# IE 616: Decision Analysis and Game Theory

## Test 1

Time: 80 minutes           Max. marks 40

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**Exercise 1.** Find the best response function of each player and using that, find all the pure NEs of the game. Explain.          **Marks 2+1**

|   | B | S |
|---|---|---|
| B | 4,3 | 0,0 |
| S | 0,0 | 3,7 |

**Exercise 2.** State true or false for the following statements:          **Marks 4**

(a) Every dominant strategy equilibrium is an NE.

(b) A Pure Strategy NE provides insurance for a player against unilateral deviations by any other player.

(c) Any game with finite players and finite action sets always has a pure strategy NE.

(d) A two player game with each player having two possible strategies can have five pure strategy NEs.

**Exercise 3.** Construct an example of a two-player game with two possible actions, where

(a) no pure strategy NE exists          **Marks 1**

(b) exactly one pure strategy NEs exists          **Marks 1**

(c) exactly two pure strategy NEs exists          **Marks 1**

(d) exactly three pure strategy NEs exists          **Marks 1.5**

(e) exactly four pure strategy NEs exists          **Marks 1.5**

**Exercise 4.** Consider the two-player game given by the following matrix.    **Marks 1+1+1**

(a) Find all the NEs for this game.

(b) Find the only dominant strategy in the game.

(c) There is a weakly dominant strategy for this game. We still have two NEs. Explain!

Player 2

|   |   | L | R |
|---|---|---|---|
| Player 1 | T | 0,0 | 2,1 |
|          | B | 3,2 | 1,2 |

**Exercise 5.** There are two players, each simultaneously make a binary choice whether to contribute to the public good. If one or more players contribute to the public good, then all the players receive a reward 1. Additionally, each player that contributes to the public good incurs a cost of $c > 0$. **Marks 2+1+2+2**

(a) Model the above scenario as a strategic form game. State the players, their actions, and utilities clearly.

(b) Are any strategies weakly dominant for either player? Justify the answer.

(c) Which outcome(s) have the highest total payoff across both players? Provide an intuitive explanation of the answer.

(d) Find all Pure Strategy Nash Equilibrium in this game. Justify the answer.

**Exercise 6.** Consider the following Prisoner's dilemma game. Suppose both the players are rational, but this information about rationality is not common knowledge. Find all the pure strategy NEs of the game. While choosing their equilibrium strategy, do players need to know

|   | C   | D   |
|---|-----|-----|
| C | 4,4 | 0,5 |
| D | 5,0 | 1,1 |

whether the other player is rational or not? Justify your answer. **Marks 3**
Is the same true for the following game? Explain! **Marks 2**

|   | L   | M   | R   |
|---|-----|-----|-----|
| U | 1,0 | 1,2 | 0,1 |
| D | 0,3 | 0,1 | 2,0 |

**Exercise 7.** Suppose you want to purchase a bicycle. Our TAs, Prem and Tushar, each own an identical bicycle and are interested in selling them. They are free to set any price $p$ within the range of 1000 to 5000. You decide to buy from the seller who quotes the lowest price. If both offer the same price, you decide not to buy the cycle. Prem values his cycle to be of value 2000, while Tushar values his cycle to be of value 3000. Note that after selling the cycle, the seller loses the value of the cycle in exchange to the price gained.
**Marks 2+2 = 4**

(a) Model the above scenario as a strategic form game. State the players, their actions, and utilities clearly.

(b) Identify all pure NEs of this game if there exists some. If pure Nash equilibria do not exist, explain why!

**Exercise 8.** Suppose a strategic form game $\langle N, (S_i), (u_i) \rangle$ has a mixed strategy NE $(\sigma_1^*, \cdots, \sigma_n^*)$. Then, show that
$$u_i(\sigma_1^*, \cdots, \sigma_n^*) \geq \bar{v}_i \geq \underline{v}_i \quad \forall i \in N,$$

where $\bar{v}_i$ is the min-max value in mixed strategies of player $i$ and $\underline{v}_i$ is the max-min value in mixed strategies of player $i$ (defined exactly as with pure strategies). **Marks 8**