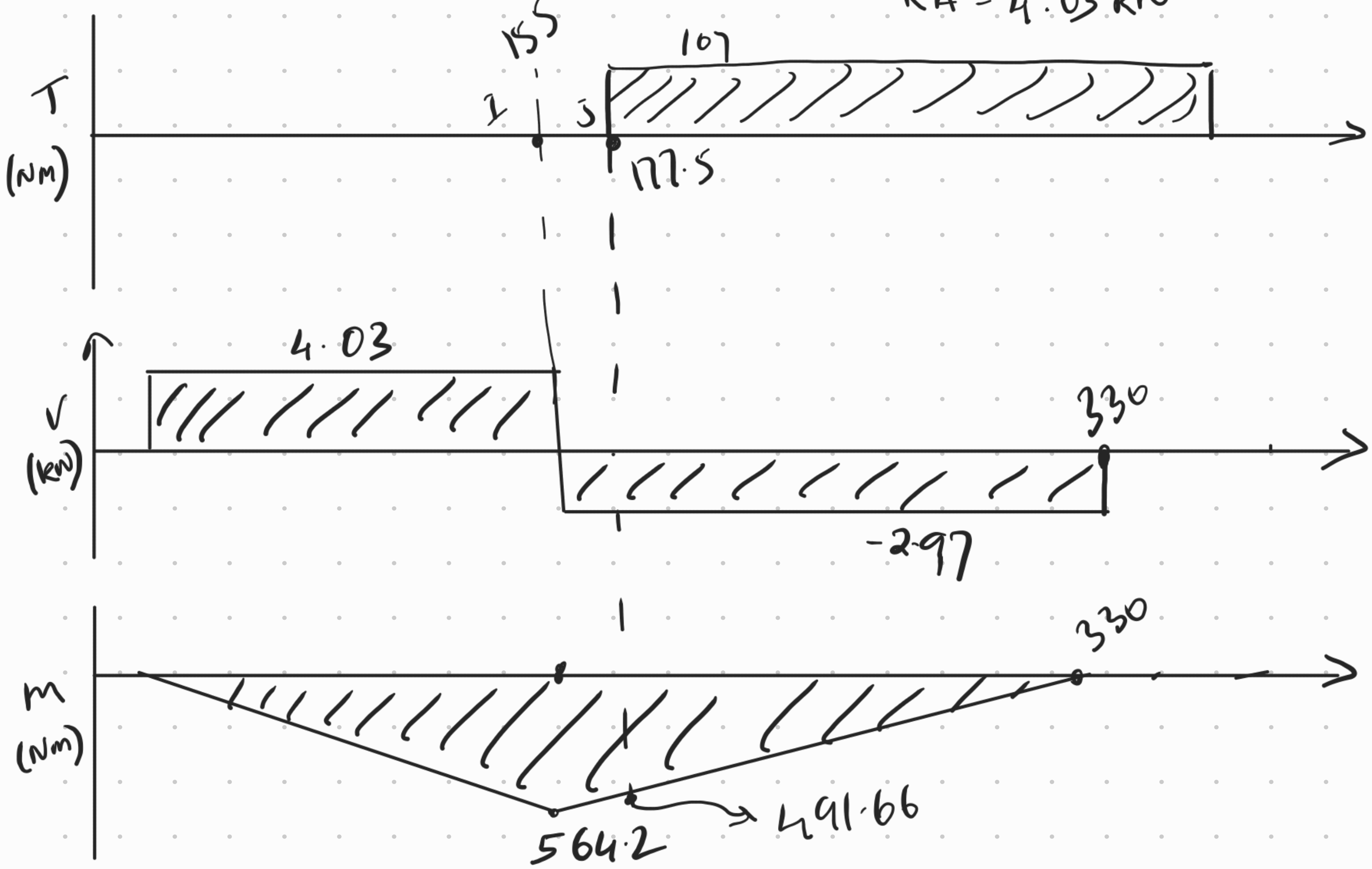
4. An AISI 1020 cold-drawn steel shaft with the geometry shown in the figure carries a transverse load of 7 kN and a torque of 107 Nm. Examine the shaft for strength and deflection. If the largest allowable slope at the bearings is 0.001 rad and at the gear mesh is 0.0005 rad, what is the factor of safety guarding against damaging distortion? What is the factor of safety guarding against a fatigue failure assuming the Goodman criterion? If the shaft turns out to be unsatisfactory, what would you recommend to correct the problem?



n=0 All fillets 2 mm

$\Sigma M = 0 \qquad 7(155-15) = R_B(375-45) \qquad R_B = 2.97\ kN$

$R_A = 4.03\ kN$



J seems to be a critical pt.

$M_a = 491.66\ Nm$ $T_a = 0$
$M_m = 0$ $T_m = 107\,Nm$

For keyway, $K_f = 2.14$ bending $K_t = 3$ torsion.

$$\sigma_{xxa} = \frac{32 K_f M_a}{\pi d^3} = 117.67\ MPa$$

$$\tau_{xym} = \frac{16 K_t T_m}{\pi d^3} = 17.95\ MPa$$

$\sigma_a = 117.67\,MPa$
$\sigma_m = \sqrt{3}\,17.95 = 31.09\,MPa$

$\sigma_{ult} = 420\,MPa \Rightarrow \sigma_e = 210\,MPa$

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{117.67}{210} + \frac{31.09}{420} = 0.632 = \frac{1}{n}$$

$$n = 1.58 \text{ (FoS)}$$

$\sigma_{xxmax} = 117.67\,MPa$ $\tau_{xymax} = 17.95\,MPa$ $\sigma_{eq} = 121.7\,MPa$

$\sigma_{eq} < \sigma_y \Rightarrow$ no yielding.

Assuming pin joint at both bearing and roller at right (simply supported beam).

$$u = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$$

Since I, J are almost in the middle they will have lesser slope than K.

slope @ K $\leq 5 \times 10^{-4}$ rad.

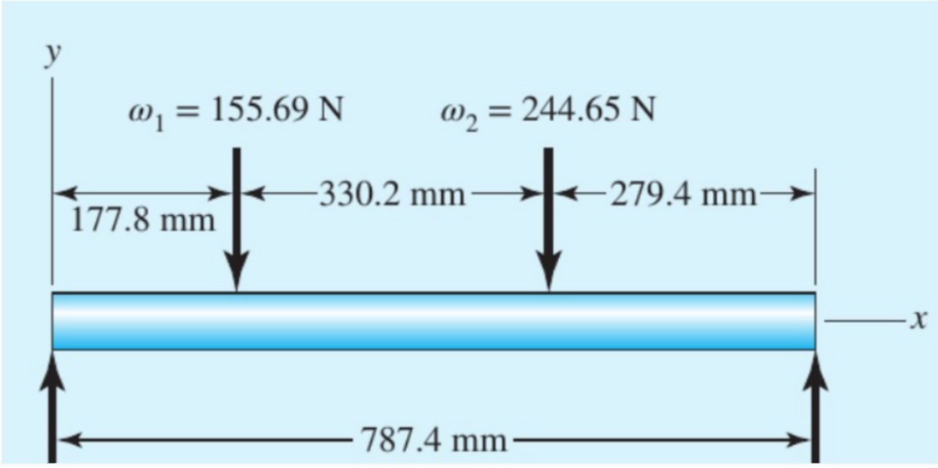
slope @ A $\leq 10^{-3}$ rad.

Since shaft is almost symmetric, slope @ A ≃ B

slope @ K = slope @ B.

$\Rightarrow \boxed{\theta(0) \leq 5 \times 10^{-4}}$

5. Consider a simply supported steel shaft as depicted in with 25.4 mm diameter and a 787.4 mm span between bearings, carrying two gears weighing 155.69 and 244.65 N. Estimate the first critical speed in rpm using Rayleigh's method.



$$u = -\frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$$

$$u_{11} = -\frac{w_1 (609.6)(177.8)}{6EIL}(787.4^2 - 609.6^2 - 177.8^2)$$

$$u_{12} = -\frac{w_1 (609.6)(508)}{6EIL}(787.4^2 - 609.6^2 - 508^2)$$

$$u_{21} = -\frac{w_2 (279.4)(177.8)}{6EIL}(787.4^2 - 279.4^2 - 177.8^2)$$

$$u_{22} = -\frac{w_2 (279.4)(508)}{6EIL}(787.4^2 - 279.4^2 - 508^2)$$

$u_{11} = \frac{-3.66}{6EIL}$ $\quad u_{12} = \frac{0.47}{6EIL}$ $\quad u_{21} = \frac{-6.2}{6EIL}$ $\quad u_{22} = \frac{9.86}{6EIL}$

$u_1 = u_{11} + u_{21} = -\frac{9.86}{6EIL}$ $\quad u_2 = u_{12} + u_{22} = \frac{-9.39}{6EIL}$

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$$\omega_1^2 = \frac{g\sum m_i \delta_i}{\sum m_i \delta_i^2} = \frac{g\sum w_i \delta_i}{\sum w_i \delta_i^2}$$

$$= \frac{6EILg\,(155.69(-9.86) + (244.65)(-9.39))}{155(9.86)^2 + 244.65\,(9.39)^2}$$

$$= \frac{6EILg\,(-3832.37)}{(36707)}$$

$$= 0.62\,EIgL$$

$E = 210\,GPa$ $\quad I = \frac{\pi d^3}{64}$ $\quad g = 10$ $\quad L = 787.4\,mm$

$$= \frac{\pi (25.4)^3}{64} = 804\,mm^3$$

$$\omega_1^2 = 210 \times 0.62 \times 804 \times 10 \times 787.4 \times 10^9$$

$$\boxed{\omega_1 = 9\,GHz}$$