EXAMPLE 5.5

A solid steel shaft AB shown in Fig. 5–14 is to be used to transmit 3750 W from the motor M to which it is attached. If the shaft rotates at $\omega = 175$ rpm and the steel has an allowable shear stress of $\tau_{allow} = 100$ MPa, determine the required diameter of the shaft to the nearest mm.

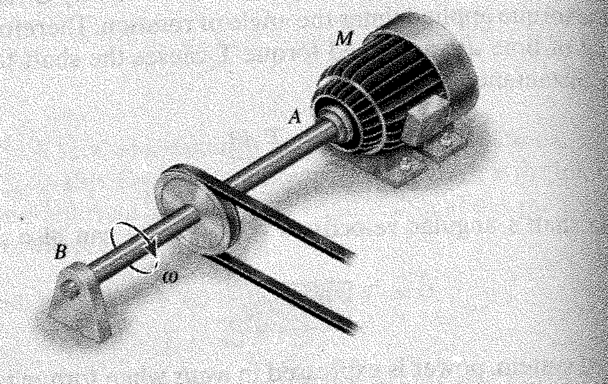


Fig. 5–14

Solution

The torque on the shaft is determined from Eq. 5–10, that is, $P = T\omega$. Expressing P in Newton-meters per second and ω in radians/second, we have

$$P = 3750 \text{ N·m/s}$$

$$\omega = \frac{175 \text{ rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 18.33 \text{ rad/s}$$

Thus,

$$P = T\omega; \qquad 3750 \text{ N·m/s} = T(18.33) \text{ rad/s}$$

$$T = 204.6 \text{ N·m}$$

Applying Eq. 5–12 yields

$$\frac{J}{c} = \frac{\pi}{2}\frac{c^4}{c} = \frac{T}{\tau_{allow}}$$

$$c = \left(\frac{2T}{\pi\tau_{allow}}\right)^{1/3} = \left(\frac{2(204.6 \text{ N·m})(1000 \text{ mm/m})}{\pi(100 \text{ N/mm}^2)}\right)^{1/3}$$

$$c = 10.92 \text{ mm}$$

Since $2c = 21.84$ mm, select a shaft having a diameter of

$$d = 22 \text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 5.6

A tubular shaft, having an inner diameter of 30 mm and an outer diameter of 42 mm, is to be used to transmit 90 kW of power. Determine the frequency of rotation of the shaft so that the shear stress will not exceed 50 MPa.

Solution

The maximum torque that can be applied to the shaft is determined from the torsion formula.

$$\tau_{max} = \frac{Tc}{J}$$

$$50(10^6) \text{ N/m}^2 = \frac{T(0.021 \text{ m})}{(\pi/2)[(0.021 \text{ m})^4 - (0.015 \text{ m})^4]}$$

$$T = 538 \text{ N·m}$$

Applying Eq. 5–11, the frequency of rotation is

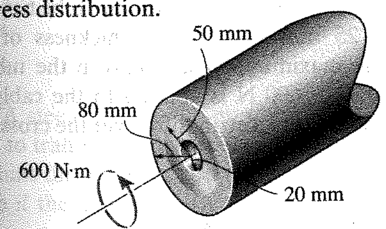
$$P = 2\pi f T$$

$$90(10^3) \text{ N·m/s} = 2\pi f(538 \text{ N·m})$$

$$f = 26.6 \text{ Hz} \qquad \textit{Ans.}$$

PROBLEMS

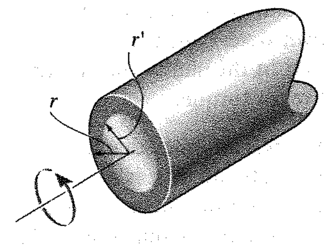
5-1. The tube is subjected to a torque of 600 N·m. Determine the amount of this torque that is resisted by the shaded section. Solve the problem two ways: (a) by using the torsion formula; (b) by finding the resultant of the shear-stress distribution.



Prob. 5–1

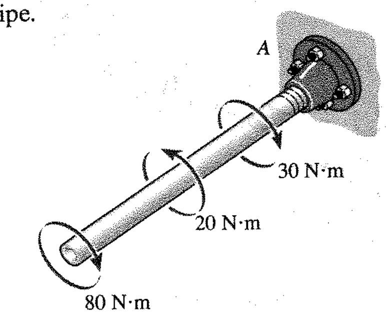
5-2. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque ($T/2$). Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

5-3. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque ($T/4$). Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



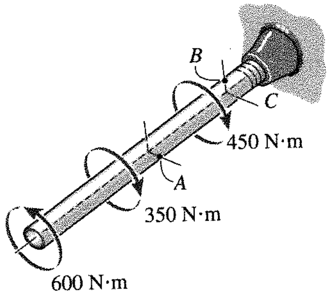
Probs. 5–2/3

***5-4.** The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.



Prob. 5–4

;. The copper pipe has an outer diameter of 50 mm d an inner diameter of 46 mm. If it is tightly secured the wall at C and three torques are applied to it as ɔwn, determine the shear stress developed at points A d B. These points lie on the pipe's outer surface. Sketch ɜ shear stress on volume elements located at A and B.

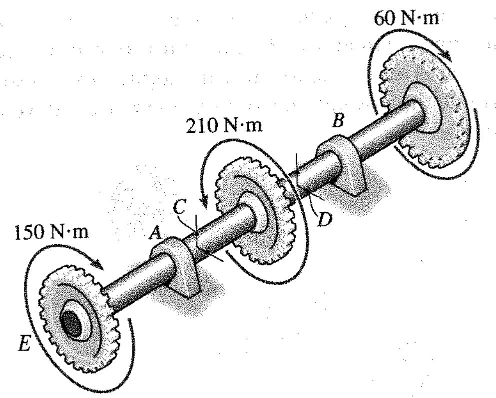


Prob. 5–5

·6. The solid 25-mm-diameter shaft is used to transmit ıe torques applied to the gears. If it is supported by nooth bearings at A and B, which do not resist torque, etermine the shear stress developed in the shaft at points ' and D. Indicate the shear stress on volume elements ɔcated at these points.

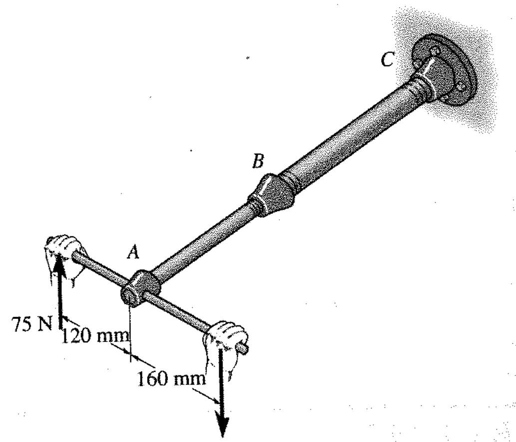
-7. The shaft has an outer diameter of 25 mm and an ıner diameter of 20 mm. If it is subjected to the applied ɔrques as shown, determine the absolute maximum shear tress developed in the shaft. The smooth bearings at A .nd B do not resist torque.

·5-8. The shaft has an outer diameter of 25 mm and an nner diameter of 20 mm. If it is subjected to the applied orques as shown, plot the shear-stress distribution acting ılong a radial line lying within region EA of the shaft. The smooth bearings at A and B do not resist torque.



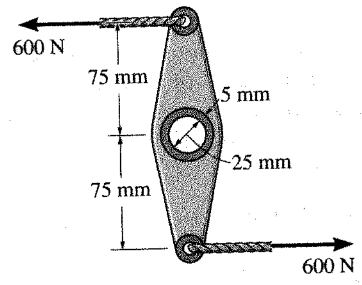
Probs. 5–6/7/8

5-9. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 15 mm and an inner diameter of 13 mm, whereas the larger pipe has an outer diameter of 20 mm and an inner diameter of 17 mm. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



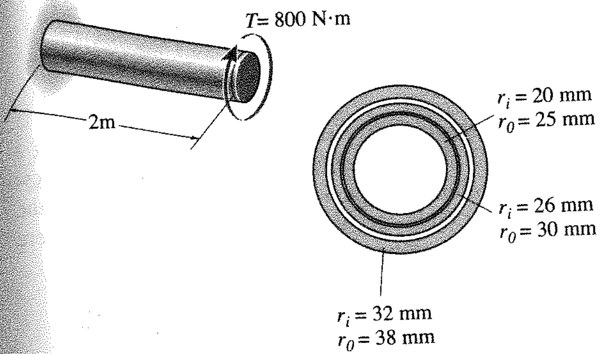
Prob. 5–9

5-10. The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.



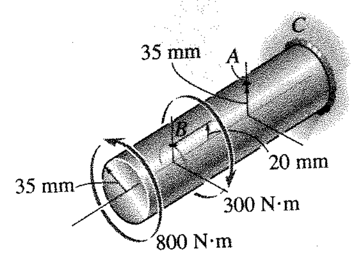
Prob. 5–10

5-11. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800\ \text{N}\cdot\text{m}$ is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.



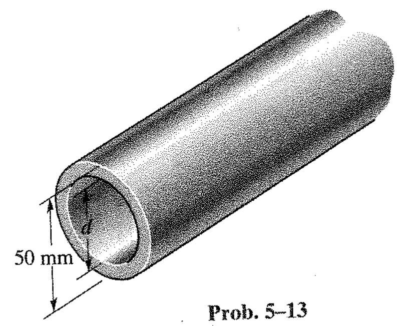
Prob. 5–11

***5-12.** The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



Prob. 5–12

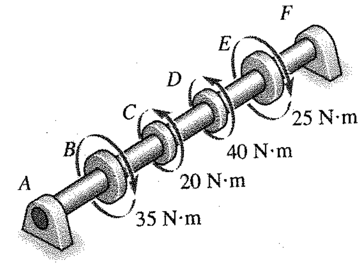
5-13. A steel tube having an outer diameter of 50 mm is used to transmit 25 kW when turning at 2700 rev/min. Determine the inner diameter d of the tube to the nearest 0.1 mm if the allowable shear stress is $\tau_{\text{allow}} = 70$ MPa.



Prob. 5–13

5-14. The solid shaft has a diameter of 15 mm. If it is subjected to the torques shown, determine the maximum shear stress developed in regions BC and DE of the shaft. The bearings at A and F allow free rotation of the shaft.

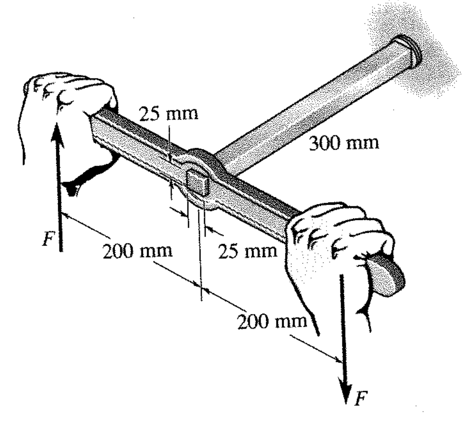
5-15. The solid shaft has a diameter of 15 mm. If it is subjected to the torques shown, determine the maximum shear stress developed in regions CD and EF of the shaft. The bearings at A and F allow free rotation of the shaft.



Probs. 5–14/15

***5-16.** The steel shaft has a diameter of 25 mm and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 55$ MPa.

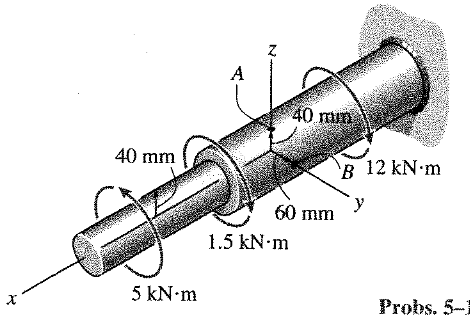
5-17. The steel shaft has a diameter of 25 mm and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft if the couple forces have a magnitude of $F = 150$ N.



Probs. 5–16/17

5-18. The steel shaft is subjected to the torsional loading shown. Determine the shear stress developed at points A and B and sketch the shear stress on volume elements located at these points. The shaft where A and B are located has an outer radius of 60 mm.

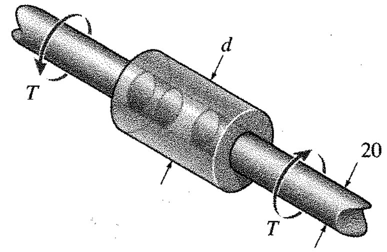
5-19. The steel shaft is subjected to the torsional loading shown. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line where it is maximum.



Probs. 5–18/19

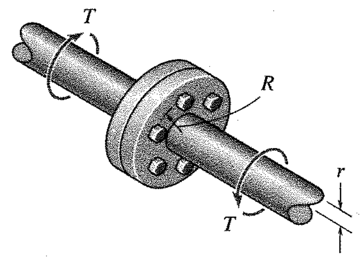
***■5-20.** The 20-mm-diameter steel shafts are connected using a brass coupling. If the yield point for the steel is $(\tau_Y)_{st} = 100$ MPa and for the brass $(\tau_Y)_{br} = 250$ MPa, determine the required outer diameter d of the coupling so that the steel and brass begin to yield at the same time when the assembly is subjected to a torque **T**. Assume that the coupling has an inner diameter of 20 mm.

5-21. The 20-mm-diameter steel shafts are connected using a brass coupling. If the yield point for the steel is $(\tau_Y)_{st} = 100$ MPa, determine the applied torque **T** necessary to cause the steel to yield. If $d = 40$ mm, determine the maximum shear stress in the brass. The coupling has an inner diameter of 20 mm.



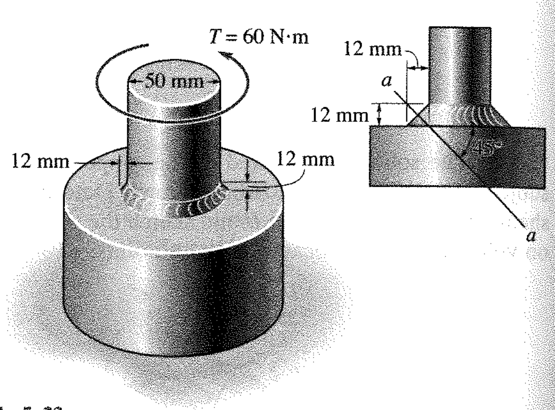
Probs. 5–20/21

5-22. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter d.



Prob. 5–22

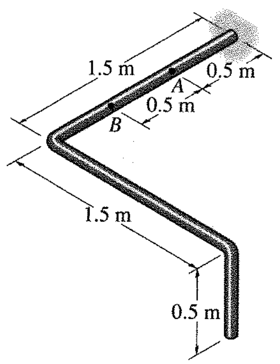
5-23. The steel shafts are connected together using a fillet weld as shown. Determine the average shear stress in the weld along section a–a if the torque applied to the shafts is $T = 60$ N · m. *Note:* The critical section where the weld fails is along section a–a.



Prob. 5–23

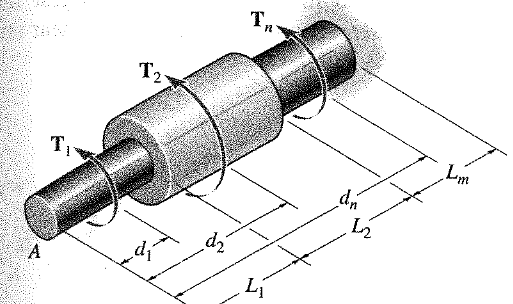
***5-24.** The rod has a diameter of 12 mm and a weight of 75 N/m. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.

5-25. Solve Prob. 5–24 for the maximum torsional stress at B.



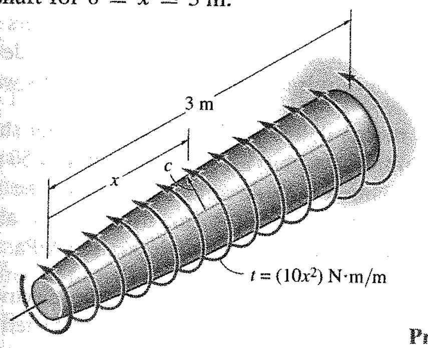
Probs. 5–24/25

■5-26. Consider the general problem of a circular shaft made from m segments each having a radius of c_m. If there are n torques on the shaft as shown, write a computer program that can be used to determine the maximum shearing stress at any specified location x along the shaft. Show an application of the program using the values $L_1 = 0.6$ m, $c_1 = 50$ mm, $L_2 = 1.2$ m, $c_2 = 25$ mm, $T_1 = 1200$ N·m, $d_1 = 0$, $T_2 = -900$ N·m, $d_2 = 1.5$ m.



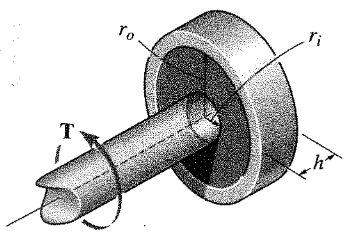
Prob. 5–26

5-27. The shaft is subjected to a distributed torque along its length of $t = (10x^2)$ N · m/m, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \leq x \leq 3$ m.



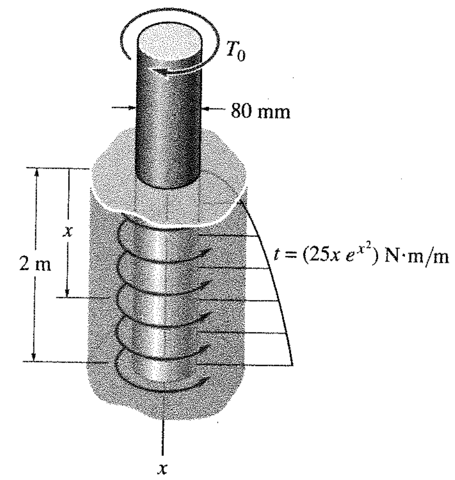
Prob. 5–27

***5-28.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.



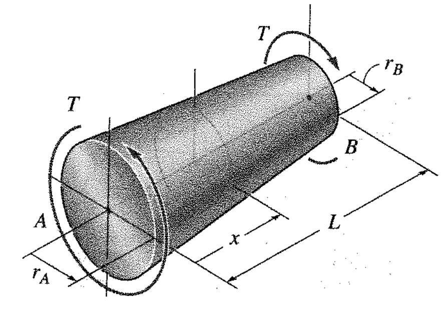
Prob. 5–28

■5-29. The shaft has a diameter of 80 mm and due to friction at its surface within the hole, it is subjected to a variable torque described by the function $t = (25xe^{x^2})$ N · m/m, where x is in meters. Determine the minimum torque T_0 needed to overcome friction and cause it to twist. Also, determine the absolute maximum stress in the shaft.



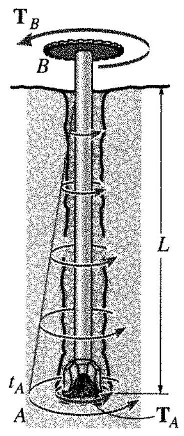
Prob. 5–29

5-30. The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.



Prob. 5–30

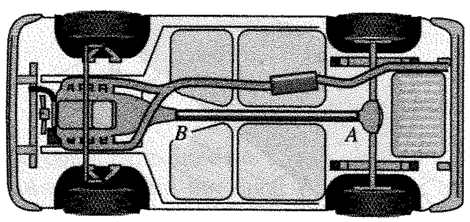
5-31. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



Prob. 5–31

***5-32.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{allow} = 55$ MPa. If the outer diameter of the shaft is 60 mm and the engine delivers 150 kW to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.

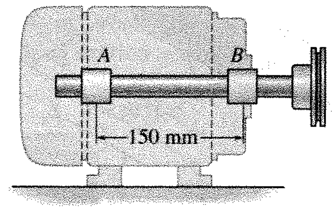
5-33. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 115 kW when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 60 mm. The material has an allowable shear stress of $\tau_{allow} = 50$ MPa.



Probs. 5–32/33

5-34. The drive shaft of a tractor is to be designed as a thin-walled tube. The engine delivers 150 kW when the shaft is turning at 1200 rev/min. Determine the minimum thickness of the wall of the shaft if the shaft's outer diameter is 75 mm. The material has an allowable shear stress of $\tau_{allow} = 50$ MPa.

5-35. A motor delivers 375 kW to the steel shaft AB, which is tubular and has an outer diameter of 50 mm. If it is rotating at 200 rad/s, determine its largest inner diameter to the nearest mm if the allowable shear stress for the material is $\tau_{allow} = 175$ MPa.

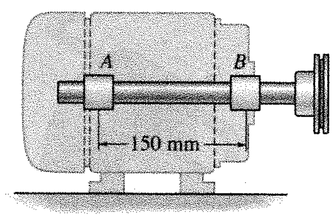


Probs. 5–34/35

***5-36.** The drive shaft of a tractor is made of a steel tube having an allowable shear stress of $\tau_{allow} = 40$ MPa. If the outer diameter is 75 mm and the engine delivers 130 kW to the shaft when it is turning at 1250 rev/min, determine the minimum required thickness of the shaft's wall.

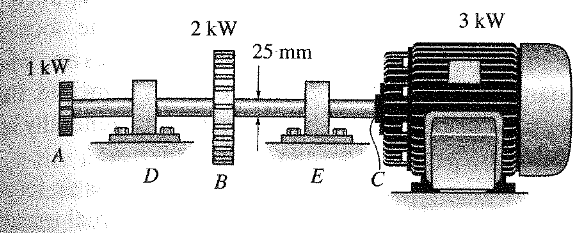
5-37. A motor delivers 375 kW to the steel shaft AB, which is tubular and has an outer diameter of 50 mm and an inner diameter of 46 mm. Determine the *smallest* angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 175$ MPa.

5-38. The 16-mm-diameter shaft for the electric motor develops 375 W and runs at 1740 rev/min. Determine the torque produced and compute the maximum shear stress in the shaft. The shaft is supported by ball bearings at A and B.



Probs. 5–36/37/38

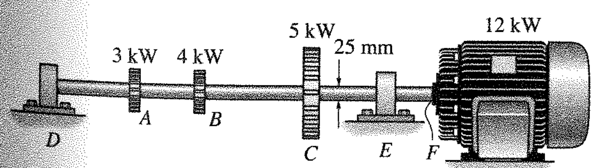
5-39. The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



Prob. 5–39

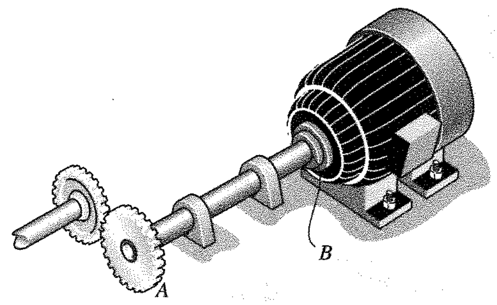
***5-40.** The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.

5-41. Determine the absolute maximum shear stress developed in the shaft in Prob. 5–40.



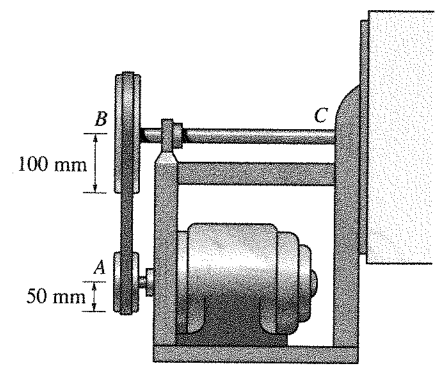
Probs. 5–40/41

5-42. The motor delivers 375 kW to the steel shaft AB, which is tubular and has an outer diameter of 50 mm and an inner diameter of 46 mm. Determine the *smallest* angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 175$ MPa.



Prob. 5–42

5-43. The motor delivers 37.5 kW while turning at a constant rate of 1350 rpm at A. Using the belt and pulley system this loading is delivered to the steel blower shaft BC. Determine to the nearest mm the smallest diameter of this shaft if the allowable shear stress for the steel is $\tau_{allow} = 85$ MPa.



Prob. 5–43

EXAMPLE 5.10

The tapered shaft shown in Fig. 5–23*a* is made of a material having a shear modulus G. Determine the angle of twist of its end B when subjected to the torque.

Solution

Internal Torque. By inspection or from the free-body diagram of a section located at the arbitrary position x, Fig. 5–23*b*, the internal torque is T.

Angle of Twist. Here the polar moment of inertia varies along the shaft's axis and therefore we must express it in terms of the coordinate x. The radius c of the shaft at x can be determined in terms of x by proportion of the slope of line AB in Fig. 5–23*c*. We have

$$\frac{c_2 - c_1}{L} = \frac{c_2 - c}{x}$$

$$c = c_2 - x\left(\frac{c_2 - c_1}{L}\right)$$

Thus, at x,

$$J(x) = \frac{\pi}{2}\left[c_2 - x\left(\frac{c_2 - c_1}{L}\right)\right]^4$$

Applying Eq. 5–14, we have

$$\phi = \int_0^L \frac{T\,dx}{\left(\frac{\pi}{2}\right)\left[c_2 - x\left(\frac{c_2 - c_1}{L}\right)\right]^4 G} = \frac{2T}{\pi G}\int_0^L \frac{dx}{\left[c_2 - x\left(\frac{c_2 - c_1}{L}\right)\right]^4}$$

Performing the integration using an integral table, the result becomes

$$\phi = \left(\frac{2T}{\pi G}\right)\frac{1}{3\left(\frac{c_2 - c_1}{L}\right)\left[c_2 - x\left(\frac{c_2 - c_1}{L}\right)\right]^3}\Bigg|_0^L$$

$$= \frac{2T}{\pi G}\left(\frac{L}{3(c_2 - c_1)}\right)\left(\frac{1}{c_1^3} - \frac{1}{c_2^3}\right)$$

Rearranging terms yields

$$\phi = \frac{2TL}{3\pi G}\left(\frac{c_2^2 + c_1 c_2 + c_1^2}{c_1^3 c_2^3}\right) \qquad \textit{Ans.}$$

To partially check this result, note that when $c_1 = c_2 = c$, then

$$\phi = \frac{TL}{[(\pi/2)c^4]G} = \frac{TL}{JG}$$

which is Eq. 5–15.

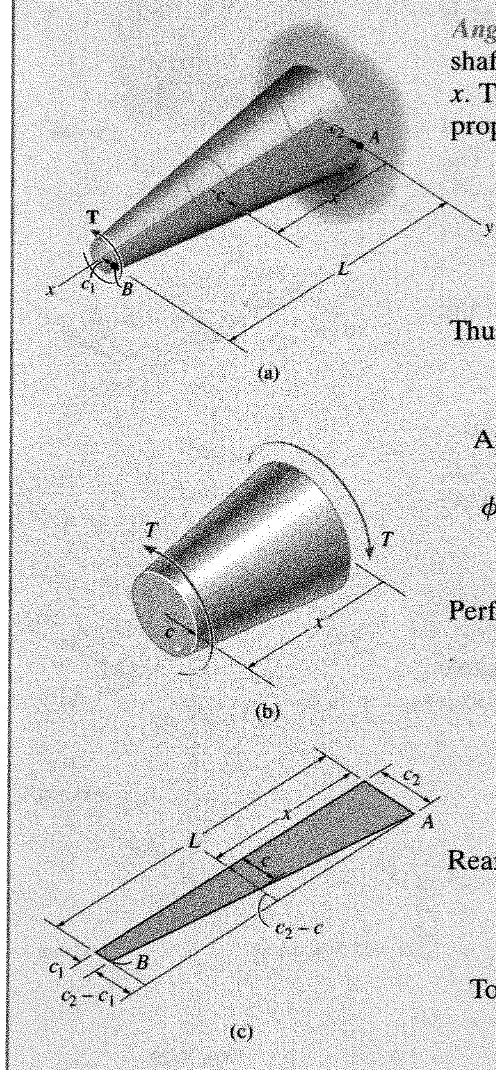
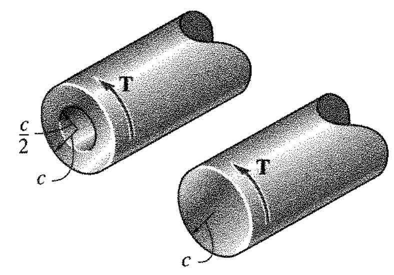


Fig. 5–23

PROBLEMS

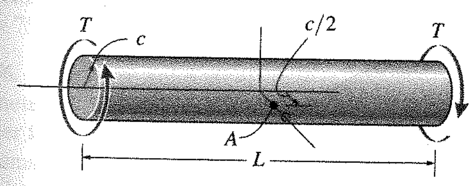
***5–44.** The propellers of a ship are connected to an A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist. $G = 75$ GPa.

5–45. A shaft is subjected to a torque **T**. Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius c. To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



Probs. 5–44/45

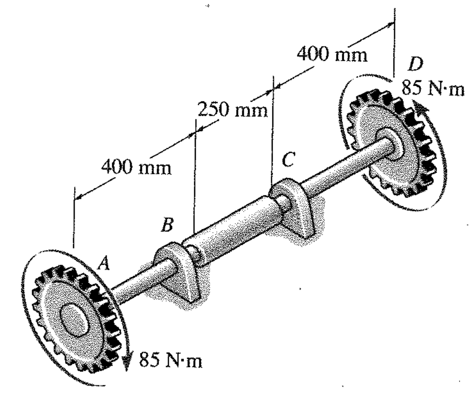
5–46. The solid shaft of radius c is subjected to a torque **T** at its ends. Show that the maximum shear strain developed in the shaft is $\gamma_{max} = Tc/JG$. What is the shear strain on an element located at point A, $c/2$ from the center of the shaft? Sketch the strain distortion of this element.



Prob. 5–46

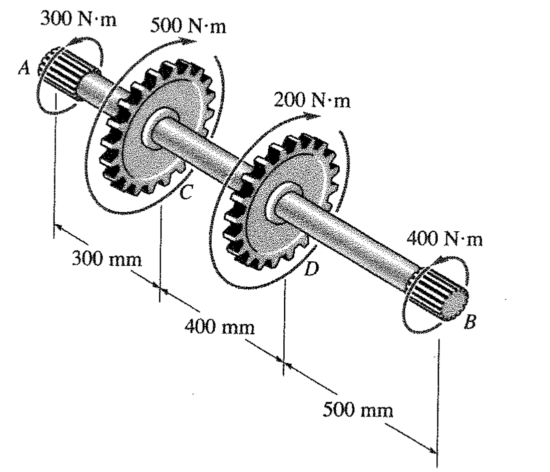
5–47. The A-36 steel axle is made from tubes AB and CD and a solid section BC. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of gear A relative to gear D. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm. $G = 75$ GPa.

***5–48.** The A-36 steel axle is made from tubes AB and CD and a solid section BC. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of the end B of the solid section relative to end C. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm. $G = 75$ GPa.



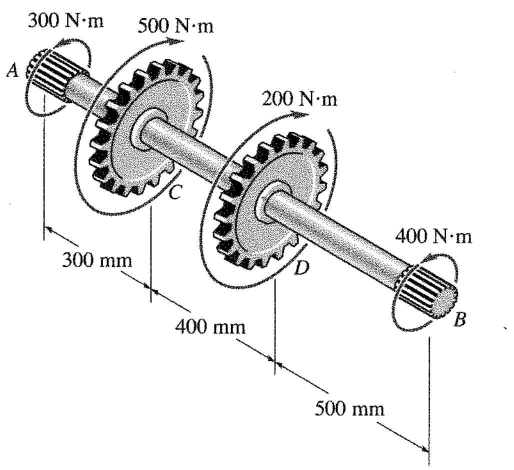
Probs. 5–47/48

5–49. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm. $G = 75$ GPa.



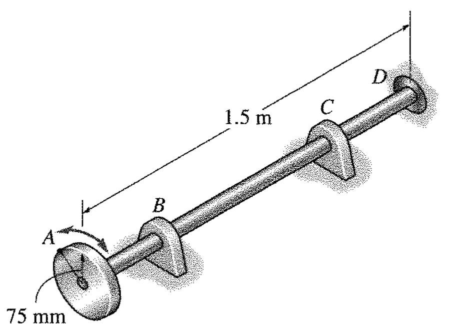
Prob. 5–49

5-50. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of gear C with respect to gear D. The shaft has a diameter of 40 mm. $G = 75$ GPa.



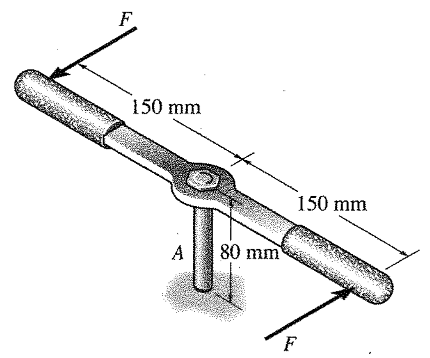
Prob. 5–50

5-51. The rotating flywheel-and-shaft, when brought to a sudden stop at D, begins to oscillate clockwise-counterclockwise such that a point A on the outer edge of the flywheel is displaced through a 6-mm arc. Determine the maximum shear stress developed in the tubular A-36 steel shaft due to this oscillation. The shaft has an inner diameter of 24 mm and an outer diameter of 32 mm. The bearings at B and C allow the shaft to rotate freely, whereas the support at D holds the shaft fixed. $G = 75$ GPa.



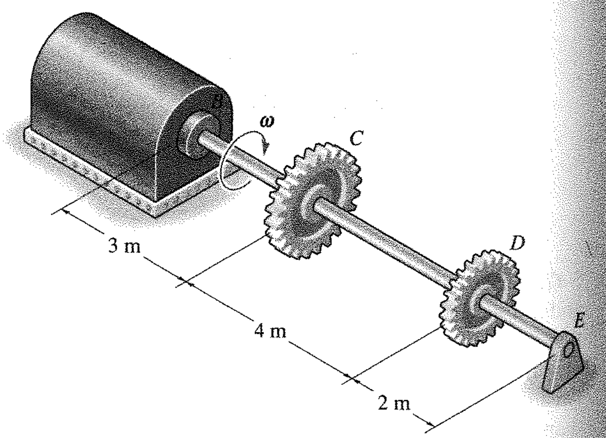
Prob. 5–51

***5-52.** The 8-mm-diameter A-36 bolt is screwed tightly into a block at A. Determine the couple forces F that should be applied to the wrench so that the maximum shear stress in the bolt becomes 18 MPa. Also, compute the corresponding displacement of each force F needed to cause this stress. Assume that the wrench is rigid. $G = 75$ GPa.



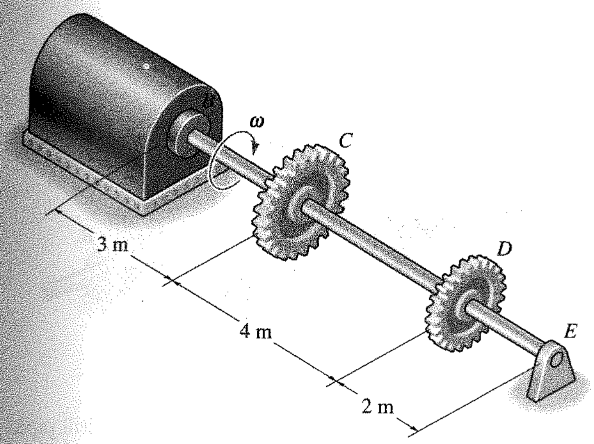
Prob. 5–52

5-53. The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis. $G = 75$ GPa.



Prob. 5–53

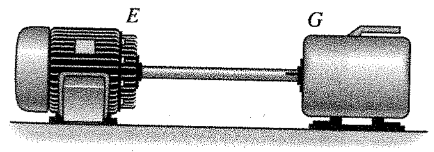
5-54. The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at C allows the shaft to turn freely about its axis. $G = 75$ GPa.



Prob. 5–54

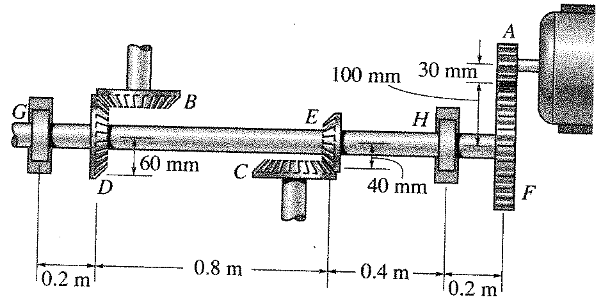
5-55. The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{allow} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad. $G = 75$ GPa.

***5-56.** The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity the shaft can have if it is restricted not to twist more than 1°. $G = 75$ GPa.



Probs. 5–55/56

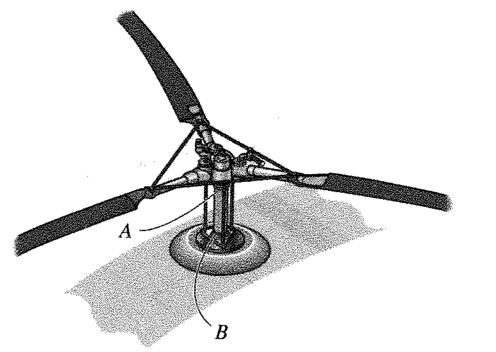
5-57. The motor produces a torque of $T = 20$ N·m on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G and H. Take $G = 75$ GPa.



Prob. 5–57

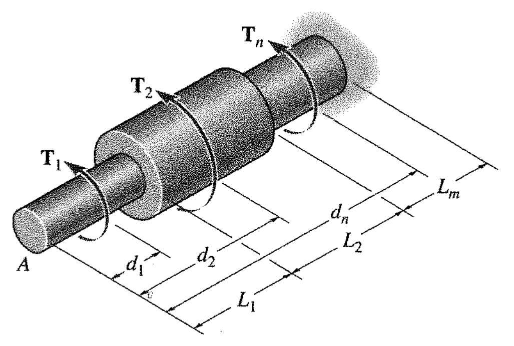
5-58. The engine of the helicopter is delivering 450 kW to the rotor shaft AB when the blades are rotating at 1200 rev/min. Determine to the nearest mm the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 55$ MPa and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 0.6 m long and made from L2 steel. $G = 75$ GPa.

5-59. The engine of the helicopter is delivering 450 kW to the rotor shaft AB when the blades are rotating at 1200 rev/min. Determine to the nearest mm the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 74$ MPa and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 0.6 m long and made from L2 steel. $G = 75$ GPa.



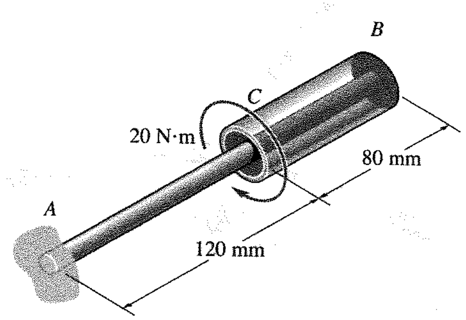
Probs. 5–58/59

*■5-60. Consider the general problem of a circular shaft made from m segments, each having a radius of c_m and shearing modulus G_m. If there are n torques on the shaft as shown, write a computer program that can be used to determine the angle of twist of its end A. Show an application of the program using the values $L_2 = 1.5$ m, $c_1 = 0.02$ m, $G_1 = 30$ GPa, $L_2 = 1.5$ m, $c_2 = 0.05$ m, $G_2 = 15$ GPa, $T_1 = -455$ N · m, $d_1 = 0.25$ m, $T_2 = 600$ N · m, $d_2 = 0.8$ m.



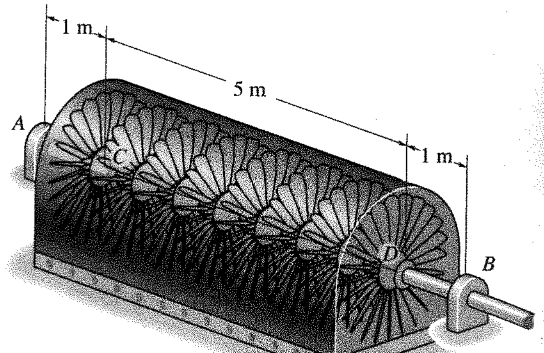
Prob. 5–60

5-61. The A-36 steel assembly consists of a tube having an outer radius of 20 mm and a wall thickness of 2.5 mm. Using a rigid plate at B, it is connected to the solid 20-mm-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 20 N·m is applied to the tube at this end. The end A of the shaft is fixed-supported. $G = 76$ GPa.



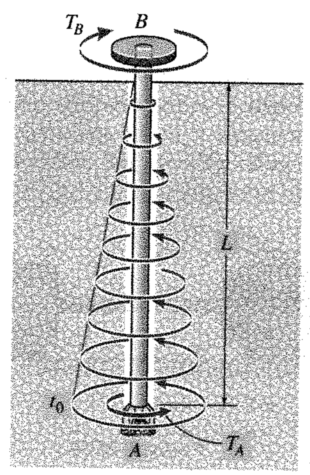
Prob. 5–61

5-62. The 250-mm-diameter L-2 steel shaft on the turbine is supported on journal bearings at A and B. If C is held fixed and the turbine blades create a torque on the shaft that increases linearly from zero at C to 5 kN·m at D, determine the angle of twist of the shaft of end D relative to end C. Also, compute the absolute maximum shear stress in the shaft. Neglect the size of the blades. $G = 75$ GPa.



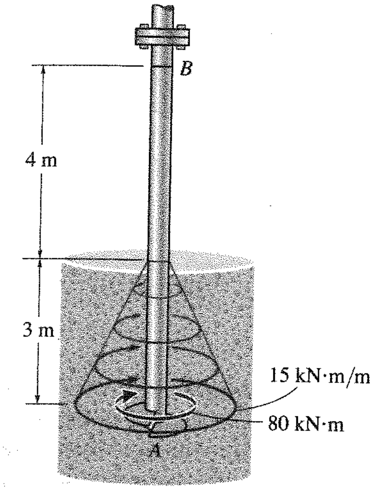
Prob. 5–62

5-63. When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance T_A. Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface B to t_0 at A. Determine the necessary torque T_B that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius r_o and an inner radius r_i. The shear modulus is G.



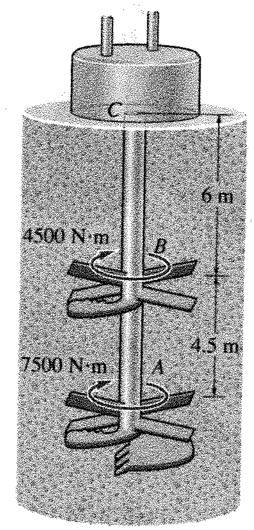
Prob. 5–63

*5-64. The A-36 steel posts are "drilled" at constant angular speed into the soil using the rotary installer. If the post has an inner diameter of 200 mm and an outer diameter of 225 mm, determine the relative angle of twist of end A of the post with respect to end B when the post reaches the depth indicated. Due to soil friction, assume the torque along the post varies linearly as shown, and a concentrated torque of 80 kN · m acts at the bit. $G = 75$ GPa.



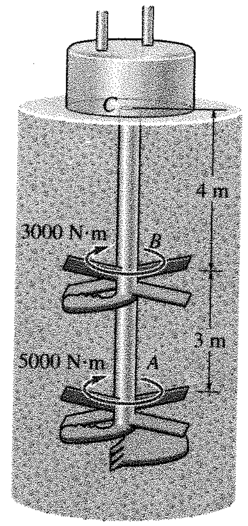
Prob. 5–64

5-65. The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 60 mm and an outer diameter of 90 mm, determine the angle of twist of the shaft of A relative to B and the absolute maximum shear stress in the shaft if each mixing blade is subjected to the torques shown. $G = 80$ GPa.



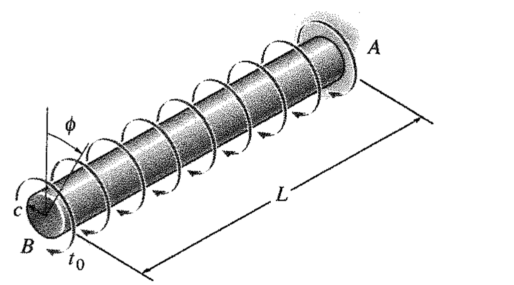
Prob. 5–65

5-66. The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 60 mm and an outer diameter of 90 mm, determine the angle of twist of the shaft of A relative to C if each mixing blade is subjected to the torques shown. $G = 70$ GPa.



Prob. 5–66

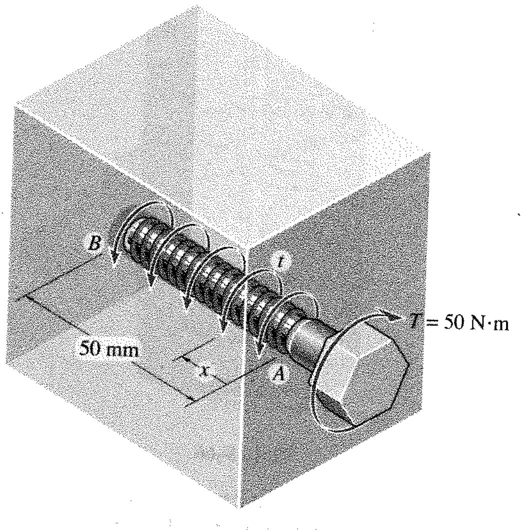
5-67. The shaft has a radius c and is subjected to a torque per unit length of t_0, which is distributed uniformly over the shaft's entire length L. If it is fixed at its far end A, determine the angle of twist ϕ of end B. The shear modulus is G.



Prob. 5–67

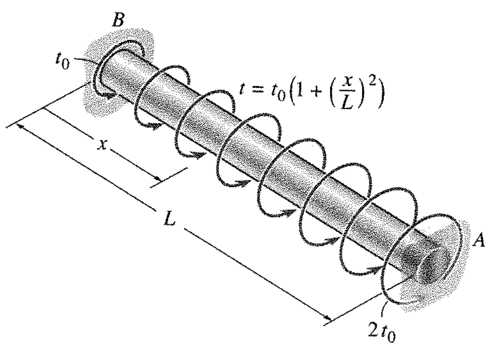
***5-68.** The A-36 bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)$ N·m/m, where x is in meters. If a torque of $T = 50$ N·m is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm. $G = 75$ GPa.

5-69. Solve Prob. 5–68 if the distributed torque is $t = (kx^{2/3})$ N·m/m.



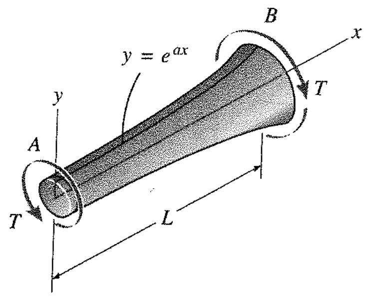
Probs. 5–68/69

5-70. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the angle of twist at end A. The shear modulus is G.



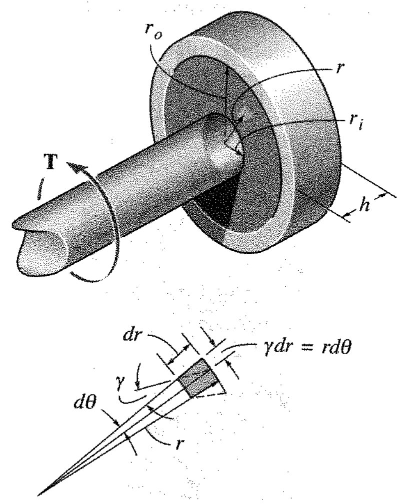
Prob. 5–70

5-71. The contour of the surface of the shaft is defined by the equation $y = e^{ax}$, where a is a constant. If the shaft is subjected to a torque T at its ends, determine the angle of twist of end A with respect to end B. The shear modulus is G.



Prob. 5–71

***5-72.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is G. *Hint:* As shown in the figure, the deformation of the element at radius r can be determined from $r\,d\theta = dr\,\gamma$. Use this expression along with $\tau = T/(2\pi r^2 h)$, from Prob. 5–28, to obtain the result.



Prob. 5–72

5.5 Statically Indeterminate Torque-Loaded Members

A torsionally loaded shaft may be classified as statically indeterminate if the moment equation of equilibrium, applied about the axis of the shaft, is not adequate to determine the unknown torques acting on the shaft. An example of this situation is shown in Fig. 5–24a. As shown on the free-body diagram, Fig. 5–24b, the reactive torques at the supports A and B are unknown. We require that

$$\Sigma M_x = 0; \qquad T - T_A - T_B = 0$$

Since only one equilibrium equation is relevant and there are two unknowns, this problem is statically indeterminate. In order to obtain a solution, we will use the method of analysis discussed in Sec. 4.4.

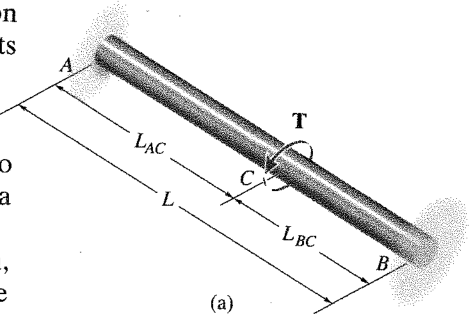
The necessary condition of compatibility, or the kinematic condition, requires the angle of twist of one end of the shaft with respect to the other end to be equal to zero, since the end supports are fixed. Therefore,

$$\phi_{A/B} = 0$$

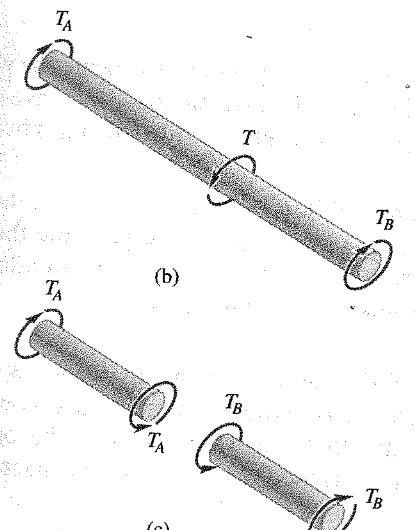
In order to write this equation in terms of the unknown torques, we will assume that the material behaves in a linear-elastic manner, so that the load-displacement relationship is expressed by $\phi = TL/JG$. Realizing that the internal torque in segment AC is $+T_A$ and that in segment CB the internal torque is $-T_B$, Fig. 5–24c, the above compatibility equation can be written as

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0$$

Here JG is assumed to be constant.



(a)



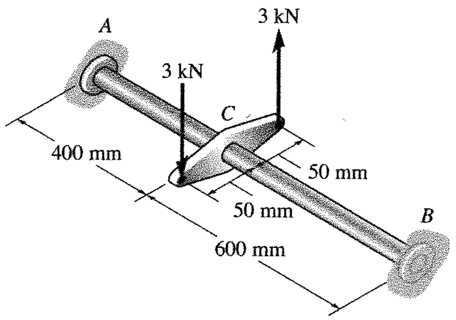
(b)

(c)

Fig. 5–24

PROBLEMS

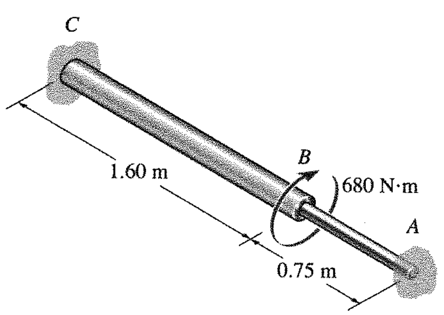
5-73. The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 80$ GPa.



Prob. 5–73

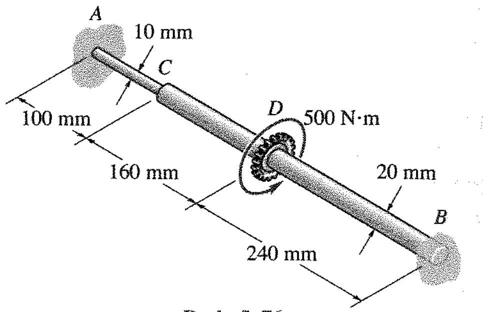
5-74. A rod is made from two segments: AB is steel and BC is brass. It is fixed at its ends and subjected to a torque of $T = 680$ N·m. If the steel portion has a diameter of 30 mm, determine the required diameter of the brass portion so the reactions at the walls will be the same. $G_{st} = 75$ GPa, $G_{br} = 39$ GPa.

5-75. Determine the absolute maximum shear stress in the shaft of Prob. 5–74.



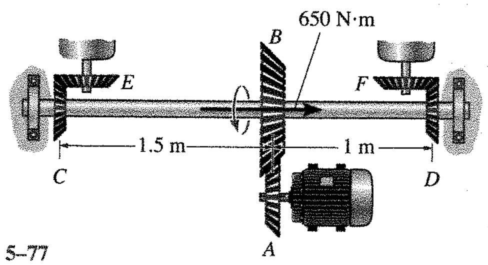
Probs. 5–74/75

5-76. The steel shaft is made from two segments: AC has a diameter of 10 mm, and CB has a diameter of 20 mm. If it is fixed at its ends A and B and subjected to a torque of 500 N · m, determine the maximum shear stress in the shaft.



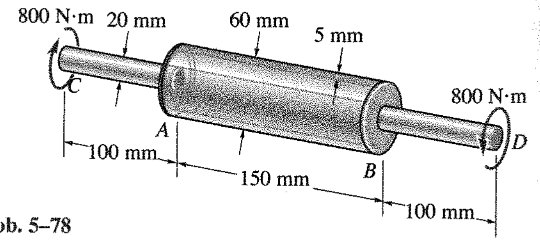
Prob. 5–76

5-77. The motor A develops a torque at gear B of 650 N·m which is applied along the axis of the 50-mm-diameter steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft and do not resist torque. $G_{st} = 85(10^3)$ MPa.



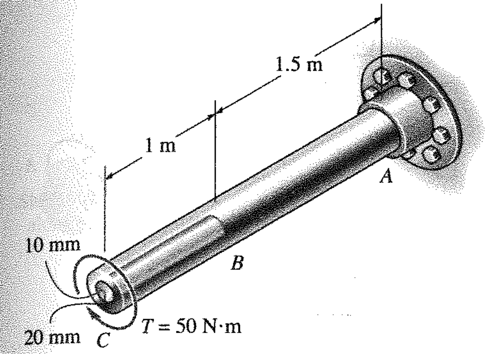
Prob. 5–77

5-78. The composite shaft consists of a mid-section that includes the 20-mm-diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 N·m. The material is A-36 steel. $G = 70$ GPa.



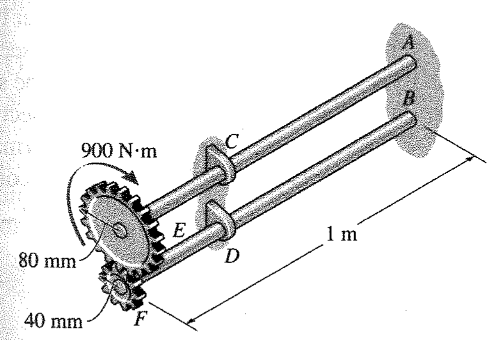
Prob. 5–78

5-79. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50$ N · m is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear and maximum shear strain in the brass and steel. Take $G_{st} = 80$ GPa, $G_{br} = 40$ GPa.



Prob. 5–79

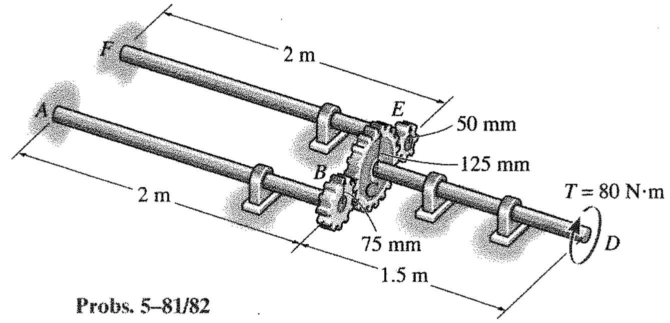
*5-80. The two 1-m-long shafts are made of 2014-T6 aluminum. Each has a diameter of 30 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 900 N·m is applied to the top gear as shown, determine the maximum shear stress in each shaft.



Prob. 5–80

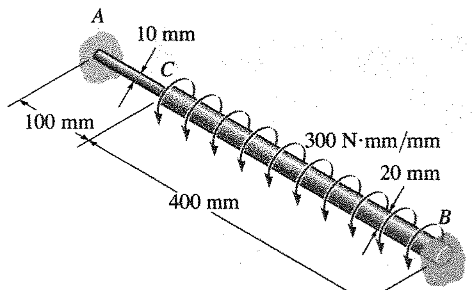
5-81. The two shafts AB and EF are fixed at their ends and fixed connected to gears that are in mesh with a common gear at C, which is fixed connected to shaft CD. If a torque of $T = 80$ N · m is applied to end D, determine the angle of twist of end D. Each shaft has a diameter of 20 mm and is made from A-36 steel. $G = 75$ GPa.

5-82. The two shafts AB and EF are fixed at their ends and fixed connected to gears that are in mesh with a common gear at C, which is fixed connected to shaft CD. If a torque of $T = 80$ N · m is applied to end D, determine the torque at A and F. Each shaft has a diameter of 20 mm and is made from A-36 steel. $G = 75$ GPa.



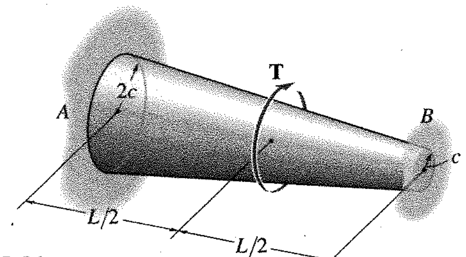
Probs. 5–81/82

5-83. The A-36 steel shaft is made from two segments: AC has a diameter of 10 mm and CB has a diameter of 20 mm. If the shaft is fixed at its ends A and B and subjected to a uniform distributed torque of 300 N·mm/mm along segment CB, determine the absolute maximum shear stress in the shaft. $G = 75$ GPa.



Prob. 5–83

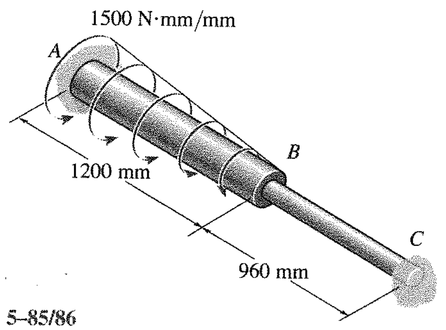
*5-84. The tapered shaft is confined by the fixed supports at A and B. If a torque $\mathbf{T}$ is applied at its mid-point, determine the reactions at the supports.



Prob. 5–84

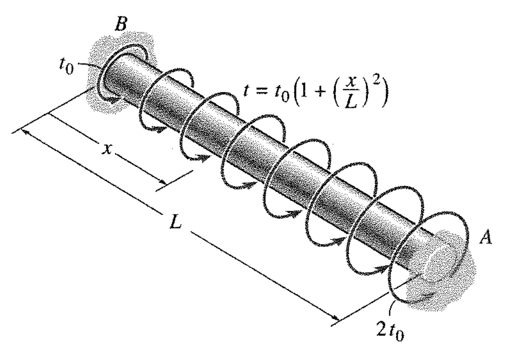
5-85. A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports A and C. Segment AB has a diameter of 30 mm and segment BC has a diameter of 15 mm.

5-86. Determine the rotation of joint B and the absolute maximum shear stress in the shaft in Prob. 5-85. $G = 75$ GPa.



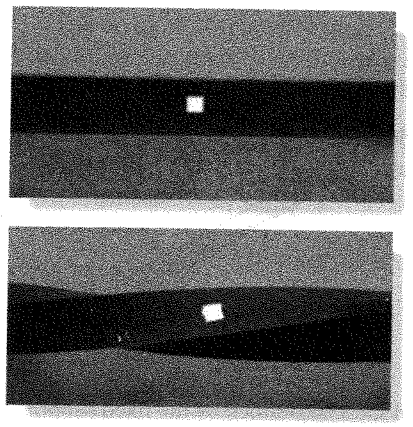
Probs. 5–85/86

5-87. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



Prob. 5–87

*5.6 Solid Noncircular Shafts



Notice the deformation that occurs to the square element when this rubber bar is subjected to a torque.

It was demonstrated in Sec. 5.1 that when a torque is applied to a shaft having a circular cross section—that is, one that is axisymmetric—the shear strains vary linearly from zero at its center to a maximum at its outer surface. Furthermore, due to the uniformity of the shear strain at all points on the same radius, the cross section does not deform, but rather remains plane after the shaft has twisted. Shafts that have a noncircular cross section, however, are *not* axisymmetric, and because the shear stress over their cross section is distributed in a very complex manner, their cross sections will ***bulge*** or ***warp*** when the shaft is twisted. Evidence of this can be seen from the way grid lines deform on a shaft having a square cross section when the shaft is twisted, Fig. 5–27. As a consequence of this deformation, the torsional analysis of *noncircular* shafts becomes considerably complicated and will not be considered in this text.

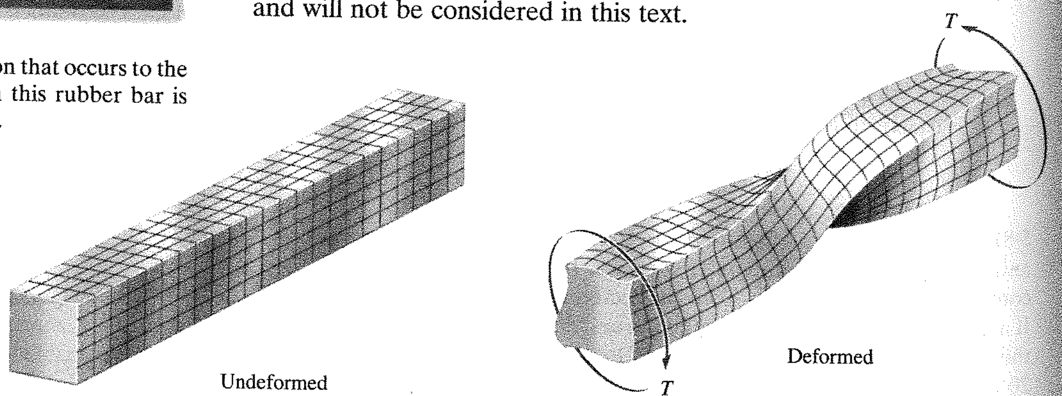
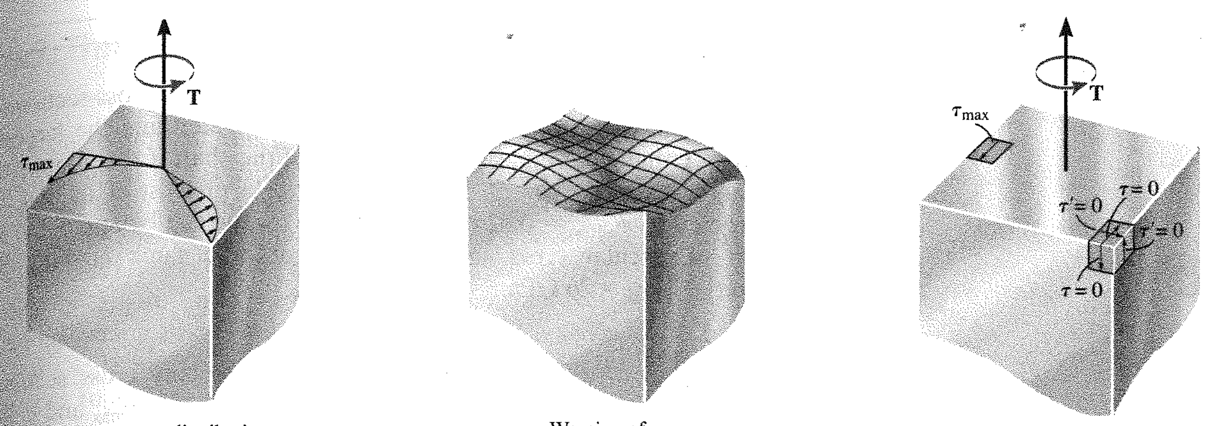


Fig. 5–27



Shear stress distribution along two radial lines

(a)

Warping of cross-sectional area

(b)

(c)

Fig. 5–28

Using a mathematical analysis based on the theory of elasticity, however, it is possible to determine the shear-stress distribution within a shaft of square cross section. Examples of how this shear stress varies along two radial lines of the shaft are shown in Fig. 5–28a. Because these shear-stress distributions vary in a complex manner, the shear strains they create will *warp* the cross section as shown in Fig. 5–28b. Notice that the corner points of the shaft will be subjected to zero shear stress and therefore zero shear strain. The reason for this can be shown by considering an element of material located at one of these points, Fig. 5–28c. One would expect the top face of this element to be subjected to a shear stress in order to aid in resisting the applied torque **T**. This, however, is *not* the case, since the shear stresses τ and τ', acting on the *outer surface* of the shaft, must be *zero*, which in turn implies that the corresponding shear-stress components τ and τ' on the top face must also be equal to zero.

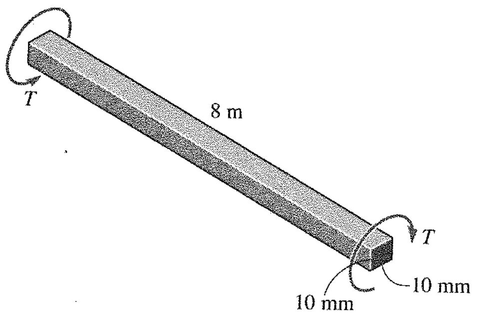
The results of the analysis for square cross sections, along with other results from the theory of elasticity, for shafts having triangular and elliptical cross sections, are reported in Table 5–1. In all cases, the *maximum shear stress* occurs at a point on the edge of the cross section that is *closest to* the center axis of the shaft. In Table 5–1, these points are indicated as "dots" on the cross sections. Also given are formulas for the angle of twist of each shaft. By extending these results to a shaft having an *arbitrary* cross section, it can also be shown that a shaft having a *circular* cross section is most efficient, since it is subjected to both a *smaller* maximum shear stress and a *smaller* angle of twist than a corresponding shaft having the same cross sectional area, but having a noncircular cross section and subjected to the same torque.

TABLE 5–1

Shape of cross section	τ_{max}	ϕ
Square (sides a)	$\dfrac{4.81\,T}{a^3}$	$\dfrac{7.10\,TL}{a^4 G}$
Equilateral triangle (sides a)	$\dfrac{20\,T}{a^3}$	$\dfrac{46\,TL}{a^4 G}$
Ellipse (semi-axes a, b)	$\dfrac{2\,T}{\pi a b^2}$	$\dfrac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$

PROBLEMS

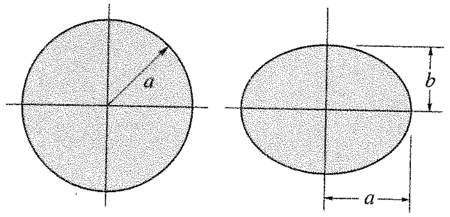
5-88. The aluminum rod has a square cross section of 0 mm by 10 mm. If it is 8 m long, determine the torque T hat is required to rotate one end relative to the other end y 90°. $G_{al} = 28$ GPa, $(\tau_Y)_{al} = 240$ MPa.



Prob. 5–88

89. Determine the amount the maximum shear stress the shaft having an elliptical cross section is increased ompared to the shaft having a circular cross section if oth shafts withstand the same torque.

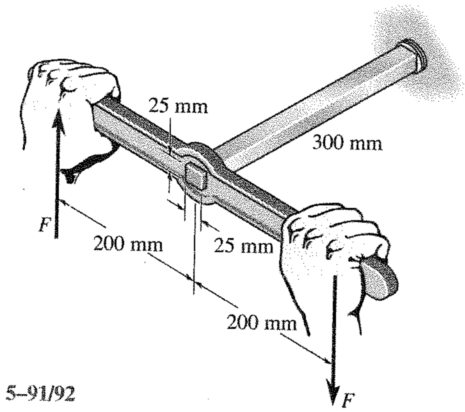
90. If $a = 25$ mm and $b = 15$ mm, determine the aximum shear stress in the circular and elliptical shafts ıen the applied torque is $T = 80$ N·m. By what rcentage is the shaft of circular cross section more icient at withstanding the torque than the shaft of iptical cross section?



Probs. 5–89/90

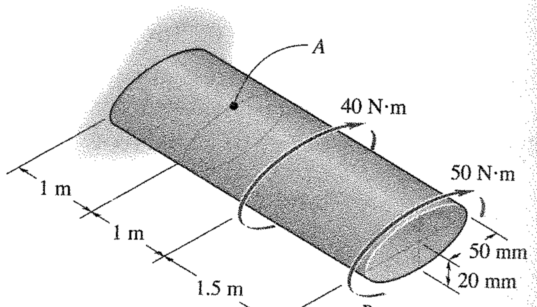
1. The steel shaft is 300 mm long and is screwed into wall using a wrench. Determine the largest couple ces F that can be applied to the shaft without causing steel to yield. $\tau_Y = 55$ MPa.

92. The steel shaft is 300 mm long and is screwed into wall using a wrench. Determine the maximum shear ess in the shaft and the amount of displacement that h couple force undergoes if the couple forces have a gnitude of $F = 150$ N. $G_{st} = 75(10^3)$ MPa.



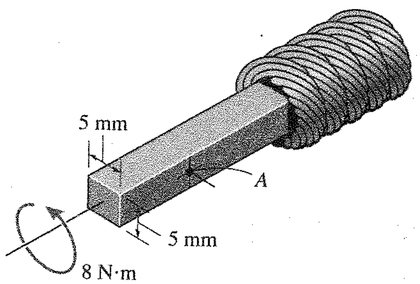
Probs. 5–91/92

5-93. The shaft is made of plastic and has an elliptical cross-section. If it is subjected to the torsional loading shown, determine the shear stress at point A and show the shear stress on a volume element located at this point. Also, determine the angle of twist ϕ at the end B. $G_p = 15$ GPa.



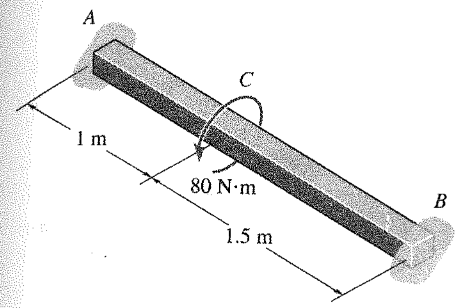
Prob. 5–93

5-94. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of 8 N·m, determine the shear stress in the shaft at point A. Sketch the shear stress on a volume element located at this point.



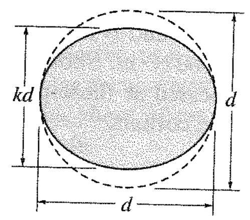
Prob. 5–94

5-95. The aluminum strut is fixed between the two walls at A and B. If it has a 50 mm by 50 mm square cross section, and it is subjected to the torque of 80 N·m at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 26$ GPa.



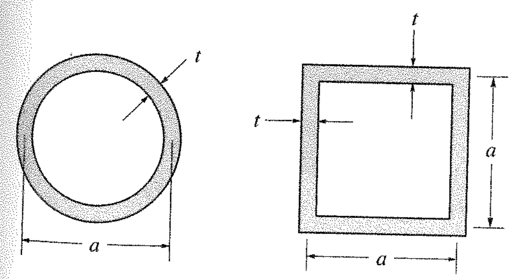
Prob. 5–95

***5-96.** It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



Prob. 5–96

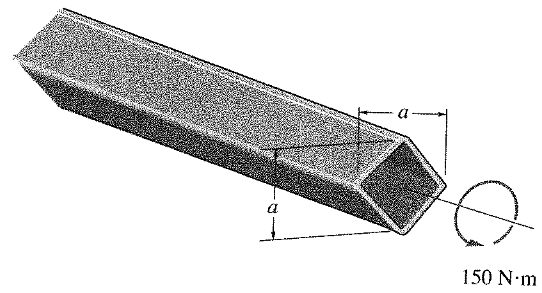
5-97. A torque **T** is applied to two tubes having the cross-sections shown. Compare the shear flow developed in each tube.



Prob. 5–97

5-98. The plastic tube is subjected to a torque of 150 N·m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.

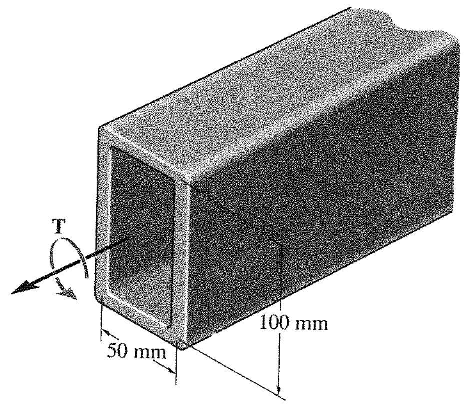
5-99. The plastic tube is subjected to a torque of 150 N·m. Determine the average shear stress in the tube if the mean dimension $a = 200$ mm. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.



Probs. 5–98/99

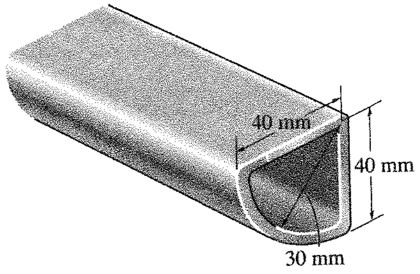
***5-100.** Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 85 MPa when a torque of $T = 2.5$ kN·m is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.

5-101. Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 85 MPa. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 3 mm.



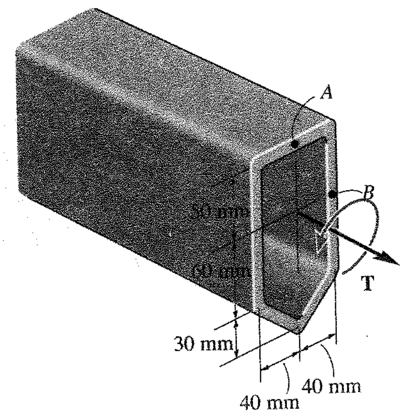
Probs. 5–100/101

5-102. A torque of 200 N·m is applied to the tube. If the wall thickness is 2 mm, determine the average shear stress in the tube.



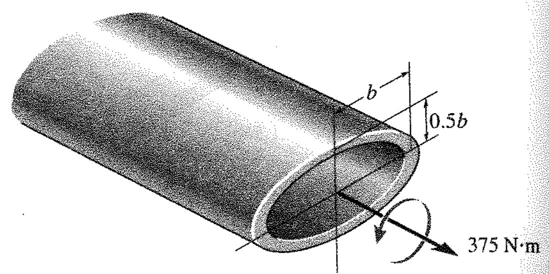
Prob. 5–102

5-103. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if it is subjected to the torque of $T = 5$ N · m. Show the shear stress on volume elements located at these points.



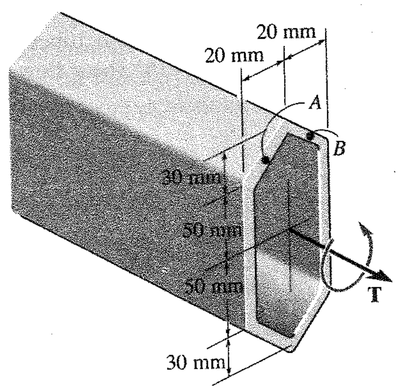
Prob. 5–103

***5-104.** The steel tube has an elliptical cross section of mean dimensions shown and a constant thickness of $t = 5$ mm. If the allowable shear stress is $\tau_{allow} = 55$ MPa, and the tube is to resist a torque of $T = 375$ N·m, determine the necessary dimension b. The mean area for the ellipse is $A_m = \pi b(0.5b)$.



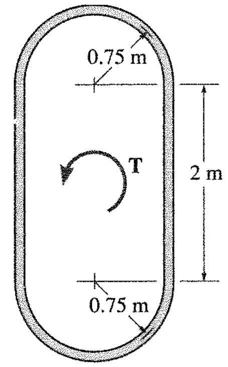
Prob. 5–104

5-105. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if the tube is subjected to the torque of $T = 500$ N · m. Show the shear stress on volume elements located at these points. Neglect stress concentrations at the corners.



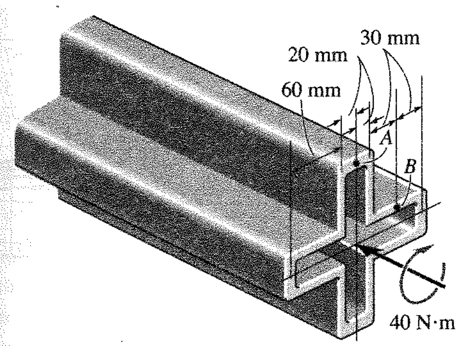
Prob. 5–105

5-106. A portion of an airplane fuselage can be approximated by the cross section shown. If the thickness of its 2014-T6-aluminum skin is 10 mm, determine the maximum wing torque **T** that can be applied if $\tau_{allow} = 4$ MPa. Also, in a 4-m long section, determine the angle of twist. $G = 27$ GPa.



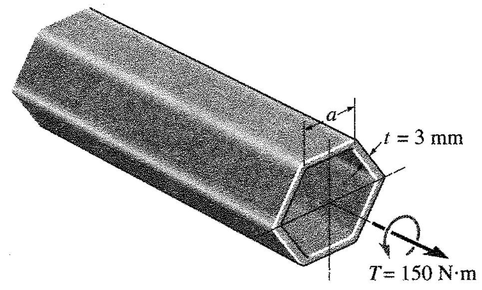
Prob. 5–106

5-107. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40$ N · m, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



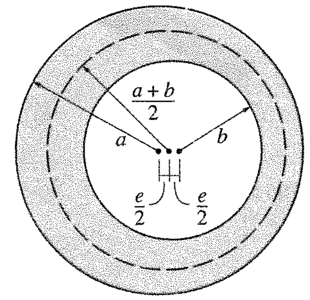
Prob. 5–107

***5-108.** The plastic hexagonal tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



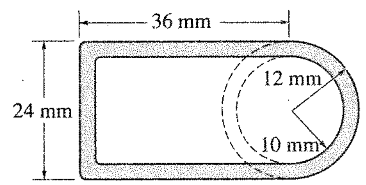
Prob. 5–108

5-109. Due to fabrication, the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



Prob. 5–109

5-110. For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 2 mm thick.



Prob. 5–110