Practice Problem Set 1

IE 708 Markov Decision Processes
IEOR, IIT Bombay

Autumn '24

1. Find the average revenue of both the systems: System 1 and System 2, each modelled as a 2-state MC.

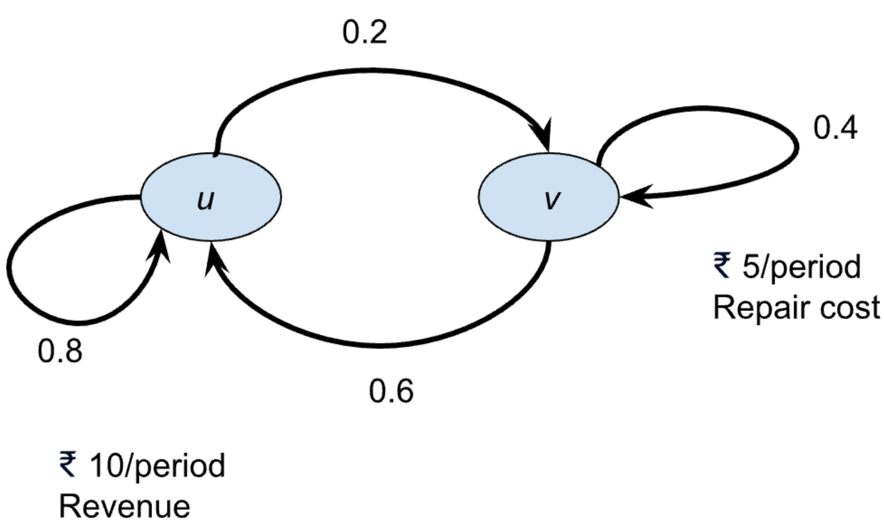

Figure 1: System 1

2. Evaluate both systems for their expected total discounted cost.
   - Take discount factor $\lambda = 0.7$
   - Take discount factor $\lambda = 0.9$
3. Write your observations about the optimal policies for discounted and average criteria.
4. Write the conditions for existence of this stationary distribution $\pi = [\pi_1, \cdots, \pi_{|\mathcal{S}|}]$; here $\mathcal{S}$ is the state space of a finite state MC.

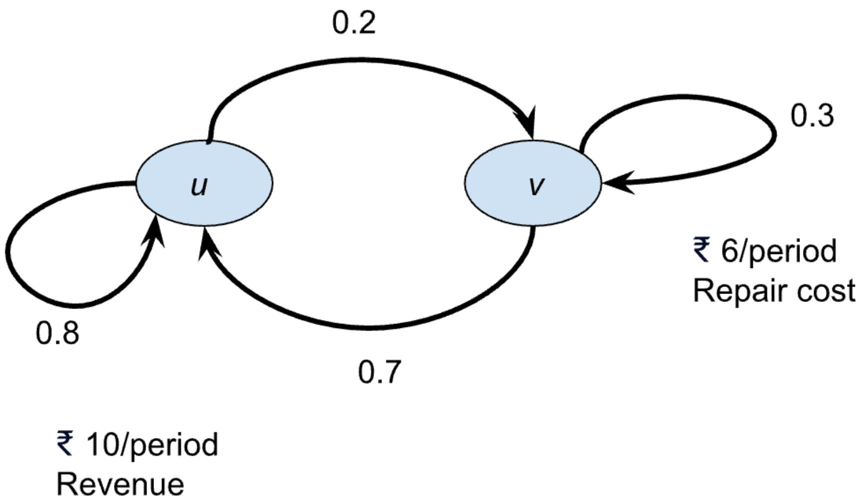

Figure 2: System 2

5. Write the system of linear equations for the stationary distribution of a (finite state) MC.

   Recall this system of linear equations, has unique solution, under mild conditions which means that it characterises the stationary distribution of MC.

6. Write at least 2 more interpretations of $\pi_i, \in \mathcal{S}$.

7. Write one more interpretation for $\{\pi_i\}_1^{|\mathcal{S}|}$, along with conditions needed, if any.

8. Now, consider a countable state MC. Write one new phenomenon for these countable state MCs, that is not present for finite state MCs. *Hint:* Consider a Markovian queue with arrival rate same as the service rate.

9. A firm wants to announce a one time discount offer for a single product in a price sensitive market. Model this as a MDP, i.e., list the elements of the 5 tuple that define a MDP (state space, action space, reward functions, transition probabilities and the planning horizon).

10. To solve a shortest past problem (given in Figure 3), the straight forward way is to write out all paths and their costs and then find the minimum of them. However, this is a combinatorial problem and the solution space explodes as the number of links and arcs increases. Listing all of them is difficult and then one can use merge sort or quick sort type algorithm for finding the minima. Use a dynamic programming, DP, recursion to

find the optimal path. Convince yourself that this far easier than explicit enumeration/search.

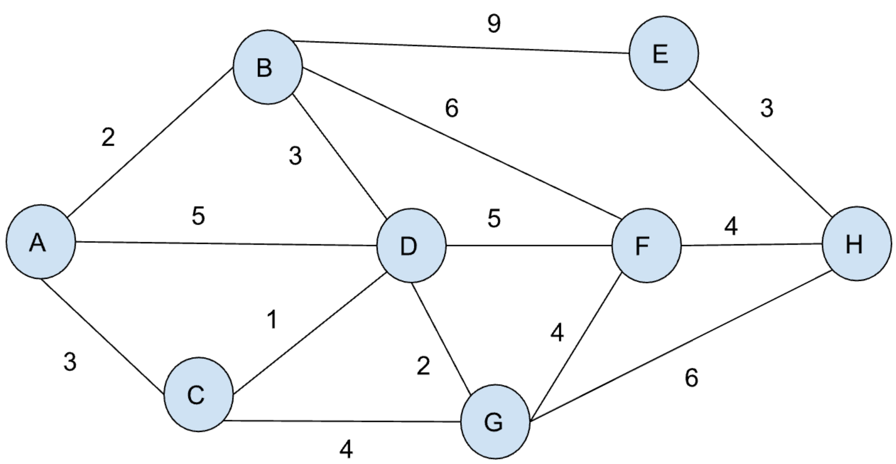

Figure 3: An instance of SSP (shortest path problem)

11. Using the above, find the optimal path between B and H.

12. What is the minimal cost between C and H? Note that while the optimal cost/value is unique, the optimal path is not.

13. Which of the following functions are superadditive, subadditive or neither?

    (a) e^{-x}

    (b) $\frac{1}{x}$ for $x > 0$

    (c) $\sqrt{x}$ for $x > 0$

    (d) e^x for $x > 1$

14. Show that the sum of superadditive functions is superadditive. Is the product also superadditive? (Pbm 4.14 Martin Puterman)

15. Show that a twice differentiable function $g(x, y)$ on $R^1 \times R^1$ is superadditive (subadditive) whenever its second mixed partial derivatives are non-negative (non-positive). (Pbm 4.12 Martin Puterman)