Week1

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1 Examples

1.1 Duopoly

$$x_1(p_1, p_2) = \begin{cases} 0 & if p_2 < p_1 \\ x(p_1) & if p_1 < p_2 \\ x(p_2)/2 & if p_1 = p_2 \end{cases}$$

$max(p_1 x(p_1, p_2))$ for p_1

1.2 Auctions

values known to only agents and no sharing of information
places bids
highest bid wins

model:
$1, 2, 3, \ldots N$
$v_1, v_2, v_3, \ldots v_N \rightarrow values$
$b_1, b_2, b_3, \ldots b_N \rightarrow bids$

max bidden value wins
$\rightarrow b_1^* = max(b_1, b_2, \ldots b_N)$

but has to pay second max bid
$\hat{b^*} \rightarrow$ second best bid

1.3 NCG

$N = \{1, 2, \ldots, n\} \rightarrow$ set of players
$S = \{S_{i,i \in N}\} \rightarrow$ set of actions
$U = \{U_{i,i \in N}\} \rightarrow$ set of utilities
$U_i(a_i, a_{-i}) \forall a_i \in S_i, a_{-i} \in S_{-i} = S_1 \times S_2 \times \ldots S_{i+1} \ldots$

For 2 players: N= 2

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ x(p_1) & a_1 = 2 \& a_{-1} \in S_{-i} \\ x(p_2)/2 & a_2 = 2 \end{cases}$$

For N players: N= N

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ -\frac{n_2(a_{-i})+1}{N} & a_1 = 2 \& a_{-1} \in S_{-i} \end{cases}$$

1.4 Hotelling Game

$N = \{H_1, H_2\}$
$S_1 = S_2$

People are lazy, will try to go to the nearest hotel

$$U_1(a_1, a_{-1}) = \begin{cases} \frac{a_1 + a_{-1}}{2} & a_1 < a_{-1} \\ 0 & a_1 = a_{-1} \\ L - \frac{a_1 + a_{-1}}{2} & a_1 > a_{-1} \end{cases}$$

- Order of decisions
- Best Response

BR → BEST Response
$BR_i(a_{-i}) = arg \max_{a_i} U_i(a_i, a_{-i})$
Situations where players dont know each others actions
Best strategy is to anticiapate ie assume $a_{-i} = \tilde{a}$
Then, play $BR_i = BR_i(\tilde{a})$

1.5 Free Riding

- Simultaneous move games (assumption)

eg. N people in area, decide whether to participate in a cleaniong activity or not

- n_p participates
- Every person who participates gets a negative utility of $-v$
- Everybody in area gets positive reward of $g(n_p)$

Num of people participated,
$n_p(a_{-i}) = \sum_{j!=i}^{N} 1_{a_j=1} = \sum_{j!=i} a_j$

Utility:
$U_1(a_i, a_{-i}) = -a_i.v + g(n_p(a_{-i}) + a_i)$