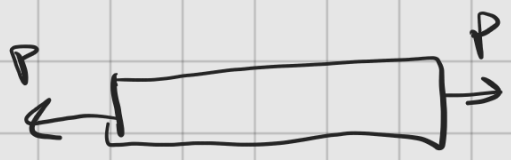
HW-3

2010008?

Q.1 (a) $S_{\text{light struts}} > S^*$



$S > S^*$

$S = \frac{PL}{AE} \Rightarrow S = \frac{AE}{L}$

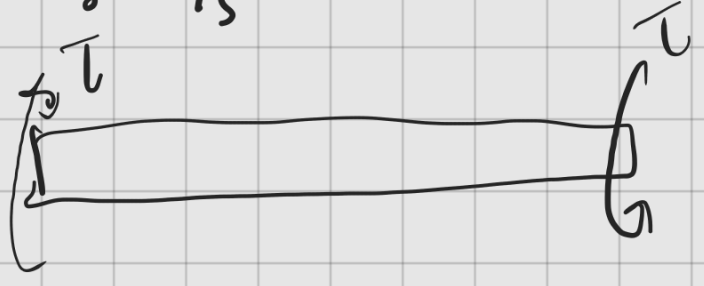
$\frac{AE}{L} > S^*$

$\frac{E}{L}\left(\frac{M}{\rho L}\right) > S^*$

$m > S^* L^2 \left(\frac{\rho}{E}\right)$ → material index

minimize $\frac{E}{\rho}$

(b)



$\frac{T}{\theta} = \frac{GJ}{L} = S_{\text{beam}}$

$S_{\text{beam}} > S^*$

$\frac{GJ}{L} > S^*$ $\qquad \frac{G\pi d^4}{32L} > S^*$

$S^* < \frac{G}{2\pi L}\left(\frac{\pi d^2}{4}\right)^2 = \frac{G}{2\pi L} A^2 = \frac{G}{2\pi L}\left(\frac{m}{\rho L}\right)^2$

$$s^* < \frac{G m^2}{2\pi s^2 L^3}$$

$$m > \sqrt{2\pi s^*}\, L^{3/2}$$

maximise $\sqrt{G}/s$ → material index

Q.2

$$K = \frac{GJ}{L} = \frac{G\pi d^4}{32L}$$

$$A_{NS} = A$$

$$\frac{\pi d_{NS}^2}{4} = \pi (R_o^2 - R_i^2)$$

$$d_{NS}^2 = 4(R_o^2 - R_i^2)$$

$$K_{NS} = \frac{\pi G}{32L} \times 16 (R_o^2 - R_i^2)^2$$

$$K = \frac{\pi G}{32L} \times 16 (R_o^4 - R_i^4)$$

$$\phi = \frac{K}{K_{NS}} = \frac{R_o^4 - R_i^4}{(R_o^2 - R_i^2)^2} = \frac{(R_o^2 - R_i^2)(R_o^2 + R_i^2)}{(R_o^2 - R_i^2)^2}$$

$$\boxed{\phi = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}}$$

Q.3

(a) $\sigma = \frac{M}{I/y} < \sigma_f$

$Z = \frac{I}{y} > \frac{M}{\sigma_f}$

$Q_B^6 = \frac{6Z}{A^{3/2}} \Rightarrow Z = \frac{Q_B^6 A^{3/2}}{6}$

given moment

$\frac{M}{\sigma_f} < \frac{Q_B^6}{6} A^{3/2} = \frac{Q_B^6}{6} \left(\frac{m}{\rho L}\right)^{3/2}$

$m^{3/2} > (\rho L)^{3/2} \frac{6M}{\sigma_f Q_B^6}$

mass

$m > (6M)^{2/3} L \frac{\rho}{(\sigma_f Q_B^6)^{2/3}}$

performance index

minimize $\frac{(\sigma_f Q_B^6)^{2/3}}{\rho}$

$$\phi_s^e = \frac{G_s J/L}{(G_s J/L)_{NS}} = \frac{J}{J_{NS}} = \frac{2J}{\pi r_{NS}^4} = \frac{2\pi J}{A^2}$$

$$\phi_s^e = \frac{2\pi J}{A^2} \Rightarrow J = \frac{\phi_s^e A^2}{2\pi}$$

$$S = \frac{G_s J}{L} = \frac{G_s \phi_s^e A^2}{2\pi} = \frac{G_s \phi_s^e}{2\pi} \left(\frac{m}{\rho L}\right)^2$$

$$S > S^*$$

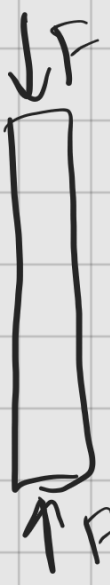
$$\frac{G_s \phi_s^e m^2}{2\pi \rho^2 L^2} > S^*$$

$$m > \sqrt{2\pi S^*}\, L \frac{\rho}{\sqrt{G_s \phi_s^e}}$$

performance index

maximise $\frac{\sqrt{\phi_s^e G_s}}{\rho}$

Q.4



column ⇒ double cantilever

$$F_{buckling} = \frac{4\pi^2 EI}{L^2}$$

$$F < F_{buckling}$$

$$F < \frac{4\pi^2 E}{L^2} I = \frac{4\pi^2 E}{L^2} \times \frac{\pi R^4}{4}$$

$$F < \pi^3 E \left(\frac{R}{L}\right)^4 L^2$$

L is given for a column under buckling

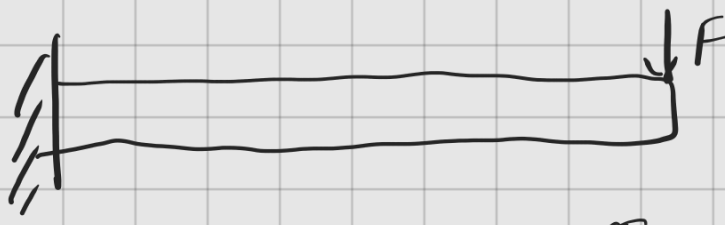
$$\left(\frac{L}{R}\right)^4 < \frac{\pi^3 L^2 E}{F}$$

$$\frac{L}{R} < \left(\frac{\pi}{F}\right)^{3/4} \sqrt{L} \; E^{1/4}$$

$E^{1/4}$ → material index.

maximise $E^{1/4}$

Q.5.



$$\delta \propto \frac{F}{EI} \qquad \delta = \frac{CF}{EI}$$

$$\delta > \delta^*$$

$$\delta^* < \frac{CF}{EI} = \frac{12CF}{EA^2\phi_B^e} \qquad C = C_m \rho A L$$

$$= \frac{12C F}{E\left(\frac{C}{C_m \rho L}\right)^2 \phi_B^e}$$

$$\frac{C}{C_m \rho L} < \sqrt{\frac{12CF}{\delta^*}} \frac{1}{\sqrt{E \phi_B^e}}$$

$$C < \sqrt{\frac{12CF}{\delta^*}} L \frac{C_m \rho}{\sqrt{E \phi_B^e}}$$

For $\phi_B^e = 1$

material index $= \frac{\sqrt{E}}{C_m \rho}$

maximise $\frac{\sqrt{E}}{C_m \rho}$

Q.6.

$$\sigma_{max} < \sigma_b$$

$$\frac{(3+\nu)}{8}\rho R^2\omega^2 < \sigma_f$$

$$E = \frac{I\omega^2}{2} = \frac{mR^2\omega^2}{4}$$

$$\frac{E}{m} = \frac{R^2\omega^2}{4} = E_m$$

$$\frac{(3+\nu)}{8}\rho\,(4E_m)^2 < \sigma_f$$

$$E_m < \sqrt{\frac{\sigma_f}{2}}\,\frac{1}{\sqrt{\rho(3+\nu)}}$$

material index

minimize $\sqrt{\rho\,(3+\nu)}$