1. Estimate σ'_e in MPa for the following materials:
(a) AISI 1035 CD steel.
(b) AISI 1050 HR steel.
(c) 2024 T4 aluminum.
(d) AISI 4130 steel heat-treated to a tensile strength of 1620 MPa.

1-

a) $S_{ut} = 550$ MPa

$\sigma_e' = 0.5\, S_{ut} = 225$ MPa

b) $S_{ut} = 620$ MPa

$\sigma_e' = 0.5\, S_{ut} = 310$ MPa

c) Al. has no endurance limit

d) $S_{ut} = 1620$ MPa

$\because S_{ut} > 1400$ Ma

$\therefore \sigma_e' = 700$ MPa

2. A steel rotating-beam test specimen has an ultimate strength of 830 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 480 MPa.

$S_{ut} = 830$ MPa $\qquad \sigma_{rev} = 480$

1 MPa = 0.145 Kpsi

$S_{ut} = 170.35$ Kpsi

$f = 0.82$

$\sigma_e' = 415$ MPa

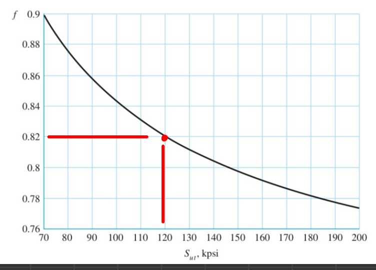


Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.

$$a = \frac{(f S_{ut})^2}{\sigma_e'} = 1116.184 \text{ MPa}$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} = 131413.2 \text{ cycles}$$

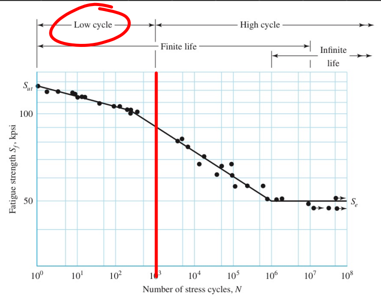
$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{\sigma_e'}\right) = -0.0716$$

3. A steel rotating-beam test specimen has an ultimate strength of 1030 MPa and a yield strength of 930 MPa. It is desired to test low-cycle fatigue at approximately 500 cycles. Check if this is possible without yielding by determining the necessary reversed stress amplitude.

$S_{ut} = 1030$ MPa
$S_{yt} = 930$ MPa
$N = 500$

Figure 6–10

An *S-N* diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel, normalized; S_{ut} = 116 kpsi; maximum S_{ut} = 125 kpsi. *(Data from NACA Tech. Note 3866, December 1966.)*



$S_{ut} = 149.35$ kpsi

$f = 0.79$

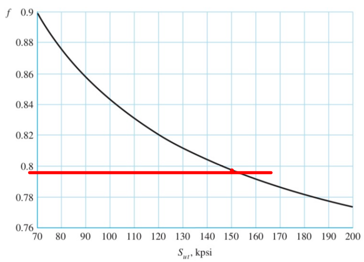
$$S_f = S_{ut} N^{(\log f)/3}$$

$$= 1030(500)^{(\log_{10} 0.79)/3}$$

$$= 833.17 \text{ MPa}$$

Figure 6–18

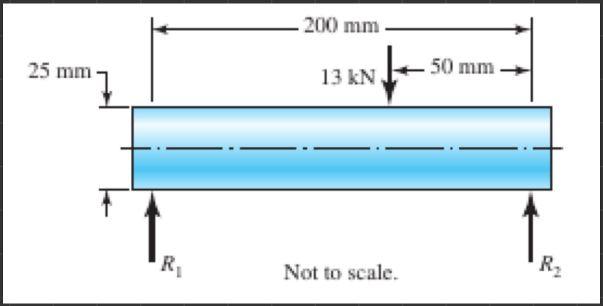
Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S_e' = 0.5S_{ut}$ at 10^6 cycles.



$\therefore S_f < S_{yt}$

Yes it is possible w/o yielding

4. A rotating shaft of 25-mm diameter is simply supported by bearing reaction forces R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine
(a) the minimum static factor of safety based on yielding.
(b) the endurance limit, adjusted as necessary with correction (Marin) factors.
(c) the minimum fatigue factor of safety based on achieving infinite life.
(d) If the fatigue factor of safety is less than 1, then estimate the life of the part in number of rotations of rotations.



$S_{ut} = 570$ MPa
$S_{yt} = 310$ MPa

$\Sigma M_{R_1} = 0$

$R_2 (200) - 13(150) = 0$

$R_2 = 9.75$ kN

$\Sigma F_y = 0 \Rightarrow R_1 + R_2 = 13 \Rightarrow R_1 = 3.25$ kN

$M_{max} = (9.75)(50) = 487.5$ kN-mm

$$\sigma_{rev} = \frac{M_m c}{I} = \frac{32M}{\pi d^3} = 317.8 \text{ MPa}$$

a) Min. static fact of safety (yielding)

$$n_y = \frac{S_{yT}}{\sigma_{rev}} = \frac{310}{317.8} = 0.97$$

b) $S_e' = 0.5\, S_{ut} = 0.5 \times 570 = 285$ MPa

$$K_a = a S_{ut}^{\,b} = 4.51 \times 570^{-0.265} = 0.839$$

$$K_b = 1.24 d^{-0.107} = 0.878$$

$K_c = 1$

$S_e = K_a K_b K_c \; S_e'$

$= 0.839 \times 0.878 \times 1 \times 285$

$S_e = 209.9$ MPa

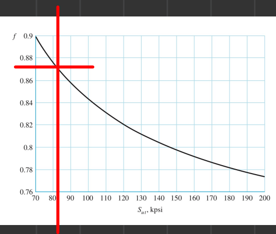
c) Min. fat. fos

$n_f = \dfrac{S_e}{\sigma_{rev}} = \dfrac{209.9}{317.8} = 0.66 \Rightarrow$ Infinite life is not predicted

d) $S_{ut} = 570 \times 0.145 = 82.65$

$f = 0.87$

Figure 6–18
Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S_e' = 0.5S_{ut}$ at 10^6 cycles.



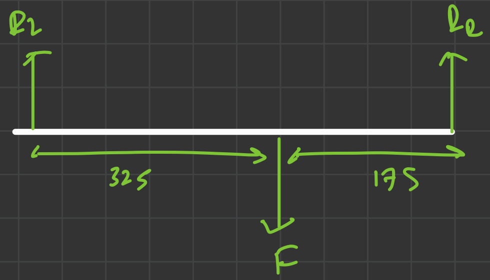
$a = \dfrac{(0.87 \times 570)^2}{209.9} = \dfrac{(fS_{ut})^2}{S_e}$

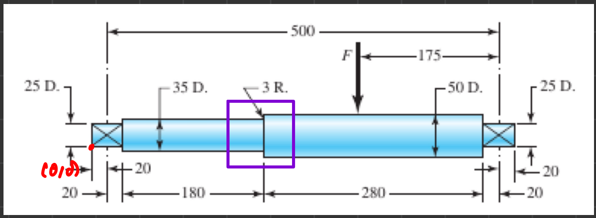
$a = 1171.59$

$b = -\dfrac{1}{3}\log\left(\dfrac{fS_{ut}}{S_e}\right) = -0.124$

$N = \left(\dfrac{\sigma_{rev}}{a}\right)^{1/b} = \left(\dfrac{317.8}{1171.59}\right)^{\frac{1}{-0.124}} = 3.7 \times 10^4$ cycles

5. The rotating shaft shown in the figure is machined from AISI 1020 CD (cold rolled) steel. It is subjected to a force of F = 6 kN. Find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding. All the dimensions are in mm.





$R_l + R_r = F \quad (\Sigma F_y = 0) \quad \rightarrow \Sigma M_z = 0$

$R_l + R_r = 6$

$R_l \times 325 = R_r \times 175$

$R_r = 1.86\, R_l$

$R_l = \dfrac{6}{1 + 1.86} = 2.1 \text{ kN}$

$S_{yt} = 390 \text{ MPa}$

$S_{ut} = 470 \text{ MPa}$

$R_r = 3.9 \text{ kN}$

Critical loc^n => B/w 35mm & 50mm

$M = (2.1)(200) = 420 \text{ kN·mm}$

$\sigma_{rev} = \left(\dfrac{Mc}{I}\right) = \dfrac{420/(35/2)}{\frac{\pi}{64}(35)^4} = 99.8 \text{ MPa} < S_{yt}$

No yielding

$\dfrac{r}{d} = 3/35 = 0.086$

$D/d = 50/35 = 1.43$

$K_t = 1.7$

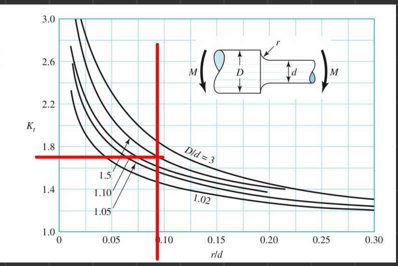


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

$S_{ut} = 68$ kpsi

$r = 3mm = 0.118$ in

$\sqrt{a} = 0.246 - 3.08 \times 10^{-3} \times 68 + 1.51 \times 10^{-5} \times 68^2 - 2.67 \times 10^{-8} (68^3)$

$\sqrt{a} = 0.0979$

$a = 9.6 \times 10^{-3}$

$q = \dfrac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \dfrac{1}{\dfrac{0.0979}{\sqrt{0.118}} + 1} = 0.778$

$K_f = 1 + q(K_t - 1) = 1 + (0.778)(1.7 - 1) = 1.54$

$S_e' = 0.5 \times 470 = 235$ MPa

$K_a = a S_{ut}^b \Rightarrow a = 4.51$

$b = -0.265$

Surface Finish	Factor a S_{ut}, kpsi	S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

$= 0.88$

$d = 35$ mm

$K_b = 1.24(35)^{-0.107}$

$\boxed{K_b = 0.8476}$

$K_c = 1$ {bending}

$S_e = K_a K_b K_c S_e' = 175.28$ MPa

$n_f = \dfrac{S_e}{K_f \sigma_{rev}} = \dfrac{175.28}{1.54 \times 99.8} = 1.14 \Rightarrow$ Infinite Life

6. A steel part is loaded with a combination of bending, axial, and torsion such that the following stresses are created at a particular location:

Bending: Completely reversed, with a maximum stress of 60 MPa

Axial: Constant stress of 20 MPa

Torsion: Repeated load, varying from 0 MPa to 70 MPa

Assume the varying stresses are in phase with each other. The part contains a notch such that $(K_f)_{\text{bending}} = 1.4$, $(K_f)_{\text{axial}} = 1.1$, and $(K_f)_{\text{torsion}} = 2.0$. The material properties are $\sigma_y = 300$ MPa and $\sigma_{ult} = 400$ MPa. The completely adjusted (corrected) endurance limit is found to be $\sigma_e = 160$ MPa. Find the factor of safety for fatigue based on infinite life, using the Goodman criterion (assume proportional loading). If the life is not infinite, estimate the number of cycles, using the SWT criterion to find the equivalent completely reversed stress. Be sure to check for yielding.

$K_{f,b} = 1.4$, $K_{f,t} = 2$, $S_y = 300$, $S_{ut} = 400$

$K_{f,a} = 1.1$

$S_e = \frac{1}{2}(400) = 200$

Bending , $\sigma_m = 0$, $\sigma_a = 60$

Axial , $\sigma_m = 20$, $\sigma_a = 0$

torsion, $\tau_m = 25$, $\tau_a = 25$

$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}]^2\right\}^{1/2} \tag{6–55}$$

$$\sigma'_m = \{[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}]^2\}^{1/2} \tag{6–56}$$

$$\sigma_a' = \left((1.4\times 60 + 0)^2 + 3(2\times 25)^2\right)^{1/2} = 120.65 \text{ MPa}$$

$$\sigma_m' = \left((1.4\times 20)^2 + 3(50)^2\right)^{1/2} = 89.35 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{120.6}{200} + \frac{89.35}{400} \Rightarrow n_f = 1.21$$

$$\sigma'_{max} = \sigma_a' + \sigma_m' = 209.95 < S_{yt}$$

$$n_y = \frac{S_y}{\sigma'_{max}}$$

$$= \frac{300}{209.95} = 1.43$$

7. A machine part will be cycled at ±350 MPa for $5x10^3$ cycles. Then the loading will be changed to ±260 MPa for $5x10^4$ cycles. Finally, the load will be changed to ±225 MPa. Using the Miner's rule, estimate the of cycles of operation that can be expected at this stress level before the part fails? For the part, $\sigma_{ult} = 530$ MPa, $f = 0.9$, and has a fully corrected endurance strength of $\sigma_e = 210$ MPa.

$S_{ut} = 530$ MPa ; $S_e = 210$
$f = 0.9$

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.9 \times 530)^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S_e}\right) = -0.118$$

$$\sigma_1 = 350, \; N_1 = \left(\frac{\sigma_1}{a}\right)^{1/b} = 14417$$

$$\sigma_2 = 260, \; N_2 = 179024$$

$$\sigma_3 = 225, \; N_3 = 609589$$

$$\frac{5000}{N_1} + \frac{50000}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{14417} + \frac{5 \times 10^4}{179024} + \frac{n_3}{609589} = 1$$

$$0.34 + 0.279 + \frac{n_3}{N_3} = 1$$

$$n_3 = (N_3)(0.381)$$

$$n_3 = 232253 \text{ cycles}$$

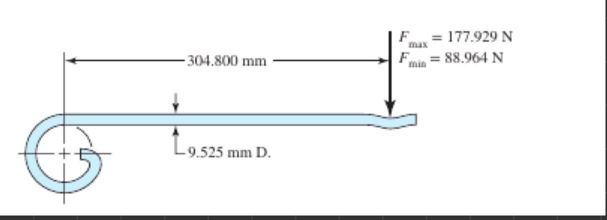
8. The figure shows a formed round-wire cantilever spring subjected to a varying force. The inner radius of the bend is 20 mm. The hardness tests made on 50 springs gave a minimum hardness of 400 Brinell. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. Estimate the number of cycles to likely to cause failure using the Goodman criterion.

1. If the curvature effects on the bending stress are ignored.
2. If the curvature effects on the bending stress are not ignored

1

$S_{ut} = 3.4\, H_B$

$= 1360$ MPa



$S_e' = 0.5\,(1360) = 680$ MPa

$a = 57.7$

$b = -0.718$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

$K_a = 57.7\,(1360)^{-0.718} = 0.324$

$d_e = 0.37\, d = (0.37)(9.525) = 3.524$ mm

$K_b = 1.24 \times (3.524)^{-0.107} = 1.08$

$\hookrightarrow K_b = 1$

$S_e = (0.324)(1)(680) = 220.32$

$F_a = 44.4825 = \frac{F_1 - F_2}{2}$

$F_b = \frac{F_1 + F_2}{2} = 133.44$

$\sigma_a = \frac{32\, M_a}{\pi d^3} = \frac{(32)(44.48)(304.8)}{\pi \left(\frac{9.525}{1000}\right)^3 \quad 1000} = 159.8$ MPa

$$\sigma_m = \frac{32 M_m}{\pi d^3} = 479.4 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{159.8}{270.37} + \frac{479.4}{1360} = 1.0778$$

$n_f = 0.93 \Rightarrow$ Not infinite life

$$\sigma_{rev} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{S_{ut}}\right)} = \frac{159.8}{1 - (479.4/1360)} = 246.8 \text{ MPa}$$

$f = 0.775$

$$a = \frac{(f S_{ut})^2}{S_e} = 5050$$

$$b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S_e}\right) = -0.2268$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} = 602817$$

Ganesh Iyer ME-423: Tut-1

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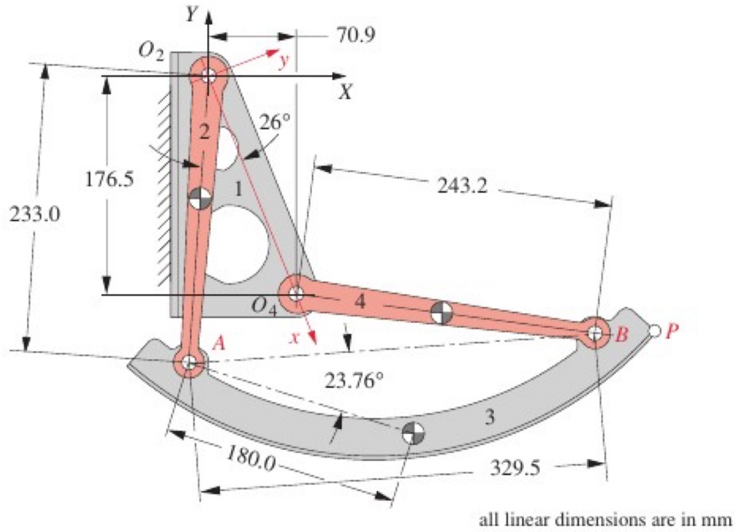
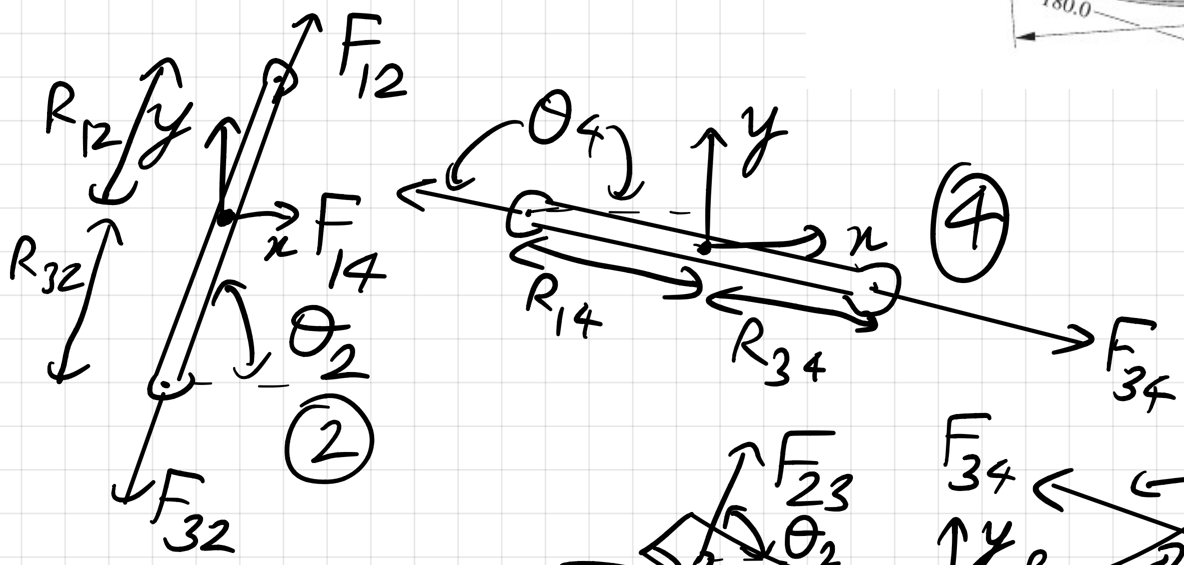
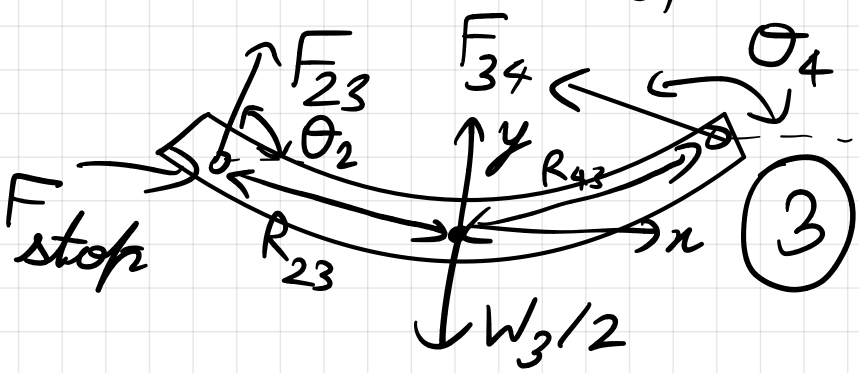
2. [Norton, Chapter 3] Figure shows an aircraft overhead bin mechanism in end view. For the position shown, draw free-body diagrams of links 2 and 4 and the door (3). There are stops that prevent further clockwise motion of link 2 (and the identical link behind it at the other end of the door) resulting in horizontal forces being applied to the door at points A. Assume that the mechanism is symmetrical so that each set of links 2 and 4 carry one half of the door weight. Ignore the weight of links 2 and 4 as they are negligible.
Also determine the pin forces on the door (3), and links 2 & 4 and the reaction force on each of the two stops. Available data:

R_{23}	180 mm @ 160.345°
R_{43}	180 mm @ 27.862°
W_3	45 N
θ_2	85.879°
θ_4	172.352°



all linear dimensions are in mm





Eqbm for 3 :

$$\Sigma F_x = 0 \Rightarrow F_{stop} + F_{23,x} + F_{43,x} = 0$$

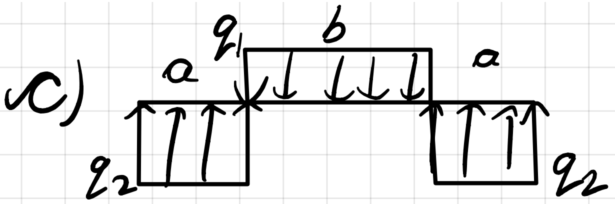
$$\Sigma F_y = 0 \Rightarrow F_{23,y} + F_{43,y} - W_3/2 = 0$$

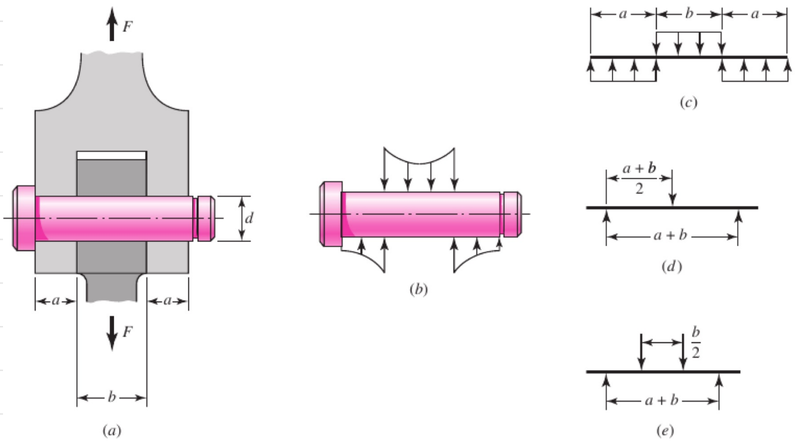
$$\Sigma M = 0 \Rightarrow -F_{stop} \cdot R_{23,x} + F_{23,y} \cdot R_{23,x} - F_{23,x} \cdot R_{23,y} + F_{43y} \cdot R_{43x} - R_{43,y} \cdot F_{43,x} = 0$$

We can solve for the variables, and get pin force F_{23}, F_{43}, F_{stop}, and reaction on 2, 4 $-F_{23}, -F_{43}$.

3. [Shigley, Chapter 3] A pin in a knuckle joint carrying a tensile load F deflects somewhat on account of this loading, making the distribution of reaction and load as shown in part (b) of the figure. A common simplification is to assume uniform load distributions, as shown in part (c). To further simplify, designers may consider replacing the distributed loads with point loads, such as in the two models shown in parts d and e. If a = 1.20 cm, b = 1.8 cm, d = 1.20 cm, and F = 4500 N, estimate the maximum bending stress and the maximum shear stress due to V for the three simplified models. Compare the three models from a designer's perspective in terms of accuracy, safety, and modeling time.

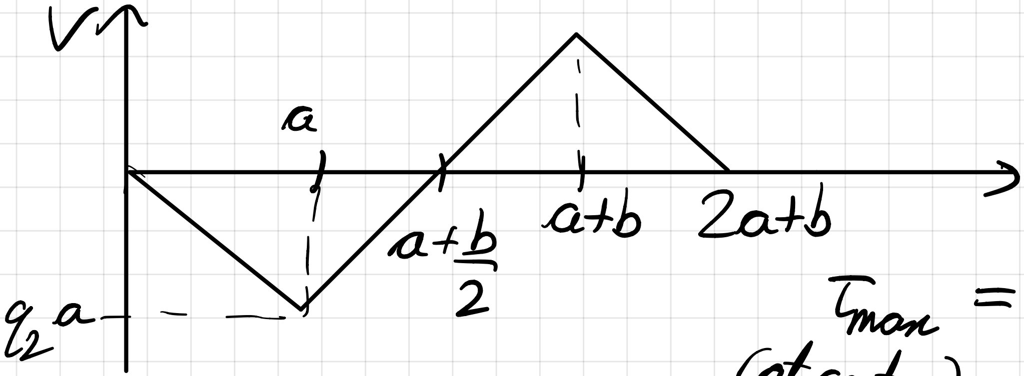
c)





$$q_1 b = F \Rightarrow q_1 = \frac{\cancel{4500}\times10^3}{\cancel{18}} = 250\ KN/m$$

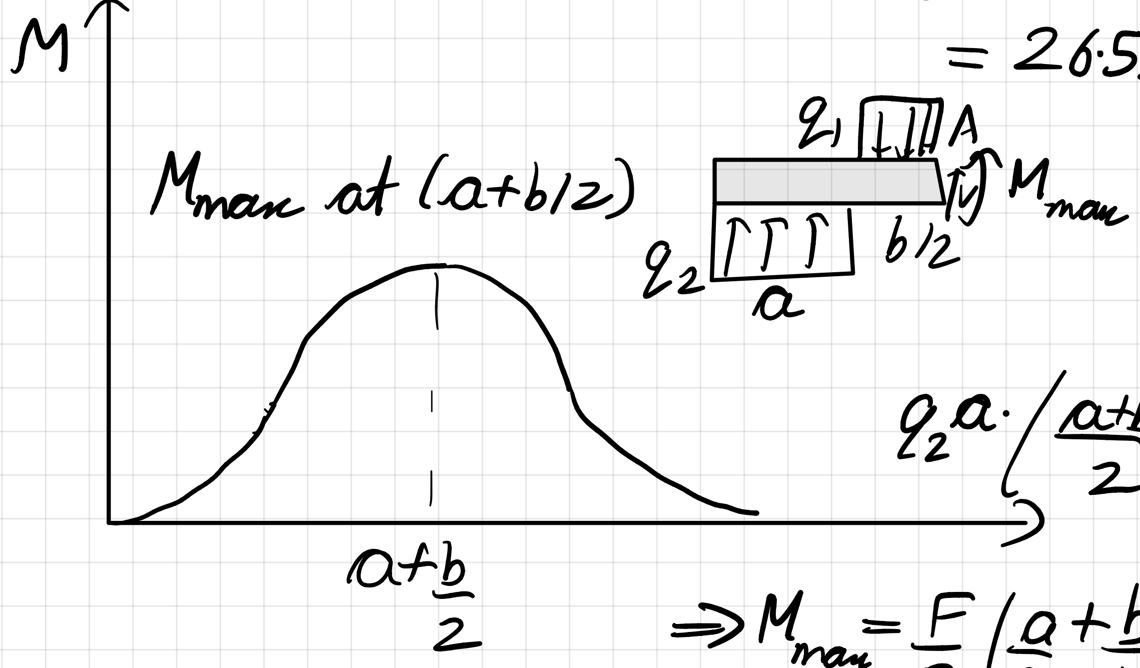
$$2q_2 a = F \Rightarrow q_2 = \frac{\cancel{4500}\times10^3}{2\times\cancel{12}} = 187.5\ KN/m$$



$V_{max} = q_2 a = F/2$

For circular beam,

$$\tau_{max} \text{ (at centre)} = \frac{4V_{max}}{3A} = \frac{4\times F/2}{3\times\pi d^2/4} = 26.52\ MPa$$



Balance T about A,

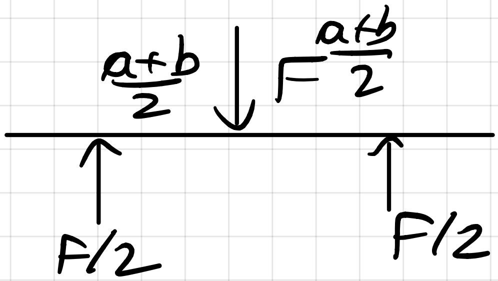
$$q_2 a\cdot\left(\frac{a+b}{2}\right) - \frac{q_1 b}{2}\cdot\frac{b}{4} = M_{max}$$

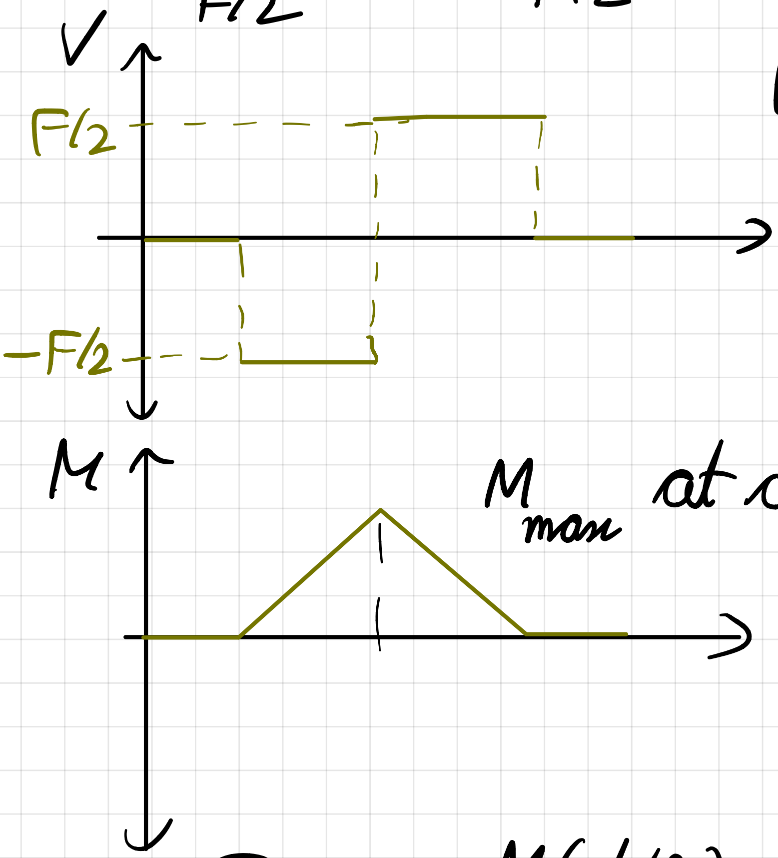
$$\Rightarrow M_{max} = \frac{F}{2}\left(\frac{a}{2}+\frac{b}{4}\right) = 23.625\ N\text{-}m$$

$$I = \frac{\pi d^4}{64} = 1.018\times10^{-9}$$

$$\sigma_{max} = \frac{M(d/2)}{I} = \frac{23.625\times 0.6\times10^{-2}}{1.018\times10^{-9}} = 139.26\ MPa$$

d)

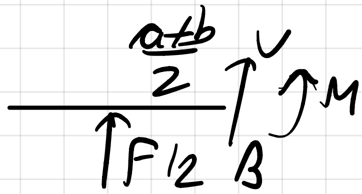




$V_{max} = F/2$

$$\tau_{max} = \frac{4\times F/2}{3A} = 26.52\ MPa$$

M_{max} at centre

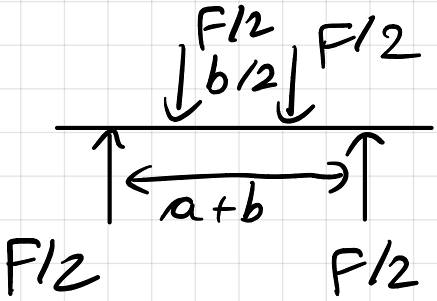


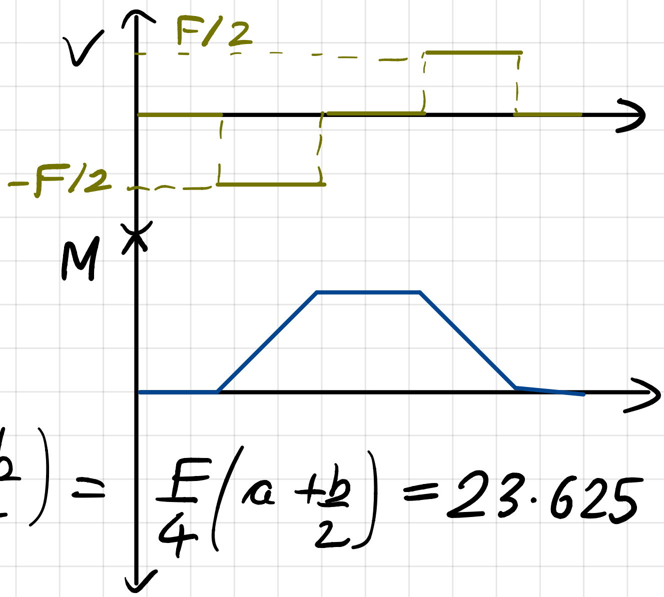
About B,

$$M = \frac{F}{2}\left(\frac{a+b}{2}\right) = 33.75\ N\text{-}m$$

$$\sigma_{max} = \frac{M(d/2)}{I} = 198.91\ MPa$$

e)





$V_{max} = F/2$

$$M_{max}\ (\text{around centre}) = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{b}{4}\right) = \frac{F}{4}\left(a + \frac{b}{2}\right) = 23.625\ N\text{-}m$$

$$\tau_{max} = \frac{4 \times F/2}{3A} = 26.52\ MPa$$

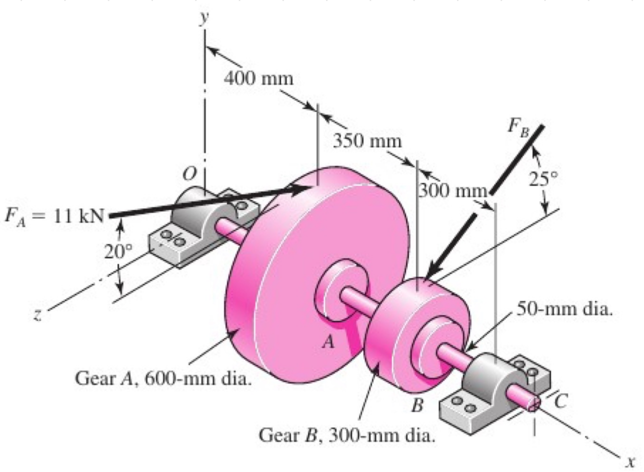
$$\sigma_{max} = \frac{M(d/2)}{I} = 139.26\ MPa$$

Distributed Load most accurate, since actual loads are not point loads.

τ_{max} same in all, but σ_{max} highest in 3-point loading, most conservative, good from safety point of view.

From modelling perspective, 3-point loading has min no. of loads so easiest to analyze.

5. [Shigley, Chapter 3]. A gear reduction unit uses the countershaft shown in the figure. Gear A receives power from another gear with the transmitted force F_A applied at the 20° pressure angle as shown. The power is transmitted through the shaft and delivered through gear B through a transmitted force F_B at the pressure angle shown.
(a) Determine the force F_B, assuming the shaft is running at a constant speed.
(b) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.
(c) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
(d) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
(e) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

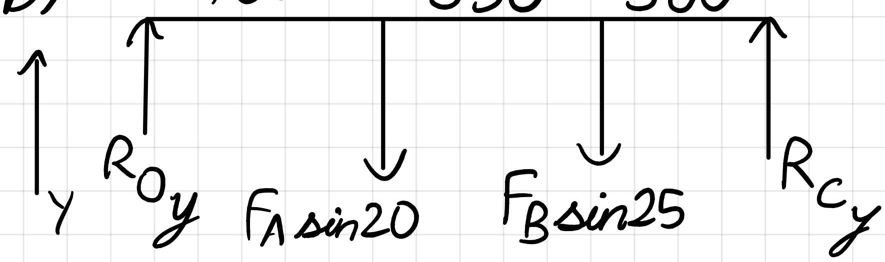


a) Torque Balance about axis,

$$F_A \cos 20 \times r_A = F_B \cos 25\, r_B$$

$$\Rightarrow F_B = 22.81\ KN$$

b)

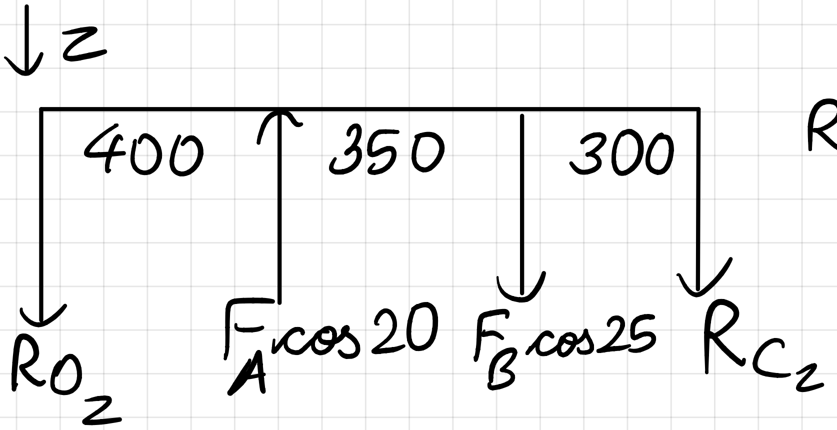


$R_{O_y} + R_{C_y} = 13.4$ KN

$F_A \sin 20 \times 400 + F_B \sin 25 \times 750 = R_{C_y} \times 1050$

$R_{C_y} = 8.32$ KN

$R_{O_y} = 5.08$ KN



$R_{O_z} + R_{C_z} = -10.33$

$F_A \cos 20 \times 400 = F_B \cos 25 \times 750 + R_{C_z} \times 1050$

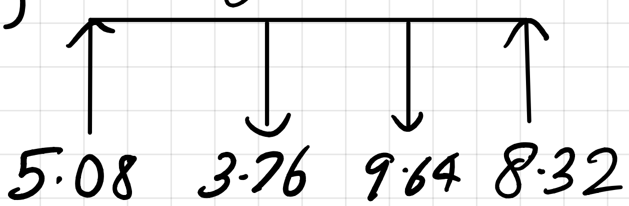
$R_{C_z} = -10.83$ KN

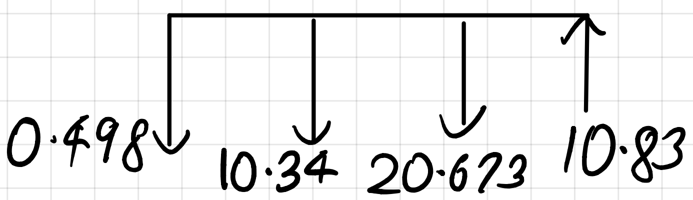
$R_{O_z} = 0.498$ KN

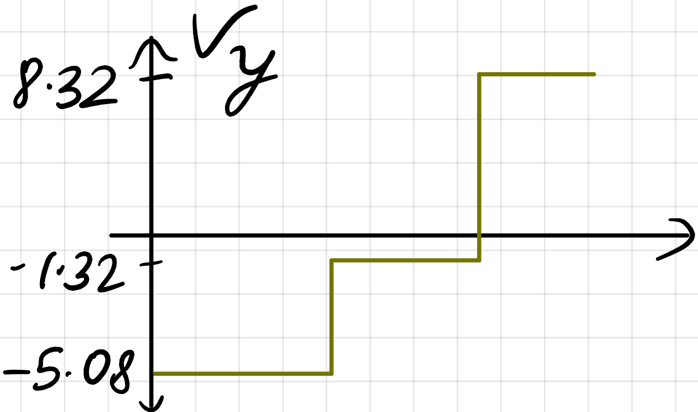
$R_C = \sqrt{R_{C_y}^2 + R_{C_z}^2} = 13.657$ KN

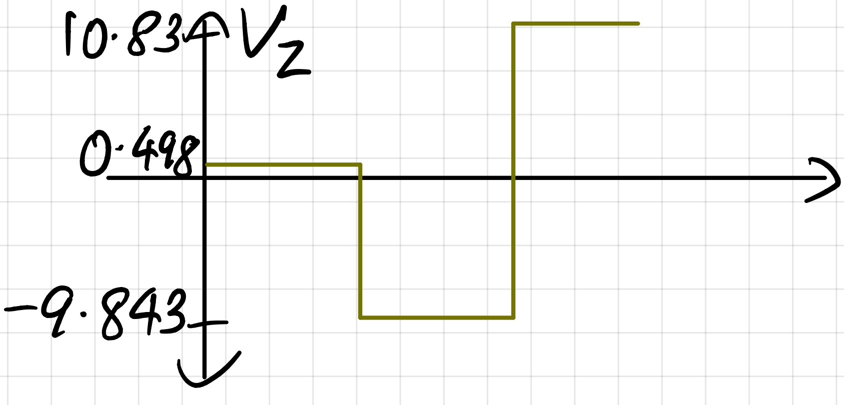
$R_O = \sqrt{R_{O_y}^2 + R_{O_z}^2} = 5.1$ KN

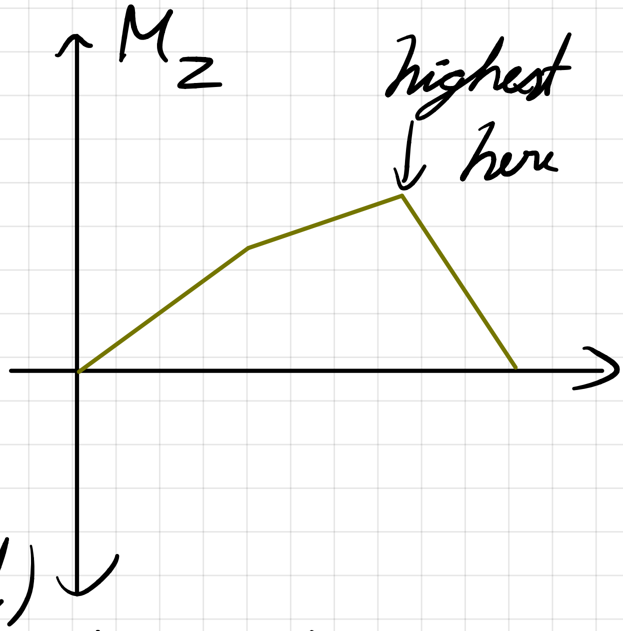
c)

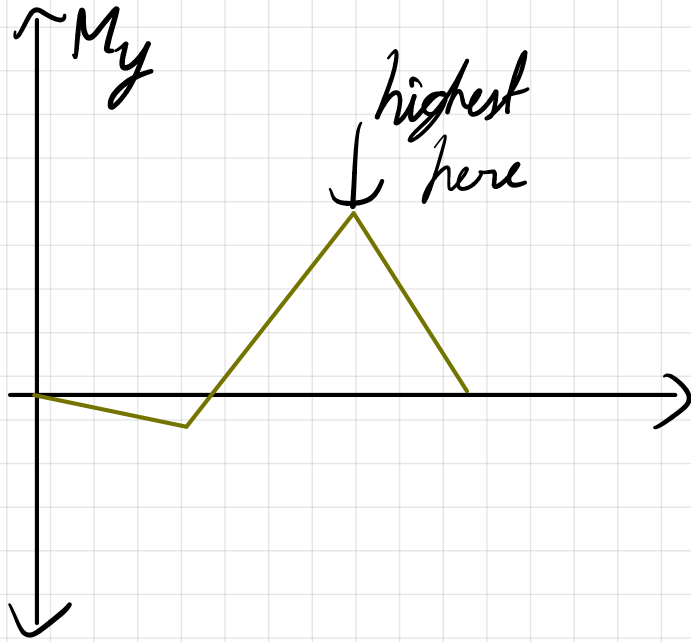












d)

Critical location at B, max bending moment.

$$M_z = 300 \times R_{C_y} = 2.5 \text{ KN-m}$$

$$M_y = 300 \times R_{C_z} = 3.25 \text{ KN-m}$$

$$M = \sqrt{M_y^2 + M_z^2} = 4.1 \text{ KN-m}$$

$$T = 11 \cos 20 \times r_A = 3.1 \text{ KN-m}$$

$$\sigma = \frac{Mr}{I} = \frac{Md}{2 \times \frac{\pi d^{\cancel{4}3}}{\cancel{64}32}} = \frac{32M}{\pi d^3} = \frac{32 \times 4.1}{\pi (0.05)^3} = 334 \text{ MPa}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = 126.3 \text{ MPa}$$

e) $$\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$= 376, -42.4 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 209 \text{ MPa}$$

ME-423: Tut-2

Ganesh Iyer,
210100059

1. Estimate σ_e' in MPa for the following materials:
(a) AISI 1035 CD steel.
(b) AISI 1050 HR steel.
(c) 2024 T4 aluminum.
(d) AISI 4130 steel heat-treated to a tensile strength of 1620 MPa.

a) $\sigma_{ult} = 552\ \text{MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 276\ \text{MPa}$

b) $\sigma_{ult} = 621\ \text{MPa} \Rightarrow \sigma_e' = 0.5\sigma_{ult} = 310.5\ \text{MPa}$

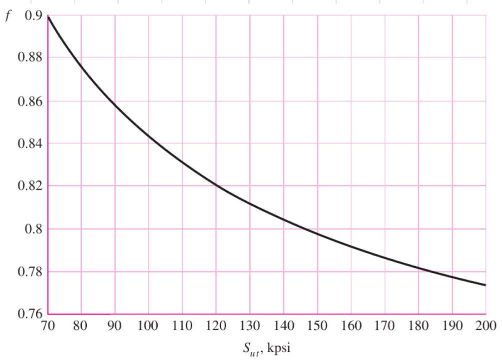
c) $\sigma_{ult} = 441\ \text{MPa (Al)} \Rightarrow \sigma_{f @ 5\times10^8} = 130\ \text{MPa} \left(\sigma_{ult} > 300\right)$

d) $\sigma_{ult} = 1620\ \text{MPa} (> 1400\ \text{MPa}) \Rightarrow \sigma_e' = 700\ \text{MPa}$

2. A steel rotating-beam test specimen has an ultimate strength of 830 MPa. Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 480 MPa.

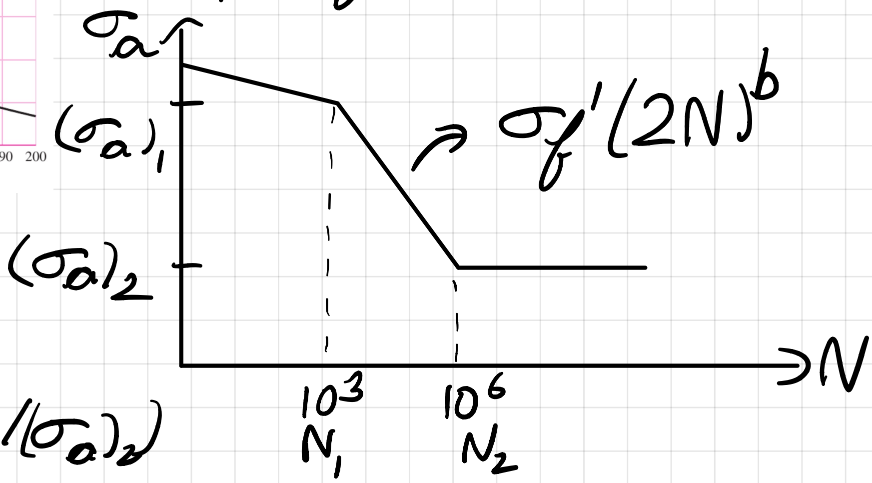
2. $\sigma_{ult} = 830\ \text{MPa} \qquad \sigma_e' \approx 0.5\sigma_{ult} = 415\ \text{MPa} \rightarrow (\sigma_a)_2$

$= \dfrac{830}{6.894}\ \text{Kpsi} = 120.38\ \text{Kpsi}$



$f \approx 0.82$

$(\sigma_a)_1 \approx f\sigma_{ult} = 680.6\ \text{MPa}$



$\Rightarrow b = \dfrac{\ln\left((\sigma_a)_1/(\sigma_a)_2\right)}{\ln(N_1/N_2)} = -0.0716, \quad \sigma_f' = \dfrac{(\sigma_a)_2}{(2N_2)^b} = 1172.99 \approx 1173\ \text{MPa}$

N for $\sigma_a = 480\ \text{MPa}$

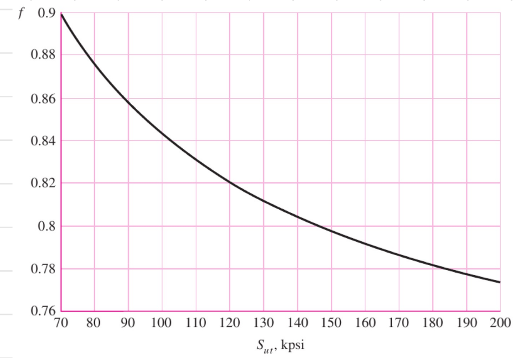
$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 131431.81$$

$$\Rightarrow N \approx 131400 \text{ cycles}$$

3. A steel rotating-beam test specimen has an ultimate strength of 1030 MPa and a yield strength of 930 MPa. It is desired to test low-cycle fatigue at approximately 500 cycles. Check if this is possible without yielding by determining the necessary reversed stress amplitude.

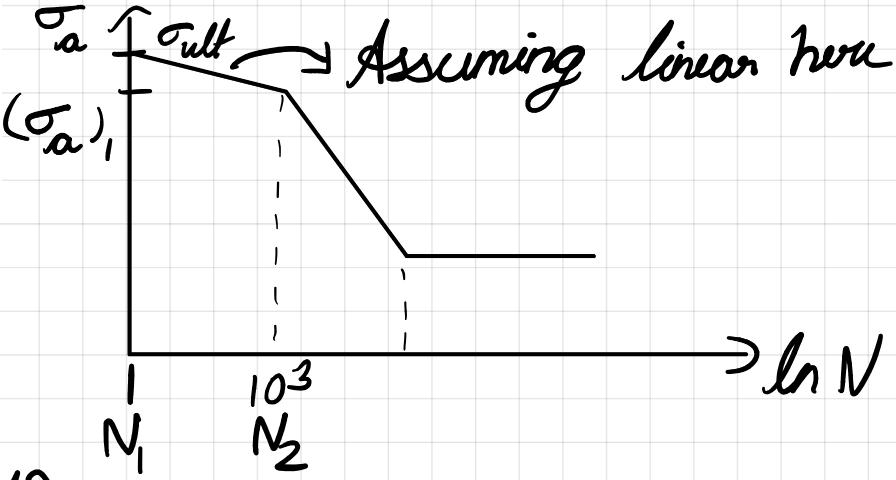
3. $\sigma_{ult} = 1030$ MPa $= \frac{1030}{6.894}$ Kpsi $= 149.4$ Kpsi

$\sigma_e' \approx 0.5\sigma_{ult} = 515$ MPa $\rightarrow (\sigma_a)_2$



$f \approx 0.8$

$(\sigma_a)_1 \approx f\sigma_{ult} = 824$ MPa



$(\sigma_a)_1 \approx f\sigma_{ult}$, $f \sim 0.8$

$\Rightarrow (\sigma_a)_1 = 824$ MPa

Taking relation as $\sigma_f = aN^b$,

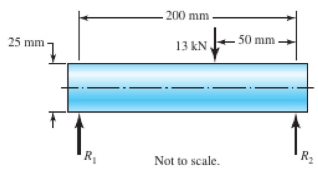
$\sigma_{ult} = aN_1^b \Rightarrow 1030 = a(1)^b \Rightarrow a = 1030$

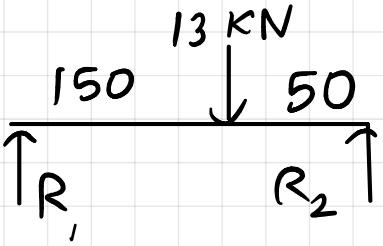
$824 = 1030 \times (10^3)^b \Rightarrow b = -0.032$

At $N = 500$, $\sigma_a = 1030 \times 500^{-0.032}$
$= 844.25$ MPa

$\sigma_a < \sigma_y = 930$ MPa $\Rightarrow$ Does not yield

4. A rotating shaft of 25-mm diameter is simply supported by bearing reaction forces R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine
(a) the minimum static factor of safety based on yielding.
(b) the endurance limit, adjusted as necessary with correction (Marin) factors.
(c) the minimum fatigue factor of safety based on achieving infinite life.
(d) If the fatigue factor of safety is less than 1, then estimate the life of the part in number of rotations of rotations.



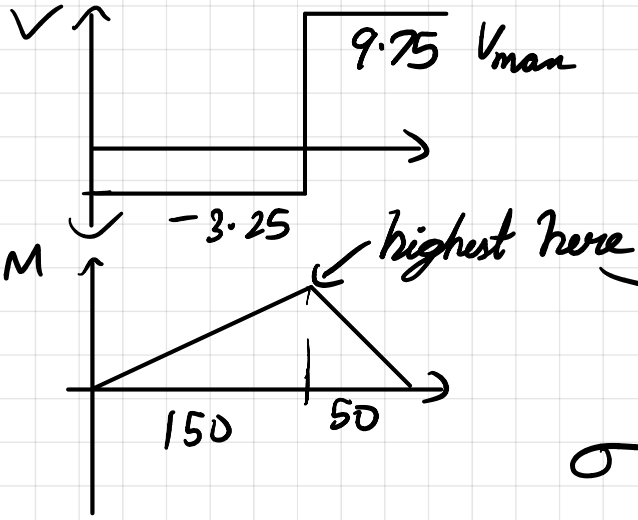


$3R_1 = R_2$ (moment balance)

$R_1 + R_2 = 13$

$\Rightarrow R_1 = \frac{13}{4}, \; R_2 = \frac{39}{4}$ KN

a)



$M = \frac{50}{1000} \times R_2 = 487.5$ N-m

$\sigma = \frac{M \times d/2}{\pi d^4/64} = \frac{32M}{\pi d^3}$

$\sigma = 317.8$ MPa

Top, bottom

$\tau = \frac{4V_{max}}{3A} = \frac{4V_{max}}{3(\pi d^2/4)} = 26.48$ MPa

At center

$\|\tau\| << \|\sigma\|$. Hence critical points top/bottom.

σ_y (1045 HR Steel) = 310 MPa

σ_{ult} " $= 565$ MPa

$N = \sigma_y / \sigma = 0.975$

$\Rightarrow$ It will fail by yielding

b) $\sigma_e' = 0.5\,\sigma_{ult} = 0.5 \times 565 = 282.5$ MPa

K_a (Surface) $= a\,\sigma_{ult}^{b}$, For machining,

$= 4.51(565)^{-0.265} = 0.8411$

$K_b(\text{Size}) = 1.24\, d^{-0.107} = 0.8787$

$K_c(\text{Load}) = 1,\ K_d, K_e, K_f = 1$ (No info given)

$\Rightarrow \sigma_e = 0.8411 \times 0.8787\, \sigma_e' = 208.8 \ \text{MPa}$

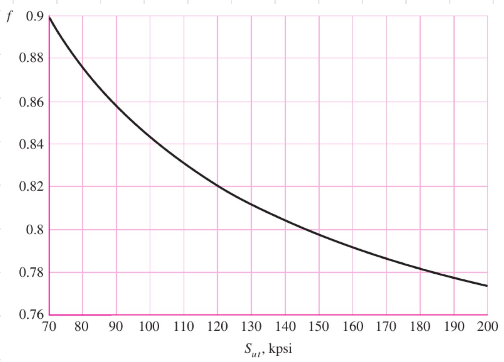
c) $N_f = \dfrac{\sigma_e}{\sigma} = \dfrac{208.8}{317.8}$

$\Rightarrow N_f = 0.657$ Fatigue factor of safety

d) As $N_f < 1$, finite life

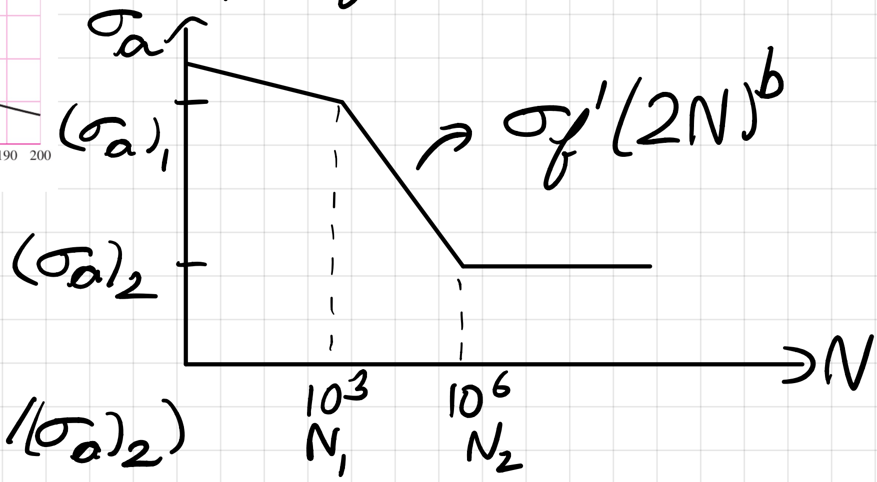
$\sigma_{ult} = 565 \ \text{MPa} = \dfrac{565}{6.894} \ \text{Kpsi} = 81.97 \ \text{Kpsi}$

$(\sigma_a)_2 = \sigma_e = 208.8 \ \text{MPa}$



$f \approx 0.875$

$(\sigma_a)_1 \approx f\,\sigma_{ult} = 494.375 \ \text{MPa}$



$$\Rightarrow b = \frac{\ln\left((\sigma_a)_1/(\sigma_a)_2\right)}{\ln(N_1/N_2)} = -0.12477$$

$$\sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 1276.27 \ \text{MPa}$$

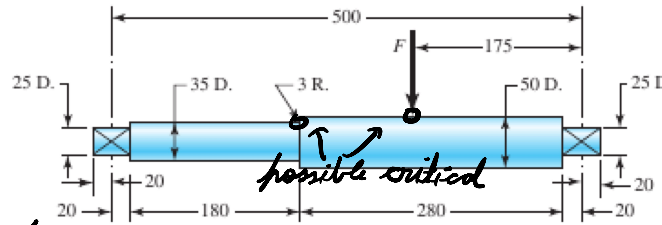
For $\sigma_a = 317.8$ MPa,

$$N = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = 34528.96$$

$\Rightarrow N \approx 34500$ cycles

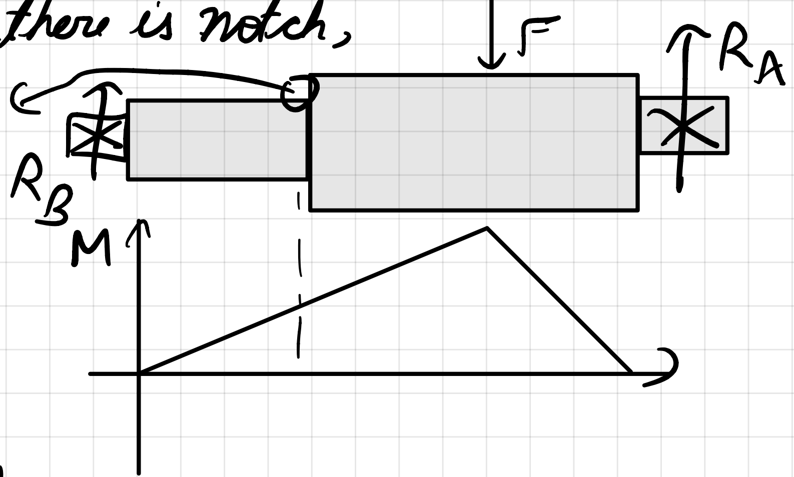
But as $\sigma_a > \sigma_y$, It will yield before fatigue.

5. The rotating shaft shown in the figure is machined from AISI 1020 CD (cold rolled) steel. It is subjected to a force of F = 6 kN. Find the minimum factor of safety for fatigue based on infinite life. If the life is not infinite, estimate the number of cycles. Be sure to check for yielding. All the dimensions are in mm.



$$\sigma_0 = \frac{32M}{\pi d^3}$$

As $\sigma_0 \propto 1/d^3$, and there is notch, it would be higher at



$R_A + R_B = F = 6$

$\circlearrowright$ Balance,

$R_B \times 500 = F \times 175$

$\Rightarrow R_B = \frac{7}{20} F = 2.1$ kN $\Rightarrow R_A = 3.9$ kN

M at critical point : $M = R_A \times 300 - F(125)$

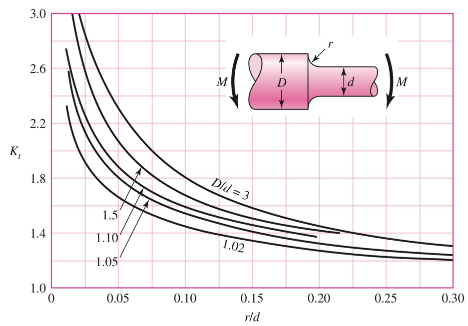
$= 420$ N-m

$$\sigma_0 = \frac{32 \times 420}{\pi(35\times10^{-3})^3} = 99.78 \text{ MPa}$$

Notch (3mm radius) $\Rightarrow$ $\frac{r}{d} = \frac{3}{35} = 0.085$, $\frac{D}{d} = \frac{50}{35} \approx 1.5$

Figure A–15–9

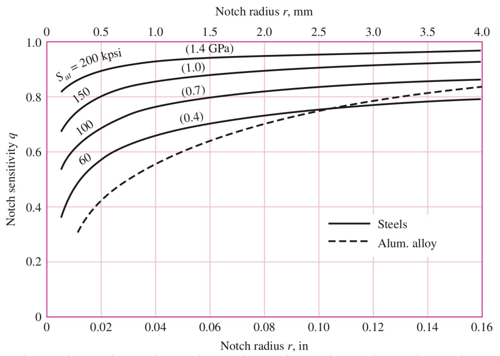
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$\Rightarrow K_t \approx 1.7$

Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



$\sigma_{ult} = 60.9$ kpsi

$\Rightarrow q \approx 0.78$

$$K_f = 1 + q(K_t - 1) = 1.546$$

$$\sigma_{max} = K_f \sigma_0 = 154.26 \text{ MPa}$$

$$\sigma_c' = 0.5\sigma_{ult} = 0.5 \times 420 = 210 \text{ MPa}$$

Taking highest D for lowest size factor K_b,

$$K_b = 1.24 \times 50^{-0.107} = 0.816$$

$$K_a = 4.51 \times 420^{-0.265} = 0.91$$

(Machined)

$$\Rightarrow \sigma_c = K_a \cdot K_b \sigma_c' = 155.93 \text{ MPa}$$

As $\sigma_{max} < \sigma_c$, infinite life

$\sigma_y = 350$ MPa, $\sigma_{max} < \sigma_y$, No yielding

6. A steel part is loaded with a combination of bending, axial, and torsion such that the following stresses are created at a particular location:

Bending: Completely reversed, with a maximum stress of 60 MPa

Axial: Constant stress of 20 MPa

Torsion: Repeated load, varying from 0 MPa to 70 MPa

Assume the varying stresses are in phase with each other. The part contains a notch such that $(K_f)_{\text{bending}} = 1.4$, $(K_f)_{\text{axial}} = 1.1$, and $(K_f)_{\text{torsion}} = 2.0$. The material properties are $\sigma_y = 300$ MPa and $\sigma_{ult} = 400$ MPa. The completely adjusted (corrected) endurance limit is found to be $\sigma_e = 160$ MPa. Find the factor of safety for fatigue based on infinite life, using the Goodman criterion (assume proportional loading). If the life is not infinite, estimate the number of cycles, using the SWT criterion to find the equivalent completely reversed stress. Be sure to check for yielding.

(Complete Reverse)

$\sigma_{bending} = (K_f)_{bending} \times \sigma_0$

$= 1.4 \times 60 = 84$ MPa

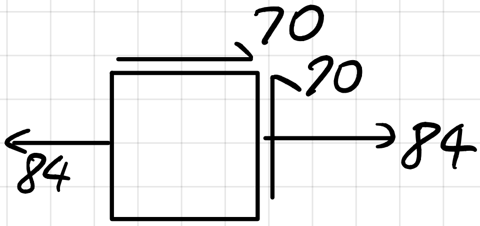
$\sigma_{axial} = 1.1 \times 20$

$= 22$ MPa

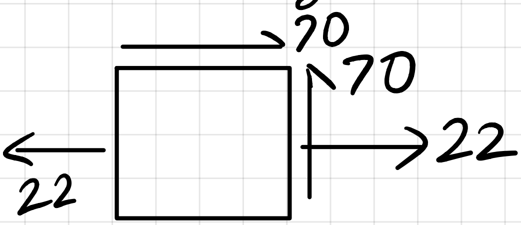
$\sigma_{torsion} = 0$ to $\underbrace{70 \times 2}$

140 MPa

i.e mean 70 MPa, alternating 70 MPa



Alternating



Mean

Von-Mises:

$\sigma_{vm,a} = (84^2 + 3 \times 70^2)^{1/2}$

$= 147.5$ MPa

$\sigma_{vm,m} = (22^2 + 3 \times 70^2)^{1/2}$

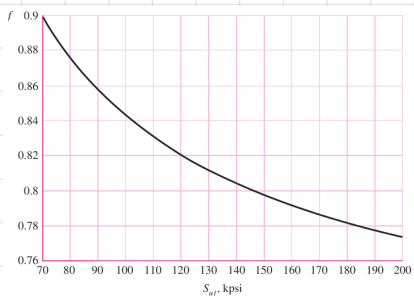
$= 123.22$ MPa

$$\frac{\sigma_{vm,a}}{\sigma_e} + \frac{\sigma_{vm,m}}{\sigma_{ult}} = \frac{147.5}{160} + \frac{123.22}{400} = 1.23 > 1$$

$\Rightarrow$ Point lies above Goodman line, finite life.

SWT: $\sigma_a' = \sqrt{(\sigma_m + \sigma_a)\sigma_a} = \sqrt{(123.22 + 147.5)(147.5)}$

$= 199.83$ MPa



$\sigma_{ult} = 53$ Kpsi $\Rightarrow f \approx 1$

$(\sigma_a)_1 = 400$ MPa, $(\sigma_a)_2 = \sigma_e = 160$ MPa

$b = \ln(400/160) / \ln(10^3/10^6) = -0.1326$

$\sigma_f' = (\sigma_a)_1 / (2000)^b = 1096.3$ MPa

For equivalent σ_a',

$$\Rightarrow N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{199.83}{1096.3}\right)^{1/b} \approx 188000 \text{ cycles}$$

$$\sigma_{max} = \sigma_m + \sigma_a = 270.72 < \sigma_y = 300 \text{ MPa}$$

(Not yielded)

7. A machine part will be cycled at ±350 MPa for $5\text{x}10^3$ cycles. Then the loading will be changed to ±260 MPa for $5\text{x}10^4$ cycles. Finally, the load will be changed to ±225 MPa. Using the Miner's rule, estimate the of cycles of operation that can be expected at this stress level before the part fails? For the part, σ_{ult} = 530 MPa, f = 0.9, and has a fully corrected endurance strength of σ_e = 210 MPa.

$(\sigma_a)_1 = f\sigma_{ult} = 477 \text{ MPa}$, $(\sigma_a)_2 = \sigma_e = 210 \text{ MPa}$

$N_1 = 10^3$, $N_2 = 10^6$ cycles

$$\sigma_a = \sigma_f'(2N)^b \Rightarrow b = \frac{\ln((\sigma_a)_1/(\sigma_a)_2)}{\ln(N_1/N_2)} = -0.1187$$

$$\Rightarrow \sigma_f' = \frac{(\sigma_a)_1}{(2N_1)^b} = 1176.44 \text{ MPa}$$

Now, $(\sigma_a)_A = 350 \text{ MPa}$, $n_A = 5\times10^3$

$$N_A = \frac{1}{2}\left(\frac{(\sigma_a)_A}{\sigma_f'}\right)^{1/b} = 13553.7$$

$$\Rightarrow N_A \approx 13500 \text{ cycles}$$

$(\sigma_a)_B = 260 \text{ MPa}$, $n_A = 5\times10^4$

$$N_B = \frac{1}{2}\left(\frac{(\sigma_a)_B}{\sigma_f'}\right)^{1/b} = 165585.2$$

$$\Rightarrow N_A \approx 165500 \text{ cycles}$$

So that estimate for n_c is low, conservative

$(\sigma_a)_C = 225 \text{ MPa}$

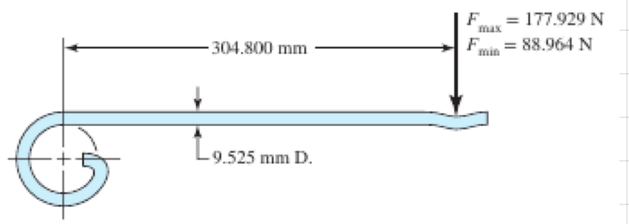
$$N_C = \frac{1}{2}\left(\frac{(\sigma_a)_C}{\sigma_f'}\right)^{1/b} = 559388.5$$

$$\approx 559300 \text{ cycles}$$

$$n_C = N_C\left(1 - \frac{n_A}{N_A} - \frac{n_B}{N_B}\right)$$

$$n_C = 183179 \approx 183100 \text{ cycles}$$

8. The figure shows a formed round-wire cantilever spring subjected to a varying force. The inner radius of the bend is 20 mm. The hardness tests made on 50 springs gave a minimum hardness of 400 Brinell. A visual inspection of the springs indicates that the surface finish corresponds closely to a hot-rolled finish. Estimate the number of cycles to likely to cause failure using the Goodman criterion.
1. If the curvature effects on the bending stress are ignored.
2. If the curvature effects on the bending stress are not ignored



$$\sigma_{ult} \approx 3.1\ HB = 1240\ MPa \quad , \quad F_m = 133.4465\,N$$
$$F_a = 44.825\,N$$

1) 

$$M = 0.3048 \times F, \quad \sigma = \frac{32M}{\pi d^3} = 3592693 \times F\ Pa$$

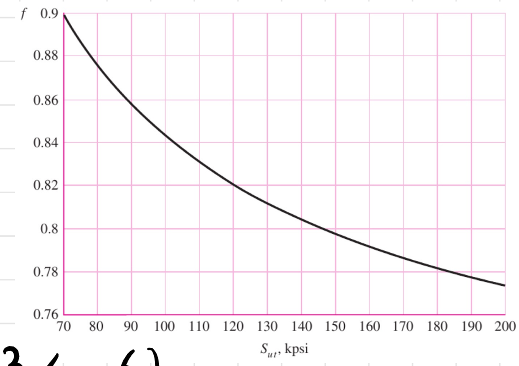
$$\sigma_a = 159.811\ MPa \quad , \quad \sigma_a' = \frac{\sigma_a}{\left(1 - \frac{\sigma_m}{\sigma_{ult}}\right)}$$

$$\sigma_m = 475.77\ MPa$$

$$\sigma_a' = 259.3\ MPa$$

$$\sigma_{ult} = 1240\ MPa = 179.86\,Kpsi$$
$$\sigma_e' = 0.5\,\sigma_{ult} = 620\,MPa$$
$$K_a = 57.7(1240)^{-0.718} \text{ Hot rolled}$$
$$= 0.3468$$
$$K_b = 1.24(9.525)^{-0.107} = 0.974$$
$$\sigma_e = K_a \cdot K_b \cdot \sigma_e' = 209.485\,MPa$$

$$\sigma_a' > \sigma_e \Rightarrow \text{Finite Life}$$



$f \approx 0.78$

$(\sigma_a)_1 = f\sigma_{ult} = 967.2$ MPa

$(\sigma_a)_2 = \sigma_c = 209.485$ MPa

$b = \ln((\sigma_a)_1/(\sigma_a)_2)/\ln(10^3/10^6)$

$= -0.2214$

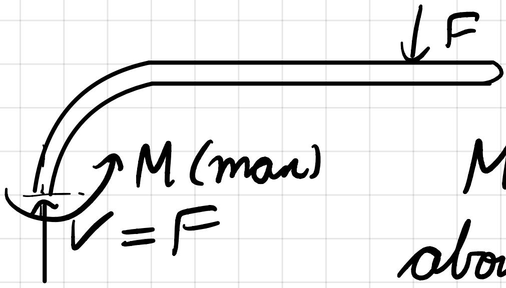
$\sigma_f' = \dfrac{(\sigma_a)_1}{(2N_1)^b} = 5204.32$ MPa

At $\sigma_a' = 259.3$ MPa,

$N = \dfrac{1}{2}\left(\dfrac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 382907$

≈ 382900 cycles

2)



$M = (304.8 + 20 + \frac{1}{2} \times 9.525) \times F$ about neutral axis

$= 0.3295625\,F$ Nm

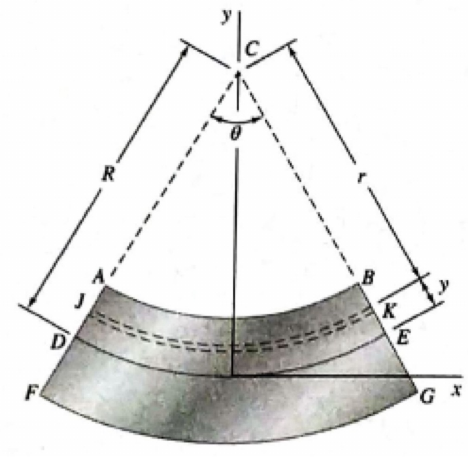
$\bar{r} = 20 + \dfrac{1}{2} \times 9.525$

$= 24.7625$, $c = 0.5 \times 9.525 = 4.7625$ mm

$R = \dfrac{A}{2\pi(\bar{r} - \sqrt{\bar{r}^2 - c^2})} = 24.53$ mm

$\sigma_{\theta\theta} = \dfrac{-My}{A(R-y)e}$, $e = \bar{r} - R = 0.2325$ mm, $y = R - r$

$$\Rightarrow \sigma_{\theta\theta}(r) = \frac{-M(R-r)}{\pi c^2 r e} = -\frac{M}{\pi c^2 e}\left(\frac{R}{r} - 1\right)$$



Highest at $r = r_i = 20\,mm$

$$\sigma_{\theta\theta,max} = 4.50057\,F \text{ MPa}$$

$$\sigma_m = 4.50057\,F_m = 601.27\,\text{MPa}$$

$$\sigma_a = 201.97\,\text{MPa}$$

$$\sigma_a' = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_{ult}}} = 392.091\,\text{MPa}$$

At $\sigma_a' = 392.091$ MPa, (Basquin found in (1))

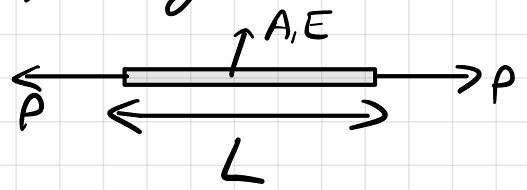
$$N = \frac{1}{2}\left(\frac{\sigma_a'}{\sigma_f'}\right)^{1/b} = 60591.62$$

$$\approx 60500 \text{ cycles}$$

Ganesh Iyer, 210100059 ME-423: Tutorial-3

1. Derive the material indices for the following cases:
   a. A light truss with stiffness greater than S^*
   b. A light shaft with stiffness greater than S^*

a) Objective: min m Constraint: $S \geq S^*$



$$\delta = \frac{PL}{EA}$$ Truss → Axial

$$S = \frac{P}{\delta} = \frac{EA}{L} \geq S^*$$

$$m = \rho A L \Rightarrow A = \frac{m}{\rho L}$$

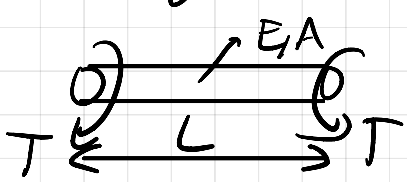
$$\Rightarrow \frac{E}{L} \times \frac{m}{\rho L} \geq S^* \Rightarrow m \geq S^* \times \left(\frac{\rho}{E}\right) \times L^2$$

Material

$$\min(m) \Rightarrow \max\left(\frac{E}{\rho}\right)$$

$\frac{E}{\rho}$: material index

b) Objective: min m Constraint: $S \geq S^*$



$$\theta = \frac{TL}{GJ}$$ Shaft → Torsion

$$S = \frac{T}{\theta} = \frac{GJ}{L} \geq S^*$$

$$m = \rho A L \Rightarrow A = \frac{m}{\rho L}$$ Let us take circular c/s

$$\Rightarrow A = \pi d^2, \quad J = \frac{\pi d^4}{32} \quad \Rightarrow \quad J = \frac{A^2}{32\pi}$$

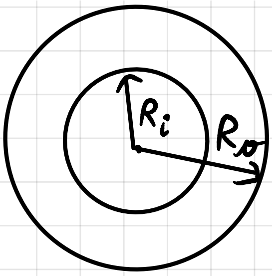
$$\Rightarrow J = \frac{m^2}{32\pi \rho^2 L^2}$$

Now, $$\frac{G}{L}\left(\frac{m^2}{32\pi\rho^2 L^2}\right) \geqslant S^*$$

$$\Rightarrow m \geqslant S^{*1/2} \times \left(\frac{\rho}{\sqrt{G}}\right) \times (32\pi L^3)^{1/2}$$

$\min(m) \Rightarrow \max\left(\frac{G^{1/2}}{\rho}\right)$ material index

2. Derive the shape factor for annular cross-section with inner radius R_i and outer radius R_o for torsional stiffness.



$$J = \frac{\pi(R_o^4 - R_i^4)}{2}$$

Reference c/s : $\cancel{\pi} R^2 = \cancel{\pi}(R_o^2 - R_i^2)$

$$J_{ref} = \frac{\pi R^4}{2} = \frac{\pi(R_o^2 - R_i^2)^2}{2}$$

$$S_{tors.} = \frac{GJ}{L} \propto J$$

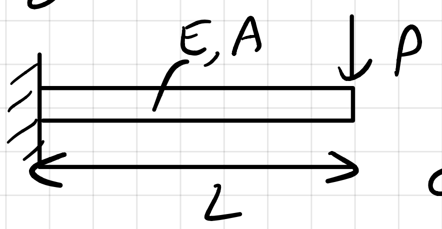
$$\Rightarrow \varphi = \frac{J}{J_{ref}} = \frac{R_o^4 - R_i^4}{(R_o^2 - R_i^2)^2} = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}$$

3. Derive the performance indices for the following cases:

a. A light beam with maximum stress less than or equal to σ_f (material property)

b. A light hollow shaft with stiffness greater than S^*

a) Objective : min m , Constraint: $\sigma_{max} \leq \sigma_f$



$$M_{max} = PL$$

$$\sigma_{max} = \frac{M_{max}\,y}{I} = \frac{M_{max}}{Z}$$

For bending strength, shape factor

$$\varphi_B^f = 6Z/A^{3/2} \Rightarrow Z = \varphi_B^f \cdot A^{3/2}/6$$

$$\Rightarrow \sigma_{max} = \frac{6\,M_{max}}{\varphi_B^f A^{3/2}}, \quad A = \frac{m}{\rho L} \Rightarrow \sigma_{max} = \frac{6(PL)(\rho L)^{3/2}}{\varphi_B^f m^{3/2}}$$

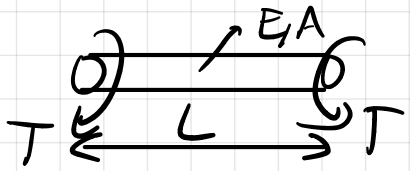
$$\sigma_{max} \leq \sigma_f$$

$$\Rightarrow \frac{6P\rho^{3/2}L^{5/2}}{\varphi_B^f m^{3/2}} \leq \sigma_f$$

$$\Rightarrow m \geq (6P)^{2/3} \cdot (L)^{5/3} \cdot \left(\frac{\rho}{\varphi_B^{2/3}\,\sigma_f^{2/3}}\right)$$

$$\min(m) \Rightarrow \max\left(\frac{\sigma_f^{2/3} \cdot \varphi_B^{2/3}}{\rho}\right) : \text{Material Index}$$

b) Objective: min m Constraint: $S \geq S^*$



$$\theta = \frac{TL}{GJ}$$ Shaft → Torsion

$$S = \frac{T}{\theta} = \frac{GJ}{L} \geq S^* \qquad m = \rho A L$$

For shaft, $\varphi_T^e = \frac{2\pi J}{A^2}$ from eq.(2).

$$J = \frac{\varphi_T^e A^2}{2\pi} = \frac{\varphi_T^e (m/\rho L)^2}{2\pi} = \frac{\varphi_e^T m^2}{2\pi \rho^2 L^2}$$

$$\Rightarrow \frac{G}{L} \times \left(\frac{\varphi_e^T m^2}{2\pi \rho^2 L^2} \right) \geq S^*$$

$$\Rightarrow m \geq (2\pi S^*)^{1/2} \times (L)^{3/2} \times \left(\frac{\rho}{\sqrt{G \varphi_e^T}} \right)$$

Performance Index: $\dfrac{\sqrt{G \varphi_e^T}}{\rho}$

4. Derive the material index to maximize the slenderness ratio (L/r) of a column with circular c/s subject to the constraint that it must not buckle under a given load F.

$$P_{cr} = c\,\pi^2 \frac{EI}{L^2}$$ c depends on B.Cs, some constant

Objective: $\max\left(\frac{L}{r}\right)$ Constraint: $P_{cr} \geq F$

$$c\pi^2 \frac{EI}{L^2} \geq F \Rightarrow \frac{c\pi^3 E r^4}{4L^2} \geq F$$

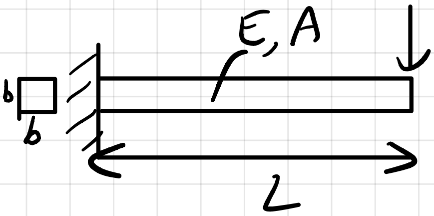
$$\Rightarrow \frac{c\pi^3 E L^2}{4}\left(\frac{r}{L}\right)^4 \geq F$$

$$\Rightarrow \frac{L}{r} \leq \left(\frac{c\pi^3}{4}\right)^{1/4} E^{1/4} \times \frac{1}{F^{1/4}} \times L^{1/2}$$

$$\max\left(\frac{L}{r}\right) \Rightarrow \max\left(E^{1/4}\right)$$ material index

5. Derive the material index to minimize the cost of a beam with stiffness greater than S^*. Note that the cost of the beam, C, can be assumed to be directly proportional to the mass of the beam, i.e. $C = C_m \boldsymbol{m}$ where C_m is the cost per unit mass and is a material property. Your material index will now include E, ρ and C_m.

Objective: min C, Constraint: $S \geq S^*$



$$\delta = \frac{PL^3}{3EI}, \quad S = \frac{P}{\delta} = \frac{3EI}{L^3}$$

$$S = \frac{3Eb^4}{12L^3} = \frac{EA^2}{4L^3} \quad (A = b^2)$$

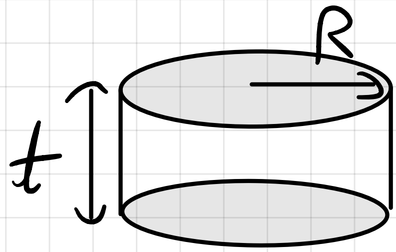
$$C = C_m \times m = C_m \rho A L \Rightarrow A = C/(C_m \rho L)$$

$$S = \frac{EC^2}{4L^5 C_m^2 \rho^2} \geqslant S^*$$

$$\Rightarrow C \geqslant (4L^5)^{1/2} \times \left(\frac{C_m \rho}{E^{1/2}}\right)$$

$$\min(C) \Rightarrow \max\left(\frac{E^{1/2}}{C_m \rho}\right) \leftarrow \text{index}$$

6. Derive the material index to maximize the energy stored per unit mass in a flywheel of fixed outer radius R, radius t and rotating with angular speed ω. Note that the maximum stress induced in the flywheel should be less than or equal to the failure stress σ_f, a material property. Note that at this stress the flywheel bursts. The maximum principal stress in a spinning disk of radius R with uniform thickness is $\sigma_{max} = \frac{3+\nu}{8}\rho R^2\omega^2$.



$$E = \frac{1}{2} I \omega^2 = \frac{1}{4} m R^2 \omega^2$$

$$E/m = \frac{1}{4} R^2 \omega^2$$

$$\sigma_{max} = \frac{3+\nu}{8} \rho R^2 \omega^2 \leqslant \sigma_f$$

$$\Rightarrow \frac{3+\nu}{2} \rho \left(\frac{E}{m}\right) \leqslant \sigma_f$$

$$\Rightarrow \frac{E}{m} \leqslant \frac{2\sigma_f}{\rho(3+\nu)} \leftarrow \text{material index}$$

tut4

EXAMPLE 7–5 Consider a simply supported steel shaft as depicted in Fig. 7–14, with 1 in diameter and a 31-in span between bearings, carrying two gears weighing 35 and 55 lbf.

(*a*) Find the influence coefficients.

(*b*) Find $\sum wy$ and $\sum wy^2$ and the first critical speed using Rayleigh's equation, Eq. (7–23).

(*c*) From the influence coefficients, find ω_{11} and ω_{22}.

(*d*) Using Dunkerley's equation, Eq. (7–32), estimate the first critical speed.

(*e*) Use superposition to estimate the first critical speed.

(*f*) Estimate the shaft's intrinsic critical speed. Suggest a modification to Dunkerley's equation to include the effect of the shaft's mass on the first critical speed of the attachments.

Solution (*a*)

$$I = \frac{\pi d^4}{64} = \frac{\pi(1)^4}{64} = 0.049\,09 \text{ in}^4$$

$$6EIl = 6(30)10^6(0.049\,09)31 = 0.2739(10^9) \text{ lbf} \cdot \text{in}^3$$

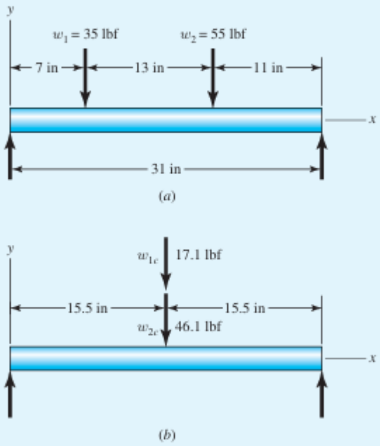
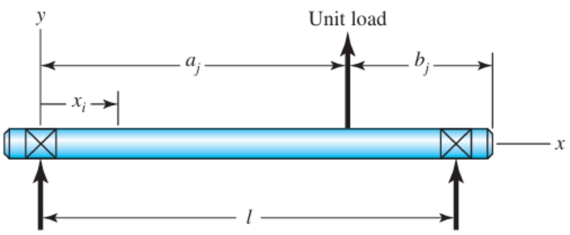


Figure 7–14

(*a*) A 1-in uniform-diameter shaft for Ex. 7–5.

(*b*) Superposing of equivalent loads at the center of the shaft for the purpose of finding the first critical speed.



Computer assistance is often used to lessen the difficulty in finding transverse deflections of a stepped shaft. Rayleigh's equation overestimates the critical speed.

To counter the increasing complexity of detail, we adopt a useful viewpoint. Inasmuch as the shaft is an elastic body, we can use *influence coefficients*. An influence coefficient is the transverse deflection at location i on a shaft due to a unit load at location j on the shaft. From Table A–9–6 we obtain, for a simply supported beam with a single unit load as shown in Fig. 7–13,

$$\delta_{ij} = \begin{cases} \dfrac{b_j x_i}{6EIl}(l^2 - b_j^2 - x_i^2) & x_i \le a_i \\ \dfrac{a_j(l - x_i)}{6EIl}(2lx_i - a_j^2 - x_i^2) & x_i > a_i \end{cases} \tag{7–24}$$

From Eq. set (7–24),

$$\delta_{11} = \frac{24(7)(31^2 - 24^2 - 7^2)}{0.2739(10^9)} = 2.061(10^{-4})\ \text{in/lbf}$$

$$\delta_{22} = \frac{11(20)(31^2 - 11^2 - 20^2)}{0.2739(10^9)} = 3.534(10^{-4})\ \text{in/lbf}$$

$$\delta_{12} = \delta_{21} = \frac{11(7)(31^2 - 11^2 - 7^2)}{0.2739(10^9)} = 2.224(10^{-4})\ \text{in/lbf}$$

Answer

	j	
i	**1**	**2**
1	$2.061(10^{-4})$	$2.224(10^{-4})$
2	$2.224(10^{-4})$	$3.534(10^{-4})$

$$y_1 = w_1\delta_{11} + w_2\delta_{12} = 35(2.061)10^{-4} + 55(2.224)10^{-4} = 0.019\,45\ \text{in}$$

$$y_2 = w_1\delta_{21} + w_2\delta_{22} = 35(2.224)10^{-4} + 55(3.534)10^{-4} = 0.027\,22\ \text{in}$$

(*b*)

$$\sum w_i y_i = 35(0.019\,45) + 55(0.027\,22) = 2.178\ \text{lbf} \cdot \text{in}$$

Answer

$$\sum w_i y_i^2 = 35(0.019\,45)^2 + 55(0.027\,22)^2 = 0.053\,99\ \text{lbf} \cdot \text{in}^2$$

Answer

$$\omega = \sqrt{\frac{386.1(2.178)}{0.053\,99}} = 124.8\ \text{rad/s, or } 1192\ \text{rev/min}$$

(*c*)

Answer

$$\frac{1}{\omega_{11}^2} = \frac{w_1}{g}\delta_{11}$$

$$\omega_{11} = \sqrt{\frac{g}{w_1\delta_{11}}} = \sqrt{\frac{386.1}{35(2.061)10^{-4}}} = 231.4\ \text{rad/s, or } 2210\ \text{rev/min}$$

Answer

$$\omega_{22} = \sqrt{\frac{g}{w_2\delta_{22}}} = \sqrt{\frac{386.1}{55(3.534)10^{-4}}} = 140.9\ \text{rad/s, or } 1346\ \text{rev/min}$$

(*d*)

$$\frac{1}{\omega_1^2} \approx \sum \frac{1}{\omega_{ii}^2} = \frac{1}{231.4^2} + \frac{1}{140.9^2} = 6.905(10^{-5}) \qquad (1)$$

Answer

$$\omega_1 \approx \sqrt{\frac{1}{6.905(10^{-5})}} = 120.3\ \text{rad/s, or } 1149\ \text{rev/min}$$

which is less than part *b*, as expected.

(*e*) From Eq. (7–24),

$$\delta_{cc} = \frac{b_{cc}x_{cc}(l^2 - b_{cc}^2 - x_{cc}^2)}{6EIl} = \frac{15.5(15.5)(31^2 - 15.5^2 - 15.5^2)}{0.2739(10^9)}$$

$$= 4.215(10^{-4})\ \text{in/lbf}$$

From Eq. (7–33),

$$w_{1c} = w_1\frac{\delta_{11}}{\delta_{cc}} = 35\frac{2.061(10^{-4})}{4.215(10^{-4})} = 17.11 \text{ lbf}$$

$$w_{2c} = w_2\frac{\delta_{22}}{\delta_{cc}} = 55\frac{3.534(10^{-4})}{4.215(10^{-4})} = 46.11 \text{ lbf}$$

Answer
$$\omega = \sqrt{\frac{g}{\delta_{cc}\sum w_{ic}}} = \sqrt{\frac{386.1}{4.215(10^{-4})(17.11 + 46.11)}} = 120.4 \text{ rad/s, or } 1150 \text{ rev/min}$$

which, except for rounding, agrees with part d, as expected.
(f) For the shaft, $E = 30(10^6)$ psi, $\gamma = 0.282$ lbf/in^3, and $A = \pi(1^2)/4 = 0.7854$ in^2. Considering the shaft alone, the critical speed, from Eq. (7–22), is

Answer
$$\omega_s = \left(\frac{\pi}{l}\right)^2\sqrt{\frac{gEI}{A\gamma}} = \left(\frac{\pi}{31}\right)^2\sqrt{\frac{386.1(30)10^6(0.049\,09)}{0.7854(0.282)}}$$
$$= 520.4 \text{ rad/s, or } 4970 \text{ rev/min}$$

We can simply add $1/\omega_s^2$ to the right side of Dunkerley's equation, Eq. (1), to include the shaft's contribution,

Answer
$$\frac{1}{\omega_1^2} \approx \frac{1}{520.4^2} + 6.905(10^{-5}) = 7.274(10^{-5})$$
$$\omega_1 \approx 117.3 \text{ rad/s, or } 1120 \text{ rev/min}$$

which is slightly less than part d, as expected.

The shaft's first critical speed ω_s is just one more single effect to add to Dunkerley's equation. Since it does not fit into the summation, it is usually written up front.

Answer
$$\frac{1}{\omega_1^2} \approx \frac{1}{\omega_s^2} + \sum_{i=1}^{n}\frac{1}{\omega_{ii}^2} \tag{7–34}$$

Common shafts are complicated by the stepped-cylinder geometry, which makes the influence-coefficient determination part of a numerical solution.

1. A shaft is loaded in bending and torsion such that $M_a = 70$ Nm, $T_a = 45$ Nm, $M_m = 55$ Nm, and $T_m = 35$ Nm. For the shaft, ultimate strength = 700 MPa, yield strength = 560 MPa, and a fully corrected endurance limit of = 210 MPa is assumed. Let $K_f = 2.2$ and Kfs = 1.8. For a factor of safety of 2.0 determine the minimum acceptable diameter of the shaft. Use the Goodman criterion. Clearly mention any assumptions that you make.

7-1 **(a)** DE-Gerber, Eq. (7-10):

$$A=\sqrt{4\left(K_f M_a\right)^2+3\left(K_{fs}T_a\right)^2}=\sqrt{4[(2.2)(70)]^2+3[(1.8)(45)]^2}=338.4\ \text{N}\cdot\text{m}$$

$$B=\sqrt{4\left(K_f M_m\right)^2+3\left(K_{fs}T_m\right)^2}=\sqrt{4[(2.2)(55)]^2+3[(1.8)(35)]^2}=265.5\ \text{N}\cdot\text{m}$$

$$d=\left\{\frac{8(2)(338.4)}{\pi(210)(10^6)}\left[1+\left(1+\left[\frac{2(265.5)(210)(10^6)}{338.4(700)(10^6)}\right]^2\right)^{1/2}\right]\right\}^{1/3}$$

$d = 25.85\ (10^{-3})$ m = 25.85 mm *Ans.*

(b) DE-elliptic, Eq. (7-12) can be shown to be

$$d=\left(\frac{16n}{\pi}\sqrt{\frac{A^2}{S_e^2}+\frac{B^2}{S_y^2}}\right)^{1/3}=\left(\frac{16(2)}{\pi}\sqrt{\frac{(338.4)^2}{\left[(210)(10^6)\right]^2}+\frac{(265.5)^2}{\left[(560)(10^6)\right]^2}}\right)^{1/3}$$

$d = 25.77\ (10^{-3})$ m = 25.77 mm *Ans.*

(c) DE-Soderberg, Eq. (7-14) can be shown to be

$$d=\left[\frac{16n}{\pi}\left(\frac{A}{S_e}+\frac{B}{S_y}\right)\right]^{1/3}=\left[\frac{16(2)}{\pi}\left(\frac{338.4}{210(10^6)}+\frac{265.5}{560(10^6)}\right)\right]^{1/3}$$

$d = 27.70\ (10^{-3})$ m = 27.70 mm *Ans.*

(d) DE-Goodman: Eq. (7-8) can be shown to be

$$d=\left[\frac{16n}{\pi}\left(\frac{A}{S_e}+\frac{B}{S_{ut}}\right)\right]^{1/3}=\left[\frac{16(2)}{\pi}\left(\frac{338.4}{210(10^6)}+\frac{265.5}{700(10^6)}\right)\right]^{1/3}$$

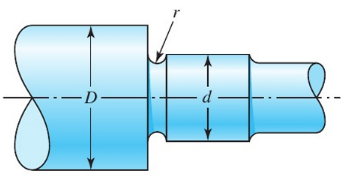
$d = 27.27\ (10^{-3})$ m = 27.27 mm *Ans.*

Criterion	*d* (mm)	Compared to DE-Gerber	
DE-Gerber	25.85		
DE-Elliptic	25.77	0.31% Lower	Less conservative
DE-Soderberg	27.70	7.2% Higher	More conservative
DE-Goodman	27.27	5.5% Higher	More conservative

2. The section of shaft shown in the figure is to be designed to approximate relative sizes of $d = 0.75D$ and $r = D/20$ with diameter d conforming to that of standard rolling-bearing bore sizes. The shaft is to be made of SAE 2340 steel, heat-treated to obtain minimum strengths in the shoulder area of 1200 MPa ultimate tensile strength and 1100 MPa yield strength with a Brinell hardness not less than 370. At the shoulder the shaft is subjected to a completely reversed bending moment of 67.800 Nmm, accompanied by a steady torsion of 45.200 Nmm. Use a facor of safety of 2.5 and size the shaft for an infinite life using the DE-Goodman criterion.

Problem 7–3

Section of a shaft containing a grinding-relief groove. Unless otherwise specified, the diameter at the root of the groove $d_r = d - 2r$, and though the section of diameter d is ground, the root of the groove is still a machined surface.



Eq. (6-19), p. 295: $k_a = 2.70(175)^{-0.265} = 0.69$

Trial #1: Choose $d_r = 0.75$ in

Eq. (6-20), p. 296: $k_b = 0.879(0.75)^{-0.107} = 0.91$

Eq. (6-8), p.290: $S_e' = 0.5S_{ut} = 0.5(175) = 87.5$ kpsi

Eq. (6-18), p. 295: $S_e = 0.69\ (0.91)(87.5) = 54.9$ kpsi

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$
$$D = \frac{d_r}{0.65} = \frac{0.75}{0.65} = 1.15 \text{ in}$$
$$r = \frac{D}{20} = \frac{1.15}{20} = 0.058 \text{ in}$$

Fig. A-15-14:

$$d = d_r + 2r = 0.75 + 2(0.058) = 0.808 \text{ in}$$
$$\frac{d}{d_r} = \frac{0.808}{0.75} = 1.08$$
$$\frac{r}{d_r} = \frac{0.058}{0.75} = 0.077$$

$K_t = 1.9$

Fig. 6-20, p. 296: $r = 0.058$ in, $q = 0.90$

Eq. (6-32), p. 303: $K_f = 1 + 0.90\ (1.9 - 1) = 1.81$

Fig. A-15-15: $K_{ts} = 1.5$

Fig. 6-21, p. 304: $r = 0.058$ in, $q_s = 0.92$

Eq. (6-32), p. 303: $K_{fs} = 1 + 0.92\ (1.5 - 1) = 1.46$

We select the DE-ASME Elliptic failure criteria, Eq. (7-12), with d as d_r, and $M_m = T_a = 0$,

$$d_r = \left\{ \frac{16(2.5)}{\pi} \left[4\left(\frac{1.81(600)}{54.9(10^3)} \right)^2 + 3\left(\frac{1.46(400)}{160(10^3)} \right)^2 \right]^{1/2} \right\}^{1/3}$$

$d_r = 0.799$ in

Trial #2: Choose $d_r = 0.799$ in.

$$k_b = 0.879(0.799)^{-0.107} = 0.90$$
$$S_e = 0.69\ (0.90)(0.5)(175) = 54.3 \text{ kpsi}$$
$$D = \frac{d_r}{0.65} = \frac{0.799}{0.65} = 1.23 \text{ in}$$
$$r = D/20 = 1.23/20 = 0.062 \text{ in}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 0.799 + 2(0.062) = 0.923 \text{ in}$$
$$\frac{d}{d_r} = \frac{0.923}{0.799} = 1.16$$
$$\frac{r}{d_r} = \frac{0.062}{0.799} = 0.078$$

With these ratios only slightly different from the previous iteration, we are at the limit of readability of the figures. We will keep the same values as before.

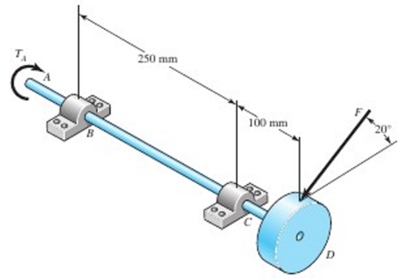
$$K_t = 1.9, \quad K_{ts} = 1.5, \quad q = 0.90, \quad q_s = 0.92$$
$$\therefore K_f = 1.81, \quad K_{fs} = 1.46$$

Using Eq. (7-12) produces $d_r = 0.802$ in. Further iteration produces no change. With $d_r = 0.802$ in,

$$D = \frac{0.802}{0.65} = 1.23 \text{ in}$$
$$d = 0.75(1.23) = 0.92 \text{ in}$$

A look at a bearing catalog finds that the next available bore diameter is 0.9375 in. In nominal sizes, we select $d = 0.94$ in, $D = 1.25$ in, $r = 0.0625$ in *Ans.*

3. The rotating solid steel shaft is simply supported by bearings at points B and C and is driven by a gear (not shown) which meshes with the spur gear at D, which has a 150-mm pitch diameter. The force F from the drive gear acts at a pressure angle of 20°. The shaft transmits a torque to point A of TA = 340 Nm. The shaft is machined from steel with yield strength = 420 MPa and ultimate tensile strength = 560 MPa. Using a factor of safety of 2.5, determine the minimum allowable diameter of the 250-mm section of the shaft based on (a) a static yield analysis using the distortion energy theory and (b) a fatigue-failure analysis. Assume sharp fillet radii at the bearing shoulders for estimating stress-concentration factors.



7-3 $F \cos 20°(d/2) = T_A$, $F = 2\,T_A/(d \cos 20°) = 2(340)/(0.150 \cos 20°) = 4824$ N.
The maximum bending moment will be at point C, with $M_C = 4824(0.100) = 482.4$ N·m.
Due to the rotation, the bending is completely reversed, while the torsion is constant.
Thus, $M_a = 482.4$ N·m, $T_m = 340$ N·m, $M_m = T_a = 0$.

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$. Examining Figs. 6-20 and 6-21 (pp. 303 and 304 respectively) with $S_{ut} = 560$ MPa, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $K_f = 1 + 0.8(2.7 - 1) = 2.4$

$K_{fs} = 1 + 0.9(2.2 - 1) = 2.1$

(a) We will choose to include fatigue stress concentration factors even for the static analysis to avoid localized yielding.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

Eq. (7-16): $$n = \frac{S_y}{\sigma'_{\max}} = \frac{\pi d^3 S_y}{16}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs}T_m\right)^2\right]^{-1/2}$$

Solving for d,

$$d = \left\{\frac{16n}{\pi S_y}\left[4(K_f M_a)^2 + 3(K_{fs}T_a)^2\right]^{1/2}\right\}^{1/3}$$

$$= \left(\frac{16(2.5)}{\pi(420)(10^6)}\left\{4\left[(2.4)(482.4)\right]^2 + 3\left[(2.1)(340)\right]^2\right\}^{1/2}\right)^{1/3}$$

$d = 0.0430$ m $= 43.0$ mm *Ans.*

(b) $k_a = 4.51(560)^{-0.265} = 0.84$

Assume $k_b = 0.85$ for now. Check later once a diameter is known.

$S_e = 0.84(0.85)(0.5)(560) = 200$ MPa

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(482.4)}{200(10^6)}\right)^2 + 3\left(\frac{2.1(340)}{420(10^6)}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= 0.0534 \text{ m} = 53.4 \text{ mm}$$

With this diameter, we can refine our estimates for k_b and q.

Eq. (6-20): $k_b = 1.51d^{-0.157} = 1.51(53.4)^{-0.157} = 0.81$

Assuming a sharp fillet radius, from Table 7-1, $r = 0.02d = 0.02\,(53.4) = 1.07$ mm.

Fig. (6-20): $q = 0.72$
Fig. (6-21): $q_s = 0.77$

Iterating with these new estimates,

Eq. (6-32): $K_f = 1 + 0.72\,(2.7 - 1) = 2.2$
$K_{fs} = 1 + 0.77\,(2.2 - 1) = 1.9$
Eq. (6-18): $S_e = 0.84(0.81)(0.5)(560) = 191$ MPa
Eq. (7-12): $d = 53$ mm *Ans.*

Further iteration does not change the results.

Comparing these values to the recommended limits in Table 7-2, we find that they are all within the recommended range.

7-24 Shaft analysis software or finite element software can be utilized if available. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

Deflection: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

Statics: Left support: $R_1 = 7(315-140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

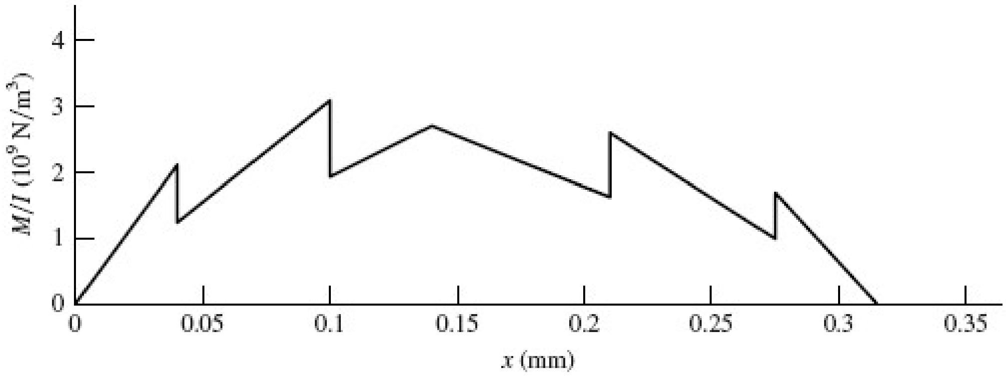
Determine the bending moment at each step.

x(mm)	0	40	100	140	210	275	315
M(N · m)	0	155.56	388.89	544.44	326.67	124.44	0

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})\ \text{m}^4$, $I_{40} = 1.257(10^{-7})\ \text{m}^4$, $I_{45} = 2.013(10^{-7})\ \text{m}^4$

Plot M/I as a function of x.

x(m)	M/I (10^9 N/m^3)	Step	Slope	ΔSlope
0	0		52.8	
0.04	2.112			
0.04	1.2375	–0.8745	30.942	–21.86
0.1	3.094			
0.1	1.932	–1.162	19.325	–11.617
0.14	2.705			
0.14	2.705	0	–15.457	–34.78
0.21	1.623			
0.21	2.6	0.977	-24.769	-9.312
0.275	0.99			
0.275	1.6894	0.6994	-42.235	-17.47
0.315	0			



The steps and the change of slopes are evaluated in the table. From these, the function *M/I* can be generated:

$$M\ /\ I = \Big[52.8x - 0.8745\langle x-0.04\rangle^0 - 21.86\langle x-0.04\rangle^1 - 1.162\langle x-0.1\rangle^0$$
$$-11.617\langle x-0.1\rangle^1 - 34.78\langle x-0.14\rangle^1 + 0.977\langle x-0.21\rangle^0$$
$$-9.312\langle x-0.21\rangle^1 + 0.6994\langle x-0.275\rangle^0 - 17.47\langle x-0.275\rangle^1\Big]10^9$$

Integrate twice:

$$E\frac{dy}{dx} = \Big[26.4x^2 - 0.8745\langle x-0.04\rangle^1 - 10.93\langle x-0.04\rangle^2 - 1.162\langle x-0.1\rangle^1$$
$$-5.81\langle x-0.1\rangle^2 - 17.39\langle x-0.14\rangle^2 + 0.977\langle x-0.21\rangle^1$$
$$-4.655\langle x-0.21\rangle^2 + 0.6994\langle x-0.275\rangle^1 - 8.735\langle x-0.275\rangle^2 + C_1\Big]10^9 \quad (1)$$

$$Ey = \Big[8.8x^3 - 0.4373\langle x-0.04\rangle^2 - 3.643\langle x-0.04\rangle^3 - 0.581\langle x-0.1\rangle^2$$
$$-1.937\langle x-0.1\rangle^3 - 5.797\langle x-0.14\rangle^3 + 0.4885\langle x-0.21\rangle^2$$
$$-1.552\langle x-0.21\rangle^3 + 0.3497\langle x-0.275\rangle^2 - 2.912\langle x-0.275\rangle^3 + C_1x + C_2\Big]10^9$$

Boundary conditions: $y = 0$ at $x = 0$ yields $C_2 = 0$;
$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\ 25$ N/m^2.

Equation (1) with $C_1 = -0.295\ 25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the results of a full model which models the 35 and 55 mm diameter steps.

x (mm)	θ (rad)	F.E. Model	Full F.E. Model
0	–0.001 4260	–0.001 4270	–0.001 4160
140	–0.000 1466	–0.000 1467	–0.000 1646
315	0.001 3120	0.001 3280	0.001 3150

The main discrepancy between the results is at the gear location (x = 140 mm). The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max}$ = (0.001/0.001 426)7 = 4.91 kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, apply Eq. (7-18) to the most offending deflection (at x = 0) to determine a multiplier to be used for all diameters.

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d \left(dy/dx \right)_{\text{old}}}{\left(\text{slope} \right)_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(0.0014260)}{0.001} \right|^{1/4} = 1.093$$

Form a table:

Old d, mm	20.00	30.00	35.00	40.00	45.00	55.00
New ideal d, mm	21.86	32.79	38.26	43.72	49.19	60.12
Rounded up d, mm	22.00	34.00	40.00	44.00	50.00	62.00

Repeating the full finite element model results in

x = 0: $\theta = -9.30 \times 10^{-4}$ rad
x = 140 mm: $\theta = -1.09 \times 10^{-4}$ rad
x = 315 mm: $\theta = 8.65 \times 10^{-4}$ rad

This is well within our goal. Have the students try a goal of 0.0005 rad at the gears.

Strength: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

x (mm)	0	15	40	100	110	140	210	275	300	330
σ (MPa)	0	22.0	37.0	61.9	47.8	60.9	52.0	39.6	17.6	0
τ (MPa)	0	0	0	0	0	6	8.5	12.7	20.2	68.1
σ' (MPa)	0	22.0	37.0	61.9	47.8	61.8	53.1	45.3	39.2	118.0

Table A-20 for AISI 1020 CD steel: S_{ut} = 470 MPa, S_y = 390 MPa

At x = 210 mm:
Eq. (6-19): $k_a = 4.51(470)^{-0.265} = 0.883$

Eq. (6-20): $k_b = (40 / 7.62)^{-0.107} = 0.837$

Eq. (6-18): $S_e = 0.883\ (0.837)(0.5)(470) = 174$ MPa

$D / d = 45 / 40 = 1.125, \quad r / d = 2 / 40 = 0.05$

Fig. A-15-8: $K_{ts} = 1.4$

Fig. A-15-9: $K_t = 1.9$

Fig. 6-20: $q = 0.75$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_f = 1 + 0.75(1.9 - 1) = 1.68$

$K_{fs} = 1 + 0.79(1.4 - 1) = 1.32$

Choosing DE-ASME Elliptic to inherently include the yield check, from Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi\left(0.04^3\right)}\left\{4\left[\frac{1.68(326.67)}{174\left(10^6\right)}\right]^2 + 3\left[\frac{1.32(107)}{390\left(10^6\right)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm:

The von Mises stress is the highest but it comes from the steady torque only.

$D / d = 30 / 20 = 1.5, \quad r / d = 2 / 20 = 0.1$

Fig. A-15-9: $K_{ts} = 1.42$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_{fs} = 1 + 0.79(1.42 - 1) = 1.33$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi\left(0.02^3\right)}\left(\sqrt{3}\right)\left[\frac{1.33(107)}{390\left(10^6\right)}\right]$$

$$n = 2.49$$

Note that since there is only a steady torque, Eq. (7-11) reduces to essentially the equivalent of the distortion energy failure theory.

Check the other locations.

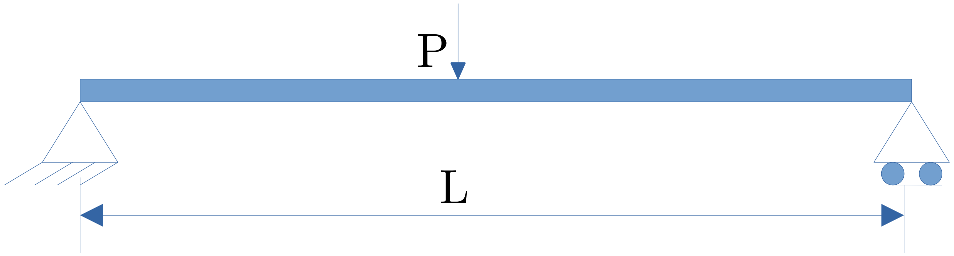
If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

7-25 and 7-26 With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints – explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.

Influence of c/s Shape on the Performance

Problem: Want to design a light stiff simply supported beam



$$\delta = \frac{PL^3}{48EI} \qquad S = \frac{P}{\delta} = \frac{48EI}{L^3}$$

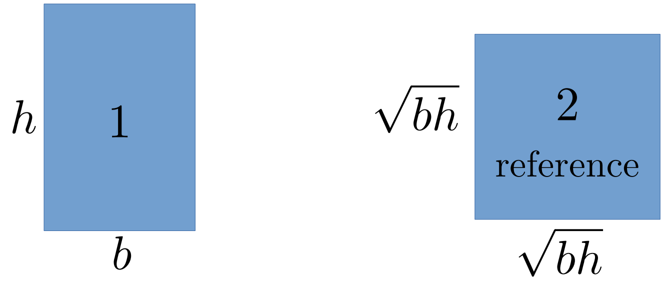
$$S \propto EI$$

Material property   Material distribution

For two beams with the same material

$$S \propto I$$

Consider two c/ having the same area



$$I_1 = \frac{bh^3}{12} \qquad I_2 = \frac{(\sqrt{bh})(\sqrt{bh})^3}{12} = \frac{b^2h^2}{12}$$

$$\frac{I_1}{I_2} = \frac{h}{b}$$

For $h > b, \quad I_1 > I_2$

For the same material, we get a better performance with a rectangular c/s as compared with the square c/s

Influence of c/s Shape on the Performance

- Shape is taken into account using **shape factors**
- Shape factor measures the efficiency of the material usage and is independent of the material
- Shape factor is a dimensionless quantity and is independent of scale
- The same c/s can have different shape factors depending on the type of loading conditions
- **Shape factor for stiffness (bending or torsional)**

$$\phi^e = \frac{\text{stiffness of the shaped c/s}}{\text{stiffness of the neutral or reference c/s}}$$

- **Shape factor for strength (bending or torsional)**

$$\phi^f = \frac{\text{strength of the shaped c/s}}{\text{strength of the neutral or reference c/s}}$$

- Neutral or reference c/s: solid c/s section with the same c/s area as that of the shaped section
- For bending, the reference c/s is a square c/s with the same area
- For torsion, the reference c/s is a circular c/s with the same area

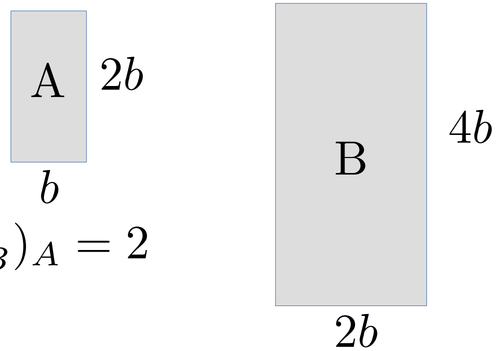
Influence of c/s Shape on the Performance

Shape factor for Bending Stiffness

$$\phi_B^e = \frac{I}{I_{NS}}, \; I_{NS} = \frac{b^4}{12} = \frac{A^2}{12}$$

$$\boxed{\phi_B^e = \frac{12I}{A^2}}$$

Scale Independence



$(\phi_B^e)_A = 2$

$(\phi_B^e)_B = 2$

Shape factor for Bending Strength

- Bending Strength $\sigma = \dfrac{M}{I/y} = \dfrac{M}{Z}$

Section modulus

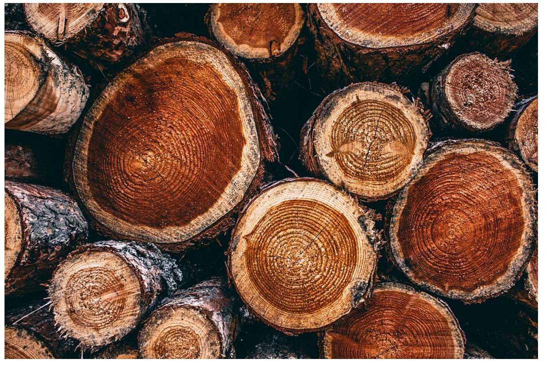
- For a given moment M, to minimize σ we need to maximize Z

$$\phi_B^f = \frac{Z}{Z_{NS}}, \; Z_{NS} = \frac{b^3}{6} = \frac{A^{3/2}}{6}$$

$$\boxed{\phi_B^f = \frac{6Z}{A^{3/2}}}$$

Simultaneous Selection of Material and Shape of c/s to Maximize Performance

- Have looked at material selection and shape selection independently up to this point
- In practice, they are not independent – Aluminum is available as thin walled tubes while wood is not.



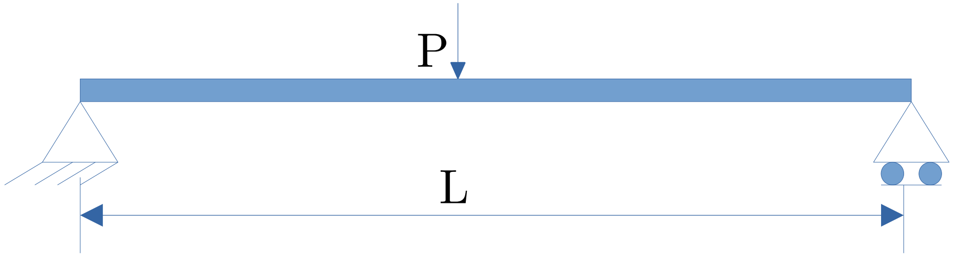
- Will now develop a **performance index** which account for both the material and c/s shape

https://alorwood.com/en/what-is-natural-wood

https://www.chaluminium.com/extruded-aluminum-tubes-manufacturing-applications-and-advantages

Example

Choose a material and c/s for a simply supported light stiff beam. The stiffness of the beam should be at least S^*



Function: The beam supports a load P

Constraint: stiffness should be at least S^*

Objective: minimize the mass of the beam

Free variable: material, shape of c/s, A

Material	(kg/m^3)	E(GPa)	ϕ_B^e
1020 Steel	7850	205	20
6061 Al	2700	70	15
Wood (oak)	900	13.5	2
GFRP	1750	28	8

$$\delta = \frac{PL^3}{48EI} \qquad \phi_B^e = \frac{12I}{A^2}$$

$$\text{Stiffness} \quad S = \frac{P}{\delta} = \frac{48EI}{L^3} = \frac{4EA^2\phi_B^e}{L^3}$$

Example

Mass $m = \rho A L$

Stiffness $S = \dfrac{4EA^2\phi_B^e}{L^3}$

$S \geq S^*$ constraint

$$\frac{4EA^2\phi_B^e}{L^3} \geq S^*$$

$$\frac{4Em^2\phi_B^e}{L^5\rho^2} \geq S^*$$

or

$$m \geq \left(\frac{1}{2}\sqrt{S^*}\right)\left(L^{5/2}\right)\left(\frac{\rho}{(E\phi_B^e)^1/2}\right)$$

Functional parameters | **G**eometric parameters | **M**aterial and c/s properties

$m \geq f_1(F) f_2(G) f_3(MS)$ separable form

To minimize the mass we therefore need to maximize the performance index $(E\phi_B^e)^{1/2}/\rho$

Material	ρ (kg/m^3)	E(GPa)	ϕ_B^e	$E^{1/2}/\rho$	$(E\phi_B^e)^{1/2}/\rho$
1020 Steel	7850	205	20	58	258
6061 Al	2700	70	15	98	**380**
Wood(oak)	900	13.5	2	**129**	183
GFRP	1750	28	8	96	270

Select this

End

Curved Beams

We will determine the bending stresses developed in a member which is initially curved.

(a) Crane Hook

(b) Chain Link

Here the radius of curvature is of the same order as the dimensions of the cross-sections. The method of analysis was first presented by E. Wrinkler.

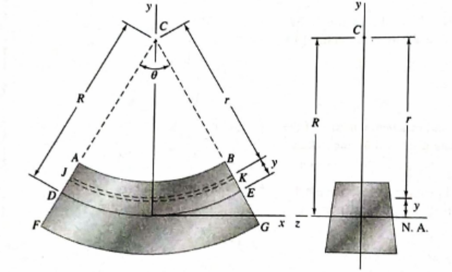
Curved Beams

Assumptions:

- The area of the cross-section is constant along the length of the beam.
- The cross-section is symmetric about the loading plane.
- Cross-sections remain plane after loading.
- All strains are small.
- Material is linear elastic, homogeneous and isotropic.

Curved Beams

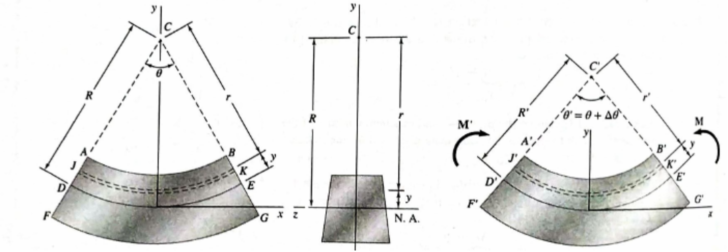
- Consider the undeformed curved beam shown below:



- There are two radii shown in the figure:
  - R: radius of the neutral axis (unknown)
  - r: location of arbitrary point in the cross-section.
- The vertical xy plane intersects the upper and the lower surfaces along the arc of circles AB and FG centered at C.
- The vertical xy plane intersects the neutral surface along the arc of the circle DE.
- Arc JK represents the intersections of the vertical plane with a surface situated at a distance y from the neutral axis.

Curved Beams

- The length of arc JK in the undeformed beam is $l\left(JK\right) = r\theta$.
- Now apply equal and opposite moments and the two ends.



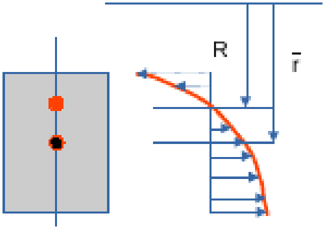
- The length of arc in the JK in the deformed configuration is $l\left(J'K'\right) = r'\left(\theta + \Delta\theta\right)$
- The strain $\epsilon_{\theta\theta}$ in the circumferential direction are given by

$$\epsilon_{\theta\theta} = \frac{l(J'K') - l(JK)}{l(JK)}$$
$$= \frac{r'\left(\theta + \Delta\theta\right) - r\theta}{r\theta}$$
$$= \frac{\left(R' - y\right)\left(\theta + \Delta\theta\right) - \left(R - y\right)\theta}{r\theta}$$
$$= -\frac{y\Delta\theta}{r\theta}$$
$$= -\frac{\left(R - r\right)\Delta\theta}{r\theta}$$
$$= -k\frac{\left(R - r\right)}{r}, \; k = \Delta\theta/\theta$$

Curved Beams

- We have
$$\epsilon_{\theta\theta} = -k\frac{(R-r)}{r},\ k = \Delta\theta/\theta$$
The strain does not vary linearly with distance from the neutral axis.

- For a linear elastic material (assuming all the other stress components are negligible) we get
$$\sigma_{\theta\theta} = E\epsilon_{\theta\theta} = -Ek\frac{(R-r)}{r}$$
The stress does not vary linearly with distance from the neutral axis.



Curved Beams

- To obtain R, the location of the neutral axis, we use the condition that the normal force acting on the cross-section is zero.

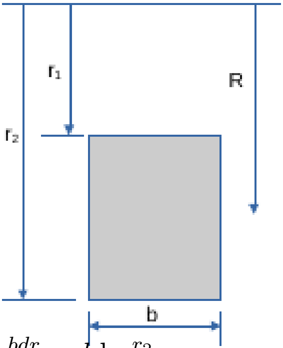
$$\int_A \sigma_{\theta\theta} dA = 0$$

$$\text{or } -\int_A Ek\frac{(R-r)}{r}dA = 0$$

$$\text{or } R\int_A \frac{dA}{r} - \int_A dA = 0$$

$$\text{or } R = \frac{A}{\int_A \frac{dA}{r}}$$

- To find the position of the neutral axis for the rectangular c/s



- We have $A = b(r_2 - r_1)$. Also $\int_A \frac{dA}{r} = \int_{r_1}^{r_2} \frac{bdr}{r} = b\ln\frac{r_2}{r_1}$

- Therefore

$$R = \frac{r_2 - r_1}{\ln\frac{r_2}{r_1}}$$

Curved Beams

The relation between the internal resisting moment and the developed stress is obtained as follows:

- We have

$$
\begin{aligned}
M &= -\int_A \sigma_{\theta\theta}\,(R-r)\,dA \\
&= kE\int_A \frac{(R-r)^2}{r}dA \\
&= Ek\left(R^2\int_A \frac{dA}{r} - 2R\int_A dA + \int_A r dA\right) \\
&= Ek\left(R^2\cdot\frac{A}{R} - 2RA + \bar{r}A\right) \\
\therefore M &= EkA\,(\bar{r}-R) \qquad \text{(A)}
\end{aligned}
$$

- We also have

$$\sigma_{\theta\theta} = -Ek\frac{(R-r)}{r} \qquad \text{(B)}$$

Curved Beams

- From Eqns A and B we get

$$\sigma_{\theta\theta} = -\frac{M\left(R-r\right)}{Ar\left(\bar{r}-R\right)}$$

- Let $y = R - r$ and $e = \bar{r} - R$. Here e is called the **eccentricity**. The stress can then be written as:

$$\sigma_{\theta\theta} = -\frac{My}{A\left(R-y\right)e}$$

For beams with circular c/s with radius c

$$R = \frac{A}{2\pi(\bar{r} - \sqrt{\bar{r}^2 - c^2})}$$

Curved Beams

Compare the stresses in a 50 mm $\times$ 50 mm square beam subjected to end moment of 2083 Nm in the following three cases: 1. straight beam, 2. curved beam with $\bar{r} = 250$ mm and 3. curved beam with $\bar{r} = 75$ mm.

- For a straight beam:

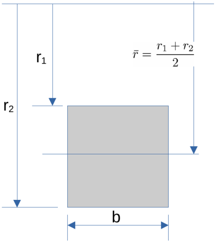
$$\sigma = -\frac{My}{I}$$

- $I = \frac{1}{12}\left(50 \times 10^{-3}\right)\left(50 \times 10^{-3}\right)^3 \ m^4$ and $y = 25 \times 10^{-3} \ m$.

- Therefore $\sigma_T = 100$ MPa and $\sigma_C = -100$ MPa

- For a curved beam

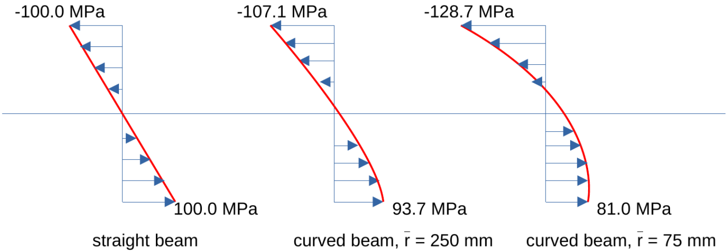
$$\sigma = -\frac{M\left(R - r\right)}{Ar\left(\bar{r} - R\right)}$$



Curved Beams

- For a beam with rectangular c/s
$$R = \frac{r_2 - r_1}{\ln \frac{r_2}{r_1}}$$
- Case 2: $\bar{r} = 250$ mm
We have
$$r_1 = \bar{r} - 25 \times 10^{-3} = 225 \times 10^{-3} m$$
$$r_2 = \bar{r} + 25 \times 10^{-3} = 275 \times 10^{-3} m$$
- Hence $R = 249.164$ mm.
- Therefore $\sigma_T = 93.7$ MPa
and $\sigma_C = -107.1$ MPa
- Case 3: $\bar{r} = 75$ mm
We have
$$r_1 = \bar{r} - 25 \times 10^{-3} = 50 \times 10^{-3} m$$
$$r_2 = \bar{r} + 25 \times 10^{-3} = 100 \times 10^{-3} m$$
- Hence $R = 72.134$ mm.
- Therefore $\sigma_T = 81.0$ MPa
and $\sigma_C = -128.7$ MPa

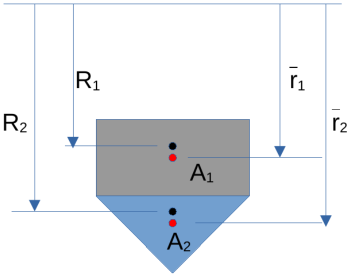
Curved Beams



- Let h denote the depth of the beam. Here $h = 50$ mm. As the ratio $\bar{r}/h$ increases, the results of the curved beam approach those of the straight beam.

Curved Beams

Find R of the c/s made of two areas A_1 and A_2 shown below



- We have

$$R = \frac{A}{\int_A \frac{dA}{r}}$$

- Now

$$\int_A \frac{dA}{r} = \int_{A_1} \frac{dA}{r} + \int_{A_2} \frac{dA}{r}$$

- Also,

$$R_1 = \frac{A_1}{\int_{A_1} \frac{dA}{r}}, \quad R_2 = \frac{A_2}{\int_{A_2} \frac{dA}{r}}$$

- Therefore

$$\int_A \frac{dA}{r} = \frac{A_1}{R_1} + \frac{A_2}{R_2}$$

- Hence

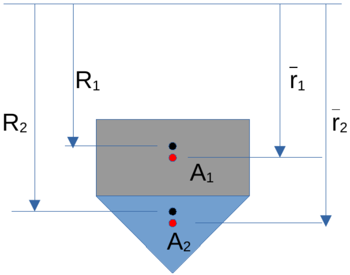
$$R = \frac{A}{\frac{A_1}{R_1} + \frac{A_2}{R_2}} = \frac{A_1 + A_2}{\frac{A_1}{R_1} + \frac{A_2}{R_2}}$$

- In general, for a c/s made up of n areas, $A_i, i = 1, 2, \ldots, n$

$$R = \frac{\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} \left(\frac{A_i}{R_i}\right)}$$

Curved Beams

Find $\bar{r}$ of the c/s made of two areas A_1 and A_2 shown below



- We have

$$\bar{r} = \frac{\int_A r dA}{A}$$

- Now

$$\int_A r dA = \int_{A_1} r dA + \int_{A_2} r dA$$

- Also,

$$\bar{r}_1 = \frac{\int_{A_1} r dA}{A_1}, \ \bar{r}_2 = \frac{\int_{A_2} r dA}{A_2}$$

- Therefore

$$\bar{r} = \frac{\bar{r}_1 A_1 + \bar{r}_2 A_2}{A} = \frac{\bar{r}_1 A_1 + \bar{r}_2 A_2}{A_1 + A_2}$$

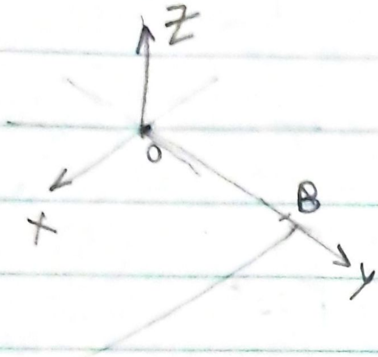
- In general, for a c/s made up of n areas, $A_i, i = 1, 2, \ldots, n$

$$\bar{r} = \frac{\sum_{i=1}^{n} \bar{r}_i A_i}{\sum_{i=1}^{n} A_i}$$

Problem 1

ME 423 - Quiz 1

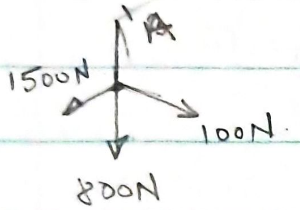
1a. The internal resisting forces and moments at different locations are calculated using:



$$\underline{F}_R + \underline{F}_A = \underline{0}$$

$\underline{F}_R$ & $\underline{M}_R$ are internal resisting forces & moments

$$\underline{M}_R + \underline{r} \times \underline{F}_A = \underline{0}$$



$$\underline{F}_A = 1500\hat{i} + 100\hat{j} - 800\hat{k}$$

$$\underline{F}_R = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\underline{M}_R = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

	F_x (N)	F_y (N)	F_z (N)	M_x (Nm)	M_y (Nm)	M_z (Nm)
A	−1500	−100	800	−5 (T)	75	0
B	−1500	−100	800	−5	−525 (T)	−75
O	−1500	−100	800	235	−525 (T)	375

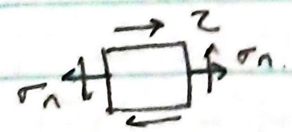
From the table it is seen that the most critical section is at pt O.

We will ignore the effect of direct shear due to F_x & F_z.

M_y represent the torque @ Y-axis. M_x represents bending moment @ x-axis & M_z represents bending @ z axis. The resultant bending moment is $\tilde{M} = \sqrt{M_y^2 + M_x^2}$

$$\sigma_b|_{max} = \frac{32\tilde{M}}{\pi d^3} \qquad \sigma_n|_{axial} = \frac{4F_y}{\pi d^2}$$

$$\sigma_n = \sigma_b|_{max} + \sigma_n|_{axial} = \frac{32\tilde{M}}{\pi d^3} + \frac{4F_y}{\pi d^2}.$$



$$\tau_{max} = \frac{16 M_y}{\pi d^3}$$

$$\sigma_{vm} = (\sigma_n^2 + 3\tau_{max}^2)^{1/2}.$$

To find the minimum required diameter

$$\sigma_{vm} = \frac{\sigma_Y}{FOS} \quad \text{or} \quad \sigma_{vm}^2 = \frac{\sigma_Y^2}{FOS^2}$$

$$\therefore \sigma_n^2 + 3\tau_{max}^2 = \frac{\sigma_Y^2}{FOS^2}$$

$$\therefore \left(\frac{32\tilde{M}}{\pi d^3} + \frac{4P}{\pi d^2}\right)^2 + 3\left(\frac{16 M_y}{\pi d^3}\right)^2 = \frac{\sigma_Y^2}{FOS^2} \quad (A)$$

Solve the above equation$^{(A)}$ for d. It is not easy to solve unless you have the "right" calculator. To simplify, we assume that $\frac{4P}{\pi d^2} << \frac{32\tilde{M}}{\pi d^3}$. We will verify the assumption at the end. Then we get.

(can be solved using any calculator).

$$\left(\frac{32\tilde{M}}{\pi d^3}\right)^2 + 3\left(\frac{16 M_y}{\pi d^3}\right)^2 = \frac{\sigma_Y^2}{FOS^2} \quad (B).$$

Solving B (approx): $d = 33.3012$ mm ≈ 33.3 mm

Solving A (exact): $d_2 = 33.3063$ mm ≈ 33.3 mm

We see that the difference in answer is about 0.02%.

$d = 33.3063$ mm.

For $\sigma_{axial} = \pm 0.11$ MPa

$\sigma_b = 122.00$ MPa.

$\tau = 72.37$ MPa

$\therefore \sigma_{axial} << \sigma_b$

$\sigma_{axial} << \tau$.

Hence our assumption of neglecting σ_a is justified. This is true in almost all problems of combined loading.

Problem 2

2) ① $(\sigma_{max})_1 = 340$ MPa, $(\sigma_{min})_1 = 160$ MPa, $n_1 = 8\times10^4$.
② $(\sigma_{max})_2 = 320$ MPa, $(\sigma_{min})_2 = -200$ MPa $n_2 = ?$
$\sigma_y = 350$ MPa, $\sigma_{ult} = 420$ MPa, $f = 0.9$ $\sigma_e = 175$ MPa.

Miners rule $\dfrac{n_1}{N_1} + \dfrac{n_2}{N_2} = 1$

$N_i \rightarrow$ number of cycles for failure at fully reversed (ie $\sigma_m = 0$) cycle with amplitude σ_{a_i}

Equivalent fully reversed stress amplitude for ①.

$$\sigma_{a_1}' = \frac{\sigma_{a_1}}{1 - \dfrac{\sigma_{m_1}}{\sigma_{ult}}} \qquad \sigma_{a_1} = 90 \text{ MPa} \quad \sigma_{m_1} = 250 \text{ MPa}.$$

$$= 222.35 \text{ MPa}$$

$$\sigma_{a_2}' = \frac{\sigma_{a_2}}{1 - \dfrac{\sigma_{m_2}}{\sigma_{ult}}} \qquad \sigma_{a_2} = 260 \text{ MPa} \quad \sigma_{m_2} = 60 \text{ MPa}.$$

$$= 303.33 \text{ MPa}$$

Need to find N_1 & N_2 corresponding to σ_{a_1}' & σ_{a_2}' from the Basquin eqn.

$$\sigma_a = \sigma_f' (2N)^b.$$

$$b = \frac{\ln(\sigma_a'/\sigma_e)}{\ln(10^3/10^6)} = \frac{\ln(f\sigma_{ult}/\sigma_e)}{\ln(10^3/10^6)} = -0.11148.$$

$$\sigma_f = \frac{\sigma_e}{(2\times10^6)^b} = 882.07 \text{ MPa.}$$

$$\therefore N_1 = \frac{1}{2}\left(\frac{\sigma'_{a_1}}{\sigma_f}\right)^{\frac{1}{b}} = 116705.75$$

$$N_2 = \frac{1}{2}\left(\frac{\sigma'_{a_2}}{\sigma_f}\right)^{\frac{1}{b}} = 7198.8558$$

Now $n_2 = N_2\left(1 - \frac{n_1}{N_1}\right) \approx 2264$ cycles.

Note that if you had used $(\sigma_{max})_1 = 360$ MPa, then $\frac{n_1}{N_1} > 1$.

In fact $(\sigma_{max})_1 > \sigma_y$ and the component would

Problem 3

3. For a cantilever beam: $\delta = \frac{PL^3}{3EI}$. For a rectangular c/s $I = \frac{bh^3}{12}$

$$\therefore \delta = \frac{4PL^3}{Ebh^3}$$

or $$P = \frac{Ebh^3\delta}{4L^3}$$

$$P_{max} = \frac{Ebh^3\delta_{max}}{4L^3}, \quad P_{min} = \frac{Ebh^3\delta_{min}}{4L^3}$$

$$\sigma_{max} = \frac{M_{max}\,h/2}{I} = \frac{(P_{max}L)\,h/2}{I} = \frac{3}{2}\frac{Eh\delta_{max}}{L^2}$$

Similarly, $$\sigma_{min} = \frac{3}{2}\frac{Eh}{L^2}\delta_{min}.$$

$$\sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max}-\delta_{min})$$

$$\sigma_m = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max}+\delta_{min})$$

Substituting $E = 200\ GPa$, $h = 4mm$, $L = 300mm$, $\delta_{max} = 20mm$, $\delta_{min} = 10\ mm$

$$\sigma_a = 66.66\ MPa, \quad \sigma_m = 200\ MPa.$$

Goodman criterion for infinite life.

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n}$$

Substituting the above values with $\sigma_e = 0.5\,\sigma_{ult}$, $n = 1.4 > 1$

$\therefore$ Hence the factor of safety based on infinite life as per Goodman's criterion is 1.4.

Note: No correction factor is assumed for the endurance limit but not was mentioned/given. In practice, one does need to use the corrections factors to modify the uncorrected endurance strength

Ganesh Iyer, 210100059 ME-423: HW-5

1. A 50 mm diameter shaft is made of steel with yield strength = 400 MPa. A parallel key of size 16 mm width and thickness 10 mm is to be used. The shearing and the crushing stresses for the key material are 42 MPa and 70 MPa, respectively. Find the required length of the key if the shaft is loaded to transmit maximum torque. Assume factor of safety to be 2.0.

$W = 16mm, T = 10mm$

L unknown

$D_S = 50\,mm, \sigma_y = 400\,MPa$

$T \to F \to \tau_{shear}, \sigma_b$ (For key)

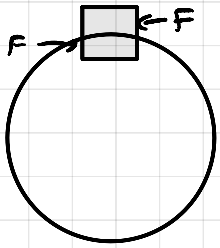
$\tau = \frac{Tr}{J}$, $\tau = (\sigma_y/2)$ → Tresca, σ_y ← Shaft

$= 200\,MPa$

No FOS for shaft, already designed

$\Rightarrow T_{max} = \frac{\tau J}{r} = \frac{\pi r^3 \tau}{2} = 4908.74$ N-m

$\Rightarrow \underbrace{F}_{\text{on key}} = \frac{T_{max}}{r_{shaft}} = 196349.54N$



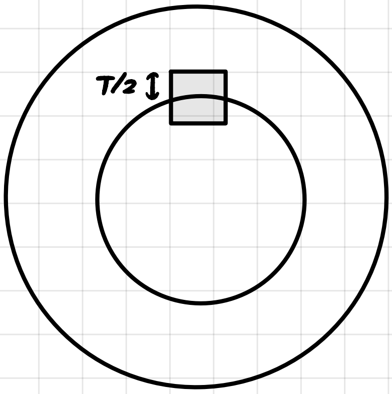
$\frac{\sigma_b}{2_{FOS}} = \frac{F}{\frac{T}{2} \times L}$, $\frac{\tau}{2_{FOS}} = \frac{F}{W \times L}$

$L \geq \frac{4F}{T\sigma_{b,max}}$, $\sigma_{b,max} = 70MPa$, $L \geq \frac{2F}{W\tau_{max}}$, $\tau_{max} = 42\,MPa$

$L \geq 1.122\,m$, $L \geq 0.5843\,m$

$\Rightarrow L \geq 1.122\,m$

2. A 63.5 mm diameter shaft has a key 16 mm $\times$ 16 mm. The yield point in tension of the shaft material = 400 MPa while its yield point in shear = 200 MPa. The factor of safety is 2. The shaft fits into a cast iron hub for which the working stress in compression is 125 MPa. What length of the key in the hub material will be required to carry the torque of the solid shaft ? The key material is assumed to be sufficiently strong.



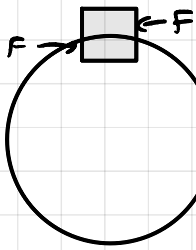
Length of key = Length of Hub

$\sigma_b = 125$ MPa , $N = 2$ (Hub)

$\tau = \frac{Tr}{J}$, $\tau = \tau_y = 200$ MPa

$\Rightarrow T_{max} = \frac{\tau J}{r} = \frac{\pi r^3 \tau}{2} = 10054.97$ N-m

$\Rightarrow \underset{\text{on key}}{F} = \frac{T_{max}}{r_{shaft}} = 316692.17$ N



$\underset{FOS}{\frac{\sigma_b}{2}} = \frac{F}{\frac{T}{2} \times L}$

$L \geqslant \frac{4F}{T\sigma_{b,max}} = 0.633$ m

$= 125$ MPa

3. A steel shaft has diameter 30 mm. The shaft rotates at a speed of 6000 rpm and transmits 30kW of power through a gear. The yield strength of shaft in tension and shear are 650 MPa and 353 MPa, respectively. The shaft and the key are made from the same material. Assume a factor of safety to be 3. Select a suitable parallel key for the gear, i.e. find the length, width and the thickness of the key that attaches the gear to the shaft.

$D_s = 30\,mm \Rightarrow$ From table : $W = 8\,mm,\ H = 7\,mm$

$$P = T \times \frac{2\pi N}{60} \Rightarrow T = \frac{60 \times 30 \times 10^3}{2\pi \times 6000}$$

$$\Rightarrow T = 47.746\ N\text{-}m$$

$$F_{\text{on key}} = T/r = 3183.1\ N$$

$$\frac{\sigma_b}{\underset{\text{FOS}}{3}} = \frac{F}{\frac{H}{2} \times L}, \quad \frac{\tau}{\underset{\text{FOS}}{3}} = \frac{F}{W \times L}$$

$$L \geq \frac{6F}{H\sigma_{b,max}}, \quad L \geq \frac{3F}{W\tau_{max}}$$

$\sigma_{b,max} = 650\ MPa$, $\tau_{max} = 353\ MPa$

$$L \geq 4.197\ mm, \quad L \geq 3.381\ mm$$

$$\Rightarrow L \geq 4.197\ mm$$

4. A 200 mm diameter steel shaft is to have a press fit in a 500 mm diameter cast iron disk. The maximum tangential stress in the disk cannot exceed 50 MPa. The modulus of elasticity of the steel is 200 GPa while that of cast iron is 100 GPa. Poisson's ratio is 0.3 for both the materials.

- Find the required diametral interference.
- If the disk is 25 mm thick in the axial direction, find the force required to press the parts together if the coefficient of friction is 0.12.
- Find the maximum torque which the joint can carry because of the shrink fit.
- Find the stress concentration factor for the shaft if it carries an alternating bending moment of 40000 Nm. Find the corresponding stress.

$a = 100\,mm,\ b = 250\,mm$

$\nu_1 = \nu_2 = 0.3$

$\mu_1 = \dfrac{200\text{ GPa}}{1-2\nu_1},\ \mu_2 = \dfrac{100\text{ GPa}}{1-2\nu_2}$

$\Rightarrow \mu_1 = 500\text{ GPa},\ \mu_2 = 250\text{ GPa}$

1) $\sigma_{\theta\theta}(r) = \dfrac{pa^2}{b^2-a^2}\left(1+\dfrac{b^2}{r^2}\right)$, max at $r=a$

$$\sigma_{\theta\theta,max} = \left(\frac{b^2+a^2}{b^2-a^2}\right)p \leq 50\text{ MPa}$$

$\Rightarrow p \leq 36.207\text{ MPa} = p_{max}$

$$p = \frac{\mu_1 \delta / a}{(1-2\nu_1) + \dfrac{\mu_1}{\mu_2}\left(\dfrac{b^2+a^2(1-2\nu_2)}{b^2-a^2}\right)} = \frac{500\times10^9 \times \delta \times 1/0.1}{0.4 + 2\left(\dfrac{6.25+0.4}{6.25-1}\right)}$$

$\Rightarrow p = 1.7045\times10^{12}\,\delta \leq 36.207\times10^6$

$\Rightarrow \delta \leq 21.241\,\mu m$

Diametrical Interference

2) $F_{axial} \geq \underbrace{p_{max} \times 2\pi a L}_{\text{Radial force}} \times f = \text{Friction}$ (coeff. of fric.)

$= 68.248\text{ KN}$

3) $T_{max} \leq \underbrace{p_{max} \times 2\pi a L \times f}_{\text{Friction along circumferential direction}} \times a$

$= 6.8248\text{ KN-m}$

4) $p = p_{max} = 36.207\,MPa$

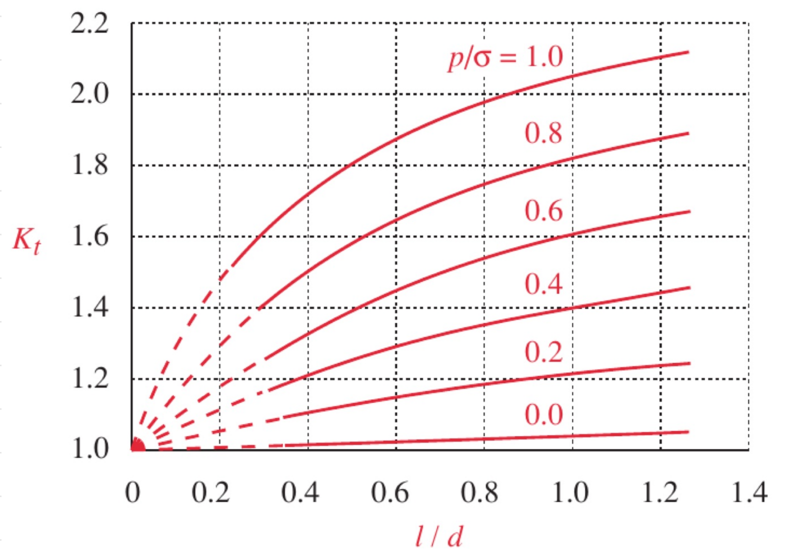
$l = 25mm$

$d = 200m$

$$\sigma = \frac{32M}{\pi d^3} = 50.93\,MPa$$

$$\frac{l}{d} = 0.125$$

$$\frac{p}{\sigma} = 0.711$$



$\Rightarrow K_t \approx 1.1$

5. A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to a stationary equivalent load of 3kN for 10% of the time, 2kN for 20% of the time, 1kN for 30% of the time and no load for remaining time. If the total expected time of the bearing is 20 × 10^6 revolutions at 90% reliability, calculate the dynamic load rate of the bearing. Repeat the calculation for 95% reliability.

i	α	F (KN)
1	0.1	3
2	0.2	2
3	0.3	1
4	0.4	0

$$F_c = \left(\sum_{i=1}^{4} \alpha_i F_i^3\right)^{1/3}$$

$$= \left(0.1\times 27 + 0.2\times 8 + 0.3\times 1 + 0\right)^{1/3}$$

$$= 4.6^{1/3} \text{ KN}$$

$$\underbrace{L_{10}}_{10^6 \text{ (At 90\% reliability)}} \times C_{10}^3 = \underbrace{L}_{20\times 10^6} \times F_c^3 \Rightarrow C_{10} = 4.514 \text{ KN}$$

At 95% reliability, $L_{10} = 0.64 \times 10^6$

$$\Rightarrow C_{10} = \left(\frac{20}{0.64} \times 4.6\right)^{1/3} \text{KN} = 5.238 \text{KN}$$

6. Rolling contact bearings are to be selected to support a countershaft. The shaft speed is 720 rpm. The bearings are expected to have 99% reliability corresponding to 24000 hours. The bearing is subject to an equivalent radial load 1kN. Find the dynamic load rating of the bearings specified at 90% reliability.

$$L_{10} = 10^6 \text{ rev},\ L = \underbrace{0.25}_{99\% \text{ reliability}} \times 720 \times 24000 \times 60 = 259.2 \times 10^6 \text{ rev}, F = 1\text{KN}$$

$$L_{10} C_{10}^{10/3} = L F^{10/3} \text{ (Roller bearing)}$$

$$\Rightarrow C_{10} = \left(\frac{L}{L_{10}}\right)^{0.3} F = 5.2977 \text{KN}$$

7. Select a single row deep groove ball bearing with operating cycles listed below, which will have a life of 15000 hours:

Fraction of time	Load factor	Radial Load (N)	Thrust load (N)	Speed (rpm)
1/10	3.0	2000	1200	400
1/10	1.5	1500	1000	500
2/10	2.0	1000	1500	600
6/10	1.0	1200	2000	800

Assume X = 1.0, Y = 1.5 and V = 1.0. (parameters to calculate the equivalent radial load)

$V=1, \quad \frac{F_e}{F_r} = X + Y\frac{F_a}{F_r}$

$\Rightarrow F_e = XF_r + YF_a$

Time fraction	rpm	N_{rev}	α	F_e(N)	(factor) a	$\alpha(aF_e)^3$
0.1	400	40	4/69	3800	3	85.886×10^9
0.1	500	50	5/69	1750	1.5	1.3107×10^9
0.2	600	120	4/23	2400	2	19.233×10^9
0.6	800	480	16/23	3200	1	22.795×10^9

$N_{tot} = 690$

$$F_e = \left(\sum_{i=1}^{4} \alpha_i (a_i F_{e,i})^3\right)^{1/3} = 5.0557 \text{ KN}$$

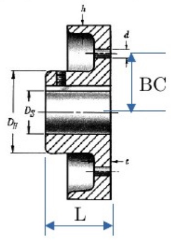
$N_{tot} = 690$ rev in 1min

$\Rightarrow L = 690 \times 15000 \times 60 \text{ rev} = 621 \times 10^6 \text{ rev}$

$$L F_e^3 = \underbrace{L_{10}}_{10^6} C_{10}^3 \quad \Rightarrow C_{10} = 43.133 \text{ KN}$$

Ganesh Iyer, 210100059 ME-423: HW-6

1. Consider a cast iron rigid coupling shown in the figure.



It is used to transmit 15 kW at 900 rpm from an electric motor to a compressor. The design factor can be assumed to be 1.35. The shaft is designed to carry maximum permissible torque. The coupling is designed as per the following specification: $D_H = 2D_S$, $L = 1.5D_S$, $t = 0.5D_S$, $BC = 3D_S$. Number of bolts = 4, diameter of the bolt, $d =$ 6 mm. Here D_S denotes the diameter of the shaft. Identify the potential failure locations and verify whether the design is safe of not by finding the factor of safety at the critical locations. If the design is safe, identify the part that is most likely to fail first. Use the following material properties:

Yield strength in tension/compression for shaft, bolt and key material = 200 MPa

Shear stress for cast iron = 30 MPa

$$T = P/\omega = 15000 \Big/ \left(\frac{2\pi \times 900}{60}\right)$$

$$T = 159.155 \text{ N-m}$$

$$T = \underset{\text{Design factor}}{1.35} \times T = 214.86 \text{ N-m}$$

Shaft : Steel Hub, Flange - cast iron

$$\tau = \frac{16T}{\pi D_s^3} \leq \tau_{max} = \frac{\sigma_y}{2} \text{ (Tresca)}$$

$$\Rightarrow D_s \geq \left(\frac{32T}{\pi\sigma_y}\right)^{1/3} = 22.2 \text{ mm}$$

$$D_H = 2D_s = 44.4 \text{ mm}, \ L = 1.5D_s = 33.3 \text{ mm}$$
$$t = 0.5D_s = 11.1 \text{ mm}, \ BC = 3D_s = 66.6 \text{ mm}$$

Potential failure :

1) Key : $F = \dfrac{T}{(D_s/2)} = \dfrac{214.86}{11.1\times10^{-3}} = 19356.75 \text{ N}$

$D_s = 22.2 \text{ mm} \Rightarrow W = 8 \text{ mm}, \ H = 7 \text{ mm}, \ L = 33.3 \text{ mm}$ (given)

$$\Rightarrow \sigma_{shear\,key} = \frac{F}{W \cdot L} = 72.66 \text{ MPa}$$

$$FOS_1 = \tau_y / \sigma_{shear} = 100/72.66 = 1.376$$

$$\Rightarrow \sigma_{bearing} = \frac{F}{\frac{L}{2}\times H} = 166.081 \Rightarrow FOS_2 = \frac{100}{166.081} = 0.602$$

2) Bolt: Force on each bolt $F_b = \dfrac{T}{4 \times BC} = 806.53$ N-m

$$\sigma_{shear} = \frac{F_b}{(\pi d^2/4)} = 28.52 \text{ MPa}$$

$$FOS_3 = \frac{\sigma_{shear,y}}{\sigma_{shear}} = \frac{30}{28.52} = 1.0517$$

$$\sigma_{bearing} = \frac{F_b}{t \cdot d} = \frac{806.53}{11.1 \times 6 \times 10^{-6}} = 12.11 \text{ MPa}$$

$$FOS_4 = \frac{\sigma_y}{\sigma_{bearing}} = \frac{200}{12.11} = 16.515$$

3) Flange:

$$\sigma_{shear} = \frac{(T/D_H/2)}{\pi \times D_H \times t} = \frac{2T}{\pi D_H^2 \times t} = 23.464 \text{ MPa}$$

$$FOS_5 = \frac{\sigma_{shear,y}}{\sigma_{shear}} = \frac{30}{23.464} = 1.278$$

4) Hub:

$$\tau_{max} = \frac{T(D_H/2)}{\frac{\pi}{32}(D_H^4 - D_S^4)} = \frac{32T}{15\pi D_S^3} = 50.057 \text{ MPa}$$

$$\Rightarrow FOS_6 = \tau_{hub}/50.057 = 30/50.057 = 0.5993$$

Both key and hub have FOS < 1 each, but hub has lesser ⇒ Failure at hub.

2. One would like to choose a *flexible coupling* made from cast iron to transmit 15 kW at 900 rpm from an electric motor to a compressor. The design factor can be assumed to be 1.35. From the catalog provided, identify the coupling which suits the requirements. Verify whether the coupling is safe by identifying the potential failure locations and finding the corresponding factor of safety. Use the following material properties:

Yield strength in tension/compression for shaft, bolt and key material = 200 MPa

Shear stress for cast iron = 40 MPa

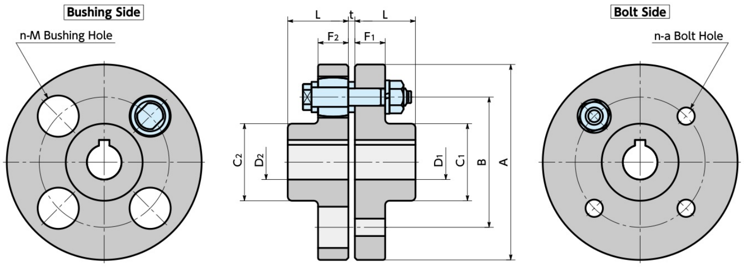
Compressive strength of bushing (Nitrile Butadiene Rubber) = 6 MPa

$$T = \underset{\text{Design factor}}{1.35} \times \frac{15000}{\frac{2\pi \times 900}{60}} = 214.86 \text{ N-m}$$

From prev q, $D_s = 22.2$ mm

Performance

Part Number	Max. Bore Diameter (mm) D_s		Maximum Torque (N•m) T
	D_1	D_2	
FCL-90	20	20	15
FCL-100	25	25	29
FCL-112	28	28	33
FCL-125	32	28	73
FCL-140	38	35	130
FCL-160	45	45	200
FCL-180	50	50	230
FCL-200	56	56	440
FCL-224	63	63	510



• The bolt hole positions are roughly arranged for the keyway.

Dimensions

Unit : mm

Part Number	A	Prepared hole diameter	L	C		B	F		n (item)	a	M	t	Bolt Draft	Coupling bolt Part Number
				C_1	C_2		F_1	F_2						
FCL-90	90	–	28	35.5	35.5	60	14	14	4	8	19	3	50	F1
FCL-100	100	–	35.5	42.5	42.5	67	16	16	4	10	23	3	56	F2
FCL-112	112	–	40	50	50	75	16	16	4	10	23	3	56	F2
FCL-125	125	–	45	56	50	85	18	18	4	14	32	3	64	F3
FCL-140	140	–	50	71	63	100	18	18	6	14	32	3	64	F3
FCL-160	160	–	56	80	80	115	18	18	8	14	32	3	64	F3
FCL-180	180	–	63	90	90	132	18	18	8	14	32	3	64	F3
FCL-200	200	18	71	100	100	145	22.4	22.4	8	20	41	4	85	F4
FCL-224	224	18	80	112	112	170	22.4	22.4	8	20	41	4	85	F4

Choose FCL-180 $\Rightarrow L = 63$ mm, $D_H = C_1 = C_2 = 90$ mm

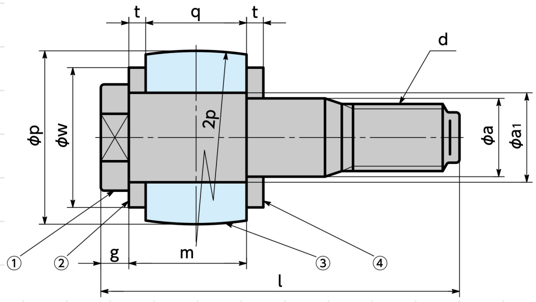
$BC = B/2 = 66$ mm, $t = F_1 = F_2 = 18$ mm

F3 coupling bolt part

Bolt Set Dimensions

Unit : mm

Bolt set Part Number	Bush Part Number	Nominal a×l	1) Bolt							2), 4) Washer		3) Bush		Tightening torque (N•m)
			d	a_1	a	f	g	m	l	w	t	p	q	
F1-SET	F1-G	8×50	M8	9	8	10	4	17	50	14	3	18	14	11
F2-SET	F2-G	10×56	M10	12	10	13	4	19	56	18	3	22	16	22
F3-SET	F3-G	14×64	M12	16	14	17	5	21	64	25	3	31	18	39
F4-SET	F4-G	20×85	M20	22.4	20	24	5	26.4	85	32	4	40	22.4	190
F5-SET	F5-G	25×100	M24	28	25	30	6	32	100	40	4	50	28	330
F6-SET	F6-G	28×116	M24	31.5	28	32	6	44	116	45	4	56	40	330
F7-SET	F7-G	35.5×150	M30	40	35.5	41	8	61	150	56	5	71	56	650
F7L-SET	F7-G	35.5×174	M30	40	35.5	41	8	61	174	56	5	71	56	650
F8-SET	F8-G	45×240	M42	50	45	50	10	87	240	71	7	85	80	1800



$d_{bolt} = a = 14$ mm, and taking $n_{bolts} = 8$.

$d_{bush} = 2 \times p = 6$ mm

Potential failure:

1) Key : $F = \dfrac{T}{(D_s/2)} = \dfrac{214.86}{11.1 \times 10^{-3}} = 19356.75$ N

$D_s = 22.2$ mm $\Rightarrow W = 8$ mm, $H = 7$ mm, $L = 63$ mm

$$\Rightarrow \sigma_{shear} = \frac{F}{W \cdot L} = 38.4 \text{ MPa}$$

$$FOS_1 = \tau_y / \sigma_{shear} = 100/38.4 = 2.603$$

$$\Rightarrow \sigma_{bearing} = \frac{F}{(L/2) \times H} = 87.78 \text{ MPa}$$

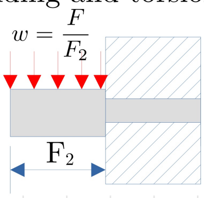
$$\Rightarrow FOS_2 = \frac{100}{87.78} = 1.14$$

2) Bolt: Force on each bolt $F_b = \frac{T}{8 \times BC} = 406.932$ N-m

$$\sigma_{bearing} = \frac{F_b}{t \cdot d} = \frac{406.932}{18 \times 66 \times 10^{-6}} = 0.342 \text{ MPa}$$

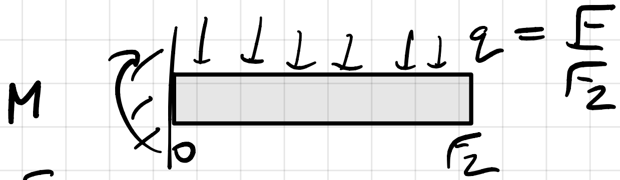
$$FOS_3 = \frac{\sigma_y}{\sigma_{bearing}} = \frac{200}{0.342} = 583.88$$

Bolt or Pin failure – check for combined bending and torsion



Cantilever beam with uniformly distributed load

Combined torsion+bending



$$M = \int_0^{F_2} \frac{F}{F_2} \, dx \times x = \frac{F \times F_2^2}{2F_2} = \frac{F \times F_2}{2}$$

$$\Rightarrow M = \frac{F \times t}{2} = \frac{406.932 \times 18 \times 10^{-3}}{2} = 3.662 \text{ N-m}$$

$$\sigma_{bend} = \frac{My}{I} = \frac{32M}{\pi d_{bolt}^3} = \frac{32 \times 3.662}{\pi \times (14 \times 10^{-3})^3} = 13.595 \text{ MPa}$$

$$\sigma_{shear} = \frac{F_b}{(\pi d_{bolt}^2/4)} = 2.643 \text{ MPa}$$

$$\sigma_{vm} = (\sigma_{bend}^2 + 3\sigma_{shear}^2)^{1/2} = 14.345 \text{ MPa}$$

$$FOS_4 = \frac{\sigma_y}{\sigma_{vm}} = \frac{200}{14.345} = 13.942$$

3) Flange :

$$\sigma_{shear} = \frac{(T/D_H/2)}{\pi \times D_H \times t} = \frac{2T}{\pi D_H^2 \times t} = 0.938 \text{ MPa}$$

$$FOS_5 = \frac{\sigma_{shear,y}}{\sigma_{shear}} = \frac{40}{0.938} = 42.64$$

4) Hub :

$$\tau_{max} = \frac{T(D_H/2)}{\frac{\pi}{32}(D_H^4 - D_s^4)} = \frac{16TD_H}{\pi(D_H^4 - D_s^4)} = 1.506 \text{ MPa}$$

$$\Rightarrow FOS_6 = \tau_{hub}/1.506 = 40/1.506 = 26.55$$

5) Bush :

$$p_{bush} = \frac{F_{bolt}}{F_2 \times d_{bush}} = \frac{406.932}{18 \times 31 \times 10^{-6} \times 2} = 0.365 \text{ MPa}$$

t

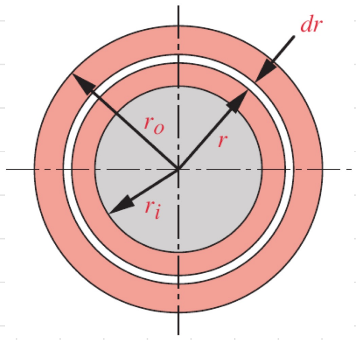
$$FOS_7 = \frac{\sigma_{comp}}{p_{bush}} = \frac{6}{0.365} = 16.44 \text{ MPa}$$

Lowest FOS for key, with bearing failure, FOS = 1.14

3. Determine the maximum, minimum and average pressure and the torque carrying capacity of a disk clutch when the axial force is 4500 N, the inside radius of contact is 50 mm, the outside radius of contact is 100 mm, coefficient of friction = 0.10 for the following two conditions:

a. Uniform pressure.

b. Uniform wear



a) Max = Min = Avg $p = p_0$

$$dN = 2\pi p_0 r\,dr$$

$$dT = dF \times r = \mu\,dN\,r = 2\pi\mu p_0 r^2\,dr$$

$$T = \int_{r_i}^{r_o} 2\pi\mu p_0 r^2\,dr = \frac{2\pi\mu p_0}{3}(r_o^3 - r_i^3)$$

$$N = \int_{r_i}^{r_o} 2\pi p_0 r\,dr = \pi p_0 (r_o^2 - r_i^2) \Rightarrow p_0 = \frac{N}{\pi(r_o^2 - r_i^2)} = 0.19\text{ MPa}$$

$$\Rightarrow T = \frac{2\pi \times 0.1 \times 0.19 \times 10^6 \times (0.1^3 - 0.05^3)}{3}$$

$$\Rightarrow T = 35\text{ N-m}$$

b) $p(r) = C/r$, for constant wear

$$dN = 2\pi \times \frac{C}{\cancel{r}} \times \cancel{r}\,dr = 2\pi C\,dr$$

$$dT = \mu\,dN \times r = 2\pi\mu C r\,dr$$

$$\Rightarrow N = 2\pi C(r_o - r_i),\quad T = \pi\mu C(r_o^2 - r_i^2)$$

$$C = p_{max} \times r_i \Rightarrow p_{max} = \frac{N}{2\pi r_i (r_o - r_i)} = 0.286\text{ MPa}$$

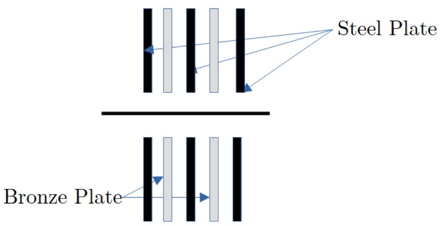
$$T = \pi\mu p_{max} r_i (r_o^2 - r_i^2) = 33.75\text{ N-m}$$

$$p_{min} = \frac{p_{max} \times r_i}{r_o} = \frac{p_{max}}{2} = 0.143 \text{ MPa}$$

$$p_{avg} = \frac{\int_{r_i}^{r_o} \cancel{2\pi} p(r)\, r\, dr}{\int_{r_i}^{r_o} \cancel{2\pi} r\, dr} = \frac{p_{max} \times r_i \times (r_o - r_i)}{\frac{1}{2}(r_o^2 - r_i^2)} = \frac{2 p_{max} r_i}{(r_o + r_i)}$$

$$= \frac{2}{3} p_{max} = 0.19 \text{ MPa}$$

4. A multiple disk clutch is composed of 3 steel and 2 bronze circular disks arranged as shown below:



It is required to transmit a torque of 20 Nm. If the inner diameter is to be 50 mm, determine the necessary outer diameter of the disks and the required outward diameter of the disks. Assume that the coefficient of friction is 0.1.

Constant wear more conservative.

$20 = 4 \times T_{\text{for each clutch}}$

$\Rightarrow T = 5 \text{ N-m}$

$$T = \pi p_{max} \mu_f r_i (r_o^2 - r_i^2)$$

If p_{max} known, can get r_o.

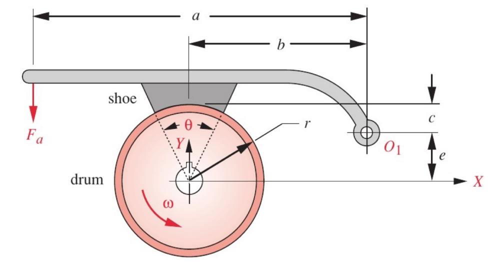
from book, p_{max} for bronze over steel $= 0.4$ MPa

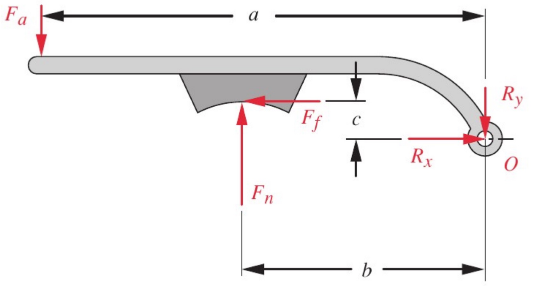
$$\Rightarrow r_o = \sqrt{r_i^2 + \frac{T}{\pi p_{max} \mu_f r_i}}$$

$\Rightarrow r_o = 47.08$ mm

$\Rightarrow d_o = 94.16$ mm

5. Figure shows a single short-shoe drum brake. Find its torque capacity and required actuating force for a = 100, b = 70, e = 20, r = 30, w = 50 mm, and θ = 35°. What value of c will make it self-locking? Assume pmax = 1.3 MPa and μ = 0.3.





Free body diagram

$$a F_a + c \underbrace{F_f}_{\mu F_n} - b F_n = 0 \quad \Rightarrow F_a = F_n\left(\frac{b - c\mu_f}{a}\right)$$

Assuming constant pres. over θ,

$$F_n = p \times r w \theta \leq p_{max} \times r w \theta = 1191.18\,\text{N}$$

Torque on drum : $T = F_f \times r = \mu F_n r$

$$\Rightarrow T_{max} = \mu p_{max} r^2 w \theta = 10.72\ \text{N-m}$$

$$c = r - e = 10\,\text{mm}$$

$$\Rightarrow F_a = 1191.18\left(\frac{70 - 0.3\times 10}{100}\right) = 798.1\,\text{N}$$

For self-locking, $F_a \leq 0 \Rightarrow c \geq \dfrac{b}{\mu_f} = 233.33\,\text{mm}$

6. Repeat Problem 5 with the drum rotating clockwise.

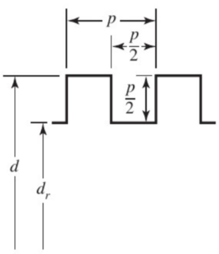
F_f changes direction, rest remain same.

$$\Rightarrow \quad F_a \text{ changes} = F_n\left(\frac{b + c\mu_f}{a}\right) = 870.014\,\text{N}$$

No self locking as $b + c\mu_f \geq 0$ always.

Tutorial 7

1. A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in the following figure.



The given data include $\mu = \mu_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.
(a) Find the thread depth, thread width, mean diameter, minor diameter, and lead.
(b) Find the torque required to raise and lower the load.
(c) Find the efficiency during lifting the load.
(d) Find the body stresses, torsional and compressive.
(e) Find the bearing stress on the first thread.
(f) Find the thread bending stress at the root of the first thread.
(g) Determine the von Mises stress at the critical stress element where the root of the first thread interfaces with the screw body.

(a)

$d_m = d - \frac{p}{2} = 30\,\text{mm}$

$d_r = d - p = 28\,\text{mm}$

thread depth = 2mm

—||— width = 2mm

Lead = 2p = 8mm

(b) $$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi \mu d_m}{\pi d_m - \mu l}\right) + \frac{F\mu_c d_c}{2}$$

$$\boxed{T_R = 26.18\,\text{Nm}}$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi \mu d_m - l}{\pi d_m + \mu l}\right) + \frac{\mu_c F d_c}{2}$$

$$\boxed{T_L = 9.77\,\text{Nm}}$$

(c) $$\text{efficiency} = \frac{Fl}{2\pi T_R} = \boxed{0.311}$$

(d) Shear stress due to torsion $= \dfrac{16T_R}{\pi d_r^3} = 6.07\,MPa$

Axial stress due to compression $= \dfrac{4F}{\pi d_r^2} = -10.39\,MPa$

(e) Bearing stress carried by first thread

$$\sigma_b = -\frac{2(0.38F)}{\pi d_m P} = -12.4\,MPa$$

(f) Bending stress at root

$$\sigma_b = \frac{6(0.38F)}{\pi d_r P} = 41.5\,MPa.$$

(g) $\sigma_{xx} = 41.5\,MPa$ $\quad \tau_{xy} = 0$

$\sigma_{yy} = -10.39\,MPa$ $\quad \tau_{yz} = 6.07\,MPa$

$\sigma_{zz} = 0$ $\quad \tau_{zx} = 0$

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\Big[(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{xx})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\Big]^{1/2}$$

$\sigma_{vm} = 48.7\,MPa.$

2. Find the bolt spring rate of M12 $\times$ 1.25 $\times$ 38.1 mm (property class 8.8) bolt.

From the following table

Nominal Major Diameter d mm	Coarse-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²	Fine-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

$A_t = 92.1\ mm^2$

$A_r = 86\ mm^2$

$A_d = \frac{\pi d^2}{4} = 113.1\ mm^2$

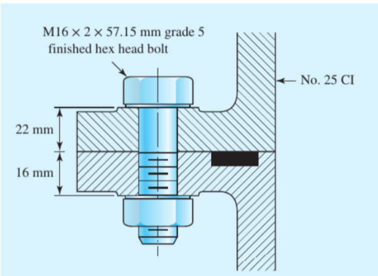
$l_t = 38.1\ mm$

$l_d = 0\ mm$

$\therefore k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{A_t E}{l_t}$

3

3. The figure shown below is a cross section of a grade 25 cast-iron pressure vessel.



A total of N bolts are to be used to resist a separating force of 160 kN.

(a) Determine k_b, k_m, and C. You can estimate k_m using the following equation presented by Wileman et al.

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

This equation is valid only if the entire joint is made of the same material. Here E is the Young's modulus, d is the mean bolt diameter and l is the grip length. For the given joint: $E = 100$ GPa, A = 0.77871, B = 0.61616.

(b) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

(c) With the number of bolts obtained in part (b), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

(a)

$l = 22 + 16 = 38\ mm$

bolt thickness = 14.8 mm

2 threads = 4 mm

$\therefore$ Total length = 56.8 mm

$\therefore L = 60\ mm$

$L_T = 2(D) + 6 = 2(16) + 6$
$= 38\ mm$

$l_D = L - L_T = 22\ mm$

$l_T = l - l_d = 38 - 22 = 16\ mm$

Also $A_t = 157\ mm^2$

$A_d = \frac{\pi (16)^2}{4} = 201\ mm^2$

$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$

$$\therefore k_b = \frac{201 \times 157 \times 207 \times 10^{-3}}{(201)(16) + (157)(22)} = \boxed{0.976 \text{ MN/mm}}$$

$$k_m = E d A \exp\left(\frac{Bd}{l}\right)$$

$$= 100 \times 10^9 \times (16 \times 10^{-3})(0.77871) \exp\left(\frac{0.61616 \times 16}{38}\right)$$

$$\boxed{k_m = 1.615 \text{ MN/mm}}$$

$$C = \frac{k_b}{k_m + k_b} = 0.3818$$

(b) $F_i = 0.75 A_t S_p \quad S_p = 600 \text{ MPa}$

$\therefore F_i = 70.65 \text{ kN}$ $\quad \underset{\text{Load Factor}}{n_L} = \dfrac{S_p A_t - F_i}{C(P/N)}$ $\quad N \to$ No. of bolts

$$\therefore N = \frac{C n_L P_{total}}{S_p A_t - F_i} = \frac{(0.3818)(2)(160)}{600(157 \times 10^{-3}) - 70.65} = 5.18$$

$\Rightarrow$ 6 bolts required.

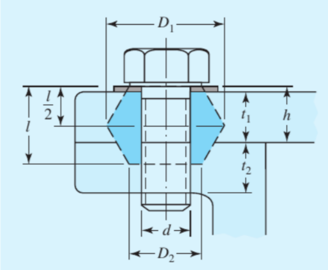
(c) For 6 bolts

$$n_L = \frac{600(157 \times 10^{-3}) - 70.65}{0.3818(160/6)} = 2.31$$

$$n_p = \frac{S_p A_t}{C(P/N) + F_i} = 1.17$$

Load factor against joint separation $= n_0 = \dfrac{F_i}{(P/N)(1-C)} = 4.29$

4. Figure shows a connection using cap screws.



The joint is subjected to a fluctuating force whose maximum value is 22.24 kN per screw. The data for the cap screw is M16 × 2, (property class 8.8). The bolt stiffness k_b is 1.28 MN/mm while the member stiffness, k_m, is 3.07 MN/mm.

(a) Find all factors of safety – yield, overload, separation, fatigue.

$$C = \frac{k_b}{k_b + k_m} = 0.294$$

Preload

$$F_i = 0.75 A_t S_p$$

$$= 0.75(157)(600 \times 10^{-3})$$

$$= 70.65 \text{ kN}$$

Yield FOS $n_p = \dfrac{S_p A_t}{CP + F_i} = 1.22$

Overload FOS $n_L = \dfrac{S_p A_t - F_i}{CP} = 3.60$

Separation FOS $n_0 = \dfrac{F_i}{P(1-C)} = 4.50$

For fatigue FOS

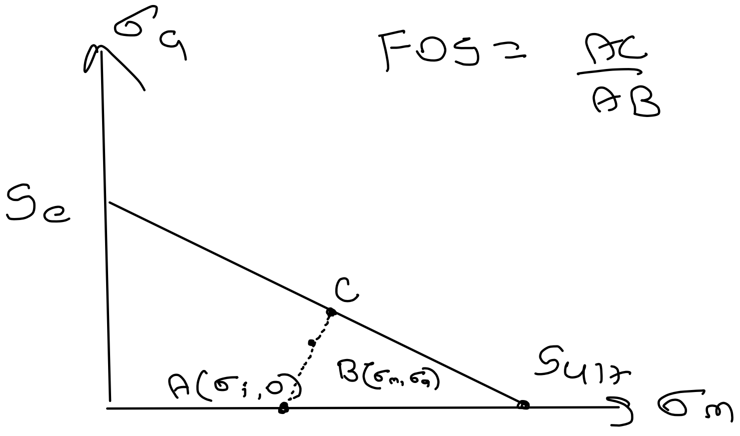
$$\sigma_i = \frac{F_i}{A_t} = 450 \text{ MPa.}$$

$$\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t} = \frac{CP}{2A_t} = 20.81 \text{ MPa}$$

$P_{max} = 22.24$ kN $\quad P_{min} = 0$

$$\sigma_m = \sigma_a + \sigma_i = 470.81 \text{ MPa.}$$

$$FOS = \frac{AC}{AB}$$



$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)}$$

$$= 2.43$$

EXAMPLE 8–6

Two 25.4 by 101.6 mm 1018 cold-rolled steel bars are butt-spliced with two 12.7 by 101.6 mm 1018 cold-rolled splice plates using four M20 × 1.5 mm grade 5 bolts as depicted in Figure 8–26. For a design factor of $n_d = 1.5$ estimate the static load F that can be carried if the bolts lose preload.

Solution

From Table A–20, minimum strengths of $S_y = 370$ MPa and $S_{ut} = 440$ MPa are found for the members, and from Table 8–11 minimum strengths of $S_p = 600$ MPa, $S_y = 660$ MPa, and $S_{ut} = 830$ MPa for the bolts are found.

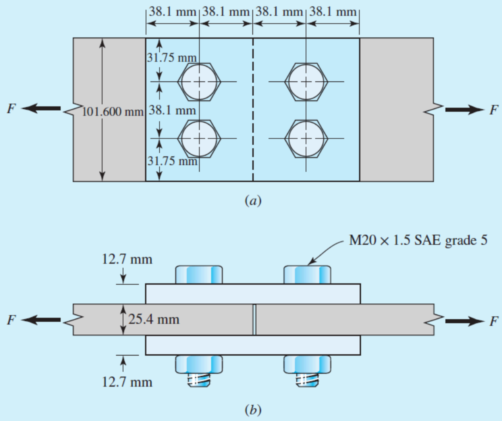


Figure 8–26

$F/2$ is transmitted by each of the splice plates, but since the areas of the splice plates are half those of the center bars, the stresses associated with the plates are the same. So for stresses associated with the plates, the force and areas used will be those of the center plates.

Bearing in bolts, all bolts loaded:

$$\sigma = \frac{F}{2td} = \frac{S_y}{n_d}$$

$$F = \frac{2td\,S_y}{n_d} = \frac{2(25.4)(20)(660)}{1.5} = 447 \text{ kN}$$

Bearing in members, all bolts active:

$$\sigma = \frac{F}{2td} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{2td(S_y)_{\text{mem}}}{n_d} = \frac{2(25.4)(20)(370)}{1.5} = 250.61 \text{ kN}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$\tau = \frac{F}{4\pi d^2/4} = 0.577\frac{S_y}{n_d}$$

$$F = 0.577\pi d^2\frac{S_y}{n_d} = 0.577\pi(20)^2\frac{660}{1.5} = 319 \text{ kN}$$

If the bolt threads extend into a shear plane:

$$\tau = \frac{F}{4A_r} = 0.577\frac{S_y}{n_d}$$

$$F = \frac{0.577(4)A_r S_y}{n_d} = \frac{0.577(4)259(660)}{1.5} = 263 \text{ kN}$$

Edge shearing of member at two margin bolts: From Figure 8–27,

$$\tau = \frac{F}{4at} = \frac{0.577(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{4at0.577(S_y)_{\text{mem}}}{n_d} = \frac{4(28.575)(25.4)0.577(370)}{1.5} = 413 \text{ kN}$$

Tensile yielding of members across bolt holes:

$$\sigma = \frac{F}{[4 - 2(\frac{3}{4})]t} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{[4 - 2(\frac{3}{4})]t(S_y)_{\text{mem}}}{n_d} = \frac{[4 - 2(\frac{3}{4})](25.4)370}{1.5} = 386 \text{ kN}$$

the limiting value of the force is 250 kN, assuming the bearing stress into a member. However, it would be poor design to allow the threads to extend into a shear plane. So, assuming a *good* design based on bolt shear, the limiting value of the force is 319 kN. For the members, the threads extend limits the load to 263 kN.

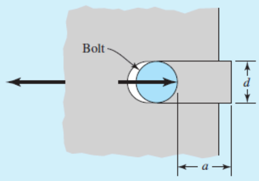


Figure 8–27

Edge shearing of member.

6

6. A M20 $\times$ 1.5 $\times$ 63.5 mm (property class 5.8) bolt is subjected to a load P of 26.69 kN in a tension joint. The initial bolt tension is $F_i = 111.205$ kN. The bolt and joint stiffnesses are $k_b = 1.14$ and $k_m = 2.42$ MN/mm, respectively.

(a) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(b) Specify the torque necessary to develop the preload. Specify the torque necessary to develop the preload.

(a) $\sigma_i = \frac{F_i}{A_t} = \frac{111.205}{240.645} = 426.11$ MPa

$C = \frac{k_b}{k_b + k_m} = 0.321$

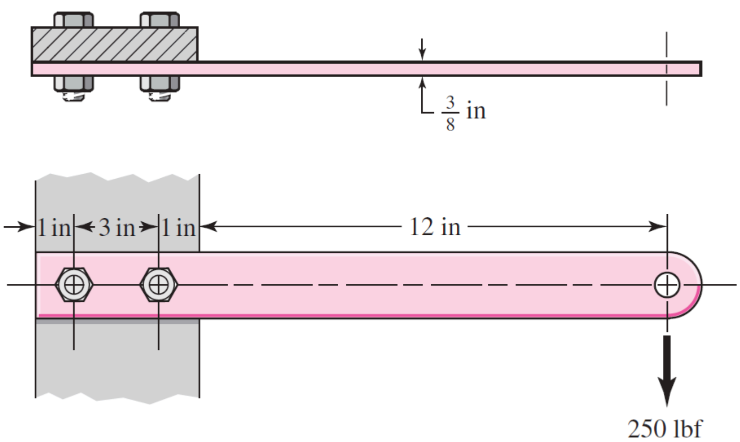
Service load stress $= \frac{CP + F_i}{A_t} = \frac{CP}{A_t} + \sigma_i = 497.66$ MPa

$S_p = 586.075$ MPa $\therefore S_p > \sigma_b$

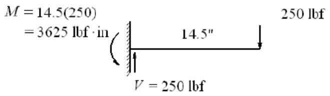
(b) $T = K F_i d = (0.2)(111.205)(20) = 444820$ N.mm

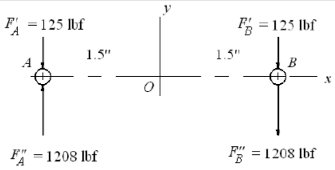
7

8–77 A $\frac{3}{8}$- $\times$ 2-in AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 250 lbf as illustrated. The bar is secured to the support using two $\frac{3}{8}$ in-16 UNC SAE grade 4 bolts. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.



blem 8–77





$$F''_A = F''_B = \left(\frac{3625}{3}\right) = 1208 \text{ lbf}$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}, \qquad F_B = 1208 + 125 = 1333 \text{ lbf}$$

Bolt shear:

$$A_s = (\pi/4)(0.375^2) = 0.1104 \text{ in}^2$$

$$\tau_{\max} = \frac{F_{\max}}{A_s} = \frac{1333}{0.1104} = 12\,070 \text{ psi}$$

From Table 8-10, $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{57.7}{12.07} = 4.78 \qquad Ans.$$

Bearing on bolt: Bearing area is $A_b = td = 0.375\ (0.375) = 0.1406 \text{ in}^2$.

$$\sigma_b = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\,481 \text{ psi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{9.481} = 10.55 \qquad Ans.$$

Bearing on member: From Table A-20, $S_y = 54$ kpsi. Bearing stress same as bolt

$$n = \frac{S_y}{|\sigma_b|} = \frac{54}{9.481} = 5.70 \qquad Ans.$$

Bending of member: At B, $M = 250(13) = 3250$ lbf·in

$$I = \frac{1}{12}\left(\frac{3}{8}\right)\left[2^3 - \left(\frac{3}{8}\right)^3\right] = 0.2484 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\,080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13 \qquad Ans.$$

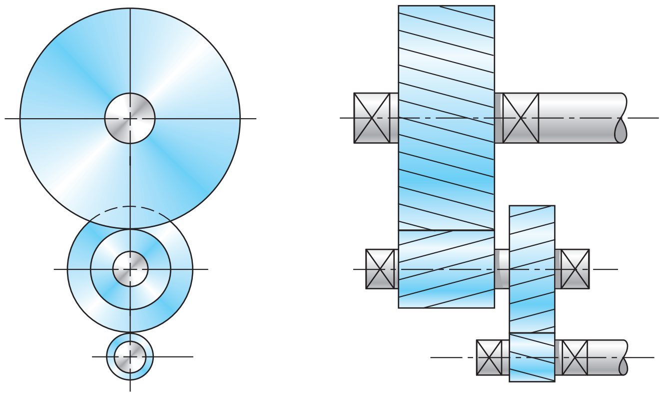


Figure 13–24

A two-stage compound gear train.

problems by compounding additional pairs of gears. A two-stage compound gear train, such as shown in Figure 13–24, can obtain a train value of up to 100 to 1.

The design of gear trains to accomplish a specific train value is straightforward. Since numbers of teeth on gears must be integers, it is better to determine them first, and then obtain pitch diameters second. Determine the number of stages necessary to obtain the overall ratio, then divide the overall ratio into portions to be accomplished in each stage. To minimize package size, keep the portions as evenly divided between the stages as possible. In cases where the overall train value need only be approximated, each stage can be identical. For example, in a two-stage compound gear train, assign the square root of the overall train value to each stage. If an exact train value is needed, attempt to factor the overall train value into integer components for each stage. Then assign the smallest gear(s) to the minimum number of teeth allowed for the specific ratio of each stage, in order to avoid interference (see Section 13–7). Finally, applying the ratio for each stage, determine the necessary number of teeth for the mating gears. Round to the nearest integer and check that the resulting overall ratio is within acceptable tolerance.

EXAMPLE 13–3

A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate tooth numbers.

Solution

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–24, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Equation (13–11). The number of teeth necessary for the mating gears is

Answer

$$16\sqrt{30} = 87.64 \approx 88$$

From Equation (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

EXAMPLE 13–4

A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \qquad \text{and} \qquad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Equation (13–11) gives the minimum as 16.

Then

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_4 = 5N_5 = 5(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Figure 13–25. This configuration is called a *compound reverted gear train*. This requires the distances between the shafts to be the same for both stages of the train, which adds to the complexity of the design task. The distance constraint is

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

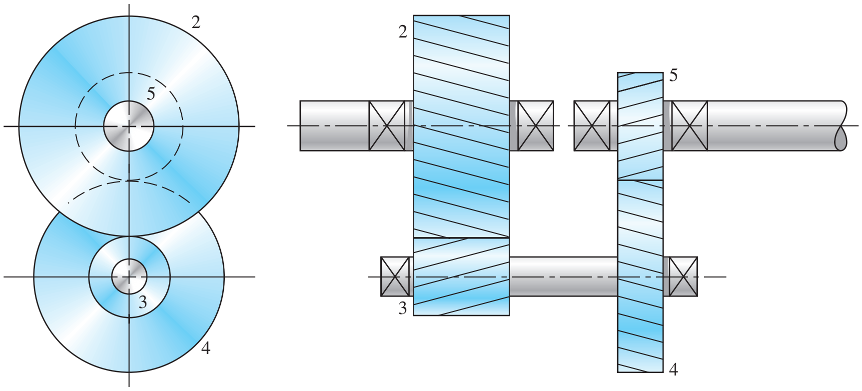


Figure 13–25

A compound reverted gear train.

EXAMPLE 13–5

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution

The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$. Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$

$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$

$$126 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 126/6 = 21$$

$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

Answer

$$N_2 = 108$$

$$N_3 = 18$$

$$N_4 = 105$$

$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$

$$108 + 18 = 105 + 21$$

$$126 = 126$$

Unusual effects can be obtained in a gear train by permitting some of the gear axes to rotate about others. Such trains are called *planetary,* or *epicyclic, gear trains.* Planetary trains always include a *sun gear,* a *planet carrier* or *arm,* and one or more *planet gears,* as shown in Figure 13–26. Planetary gear trains are unusual mechanisms because they have two degrees of freedom; that is, for constrained motion, a planetary train must have two inputs. For example, in Figure 13–26 these two inputs could be the motion of any two of the elements of the train. We might, say, specify that the sun gear rotates at 100 rev/min clockwise and that the ring gear rotates at 50 rev/min counterclockwise; these are the inputs. The output would be the motion of the arm. In most planetary trains one of the elements is attached to the frame and has no motion. Figure 13–27 shows a planetary train composed of a sun gear 2, an arm or carrier 3, and planet gears 4 and 5. The angular velocity of gear 2 relative to the arm in rev/min is

$$n_{23} = n_2 - n_3 \tag{b}$$

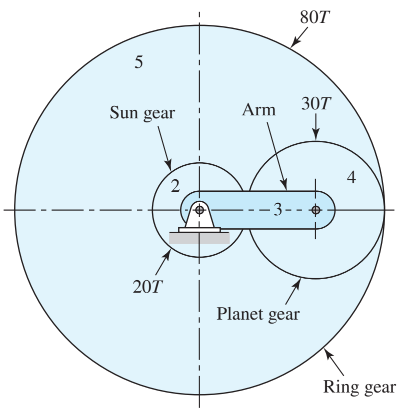


Figure 13–26

A planetary gear train.

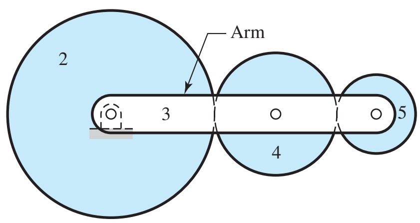


Figure 13–27

A gear train on the arm of a planetary gear train.

EXAMPLE 14–4

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 2980 watts to a 52-tooth disk gear. The module is 2.5 mm, the face width 38 mm, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is 207 GPa. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.
(*a*) Find the factor of safety of the gears in bending.
(*b*) Find the factor of safety of the gears in wear.
(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution

There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_p(m_d) = 17(2.5) = 42.5 \qquad d_G = 52(2.5) = 130 \text{ mm}$$

$$V = \pi d_P n_P = \pi(42.5)1800 = 240331.84 \text{ mm/min} = 4 \text{ m/s}$$

$$W^t = \frac{H}{V} = \frac{2980}{4} = 744 \text{ N}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{200(4)}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 38$ mm,

$$(K_s)_P = 1.192\left(\frac{38\sqrt{0.303}}{25.4^2}\right)^{0.0535} = 1.04$$

$$(K_s)_G = 1.192\left(\frac{38\sqrt{0.412}}{25.4^2}\right)^{0.0535} = 1.05$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 38/[10(42.5)] - 0.0375 + 0.0125(38/25.4) = 0.0706$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0706(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14–10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20° \sin 20°}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 191\sqrt{\text{MPa}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 0.5330(240) + 88.3000 = 216.22 \text{ MPa}$$

$$(S_t)_G = 0.5330(200) + 88.3000 = 194.9 \text{ MPa}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 2.2200(240) + 200 = 732.8 \text{ MPa}$$

$$(S_c)_G = 2.2200(200) + 200 = 644 \text{ MPa}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$
$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{K_m K_B}{FmJ}\right)_P = 743.97(1)1.377(1.043)\frac{1.22(1)}{38(2.5)(1000)0.30}$$
$$= 45725.69 \text{ kPa}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer $$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{216.22(1000)(0.977)/[1(0.85)]}{45725.69} = 5.43$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 743.97(1)1.377(1.052)\,\frac{1.22(1)}{38(2.5)(10^{-3})0.40} = 34577 \text{ kPa}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer
$$(S_F)_G = \frac{194.9(1000)(0.996)/[1(0.85)]}{34577} = 6.6$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_P\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 191(1000)\left[743.97(1)1.377(1.043)\frac{1.22}{42.5(38)}\,\frac{1}{0.121}\right]^{1/2} = 493024 \text{ kPa}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer
$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{732.8(1000)(0.948)/[1(0.85)]}{493024} = 1.66$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.05}{1.04}\right)^{1/2} 493024(1000) = 495055 \text{ kPa}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

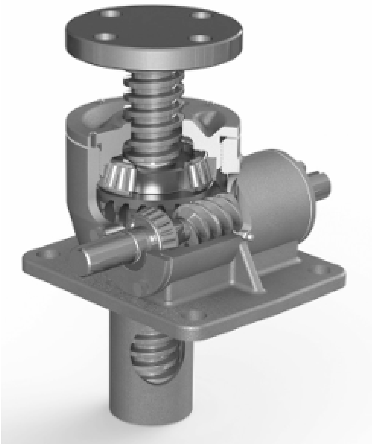
Answer
$$(S_H)_G = \frac{644(1000)(0.973)1.005/[1(0.85)]}{495055} = 1.50$$

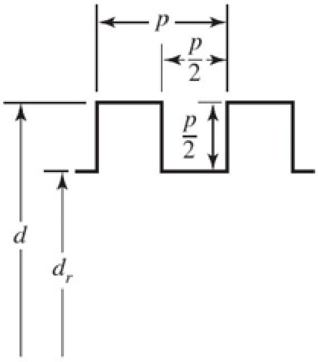
(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.43 with $1.66^2 = 2.75$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.6 with $1.5^2 = 2.24$, so the threat in the gear is also from wear.

There are perspectives to be gained from Example 14–4. First, the pinion is overly strong in bending compared to wear. The performance in wear can be improved by surface-hardening techniques, such as flame or induction hardening, nitriding, or carburizing and case hardening, as well as shot peening. This in turn permits the gearset to be made smaller. Second, in bending, the gear is stronger than the pinion, indicating that both the gear core hardness and tooth size could be reduced; that is, we may increase P and reduce the diameters of the gears, or perhaps allow a cheaper material. Third, in wear, surface strength equations have the ratio $(Z_N)/K_R$. The values of $(Z_N)_P$ and $(Z_N)_G$ are affected by gear ratio m_G. The designer can control strength by specifying surface hardness. This point will be elaborated later.

Having followed a spur-gear analysis in detail in Example 14–4, it is timely to analyze a helical gearset under similar circumstances to observe similarities and differences.

1. A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in the following figure.





The given data include $\mu = \mu_c = 0.08$, d_c = 40 mm, and F = 6.4 kN per screw.

(a) Find the thread depth, thread width, mean diameter, minor diameter, and lead.

(b) Find the torque required to raise and lower the load.

(c) Find the efficiency during lifting the load.

(d) Find the body stresses, torsional and compressive.

(e) Find the bearing stress on the first thread.

(f) Find the thread bending stress at the root of the first thread.

(g) Determine the von Mises stress at the critical stress element where the root of the first thread interfaces with the screw body.

EXAMPLE 8–1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. 8–4. The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

(*a*) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
(*b*) Find the torque required to raise and lower the load.
(*c*) Find the efficiency during lifting the load.
(*d*) Find the body stresses, torsional and compressive.
(*e*) Find the bearing stress.
(*f*) Find the thread bending stress at the root of the thread.
(*g*) Determine the von Mises stress at the root of the thread.
(*h*) Determine the maximum shear stress at the root of the thread.

Solution

(*a*) From Fig. 8–3*a* the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

Answer

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

(*b*) Using Eqs. (8–1) and (8–6), the torque required to turn the screw against the load is

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right) + \frac{Ff_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)}\right] + \frac{6.4(0.08)40}{2}$$

Answer

$$= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m}$$

Using Eqs. (8–2) and (8–6), we find the load-lowering torque is

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right) + \frac{Ff_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)}\right] + \frac{6.4(0.08)(40)}{2}$$

Answer

$$= -0.466 + 10.24 = 9.77 \text{ N} \cdot \text{m}$$

The minus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(*c*) The overall efficiency in raising the load is

Answer

$$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

(*d*) The body shear stress τ due to torsional moment T_R at the outside of the screw body is

Answer

$$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

Answer

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(*e*) The bearing stress σ_B is, with one thread carrying $0.38F$,

Answer

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(*f*) The thread-root bending stress σ_b with one thread carrying $0.38F$ is

Answer

$$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$

(*g*) The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (*d*) of 6.07 MPa. The three-dimensional stresses, after Fig. 8–8, noting the y coordinate is into the page, are

$$\sigma_x = 41.5 \text{ MPa} \qquad \tau_{xy} = 0$$

$$\sigma_y = -10.39 \text{ MPa} \qquad \tau_{yz} = 6.07 \text{ MPa}$$

$$\sigma_z = 0 \qquad \tau_{zx} = 0$$

For the von Mises stress, Eq. (5–14) of Sec. 5–5 can be written as

Answer

$$\sigma' = \frac{1}{\sqrt{2}}\{(41.5 - 0)^2 + [0 - (-10.39)]^2 + (-10.39 - 41.5)^2 + 6(6.07)^2\}^{1/2}$$

$$= 48.7 \text{ MPa}$$

Alternatively, you can determine the principal stresses and then use Eq. (5–12) to find the von Mises stress. This would prove helpful in evaluating $\tau_{\max}$ as well. The principal stresses can be found from Eq. (3–15); however, sketch the stress element and note that there are no shear stresses on the x face. This means that σ_x is a principal stress. The remaining stresses can be transformed by using the plane stress equation, Eq. (3–13). Thus, the remaining principal stresses are

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79,\ -13.18\ \text{MPa}$$

Ordering the principal stresses gives $\sigma_1, \sigma_2, \sigma_3 = 41.5, 2.79, -13.18$ MPa. Substituting these into Eq. (5–12) yields

Answer

$$\sigma' = \left\{\frac{[41.5 - 2.79]^2 + [2.79 - (-13.18)]^2 + [-13.18 - 41.5]^2}{2}\right\}^{1/2}$$
$$= 48.7\ \text{MPa}$$

(h) The maximum shear stress is given by Eq. (3–16), where $\tau_{\max} = \tau_{1/3}$, giving

Answer

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{41.5 - (-13.18)}{2} = 27.3\ \text{MPa}$$

2. Find the bolt spring rate of $\text{M12} \times 1.25 \times 38.1$ mm (property class 8.8) bolt.

Table 8–11

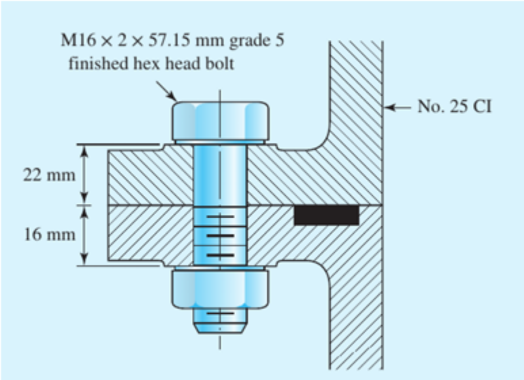
Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs

Property Class	Size Range, Inclusive	Minimum Proof Strength,* MPa	Minimum Tensile Strength,* MPa	Minimum Yield Strength,* MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6–M16	310	420	340	Low or medium carbon	4.8
5.8	M5–M24	380	520	420	Low or medium carbon	5.8
8.8	M16–M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	12.9

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Nominal Major Diameter d mm	Coarse-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²	Fine-Pitch Series Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

3. The figure shown below is a cross section of a grade 25 cast-iron pressure vessel.



A total of N bolts are to be used to resist a separating force of 160 kN.

(a) Determine k_b, k_m, and C. You can estimate k_m using the following equation presented by Wileman et al.

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

This equation is valid only if the entire joint is made of the same material. Here E is the Young's modulus, d is the mean bolt diameter and l is the grip length. For the given joint: E = 100 GPa, A = 0.77871, B = 0.61616.

(b) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

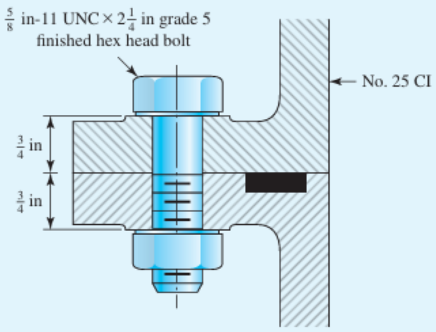
(c) With the number of bolts obtained in part (b), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

Figure 8–19 is a cross section of a grade 25 cast-iron pressure vessel. A total of N bolts are to be used to resist a separating force of 36 kip.
(*a*) Determine k_b, k_m, and C.
(*b*) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.
(*c*) With the number of bolts obtained in part (*b*), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

(*a*) The grip is l = 1.50 in. From Table A–31, the nut thickness is $\frac{35}{64}$ in. Adding two threads beyond the nut of $\frac{2}{11}$ in gives a bolt length of

$$L = \frac{35}{64} + 1.50 + \frac{2}{11} = 2.229 \text{ in}$$

From Table A–17 the next fraction size bolt is $L = 2\frac{1}{4}$ in. From Eq. (8–13), the thread length is $L_T = 2(0.625) + 0.25 = 1.50$ in. Thus, the length of the unthreaded portion



in the grip is $l_d = 2.25 - 1.50 = 0.75$ in. The threaded length in the grip is $l_t = l - l_d = 0.75$ in. From Table 8–2, $A_t = 0.226$ in^2. The major-diameter area is $A_d = \pi(0.625)^2/4 = 0.3068$ in^2. The bolt stiffness is then

Answer

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068(0.226)(30)}{0.3068(0.75) + 0.226(0.75)}$$
$$= 5.21 \text{ Mlbf/in}$$

From Table A–24, for no. 25 cast iron we will use $E = 14$ Mpsi. The stiffness of the members, from Eq. (8–22), is

Answer

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(14)(0.625)}{2\ln\left[5\dfrac{0.5774(1.5) + 0.5(0.625)}{0.5774(1.5) + 2.5(0.625)}\right]}$$
$$= 8.95 \text{ Mlbf/in}$$

If you are using Eq. (8–23), from Table 8–8, $A = 0.778\,71$ and $B = 0.616\,16$, and

$$k_m = EdA\exp(Bd/l)$$
$$= 14(0.625)(0.778\,71)\exp[0.616\,16(0.625)/1.5]$$
$$= 8.81 \text{ Mlbf/in}$$

which is only 1.6 percent lower than the previous result.

From the first calculation for k_m, the stiffness constant C is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368$$

(*b*) From Table 8–9, $S_p = 85$ kpsi. Then, using Eqs. (8–31) and (8–32), we find the recommended preload to be

$$F_i = 0.75A_t S_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

For N bolts, Eq. (8–29) can be written

$$n_L = \frac{S_p A_t - F_i}{C(P_{\text{total}}/N)} \tag{1}$$

or

$$N = \frac{C n_L P_{\text{total}}}{S_p A_t - F_i} = \frac{0.368(2)(36)}{85(0.226) - 14.4} = 5.52$$

Answer Six bolts should be used to provide the specified load factor.

(*c*) With six bolts, the load factor actually realized is

Answer

$$n_L = \frac{85(0.226) - 14.4}{0.368(36/6)} = 2.18$$

From Eq. (8–28), the yielding factor of safety is

Answer

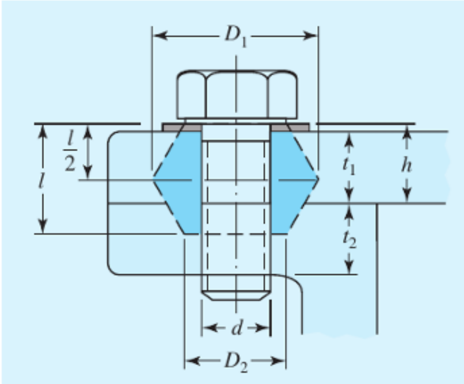
$$n_p = \frac{S_p A_t}{C(P_{\text{total}}/N) + F_i} = \frac{85(0.226)}{0.368(36/6) + 14.4} = 1.16$$

From Eq. (8–30), the load factor guarding against joint separation is

Answer

$$n_0 = \frac{F_i}{(P_{\text{total}}/N)(1 - C)} = \frac{14.4}{(36/6)(1 - 0.368)} = 3.80$$

4. Figure shows a connection using cap screws.



The joint is subjected to a fluctuating force whose maximum value is 22.24 kN per screw. The data for the cap screw is M16 × 2, (property class 8.8). The bolt stiffness k_b is 1.28 MN/mm while the member stiffness, k_m, is 3.07 MN/mm.

(a) Find all factors of safety – yield, overload, separation, fatigue.

EXAMPLE 8–5

Figure 8–21 shows a connection using cap screws. The joint is subjected to a fluctuating force whose maximum value is 5 kip per screw. The required data are: cap screw, 5/8 in-11 UNC, SAE 5; hardened-steel washer, $t_w = \frac{1}{16}$ in thick; steel cover plate, $t_1 = \frac{5}{8}$ in, $E_s = 30$ Mpsi; and cast-iron base, $t_2 = \frac{5}{8}$ in, $E_{ci} = 16$ Mpsi.

(*a*) Find k_b, k_m, and C using the assumptions given in the caption of Fig. 8–21.

(*b*) Find all factors of safety and explain what they mean.

Solution

(*a*) For the symbols of Figs. 8–15 and 8–21, $h = t_1 + t_w = 0.6875$ in, $l = h + d/2 = 1$ in, and $D_2 = 1.5d = 0.9375$ in. The joint is composed of three frusta; the upper two frusta are steel and the lower one is cast iron.

For the upper frustum: $t = l/2 = 0.5$ in, $D = 0.9375$ in, and $E = 30$ Mpsi. Using these values in Eq. (8–20) gives $k_1 = 46.46$ Mlbf/in.

For the middle frustum: $t = h - l/2 = 0.1875$ in and $D = 0.9375 + 2(l - h) \tan 30° = 1.298$ in. With these and $E_s = 30$ Mpsi, Eq. (8–20) gives $k_2 = 197.43$ Mlbf/in.

The lower frustum has $D = 0.9375$ in, $t = l - h = 0.3125$ in, and $E_{ci} = 16$ Mpsi. The same equation yields $k_3 = 32.39$ Mlbf/in.

Substituting these three stiffnesses into Eq. (8–18) gives $k_m = 17.40$ Mlbf/in. The cap screw is short and threaded all the way. Using $l = 1$ in for the grip and $A_t = 0.226\ \text{in}^2$ from Table 8–2, we find the stiffness to be $k_b = A_tE/l = 6.78$ Mlbf/in. Thus the joint constant is

Answer

$$C = \frac{k_b}{k_b + k_m} = \frac{6.78}{6.78 + 17.40} = 0.280$$

Figure 8–21

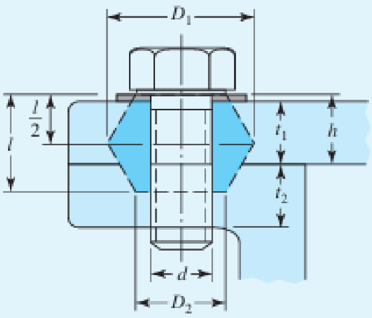
Pressure-cone frustum member model for a cap screw. For this model the significant sizes are

$$l = \begin{cases} h + t_2/2 & t_2 < d \\ h + d/2 & t_2 \geq d \end{cases}$$

$D_1 = d_w + l \tan\alpha = 1.5d + 0.577l$

$D_2 = d_w = 1.5d$

where l = effective grip. The solutions are for $\alpha = 30°$ and $d_w = 1.5d$.



(*b*) Equation (8–30) gives the preload as

$$F_i = 0.75F_p = 0.75A_tS_p = 0.75(0.226)(85) = 14.4 \text{ kip}$$

where from Table 8–9, $S_p = 85$ kpsi for an SAE grade 5 cap screw. Using Eq. (8–28), we obtain the load factor as the yielding factor of safety is

Answer
$$n_p = \frac{S_pA_t}{CP + F_i} = \frac{85(0.226)}{0.280(5) + 14.4} = 1.22$$

This is the traditional factor of safety, which compares the maximum bolt stress to the proof strength.

Using Eq. (8–29),

Answer
$$n_L = \frac{S_pA_t - F_i}{CP} = \frac{85(0.226) - 14.4}{0.280(5)} = 3.44$$

This factor is an indication of the overload on P that can be applied without exceeding the proof strength.

Next, using Eq. (8–30), we have

Answer
$$n_0 = \frac{F_i}{P(1 - C)} = \frac{14.4}{5(1 - 0.280)} = 4.00$$

If the force P gets too large, the joint will separate and the bolt will take the entire load. This factor guards against that event.

For the remaining factors, refer to Fig. 8–22. This diagram contains the modified Goodman line, the Gerber line, the proof-strength line, and the load line. The intersection

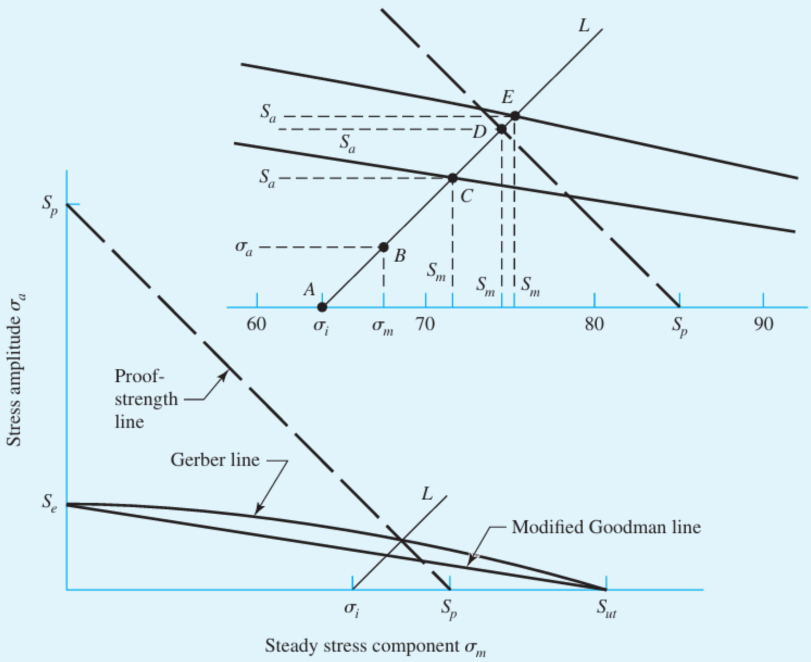


Figure 8–22

Designer's fatigue diagram for preloaded bolts, drawn to scale, showing the modified Goodman line, the Gerber line, and the Langer proof-strength line, with an exploded view of the area of interest. The strengths used are $S_p = 85$ kpsi, $S_e = 18.6$ kpsi, and $S_{ut} = 120$ kpsi. The coordinates are A, $\sigma_i = 63.72$ kpsi; B, $\sigma_a = 3.10$ kpsi, $\sigma_m = 66.82$ kpsi; C, $S_a = 7.55$ kpsi, $S_m = 71.29$ kpsi; D, $S_a = 10.64$ kpsi, $S_m = 74.36$ kpsi; E, $S_a = 11.32$ kpsi, $S_m = 75.04$ kpsi.

of the load line L with the respective failure lines at points C, D, and E defines a set of strengths S_a and S_m at each intersection. Point B represents the stress state σ_a, σ_m. Point A is the preload stress σ_i. Therefore the load line begins at A and makes an angle having a unit slope. This angle is 45° only when both stress axes have the same scale.

The factors of safety are found by dividing the distances AC, AD, and AE by the distance AB. Note that this is the same as dividing S_a for each theory by σ_a.

The quantities shown in the caption of Fig. 8–22 are obtained as follows:

Point A

$$\sigma_i = \frac{F_i}{A_t} = \frac{14.4}{0.226} = 63.72 \text{ kpsi}$$

Point B

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.280(5)}{2(0.226)} = 3.10 \text{ kpsi}$$

$$\sigma_m = \sigma_a + \sigma_i = 3.10 + 63.72 = 66.82 \text{ kpsi}$$

Point C

This is the modified Goodman criteria. From Table 8–17, we find $S_e = 18.6$ kpsi. Then, using Eq. (8–45), the factor of safety is found to be

Answer

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{18.6(120 - 63.72)}{3.10(120 + 18.6)} = 2.44$$

Point D

This is on the proof-strength line where

$$S_m + S_a = S_p \tag{1}$$

In addition, the horizontal projection of the load line AD is

$$S_m = \sigma_i + S_a \tag{2}$$

Solving Eqs. (1) and (2) simultaneously results in

$$S_a = \frac{S_p - \sigma_i}{2} = \frac{85 - 63.72}{2} = 10.64 \text{ kpsi}$$

The factor of safety resulting from this is

Answer

$$n_p = \frac{S_a}{\sigma_a} = \frac{10.64}{3.10} = 3.43$$

which, of course, is identical to the result previously obtained by using Eq. (8–29).

A similar analysis of a fatigue diagram could have been done using yield strength instead of proof strength. Though the two strengths are somewhat related, proof strength is a much better and more positive indicator of a fully loaded bolt than is the yield strength. It is also worth remembering that proof-strength values are specified in design codes; yield strengths are not.

We found $n_f = 2.44$ on the basis of fatigue and the modified Goodman line, and $n_p = 3.43$ on the basis of proof strength. Thus the danger of failure is by fatigue, not by overproof loading. These two factors should always be compared to determine where the greatest danger lies.

Point E

For the Gerber criterion, from Eq. (8–46), the safety factor is

Answer

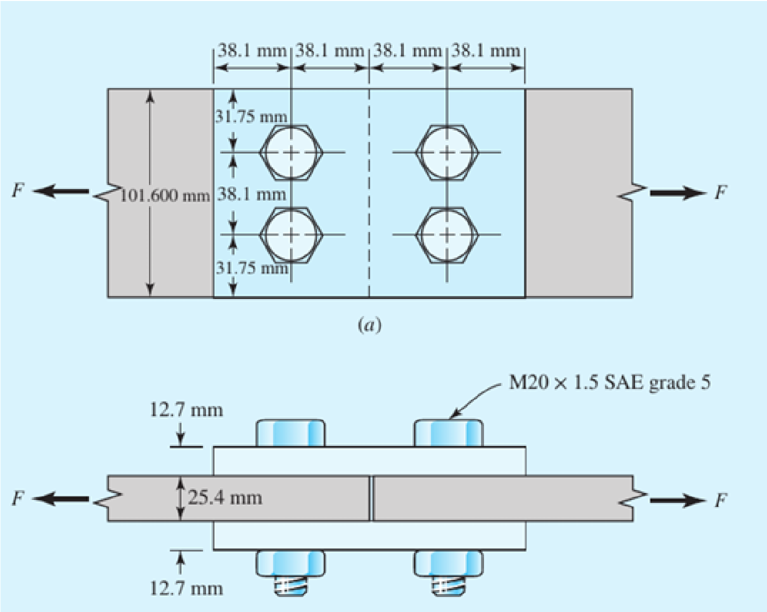
$$n_f = \frac{1}{2\sigma_a S_e}[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e]$$

$$= \frac{1}{2(3.10)(18.6)}[120\sqrt{120^2 + 4(18.6)(18.6 + 63.72)} - 120^2 - 2(63.72)(18.6)]$$

$$= 3.65$$

which is greater than $n_p = 3.43$ and contradicts the conclusion earlier that the danger of failure is fatigue. Figure 8–22 clearly shows the conflict where point *D* lies between points *C* and *E*. Again, the conservative nature of the Goodman criterion explains the discrepancy and the designer must form his or her own conclusion.

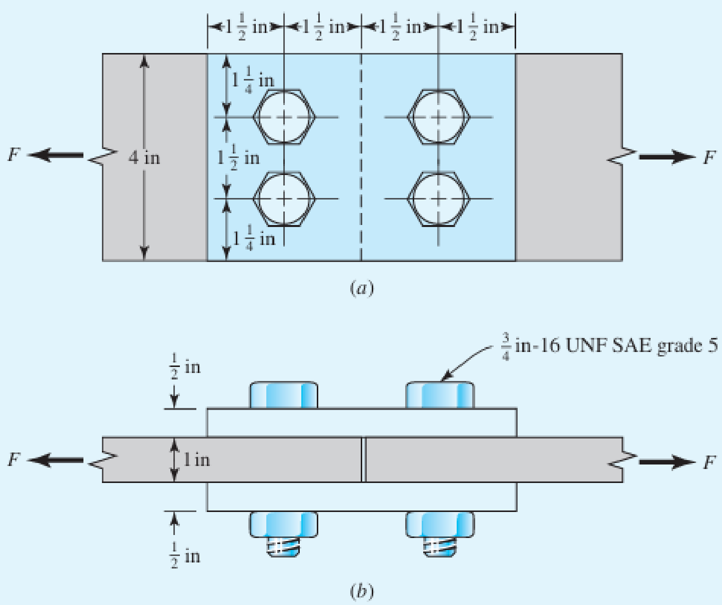
5. Two 25.4 by 101.6 mm 1018 cold-rolled steel bars are butt-spliced with two 12.7 by 101.6 mm 1018 cold-rolled splice plates using four M20 × 1.5 mm (property class 8.8) bolts as depicted in Figure 8–26. For a design factor of nd = 1.5 estimate the static load F that can be carried if the bolts lose preload.



EXAMPLE 8–6 Two 1- by 4-in 1018 cold-rolled steel bars are butt-spliced with two $\frac{1}{2}$- by 4-in 1018 cold-rolled splice plates using four $\frac{3}{4}$ in-16 UNF grade 5 bolts as depicted in Fig. 8–24. For a design factor of $n_d = 1.5$ estimate the static load F that can be carried if the bolts lose preload.

Solution From Table A–20, minimum strengths of $S_y = 54$ kpsi and $S_{ut} = 64$ kpsi are found for the members, and from Table 8–9 minimum strengths of $S_p = 85$ kpsi, $S_y = 92$ kpsi, and $S_{ut} = 120$ kpsi for the bolts are found.

Figure 8–24



$F/2$ is transmitted by each of the splice plates, but since the areas of the splice plates are half those of the center bars, the stresses associated with the plates are the same. So for stresses associated with the plates, the force and areas used will be those of the center plates.

Bearing in bolts, all bolts loaded:

$$\sigma = \frac{F}{2td} = \frac{S_y}{n_d}$$

$$F = \frac{2td\,S_y}{n_d} = \frac{2(1)(\frac{3}{4})92}{1.5} = 92 \text{ kip}$$

Bearing in members, all bolts active:

$$\sigma = \frac{F}{2td} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{2td(S_y)_{\text{mem}}}{n_d} = \frac{2(1)(\frac{3}{4})54}{1.5} = 54 \text{ kip}$$

Shear of bolt, all bolts active: If the bolt threads do not extend into the shear planes for four shanks:

$$\tau = \frac{F}{4\pi d^2/4} = 0.577\frac{S_y}{n_d}$$

$$F = 0.577\pi d^2\frac{S_y}{n_d} = 0.577\pi(0.75)^2\frac{92}{1.5} = 62.5 \text{ kip}$$

If the bolt threads extend into a shear plane:

$$\tau = \frac{F}{4A_r} = 0.577\frac{S_y}{n_d}$$

$$F = \frac{0.577(4)A_r S_y}{n_d} = \frac{0.577(4)0.351(92)}{1.5} = 49.7 \text{ kip}$$

Edge shearing of member at two margin bolts: From Fig. 8–25,

$$\tau = \frac{F}{4at} = \frac{0.577(S_y)_{\text{mem}}}{n_d}$$

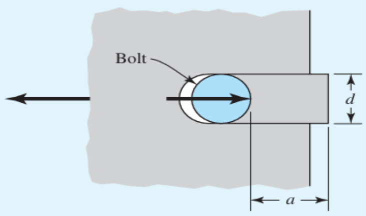
$$F = \frac{4at0.577(S_y)_{\text{mem}}}{n_d} = \frac{4(1.125)(1)0.577(54)}{1.5} = 93.5 \text{ kip}$$

Tensile yielding of members across bolt holes:

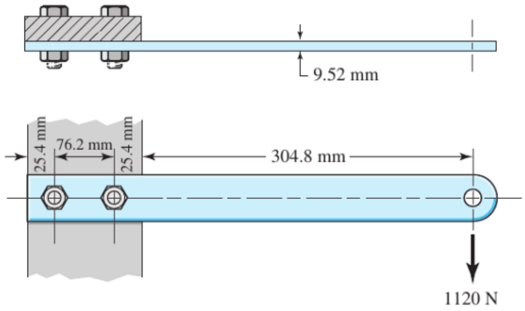
$$\sigma = \frac{F}{[4 - 2(\frac{3}{4})]t} = \frac{(S_y)_{\text{mem}}}{n_d}$$

$$F = \frac{[4 - 2(\frac{3}{4})]t(S_y)_{\text{mem}}}{n_d} = \frac{[4 - 2(\frac{3}{4})](1)54}{1.5} = 90 \text{ kip}$$

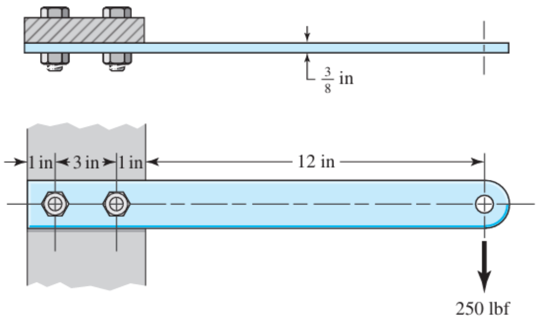
On the basis of bolt shear, the limiting value of the force is 49.7 kip, assuming the threads extend into a shear plane. However, it would be poor design to allow the threads to extend into a shear plane. So, assuming a *good* design based on bolt shear, the limiting value of the force is 62.5 kip. For the members, the bearing stress limits the load to 54 kip.



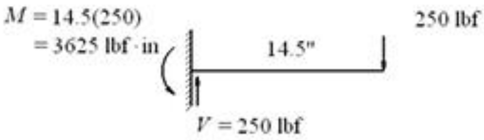
7. A 9.52 × 50.8 mm AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 1120 N as illustrated. The bar is secured to the support using two M10x1.5 (property class 5.8). Assume the bolt threads do not extend into the joint. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.

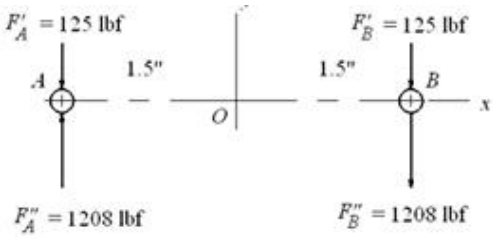


8–77 A $\frac{3}{8}$- × 2-in AISI 1018 cold-drawn steel bar is cantilevered to support a static load of 250 lbf as illustrated. The bar is secured to the support using two $\frac{3}{8}$ in-16 UNC SAE grade 4 bolts. Assume the bolt threads do not extend into the joint. Find the factor of safety for the following modes of failure: shear of bolt, bearing on bolt, bearing on member, and strength of member.



Problem 8–77





$$F''_A = F''_B = \left(\frac{3625}{3}\right) = 1208 \text{ lbf}$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}, \qquad F_B = 1208 + 125 = 1333 \text{ lbf}$$

Bolt shear:

$$A_s = (\pi/4)(0.375^2) = 0.1104 \text{ in}^2$$

$$\tau_{\max} = \frac{F_{\max}}{A_s} = \frac{1333}{0.1104} = 12\ 070 \text{ psi}$$

From Table 8-10, $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{57.7}{12.07} = 4.78 \qquad \textit{Ans.}$$

Bearing on bolt: Bearing area is $A_b = td = 0.375\ (0.375) = 0.1406 \text{ in}^2$.

$$\sigma_b = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\ 481 \text{ psi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{9.481} = 10.55 \qquad \textit{Ans.}$$

Bearing on member: From Table A-20, $S_y = 54$ kpsi. Bearing stress same as bolt

$$n = \frac{S_y}{|\sigma_b|} = \frac{54}{9.481} = 5.70 \qquad \textit{Ans.}$$

Bending of member: At B, $M = 250(13) = 3250$ lbf·in

$$I = \frac{1}{12}\left(\frac{3}{8}\right)\left[2^3 - \left(\frac{3}{8}\right)^3\right] = 0.2484 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\ 080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13 \qquad \textit{Ans.}$$

EXAMPLE 8–3 A $\frac{3}{4}$ in-16 UNF $\times$ $2\frac{1}{2}$ in SAE grade 5 bolt is subjected to a load P of 6 kip in a tension joint. The initial bolt tension is $F_i = 25$ kip. The bolt and joint stiffnesses are $k_b = 6.50$ and $k_m = 13.8$ Mlbf/in, respectively.

(*a*) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(*b*) Specify the torque necessary to develop the preload, using Eq. (8–27).

(*c*) Specify the torque necessary to develop the preload, using Eq. (8–26) with $f = f_c = 0.15$.

6. A M20 $\times$ 1.5 $\times$ 63.5 mm (property class 5.8) bolt is subjected to a load P of 26.69 kN in a tension joint. The initial bolt tension is $F_i = 111.205$ kN. The bolt and joint stiffnesses are $k_b = 1.14$ and $k_m = 2.42$ MN/mm, respectively.

(a) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.

(b) Specify the torque necessary to develop the preload. Specify the torque necessary to develop the preload.

Solution From Table 8–2, $A_t = 0.373$ in^2.

(*a*) The preload stress is

Answer

$$\sigma_i = \frac{F_i}{A_t} = \frac{25}{0.373} = 67.02 \text{ kpsi}$$

The stiffness constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.320$$

From Eq. (8–24), the stress under the service load is

Answer

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = C\frac{P}{A_t} + \sigma_i$$

$$= 0.320\frac{6}{0.373} + 67.02 = 72.17 \text{ kpsi}$$

From Table 8–9, the SAE minimum proof strength of the bolt is $S_p = 85$ kpsi. The preload and service load stresses are respectively 21 and 15 percent less than the proof strength.

(*b*) From Eq. (8–27), the torque necessary to achieve the preload is

$$T = KF_i d = 0.2(25)(10^3)(0.75) = 3750 \text{ lbf} \cdot \text{in}$$

(*c*) The minor diameter can be determined from the minor area in Table 8–2. Thus $d_r = \sqrt{4A_r/\pi} = \sqrt{4(0.351)/\pi} = 0.6685$ in. Thus, the mean diameter is $d_m = (0.75 + 0.6685)/2 = 0.7093$ in. The lead angle is

$$\lambda = \tan^{-1}\frac{l}{\pi d_m} = \tan^{-1}\frac{1}{\pi d_m N} = \tan^{-1}\frac{1}{\pi(0.7093)(16)} = 1.6066°$$

For $\alpha = 30°$, Eq. (8–26) gives

$$T = \left\{\left[\frac{0.7093}{2(0.75)}\right]\left[\frac{\tan 1.6066° + 0.15(\sec 30°)}{1 - 0.15(\tan 1.6066°)(\sec 30°)}\right] + 0.625(0.15)\right\}25(10^3)(0.75)$$

$$= 3551 \text{ lbf} \cdot \text{in}$$

which is 5.3 percent less than the value found in part (*b*).

TUT 8

4. A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 2980 watts to a 52-tooth disk gear. The module is 2.5 mm, the face width 38 mm, and the quality standard is No. 6. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Assume $J_P = 0.30$, $J_G = 0.40$, $I = 0.121$, $(K_S)_P = (K_S)_G = 1$, $C_H = 1$, $(C_f)_P = (C_g)_G = 1$. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90.

(a) Find the factor of safety of the gears in bending.

(b) Find the factor of safety of the gears in wear.

EXAMPLE 14–4 A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\,H}{V} = \frac{33\,000(4)}{801.1} = 164.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with $N(\text{pinion}) = 10^8$ cycles and $N(\text{gear}) = 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$\begin{aligned} A' &= 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3}) \\ &= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49 \end{aligned}$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\frac{10}{1.5}\frac{1.22(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer
$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer
$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_P\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer
$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

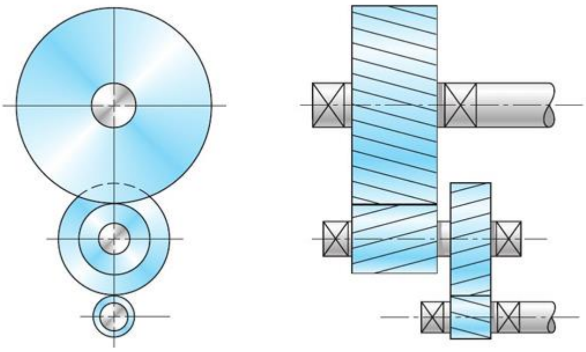
$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer
$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

1. As a rough guideline a reduction ratio (speed ratio, gear ratio) of pair of gears should not exceed 10:1. Note reduction ratio = number of teeth on the driven gear/number of teeth of the driver gear. To obtain higher reduction ratios, one uses a compounded gear train. A two-stage compounded gear train is shown below:



a. Select the appropriate number of teeth (20° pressure angle) of the a two-stage compounded gear train to obtain a reduction ratio of 30:1 (±1%). Assume that in each stage a reduction of approximately $\sqrt{30}$ is desired. Also, assume that the number of teeth on the pinion is 16. Note that the minimum number of teeth on a pinion to avoid interference for given speed ratio and pressure angle is

$$N_p = \frac{2}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right),\ m_G = N_G/N_P$$

b. Select the appropriate number of teeth of the a two-stage compounded gear train to obtain an **exact** reduction ratio of 30:1.

EXAMPLE 13–3 A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate tooth numbers.

Solution

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

Answer

$$16\sqrt{30} = 87.64 \approx 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.

EXAMPLE 13–4 A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \qquad \text{and} \qquad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Then

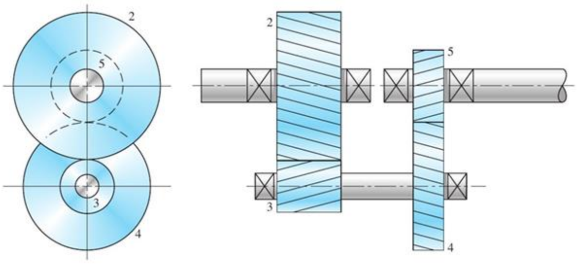
$$N_2 = 6N_3 = 6(16) = 96$$

$$N_4 = 5N_5 = 5(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

2. It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Figure. This configuration is called a **compound reverted gear train**. This requires the distances between the shafts to be the same for both stages of the train.



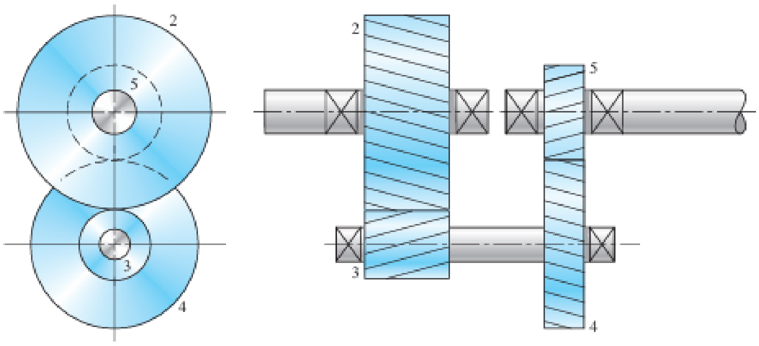
Select the appropriate number of teeth of a compound reverted gear train to obtain an **exact** reduction ratio of 30:1.

It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line, as shown in Fig. 13–29. This configuration is called a *compound reverted gear train*. This requires the distances between the shafts to be the same for both stages of the train, which adds to the complexity of the design task. The distance constraint is

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

29

erted gear



ing Design

The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) = N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

EXAMPLE 13–5 A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution The governing equations are

$$N_2/N_3 = 6$$
$$N_4/N_5 = 5$$
$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$
$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$
$$N_5 = 112/6 = 18.67$$

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$
$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$
$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$. Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$
$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$
$$126 = 5N_5 + N_5 = 6N_5$$
$$N_5 = 126/6 = 21$$
$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

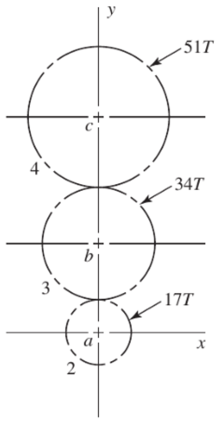
$$N_2 = 108$$
$$N_3 = 18$$
$$N_4 = 105$$
$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$
$$108 + 18 = 105 + 21$$
$$126 = 126$$

3. Shaft a in the figure has a power input of 75 kW at a speed of 1000 rev/min in the counterclockwise direction. The gears have a module of 5 mm and a 20° pressure angle. Gear 3 is an idler.



(a) Find the force F_{3b} that gear 3 exerts against shaft b.
(b) Find the torque T_{4c} that gear 4 exerts on shaft c.

13-31 (a)

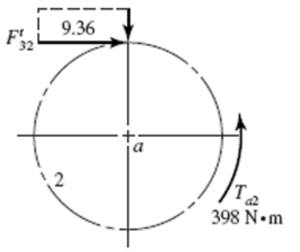
$$\omega = 2\pi n/60$$
$$H = T\omega = 2\pi Tn/60 \qquad (T \text{ in N}\cdot\text{m},\ H \text{ in W})$$

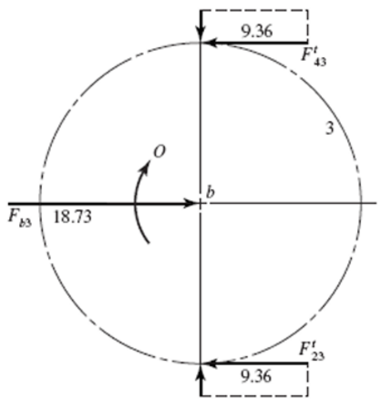
So

$$T = \frac{60H(10^3)}{2\pi n}$$
$$= 9550\ H/n \qquad (H \text{ in kW},\ n \text{ in rev/min})$$
$$T_a = \frac{9550(75)}{1800} = 398\ \text{N}\cdot\text{m}$$
$$r_2 = \frac{mN_2}{2} = \frac{5(17)}{2} = 42.5\ \text{mm}$$

So

$$F_{32}^t = \frac{T_a}{r_2} = \frac{398}{42.5} = 9.36\ \text{kN}$$

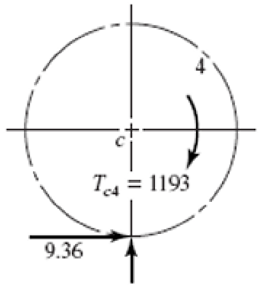




$$F_{3b} = -F_{b3} = 2(9.36) = 18.73\ \text{kN in the positive } x\text{-direction.} \qquad \textit{Ans.}$$

(b)

$$r_4 = \frac{mN_4}{2} = \frac{5(51)}{2} = 127.5\ \text{mm}$$
$$T_{c4} = 9.36(127.5) = 1193\ \text{N}\cdot\text{m ccw}$$
$$\therefore T_{4c} = 1193\ \text{N}\cdot\text{m cw} \qquad \textit{Ans.}$$



Note: The solution is independent of the pressure angle.

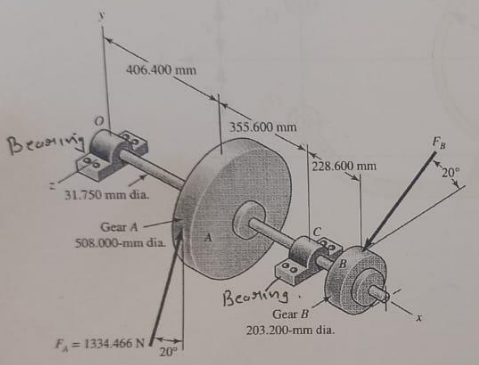
ME423 – Quiz 2

Wednesday 23rd October 2024

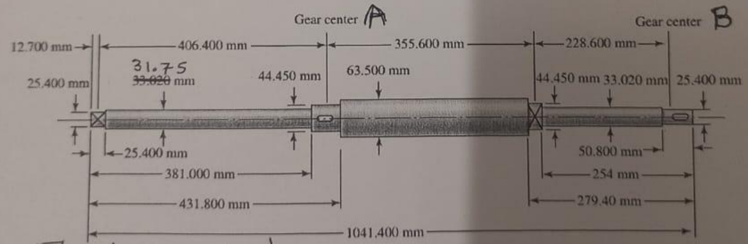
5.30 pm to 7.00 pm
Max Marks: 30

Instructions:
a. There are FOUR questions b. This is an open notes exam

1. Consider a shaft on which gears, A and B, are mounted as shown. The detailed drawing of the shaft is also shown. Select an appropriate parallel key for the gear A and gear B for a factor of safety 1.2. (you need to specify all its dimensions). The standard key sizes available are given in the table. The key material has a yield strength of 372 MPa in tension and compression (10 marks)



Shaft Diameter (mm)	Key Width x Height (mm)
$8 < d \le 10$	3 x 3
$10 < d \le 12$	4 x 4
$12 < d \le 17$	5 x 5
$17 < d \le 22$	6 x 6
$22 < d \le 30$	8 x 7
$30 < d \le 38$	10 x 8
$38 < d \le 44$	12 x 8
$44 < d \le 50$	14 x 9
$50 < d \le 58$	16 x 10
$58 < d \le 65$	18 x 11
$65 < d \le 75$	20 x 12
$75 < d \le 85$	22 x 14
$85 < d \le 95$	25 x 14



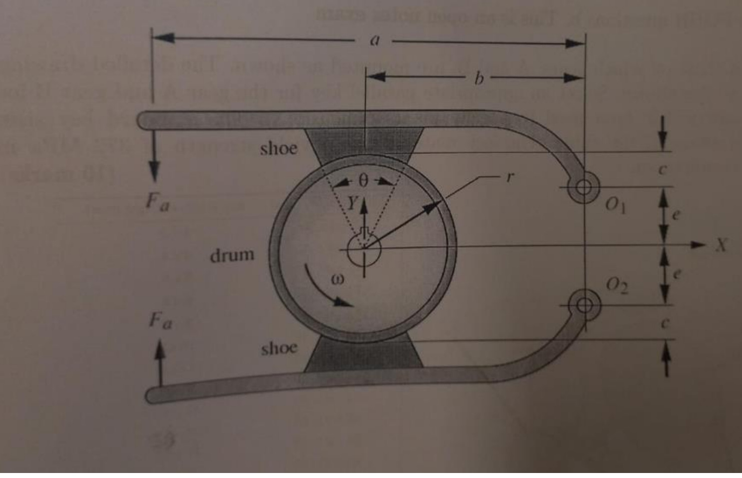
☒ – bearing location.

2. Consider problem 1. One would like to select appropriate ball bearings at the two locations from SKF's catalog with a desired life of 5000 hours for the shaft running at 1500 rpm. Note that SKF rates its bearings for 1 million cycles. Which catalog ratings (C_{10}) would one search in an SKF catalog? Assume a load application factor of 1.2. (6 marks)

3. Two ball bearings from different manufacturers are being considered for a certain application. Bearing A has a catalog rating of 2.0 kN based on a catalog rating system of 3000 hours at 500 rev/min. Bearing B has a catalog rating of 7.0 kN based on a catalog that rates at 10^6 cycles. For a given application, determine which bearing can carry the larger load. You need to give proper justification for your answer. (4 marks)

Please Turn Over

4. The figure shows a double short-shoe drum brake. Find its torque capacity and required actuating force for a = 90, b = 80, e = 30, r = 40, w = 60 mm (width), and θ = 25°. What value of c will make it self-locking? Assume p_{max} = 1.5 MPa and μ = 0.25. (10 marks)



Quiz 2

Soln - 1 For gear A

Shaft diameter $d_A = 44.45$ mm or $r_A = 22.225$ mm

from the given table we get $W \times H = 14 \text{ mm} \times 9 \text{ mm}$ (+1)

~~Torque acting on gear A = $r_A \times F_A \cos 20°$~~

~~= $0.022225 \times 1384.466 \times 0.9397$~~

Torque acting on gear A = $R_A \times F_A \cos 20°$

$T_A = 0.254 \times 1334.466 \times 0.9397$

$= 318.52$ Nm

(+1) T_A or F_{key}

Force acting on the key $F_{key} = T_A / r_A$

$= 318.52 / 0.022225$

$= 14331.61$ N

Average direct shear $= F_{key} / A_{shear} = F_{key} / L_s W$

$= 14331.61 / L_s \times 14 = 1023.69 / L_s$ N/mm²

for shear failure $\tau_{max} = \dfrac{\sigma_{ys}}{FoS}$ $\left(\begin{array}{l} \sigma_{ys} = \sigma_y / 2 \\ \sigma_{ys} = \sigma_y / \sqrt{3} \end{array} \right.$

Using maximum shear stress theory	Using distortion energy theory
$\sigma_y / 2$	$\sigma_y / \sqrt{3}$
$\dfrac{1023.69}{L_s} = \dfrac{372}{1.2 \times 2}$	$\dfrac{1023.69}{L_s} = \dfrac{372}{1.2 \times \sqrt{3}}$
(+1) L_s	
$\Rightarrow L_s = 6.6$ mm	$L_s = 5.72$ mm

Average crushing stress $\sigma_c = F_{key}/A_{crush} = F_{key}/L_c H/2$

$$\Rightarrow \sigma_c = \frac{2 \times 14331.61}{L_c \times 9} = \frac{3184.8}{L_c} \text{ N/mm}^2$$

& for crushing failure $\sigma_{c\,max} = \sigma_y / FOS$

$$\Rightarrow \cancel{318.4} \quad \frac{3184.8}{L_c} = \frac{372}{1.2}$$ +1

or $L_c = 10.27$ mm

∴ length of key for gear A = max (L_s, L_c)

= 10.27 mm ~~= 10.27 mm~~ +1

for gear B

Shaft diameter = 25.4 mm

key's $W \times H$ = 8×7 mm^2. (+1)

Torque acting would be same & = 318.5 Nm

Forces acting on the key = T / r_{shaft}

F_B = 25078.7 N (+1), [2]

Average direct shear for B = τ_B = F_B / LW

= 3134.84 / L N/mm^2

∴ for shear failure $\tau_B = \sigma_y / \sqrt{3} \times FOS$

$$\Rightarrow \frac{3134.84}{L_s} = \frac{372}{\sqrt{3} \times 1.2}$$

(+1), [3]

L_s = 17.51 mm

using $\left(\frac{\sigma_y}{2}\right)$ L_s = 20.22 mm

Average crushing stress = $\sigma_{B\,crush}$ = $2F_B / LH$

= 7165.34 / L N/mm^2

∴ for crushing failure $\sigma_{B\,crush} = \sigma_y /$ FOS

$$\Rightarrow \quad \frac{7165.34}{L_c} = \frac{372}{1.2}$$

$$\text{or} \quad L_c = 23.11 \text{ mm} \quad +1$$

$\therefore$ length of key for gear B = $\max(L_s, L_c)$

$= 23.11$ mm. +1

ME 423 Quiz 2

2) Given :- Desired life $= L_{10h} = 5000$ hrs

Speed $= N = 1500$ rpm

Load Application factor $= K_a = 1.2$

$F_A = 1334.466$ N

To find:- C_{10} for bearing at 'O' and 'C'

Solution:- For given shaft system.

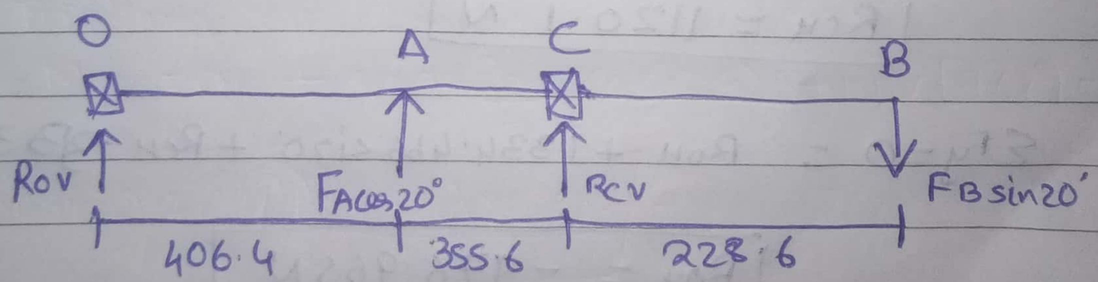
Torque A + Torque B = 0

$$F_A \cos 20^\circ \times \frac{508}{2} - F_B \times \cos 20^\circ \times \frac{203.2}{2} = 0$$

$$F_B = \frac{F_A \times 508}{203.2} = \frac{1334.466 \times 508}{203.2}$$

$$\boxed{F_B = 3336.165 \text{ N}}$$

Now vertical loading diagram.



$$\Sigma M_O = F_A \cos 20^\circ \times 406.4 + R_{cv}(762) - F_B \sin 20 (990.6) = 0$$

$$R_{cv} = \frac{3336.165 \times \sin 20 \times 990.6 - 13334.466 \times \cos 20^\circ \times 406.4}{762}$$

$$\boxed{R_{cv} = 3669.018 \text{ N}}$$

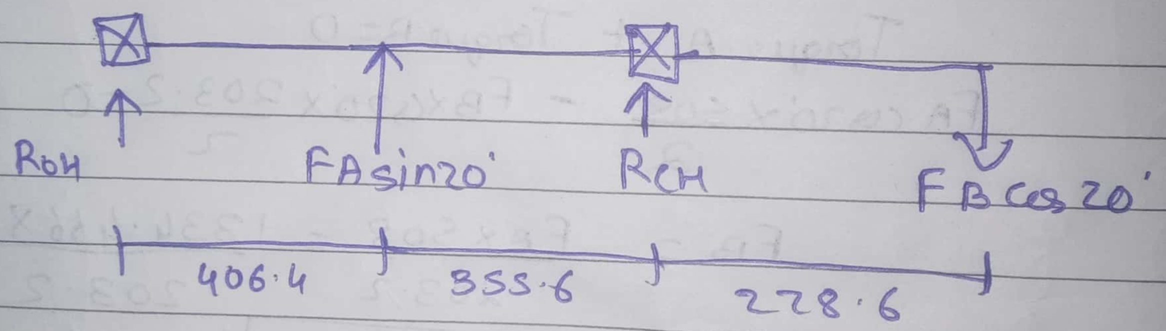
Date

$\Sigma F_y = 0$ $\quad R_{OV} + F_A \cos 20^\circ + R_{CV} - F_B \sin 20^\circ = 0$

$R_{OV} = 3336.165 \sin 20^\circ - 1334.466 \cos 20^\circ$
$- 3669.02$

$\boxed{R_{OV} = -1167.85 \text{ N}}$

Horizontal loading diagram:-



$\Sigma M_O = 0 = 1334.466 \times \sin 20^\circ \times 406.4 + R_{CH} \times 762$
$- 3336.165 \times 990.6 = 0$
$\cos 20^\circ$

$\boxed{R_{CH} = 1120.1 \text{ N}}$

$\Sigma F_y = 0 = R_{OH} + 1334.466 \sin 20^\circ + R_{CH} - 3336.165 \cos 20^\circ = 0$

$\boxed{R_{OH} = -976.965 \text{ N}}$

$\therefore$ Radial load at 'O' $= \sqrt{R_{OH}^2 + R_{OV}^2}$

$= \sqrt{(-976.965)^2 + (-1167.85)^2}$

$$\boxed{R_O = 1522.61 \text{ N}}$$

Radial load at 'C' $= \sqrt{R_{CH}^2 + R_{CV}^2}$

$= \sqrt{(1120.1)^2 + (3669.018)^2}$

$$\boxed{R_C = 3836.185 \text{ N}}$$

$\therefore$ Equivalent ~~eq.~~ dynamic load on Bearing 'O', 'C'

$P_{eO} = K_a R_O = 1.2 \times 1522.61$
$= 1827.132$ N

$P_{eC} = K_a R_C = 1.2 \times 3836.185$
$= 4603.422$ N

Now for bearing $L_{10} = \frac{L_{10h} \times N \times 60}{10^6}$ million rev

$= \frac{5000 \times 1500 \times 60}{10^6}$

$= 450$

$L_{10} = \left(\frac{C}{P_e}\right)^3$ for ball bearing.

for Bearing 'O' $\quad 450 = \left[\frac{C}{1827.132}\right]^3 \quad C = 14.001$ kN

for Bearing 'C' $\quad 450 = \left[\frac{C}{4603.422}\right]^3 \quad C = 35.276$ kN

$a = 3$ (roller Bearing)

Q_3 **Bearing A →**

$F_A = 2\,kN$

$L_A = 3000 \times 500 \times 60$ cycles

$= 9 \times 10^7$ cycles

Bearing B

$C_B = 7\,kN$

$L_B = 10^6$ cycles

To compare the both, they need to be rated in terms of the same Catalog rating system. (10^6 cycles)

$$C_A = F_A \times \left(\frac{L_A}{L_B}\right)^{1/a} = 2 \times \left(\frac{9 \times 10^7}{10^6}\right)^{1/3} = 8.96\,kN$$

Catalogue rating of Bearing A for 10^6 cycles is $8.96\,kN$

Catalogue rating of Bearing B for 10^6 cycles is $7.0\,kN$

$\boxed{C_A > C_B}$

∴ Bearing A can carry the larger load.

→ Calculation of number of cycles → 1 mark

→ Using $L_{10} C_{10}^3 = \text{Constant}$ → 1 mark

→ Finding $(C_{10})_A$ Or $L_{10} C_{10}^3 = \boxed{\text{Constant}}$ → 1 mark

(calculation of constant)

→ Final Comparision of Bearing A & B → 1 mark

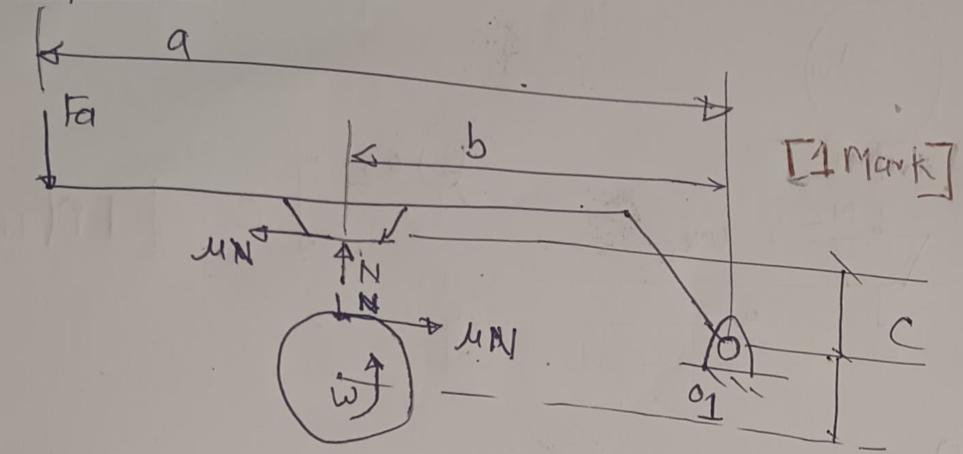
Quiz2

[Q.4]

Partial FBD

FBD

Top Shoe



given

$a = 90$

$b = 80$

$e = 30$

$r = 40$

$w = 60$

$\theta = 25^\circ$

$P_{max} = 1.5\ MPa$

$\mu = 0.25$

1. Normal reaction $N = PA = (r\theta w) = 1570.796\ (N)$ → [1 Mark]

2. $T_1 = \mu N r = 15.708\ (Nm)$ [1 Mark]

3. $c = r - e = 40 - 30 = 10$

$+\circlearrowleft \Sigma M_{O_1} = 0$

$+(Fa \cdot a) - (Nb) + \mu N c = 0$; $Fa = \dfrac{N(b - \mu c)}{a} = 1352.63\ (N)$

[1 Mark]

4. Self lock

If $\mu c \geq b$

here

$\mu c = 0.25 \times 10 = 2.5$

$b = 80$ ⇒ $\because \mu c \ngeq b \Rightarrow$ no self lock.

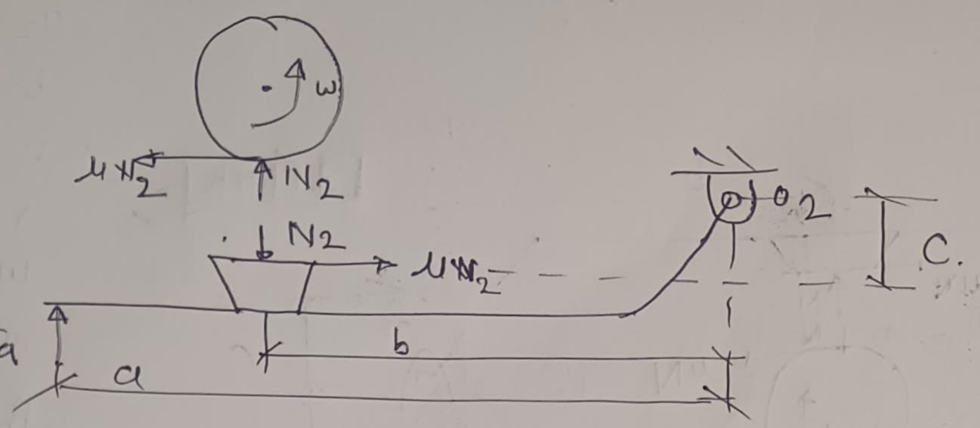
5. Value of c for self lock $= b/\mu = 80/0.25 = 320\ (mm)$

[1 Mark]

(6) bottom Shoe :-

Partial FBD

FBD



[1 Mark]

$$+\circlearrowleft \sum M_{O_2} = 0$$

$$-[F_a \cdot a] + [N_2 \cdot b] + [\mu N_2 c] = 0$$

$$F_a = \frac{N_2(b+\mu c)}{a} \Rightarrow N_2 = F_a \cdot \left[\frac{a}{b+\mu c}\right]$$

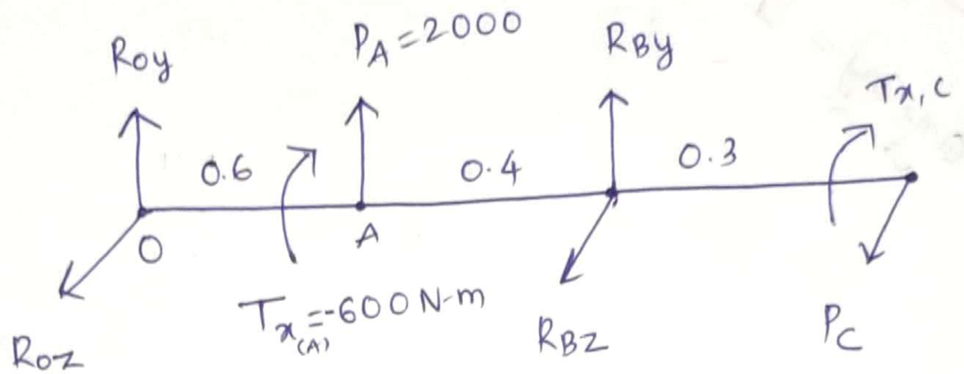
$$N_2 = 1475.596\ (N)$$ [1 Mark]

(7) $$T_2 = (\mu N_2 r) = 14.7559\ (Nm) \simeq 14.756\ (Nm)$$ [1 Mark]

(8) total torque Capacity $= T_1 + T_2 = 30.464\ (Nm)$ [2 Mark]

ME423 Midsem Q1(Sol'n)

FBD of shaft:



Finding P_c:

As constant Torque is getting transmitted (1M)

$$\sum T_x = 0 \Rightarrow T_{xA} + T_{xB} = 0 \Rightarrow -600 + P_c \times 0.15 = 0$$

$$\Rightarrow P_c = 4000\,N$$

Finding All Reaction forces:

Force Balance (Y Axis) (1M)

$$\sum F_y = 0 \Rightarrow \quad R_{oy} + 2000 + R_{By} = 0 \quad \text{—— (1)}$$

Moment Balance (Z axis at O)

from (1) & (2)

1200 N

Force Balance (z axis)

$$\sum F_z = 0 \Rightarrow R_{Oz} + R_{Bz} + 4000 = 0 \quad -③$$

Moment Balance (Y Axis w.r.t O):

$$\sum M_y\Big|_O = 0 \Rightarrow R_{Bz} \times 1 + P_c \times 1.2 = 0 \quad -④$$

from ③ & ④

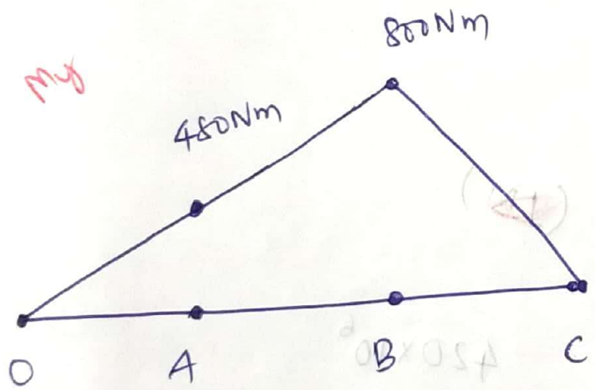
$\Rightarrow R_{Bz} = -4800N$ (1M)

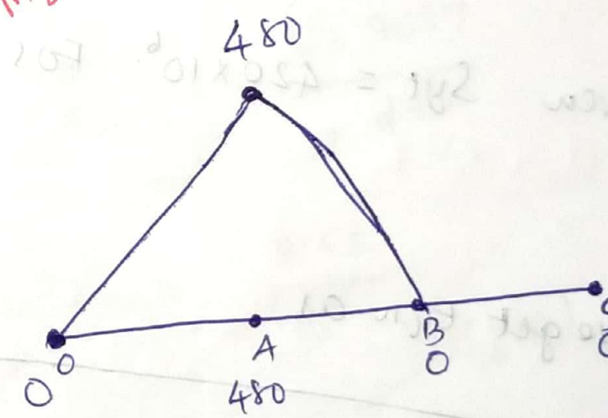
$R_{Oz} = 800 N.$

(1M)

Finding critical section:

Drawing BMD's





Maximum Bending Moment,

$$M_A = \sqrt{480^2 + 480^2} = 678.82$$

$$M_B = \sqrt{800^2 + 0^2} = 800 \text{ Nm (critical section)}.$$

(a)

Finding Equivalent σ & τ at critical section: (Static case)

(1M)

$$\sigma = \frac{32M}{\pi d^3} = \frac{32 \times 800}{\pi d^3} = \frac{8152}{d^3}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 600}{\pi d^3} = \frac{3057}{d^3}$$

if directly Using M & T formula check values of (M & T)

Using D.E.T for find Eq. stress (1M)

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{S_{yt}}{FoS}$$

given $S_{yt} = 420 \times 10^6$ $FoS = 2$

(1M)

So we get Eqn as

$$\sqrt{\left(\frac{8152}{d^3}\right)^2 + 3\left(\frac{3057}{d^3}\right)^2} = \frac{420 \times 10^6}{2}$$

Upon solving for d

we get $d = 35.9$ mm

1M

(b) Fatigue failure case:

we can get that Moment is Completely reversing and Torque is constant at Critical section

so, $M_m = 0 \quad M_a = M \quad \& \quad T_m = T$ and $T_a = 0$. (1M)

we get $\sigma_m = 0$ and $\sigma_a = \dfrac{8152}{d^3}$. (1M) 1M

and $\quad T_m = \dfrac{3057}{d^3}$ and $T_a = 0$.

Using D.E.T we get (1M)

$$(\sigma_{eq})_m = \sqrt{(\sigma_m)^2 + 3(T_m)^2} = \sqrt{3}\,T_m = \frac{\sqrt{3}\;3057}{d^3}$$

$$(\sigma_{eq})_a = \sqrt{(\sigma_a)^2 + 3(T_a)^2} = \sigma_a = \frac{8152}{d^3}$$

(1M)

Using Goodman Eqn $\quad \dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} = \dfrac{1}{FOS}$ $\quad \left(S_e = 250 \times 10^6 \text{ given} \right)$

we get Expression as $\quad \dfrac{8152}{250 \times 10^6 \times d^3} + \dfrac{\sqrt{3} \times 3057}{d^3 \times 560 \times 10^6} = \dfrac{1}{2}$.

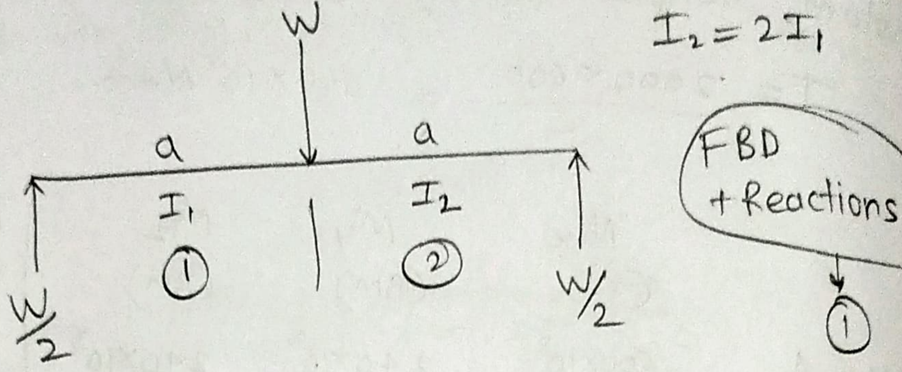
$d = 43.8\text{ mm}$ (1M)

Marking Scheme

Finding Value of P_c	1M
Finding the Reaction Forces	1M
Identifying the Correct Critical Section	1M
Writing Proper Expressions for σ and τ using Correct Values of M and T and Writing Proper expression for Equivalent stress using D.E.T **(or)** Writing Direct Expression for Equivalent D.E.T Stress with Correct Values of T and M	1M+1M
Final Value of d (Diameter)	1M
Identifying that Bending Moment (BM) is Completely Reversed, Torsion is Constant, and Writing Values for M_m, M_a, T_m, T_a	1M
Writing Respective σ and τ, Using D.E.T to Calculate Equivalent Stresses, and Correct Expression for Goodman Equation (or Combined D.E.T and Goodman Equation) with Proper Substitution of M_m, M_a, T_m, T_a	1M + 1M
Final Value of d (Diameter)	1M
	Total Marks: 10

Q.2

$I_2 = 2I_1$



$U = U_1 + U_2$

$$U = \int_0^a \frac{M_{x_1}^2\,dx}{2EI_1} + \int_a^{2a} \frac{M_{x_2}^2\,dx}{2EI_2} \quad \Big\} \; 1$$

$$\frac{\partial U}{\partial W} = \int_0^a \frac{M_{x_1}}{EI_1}\frac{\partial M_{x_1}}{\partial W}\,dx + \int_a^{2a} \frac{M_{x_2}}{EI_2}\frac{\partial M_{x_2}}{\partial W}\,dx$$

$$M_{x_1} = \frac{W}{2}\times x \qquad \frac{\partial M_{x_1}}{\partial W} = \frac{x}{2} \quad 1$$

$$M_{x_2} = \frac{W}{2}x - W(x-a) \qquad \frac{\partial M_{x_2}}{\partial W} = \frac{x}{2} - (x-a) \quad 1$$

$$\delta = \frac{\partial U}{\partial W} = \int_0^a \frac{W\left(\frac{x}{2}\right)^2 dx}{EI_1} + \int_a^{2a} \frac{W\left(\frac{x}{2}-(x-a)\right)^2 dx}{EI_2} \quad 1$$

$$\delta = \int_0^a \frac{Wx^2\,dx}{4EI_1} + \int_a^{2a} \frac{W\left(-\frac{x}{2}+a\right)^2}{2EI_1}\,dx$$

$$\delta = \frac{W}{4EI_1}\left(\frac{x^3}{3}\right)_0^a + \frac{W}{2EI_1}\left.\frac{\left(-\frac{x}{2}+a\right)^3}{-3/2}\right|_a^{2a}$$

$$\delta = \frac{Wa^3}{12EI_1} + \frac{W}{EI_1}\left[0 + \frac{a^3}{8\times 3}\right) = \frac{Wa^3}{12EI_1} + \frac{Wa^3}{24EI_1}$$

$$\boxed{\delta = \frac{Wa^3}{8EI_1}} \quad 2$$

$$W_c = \frac{2\pi \times 9000}{60} \text{ rad/s.} \longrightarrow 1$$

300π , 942.47

$$W = 50 \times 10 = 500\,N$$

$$W_c = \sqrt{g/\delta}$$

$$\left(\frac{2\pi \times 9000}{60}\right)^2 = \frac{10 \times 10^4}{\delta}$$

1

$$\Rightarrow \delta = 0.0113 \text{ mm.}$$

$$\delta = \frac{Wa^3}{8EI_1} \qquad a = 300\text{mm}, \quad E = 200 \text{ GPa},$$

$$\Rightarrow I_1 = 7.4947 \times 10^5 \text{ mm}^4$$

$$I_1 = \frac{\pi}{64} d_1^4$$

$$\Rightarrow \boxed{d_1 = 62.5 \text{ mm}}$$

1

3) a) Objective minimize $M = 2\pi r t L \rho$

s.t. $T = \pi r^2 t \sigma_y$

$N_{cr} \geq N_{cr}^*$

+1

b) We have $f = c' \sqrt{\frac{k}{m}}$

For a shaft $N_{cr} = c'' \sqrt{\frac{k_{shaft}}{m_{shaft}}}$

We have $\delta = \tilde{c} \frac{PL^3}{EI}$ $\rightarrow$ deflection of a beam / or shaft

$\therefore k_{shaft} = \frac{P}{\delta} = \tilde{c} \frac{EI}{L^3}$. Mass of the shaft $m_{shaft} = M$.

$N_{cr} = c' \sqrt{\frac{\tilde{c} EI}{ML^3}} = c \sqrt{\frac{EI}{ML^3}}$ +3

c) $N \geq N_{cr}^*$

or $N^2 \geq N_{cr}^{*2}$

$\frac{c^2 EI}{ML^3} \geq N_{cr}^{*2}$

Now $I = \pi r^3 t = \frac{\pi r^4 t^2}{rt}$

$= \frac{2 T^2 L \rho}{M \sigma_y^2}$

$\frac{c^2 E}{ML^3} \cdot \frac{2T^2 L \rho}{M \sigma_y^2} \geq N_{cr}^{*2}$ +3

$M^2 \leq 2c^2 \frac{T^2}{N_{cr}^{*2}} \frac{1}{L^2} \frac{E\rho}{\sigma_y^2}$

or $M \leq \sqrt{2} c \frac{T}{N_{cr}^*} \cdot \frac{1}{L} \underbrace{\frac{(E\rho)^{1/2}}{\sigma_y}}_{\text{material index}}$

For a fixed T, N_{cr}^*, L to minimize M we need to choose a material with as low +1 of $\frac{(E\rho)^{1/2}}{\sigma_y}$ +1 as possible.

For the given materials

	MI.
Mild steel	0.18
High strength steel	0.10
Al alloy.	0.05 ✓ Lowest MI

chosen material Al alloy.

+1 if the material index is correct

4) $\sigma_{ult} = 1000$ MPa, $\sigma_y = 800$ MPa, $\sigma_e' = 500$ MPa. $k_a = 0.679$, $k_b = 1$.

$T_m = 45$ Nm, $M_a = 70$ Nm

$d_r = d - 2r = 0.65D$

$d = 0.8D$.

$$\frac{d_r}{d} = \frac{0.65}{0.8}$$

$$\therefore \frac{d_r}{d} = 1 - \frac{2r}{d}.$$

$$\therefore \frac{r}{d} = \frac{1}{2}\left(1 - \frac{0.65}{0.8}\right) = 0.094 \approx 0.1.$$

$K_t = 1.7$ $K_t = 1.5$

$\therefore$ For the first iteration we will assume that $\underline{q = 1}$ (To know q in addition to knowing σ_{ult} & r/d, we also need r.
$\therefore [K_f = K_t$ & $K_{fs} = K_t]$.

The minimum diameter is d_r and that is the most critical regions as compared to the locations shown in the figure.

Now $\sigma_e = \underbrace{k_a k_b k_c k_d k_e k_f}_{1} \sigma_e' = 339.5$ MPa.

Also: $\sigma_a = K_f \dfrac{32 M_a}{\pi d_r^3}$, $\tau_m = K_{fs} \dfrac{16 T_m}{\pi d_r^3}$

in these formulae, the diameter to be chosen is the smallest diameter, i.e. dr

$$\sigma_{eq,a} = \frac{32 K_f M_a}{\pi d_r^3}, \quad \sigma_{eq\,m} = \frac{\sqrt{3} \cdot 16 \cdot K_{fs} T_m}{\pi d_r^3}$$

We have $\dfrac{\sigma_{eq\,a}}{\sigma_e} + \dfrac{\sigma_{eq\,m}}{\sigma_{ult}} = \dfrac{1}{FOS}$.

Solving for d_r; $d_r = 20.27$ mm., $D = 31.19$ mm
$d = 24.95$ mm, $r = 2.34$ mm.

+7 - for iteration 1. I have shown one way to solve the problem. Another way is to assume dr and then check the FOS.

For the second iteration, we calculate K_b using d_r & q using d & r. We find FOS & if it is close to 2 we accept the values of d_r, D, d & r. Else using FOS, K_b, & q we calculate new d_r & d repeat the above procedure. [In our case FOS = 1.9]
If we calculate d_r, then $d_r = 20.62$ mm. To be on the safe we can choose $d_r = 20.62$ mm ≈ 21.0 mm.

+3

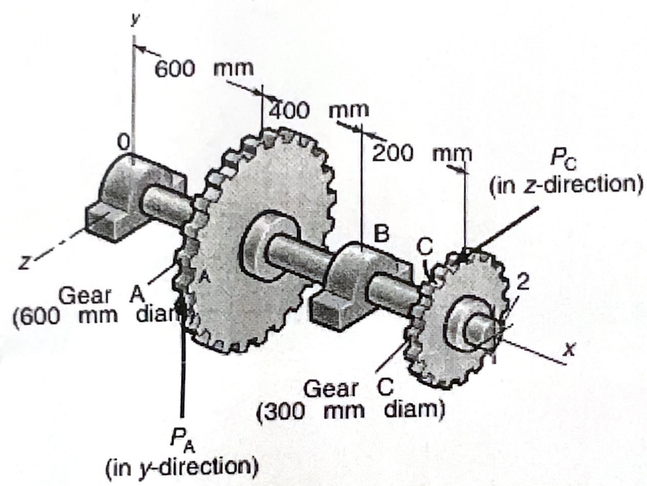
ME423 – Midsemester Exam

Saturday
21st September 2024

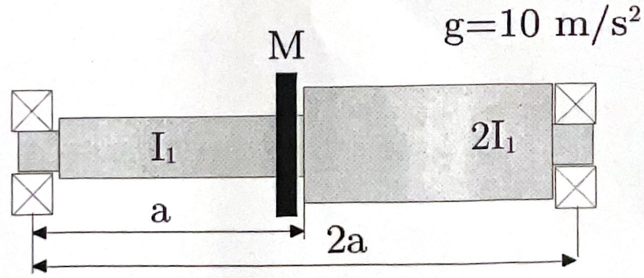
1.30 pm to 3.30 pm
Max Marks: 40

Instructions:
a. There are FOUR questions b. This is an open notes exam

1. The rotating solid steel shaft of constant diameter shown in the figure has two gears mounted on it and is supported by bearings at points O and B. The gears transmit a constant torque caused due to P_A = 2000 N acting vertically as shown. The shaft is machined from steel with yield strength = 420 MPa and ultimate tensile strength = 560 MPa. The corrected endurance limit is 250 MPa. Using a factor of safety of 2.0, determine the minimum allowable diameter of the shaft based on (a) a static yield analysis using the distortion energy theory and (b) a fatigue-failure analysis (Goodman + distortion energy theory). **(10 marks)**



2. The figure shows a stepped shaft which is simply supported and on which a pulley of mass $M = 50$ kg is mounted. The moment of inertia's of the two sections are $I_1 = \frac{\pi}{64}d^4$ and $I_2 = 2I_1$. Calculate d that ensures that the first critical speed is at least 9000 rpm.



The shaft is made of steel with E = 200 GPa. The distance a = 300 mm and the mass of the shaft is neglected. The effect of transverse shear is also neglected. You can assume $g = 10\ m/s^2$. **(10 marks)**

3. One would like to select the material for a **light** thin walled hollow circular transmission shaft (mean radius r and thickness t). The shaft has to be designed in such a way that its critical speed is at least N^*_{cr} rpm. It should also be able to transmit a torque T without yielding. To proceed: **(10 marks)**

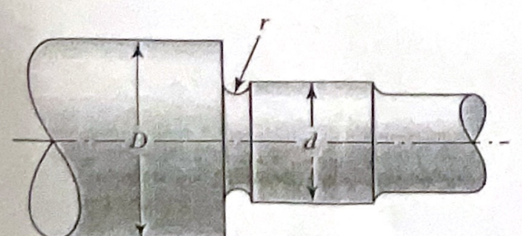
- Clearly identify the objective, the equality constraint and the inequality constraint.
- Show that the critical speed of the shaft is of the form

$$N_{cr} = c\sqrt{\frac{EI}{ML^3}}$$

Here c is a constant, E is the Young's modulus, I is the moment of inertia, M mass of the shaft and L is the length of the shaft. Hint: The natural frequency of oscillation of an elastic system is of the form $f = c'\sqrt{k/m}$ where c' is a constant, k is its stiffness and m is its mass.
- Start with the inequality constraint. Express the moment of inertia of the cross-section, I, in terms of the torque carrying capacity T, yield strength σ_y, length L of the shaft, density ρ and mass M of the shaft. Note that for a shaft of length L with a thin circular cross-section with mean radius r and thickness t, $M = 2\pi r t L\rho$, moment of inertia $I = \pi r^3 t$ and torque carrying capacity is $T = \pi r^2 t \sigma_y$.
- Use of the expression of I derived above to find the material index. Hint: This expression will be in a form of inequality which includes M, L, ρ, E, σ_y, T , N^*_{cr}
- Based on the material index derived, choose the appropriate material from table given below.

Material	E (GPa)	ρ (kg/m3)	σy (MPa)
Mild steel	210	7800	220
High strength steel	210	7800	400
Aluminum alloy	70	2700	300

4. A section of shaft shown in the figure. The shaft is made steel with σ_{ult} = 1000 MPa and σ_y = 800 MPa and is subjected to completely reversed bending and steady torsion. The uncorrected endurance limit is 500 MPa and the surface finish factor for all surfaces is $k_a = 0.679$. At the location shown, $M_a = 70\ Nm$ and $T_m = 45\ Nm$. The diameter at the root of the groove is $d_r = d - 2r$, where d is the shoulder diameter. The relative sizes of the diameters are as follows: $d_r = 0.65D$ and $d = 0.8D$. One needs to size the shaft for an infinite life using the DE-Goodman criterion with a factor of safety 2.0. **(10 marks)** Show **only the first iteration** of your calculations, i.e. find d, d_r, D: you can assume $k_b = 1$. Then clearly **describe the methodology in WORDS** to obtain the final design. You also need to highlight the changes that you anticipate that need to be made from the first iteration. In case you feel some information is missing, make appropriate assumptions and clearly state them.



Stress Concentration Factors for the first iteration

	Bending	Torsional
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5