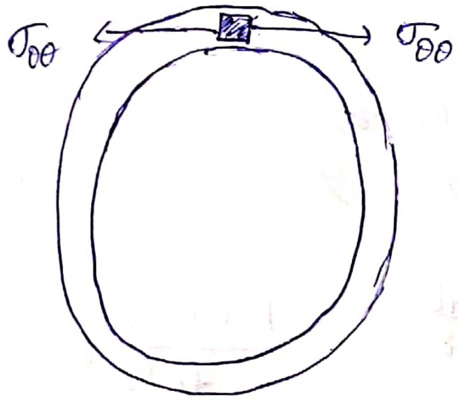
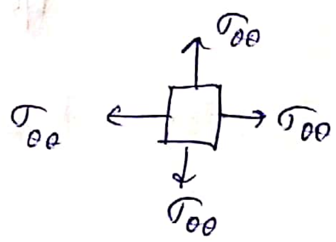
QUIZ-1

PRATMIK REDDY
17D100022

(1) cut the sphere diametrically using a plane

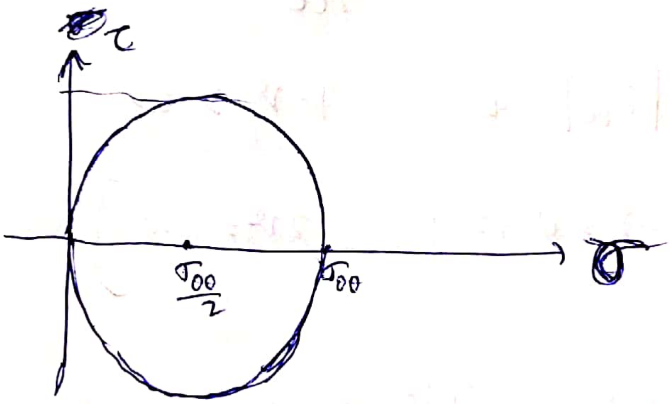


Top view:-



there is no r-r component of stress

$$\sigma_{\theta\theta} \times 2\pi r t = \pi r^2 \times p \Rightarrow \sigma_{\theta\theta} = \frac{Pr}{2t}$$



Max Tensile stress $= \sigma_{\theta\theta}$

Max shear stress $= \frac{\sigma_{\theta\theta}}{2}$

↳ In 45° plane.

Given Yield values :- In tension = 140,000 psi

In shear = 65,000 Psi

~~If it yields first :-~~

a) Shear yield stress is less than half of Tensile yield stress. But ~~the~~ Max. Tensile stress is twice the Max shear stress.

⇒ Shear yield will happen first if the failure happens by yield.

If it happens by yield, :-

$$\frac{Pr}{4t} = \left(\frac{\text{Shear yield stress}}{\text{FOS}}\right)$$

$$\Rightarrow \quad P_1 = \frac{65000}{2.75} \times \frac{4t}{r} = \frac{130000}{11} t$$

If the normal Strain limit reaches first :- Strain in rr :- $\frac{-Pr}{2tE} \times 2\nu$

Strain in $\theta\theta$:- $\frac{Pr}{2tE}(1-\nu)$

$$\Rightarrow \quad |\varepsilon_{rr}| < |\varepsilon_{\theta\theta}| \quad \text{as} \quad 1-\nu > 2\nu$$

$$\left[\because \; 1-\nu = 0.72 \; ; \quad 2\nu = 0.56\right]$$

⇒ First Brittle failure if it happens, happens through $\theta\theta$ - direction

$$\Rightarrow \quad \frac{Pr}{2tE}(1-\nu) = 1000 \times 10^{-6}$$

$$\Rightarrow \quad P = \frac{1000 \times 10^{-6} \times 2 \times E \times t}{(1-\nu) \times r}$$

$$\Rightarrow P_2 = \frac{10^{-3} \times 2 \times 30 \times 10^6}{0.72 \times 8} t = \cancel{\frac{250000}{3}} t$$

$$= \frac{31250}{3} t$$

$\cancel{P_2 > P_1}$ $\quad P_2 < P_1$

$\Rightarrow$ Failure will happen because of normal strain limit i.e; Brittle failure.

b) Min. permissible thickness :-

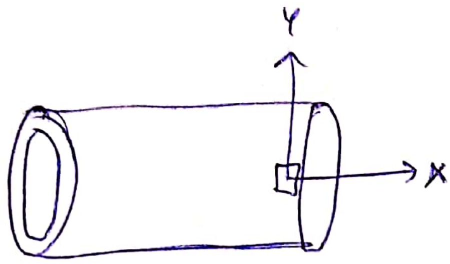
$$P_2 = \frac{31250}{3} t \Rightarrow \cancel{t = 3P_2 \times 31250}$$

$$t = \frac{3P_2}{31250} \Rightarrow t = \frac{3 \times 3000}{31250} \text{ in}$$

$$\Rightarrow t = 0.288 \text{ in}$$

②

i) Failure will occur likely at either A or B.



Y → tangential direction
X → Cylinder's Axis direction.

~~Given P = 3.5 MPa (But requires 1.5 times the operating pressure~~

$$\boxed{I = \pi r^3 t}$$

~~$\sigma = pt = 3.5 \times 1.5 = 5.25$ MPa~~

POINT A :-

$d_2 = 250$ mm ; $P = 3.5$ MPa ; $r_{out} = 125$ mm.

$r_{in} = 125$ mm $- 6$ mm $= 119$ mm.

Due to pressure :- and Bending,

$$\cancel{\sigma_x = \frac{Pr}{2t}}$$

$$\sigma_x = \frac{M r_{out}}{I} + \frac{P r_1}{2t}$$

$$= \frac{(45 \times 1000)(750 \times 10^{-3}) \times 125 \text{mm}}{(6.846 \times 10^7 \text{ mm}^4)} + \frac{(3.5 \text{MPa} \times 119 \text{mm})}{2 \times 6 \text{mm}}$$

$$= 158 \text{ MPa}.$$

~~Due to~~ ~~σ_{xy} = ...MPa~~.

$$\sigma_y = \frac{P r_1}{t} = \frac{(3.5 \text{MPa}) \times (119 \text{mm})}{6 \text{mm}}$$

$$= 69.4 \text{ MPa}.$$

$$\tau_{xy} = \frac{T r_2}{J} = \frac{\cancel{45000 \times 125}\;(45000\,N \times 125\,mm) \times (125\,mm)}{6.846 \times 10^{7}\,mm^4} = 10.27\ MPa.$$

At B there is no Bending as it is on neutral axis.

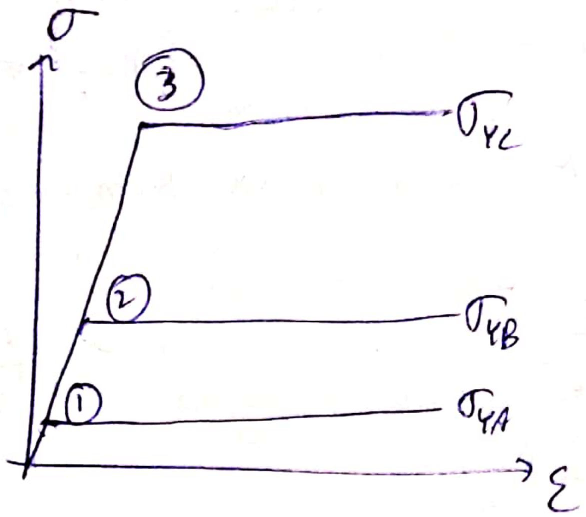
At A, there will be Maximum Bending load.

Every other stress is same between A and B

$\Rightarrow$ Stress at A > Stress at B.

$\Rightarrow$ Failure is likely to occur at A.

3 All the tie-rods have same E [Young's Modulus]



Order of yielding:-

A, B, C

Deflection will be same in all the rods.

$\delta_A = \delta_B = \delta_C \Rightarrow \varepsilon_A = \varepsilon_B = \varepsilon_C$ [As L is same].

$F_A + F_B + F_C = F$

$\sigma_A \times A_A + \sigma_B \times A_B + \sigma_C \times A_C = F$

a) when A yields, $\sigma_A = \sigma_{YA}$

until yielding, $\sigma_A = E\varepsilon_A$, $\sigma_B = E\varepsilon_B$, $\sigma_C = E\varepsilon_C$

But, $\varepsilon_A = \varepsilon_B = \varepsilon_C$

$\Rightarrow \sigma_A = \sigma_B = \sigma_C = E\varepsilon$

$\Rightarrow \varepsilon = \dfrac{\sigma_{YA}}{E} = \dfrac{50 \times 10^6 \text{ Pa}}{1.00 \times 10^9 \text{ Pa}} = 5 \times 10^{-4}$

$$\Rightarrow F = 50\times10^6 \times \left[5+3+1\right] \times 10^{-4}$$

$$= 50\times 9\times 10^2 = 45\times10^3\,N.$$

Deflection is $\delta = \varepsilon \times l$

$$= 5\times10^{-4} \times \frac{1}{10}\,m = 0.05\,mm$$

b) when B yields,

Here, $\sigma_A = \sigma_{YA}$, $\sigma_B = \sigma_{YB}$, $\sigma_C = \sigma_{YB}$
[Hasn't yielded yet]

$$\Rightarrow \varepsilon_B = \varepsilon_C = \frac{\sigma_{YB}}{E} = \varepsilon_A$$

$$F = \sigma_A A_A + \sigma_B A_B + \sigma_C A_C$$

$$\Rightarrow F = \sigma_{YA} \times A_A + (\sigma_{YB})(A_B + A_C)$$

$$\Rightarrow \varepsilon = \frac{\sigma_{YB}}{E} = \frac{100\times10^6}{100\times10^9} = 10^{-3} \Rightarrow \delta = 10^{-3}\times\frac{1}{10} = 0.1\,mm$$

$$\Rightarrow F = 50\times10^{-6}\times5\times10^{-4} + 100\times10^6(3+1)\times10^{-4}$$

$$\Rightarrow F = 65000\,N = 65\times10^3\,N.$$

c) when C yields,

$\sigma_A = \sigma_{YA}$, $\sigma_B = \sigma_{YB}$, $\sigma_C = \sigma_{YC}$

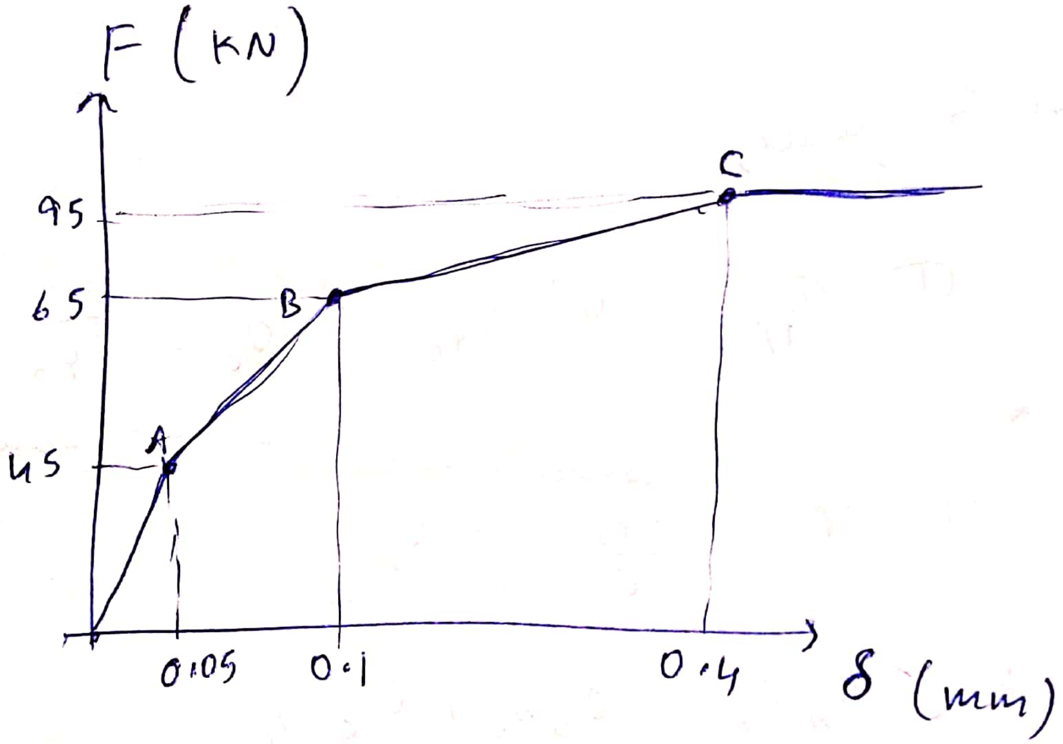
$$\Rightarrow \varepsilon_C = \frac{\sigma_{YC}}{E} = \frac{400\times10^6}{100\times10^9} = 4\times10^{-3}$$

$$\Rightarrow \delta = 4\times10^{-3}\times\frac{1}{10} = 0.4\,mm$$

$$\Rightarrow F = \sigma_A A_A + \sigma_B A_B + \sigma_C A_C$$

$$= (50\times10^{6}\times5\times10^{-4}) + (100\times10^{6}\times3\times10^{-4}) + (400\times10^{6}\times1\times10^{-4})$$

$$= 95\times10^{3}\ N$$

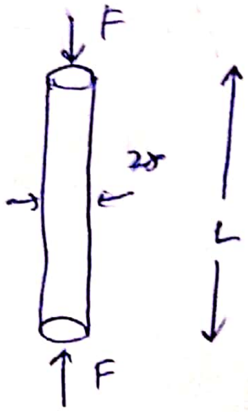


(4)

Given is the slender rod [legs of table]

Function:-	Table leg [should bear compressive load].
objective:-	* Minimizing mass * Improving slenderness (minimize $\frac{r}{L}$)
Constraints:-	* L is given. * Avoid Buckling
Free variables:-	Radius of leg, Material.

$$m = \pi r^2 L \times \rho$$



* For Buckling, critical load will be:-

$$F_{Critical} = \frac{\pi^2 E I}{L^2}$$

where $I = \frac{\pi r^4}{4}$

Given load F must be below $F_{critical}$

$$\Rightarrow \quad F < \frac{\pi^2 E}{L^2} \times \frac{\pi r^4}{4} = \frac{\pi^3 E r^4}{4L^2}$$

But, $r^2 = \frac{m}{\pi L \rho} \Rightarrow r^4 = \frac{m^2}{\pi^2 L^2 \rho^2}$

$$\Rightarrow F < \frac{\pi^3 E}{4L^2} \times \frac{m^2}{\pi^2 L^2 \rho^2}$$

$$\Rightarrow F < \frac{\pi}{4L^4} \times \frac{E}{\rho^2} m^2$$

$$\Rightarrow m > \sqrt{\frac{4F}{\pi}} L^2 \times \left(\frac{\rho}{E^{1/2}}\right)$$

$$\Rightarrow m > \sqrt{\frac{4F}{\pi}} \times L^2 \times \frac{1}{M_1} \quad \text{where} \quad \boxed{M_1 = \frac{E^{1/2}}{\rho}}$$

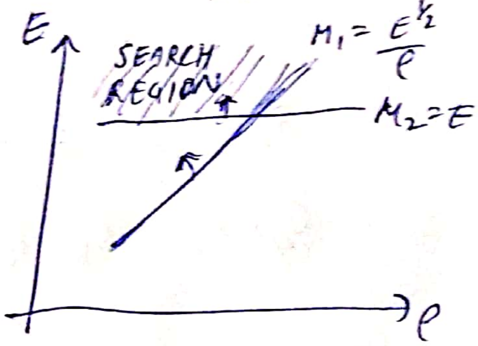
⇒ To minimize mass, M_1 needs to be maximized

Also, $F < \frac{\pi^3 E r^4}{4L^2} \Rightarrow r > \left(\frac{4FL^2}{\pi^3}\right)^{1/4} \times \left(\frac{1}{E}\right)^{1/4}$

$$\Rightarrow r > \left[\frac{4FL^2}{\pi^3}\right]^{1/4} \times \left[\frac{1}{M_2}\right]^{1/4} \quad \text{where} \quad \boxed{M_2 = E}$$

⇒ To minimize radius, M_2 needs to be maximized

From the Material selection chart, plotting two Material indices, Choices of materials can be :-



Material :-	$M_1 = \frac{E^{1/2}}{\rho}$ $\left[\frac{(GPa)^{1/2}}{Kg/m^3}\right]$	$M_2 = E$ [GPa]	Comments:
Mg Alloys	0.115	44.8	Very low M_1
Al Alloys	0.327	70	Very low M_1
CFRP	6.201	96	Best acc. to Indices but very costly.
GFRP	2.211	21	Very low M_2
Wood.	4.234	9.3	Consider cost, wood is the best material after CFRP for M_1

But choice without cost

Best choices with cost

With only Material Indices, CFRP is the best choice

With Material Indices and cost, ~~wood is the best choice.~~
wood and GFRP are good choices depending on Material Index.