ME6Φ4 Assignment #4

① i) $\vec{OP} = d_1 \hat{z}_0 + a_2 [\cos\theta_2 \hat{x}_0 + \sin\theta_2 \hat{z}_0]$

$$\frac{\partial \vec{OP}}{\partial d_1} = \hat{z}_0$$

$$\frac{\partial \vec{OP}}{\partial \theta_2} = -a_2[\sin\theta_2 \hat{x}_0 + \cos\theta_2 \hat{z}_0]$$

$$\vec{OP} = \begin{pmatrix} a_2\cos\theta_2 \\ 0 \\ d_1 + a_2\sin\theta_2 \end{pmatrix} \quad \frac{\partial \vec{OP}}{\partial d_1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \frac{\partial \vec{OP}}{\partial \theta_2} = \begin{pmatrix} -a_2\sin\theta_2 \\ 0 \\ a_2\cos\theta_2 \end{pmatrix}$$

$$^0_1v_{op} = \underbrace{\begin{pmatrix} 0 & -a_2\sin\theta_2 \\ 0 & 0 \\ 1 & a_2\cos\theta_2 \end{pmatrix}}_{(J_v)} \begin{pmatrix} \dot{d}_1 \\ \dot{\theta}_2 \end{pmatrix}$$

ii) DH params.

$$J_v = [z_0][{}^0z_1 \times ({}^0O_2 - {}^0O_1)]$$

$[z_0]$ ↑ prismatic, $[{}^0z_1 \times ({}^0O_2 - {}^0O_1)]$ ↓ Revolute

$z_0 = (0;0;1)$

${}^0z_1 = (0;-1;0)$

$${}^0O_2 - {}^0O_1 = \begin{bmatrix} a_2\cos\theta_2 \\ 0 \\ a_2\sin\theta_2 \end{bmatrix}$$

$${}^0z_1 \times ({}^0O_2 - {}^0O_1) = \begin{bmatrix} -a_2\sin\theta_2 \\ 0 \\ a_2\cos\theta_2 \end{bmatrix}$$

$$J_v = \begin{pmatrix} 0 & -a_2\sin\theta_2 \\ 0 & 0 \\ 1 & a_2\cos\theta_2 \end{pmatrix}$$

singular config of robot:

$$\begin{vmatrix} 0 & -a_2\sin\theta_2 \\ 1 & a_2\cos\theta_2 \end{vmatrix} = 0 \Rightarrow a_2\sin\theta_2 = 0$$

$$\Rightarrow \theta_2 = 0, \pi$$

② $R = R_x(\psi) R_y(\theta) R_z(\phi)$

~~$R = R_x$~~

$$S_x(\dot\phi) = S\left(\begin{bmatrix}\dot\psi\\0\\0\end{bmatrix}\right)$$

$$S_y(\dot\theta) = S\left(\begin{bmatrix}0\\\dot\theta\\0\end{bmatrix}\right)$$

$$S_z(\dot\phi) = S\left(\begin{bmatrix}0\\0\\\dot\phi\end{bmatrix}\right)$$

$$\dot R = \underbrace{S_x(\dot\phi) R_x(\phi) R_y(\theta) R_z(\phi)}_{A} + \underbrace{R_x(\phi) S_y(\dot\theta) R_y(\theta) R_z(\phi)}_{B} + \underbrace{R_x(\phi) R_y(\theta) S_z(\dot\phi) R_z(\phi)}_{C}$$

$$B = R_x(\phi) S_y(\dot\theta) R_x(\phi)^T R_x(\phi) R_y(\theta) R_z(\phi)$$

$$= S\left(R_x(\phi)\begin{bmatrix}0\\\dot\theta\\0\end{bmatrix}\right) R$$

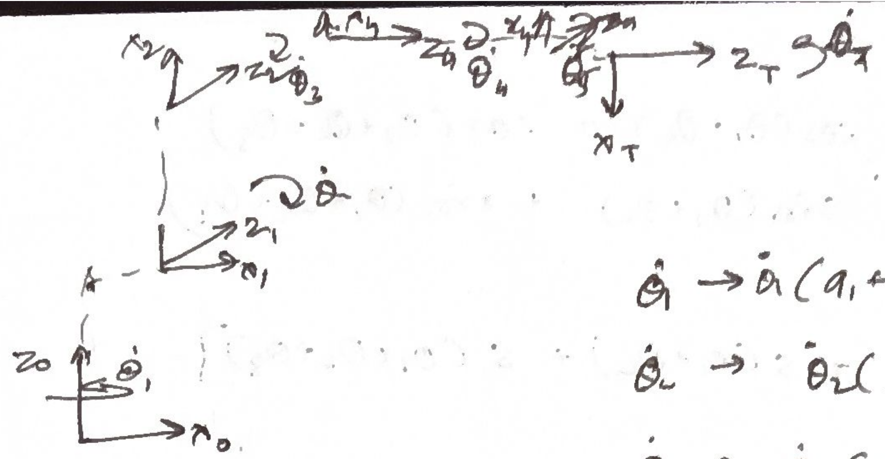
$$C = R_x(\phi) R_y(\theta) S_z(\dot\phi) (R_x(\phi) R_y(\theta))^T (R_x(\phi) R_y(\theta)) R_z(\phi)$$

$$= S\left(R_x(\phi) R_y(\theta)\begin{bmatrix}0\\0\\\dot\phi\end{bmatrix}\right) R$$

$$\Rightarrow \dot R = S\left(\begin{bmatrix}\dot\psi\\0\\0\end{bmatrix}\right) + R_x\begin{bmatrix}0\\\dot\theta\\0\end{bmatrix} + R_x R_y \begin{bmatrix}0\\0\\\dot\phi\end{bmatrix}\Big) R$$

$$= S(\omega)$$

③



$$\dot\theta_1 \to \dot\theta_1(a_1+d_4+d_6)(\hat{y}_5)$$

$$\dot\theta_2 \to \dot\theta_2(d_4+d_6)(\hat{x}_5) + \dot\theta_2(a_2+a_3)(\hat{z}_5)$$

$$\dot\theta_3 \to \dot\theta_3(d_4+d_6)\hat{x}_5 + \dot\theta_3\, a_3\, \hat{z}_5$$

$$\dot\theta_4 \to 0$$

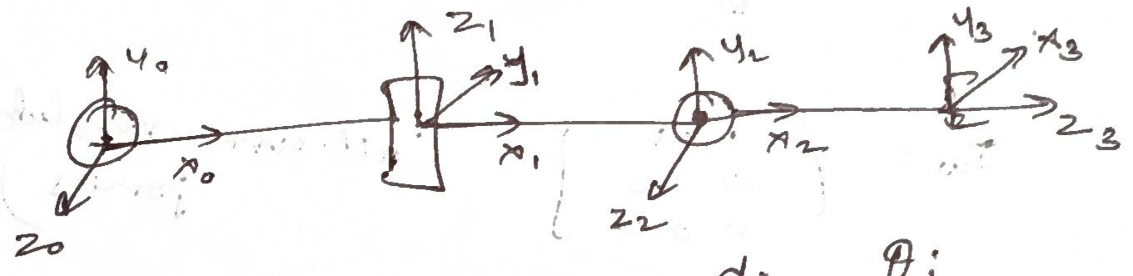
$$\dot\theta_5 \to \dot\theta_5\, d_6(\hat{x}_5)$$

$$\theta_6 \to 0$$

$$\begin{bmatrix} v_{x,T} \\ v_{y,T} \\ v_{z,T} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & d_4+d_6 & d_4+d_6 & 0 & d_6 & 0 \\ a_1+d_4+d_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2+a_3 & a_3 & 0 & 0 & 0 \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix}$$

⑤

a)



i	a_i	α_i	d_i	θ_i
1	1	0	0	θ_1
2	1	0	0	θ_2
3	1	0	0	θ_3

b)

$$T_{i-1}^{i} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & c\theta_i \\ s\theta_i & c\theta_i & 0 & s\theta_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = T_0^1\, T_1^2\, T_2^3$$

d) $x_T = \cos\theta_1 + \cos(\theta_1+\theta_2) + \cos(\theta_1+\theta_2+\theta_3)$

$y_T = \sin\theta_1 + \sin(\theta_1+\theta_2) + \sin(\theta_1+\theta_2+\theta_3)$

$$\frac{\partial x_T}{\partial \theta_1} = -[s\theta_1 + s(\theta_1+\theta_2) + s(\theta_1+\theta_2+\theta_3)]$$

$$\frac{\partial y_T}{\partial \theta_1} = [c\theta_1 + c(\theta_1+\theta_2) + c(\theta_1+\theta_2+\theta_3)]$$

$$\frac{\partial x_T}{\partial \theta_2} = -[0 + s(\theta_1+\theta_2) + \sin(\theta_1+\theta_2+\theta_3)]$$

$$\frac{\partial y_T}{\partial \theta_2} = [c(\theta_1+\theta_2) + c(\theta_1+\theta_2+\theta_3)]$$

$$J_v = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_w = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (all are revolute joints)

$$J = \begin{bmatrix} J_v \\ \hdashline J_w \end{bmatrix}$$

(6)

i	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3

$$x = a_1 c\theta_1 + a_2 c(\theta_1+\theta_2) + a_3 c(\theta_1+\theta_2+\theta_3)$$

$$y = a_1 s\theta_1 + a_2 s(\theta_1+\theta_2) + a_3 s(\theta_1+\theta_2+\theta_3)$$

$$J_v = \begin{bmatrix} \frac{\partial x}{\partial\theta_1} & \frac{\partial x}{\partial\theta_2} & \frac{\partial x}{\partial\theta_3} \\ \frac{\partial y}{\partial\theta_2} & \frac{\partial y}{\partial\theta_2} & \frac{\partial y}{\theta_3} \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad J_\omega = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ \hline J_\omega \end{bmatrix}$$

singular configuration: occurs when Jacobian loses rank

$\therefore$ $|J| = 0$

for $\theta_2 = 0, \pi$

$\theta_3 = 0$

(7)

$$x_e = L\cos\theta_1 + L\cos\theta = 2L\cos\theta \qquad \frac{\partial x_e}{\partial\theta} = -2Ls\theta_1$$

$$y_e = 2L\sin\theta \qquad \frac{\partial y_e}{\partial\theta_1} = 2Lc\theta_1$$

$$J = \begin{bmatrix} -2L\sin\theta \\ 2L\cos\theta \end{bmatrix}$$

DH parameters cannot be used as it is not open chain manipulators with independent joints.