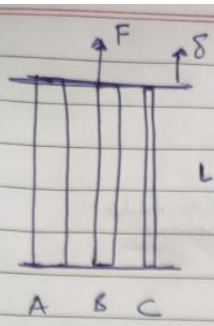
3



$L = 10$ cm

$X_A = 5 \text{ cm}^2$
$X_B = 3 \text{ cm}^2$ } cross sectional area
$X_C = 1 \text{ cm}^2$

$\delta_A = \delta_B = \delta_C$ —— (1).

$E_A = E_B = E_C = 100 \text{ GPa}$

$$\boxed{\begin{aligned}\sigma_{YC} &= 400 \text{ MPa}\\ \sigma_{YA} &= 50 \text{ MPa}\\ \sigma_{YB} &= 100 \text{ MPa}\end{aligned}}$$

For the yield inp,

σ must cross the yield values for the respective tie rods.

$\sigma_A = \dfrac{F_A}{5}$ $\quad \sigma_B = \dfrac{F_B}{3}$ $\quad \sigma_C = \dfrac{F_C}{1}$

Where $F_A + F_B + F_C = F$

Case 1 No rod yielded.

$\dfrac{\delta_A}{L} = \dfrac{\sigma_A}{E}$; $\dfrac{\delta_B}{L} = \dfrac{\sigma_B}{E}$; $\dfrac{\delta_C}{L} = \dfrac{\sigma_C}{E} = \dfrac{\delta}{L}$

$\sigma_A = \dfrac{E\delta}{L} = \sigma_B = \sigma_C$.

Since $\delta \uparrow$ so σ_{YA} is reached first.

$\sigma_A = \sigma_B = \sigma_C = 50 \text{ MPa} = E.\dfrac{\delta}{L}$

$$\delta_A = \frac{\cancel{50} \times \cancel{10^6} \times 1\cancel{0}}{\cancel{1000}\, 100 \times \cancel{1000} \times \cancel{10^9}} = 0.005 \text{ cm}.$$

$\cancel{1000}$ 200

$\sigma_A = \dfrac{F_A}{5/10^4} = 50 \text{ MPa}.$

$$\frac{F_A}{5} = 50\times100$$

$F_A = 250\times100$

$F_A = 25\ kN$

$\therefore \sigma_A = \sigma_B = \sigma_C$

$F_A = 5F_C \longrightarrow F_C = 5kN$

$F_B = 3F_C \longrightarrow F_B = 15kN$

$$\boxed{F_{tot} = F_A + F_B + F_C = 45\ kN}$$

Case 2 A has yielded.

$F = F_B + F_C + F_A$

$\sigma_B = \frac{F_B}{3}$ $\sigma_C = \frac{F_C}{1}$. $\sigma_A = 50\ MPa = \frac{F_A}{5}$

$F_A = 25kN$

$\delta = \delta_B = \delta_C = \frac{\sigma_B}{E} = \frac{\sigma_C}{E}$.

For yielding, σ_{YB} is reached next.

$$100\times10^6 = \sigma_B = E\times\frac{\delta}{L}.$$

$$\delta_B = \frac{100\times10^6\times10}{100\times100\times10^9} = 0.01\ cm$$

$\sigma_B = \frac{F_B}{3/10^4} = 100\ MPa$

$F_B = 300\times100$

$F_B = 30\ kN$

$\sigma_B = \sigma_C$

$F_B = 3F_C \longrightarrow F_C = 10\ kN.$

$$\boxed{F = F_B + F_C + F_A = 40\ kN + 25} = 65\ kN.$$

Case 3 A and B have yielded.

$\sigma_A = 50\,MPa \longrightarrow F_A = 25\,kN$

$\sigma_B = 100\,MPa \longrightarrow F_B = 30\,kN.$

$\sigma_c = \dfrac{F_c}{1}$

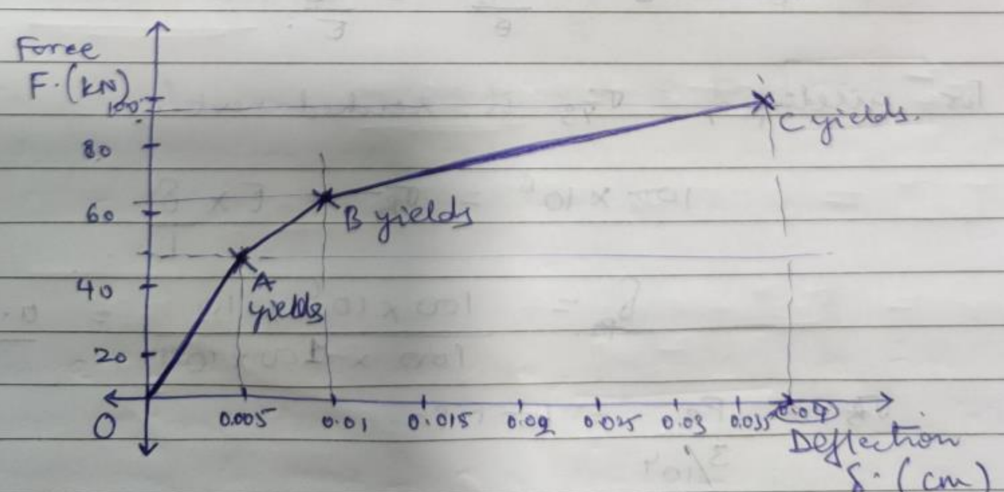
$400 \times 10^6 = \sigma_c = E \cdot \dfrac{\delta}{L}$

$$\boxed{\delta_c = \frac{400 \times 10^6 \times 10}{100 \times 100 \times 10^9}} = 0.04\ cm$$

$F_c = \dfrac{400 \times 10^6}{10^4} = 40\,kN$

$F = F_A + F_B + F_c = 40 + 30 + 25$

$$\boxed{F = 95\,kN}$$



Given

```
l = 750;
t = 6 ;
```

$d_o = 250;$

$d_i = 250 - 12;$

W = 45 000;

p = 3.5;

r = 125;

T = W r;

M = W l;

$Ib = \frac{\pi}{64}\left(d_o{}^4 - d_i{}^4\right)$

$Jt = \frac{\pi}{32}\left(d_o{}^4 - d_i{}^4\right)$

$10\,901\,676\,\pi$

$21\,803\,352\,\pi$

Polar moment of Inertia

$Jt = \frac{\pi}{32}\left(d_o{}^4 - d_i{}^4\right)$

$21\,803\,352\,\pi$

Second Area moment of Inertia

$Ib = \frac{\pi}{64}\left(d_o{}^4 - d_i{}^4\right)$

In[∘]:= N[10 901 676 π]

Out[∘]= 3.42486×10^7

Two most critical points are A and B, Let the axis be zz and hoop be θθ

At A there will be bending stress and axial stress due to pressure

$\sigma zzb = \frac{M\,r}{Ib}$

$\sigma zzp = \frac{p\,r}{2\,t}$

$\sigma\theta\theta = \frac{p\,r}{t}$

$\tau z\theta = \frac{T\,r}{Jt}$

$\frac{351\,562\,500}{908\,473\,\pi}$

36.4583

72.9167

$\frac{29\,296\,875}{908\,473\,\pi}$

$N\left[\frac{351562500}{908473\,\pi}\right]$

123.18

$N\left[\frac{29296875}{908473\,\pi}\right]$

10.265

$\sigma zz = \sigma zzb + \sigma zzp$

159.638

$$\sigma 1 = \frac{\sigma zz + \sigma\theta\theta}{2} + \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$$\sigma 2 = \frac{\sigma zz + \sigma\theta\theta}{2} - \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$\sigma 3 = 0$

160.837

71.7182

0

$$\sigma vmA = \frac{1}{\sqrt{2}}\left(\sqrt{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2}\right)$$

139.56

```
Clear["Global`*"]
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At B,

$\sigma zzb = 0$

$\sigma zzp = \frac{p\,r}{2\,t}$

$\sigma\theta\theta = \frac{p\,r}{t}$

$\tau z\theta = \frac{T\,r}{Jt}$

0

36.4583

72.9167

$\frac{29296875}{908473\,\pi}$

$\sigma zz = \sigma zzb + \sigma zzp$

$$\sigma 1 = \frac{\sigma zz + \sigma\theta\theta}{2} + \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$$\sigma 2 = \frac{\sigma zz + \sigma\theta\theta}{2} - \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$\sigma 3 = 0$

75.6081

33.7669

0

$$\sigma vmB = \frac{1}{\sqrt{2}} \left(\sqrt{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2}\right)$$

65.602907971277`

At C in the bottom,

$$\sigma zzb = -\frac{M\,r}{Ib}$$

$$\sigma zzp = \frac{p\,r}{2\,t}$$

$$\sigma\theta\theta = \frac{p\,r}{t}$$

$$\tau z\theta = \frac{T\,r}{Jt}$$

$$-\frac{351\,562\,500}{908\,473\,\pi}$$

36.4583

72.9167

$$\frac{29\,296\,875}{908\,473\,\pi}$$

$$N\left[-\frac{351\,562\,500}{908\,473\,\pi}\right]$$

-123.18

$\sigma zz = \sigma zzb + \sigma zzp$

-86.7218

$$\sigma 1 = \frac{\sigma zz + \sigma\theta\theta}{2} + \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$$\sigma 2 = \frac{\sigma zz + \sigma\theta\theta}{2} - \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$\sigma 3 = 0$

73.574

-87.3791

0

$$\sigma vmB = \frac{1}{\sqrt{2}} \left(\sqrt{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2}\right)$$

139.56

Most critical location is A The vonMises is 139.56 and the Tresca gives 160 MPa

If the hydrostatic tests 1.5 p and load is 1.25 W

```
l = 750;
t = 6 ;
```

$d_o = 250;$

$d_i = 250 - 12;$

$W = 45\,000;$

$p = 1.5 \times 3.5;$

$r = 125;$

$T = 1.25\, W\, r;$

$M = 1.25\, W\, l;$

$$Ib = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

$$Jt = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

$10\,901\,676\,\pi$

$21\,803\,352\,\pi$

$$\sigma zzb = \frac{M\, r}{Ib}$$

$$\sigma zzp = \frac{p\, r}{2\, t}$$

$$\sigma\theta\theta = \frac{p\, r}{t}$$

$$\tau z\theta = \frac{T\, r}{Jt}$$

153.975

54.6875

109.375

12.8313

$\sigma zz = \sigma zzb + \sigma zzp$

208.663

$$\sigma 1 = \frac{\sigma zz + \sigma\theta\theta}{2} + \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$$\sigma 2 = \frac{\sigma zz + \sigma\theta\theta}{2} - \sqrt{\left(\frac{\sigma zz - \sigma\theta\theta}{2}\right)^2 + \tau z\theta^2}$$

$\sigma 3 = 0$

210.294

107.744

0

$$\sigma vmA = \frac{1}{\sqrt{2}}\left(\sqrt{(\sigma 1 - \sigma 2)^2 + (\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2}\right)$$

182.139

The factor of safety is 182.13 / 139.56 = 1.30

In[•]:= $\sigma = \frac{4 F x}{\pi \left(\frac{a x}{2 l} + \frac{a}{2}\right)^3}$

Out[•]= $\frac{4 F x}{\pi \left(\frac{a}{2} + \frac{a x}{2 l}\right)^3}$

In[•]:= D[σ , x]

Out[•]= $-\frac{6 a F x}{l \pi \left(\frac{a}{2} + \frac{a x}{2 l}\right)^4} + \frac{4 F}{\pi \left(\frac{a}{2} + \frac{a x}{2 l}\right)^3}$

In[•]:= Solve[D[σ , x] == 0, x]

Out[•]= $\left\{\left\{x \to \frac{l}{2}\right\}\right\}$