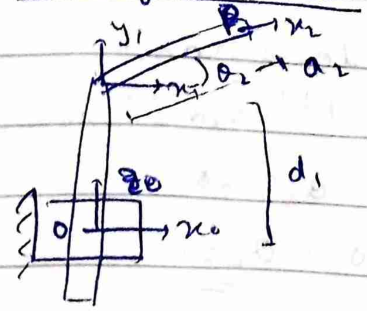
Assignment-4

1.



i) $\vec{OP} = d_1 \hat{z}_0 + a_2 [\cos\theta_2 \hat{x}_0 + \sin\theta_2 \hat{z}_0]$.

$$\Rightarrow \frac{\partial \vec{OP}}{\partial d_1} = \hat{z}_0 \; ; \; \frac{\partial \vec{OP}}{\partial \theta_2} = -a_2 \sin\theta_2 \hat{x}_0 + a_2 \cos\theta_2 \hat{z}_0$$

$$-{}^0(\vec{OP}) = \begin{pmatrix} a_2\cos\theta_2 \\ 0 \\ d_1 + a_2 \sin\theta_2 \end{pmatrix} \; ; \; \frac{\partial \vec{OP}}{\partial (d_1)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \; ; \; \frac{\partial \vec{OP}}{\partial \theta_2} = \begin{pmatrix} -a_2\sin\theta_2 \\ 0 \\ a_2\cos\theta_2 \end{pmatrix}$$

$$\Rightarrow {}^0\vec{v}_p = \underbrace{\begin{pmatrix} 0 & -a_2\sin\theta_2 \\ 0 & 0 \\ 1 & a_2\cos\theta_2 \end{pmatrix}}_{J_v} \begin{pmatrix} \dot{d}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

ii) using D-H parameters:

$$J_v = [\underbrace{z_0}_{\text{Prismatic (1)}} \mid \underbrace{{}^0z_1 \times ({}^0o_2 - {}^0o_1)}_{\text{Revolute (2)}}] \; ; \; z_0 = [0, 0, 1]^T$$

~~Revoluted~~

$${}^0z_1 = [0, -1, 0]^T$$

$${}^0o_2 - {}^0o_1 = [a_2\cos\theta_2, 0, a_2\sin\theta_2]^T$$

$$\Rightarrow {}^0z_1 \times ({}^0o_2 - {}^0o_1) = \begin{pmatrix} i & j & k \\ 0 & -1 & 0 \\ a_2\cos\theta_2 & 0 & a_2\sin\theta_2 \end{pmatrix}$$

$$= \begin{pmatrix} -a_2\sin\theta_2 \\ 0 \\ a_2\cos\theta_2 \end{pmatrix}$$

$$\Rightarrow J_v = \begin{pmatrix} 0 & -a_2\sin\theta_2 \\ 0 & 0 \\ 1 & a_2\cos\theta_2 \end{pmatrix}$$

singular configurations: J_v loses rank.

Looking at 2x2 submatrix: (1st and 3rd rows):

$$\begin{pmatrix} 0 & -a_2 \sin\theta_2 \\ 1 & \cos\theta_2 \cos\theta_2 \end{pmatrix} \Rightarrow \det = a_2 \sin\theta_2$$

$$\det = 0 \Rightarrow \underline{\underline{\theta_2 = 0}}$$

→ Singularity ⇒ $\underline{\theta_2 = 0}$ → both links align and the manipulator behaves like a single link system

Q2. $R = R_x(\psi) R_y(\theta) R_z(\phi)$

$$\dot{R} = \dot{R}_x(\psi) R_y(\theta) R_z(\phi) + R_x(\psi) \dot{R}_y(\theta) R_z(\phi) + R_x(\psi) R_y(\theta) \dot{R}_z(\phi)$$

$$= \cancel{S_x(\dot\psi) R_x(\psi)}$$

$$S_x(\dot\psi) := S\left(\begin{bmatrix} \dot\psi \\ 0 \\ 0 \end{bmatrix}\right),\quad S_y(\dot\theta) := S\left(\begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix}\right),\quad S_z(\dot\phi) := S\left(\begin{bmatrix} 0 \\ 0 \\ \dot\phi \end{bmatrix}\right)$$

$$\Rightarrow \dot{R} = \overbrace{S_x(\dot\psi) R_x(\psi) R_y(\theta) R_z(\phi)}^{A} + \overbrace{R_x(\psi) S_y(\dot\theta) R_y(\theta) R_z(\phi)}^{B} + \overbrace{R_x(\psi) R_y(\theta) S_z(\dot\phi) R_z(\phi)}^{C}$$

$$B = \underbrace{R_x(\psi) S_y(\dot\theta) R_x(\psi)^T}\, R_x(\psi) R_y(\theta) R_z(\phi)$$

$$= \underline{S\left(R_x(\psi)\begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix}\right)} R$$

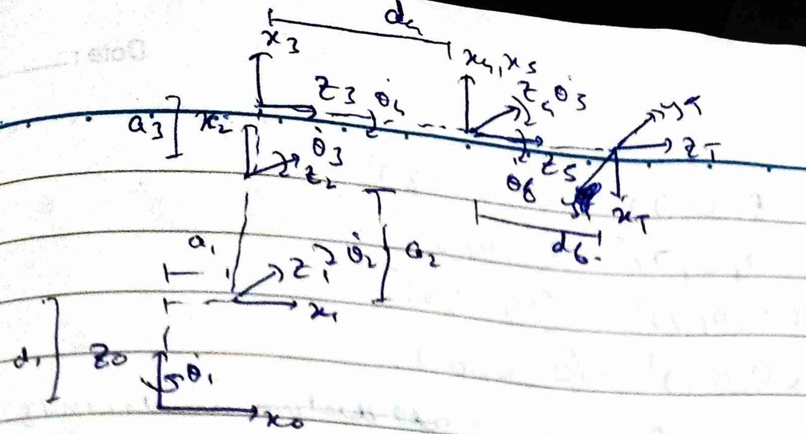
$$C = \cancel{R_x(\theta)}\, \underbrace{R_x(\psi) R_y(\theta) S_z(\dot\phi) (R_x(\psi) R_y(\theta))^T} (R_x(\psi) R_y(\theta)) R_z(\phi)$$

$$= S\left(R_x(\psi) R_y(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot\phi \end{bmatrix}\right) R$$

$$\Rightarrow \dot{R} = \underbrace{S\left(\begin{bmatrix} \dot\psi \\ 0 \\ 0 \end{bmatrix} + R_x \begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix} + R_x R_y \begin{bmatrix} 0 \\ 0 \\ \dot\phi \end{bmatrix}\right)}_{S(\omega)} R$$

$$\text{where } \omega = \begin{pmatrix} \dot\psi \\ 0 \\ 0 \end{pmatrix} + \underbrace{R_x}_{{}^0_1R} \underbrace{\begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix}}_{{}^1\omega} + \underbrace{R_x R_y}_{{}^0_1R\,{}^1_2R = {}^0_2R} \underbrace{\begin{pmatrix} 0 \\ 0 \\ \dot\phi \end{pmatrix}}_{{}^2\omega}$$

Q3



At this instant: contribu^n of each joint angular velocity to end effector linear velocity components:

$\dot\theta_1 \rightarrow \cancel{(d_4+d_6)\dot\theta_1(+\hat{y}_T)}\ \dot\theta_1(a_1+d_4+d_6)(+\hat{y}_T)$

$\dot\theta_2 \rightarrow \dot\theta_2(d_4+d_6)(\hat{x}_T) + \dot\theta_2(a_2+a_3)(\hat{z}_T)$

$\dot\theta_3 \rightarrow \dot\theta_3(d_4+d_6)(\hat{x}_T) + \dot\theta_3(a_3)(\hat{z}_T)$

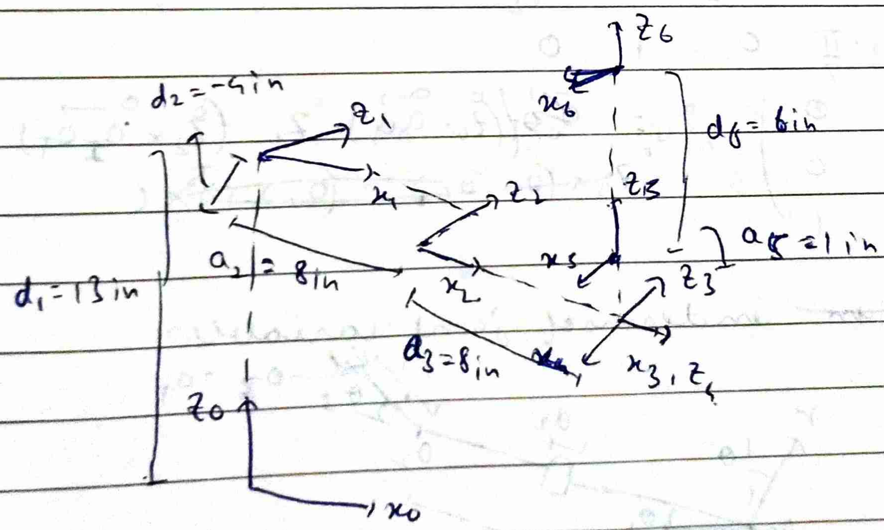
$\dot\theta_4 \rightarrow 0$

$\dot\theta_5 \rightarrow \dot\theta_5 d_6(\hat{x}_T)$

$\theta_6 \rightarrow 0.$

$$\Rightarrow \begin{pmatrix} v_{x,T} \\ v_{y,T} \\ v_{z,T} \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & (d_4+d_6) & (d_4+d_6) & 0 & d_6 & 0 \\ (a_1+d_4+d_6) & 0 & 0 & 0 & 0 & 0 \\ 0 & (a_2+a_3) & a_3 & 0 & 0 & 0 \end{bmatrix}}_{= J_v} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix}$$

4.



$$\therefore J_w = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$J_v(1) = 1^{st}$ column $= (0,0,1)^T \times (16,-4,20)^T$
$= (4, 16, 0)^T$

$J_v(2) = (0,1,0)^T \times (16,-4,7)^T = (7,0,-16)^T$

$J_v(3) = (0,1,0)^T \times (8,0,7)^T = (7,0,-8)^T$

$J_v(4) = (0,1,0)^T \times (0,0,7)^T = (7,0,0)^T$

$J_v(5) = (1,0,0)^T \times (0,0,7)^T = (0,-7,0)^T$

$J_v(6) = (0,0,1)^T \times (0,0,6)^T = (0,0,0)^T$

$$\Rightarrow J_v = \begin{pmatrix} 4 & 7 & 7 & 7 & 0 & 0 \\ 16 & 0 & 0 & 0 & -7 & 0 \\ 0 & -16 & -8 & 0 & 0 & 0 \end{pmatrix}$$; ~~all distan~~ units: in/s

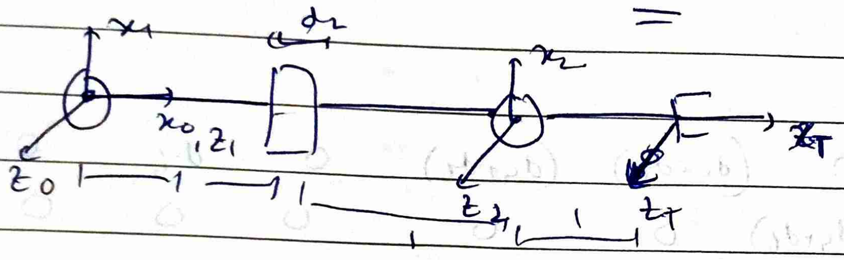
$\Rightarrow$ ω of flange $= J_\omega \cdot [1,1,1,1,1,1]^T$

$= [1,3,2]^T$ rad/s

Linear velocity of screwdriver tip: $J_v \cdot [1,1,1,1,1,1]^T$

$= (25, 9, -24)^T$ in/s

5.



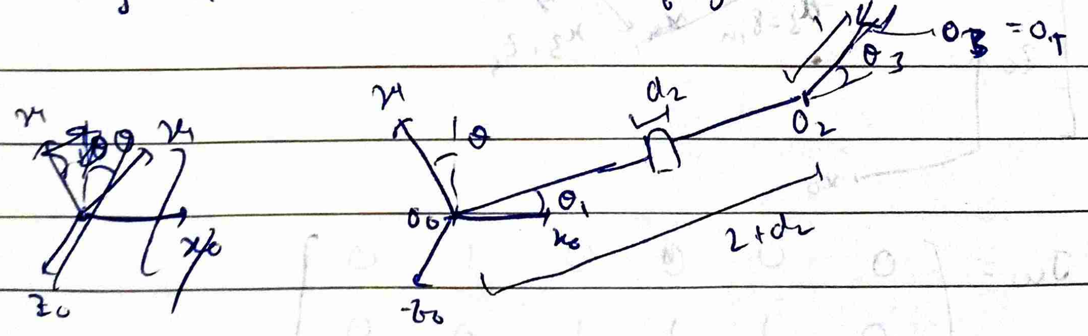
a)

i	θ_i	d_i	a_i	α_i
1	$\theta_1 + \frac{\pi}{2}$	0	0	$\frac{\pi}{2}$
2	0	$2+d_2$	0	$-\pi/2$
3	$\theta_3 - \frac{\pi}{2}$	0	1	0

$$J_\omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad J_v = \left[\cancel{{}^0\vec{O}_0\vec{O}_T}\,({}^0z_0 \times {}^0\vec{O}_0\vec{O}_T) \quad {}^0z_1 \quad ({}^0z_2 \times {}^0\vec{O}_2\vec{O}_T) \right]$$

$\cancel{{}^0z_0 \times ({}^0O_0\,{}^0O_T) = (0,0,1)^T \times (}$

writing ${}^0O_T$ ~~is in~~ in terms of joint variables:



$$\vec{O}_T = (2+d_2)\cos\theta_1\,\hat{x}_0 + (2+d_2)\sin\theta_1\,\hat{y}_0$$

$$+ 1\cdot\cos(\theta_1+\theta_3)\,\hat{x}_0 + 1\cdot\sin(\theta_1+\theta_3)\,\hat{y}_0$$

$$\Rightarrow {}^0\vec{O}_{0T} = \begin{bmatrix} (2+d_2)\cos\theta_1 + \cos(\theta_1+\theta_3) \\ (2+d_2)\sin\theta_1 + \sin(\theta_1+\theta_3) \\ 0 \end{bmatrix}$$

$$= \cancel{{}^{0}z_0 \times {}^{0}\vec{O}_{0T}} \quad {}^{0}z_0 \times ({}^{0}O_T - {}^{0}O_0) = \begin{bmatrix} -(2+d_2)\sin\theta_1 - \cancel{\cos}\sin(\theta_1+\theta_3) \\ (2+d_2)\cos\theta_1 + \cos(\theta_1+\theta_3) \\ 0 \end{bmatrix}$$

$${}^{0}O_T - {}^{0}O_2 = \begin{bmatrix} \cos(\theta_1+\theta_3) \\ \sin(\theta_1+\theta_3) \\ 0 \end{bmatrix}$$

$${}^{0}z_2 \times ({}^{0}O_T - {}^{0}O_2) = \begin{bmatrix} -\sin(\theta_1+\theta_3) \\ \cos(\theta_1+\theta_3) \\ 0 \end{bmatrix}$$

$$\Rightarrow J_v = \begin{bmatrix} -(2+d_2)\overset{\sin\theta_1}{\cancel{\cos\theta}} - \sin(\theta_1+\theta_3) & 0 & -\sin(\theta_1+\theta_3) \\ (2+d_2)\cos\theta_1 + \cos(\theta_1+\theta_3) & 0 & \cos(\theta_1+\theta_3) \\ 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_\omega \\ \hline J_v \end{bmatrix}$$