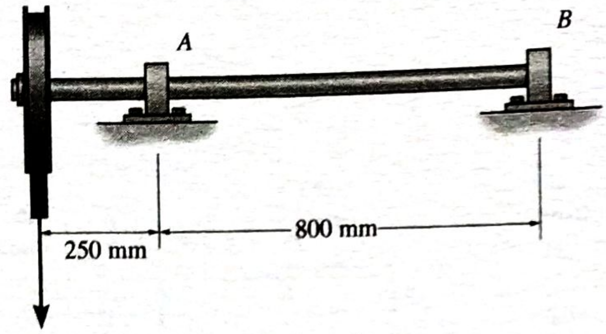
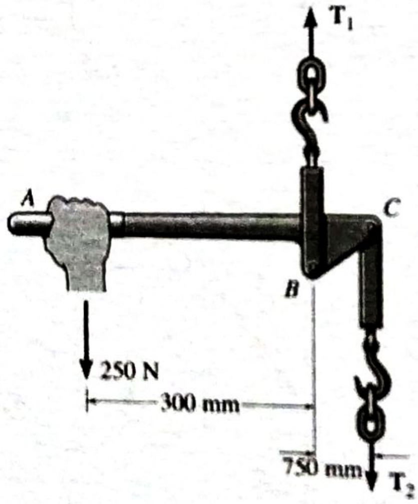
PROBLEMS

6-1. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft.



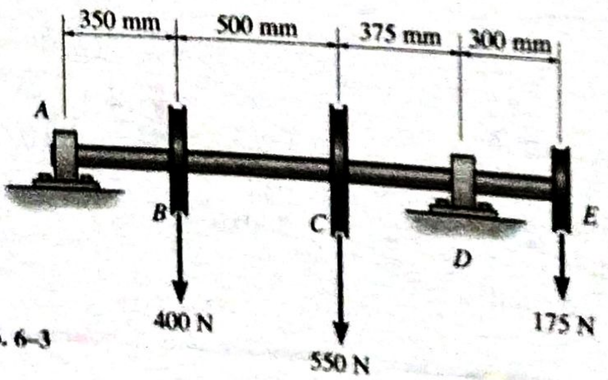
Prob. 6–1

6-2. The load binder is used to support a load. If the force applied to the handle is 250 N, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.



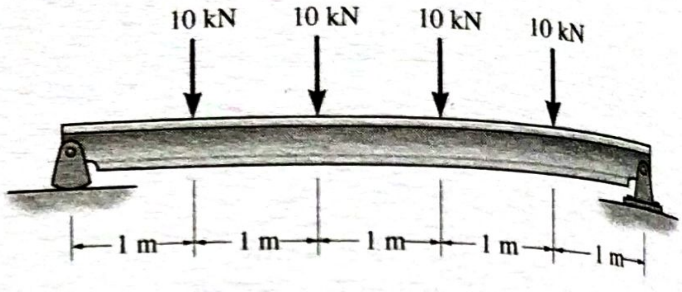
Prob. 6–2

6-3. Draw the shear and moment diagrams for the shaft. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B and C and E.



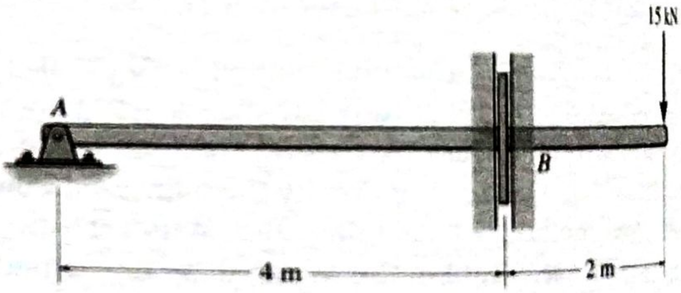
Prob. 6–3

***6-4.** Draw the shear and moment diagrams for the beam.



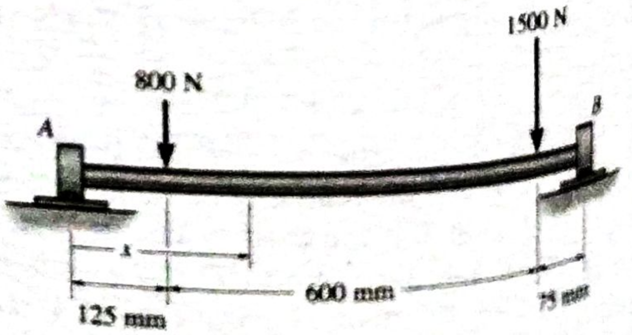
Prob. 6–4

6-5. Draw the shear and moment diagrams for the rod. It is supported by a pin at A and a smooth plate at B. The plate slides within the groove and so it cannot support a vertical force, although it can support a moment.



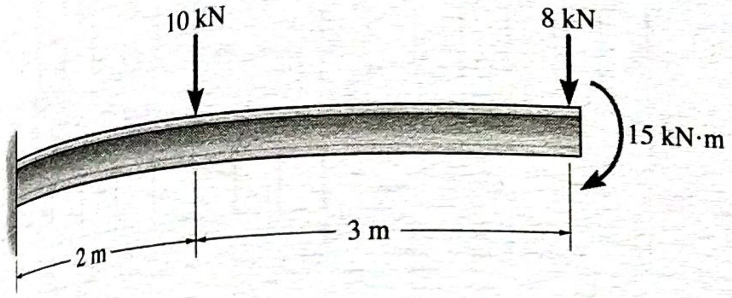
Prob. 6–5

6-6. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region 125 mm $< x <$ 725 mm.



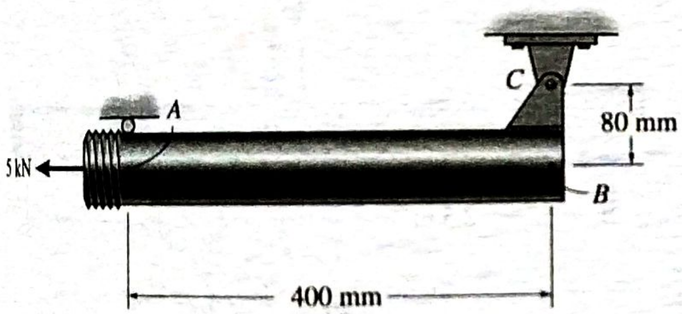
Prob. 6–6

6-7. Draw the shear and moment diagrams for the beam.



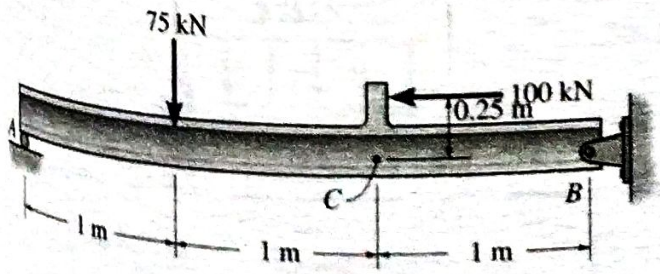
Prob. 6–7

*6-8. Draw the shear and moment diagrams for the pipe. The end screw is subjected to a horizontal force of 5 kN. *Hint:* The reactions at the pin C must be replaced by equivalent loadings at point B on the axis of the pipe.



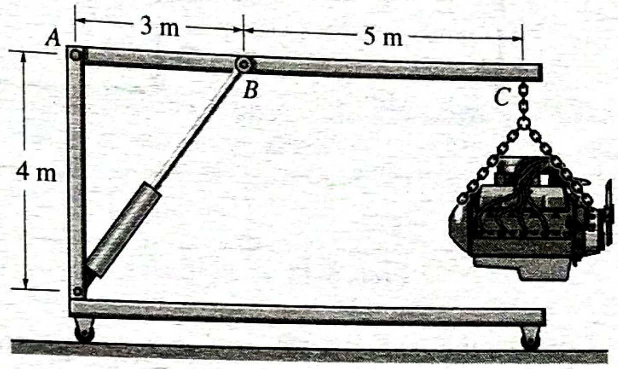
Prob. 6–8

6-9. Draw the shear and moment diagrams for the beam. *Hint:* The 100-kN load must be replaced by equivalent loadings at point C on the axis of the beam.



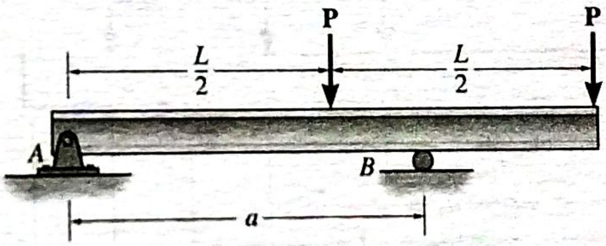
Prob. 6–9

6-10. The engine crane is used to support the engine, which has a weight of 1200 N. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position shown.



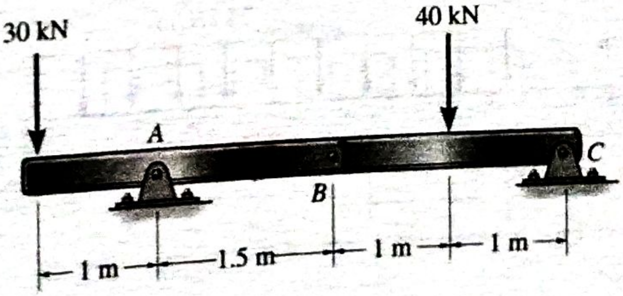
Prob. 6–10

6-11. Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



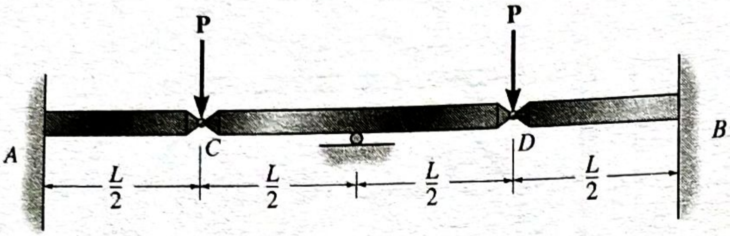
Prob. 6–11

*6-12. Draw the shear and moment diagrams for the compound beam which is pin connected at B.



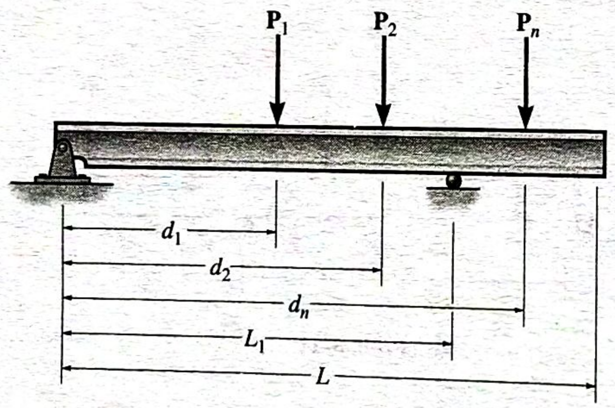
Prob. 6–12

6-13. The bars are connected by pins at C and D. Draw the shear and moment diagrams for the assembly. Neglect the effect of axial load.



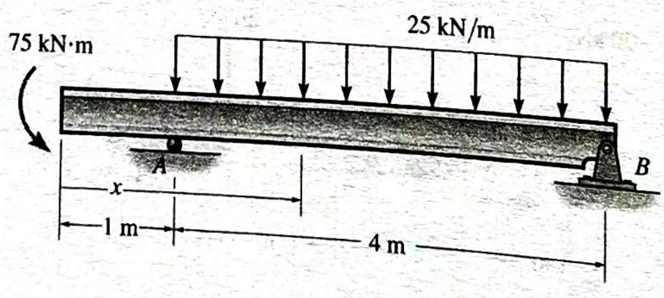
Prob. 6–13

■6-14. Consider the general problem of a simply supported beam subjected to n concentrated loads. Write a computer program that can be used to determine the internal shear and moment at any specified location x along the beam, and plot the shear and moment diagrams for the beam. Show an application of the program using the values $P_1 = 500$ kN, $d_1 = 5$ m, $P_2 = 800$ kN, $d_2 = 15$ m, $L_1 = 10$ m, $L = 15$ m.



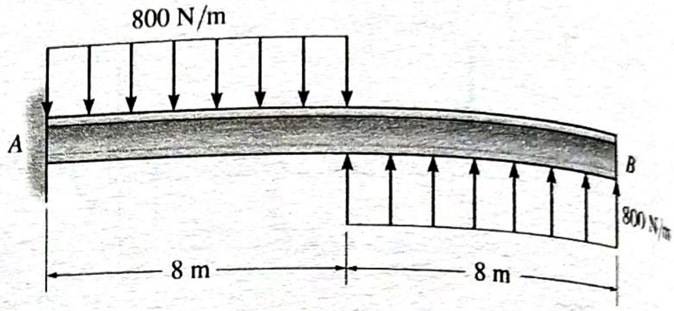
Prob. 6–14

6-15. Draw the shear and moment diagrams for the beam. Also, determine the shear and moment in the beam as functions of x, where $1 \text{ m} < x \leq 5$ m.



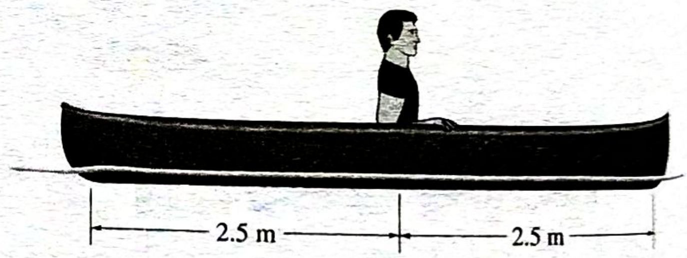
Prob. 6–15

***6-16.** Draw the shear and moment diagrams for the beam.



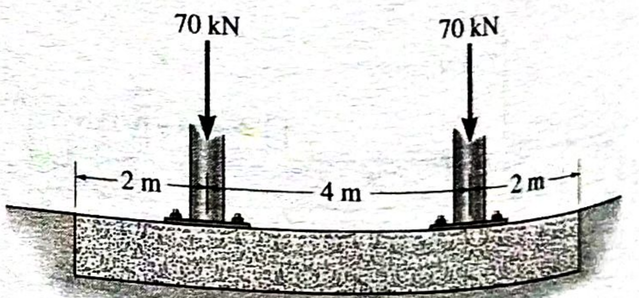
Prob. 6–16

6-17. The 750 N ($\approx$ 75 kg) man sits in the center of the boat, which has a uniform width and a weight per linear meter of 45 N/m. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.



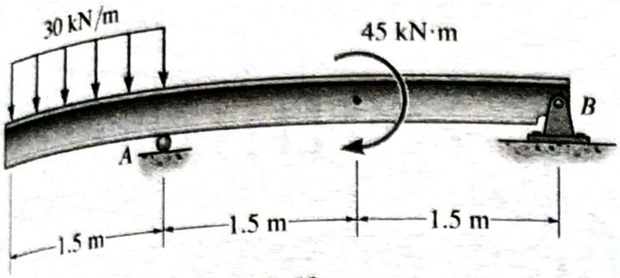
Prob. 6–17

6-18. The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the reaction of soil pressure on the footing is assumed to be uniform.



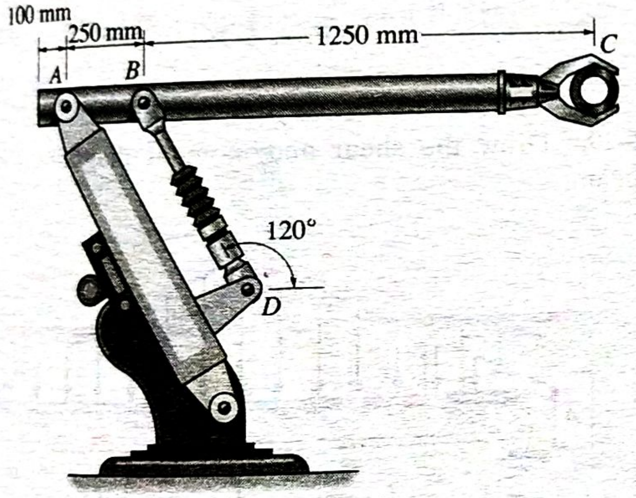
Prob. 6–18

6-19. Draw the shear and moment diagrams for the beam.



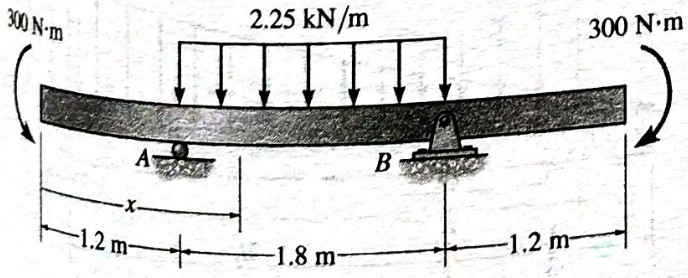
Prob. 6–19

*6-20. The industrial robot is held in the stationary position shown. Draw the shear and moment diagrams of the arm ABC if it is pin connected at A and connected to a hydraulic cylinder (two-force member) BD. Assume the arm and grip have a uniform weight of 0.3 N/mm and support the load of 200 N at C.



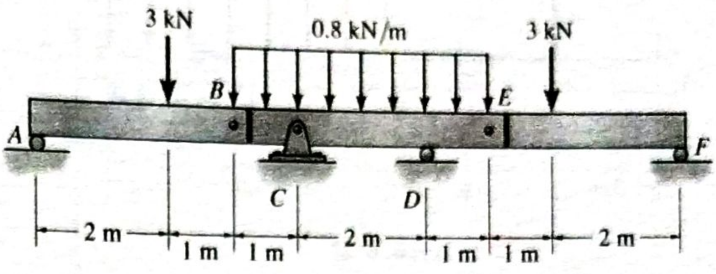
Prob. 6–20

6-21. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $1.2\text{ m} < x < 3\text{ m}$.



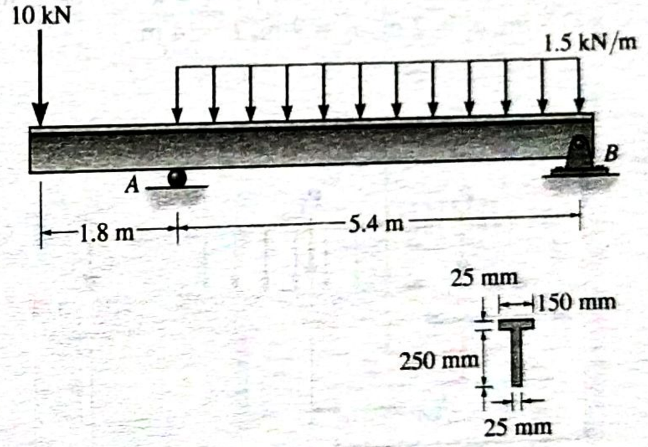
Prob. 6–21

6-22. Draw the shear and moment diagrams for the compound beam. The three segments are connected by pins at B and E.



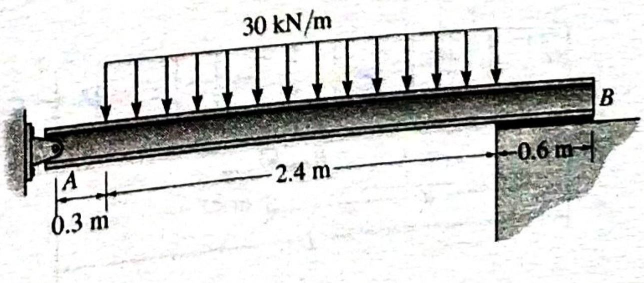
Prob. 6–22

6-23. The T-beam is subjected to the loading shown. Draw the shear and moment diagrams for the beam.



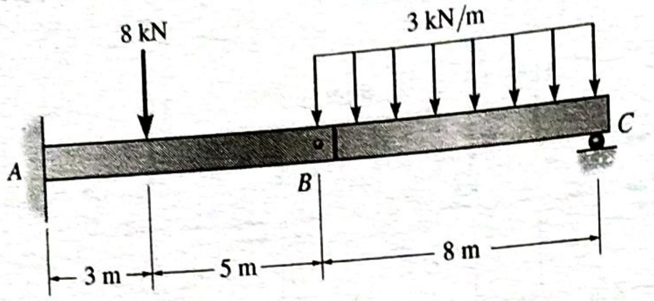
Prob. 6–23

*6-24. The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 0.6 m length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 30 kN/m.



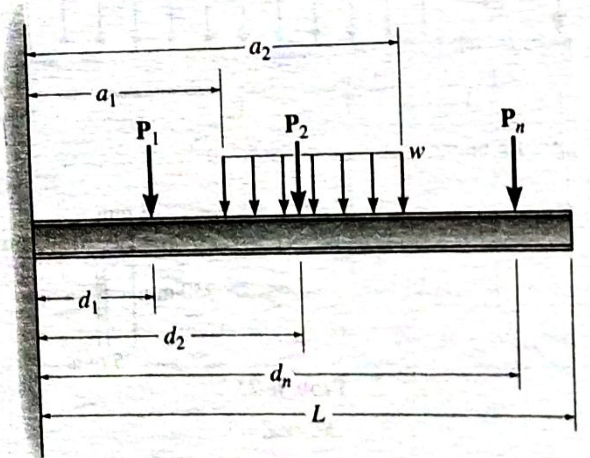
Prob. 6–24

6-25. Draw the shear and moment diagrams for the beam. The two segments are joined together at B.



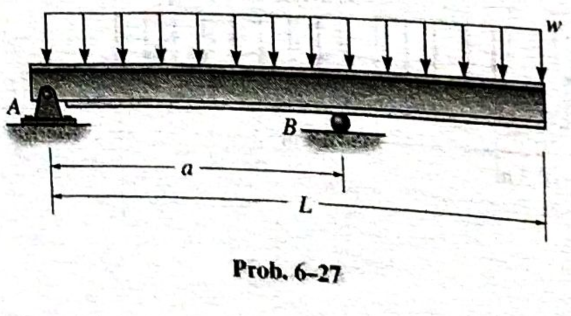
Prob. 6–25

■**6-26.** Consider the general problem of a cantilevered beam subjected to n concentrated loads and a constant distributed loading w. Write a computer program that can be used to determine the internal shear and moment at any specified location x along the beam, and plot the shear and moment diagrams for the beam. Show an application of the program using the values $P_1 = 4$ kN, $d_1 = 2$ m, $w = 800$ N/m, $a_1 = 2$ m, $a_2 = 4$ m, $L = 4$ m.



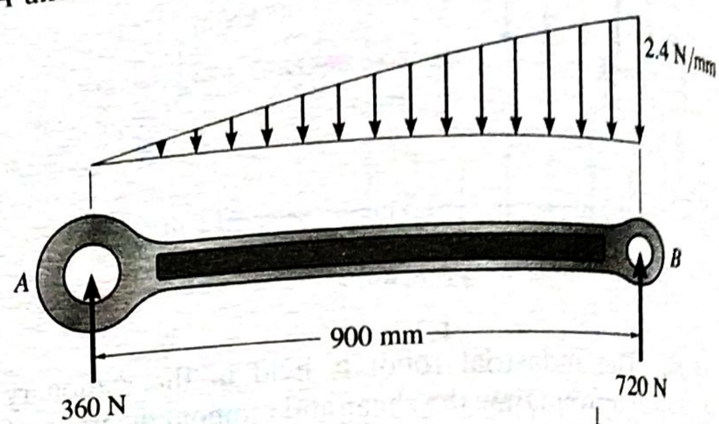
Prob. 6–26

6-27. Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



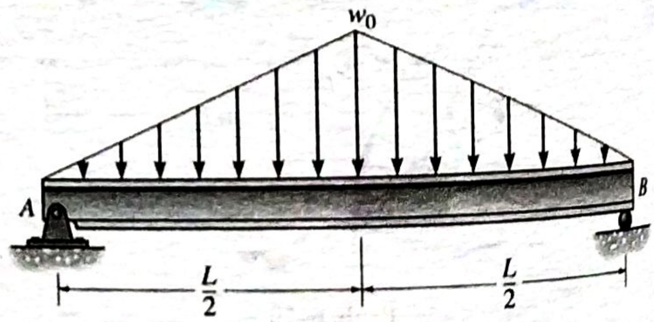
Prob. 6–27

***6-28.** Draw the shear and moment diagrams for the connecting rod. Only vertical reactions occur at its ends A and B.



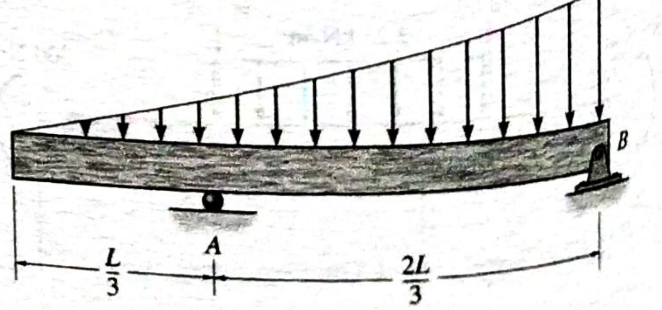
Prob. 6–28

6-29. Draw the shear and moment diagrams for the beam.



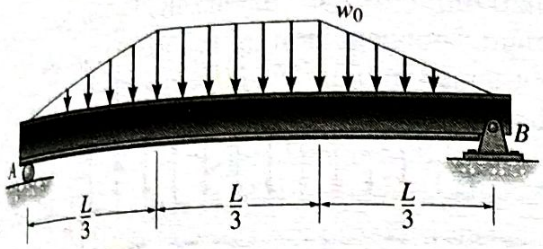
Prob. 6–29

6-30. Draw the shear and moment diagrams for the beam.



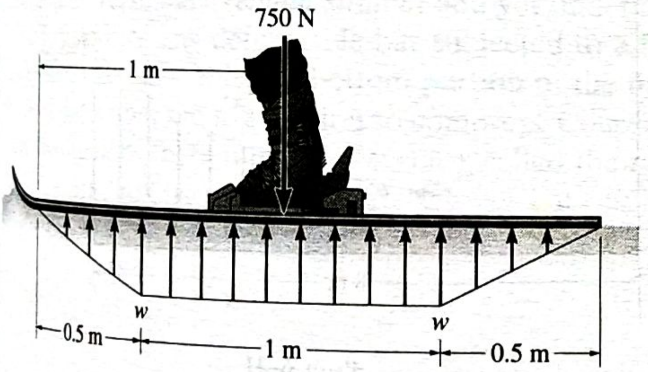
Prob. 6–30

6-31. Draw the shear and moment diagrams for the beam.



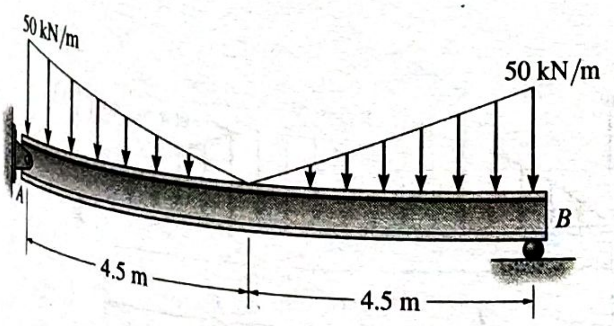
Prob. 6–31

*6-32. The ski supports the 750-N ($\approx$ 75 kg) weight of the man. If the snow loading on its bottom surface is trapezoidal as shown, determine the intensity w, and then draw the shear and moment diagrams for the ski.



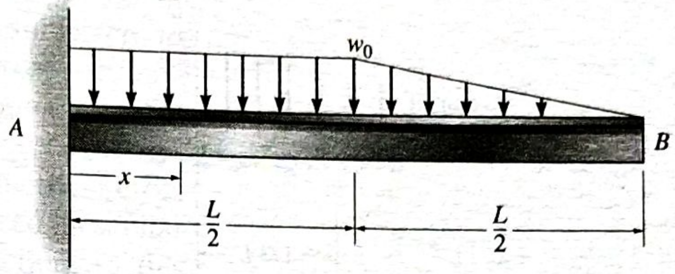
Prob. 6–32

6-33. Draw the shear and moment diagrams for the beam.



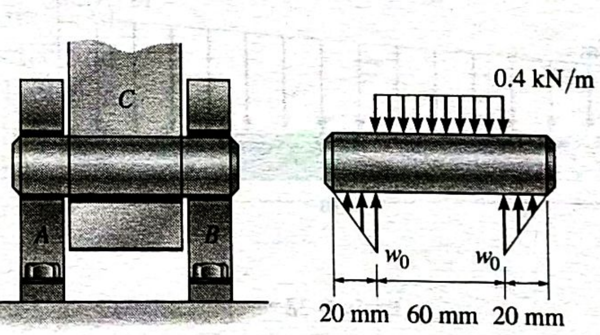
Prob. 6–33

6-34. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x.



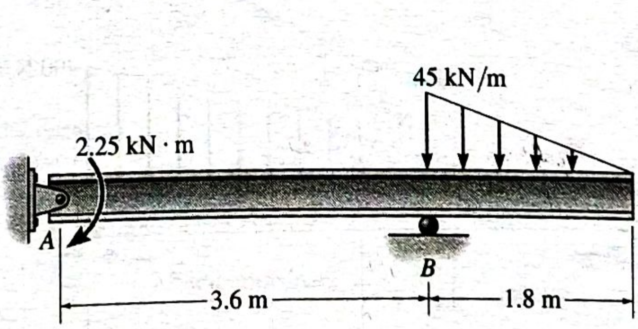
Prob. 6–34

6-35. The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagrams for the pin.



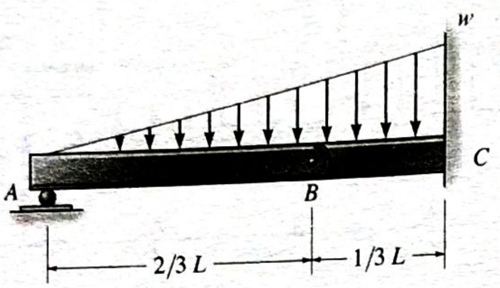
Prob. 6–35

*6-36. Draw the shear and moment diagrams for the beam.



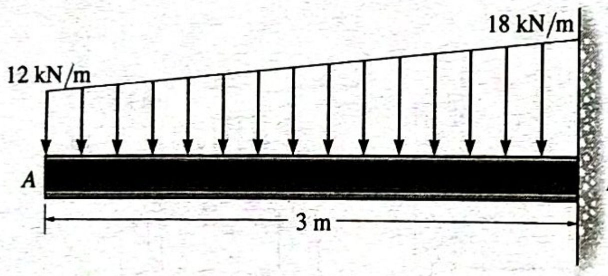
Prob. 6–36

6-37. The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.



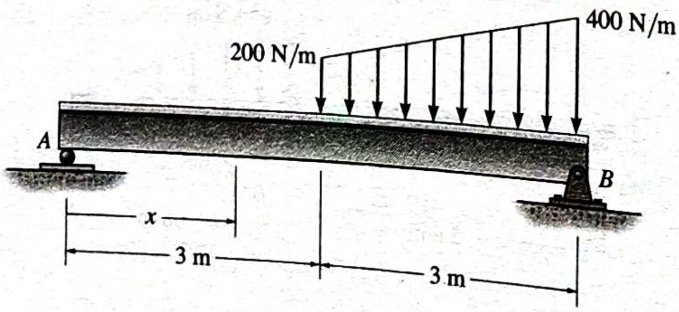
Prob. 6–37

6-38. Draw the shear and moment diagrams for the beam.



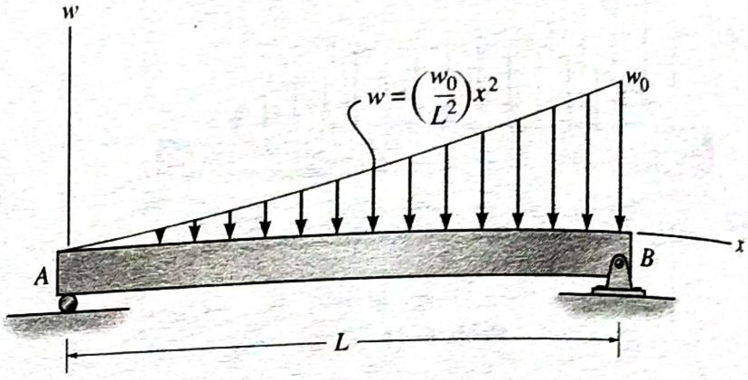
Prob. 6–38

6-39. Draw the shear and moment diagrams for the beam and determine the shear and moment as functions of x.



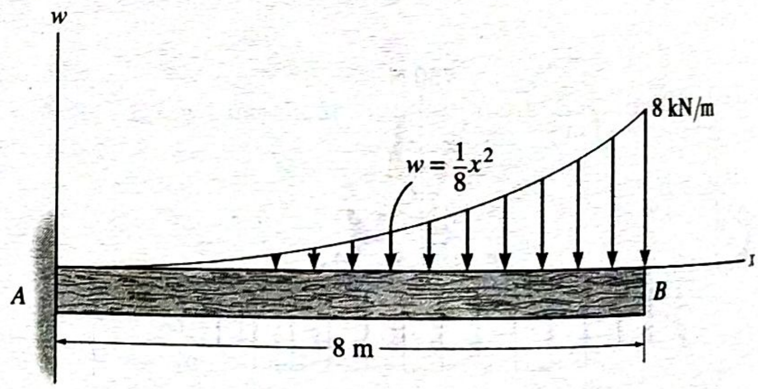
Prob. 6–39

***6-40.** Draw the shear and moment diagrams for the beam.



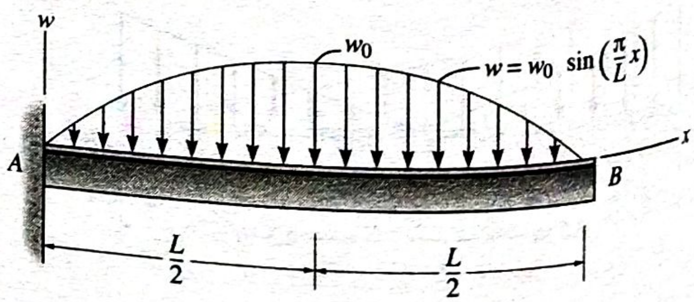
Prob. 6–40

6-41. Draw the shear and moment diagrams for the beam.



Prob. 6–41

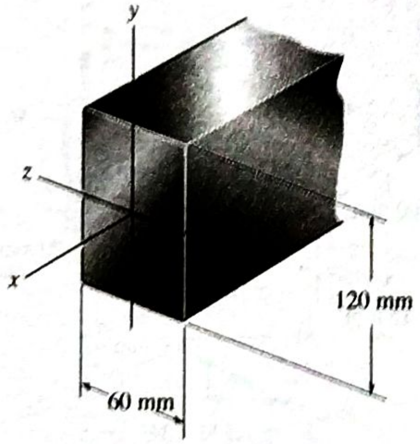
6-42. Draw the shear and moment diagrams for the beam.



Prob. 6–42

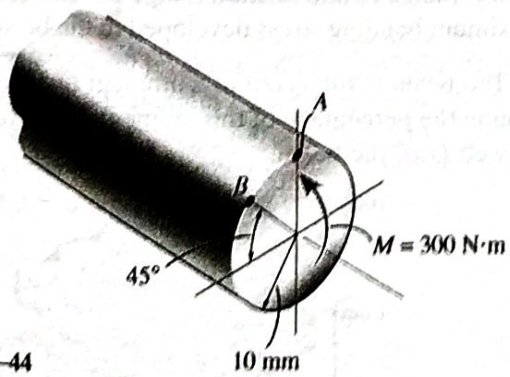
PROBLEMS

6-43. A member having the dimensions shown is to be used to resist an internal bending moment of $M = 2$ kN · m. Determine the maximum stress in the member if the moment is applied (a) about the z axis, (b) about the y axis. Sketch the stress distribution for each case.



Prob. 6–43

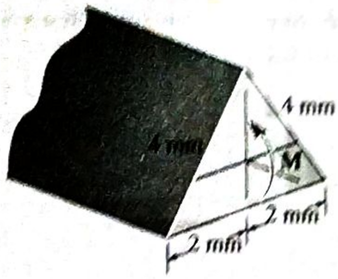
***6-44.** The steel rod having a diameter of 20 mm is subjected to an internal moment of $M = 300$ N · m. Determine the stress created at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



Prob. 6–44

6-45. A member has the triangular cross section shown. Determine the largest internal moment M that can be applied to the cross section without exceeding allowable tensile and compressive stresses of $(\sigma_{allow})_t = 154$ MPa and $(\sigma_{allow})_c = 105$ MPa, respectively.

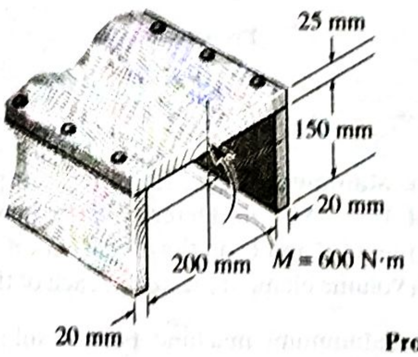
6-46. A member has the triangular cross section shown. If a moment of $M = 80$ N · mm is applied to the cross section, determine the maximum tensile and compressive bending stresses in the member. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



Probs. 6–45/46

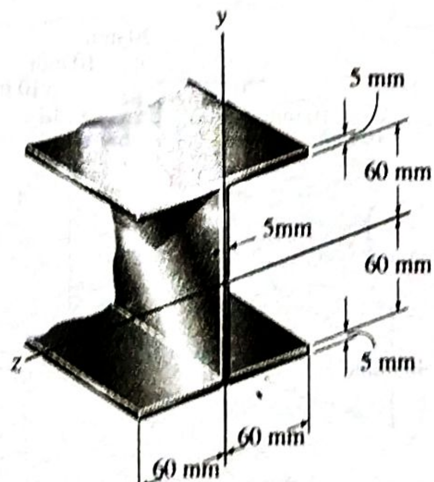
6-47. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600$ N · m, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.

***6-48.** The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600$ N · m, determine the resultant force the bending stress produces on the top board.



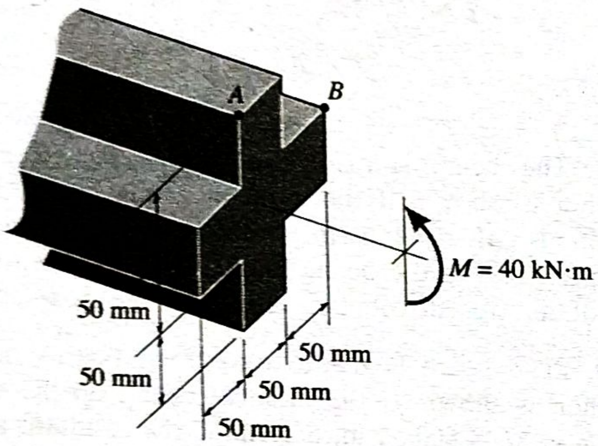
Probs. 6–47/48

6-49. A beam has the cross section shown. If it is made of steel that has an allowable stress of $\sigma_{allow} = 170$ MPa, determine the largest internal moment the beam can resist if the moment is applied (a) about the z axis, (b) about the y axis.



Prob. 6–49

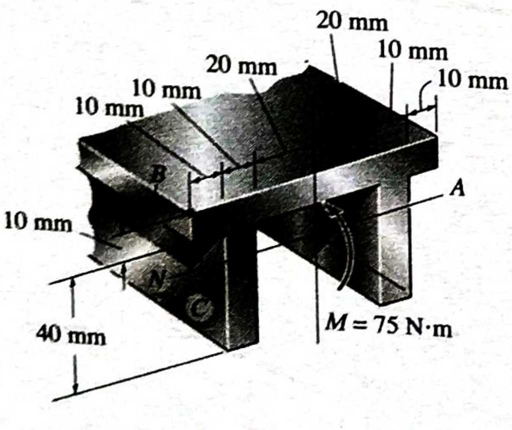
6-50. The beam is subjected to a moment of $M = 40$ kN·m. Determine the bending stress acting at points A and B. Sketch the results on a volume element acting at each of these points.



Prob. 6–50

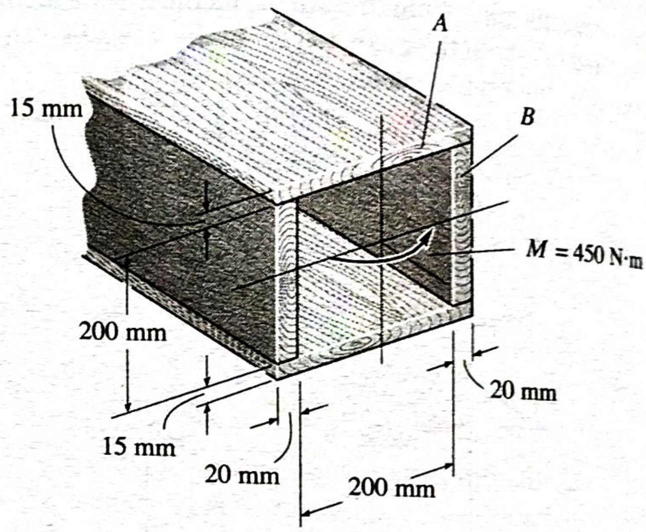
6-51. The aluminum machine part is subjected to a moment of $M = 75$ N·m. Determine the bending stress created at points B and C on the cross section. Sketch the results on a volume element located at each of these points.

***6-52.** The aluminum machine part is subjected to a moment of $M = 75$ N·m. Determine the maximum tensile and compressive bending stresses in the part.



Probs. 6–51/52

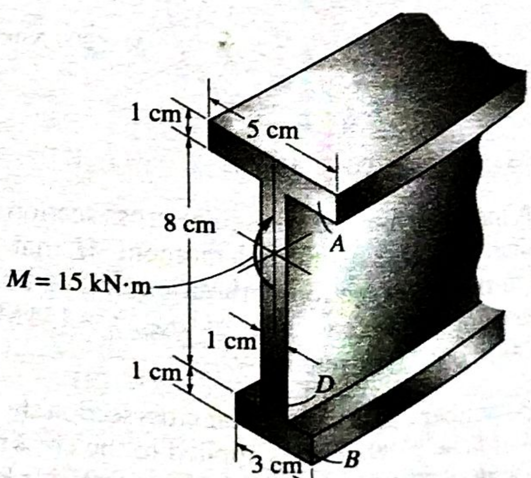
6-53. A beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross section is $M = 450$ N·m, determine the resultant force the bending stress produces on the top board A and on the side board B.



Prob. 6–53

6-54. The beam is subjected to a moment of 15 kN·m. Determine the resultant force the bending stress produces on the top flange A and bottom flange B. Also compute the maximum bending stress developed in the beam.

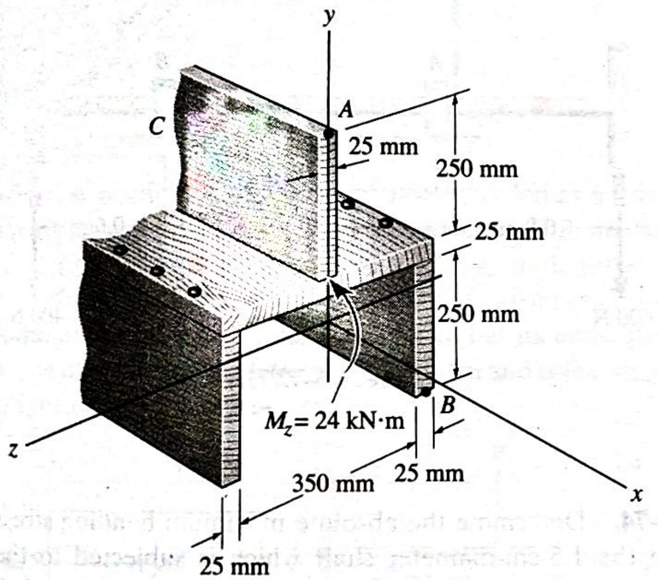
6-55. The beam is subjected to a moment of 15 kN·m. Determine the percentage of this moment that is resisted by the web D of the beam.



Probs. 6–54/55

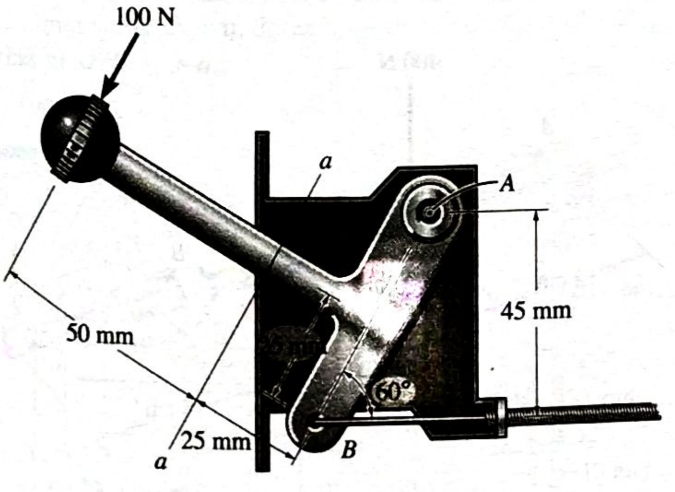
***6-56.** The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 24\ \text{kN}\cdot\text{m}$, determine the stress at points A and B. Sketch a three-dimensional view of the stress distribution.

6-57. The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 24\ \text{kN}\cdot\text{m}$, determine the resultant force the stress produces on the top board C.



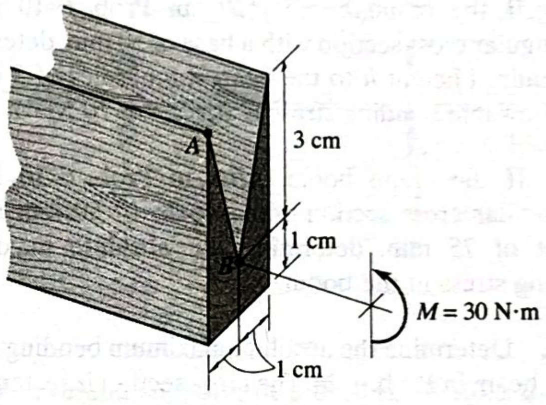
Probs. 6–56/57

6-58. The control level is used on a riding lawn mower. Determine the maximum bending stress in the lever at section $a-a$ if a force of 100 N is applied to the handle. The lever is supported by a pin at A and a wire at B. Section $a-a$ is square, 6 mm by 6 mm.



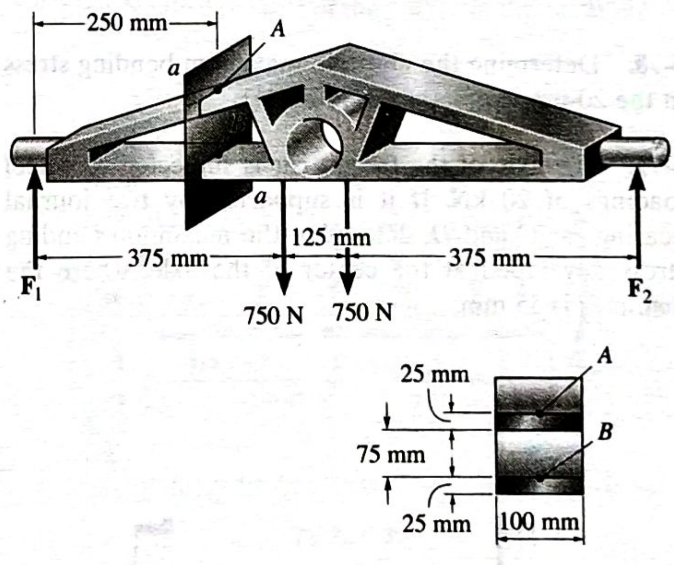
Prob. 6–58

6-59. The beam is subjected to a moment of $M = 30\ \text{N}\cdot\text{m}$. Determine the bending stress acting at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area.



Prob. 6–59

***6-60.** The tapered casting supports the loading shown. Determine the bending stress at points A and B. The cross section at section a–a is given in the figure.



Prob. 6–60

6-61. If the shaft in Prob. 6–1 has a diameter of 100 mm, determine the absolute maximum bending stress in the shaft.

6-62. If the shaft in Prob. 6–3 has a diameter of 40 mm, determine the absolute maximum bending stress in the shaft.

6-63. If the shaft in Prob. 6–6 has a diameter of 50 mm, determine the absolute maximum bending stress in the shaft.

***6-64.** If the pipe in Prob. 6–8 has an outer diameter of 30 mm and thickness of 10 mm, determine the absolute maximum bending stress in the shaft.

6-65. If the beam ACB in Prob. 6–9 has a square cross section, 150 mm by 150 mm, determine the absolute maximum bending stress in the beam.

6-66. If the crane boom ABC in Prob. 6–10 has a rectangular cross section with a base of 50 mm, determine its required height h to the nearest multiples of 5 mm if the allowable bending stress is $\sigma_{\text{allow}} = 170$ MPa.

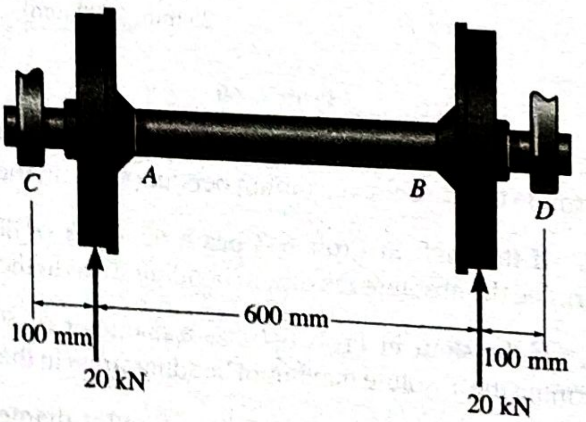
6-67. If the crane boom ABC in Prob. 6–10 has a rectangular cross section with a base of 50 mm and a height of 75 mm, determine the absolute maximum bending stress in the boom.

***6-68.** Determine the absolute maximum bending stress in the beam in Prob. 6–24. The cross section is rectangular with a base of 75 mm and height of 100 mm.

6-69. Determine the absolute maximum bending stress in the beam in Prob. 6–25. Each segment has a rectangular cross section with a base of 100 mm and height of 200 mm.

6-70. Determine the absolute maximum bending stress in the 20-mm-diameter pin in Prob. 6–35.

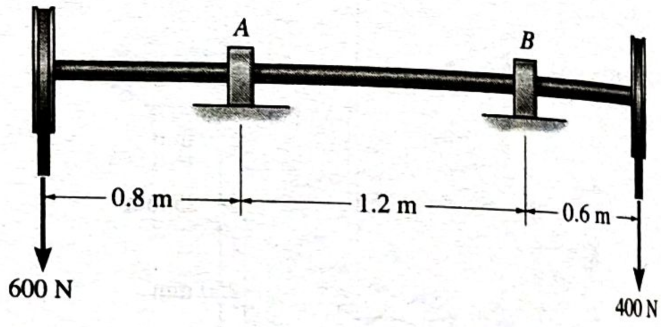
6-71. The axle of the freight car is subjected to wheel loadings of 20 kN. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 55 mm.



Prob. 6–71

***6-72.** Determine the absolute maximum bending stress in the 30-mm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

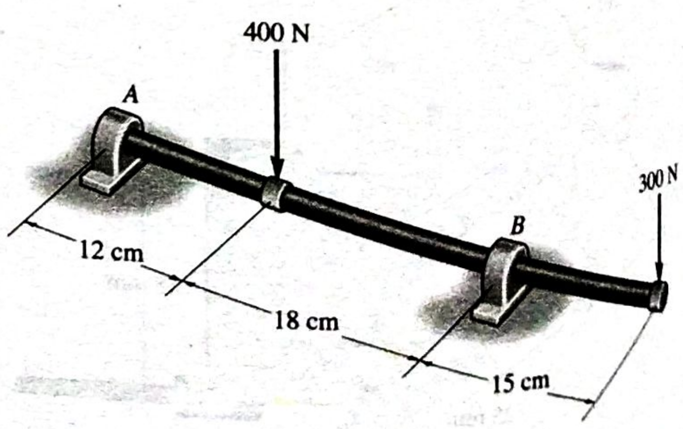
6-73. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{\text{allow}} = 160$ MPa.



Probs. 6–72/73

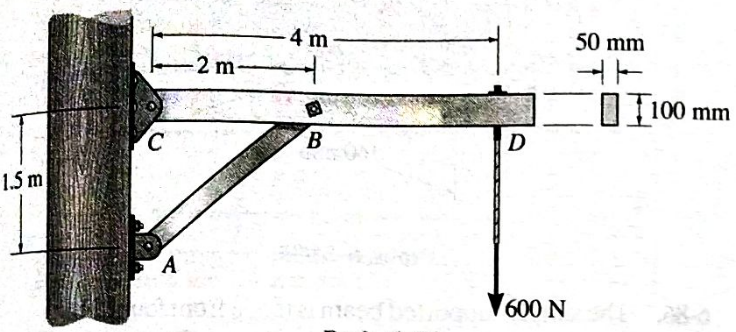
6-74. Determine the absolute maximum bending stress in the 1.5-cm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

6-75. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{\text{allow}} = 154$ MPa.



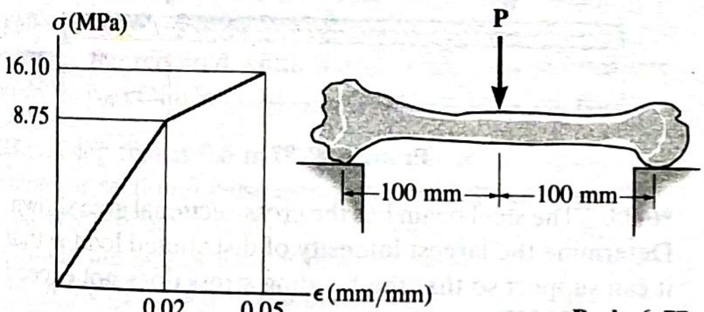
Probs. 6–74/75

***6-76.** The strut CD on the utility pole supports the cable having a weight of 600 N. Determine the absolute maximum bending stress in the strut if A, B, and C are assumed to be pinned.



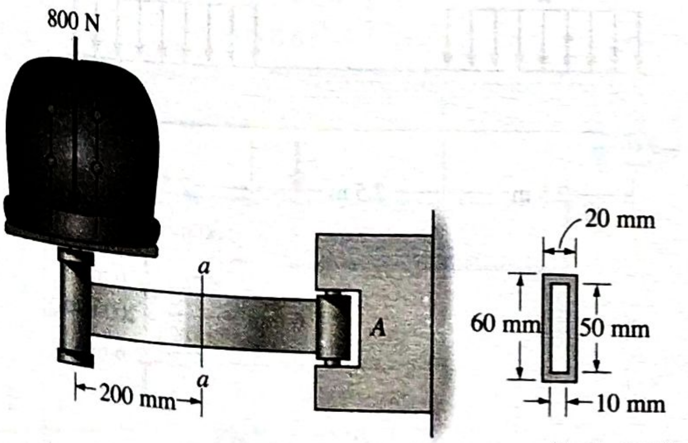
Prob. 6–76

6-77. A portion of the femur can be modeled as a tube having an inner diameter of 10 mm and an outer diameter of 32 mm. Determine the maximum elastic static force P that can be applied to its center without causing failure. Assume the bone to be roller supported at its ends. The $\sigma - \epsilon$ diagram for the bone mass is shown and is the same in tension as in compression.



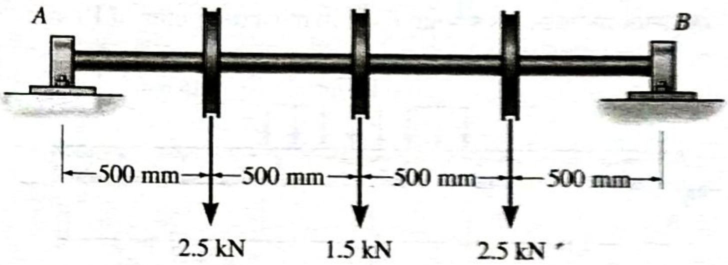
Prob. 6–77

6-78. The chair is supported by an arm that is hinged so it rotates about the vertical axis at A. If the load on the chair is 800 N and the arm is a hollow tube section having the dimensions shown, determine the maximum bending stress at section a–a.



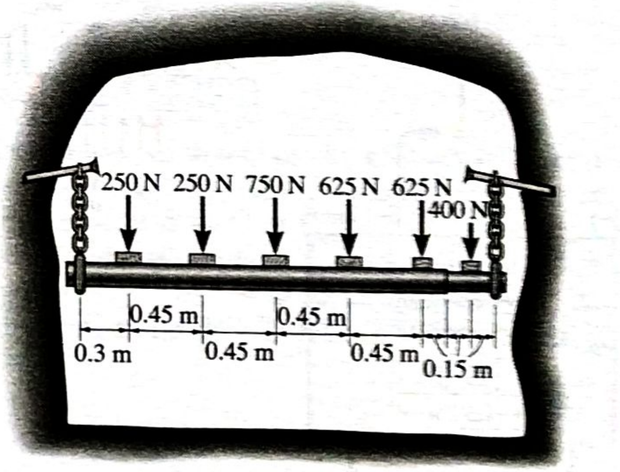
Prob. 6–78

6-79. The steel shaft has a circular cross section with a diameter of 50 mm. It is supported on smooth journal bearings A and B, which exert only vertical reactions on the shaft. Determine the absolute maximum bending stress in the shaft if it is subjected to the pulley loadings shown.



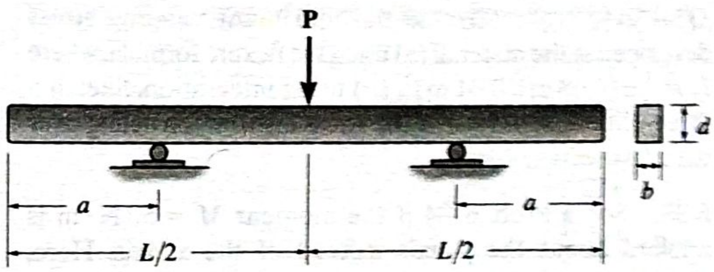
Prob. 6–79

***6-80.** The end supports of a drillers' scaffold used in coal mining consist of a suspended 80-mm-outside-diameter pipe and telescoping 60-mm-outside-diameter pipe having a length of 0.45 m. Each pipe has a thickness of 5 mm. If the end reactions of the supported planks are given, determine the absolute maximum bending stress in each pipe. Neglect the size of the planks in the calculation.



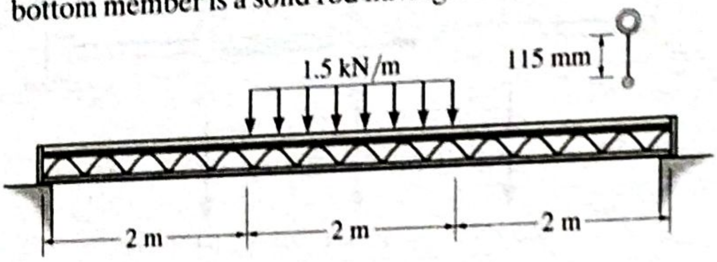
Prob. 6–80

6-81. The beam is subjected to the load **P** at its center. Determine the placement a of the supports so that the absolute maximum bending stress in the beam is as large as possible. What is this stress?



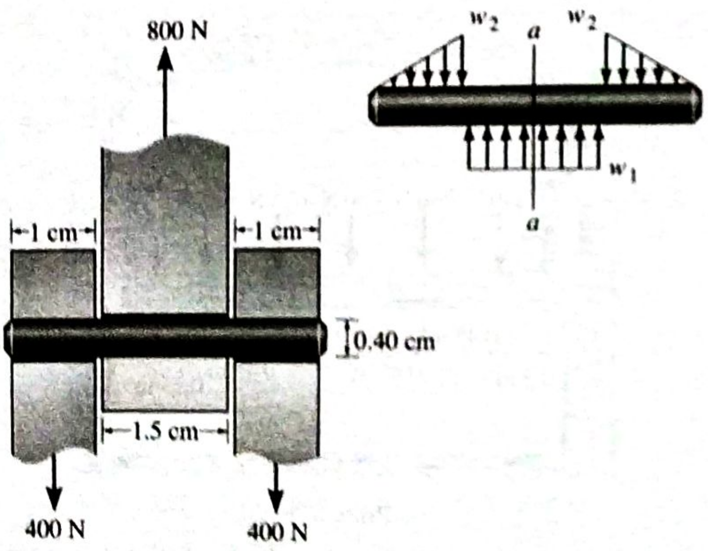
Prob. 6–81

6-82. The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 20 mm and thickness of 4 mm, and the bottom member is a solid rod having a diameter of 10 mm.



Prob. 6–82

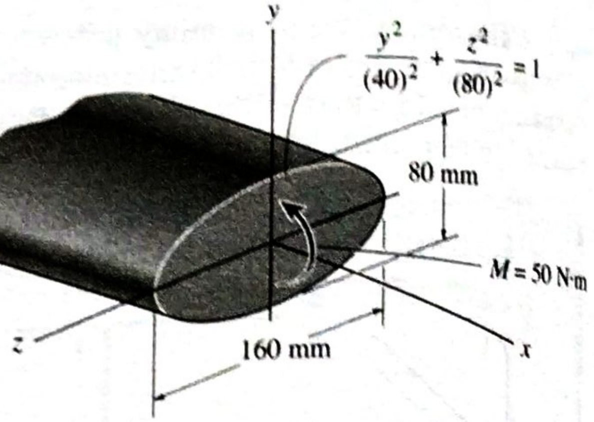
6-83. The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 0.4 cm, determine the maximum bending stress on the cross-sectional area at the center section *a–a*. For the solution it is first necessary to determine the load intensities w_1 and w_2.



Prob. 6–83

***6-84.** A shaft is made of a polymer having an elliptical cross-section. If it resists an internal moment of $M = 50\text{ N}\cdot\text{m}$, determine the maximum bending stress developed in the material (a) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area.

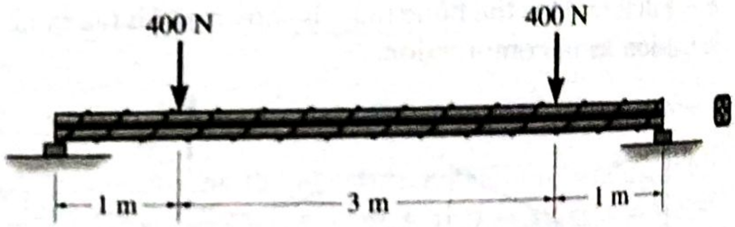
6-85. Solve Prob. 6–84 if the moment $M = 50\text{ N}\cdot\text{m}$ is applied about the y axis instead of the x axis. Here, $I_y = \frac{1}{4}\pi(0.04\text{ m})(0.08\text{ m})^3$.



Probs. 6–84/85

6-86. The simply supported beam is made from four 15-mm-diameter rods, which are bundled as shown. Determine the maximum bending stress in the beam due to the loading shown.

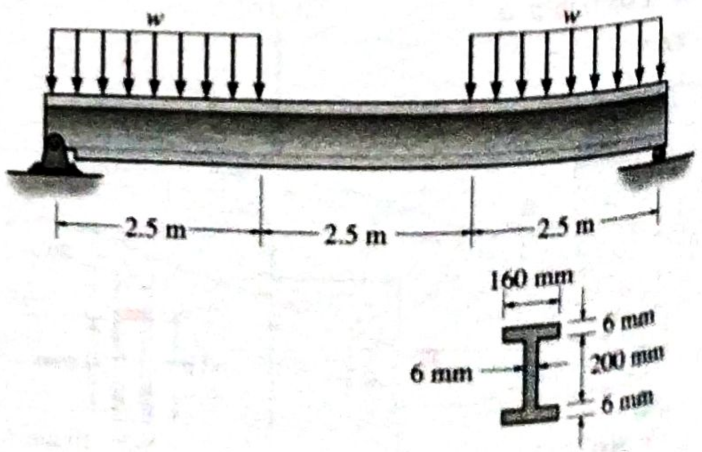
6-87. Solve Prob. 6–86 if the bundle is rotated 45° and set on the supports.



Probs. 6–86/87

***6-88.** The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w that it can support so that the bending stress does not exceed $\sigma_{max} = 154$ MPa.

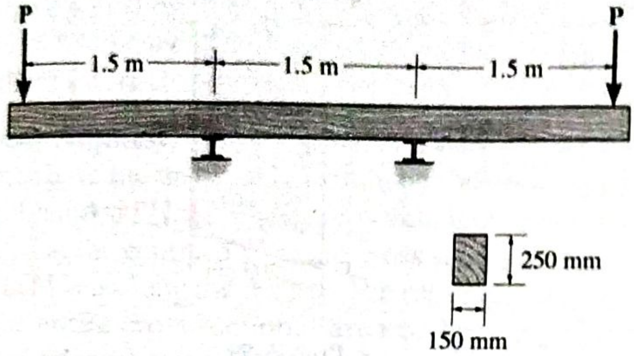
6-89. The steel beam has the cross-sectional area shown. If $w = 7.5$ kN/m, determine the absolute maximum bending stress in the beam.



Probs. 6–88/89

6-90. The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress does not exceed $\sigma_{max} = 10$ MPa.

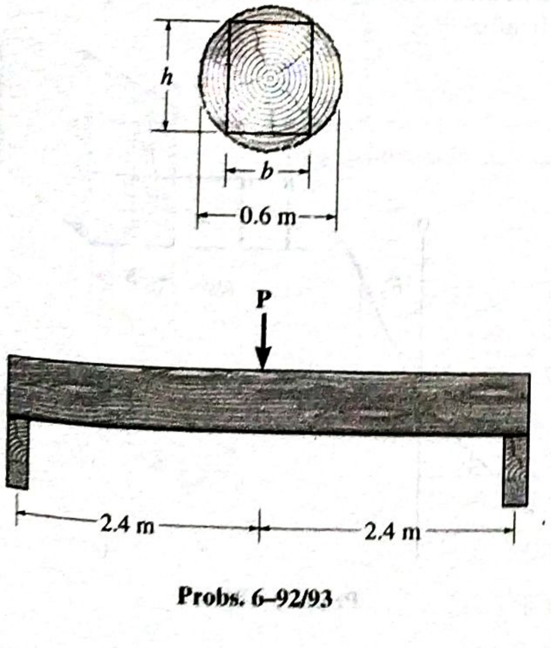
6-91. The beam has the rectangular cross section shown. If $P = 12$ kN, determine the absolute maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.



Probs. 6–90/91

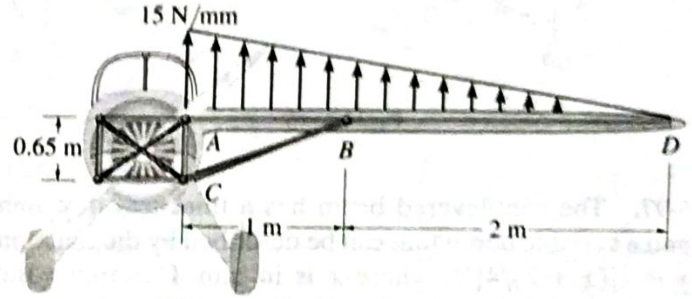
***6-92.** A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa, determine the required width b and height h of the beam that will support the largest load possible. What is this load?

6-93. A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa, determine the largest load P that can be supported if the width of the beam is $b = 200$ mm.



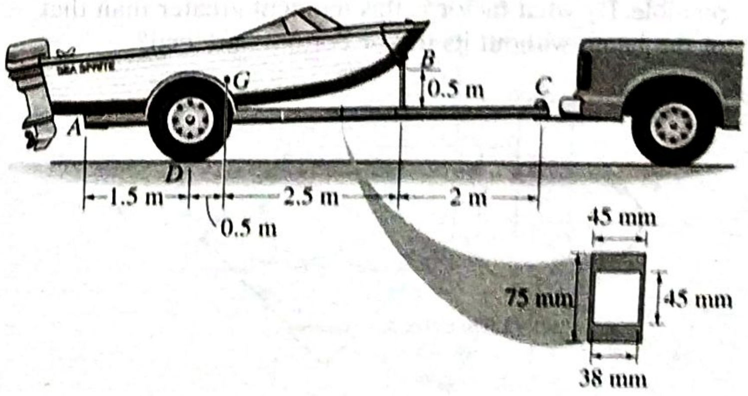
Probs. 6–92/93

6-94. The wing spar ABD of a light plane is made from 2014-T6 aluminum and has a cross-sectional area of 31.75 mm, a depth of 75 mm, and a moment of inertia about its neutral axis of 1.05×10^6 mm^4. Determine the absolute maximum bending stress in the spar if the anticipated loading is to be as shown. Assume A, B, and C are pins. Connection is made along the central longitudinal axis of the spar.



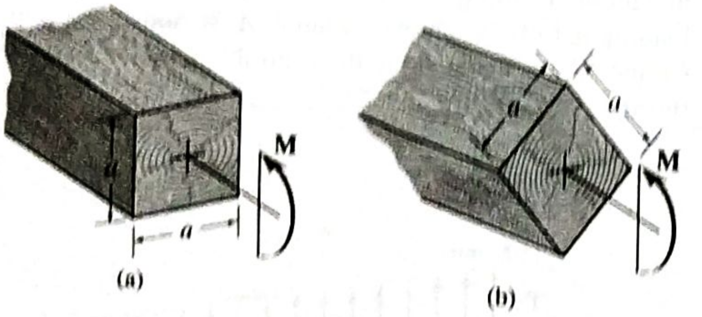
Prob. 6–94

6-95. The boat has a weight of 11.5 kN (~ 1.15 tonnes) and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.



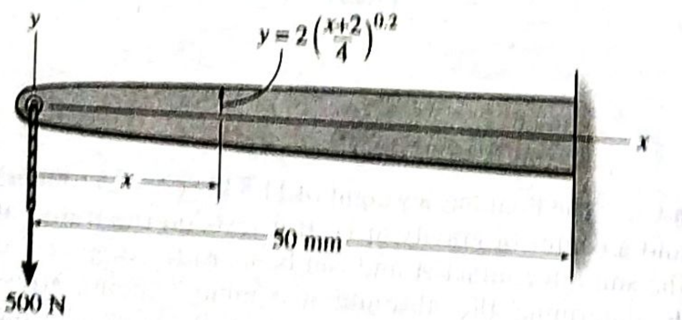
Prob. 6–95

***6-96.** A wooden beam has a square cross section as shown. Determine which orientation of the beam provides the greatest strength at resisting the moment **M**. What is the difference in the resulting maximum stress in both cases?



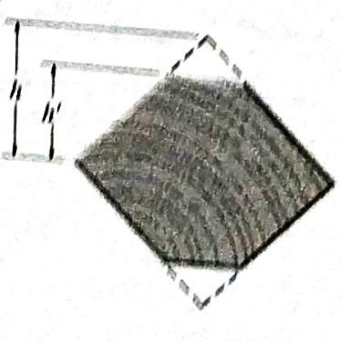
Prob. 6–96

6-97. The cantilevered beam has a thickness of 4 mm and a variable depth that can be described by the function $y = 2[(x + 2)/4]^{0.2}$, where x is in mm. Determine the maximum bending stress in the beam at its center.



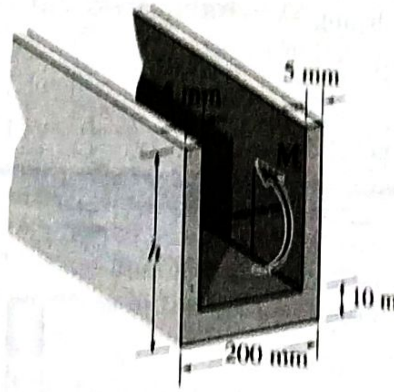
Prob. 6–97

6-98. A timber beam has a cross section which is originally square. If it is oriented as shown, determine the height h' so that it can resist the maximum moment possible. By what factor is this moment greater than that of the beam without its top or bottom flattened?



Prob. 6–98

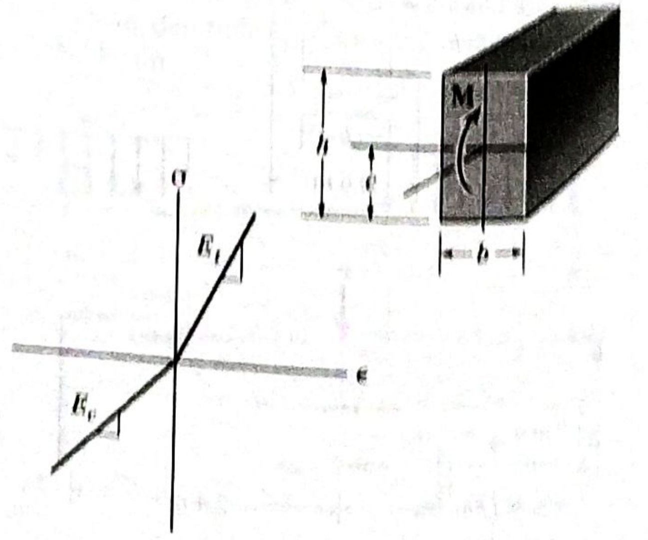
6-99. A beam is to be molded from polyethylene plastic and have the cross section shown. Determine its largest required height so that it supports the greatest moment M. What is this moment? The allowable tensile and compressive stress for the material is $(\sigma_{allow})_t = 70$ MPa and $(\sigma_{allow})_c = 210$ MPa, respectively.



Prob. 6–99

***6-100.** A beam is made of a material that has a modulus of elasticity in compression different from that given for tension. Determine the location c of the neutral axis, and derive an expression for the maximum tensile stress in the beam having the dimensions shown if it is subjected to the bending moment M.

6-101. The beam has a rectangular cross section and is subjected to a bending moment M. If the material from which it is made has a different modulus of elasticity for tension and compression as shown, determine the location c of the neutral axis and the maximum compressive stress in the beam.

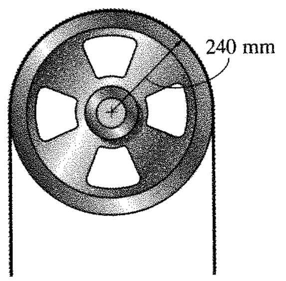


Probs. 6–100/101

PROBLEMS

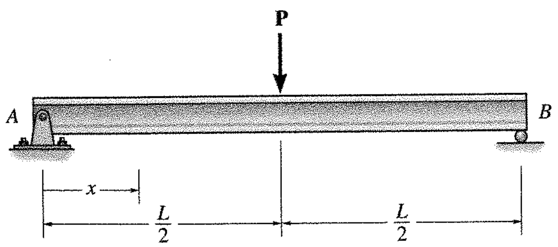
12-1. An L2 steel strap having a thickness of 2.5 mm and a width of 40 mm is bent into a circular arc of radius 12 m. Determine the maximum bending stress in the strap. $E = 200$ GPa.

12-2. The L2 steel blade of the band saw wraps around the pulley having a radius of 240 mm. Determine the maximum normal stress in the blade. The blade is made of steel having a width of 15 mm and a thickness of 1.25 mm. $E = 200$ GPa.



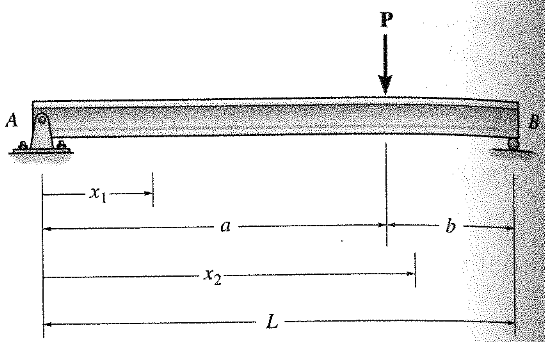
Prob. 12–2

12-3. Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



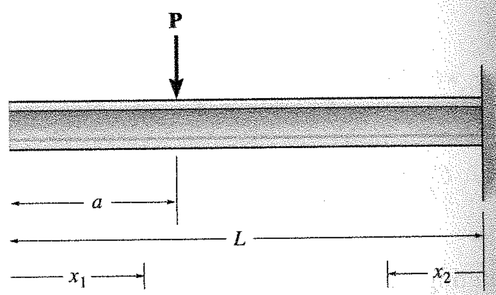
Prob. 12–3

***12-4.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



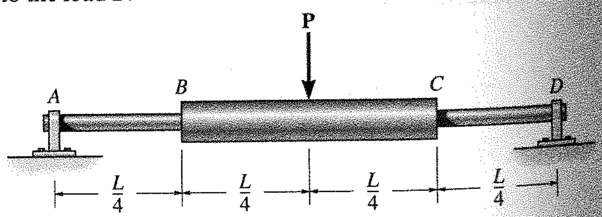
Prob. 12–4

12-5. Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



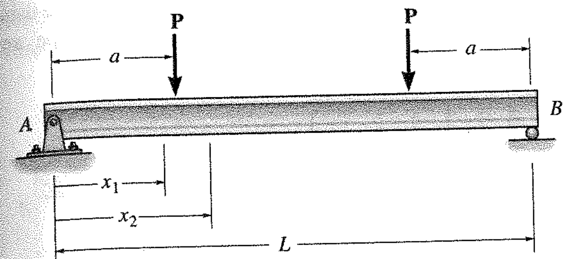
Prob. 12–5

12-6. The simply-supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the beam due to the load **P**.



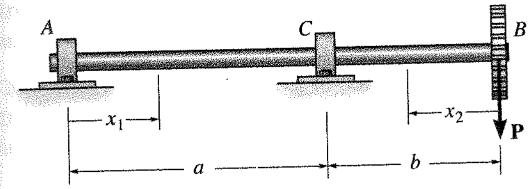
Prob. 12–6

12-7. Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant.



Prob. 12–7

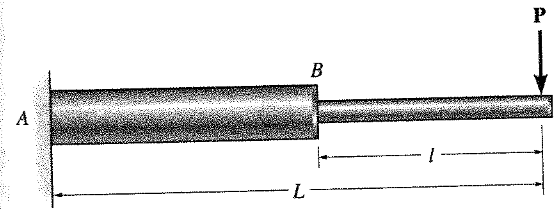
***12-8.** The shaft is supported at A by a journal bearing that exerts only vertical reactions on the shaft, and at C by a thrust bearing that exerts horizontal and vertical reactions on the shaft. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



Prob. 12–8

12-9. The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.

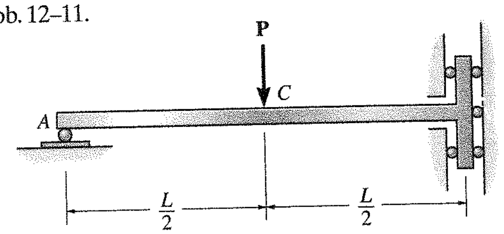
12-10. The beam is made of two rods and is subjected to the concentrated load **P**. Determine the slope at C. The moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



Probs. 12–9/10

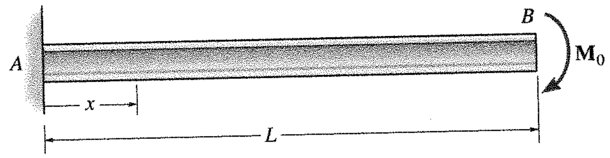
12-11. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.

***12-12.** Determine the deflection at B of the bar in Prob. 12–11.



Probs. 12–11/12

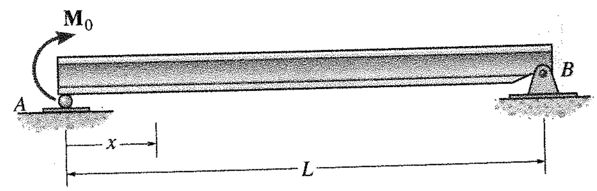
12-13. Determine the elastic curve for the cantilevered beam, which is subjected to the couple moment $\mathbf{M}_0$. Also, compute the maximum slope and maximum deflection of the beam. EI is constant.



Prob. 12–13

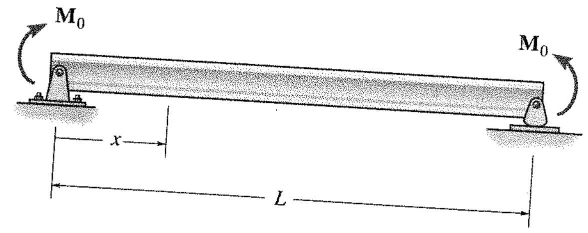
12-14. Determine the equation of the elastic curve for the beam using the x coordinate. Specify the slope at A and the maximum deflection. EI is constant.

12-15. Determine the deflection at the center of the beam and the slope at B. EI is constant.



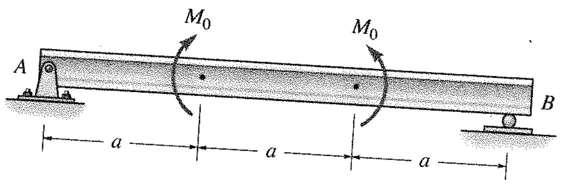
Probs. 12–14/15

***12-16.** Determine the elastic curve for the simply supported beam, which is subjected to the couple moments $\mathbf{M}_0$. Also, compute the maximum slope and the maximum deflection of the beam. EI is constant.



Prob. 12–16

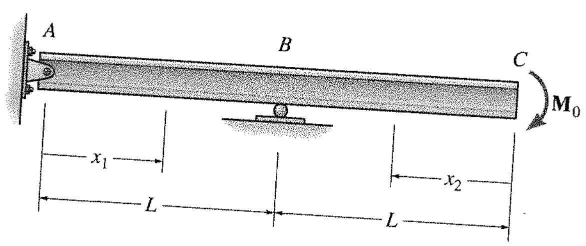
12-17. Determine the maximum deflection of the beam and the slope at A. EI is constant.



Prob. 12–17

12-18. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the deflection and slope at C. EI is constant.

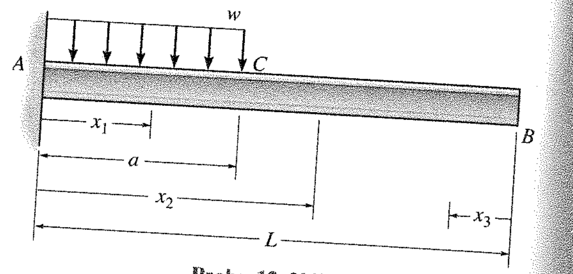
12-19. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at A. EI is constant.



Probs. 12–18/19

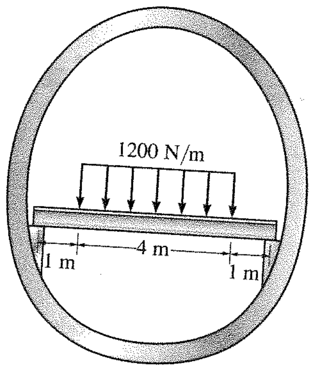
***12-20.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope and deflection at B. EI is constant.

12-21. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope and deflection at point B. EI is constant.



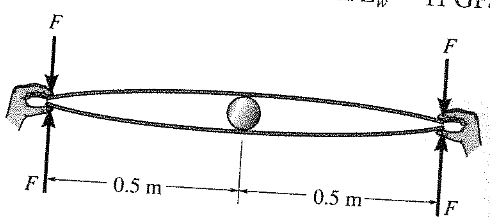
Probs. 12–20/21

12-22. The floor beam of the airplane is subjected to the loading shown. Assuming that the fuselage exerts only vertical reactions on the ends of the beam, determine the maximum deflection of the beam. EI is constant.



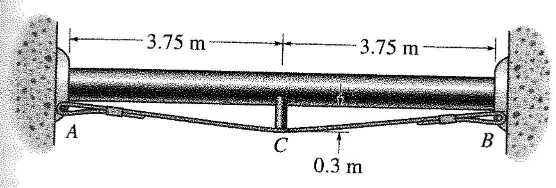
Prob. 12–22

12-23. The two wooden meter sticks are separated at their centers by a smooth rigid cylinder having a diameter of 50 mm. Determine the force F that must be applied at each end in order to just make their ends touch. Each stick has a width of 20 mm and a thickness of 5 mm. $E_w = 11$ GPa.



Prob. 12–23

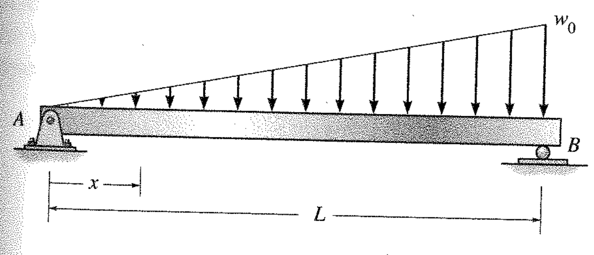
***12-24.** The pipe can be assumed roller supported at its ends and by a rigid saddle C at its center. The saddle rests on a cable that is connected to the supports. Determine the force that should be developed in the cable if the saddle keeps the pipe from sagging or deflecting at its center. The pipe and fluid within it have a combined weight of 1850 N/m. EI is constant.



Prob. 12–24

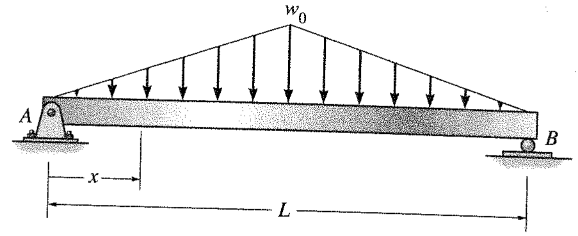
12-25. The beam is subjected to the linearly varying distributed load. Determine the maximum slope of the beam. EI is constant.

12-26. The beam is subjected to the linearly varying distributed load. Determine the maximum deflection of the beam. EI is constant.



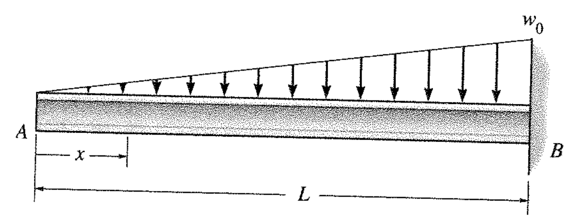
Probs. 12–25/26

12-27. Determine the elastic curve for the simply supported beam using the x coordinate $0 \leq x \leq L/2$. Also, determine the slope at A and the maximum deflection of the beam. EI is constant.



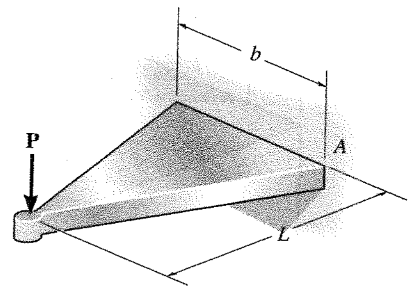
Prob. 12–27

***12-28.** Determine the elastic curve for the cantilevered beam using the x coordinate. Also, determine the maximum slope and maximum deflection. EI is constant.



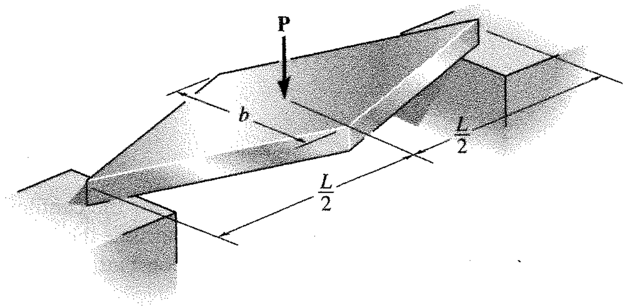
Prob. 12–28

12-29. The tapered beam has a rectangular cross section. Determine the deflection of its end in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_0 of its end.



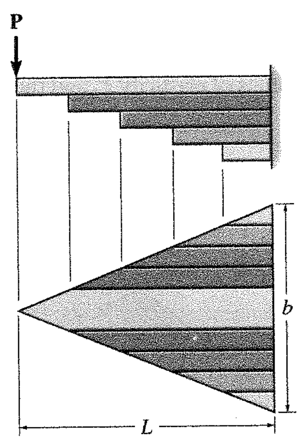
Prob. 12–29

12-30. The tapered beam has a rectangular cross section. Determine the deflection of its center in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_c of its center.



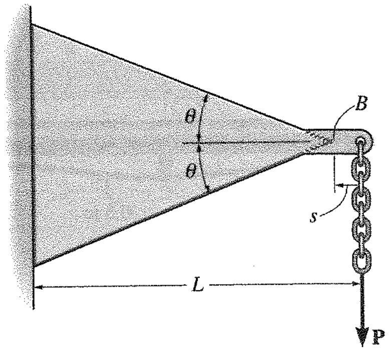
Prob. 12–30

12-31. The beam is made from a plate that has a constant thickness t and a width that varies linearly. The plate is cut into strips to form a series of leaves that are stacked to make a leaf spring consisting of n leaves. Determine the deflection at its end when loaded. Neglect friction between the leaves.



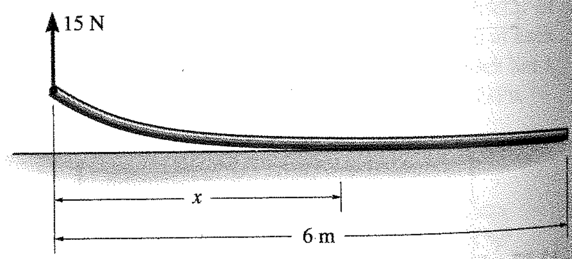
Prob. 12–31

***12-32.** The beam has a constant width b and is tapered as shown. If it supports a load **P** at its end, determine the deflection at B. The load **P** is applied a short distance s from the tapered end B, where $s \ll L$. EI is constant.



Prob. 12–32

12-33. A thin flexible 6-m-long rod having a weight of 7.5 N/m rests on the smooth surface. If a force of 15 N is applied at its end to lift it, determine the suspended length x and the maximum moment developed in the rod.

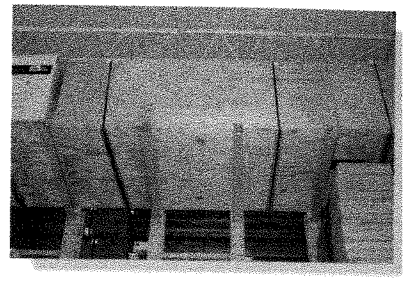


Prob. 12–33

*12.3 Discontinuity Functions

The method of integration, used to find the equation of the elastic curve for a beam or shaft, is convenient if the load or internal moment can be expressed as a continuous function throughout the beam's entire length. If several different loadings act on the beam, however, the method becomes more tedious to apply, because separate loading or moment functions must be written for each region of the beam. Furthermore, integration of these functions requires the evaluation of integration constants using boundary conditions and/or continuity conditions. For example, the beam shown in Fig. 12–14 requires four moment functions to be written. They describe the moment in regions AB, BC, CD, and DE. When applying the moment-curvature relationship, $EI\, d^2v/dx^2 = M$, and integrating each moment equation twice, we must evaluate eight constants of integration. These involve two boundary conditions that require zero displacement at points A and E, and six continuity conditions for both slope and displacement at points B, C, and D.

In this section, we will discuss a method for finding the equation of the elastic curve for a *multiply loaded beam* using a *single expression*, either formulated from the loading on the beam, $w = w(x)$, or the beam's internal moment, $M = M(x)$. If the expression for w is substituted into $EI\, d^4v/dx^4 = -w(x)$ and integrated four times, or if the expression for M is substituted into $EI\, d^2v/dx^2 = M(x)$, and integrated twice, the constants of integration will be determined only from the boundary conditions. Since the continuity equations will not be involved, the analysis will be greatly simplified.



For safety purposes, these cantilevered beams must be designed for both strength and a restricted amount of deflection.

Discontinuity Functions. In order to express the load on the beam or the internal moment within it using a single expression, we will use two types of mathematical operators known as *discontinuity functions*.

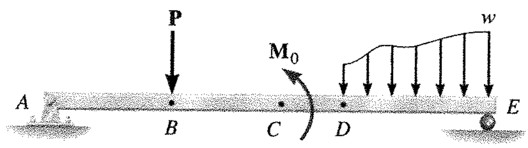


Fig. 12–14

EXAMPLE 12.16

The steel bar shown in Fig. 12–32a is supported by two springs at its ends A and B. Each spring has a stiffness of $k = 45$ kN/m and is originally unstretched. If the bar is loaded with a force of 3 kN at point C, determine the vertical displacement of the force. Neglect the weight of the bar and take $E_{st} = 200$ GPa, $I = 4.6875 \times 10^{-6}$ m^4.

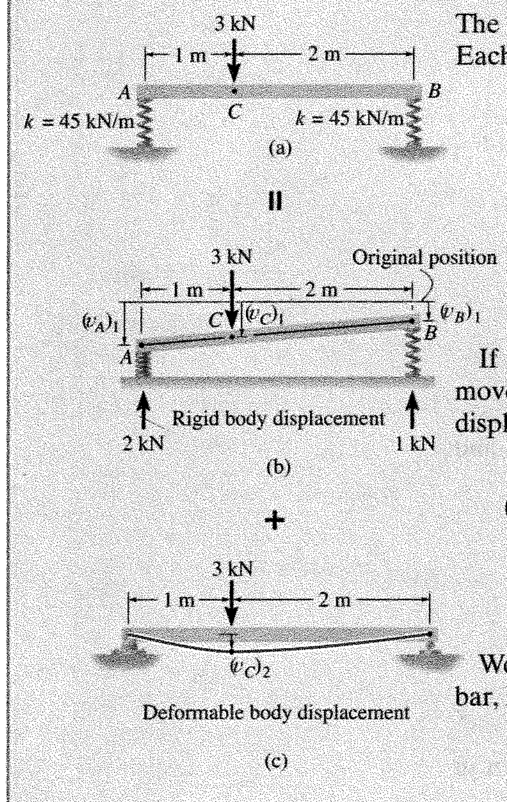


Fig. 12–32

Solution

The end reactions at A and B are computed and shown in Fig. 12–32b. Each spring deflects by an amount

$$(v_A)_1 = \frac{2 \text{ kN}}{45 \text{ kN/m}} = 0.0444 \text{ m}$$

$$(v_B)_1 = \frac{1 \text{ kN}}{45 \text{ kN/m}} = 0.0222 \text{ m}$$

If the bar is considered to be *rigid*, these displacements cause it to move into the position shown in Fig. 12–32b. For this case, the vertical displacement at C is

$$(v_C)_1 = (v_B)_1 + \frac{2 \text{ m}}{3\text{m}}[(v_A)_1 - (v_B)_1]$$

$$= 0.0222 \text{ m} + \frac{2}{3}[0.0444 \text{ m} - 0.0282 \text{ m}] = 0.0370 \text{ m} \downarrow$$

We can find the displacement at C caused by the *deformation* of the bar, Fig. 12–32c, by using the table in Appendix C. We have

$$(v_C)_2 = \frac{Pab}{6EIL}(L^2 - b^2 - a^2)$$

$$= \frac{(3 \text{ kN})(1 \text{ m})(2 \text{ m})[(3 \text{ m})^2 - (2 \text{ m})^2 - (1 \text{ m})^2]}{6(200)(10^6) \text{ kN/m}^2(4.6875)(10^{-6}) \text{ m}^4(3 \text{ m})}$$

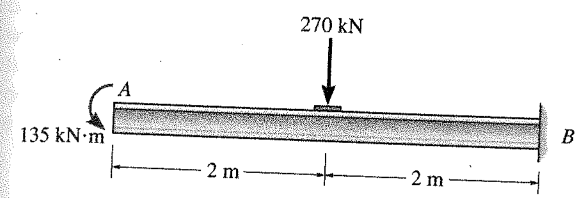
$$= 0.001422 \text{ m} = 1.422 \text{ mm} \downarrow$$

Adding the two displacement components, we get

$$(+\downarrow) \quad v_C = 0.0370 \text{ m} + 0.001422 \text{ m} = 0.0384 \text{ m} = 38.4 \text{ mm} \downarrow \qquad \textit{Ans.}$$

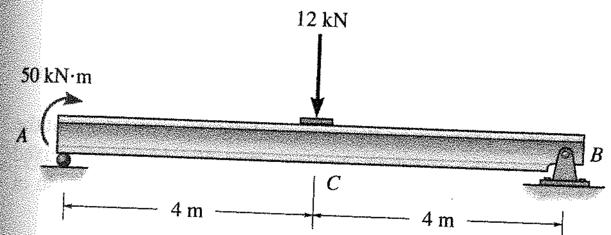
PROBLEMS

12-89. The W200 × 71 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its end A. $E = 200$ GPa.



Prob. 12–89

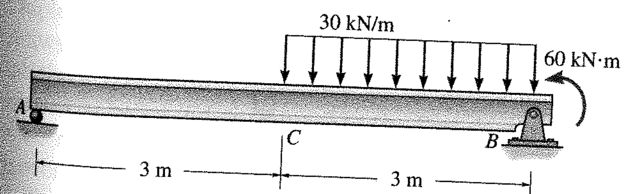
12-90. The W310 × 67 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $E = 210$ GPa.



Prob. 12–90

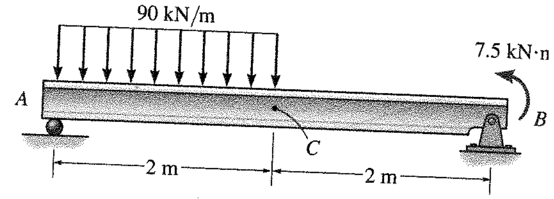
12-91. The W360 × 64 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $E = 200$ GPa.

***12-92.** The W360 × 64 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the slope at A and B. $E = 200$ GPa.



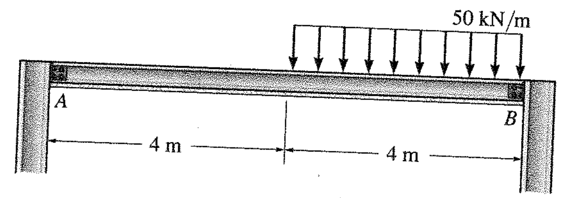
Probs. 12–91/92

12-93. The W200 × 36 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $E = 200$ GPa.



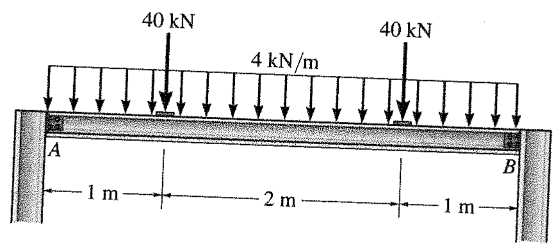
Prob. 12–93

12-94. The beam supports the loading shown. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa. Assume A is a roller and B is a pin. $E = 225$ GPa.



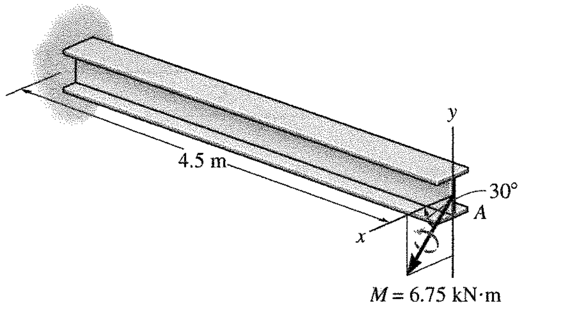
Prob. 12–94

12-95. The simply supported beam carries a uniform load of 40 kN/m. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa. Assume A is a pin and B a roller support. $E = 210$ GPa.



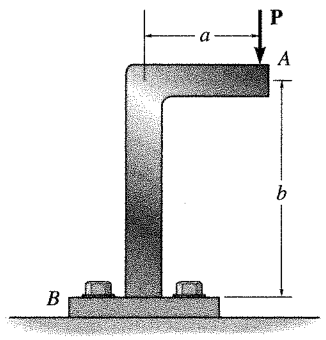
Prob. 12–95

***12-96.** The W250 × 45 cantilevered beam is made of A-36 steel and is subjected to unsymmetrical bending caused by the applied moment. Determine the deflection of the centroid at its end A due to the loading. *Hint:* Resolve the moment into components and use superposition. $E = 200$ GPa.



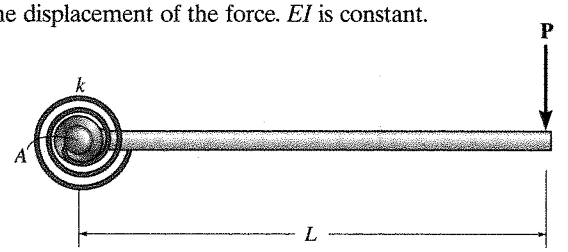
Prob. 12–96

12-97. Determine the vertical deflection at the end A of the bracket. Assume that the bracket is fixed supported at its base B and neglect axial deflection. EI is constant.



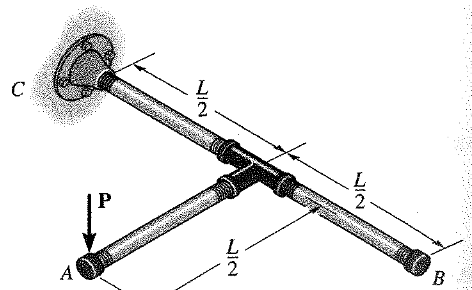
Prob. 12–97

12-98. The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force **P** is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.



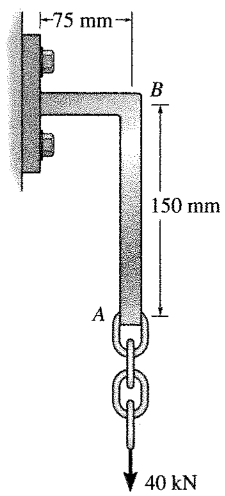
Prob. 12–98

12-99. The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at point A.



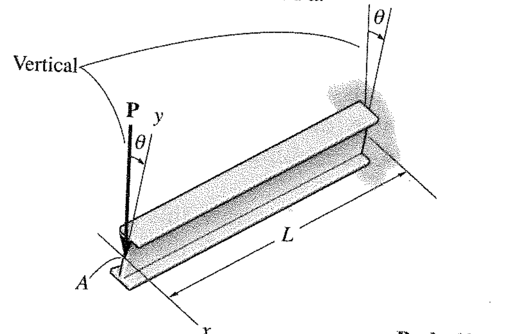
Prob. 12–99

***12-100.** Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.



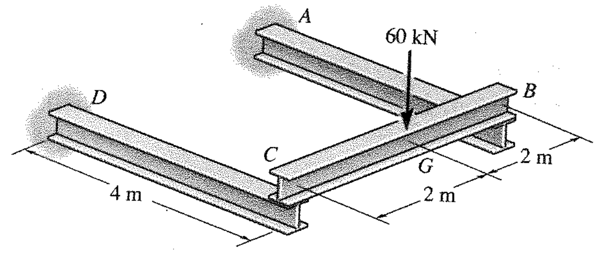
Prob. 12–100

12-101. The wide-flange beam acts as a cantilever. Due to an error, it is installed at an angle θ with the vertical. Determine the ratio of its deflection in the x direction to its deflection in the y direction at A when a load **P** is applied at this point. The moments of inertia are I_x and I_y. For the solution, resolve **P** into components and use the method of superposition. *Note:* The result indicates that large lateral deflections (x direction) can occur in narrow beams, $I_y \ll I_x$, when they are improperly installed in this manner. To show this numerically, compute the deflections in the x and y directions for an A-36 steel W250 × 22, with $P = 7.5$ kN, $\theta = 10°$, and $L = 3.6$ m. $E = 200$ GPa.



Prob. 12–101

12-102. The framework consists of two A-36 steel cantilevered beams CD and BA and a simply supported beam CB. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 46 \times 10^6$ mm^4, determine the deflection at the center G of beam CB. $E = 210$ GPa.



Prob. 12–102

12.6 Statically Indeterminate Beams and Shafts

The analysis of statically indeterminate axially loaded bars and torsionally loaded shafts has been discussed in Secs. 4.4 and 5.5, respectively. In this section, we will illustrate a general method for determining the reactions on statically indeterminate beams and shafts. Specifically, a member of any type is classified as *statically indeterminate* if the number of unknown reactions *exceeds* the available number of equilibrium equations.

The additional support reactions on the beam or shaft that are *not needed* to keep it in stable equilibrium are called *redundants*. The number of these redundants is referred to as the *degree of indeterminacy*. For example, consider the beam shown in Fig. 12–33*a*. If the free-body diagram is drawn, Fig. 12–33*b*, there will be four unknown support reactions, and since three equilibrium equations are available for solution, the beam is classified as being indeterminate to the first degree. Either $\mathbf{A}_y$, $\mathbf{B}_y$, or $\mathbf{M}_A$ can be classified as the redundant, for if any one of these reactions is removed, the beam remains stable and in equilibrium ($\mathbf{A}_x$ cannot be classified as the redundant, for if it were removed, $\Sigma F_x = 0$ would not be satisfied.) In a similar manner, the *continuous beam* in Fig. 12–34*a* is indeterminate to the second degree, since there are five unknown reactions and only three available equilibrium equations, Fig. 12–34*b*. Here the two redundant support reactions can be chosen among $\mathbf{A}_y$, $\mathbf{B}_y$, $\mathbf{C}_y$, and $\mathbf{D}_y$.

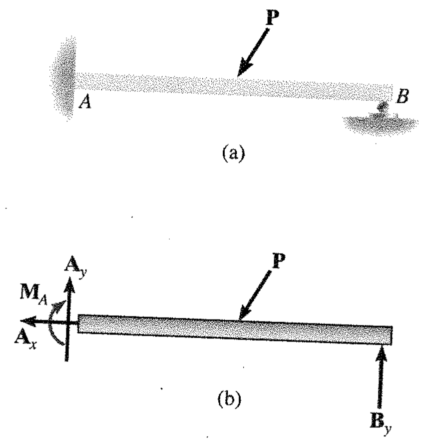


Fig. 12–33