

# Recap: 2-way ANOVA



Analyst	Thermometer			
	A	B	C	D
1	2.0	1.0	-0.5	1.5
2	1.0	0.0	-1.0	-1.0
3	1.5	1.0	1.0	0.5



## ANOVA Table

Source	SSQ	DoF	M.S.	F
Analyst	4.17	2	2.09	5.35
Thermometer	4.44	3	1.48	3.79
Grand mean	4.04	1		
Residual	2.35 (By subtraction)	6	0.39	
Total	15.00	12		

- $F_{0.95}(2, 6) = 5.143 < 5.35$
- $F_{0.95}(3, 6) = 4.757 > 3.79$
- We conclude that there are significant differences in analysts, but not in thermometers

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# ANOVA: Interaction Effects



Analyst	Thermometer			
	A	B	C	D
1	2.0	1.0	-0.5	1.5
2	1.0	0.0	-1.0	-1.0
3	1.5	1.0	1.0	0.5



- In this example, it is not unreasonable to expect that an analyst is accustomed to working with a particular type or brand of a thermometer. Therefore, his reading **may be influenced by the thermometer he is using**.
- Hence, there is a possibility that there may be an **interaction effect** between analysts and thermometers.
- The residual sum of squares we calculated could be the sum of squares due to the interaction effect plus the ‘actual’ residual sum of squares.
- In our previous problem there was no reason to suspect interaction between processes and batches. Therefore, the sum of squares due to interaction will be negligible and the residual can be calculated by subtraction.
- However, in this problem, when they are lumped together, it will decrease the F-ratios which are calculated. We might conclude that certain factors aren’t significant when they really are. **What we need then is an estimate of the residual error.**
- How do we do this? **By running more than one test under the same conditions**, we can obtain an estimate of the residual error.

# Interaction Effects



Analyst	Thermometer			
	A	B	C	D
1	2.0	1.0	-0.5	1.5
	1.5	1.5	0.5	1.5
2	1.0	0.0	-0.5	-1.0
	1.0	1.0	0.0	0.0
3	1.5	1.0	1.0	0.5
	1.0	1.5	1.0	1.0



We can find column/row means after finding mean of repeated trials first

Analyst	Thermometer				Mean
	A	B	C	D	
1	2.0 1.5	1.0 1.5	-0.5 0.5	1.5 1.5	1.125
	1.75 1.25	1.25 0.50	0.00 0.00	1.50 1.50	
2	1.0 1.0	0.0 1.0	-0.5 -0.5	-1.0 -0.50	0.125
	1.00 0.50	0.50 0.0	-0.50 -0.50	-0.50 0.0	
3	1.5 1.0	1.0 1.5	1.0 1.0	0.5 0.75	1.063
	1.25 1.25	1.25 1.00	1.00 1.00	0.75 1.0	
Mean	1.333	1.000	0.167	0.583	0.771

# Mathematical Model

$$x_{ijk} = \mu + \tau_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

Observed Value	Grand mean	Effect of factor 1	Effect of factor 2	" Effect of Interaction "	Residual Error
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here,  $i = 1, 2, \dots, a$  ✓  
(number of factor 1) ✓  
 $j = 1, 2, \dots, b$   
(number of factor 2) ✓  
 $k = 1, 2, \dots, r$   
(number of replicates) ✓

$$\underline{x_{ijk}} = \bar{x}_{...} + (\bar{x}_{i.} - \bar{x}_{...}) + (\bar{x}_{.j} - \bar{x}_{...}) + (\bar{x}_{ij.} - \bar{x}_{i.} - \bar{x}_{.j} - \bar{x}_{...})$$

"Interaction term"

$$+ (x_{ijk} - \bar{x}_{...})$$

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## Mean Values

$$\text{Grand Mean} = \bar{x}_{...} = \frac{\sum_i \sum_j \sum_k x}{N} \quad \text{here, } N = abr$$

$$\text{Means of replicates} = \bar{x}_{ij\cdot} = \frac{\sum_k x_{ij}}{r}$$

$$\text{Means of factor 1} = \bar{x}_{i\cdot} = \frac{\sum_j \bar{x}_{ij\cdot}}{b}$$

$$\text{Means of factor 2} = \bar{x}_{\cdot j} = \frac{\sum_i \bar{x}_{ij\cdot}}{a}$$

Analyst	Thermometer					Mean
	A	B	C	D		
1	2.0 1.5	1.0 1.25 1.5	-0.5 0.5 0.00	1.5	1.5 1.50	1.125
2	1.0 1.0	0.0 0.50 1.0	-0.5 -0.50 -0.5	-1.0 -0.50 0.0	-1.0 -0.50	0.125
3	1.5 1.0	1.0 1.25 1.5	1.0 1.00 1.00	0.5 0.75 1.0	0.5 0.75	1.063
Mean	1.333	1.000	0.167	0.583	0.771	

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## Sum of Squares

$$SS_T = SS_m + SS_r + SS_2 + SS_{\text{interaction}}$$

$$+ SS_{\text{Error}}$$

Total  $SS_T = \sum_i \sum_j \sum_k x_{ijk}$

Grand Mean  $SS_m = N \bar{x}_{...}^2 = abr \bar{x}_{...}^2$

found by subtraction

Factor 1  $SS_1 = (rb) \sum_{i=1}^a (\bar{x}_{i.} - \bar{x}_{...})^2$

Factor 2  $SS_2 = (ra) \sum_{j=1}^b (\bar{x}_{.j} - \bar{x}_{...})^2$

Pure error  $SS_E = \sum_i \sum_j \sum_{k=1}^r (x_{ijk} - \bar{x}_{ij.})^2$

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# ANOVA Table

$$SS_{\text{Analyst}} = 2 \times 4 [(1.125 - 0.771)^2 + (0.125 - 0.771)^2 + (1.063 - 0.771)^2] \\ = 5.021$$

$$SS_{\text{Therm}} = 6 [(1.333 - 0.771)^2 + (1.000 - 0.771)^2 + (0.167 - 0.771)^2 + (0.583 - 0.771)^2] \\ = 4.615$$

$$SS_{\text{E}} = [(2.0 - 1.75)^2 + (1.5 - 1.75)^2] + [(1.0 - 1.25)^2 + (1.5 - 1.25)^2] + \dots \\ + [(0.5 - 0.75)^2 + (1.0 - 0.75)^2]$$

Analyst	Thermometer				Mean
	A	B	C	D	
1	2.0 1.5	1.0 1.5	-0.5 0.5	1.5 1.5	1.125
2	1.0 1.0	0.0 0.50	-0.5 -0.50	-1.0 0.0	0.125
3	1.5 1.0	1.0 1.5	1.0 1.0	0.5 1.0	1.063
Mean	1.333	1.000	0.167	0.583	0.771

Source	SSQ	DoF	M.S.	F
Analyst	5.021	3-1=2	2.51	15.06
Thermometer	4.615	4-1=3	1.54	9.24
Interaction	3.354 (By subtraction)	6	0.56	3.354
Grand mean	14.26	1		
Pure Error	2.00	12	0.167	
Total	29.25	24		

- $F_{0.95}(2, 12) = 3.89$
- $F_{0.95}(3, 12) = 3.49$
- $F_{0.95}(6, 12) = 3.00$

## Conclusions

- Significant differences in analysts and thermometers
- Significant interaction effect

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CEP Short Term Course on

## **Statistical Design of Experiments and Data Analysis using Python**

### **Chapter 2.4**

## **Classical Design of Experiments Factorial Design**

# Factorial Design

- When we wish to compare two different techniques, we may either use the independent t-test, or we may use a paired t-test if a nuisance variable is to be blocked away.
- Suppose now that we want to compare several techniques, the independent t-tests would no longer be adequate and **a k-variable** analysis should be used.
- There are many experimental designs available which may be chosen to suit particular experimental situations. But these designs, e.g., k-variable analysis, involve certain assumptions and restrictions.
- We will now introduce a general and effective class of experimental design called the **Factorial Design which includes k-variable analysis**, etc. as a special case.
- **To develop a general factorial design, one would select a fixed number of “levels” for each of a number of variables (factors) and then design tests with all possible combinations.**

For example, if there are ' $L_1$ ' levels of variable 1, ' $L_2$ ' levels of variable 2, and ' $L_3$ ' levels of variable 3, ... and are ' $L_k$ ' levels of variable k; then the total number of observations required will be  $L_1 \times L_2 \times \dots \times L_k$

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# Factorial Designs

- Factorial designs are preferred over other experimental designs because :
  - They require relatively few runs per variable (factor) even though they do not explore the entire region of interest
  - They deal easily with variable interactions
  - They can indicate major trends and help determine a possible direction for further experimentation
  - They can be augmented to form composite designs
  - They form the basis for fractional factorial designs
  - The interpretation of the data produced by the designs can be easily analyzed.
- **Variables and Levels**
  - A design with 3 variables at 2 levels would require 8 tests for a  $2 \times 2 \times 2 = 2^3$  factorial design
  - A design with 3 variables at 3 levels would need 27 tests for a  $3 \times 3 \times 3 = 3^3$  factorial design.
  - A design with 5 variables at 2 levels would need 32 tests for a  $2^5$  factorial design.

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# Factorial Design: Example

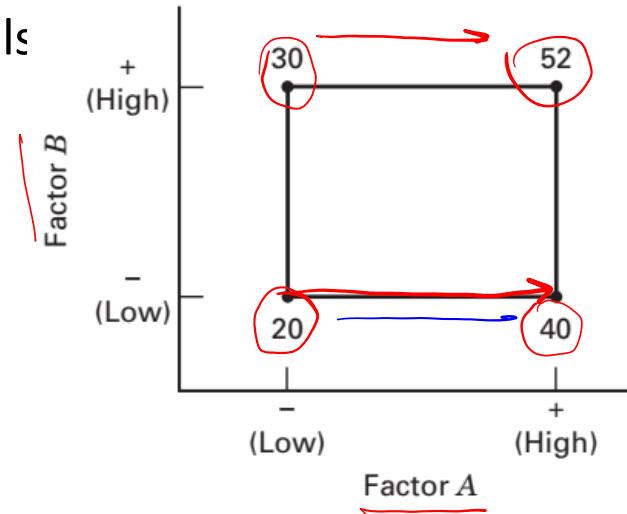
- Consider a factorial experimental design with 2 variables (A and B) each at 2 levels
- Since there are only two levels of each variable, we can call the 'low' and 'high'

What is the 'main effect' of variable A? i.e., how does the result change when you change variable A from low setting to high?

$$\begin{array}{l} \text{factor B is low} \\ \text{---} \\ + 20 \\ \text{factor B is high} \\ \text{---} \\ + 22 \end{array} ] + 2$$

What is the 'main effect' of variable B? i.e., how does the result change when you change variable B from low setting to high?

$$\begin{array}{l} \text{factor A is low} \\ \text{---} \\ + 10 \\ \text{factor A is high} \\ \text{---} \\ + 12 \end{array} ] + 11$$



The effect of a factor is defined to be the change in response produced by a change in the level of the factor. This is frequently called the **main effect** because it refers to the primary factors of interest in the experiment.

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# Interaction Effect

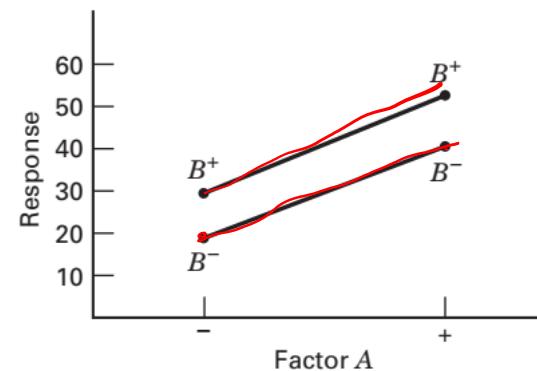
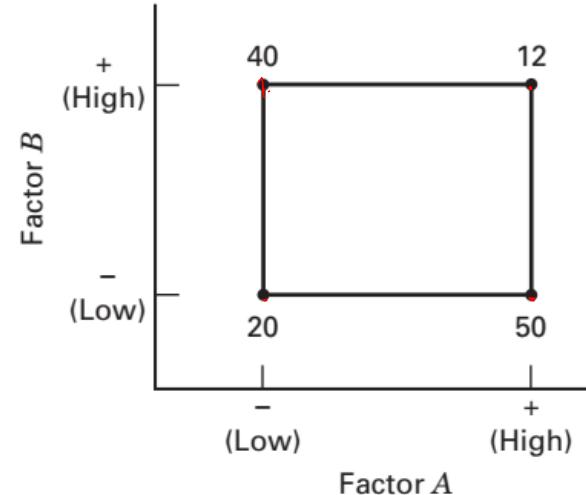
- In some experiments, we may find that the difference in **response between the levels of one factor is not the same at all levels of the other factors**. When this occurs, there is an **interaction between the factors**.

## ● Interaction Effect

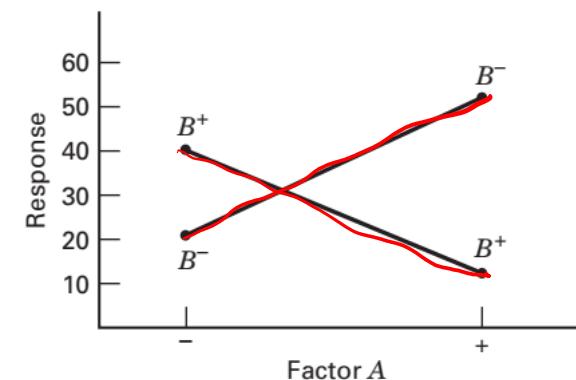
At Low level of B, effect of A is = + 30

At High level of B, effect of A is = - 28

$$\begin{array}{c} +30 \\ \quad \quad \quad ] \\ -28 \end{array} \quad +1$$



■ FIGURE 5.3 A factorial experiment without interaction



■ FIGURE 5.4 A factorial experiment with interaction

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# Example

## Welding of High Strength Steel Bars

- High carbon steel, because of its high strength and low cost, has been known to have a potential for a "good market". However, because of its high carbon content, it is not easy to weld.
- According to the code of the American Welding Society (AWS), additional steps of pre-heating and post-heating are required in order to have good quality welds and high strength steel.
- A user of this steel was interested to study whether or not these additional steps of pre-heating and post-heating were really needed.
- After a preliminary investigation by manual arc welding tests, it appeared that there were **three variables significantly affecting the ultimate tensile stress of a weld.**
  1. Ambient temperature,
  2. Wind velocity
  3. Bar size
- The evidence, however, was not decisive, and further experiments were therefore planned. **How many tests should be conducted?**

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# Example

## Example 2: Welding of High Strength Steel Bars

- Based upon available funds and time limit, it was decided that 244 tests can be run (maximum).
- In the meantime, an engineering statistician, who was called upon for consultation, suggested that 16 tests be run according to his specified sets of test conditions.
- The statistical experimental design that was formulated by the engineering statistician for this study was a two-level three-variable factorial design, simply designated as a  $2^3$  factorial design.
- The three selected variables were:
  - ✓ • Ambient temperature, denoted by T
  - ✓ • Wind velocity, denoted by V
  - ✓ • Bar size, denoted by B
- Two levels ('low' and 'high') were chosen for each variable based upon desired field conditions to be simulated.

$$2 \times 2 \times 2 = 8 = 2^3$$

↑  
repeated 2

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# Example

## Welding of High Strength Steel Bars

- The experimental variables and their two levels (low and high) are shown in the Table.

Variable	Unit	Low	High		
Ambient Temperature ( $T$ )	°F	0	70	+1	+1
Wind Velocity ( $V$ )	mph	0	20	-1	+1
Bar Size ( $B$ )	1/8 inch	4	11	-1	+1

## Transformation of Levels

- In order to adopt a notation which will be the same for all two-level factorial designs, we use transforming equations to code the variables such that
  - the high level will be denoted by +1,
  - the low level will be denoted by -1.
- By so doing, regardless of the physical conditions represented by the two levels, the basic design of any two-level factorial design becomes a simple arrangement of +1 and -1.

$$x_T = \frac{T - T_{avg}}{T_{avg}}$$

$$x_T = \frac{T - T_a}{T_a}$$

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# Example

- Construction of  $2^k$  Factorial Design

<b>Test #</b>	<b>Coded Test Conditions</b>			<b>Actual Test Conditions</b>		
	X1	X2	X3	°F	(mph)	(1/8 in)
1	-1	-1	-1	0	0	4
2	1	-1	-1	70	0	4
3	-1	1	-1	0	20	4
4	1	1	-1	70	20	4
5	-1	-1	1	0	0	11
6	1	-1	1	70	0	11
7	-1	1	1	0	20	11
8	1	1	1	70	20	11

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# Example

Results of the factorial experiment with 2 repetitions (total  $8*2 = 16$  trials)

Test #	X1	X2	X3	Test Order	Y <sub>ai</sub> (kpsi)	Test Order	Y <sub>bi</sub> (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

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# Example

Test #	X1	X2	X3	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

$$\Delta Y_{1 \rightarrow 2} = 87.3 - 87.5 = -0.2$$

$$\Delta Y_{3 \rightarrow 4} = 9.2$$

$$\Delta Y_{5 \rightarrow 6} =$$

$$\Delta Y_{7 \rightarrow 8} =$$

any

## Average effect of Temperature (E1)

- Note that between Test 1 and Test 2, Wind (X2) and Bar Size (X3) is same. The only change is that Temperature (X1) goes from LOW (-1) to HIGH (+1). Therefore, the difference in these two test results (apart from the intrinsic variation that is present) can be attributed solely to the effect of ambient temperature alone.
- Similarly, for the pairs of Test 3 and 4, 5 and 6, and 7 and 8 in Table; each pair involved similar test conditions with respect to wind velocity and bar size, but different test conditions with respect to ambient temperature. Thus, the differences in the results within each of these four pairs reflect the effect of ambient temperature alone.

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# Example

Test #	X1	X2	X3	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

$$E_1 = \frac{-0.2 + 0.2 + 18.5 + 9.1}{24}$$

$$(Test\ Nos.\ 1\ and\ 2)\ \bar{y}_2 - \bar{y}_1 = 87.3 - 87.5 = -0.2$$

$$(Test\ Nos.\ 3\ and\ 4)\ \bar{y}_4 - \bar{y}_3 = 87.0 - 77.8 = 9.2$$

$$(Test\ Nos.\ 5\ and\ 6)\ \bar{y}_6 - \bar{y}_5 = 97.6 - 79.1 = 18.5$$

$$(Test\ Nos.\ 7\ and\ 8)\ \bar{y}_8 - \bar{y}_7 = 87.7 - 78.6 = 9.1.$$

Average Effect of  
Temperature would be an  
average of these FOUR  
numbers

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# Main Effect

Test #	X1	X2	X3	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84 ✓	3	91 ✓	87.5 ✓
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

## Average effect of Temperature (E1)

- OR we can also write the average effect of ambient temperature (called 'E1') as:

$$\begin{aligned}
 E_1 &= 1/4[(\bar{y}_2 - \bar{y}_1) + (\bar{y}_4 - \bar{y}_3) + (\bar{y}_6 - \bar{y}_5) + (\bar{y}_8 - \bar{y}_7)] \\
 &= 1/4[-0.2 + 9.2 + 18.5 + 9.1] \\
 &= 9.15 \text{ units of 1000 psi} \\
 &= 9150 \text{ psi. ✓}
 \end{aligned}$$

- Note that the average effect is commonly referred to as main effect.

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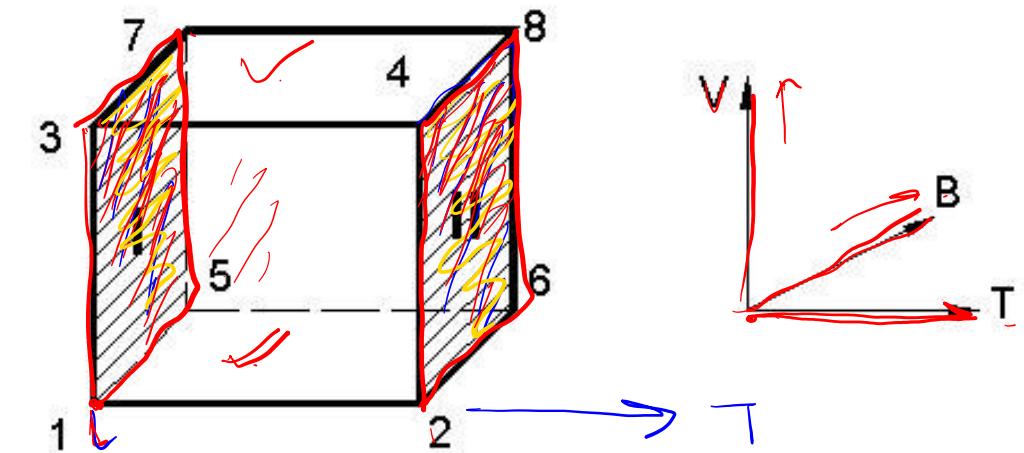
$$E_1 = 1/4 \left( \frac{y_2 + y_4 + y_6 + y_8 - y_1 - y_3 - y_5 - y_7}{4} \right)$$

# Main Effect

## Average effect of Temperature (E1)

- Geometrically, the average effect of Temperature is the difference between plane 1 (Temp setting LOW) and plane 2 (Temp setting HIGH)

Test #	X1	X2	X3	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7



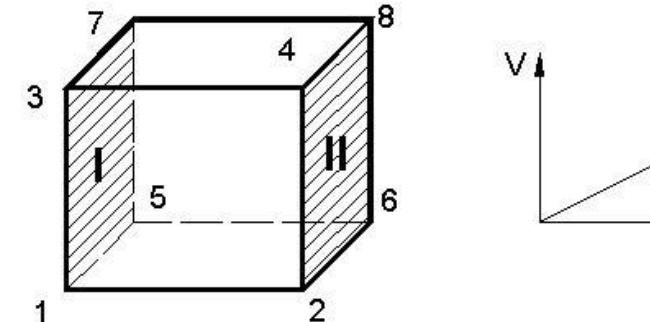
$$E_1 = \frac{1}{4} [(\bar{y}_2 + \bar{y}_4 + \bar{y}_6 + \bar{y}_8) - \underbrace{(\bar{y}_1 + \bar{y}_3 + \bar{y}_5 + \bar{y}_7)}_{\text{high level of ambient temperature}}]$$

high level of ambient  
temperature      Low level of ambient  
temperature

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# Main Effect

<b>Test #</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Test Order</b>	<b>Y<sub>ai</sub> (kpsi)</b>	<b>Test Order</b>	<b>Y<sub>bi</sub> (kpsi)</b>	<b>Average (kpsi)</b>
<b>1</b>	-1	-1	-1	6	84	3	91	87.5
<b>2</b>	1	-1	-1	8	90.6	7	84	87.3
<b>3</b>	-1	1	-1	1	69.6	5	86	77.8
<b>4</b>	1	1	-1	2	76	4	98	87
<b>5</b>	-1	-1	1	5	77.7	8	80.5	79.1
<b>6</b>	1	-1	1	3	99.7	1	95.5	97.6
<b>7</b>	-1	1	1	4	82.7	2	74.5	78.6
<b>8</b>	1	1	1	7	93.7	6	81.7	87.7



## Average effect of Temperature (E1)

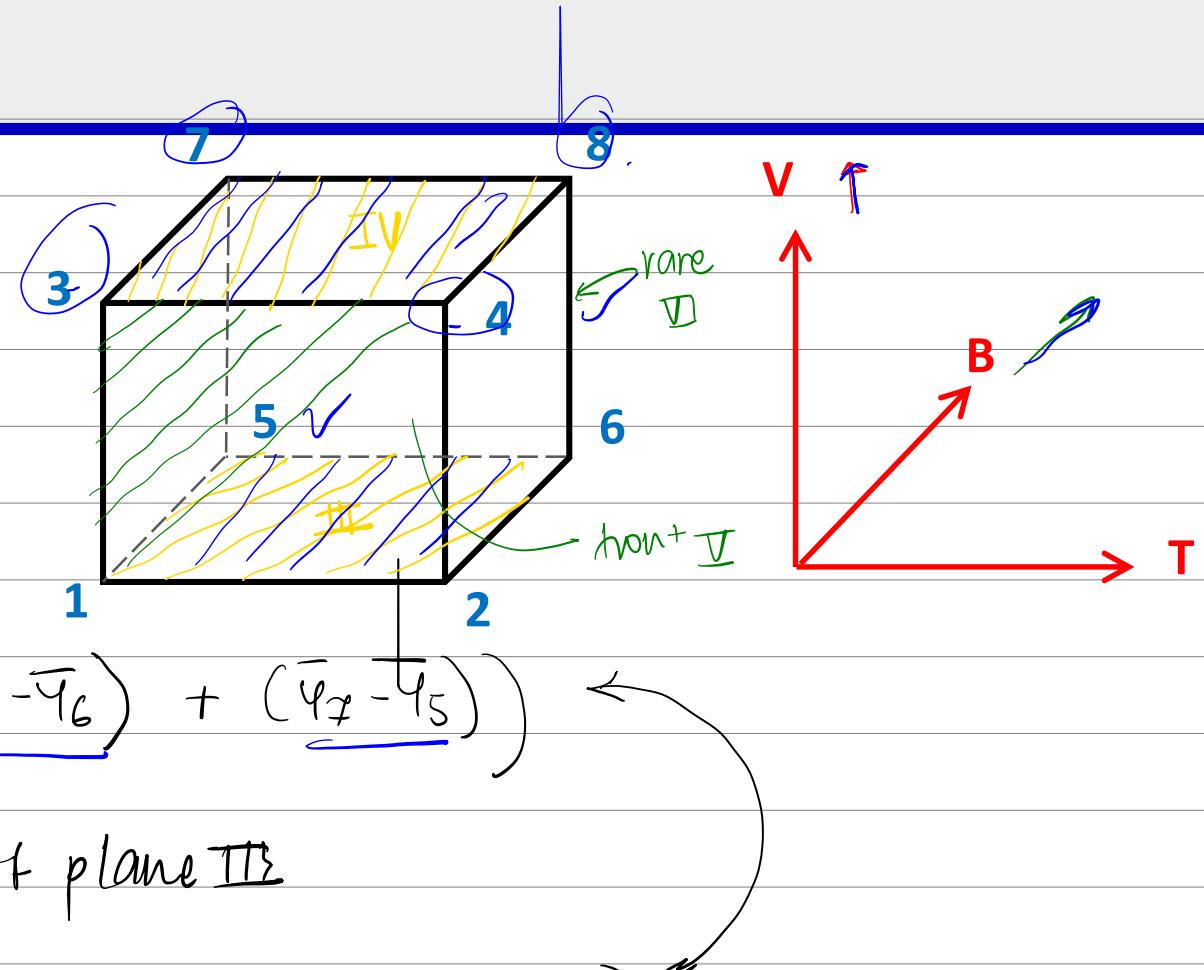
- The average effect of ambient temperature tells us that on the average, over the ranges of other variables in this investigation, the effect of changing the ambient temperature from LOW to HIGH increases the ultimate tensile stress by 9150 psi.
- But, notice that the individual differences ( $y_2 - y_1 = -200$  psi,  $y_4 - y_3 = 9200$  psi,  $y_6 - y_5 = 18000$  psi,  $y_8 - y_7 = 9100$  psi) are quite erratic.

**The average effect, therefore, must be interpreted in conjunction with the intrinsic variabilities that are present in the experimental results.**

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# Main Effect

Test #	X1	X2	X3	Test Order	Y <sub>ai</sub> (kpsi)	Test Order	Y <sub>bi</sub> (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7



$$E_2 = \frac{1}{4} \left( (\bar{Y}_3 - \bar{Y}_1) + (\bar{Y}_4 - \bar{Y}_2) + (\bar{Y}_8 - \bar{Y}_6) + (\bar{Y}_7 - \bar{Y}_5) \right)$$

$E_2 = \text{avg of plane IV} - \text{avg of plane II}$

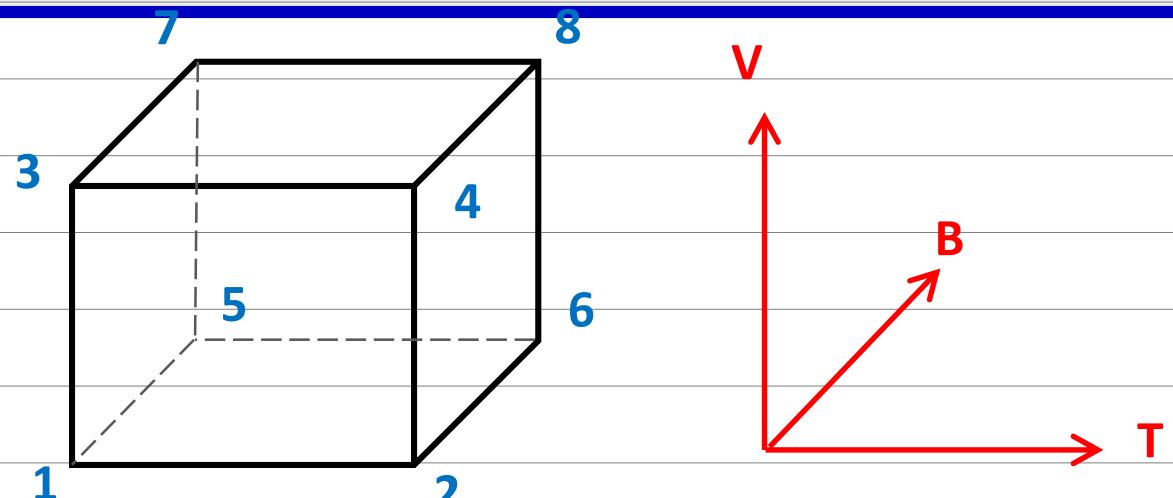
$$= \frac{1}{4} (\bar{Y}_3 + \bar{Y}_4 + \bar{Y}_7 + \bar{Y}_8) - \frac{1}{4} (\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_5 + \bar{Y}_6)$$

$E_3 = \text{avg of plane VI (rear)} - \text{avg of plane II (front)}$

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# Interaction Effect

Test #	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	Test Order	Y <sub>ai</sub> (kpsi)	Test Order	Y <sub>bi</sub> (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7



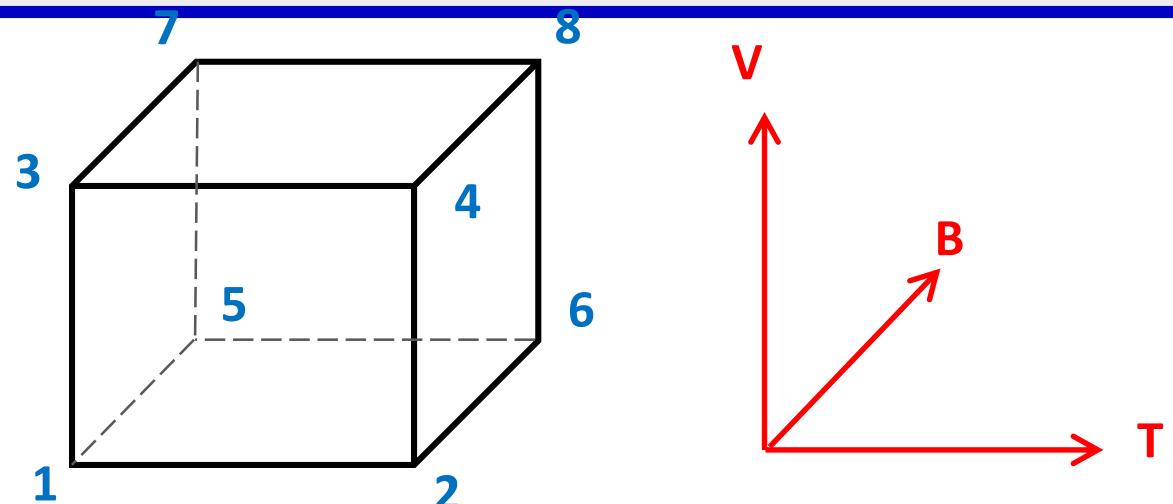
## Two-variable Interactions

- The average effects E<sub>1</sub>, E<sub>2</sub>, and E<sub>3</sub> represent the individual effects of ambient temperature, wind velocity and bar size on the ultimate tensile stress (UTS).
- But, what about the joint effect of two variables?
  - Effect of temperature AND wind velocity on the UTS?
  - Effect of wind velocity AND bar size on UTS?
  - These joint effects are indicated by the two variable interactions.

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# Two Variable Interactions

Test #	X1	X2	X3	Test Order	Y <sub>ai</sub> (kpsi)	Test Order	Y <sub>bi</sub> (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

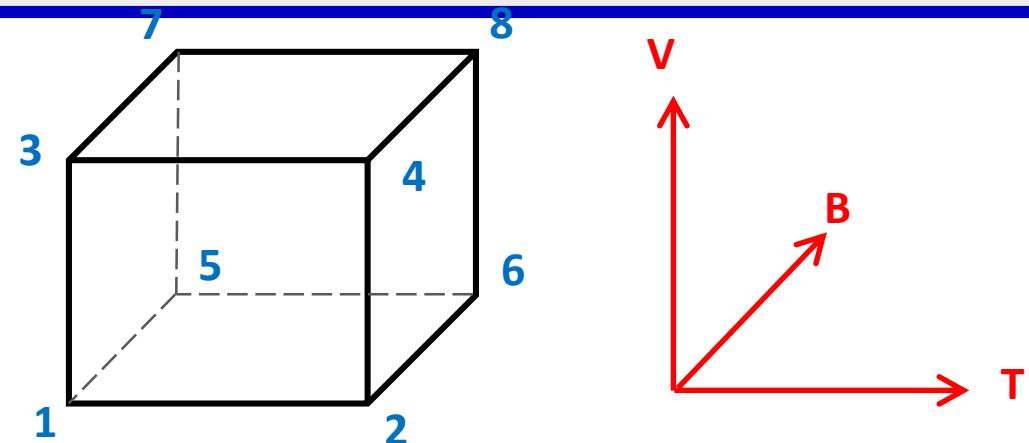


## Two-variable Interactions

- There are three two-variable interactions to be calculated,
  - Between temperature and wind velocity, denoted by  $E_{12}$
  - Between wind velocity and bar size, denoted by  $E_{23}$
  - Between bar size and temperature, denoted by  $E_{31}$  or  $E_{13}$

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<b>Test #</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Test Order</b>	<b>Y<sub>ai</sub> (kpsi)</b>	<b>Test Order</b>	<b>Y<sub>bi</sub> (kpsi)</b>	<b>Average (kpsi)</b>
<b>1</b>	-1	-1	-1	6	84	3	91	87.5
<b>2</b>	1	-1	-1	8	90.6	7	84	87.3
<b>3</b>	-1	1	-1	1	69.6	5	86	77.8
<b>4</b>	1	1	-1	2	76	4	98	87
<b>5</b>	-1	-1	1	5	77.7	8	80.5	79.1
<b>6</b>	1	-1	1	3	99.7	1	95.5	97.6
<b>7</b>	-1	1	1	4	82.7	2	74.5	78.6
<b>8</b>	1	1	1	7	93.7	6	81.7	87.7



## Two-variable Interaction between Temperature and Wind Velocity, E<sub>12</sub>

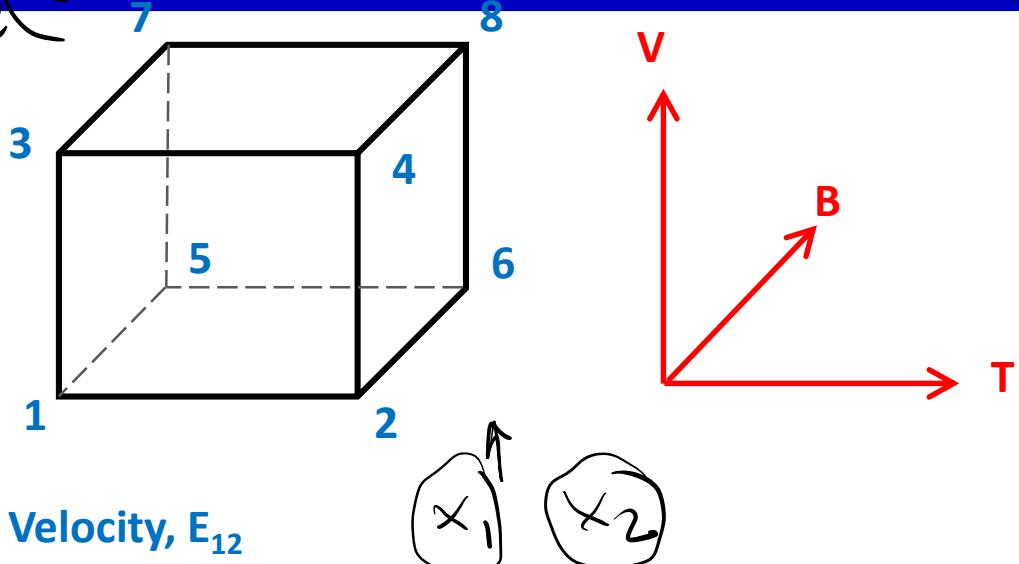
- At HIGH level of wind velocity (X2), the two differences in the result are given by  $(\bar{y}_4 - \bar{y}_3)$  and  $(\bar{y}_8 - \bar{y}_7)$
- Both these differences reflect individual effect of change in UTS due to change in temperature from LOW to HIGH.
- The average of these effects (at HIGH wind velocity) is =  $\frac{1}{2}[(\bar{y}_4 - \bar{y}_3) + (\bar{y}_8 - \bar{y}_7)]$
- Similarly, the average change due to temperature at LOW wind velocity is =  $\frac{1}{2}[(\bar{y}_2 - \bar{y}_1) + (\bar{y}_6 - \bar{y}_5)]$

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$$E_{12} = \text{avg diff bolt offset } x_1 \text{ across } x_2$$

$$= \frac{1}{2} \left( (\underline{x_2 \text{ is at -1}}) \Delta \text{Output}_{\underline{x_1}} - (\underline{x_2 \text{ is at 1}}) \Delta \text{Output}_{\underline{x_1}} \right)$$

Test #	$X_1$	$X_2$	$X_3$	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7



## Two-variable Interaction between Temperature and Wind Velocity, $E_{12}$

- The interaction between ambient temperature and wind velocity is the average difference between these two averages, i.e.,

$$E_{12} = \frac{1}{2} \left\{ \frac{1}{2} [(\bar{Y}_2 - \bar{Y}_1) + (\bar{Y}_6 - \bar{Y}_5)] - \frac{1}{2} [(\bar{Y}_4 - \bar{Y}_3) + (\bar{Y}_8 - \bar{Y}_7)] \right\}$$

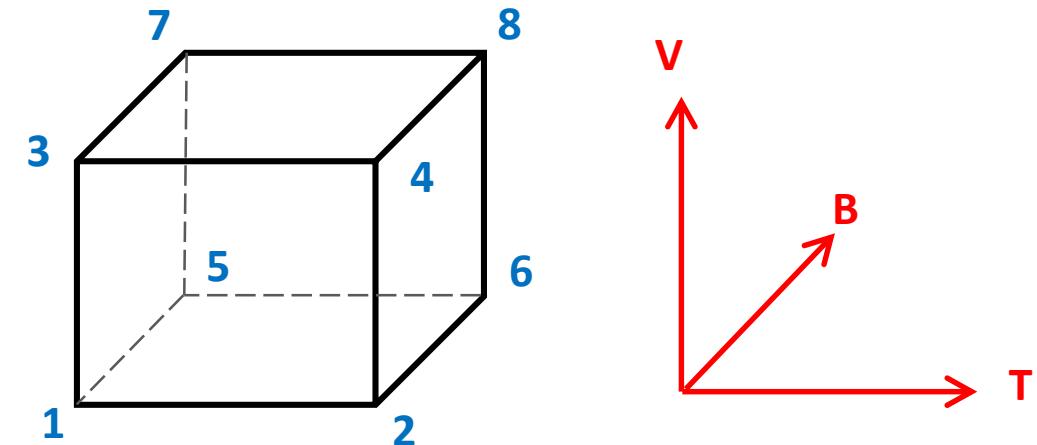
$$= \frac{1}{4} \{ (\bar{Y}_1 + \bar{Y}_5 + \bar{Y}_4 + \bar{Y}_8) - (\bar{Y}_2 + \bar{Y}_6 + \bar{Y}_3 + \bar{Y}_7) \}$$

- Note, therefore, that the interaction between ambient temperature and wind velocity tells us the average change in ultimate tensile stress that would occur due to a change from the low level to the high level in both the ambient temperature and wind velocity.

$$E_{12} = \frac{1}{4} [87.5 - 87.3 - 77.8 + 87 + 79.1 - 97.6 - 78.6 + 87.7]$$

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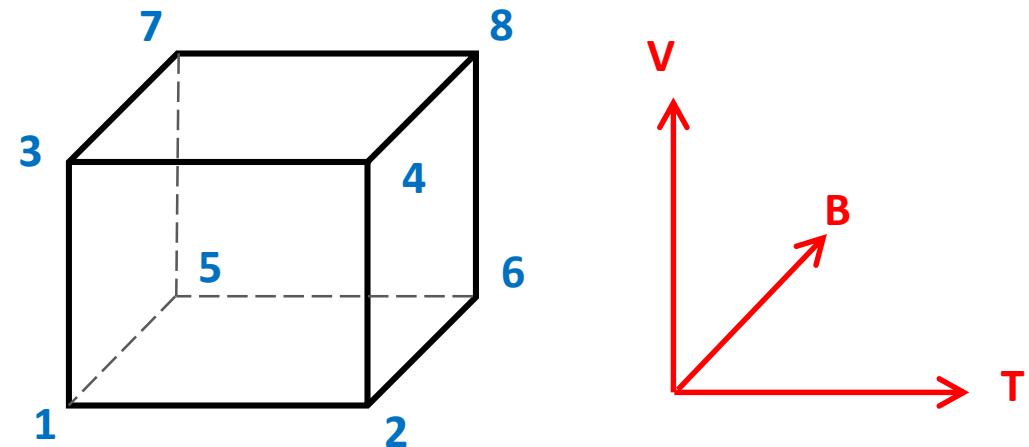
Test #	X1	X2	X3	Test Order	Y <sub>ai</sub> (kpsi)	Test Order	Y <sub>bi</sub> (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7



$$\begin{aligned}
 E_{13} &= \frac{1}{4}(\bar{Y}_1 + \bar{Y}_3 + \bar{Y}_6 + \bar{Y}_8) - \frac{1}{4}(\bar{Y}_2 + \bar{Y}_4 + \bar{Y}_5 + \bar{Y}_7) \\
 &= -4650 \text{ psi}
 \end{aligned}$$

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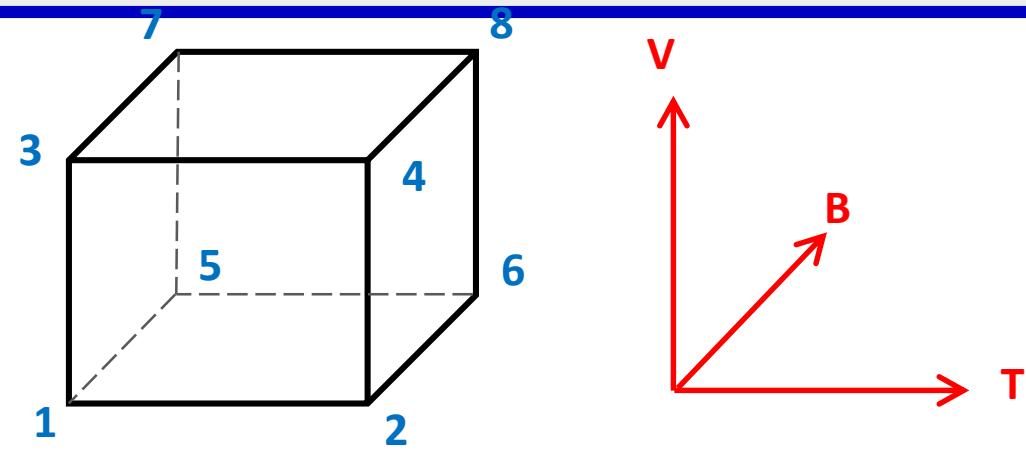
<b>Test #</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Test Order</b>	<b>Y<sub>ai</sub> (kpsi)</b>	<b>Test Order</b>	<b>Y<sub>bi</sub> (kpsi)</b>	<b>Average (kpsi)</b>
<b>1</b>	-1	-1	-1	6	84	3	91	87.5
<b>2</b>	1	-1	-1	8	90.6	7	84	87.3
<b>3</b>	-1	1	-1	1	69.6	5	86	77.8
<b>4</b>	1	1	-1	2	76	4	98	87
<b>5</b>	-1	-1	1	5	77.7	8	80.5	79.1
<b>6</b>	1	-1	1	3	99.7	1	95.5	97.6
<b>7</b>	-1	1	1	4	82.7	2	74.5	78.6
<b>8</b>	1	1	1	7	93.7	6	81.7	87.7



$$\begin{aligned}
 E_{23} &= \frac{1}{4} (\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_7 + \bar{Y}_8) - \frac{1}{4} (\bar{Y}_3 + \bar{Y}_4 + \bar{Y}_5 + \bar{Y}_6) \\
 &= \frac{1}{4} (87.5 + 87.3 + 78.6 + 87.7) - \frac{1}{4} (77.8 + 87.0 + 79.1 + 97.6) \\
 &= -0.10 \text{ Kpsi} \\
 &= -100 \text{ psi}.
 \end{aligned}$$

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Test #	X1	X2	X3	Test Order	$Y_{ai}$ (kpsi)	Test Order	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	6	84	3	91	87.5
2	1	-1	-1	8	90.6	7	84	87.3
3	-1	1	-1	1	69.6	5	86	77.8
4	1	1	-1	2	76	4	98	87
5	-1	-1	1	5	77.7	8	80.5	79.1
6	1	-1	1	3	99.7	1	95.5	97.6
7	-1	1	1	4	82.7	2	74.5	78.6
8	1	1	1	7	93.7	6	81.7	87.7

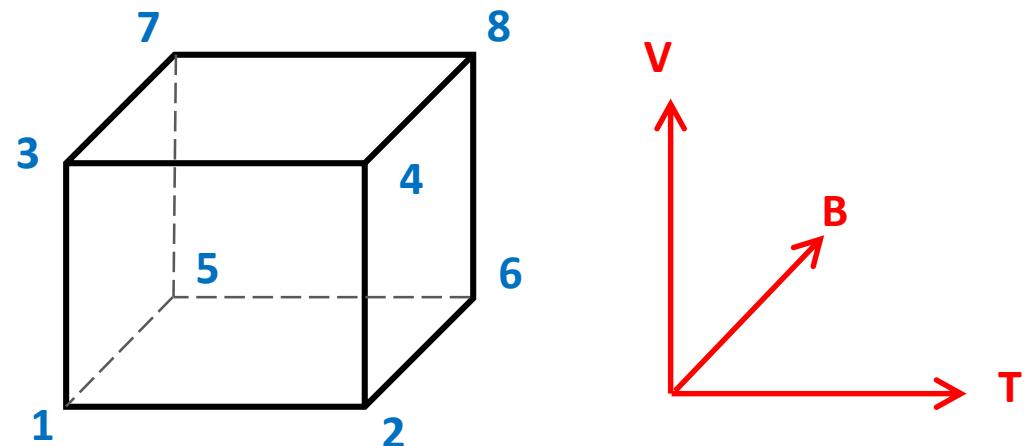


## Three-variable Interaction $E_{123}$

- At HIGH level of bar size (X3), once can calculate the interaction between temperature (X1) and wind velocity (X2)
 
$$\frac{1}{2}[(\bar{Y}_8 - \bar{Y}_6) - (\bar{Y}_7 - \bar{Y}_5)]$$
- Similarly, at LOW level of bar size, interaction between the temperature (X1) and wind velocity (X2) is
 
$$\frac{1}{2}[(\bar{Y}_4 - \bar{Y}_2) - (\bar{Y}_3 - \bar{Y}_1)].$$
- The three-variable interaction among temperature, wind velocity and bar size is the average difference between these two-variable interactions. 
$$E_{123} = \frac{1}{2} \left\{ \frac{1}{2}[(\bar{Y}_8 - \bar{Y}_6) - (\bar{Y}_7 - \bar{Y}_5)] - \frac{1}{2}[(\bar{Y}_4 - \bar{Y}_2) - (\bar{Y}_3 - \bar{Y}_1)] \right\}$$

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<b>Test #</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Test Order</b>	<b>Y<sub>ai</sub> (kpsi)</b>	<b>Test Order</b>	<b>Y<sub>bi</sub> (kpsi)</b>	<b>Average (kpsi)</b>
<b>1</b>	-1	-1	-1	6	84	3	91	87.5
<b>2</b>	1	-1	-1	8	90.6	7	84	87.3
<b>3</b>	-1	1	-1	1	69.6	5	86	77.8
<b>4</b>	1	1	-1	2	76	4	98	87
<b>5</b>	-1	-1	1	5	77.7	8	80.5	79.1
<b>6</b>	1	-1	1	3	99.7	1	95.5	97.6
<b>7</b>	-1	1	1	4	82.7	2	74.5	78.6
<b>8</b>	1	1	1	7	93.7	6	81.7	87.7



## Three-variable Interaction E<sub>123</sub>

- In common practice we *may* assume that the system is not so complicated such that the three-variable interaction will be negligible.
- Moreover, we may contribute it as a part of the intrinsic variation of the test method, or in other terms, as a part of experimental error.
- **But, this should be exercised with caution and should be checked if ever possible. (Model Adequacy Check)**

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# Calculation Matrix

Test #	X1	X2	X3	Average (kpsi)
1	-1	-1	-1	87.5
2	1	-1	-1	87.3
3	-1	1	-1	77.8
4	1	1	-1	87
5	-1	-1	1	79.1
6	1	-1	1	97.6
7	-1	1	1	78.6
8	1	1	1	87.7

Calculation Matrix

Test	Main Effects			Interactions				$\bar{y}$
	$X_1$	$X_2$	$X_3$	$X_1X_2$	$X_1X_3$	$X_2X_3$	$X_1X_2X_3$	
1	-1	-1	-1	1 ✓	1	1	-1	87.5
2	1	-1	-1	-1	-1	1	1✓	87.3
3	-1	1	-1	-1	1	-1	1✓	77.8
4	1	1	-1	1✓	-1	-1	-1	87
5	-1	-1	1	1✓	-1	-1	1✓	79.1
6	1	-1	1	-1	1	-1	-1	97.6
7	-1	1	1	-1	-1	1	-1	78.6
8	1	1	1	1✓	1	1	1✓	87.7

Main effect  $x_1$

$$\begin{aligned}
 E_1 &= \text{avg } y \text{ when } x_1 \text{ is high} - \text{avg } y \text{ when } x_1 \text{ is low} \\
 &= 1/4 (y_2 + y_4 + y_6 + y_8) - 1/4 (y_1 + y_3 + y_5 + y_7)
 \end{aligned}$$

Interaction of  $x_1$  &  $x_2$

$$\begin{aligned}
 E_{12} &= \text{avg } y \text{ when } x_1 x_2 \text{ high} - \text{avg } y \text{ when } x_1 x_2 \text{ low} \\
 &= 1/4 (y_1 + y_3 + y_5 + y_7) - 1/4 (y_2 + y_4 + y_6 + y_8)
 \end{aligned}$$

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<b>Test #</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Average (kpsi)</b>
<b>1</b>	-1	-1	-1	87.5
<b>2</b>	1	-1	-1	87.3
<b>3</b>	-1	1	-1	77.8
<b>4</b>	1	1	-1	87
<b>5</b>	-1	-1	1	79.1
<b>6</b>	1	-1	1	97.6
<b>7</b>	-1	1	1	78.6
<b>8</b>	1	1	1	87.7

## Main Effects

Ambient temperature ( $E_1$ )	9150 psi
Wind Velocity ( $E_2$ )	- 5100 psi
Bar Size ( $E_3$ )	850 psi

## Two-Variable Interactions

Ambient temperature-Wind Velocity ( $E_{12}$ )	0 psi ✓
Ambient temperature-Bar Size ( $E_{13}$ )	- 4650 psi
Wind Velocity-Bar Size ( $E_{23}$ )	-100 psi

## Three-Variable Interaction

Ambient temperature-Wind Velocity-Bar Size ( $E_{123}$ )	4700 psi
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# Significance of Main and Interaction Effects

- With 2-factor ANOVA, we can check if the factors or their interaction have a significant effect on the response
- But what about the statistical significance of main and interaction effect values?**
  - For example, if we calculate the main effect of a variable to be 500.
  - Our attitude towards this average effect of say, 500, would not be the same if the 95% confidence interval were  $500 \pm 2$  as it would be if the interval were  $500 \pm 2000$ .
  - If it is  $500 \pm 2$ , we would feel that the existence of an average effect has been rather convincingly demonstrated and we could assert with some confidence that its true magnitude is probably fairly close to 500.
  - If it is  $500 \pm 2000$ , this is not the case at all, because considerable uncertainty is associated with the effect and its magnitude.
- How to find out the uncertainty (via confidence intervals) in the calculated values of main and interaction effects?**

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# Significance of Main and Interaction Effects

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- To obtain a quantitative measure of the uncertainty in our calculated average effects and interactions, we proceed as follows:
  1. Estimate the variance  $S^2$  of an individual observation
  2. Estimate the variances associated with the average effects and interactions
  3. Calculate the appropriate 95% confidence intervals for the “true” average effects and interactions
- From the 95% confidence intervals, we may be able to interpret the significance of each average effect and interaction, and draw conclusions regarding the experimental study.

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# Significance of Main and Interaction Effects

- Recall that in the previous welding example, we had 16 observations (8 combinations with 2 replications)
- It is the variance of each of these 16 observations that we will now estimate.
- We shall assume that the true variance  $\sigma^2$  is the same for all sixteen observations and that the observations are independent.

For Test #1, the sample variance can be calculated as

$$\begin{aligned} S_1^2 &= \frac{(y_{a1} - \bar{y}_1)^2 + (y_{b1} - \bar{y}_1)^2}{(2-1)} \\ &= (84.0 - 87.5)^2 + (91.0 - 87.5)^2 \\ &= 24.50 \end{aligned}$$

Similarly, we can find 8 sample variances for 8 combinations,  $S_1^2, S_2^2, \dots, S_8^2$ , one for each test.

Test #	X1	X2	X3	$Y_{ai}$ (kpsi)	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	84	91	87.5
2	1	-1	-1	90.6	84	87.3
3	-1	1	-1	69.6	86	77.8
4	1	1	-1	76	98	87
5	-1	-1	1	77.7	80.5	79.1
6	1	-1	1	99.7	95.5	97.6
7	-1	1	1	82.7	74.5	78.6
8	1	1	1	93.7	81.7	87.7

$$S_1^2 = 24.5, S_2^2 = 21.78, S_3^2 = 134.48, S_4^2 = 242.0, S_5^2 = 3.92, S_6^2 = 8.82, S_7^2 = 33.62, S_8^2 = 72.00$$

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# Significance of Main and Interaction Effects

- Since we are assuming same true variance  $\sigma^2$  for all sixteen observations, an estimate for  $\sigma^2$  is the pooled sample variance  $S_p^2$  of the eight estimated variances  $S_1^2, S_2^2, \dots, S_8^2$ .

$$S_1^2 = 24.5, S_2^2 = 21.78, S_3^2 = 134.48, S_4^2 = 242.0, S_5^2 = 3.92, S_6^2 = 8.82, S_7^2 = 33.62, S_8^2 = 72.00$$

- Here

$$\begin{aligned} S_p^2 &= \frac{[(y_{a1} - \bar{y}_1)^2 + (y_{b1} - \bar{y}_1)^2 + \dots + (y_{a8} - \bar{y}_8)^2 + (y_{b8} - \bar{y}_8)^2]}{(2-1) + \dots + (2-1)} \\ &= \frac{24.50 + 21.78 + \dots + 72}{8} \\ &= 67.64. \end{aligned}$$

- It should be pointed out that when the number of replications are not the same for all eight tests, the pooled sample variance  $S_p^2$  has to be modified properly.

$$S_p^2 = \frac{SS}{\nu} = \frac{SS_1 + SS_2 + SS_3 + \dots + SS_8}{\nu_1 + \nu_2 + \dots + \nu_8} = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2 + \dots + (n_8-1)S_8^2}{(n_1-1) + (n_2-1) + \dots + (n_8-1)}$$

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# Significance of Main and Interaction Effects

## Estimation of the Variances Associated with the Average Effects and Interactions

- The average effect of ambient temperature,  $E_1$  is

$$\checkmark E_1 = \frac{1}{4} (\bar{y}_2 - \bar{y}_1 + \bar{y}_4 - \bar{y}_3 + \bar{y}_6 - \bar{y}_5 + \bar{y}_8 - \bar{y}_7) \checkmark$$

- But since each term  $\bar{y}_i$  is an average of two observations,

$$E_1 = \frac{1}{4} \left[ \frac{(y_{a2} + y_{b2})}{2} - \frac{(y_{a1} + y_{b1})}{2} + \dots + \frac{(y_{a8} + y_{b8})}{2} - \frac{(y_{a7} + y_{b7})}{2} \right]$$

or

$$E_1 = \frac{1}{8} [y_{a2} + y_{b2} - y_{a1} - y_{b1} + \dots - y_{a7} - y_{b7}]$$

$$\begin{aligned} V(E_1) &= \frac{1}{64} \left( V(\bar{y}_{a2}) + V(\bar{y}_{b2}) - V(\bar{y}_{a1}) - V(\bar{y}_{b1}) - \dots - V(\bar{y}_{a7}) - V(\bar{y}_{b7}) \right) \\ &= 16\sigma^2 / 64 = \sigma^2/4 \end{aligned}$$

- Thus we can show that  $V(E_1) = \sigma^2/4$

- In fact, we can show,  $V(E_1) = V(E_2) = V(E_3) = V(E_{12}) = V(E_{13}) = V(E_{23}) = V(E_{123}) = \sigma^2/4$

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# Significance of Main and Interaction Effects

## Estimation of the Variances Associated with the Average Effects and Interactions

- $V(E_1) = V(E_2) = V(E_3) = V(E_{12}) = V(E_{13}) = V(E_{23}) = V(E_{123}) = \sigma^2/4$
- Substituting pooled variance in place of true variance, we can obtain confidence intervals
- The confidence interval for the average and interaction effects can be obtained as:

$$E_i \pm t \sqrt{\frac{S_p^2}{4}} \quad i = 1, 2, \dots$$

$$E_i \pm 2.306 \sqrt{\frac{S_p^2}{4}}$$

- We already have the values of  $E_1, E_2, \dots, E_{12}, \dots$  and  $S_p^2$ ; what is left to be determined is the value of  $t$ .
- We have a total of sixteen tests, and we used up eight degrees of freedom in calculating the eight averages,
- Therefore, the appropriate  $t$ -value is the value associated with eight degrees of freedom and corresponding to a 95% confidence level, which is  $t_{8, 0.025} = 2.306$ .

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# Significance of Main and Interaction Effects

Example

$$E_i \pm t \sqrt{\frac{S_p^2}{4}} \quad i = 1, 2, \dots$$

Here,

$$S_p^2 = \frac{[(y_{a1} - \bar{y}_1)^2 + (y_{b1} - \bar{y}_1)^2 + \dots + (y_{a8} - \bar{y}_8)^2 + (y_{b8} - \bar{y}_8)^2]}{(2-1) + \dots + (2-1)} = \underline{67.64} \checkmark$$

Test #	X1	X2	X3	$Y_{ai}$ (kpsi)	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	84	91	87.5
2	1	-1	-1	90.6	84	87.3
3	-1	1	-1	69.6	86	77.8
4	1	1	-1	76	98	87
5	-1	-1	1	77.7	80.5	79.1
6	1	-1	1	99.7	95.5	97.6
7	-1	1	1	82.7	74.5	78.6
8	1	1	1	93.7	81.7	87.7

t-value is the value associated with eight degrees of freedom and corresponding to a 95% confidence level, which is  $t_{8, 0.025} = 2.306$

For example,

95% confidence interval for the “true” average effect of ambient temperature (in kpsi) is:  $E_1 \pm 9.48 = 9.15 \pm \underline{9.48}$

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# Significance of Main and Interaction Effects

## Example

$$E_i \pm t \sqrt{\frac{S_p^2}{4}} \quad i = 1, 2, \dots$$

Here,

$$S_p^2 = \frac{[(y_{a1}-\bar{y}_1)^2 + (y_{b1}-\bar{y}_1)^2 + \dots + (y_{a8}-\bar{y}_8)^2 + (y_{b8}-\bar{y}_8)^2]}{(2-1) + \dots + (2-1)} = 67.64$$

Test #	X1	X2	X3	$Y_{ai}$ (kpsi)	$Y_{bi}$ (kpsi)	Average (kpsi)
1	-1	-1	-1	84	91	87.5
2	1	-1	-1	90.6	84	87.3
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6	1	-1	1	99.7	95.5	97.6
7	-1	1	1	82.7	74.5	78.6
8	1	1	1	93.7	81.7	87.7

## Average Effects

Ambient temperature (E<sub>1</sub>)

## 95% Confidence Interval

✓ 9150 ± 9480 psi

Wind Velocity (E<sub>2</sub>)

✓ -5100 ± 9480 psi

Bar Size (E<sub>3</sub>)

✓ -850 ± 9480 psi

## Two-Variable Interactions

Ambient temperature-Wind Velocity (E<sub>12</sub>)

✓ 0 ± 9480 psi

Ambient temperature-Bar Size (E<sub>13</sub>)

✓ -4650 ± 9480 psi

Wind Velocity-Bar Size (E<sub>23</sub>)

✓ -100 ± 9480 psi

## Three-Variable Interaction

Ambient temperature-Wind Velocity-Bar Size (E<sub>123</sub>) ✓ 4700 ± 9480 psi

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