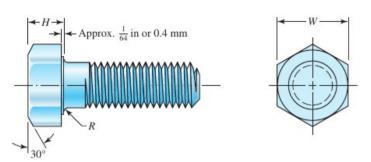
### Introduction - Screw Fasteners

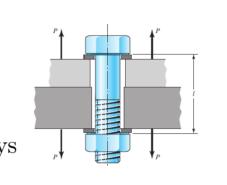
The purpose of a fastener is to clamp two or more parts together.

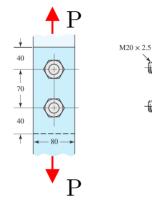


Screw fasteners are classified in different ways

- By their head style
- By their strength

• By their intended use By their thread type





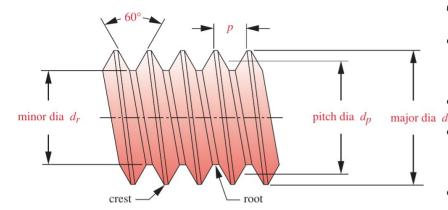
Bolted Joint in Tension Bolted Joint in Shear

Fasteners are available in wide variety of materials – steel, stainless steel, aluminum, brass, bronze and plastics

Manufacturing – Thread: cutting, rolling, Head: forming Salil S. Kulkarni ME423 - IIT Bombay

Budinas, Shigley'S Mechanical Engineering Design (SIE), 11<sup>th</sup> Ed

### Screws Fasteners



Unified National and ISO Standard Thread Form

#### Metric thread specification

 $M8 \times 1.25 - 8$ mm diameter by 1.25 mm pitch thread in ISO coarse series

$$d_p = d - 0.649519p$$
 Pitch diameter  $d_r = d - 1.226869p$  Root diameter  $p$  is the pitch in mm

Tensile strength area  $A_t = \frac{\pi}{4} \left( \frac{d_p + d_r}{2} \right)^2$  Salil S. Kulkarni

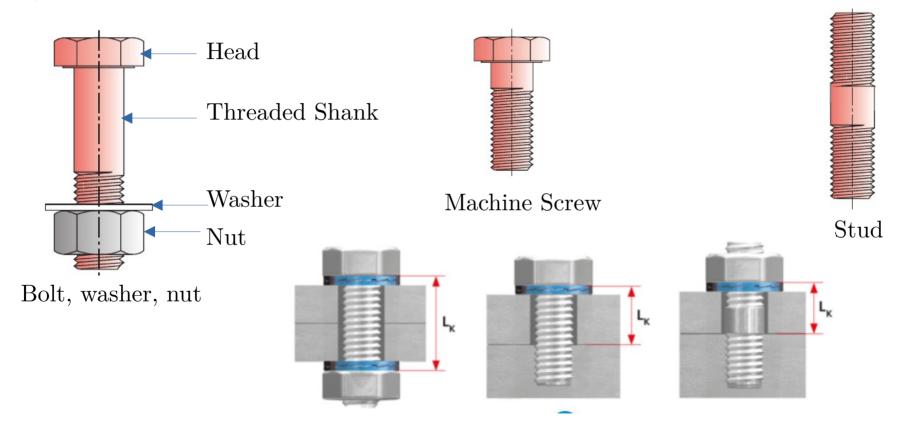
#### Terminology

- Pitch p is the distance between adjacent thread forms measured parallel to the thread axis.
- **Major diameter** d is the largest diameter of a screw thread.
- Minor (or root) diameter  $d_r$  is the smallest diameter of a screw thread.
- **Pitch diameter**  $d_p$  is a theoretical diameter between the major and minor diameters.

	•	Coarse Thread	ls	·	Fine Threads	;
Major Diameter d (mm)	Pitch p mm	Minor Diameter $d_r$ (mm)	Tensile Stress Area $A_t$ (mm <sup>2</sup> )	Pitch p mm	Minor Diameter $d_r$ (mm)	Tensile Stress Area $A_t$ (mm <sup>2</sup> )
3.0	0.50	2.39	5.03			
3.5	0.60	2.76	6.78			
4.0	0.70	3.14	8.78			
5.0	0.80	4.02	14.18			
6.0	1.00	4.77	20.12			
7.0	1.00	5.77	28.86			
8.0	1.25	6.47	36.61	1.00	6.77	39.17
10.0	1.50	8.16	57.99	1.25	8.47	61.20
12.0	1.75	9.85	84.27	1.25	10.47	92.07
14.0	2.00	11.55	115.44	1.50	12.16	124.55
16.0	2.00	13.55	156.67	1.50	14.16	167.25
18.0	2.50	14.93	192.47	1.50	16.16	216.23
20.0	2.50	16.93	244.79	1.50	18.16	271.50
22.0	2.50	18.93	303.40	1.50	20.16	333.06
24.0	3.00	20.32	352.50	2.00	21.55	384.42
27.0	3.00	23.32	459.41	2.00	24.55	495.74
30.0	3.50	25.71	560.59	2.00	27.55	621.20
33.0	3.50	28.71	693.55	2.00	30.55	760.80
36.0	4.00	31.09	816.72	3.00	32.32	864.94
39.0	4.00	34.09	975.75	n 3.00 1	35.32	1028.39

### Screw Fasteners - Classification

#### By intended use

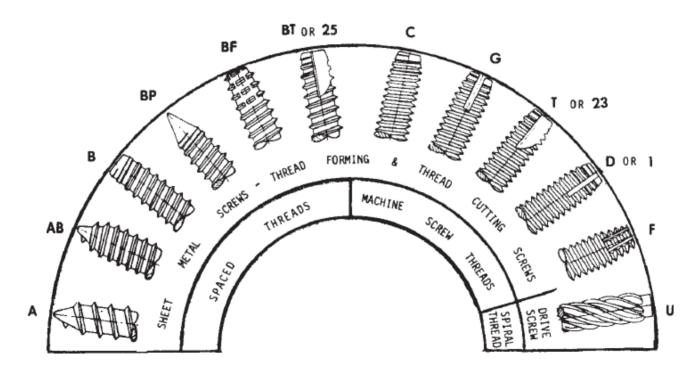


https://www.nord-lock.com/learnings/bolting-tips/2017/optimize-bolted-joint-through-clamped-length/

### Screw Fasteners - Classification

#### By thread use

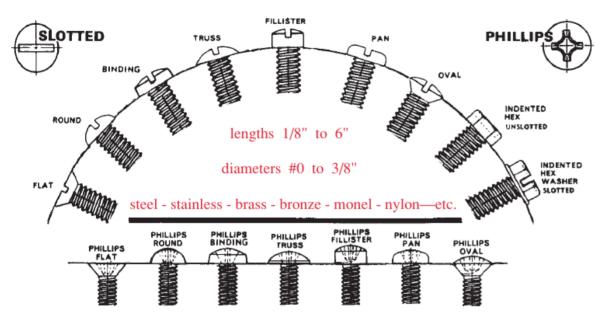
Tapping screws – Fasteners that own hole or make their own thread are called as tapping screws

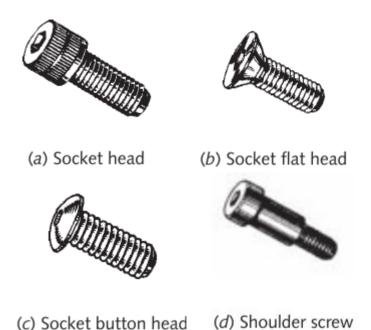


Styles of Threads on tapping screws

### Screw Fasteners - Classification

#### By Head Type





Heads Used on Small Machine Screws (small torque requirement)

Styles of Threads on tapping screws

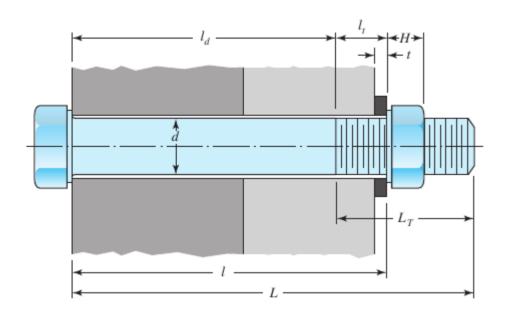
(e) Socket set screw Norton, Machine Design – An Integrated Approach

# Strength of Steel Bolts - Metric Specifications

**Proof Strength**: The stress at which the bolt begins to to take a permanent deformation/set. It is slightly lower than the yield strength

Class Number	Size Range Outside Diameter (mm)	Minimum Proof Strength (MPa)	Minimum Yield Strength (MPa)	Minimum Tensile Strength (MPa)	Material	Metric Class 4.6
4.6	M5-M36	225	240	400	low or medium carbon	4.8
4.8	M1.6-M16	310	340	420	low or medium carbon	5.8
5.8	M5-M24	380	420	520	low or medium carbon	5.8
8.8	M3-M36	600	660	830	medium carbon, Q&T	8.8
9.8	M1.6-M16	650	720	900	medium carbon, Q&T	
10.9	M5-M36	830	940	1 040	low-carbon martensite, Q&T	
12.9	M1.6-M36	970	1 100	1 220	alloy, quenched & tempered	

### Metric Bolts - Dimensions



D – bolt diameter (mm)

P - pitch (mm)

H – nut thickness (table)

t – washer thickness (table)

l – thickness of all material squeezed between the face of the bolt and face of the nut.

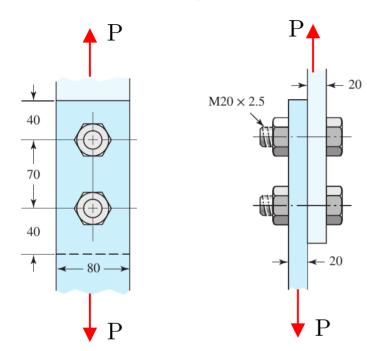
L – Bolt length. L > l + H + t

$$L_T = \begin{cases} 2d + 6 \text{ mm}, & L \le 125 \text{ mm}, d \le 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

 $l_{\rm d}$  – length of unthreaded portion in the grip.  $l_{\rm d}=L-L_{
m T}$ 

 $\mathit{l}_{t}$  – length of threaded portion in the grip.  $\mathit{l}_{t}$  =  $\mathit{l}$  -  $\mathit{l}_{d}$ 

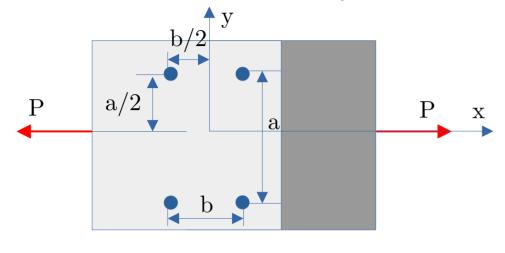
### Fasteners in Shear



### Typical Failure Modes

- Shear failure of fasteners
- Crushing failure of fasteners
- Crushing failure of plates
- Tensile failure of plates

# Fasteners in Shear – Symmetric Load



Consider two plates joined together using fasteners of equal size and subjected to loading as shown

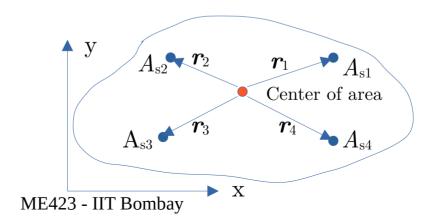
Shear stress in a fastener  $\tau = \frac{P/n}{A_s}$ n – number of fasteners

 $A_{\rm s}$  – area of fastener resisting shear The load distributes equally to all the fasteners as it Acted along the line of symmetry of the fasteners

#### Center of area

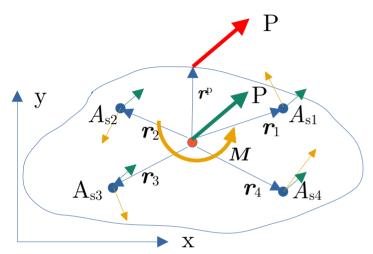
$$x_c = \frac{\sum_{i=1}^n A_{si} x_i}{\sum_{i=1}^n A_{si}}, \ y_c = \frac{\sum_{i=1}^n A_{si} y_i}{\sum_{i=1}^n A_{si}}$$

 $A_{
m si}$  – area of shear in the fastener  $x_{
m i}, y_{
m i}$  – coordinates of the center of the bolt



Salil S. Kulkarni

Line of action of force doe not pass through the center of area



The eccentric load is replaced by:
A parallel force passing through the center of the area
A moment about about the center of the area

Salil S. Kulkarni

The direction of position vector

Therefore  $\mathbf{F}_{m}^{m} + \mathbf{F}_{n}^{m}$ 

The parallel force P will produce a force in each fastener given by

$$m{F}_i^p = rac{|m{P}|}{n}\hat{m{P}} \quad \hat{m{P}} \quad ext{is a unit vector along } m{P}$$

The moment M will produce a forces  $\mathbf{F}_i^m, i = 1, \dots, n$  in the fasteners.

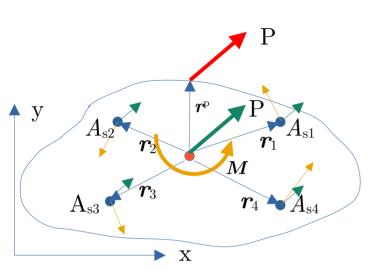
The magnitude of these forces is assumed to be proportional to the distance of the fastener center from the center of area

$$|m{F}_i^m| \propto |m{r}_i| = c|m{r}_i|$$
  $c$  is an unknown

The direction of these forces is assumed to be perpendicular to the position vector of the fastener center from the center of area  $\hat{\boldsymbol{F}}_i^m = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{r}}_i$ 

$$oldsymbol{F}_i^m = |oldsymbol{F}_i^m| \hat{oldsymbol{F}}_i^m = c |oldsymbol{r}_i| (\hat{oldsymbol{k}} imes \hat{oldsymbol{r}}_i) = c (\hat{oldsymbol{k}} imes oldsymbol{r}_i)$$

ME423 - IIT Bombay (c is an unknown)



Now 
$$\mathbf{r}^p \times \mathbf{P} = \sum_{i=1}^n \mathbf{r}_i \times \mathbf{F}_i^m$$
  
=  $c \sum_{i=1}^n \mathbf{r}_i \times (\hat{\mathbf{k}} \times \mathbf{r}_i) = c \hat{\mathbf{k}} \sum_{i=1}^n |\mathbf{r}_i|^2$ 

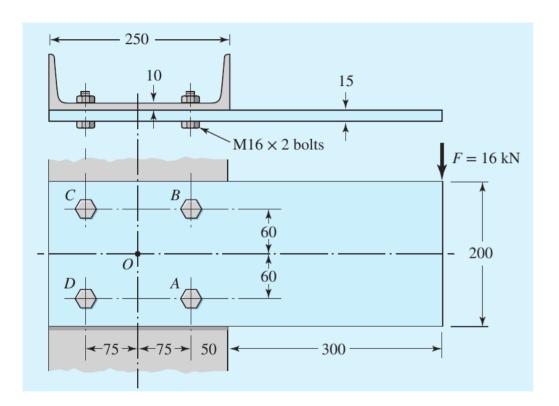
Now 
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$
  
Therefore  $c = \frac{(\mathbf{r}^p \times \mathbf{P}) \cdot \hat{\mathbf{k}}}{\sum_{i=1}^n |\mathbf{r}_i|^2}$ 

The resultant force acting on the fastener is

$$oldsymbol{F}_i = oldsymbol{F}_i^p + oldsymbol{F}_i^m, \ i=1,\ldots,n$$

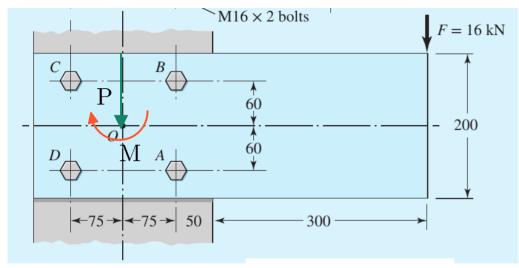
The shear stress in the fastener is

$$au_i = rac{|oldsymbol{F}_i|}{A_{si}}, \ i = 1, \dots, n$$



All dimensions in mm

Assuming that all the load is carried by the bolts, find the loads acting on each bolt

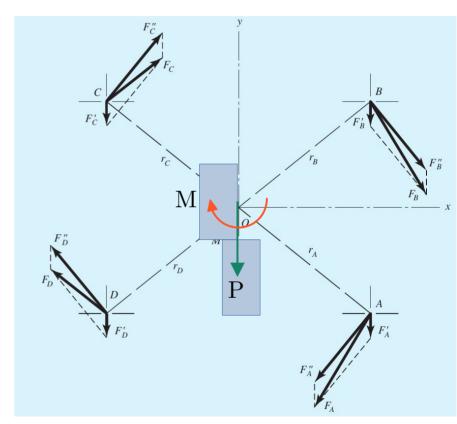


$$r^p = 425\hat{i} + 100\hat{j}$$
  $r_C = -75\hat{i} + 100\hat{j}$ 

$$r_A = 75\hat{i} - 100\hat{j}$$
  $r_D = -75\hat{i} - 100\hat{j}$ 

$$\boldsymbol{r}_B = 75\hat{\boldsymbol{i}} + 100\hat{\boldsymbol{j}}$$

### P = 16 kN, M = 6800 Nm



1	(%i20) d:16*mm		==	28		[ ]		
2	(%i21) rp:vec(425*mm,100*mm,0)	)		29		[ 0 ]		
3	(%i22) P:vec(0,(-16)*kN,0)			30	(%i37)	magFma:vecmag(Fma)		
4	(%i23) ra:vec(75*mm,(-100)*mm	,0)		31	(%o37)	13600.0		
5	(%i24) rb:vec(75*mm,100*mm,0)			32	(%i38)	uvFma:unitvec(Fma)		
6	(%i25) rc:vec((-75)*mm,100*mm	,0)		33		[ - 0.79999999 ]		
7	(%i26) rd:vec((-75)*mm,(-100)	*mm,0)		34		[ ]		
8	(%i27) ramag:vecmag(ra)			35	(%038)	[ - 0.6 ]		
9	(%i32) M:crossprod(rp,P)			36		[ ]		
10	]	0	]	37		[ 0 ]		
11	]		]	38	(%i39)	Fta:Fd+Fma		
12	(%032)	0	]	39		[ - 10880.0 ]		
13	[		]	40		[ ]		
14	[  -	5800.0	]	41	(%039)	[ - 12160.0 ]		
15	(%i33) k:vec(0,0,1)			42		[ ]		
16	(%i34) c:dotprod(M,k)/(4*ramag	g^2)		43		[ 0 ]		
17	(%034) - 10	98800.0		44	(%i40)	<pre>magFta:vecmag(Fta)</pre>		
18	(%i35) Fd:P/4			45	(%040)	16316.862		
19	[	0	]	46	(%i41)	uvFta:unitvec(Fta)		
20	[		]	47		[ - 0.66679485 ]		
21	(%035)	4000.0	]	48		[ ]		
22	[		]	49	(%041)	[ - 0.74524131 ]		
23	[	0	]	50		[ ]		
24	(%i36) Fma:c*crossprod(k,ra)			51		[ 0 ]		
25	[ - 10	0.0886	]	52	(%i42)	Fmb:c*crossprod(k,rb)		
26	]		]	53		[ 10880.0 ]		
27		160.0	]	54		[ ]		
	- 1 -					- 2 -		

55	(%042)	[ - 8160.0 ]	82	[ ]
56		[ ]	83	(%048) [ 8160.0 ]
57		[ 0 ]	84	[ ]
58	(%i43) magFmb:\	/ecmag(Fmb)	85	[ 0 ]
59	(%043)	13600.0	86	(%i49) magFmc:vecmag(Fmc)
60	(%i44) uvFmb:ur	iitvec(Fmb)	87	(%049) 13600.0
61		[ 0.7999999 ]	88	(%i50) uvFmc:unitvec(Fmc)
62		[ ]	89	[ 0.79999999 ]
63	(%044)	[ - 0.6 ]	90	[ ]
64		[ ]	91	(%050) [ 0.6 ]
65		[ 0 ]	92	[ ]
66	(%i45) Ftb:Fd+F	·mb	93	[ 0 ]
67		[ 10880.0 ]	94	(%i51) Ftc:Fd+Fmc
68		[ ]	95	[ 10880.0 ]
69	(%045)	[ - 12160.0 ]	96	[ ]
70		[ ]	97	(%051) [ 4160.0 ]
71		[ 0 ]	98	[ ]
72	(%i46) magFtb:\	/ecmag(Ftb)	99	[ 0 ]
73	(%046)	16316.862	100	(%i52) magFtc:vecmag(Ftc)
74	(%i47) uvFtb:ur	iitvec(Ftb)	101	(%052) 11648.175
75		[ 0.66679485 ]	102	(%i53) uvFtc:unitvec(Ftc)
76		[ ]	103	[ 0.93405183 ]
77	(%047)	[ - 0.74524131 ]	104	[ ]
78		[ ]	105	(%053) [ 0.35713746 ]
79		[ 0 ]	106	[ ]
80	(%i48) Fmc:c*cr	ossprod(k,rc)	107	[ 0 ]
81		[ 10880.0 ]	108	(%i54) Fmd:c*crossprod(k,rd)
		- 3 -		- 4 -

```
109
                                [ - 10880.0 ]
110
111
      (%054)
                                   8160.0
112
113
                                      0
114
      (%i55) magFmd:vecmag(Fmd)
115
      (%055)
                                  13600.0
      (%i56) uvFmd:unitvec(Fmd)
116
117
                                - 0.79999999 ]
118
119
      (%056)
                                     0.6
120
121
                                      0
122
      (%i57) Ftd:Fd+Fmd
123
                                 - 10880.0 ]
124
125
      (%057)
                                  4160.0
126
127
                                      0
      (%i58) magFtd:vecmag(Ftd)
128
129
      (%058)
                                 11648.175
130
      (%i59) uvFtd:unitvec(Ftd)
131
                               [ - 0.93405183 ]
132
133
      (%059)
                                 0.35713746
134
135
                                      0
                         - 5 -
```

## Design of Bolted Joints

### Designing of bolted joints include:

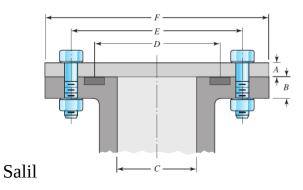
- Determining the number of bolts
- Bolt sizes
- Bolt placements/pattern
- Appropriate preload for the bolt and the torque that must be applied to achieve the desired preload.

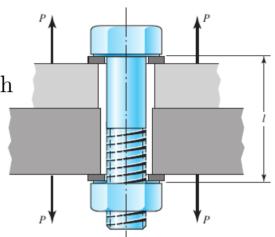
### Bolts in Tension

• Primary application of bolts and nuts is clamping parts together in situations which the applied loads put the bolts in tension

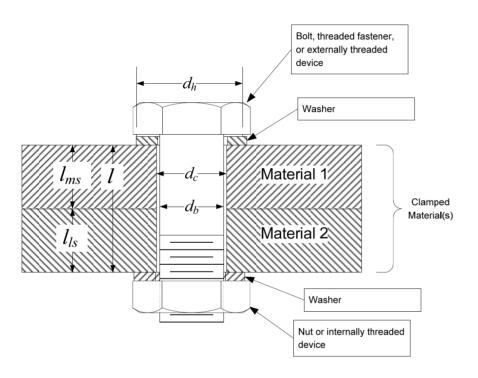
• Common practice is to preload the the joint by the tightening the bolt with sufficient torque to create tensile stress in it which approaches its proof strength

• Need to understand how the elasticities of the bolt and the clamped members interact when the bolt is tightened followed by the application of the external load





## Requirements of a Preloaded Joint



A preloaded joint must meet the following requirement:

- The bolt must have adequate strength.
- The joint must not separate at the maximum load to be applied to the joint.
- The bolt must have adequate fracture and fatigue life.

- Bolt strength is checked at maximum external load and maximum preload.
- Joint separation is checked at maximum external load and minimum preload.

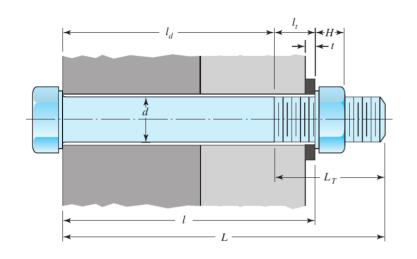
# Analytical Approach for Modeling of Bolted Joints

- Implicitly assumes an axisymmetic stress field due to a single preloaded bolt.
- Any geometric or material effects that significantly violate this assumption make the approach invalid.
  - Bolts very close together
  - Bolts near a physical boundary
  - Non axisymmetric geometries

If the bolted joint of interest does not meet these assumptions (and the additional assumptions of the approach) then it is recommended that a finite element analysis be used for the joint.

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### Bolt Stiffness





$$\delta = \delta_d + \delta_t$$

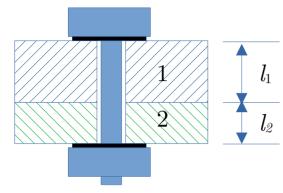
$$\frac{Pl}{AE_b} = \frac{Pl_d}{A_dE_b} + \frac{Pl_t}{A_tE_b}$$

$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}$$

 $A_{
m t}$  – tensile stress area (table)  $l_{
m t}$  – length of portion of the grip  $A_{
m d}$  – major diameter  $L_{
m d}$  – length of unthreaded portion in the grip

$$k_b = \frac{A_d A_t E_b}{A_d l_t + A_t l_d}$$

# Stiffness of Clamped Members

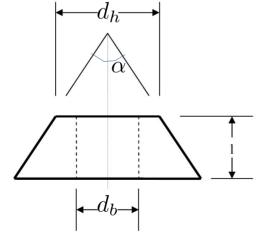


The clamped members are

e 
$$\delta = \delta_1 + \delta_2$$
s  $\frac{P}{k_m} = \frac{P}{k_1} + \frac{1}{k_2}$ 
 $\frac{1}{k_2} = \frac{1}{k_1} + \frac{1}{k_2}$ 

viewed as springs in series  $\frac{P}{k_m} = \frac{P}{k_1} + \frac{P}{k_2}$   $k_1 = \frac{A_{m1}E_1}{l_1}, \ k_2 = \frac{A_{m2}E_2}{l_2}$   $\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2}$   $A_{m1}, A_{m2}$  - effective clamped areas of members 1 and 2 areas of members 1 and 2

Shigley's Approach – the stress field is approximated as a hollow frustum of a cone



The Assumed Stress Field

 $k_i = \frac{\pi E d_b \tan(\alpha)}{\ln\left(\frac{(2l \tan \alpha + d_h - d_b)(d_h + d_b)}{(2l \tan \alpha + d_h + d_b)(d_h - d_b)}\right)}$ 

Salil

## Stiffness of Clamped Members

Morrow's approach – based on the Finite Element Analysis valid for two material joint

$$k_m = E_{eff} d_b(0.9991x_G + 0.2189n + 0.5234)$$

$$E_{eff} = \left(\frac{1}{\frac{1}{E_{ms}} + n\left(\frac{1}{E_{ls}} - \frac{1}{E_{ms}}\right)}\right), n = \frac{l_{ls}}{l}, x_G = \frac{d_b}{l}\left(\frac{d_h^2 - d_c^2}{1.25d_b^2}\right)$$

 $d_{\rm b}$  – diameter of the bolt

 $d_{\rm c}$  – diameter of the clearance hole

 $d_{\rm h}$  – Diameter of the load bearing area between the bolt head and the clamping material

 $E_{
m ms}$  – Young's modulus of the more stiff material

 $E_{\rm ls}$  – Young's modulus of the less stiff material

 $l_{\rm ls}$  – thickness of the less stiff plate

l – total thickness

# Stiffness of Clamped Members

Wileman's approach – based on the Finite Element Analysis valid for a joint made up of one material only  $k_{max}$ 

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

d – bolt diameter

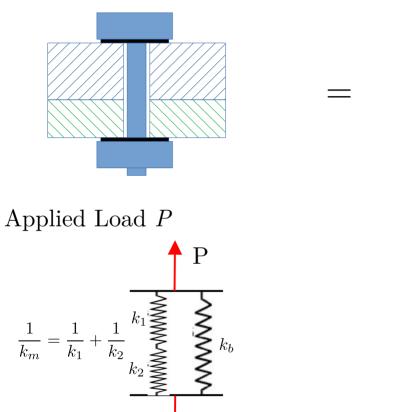
l - thickness of all material squeezed between face of bolt and face of nut

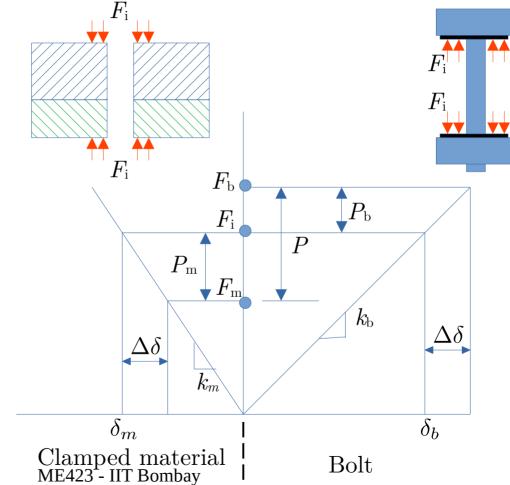
	Poisson	Elastic I	Modulus		
Material Used	Ratio	GPa	Mpsi	$\boldsymbol{A}$	В
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

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# Partitioning the Applied Tensile Load

Bolt Preload  $F_{\rm i}$ 





# Partitioning the Applied Tensile Load

Bolt Preload:  $F_i$ 

Applied load: 
$$P = P_b + P_m$$

Load in the bolt (tensile): 
$$F_b = P_b + F_i$$

Load in the members (compressive):  $F_{\rm m} = F_{\rm i} - P_{\rm m}$ 

Compatibility  $\Delta \delta = \frac{P_b}{k_b} = \frac{P_m}{k}$  or  $P_b = \frac{k_b}{k_m} P_m$ 

Part of the applied load carried by the bolt (tensile)

 $P_b = \frac{k_b}{k_b + k_m} P = CP$ , where  $C = \frac{k_b}{k_b + k_a}$ 

$$=\frac{k_b}{1-k_b}$$

C - stiffness constant of the joint C is typically less than 0.2, i.e. clamped members

Part of the applied load carried by the clamped

take 80% of the applied load

members (compressive)  $P_m = \frac{k_m}{k_m + k_b} P = (1 - C)P$ 

Total load carried by the bolt (tensile)

$$F_b = F_i + P_b$$

$$= F_i + CP$$
(compre)

Total load carried by the clamped members (compressive)  $F_m = F_i - P_m$  $=F_i-(1-C)P$ 

# Partitioning the Applied Tensile Load

Total load carried by the bolt (tensile)

$$F_b = F_i + P_b$$
$$= F_i + CP$$

Total load carried by the clamped members (compressive)

(compressive) 
$$F_m = F_i - P_m$$
$$= F_i - (1 - C)P$$

Joint separates when  $F_{\rm m}=0$ 

$$P_o = \frac{F_i}{1 - C}$$

Factor of safety against joint separation

$$(FOS)_{sep} = \frac{P_o}{P} = \frac{F_i}{P(1-C)}$$

## Bolt Pretension and Bolt Torque

• High preload increases the force required to separate the joint

• Bolt pretension

$$\mathbf{F}_i = \begin{cases} 0.75 A_t S_p & \text{nonpermanant connections} \\ 0.90 A_t S_p & \text{permanant connections} \end{cases} \quad \begin{matrix} A_{\mathrm{t}} - \text{tensile strength area (catalog)} \\ S_{\mathrm{p}} - \text{proof strength (catalog)} \end{matrix}$$

• The torque required to generate the required pretension can be estimated using

Cadmium-plated

With Bowman Anti-Seize

With Bowman-Grip nuts

Bolt ConditionKNonplated, black finish0.30Zinc-plated0.20Lubricated0.18

 $T = K d_b F_i$ 

 $d_{ extsf{b}}$  – bolt diamter (catalog)

K – nut factor

Budinas, Shigley'S Mechanical Engineering Design (SIE), 11<sup>th</sup> Ed

0.16

0.12

0.09

# Factors of Safety – Preloaded Joint

• yielding factor of safety

$$(FOS)_{yield} = \frac{S_p}{F_b/A_t} = \frac{S_p A_t}{F_i + CP}$$

• Load factor  $n_{L}$ —related to overloading. It applies only to P. Indicates the factor by which one can increase P without exceeding the proof strength

$$F_i + Cn_L P = S_p A_t$$
 or  $n_L = \frac{S_p A_t - F_i}{CP}$ 

• Factor of safety against joint separation

$$(FOS)_{sep} = \frac{P_o}{P} = \frac{F_i}{P(1 - C)}$$

## Fatigue Loading in Tension Joint

The joint is subjected to a cyclic load which varied between  $P_{\min}$  and  $P_{\max}$ The maximum and the minimum load carried by the bolt (initial pretension  $F_i$ ) is

$$F_{\text{bmin}} = CP_{\text{min}} + F_{\text{i}}$$
  
 $F_{\text{bmax}} = CP_{\text{max}} + F_{\text{i}}$ 

The alternating stress experiences by the bolt is given by

$$\sigma_a = \frac{(F_{bmax} - F_{bmin})/2}{A_t} = \frac{C(P_{max} - P_{min})}{2A_t}$$

The mean stress experiences by the bolt is given by

$$\sigma_m = \frac{(F_{bmax} + F_{bmin})/2}{A_t} = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t}$$

# Fatigue Loading in Tension Joint

Fully corrected endurance limits including stress concentration effects  $(K_{\rm f})$ 

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$ -1 in	18.6 kpsi
	$1\frac{1}{8} - 1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4} - 1\frac{1}{2}$ in	20.6 kpsi
SAE 8	$\frac{1}{4}$ -1 $\frac{1}{2}$ in	23.2 kpsi
ISO 8.8	M16–M36	129 MPa
ISO 9.8	M1.6-M16	140 MPa
ISO 10.9	M5-M36	162 MPa
ISO 12.9	M1.6-M36	190 MPa

<sup>\*</sup>Repeatedly applied, axial loading, fully corrected, including  $K_f$  as a strength reducer.

#### Goodman Criterion

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{(FOS)_{fatigue}}$$

END