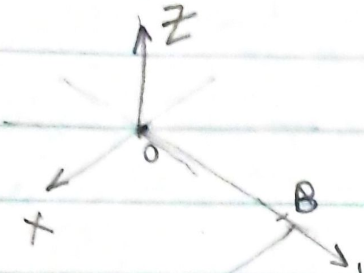
Problem 1

ME 423 - Quiz 1

1a. The internal resisting forces and moments at different locations are calculated using:



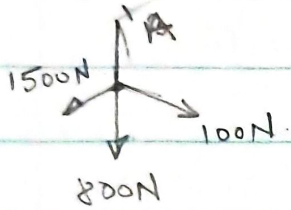
$$\underline{F}_R + \underline{F}_A = \underline{0}$$

$\underline{F}_R$ & M_R are internal resisting forces & moments

& $$\underline{M}_R + \underline{r} \times \underline{F}_A = \underline{0}$$

$$\underline{F}_A = 1500\hat{i} + 100\hat{j} - 800\hat{k}$$

$$\underline{F}_R = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$



$$\underline{M}_R = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

	F_x (N)	F_y (N)	F_z (N)	M_x (Nm)	M_y (Nm)	M_z (Nm)
A	−1500	−100	800	−5 (T)	75	0
B	−1500	−100	800	−5	−525 (T)	−75
O	−1500	−100	800	235	−525 (T)	375

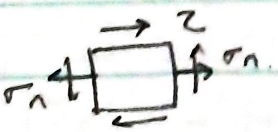
From the table it is seen that the most critical section is at pt O.

We will ignore the effect of direct shear due to F_x & F_z.

M_y represent the torque @ Y-axis. M_x represents bending moment @ x-axis & M_z represents bending @ z axis. The resultant bending moment is $\tilde{M} = \sqrt{M_y^2 + M_x^2}$

$$\sigma_b|_{max} = \frac{32\tilde{M}}{\pi d^3} \qquad \sigma_n|_{axial} = \frac{4F_y}{\pi d^2}$$

$$\sigma_n = \sigma_b|_{max} + \sigma_n|_{axial} = \frac{32\tilde{M}}{\pi d^3} + \frac{4F_y}{\pi d^2}.$$



$$\tau_{max} = \frac{16 M_y}{\pi d^3}$$

$$\sigma_{vm} = (\sigma_n^2 + 3\tau_{max}^2)^{1/2}.$$

To find the minimum required diameter

$$\sigma_{vm} = \frac{\sigma_y}{FOS} \quad \text{or} \quad \sigma_{vm}^2 = \frac{\sigma_y^2}{FOS^2}$$

$$\therefore \sigma_n^2 + 3\tau_{max}^2 = \frac{\sigma_y^2}{FOS^2}$$

$$\therefore \left(\frac{32\tilde{M}}{\pi d^3} + \frac{4P}{\pi d^2}\right)^2 + 3\left(\frac{16 M_y}{\pi d^3}\right)^2 = \frac{\sigma_y^2}{FOS^2} \quad (A)$$

Solve the above equation$^{(A)}$ for d. It is not easy to solve unless you have the "right" calculator. To simplify, we assume that $\frac{4P}{\pi d^2} << \frac{32\tilde{M}}{\pi d^3}$. We will verify the assumption at the end. Then we get.

(can be solved using any calculator).

$$\left(\frac{32\tilde{M}}{\pi d^3}\right)^2 + 3\left(\frac{16 M_y}{\pi d^3}\right)^2 = \frac{\sigma_y^2}{FOS^2} \quad (B).$$

Solving (approx) B : $d = 33.3012$ mm ≈ 33.3 mm

Solving (exact) A : $d_2 = 33.3063$ mm ≈ 33.3 mm.

We see that the difference in answer is about 0.02%.

$d = 33.3063$ mm.

For $\sigma_{axial} = \pm 0.11$ MPa.

$\sigma_b = 122.00$ MPa.

$\tau = 72.37$ MPa

$\therefore \sigma_{axial} << \sigma_b$.

$\sigma_{axial} << \tau$.

Hence our assumption of neglecting σ_a is justified. This is true in almost all problems of combined loading.

Problem 2

2) ① $(\sigma_{max})_1 = 340$ MPa, $(\sigma_{min})_1 = 160$ MPa, $n_1 = 8\times10^4$.
② $(\sigma_{max})_2 = 320$ MPa, $(\sigma_{min})_2 = -200$ MPa $n_2 = ?$
$\sigma_y = 350$ MPa, $\sigma_{ult} = 420$ MPa, $f = 0.9$ $\sigma_e = 175$ MPa.

Miners rule $\dfrac{n_1}{N_1} + \dfrac{n_2}{N_2} = 1$

$N_i \rightarrow$ number of cycles for failure at fully reversed (ie $\sigma_m = 0$) cycle with amplitude σ_{a_i}

Equivalent fully reversed stress amplitude for ①.

$$\sigma_{a_1}' = \frac{\sigma_{a_1}}{1 - \frac{\sigma_{m_1}}{\sigma_{ult}}} \qquad \sigma_{a_1} = 90 \text{ MPa} \quad \sigma_{m_1} = 250 \text{ MPa}.$$

$$= 222.35 \text{ MPa}.$$

$$\sigma_{a_2}' = \frac{\sigma_{a_2}}{1 - \frac{\sigma_{m_2}}{\sigma_{ult}}} \qquad \sigma_{a_2} = 260 \text{ MPa} \quad \sigma_{m_2} = 60 \text{ MPa}.$$

$$= 303.33 \text{ MPa}.$$

Need to find N_1 & N_2 corresponding to σ_{a_1}' & σ_{a_2}' from the Basquin eqn.

$$\sigma_a = \sigma_f' (2N)^b.$$

$$b = \frac{\ln(\sigma_a'/\sigma_e)}{\ln(10^3/10^6)} = \frac{\ln(f\sigma_{ult}/\sigma_e)}{\ln(10^3/10^6)} = -0.11148.$$

$$\sigma_f = \frac{\sigma_e}{(2\times10^6)^b} = 882.07\ MPa.$$

$$\therefore\ N_1 = \frac{1}{2}\left(\frac{\sigma'_{a_1}}{\sigma_f}\right)^{\frac{1}{b}} = 116705.75$$

$$N_2 = \frac{1}{2}\left(\frac{\sigma'_{a_2}}{\sigma_f}\right)^{\frac{1}{b}} = 7198.8558$$

Now $n_2 = N_2\left(1 - \frac{n_1}{N_1}\right) \approx 2264$ cycles.

Note that if you had used $(\sigma_{max})_1 = 360\,MPa$, then $\frac{n_1}{N_1} > 1$.

In fact $(\sigma_{max})_1 > \sigma_y$ and the component would

Problem 3

3. For a cantilever beam: $\delta = \frac{PL^3}{3EI}$ For a rectangular c/s $I = \frac{bh^3}{12}$

$$\therefore \quad \delta = \frac{4PL^3}{Ebh^3}$$

or $$P = \frac{Ebh^3\delta}{4L^3}$$

$$P_{max} = \frac{Ebh^3\delta_{max}}{4L^3}, \quad P_{min} = \frac{Ebh^3\delta_{min}}{4L^3}$$

$$\sigma_{max} = \frac{M_{max}\,h/2}{I} = \frac{(P_{max}L)\,h/2}{I} = \frac{3}{2}\frac{Eh\delta_{max}}{L^2}$$

Similarly, $$\sigma_{min} = \frac{3}{2}\frac{Eh}{L^2}\delta_{min}.$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max} - \delta_{min})$$

$$\sigma_m = \frac{3}{4}\frac{Eh}{L^2}(\delta_{max} + \delta_{min}).$$

Substituting $E = 200\,GPa$, $h = 4mm$, $L = 300mm$,
$\delta_{max} = 20mm$, $\delta_{min} = 10\,mm$

$$\sigma_a = 66.66\,MPa, \quad \sigma_m = 200\,MPa.$$

Goodman criterion for infinite life.

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ult}} = \frac{1}{n}$$

Substituting the above values with $\sigma_e = 0.5\,\sigma_{ult}$, $n = 1.4 > 1$
$\therefore$ Hence the factor of safety based on infinite life as per Goodman's criterion is 1.4.

Note: No correction factor is assumed for the endurance limit but not was mentioned/given. In practice, one does need to use the corrections factors to modify the uncorrected endurance strength