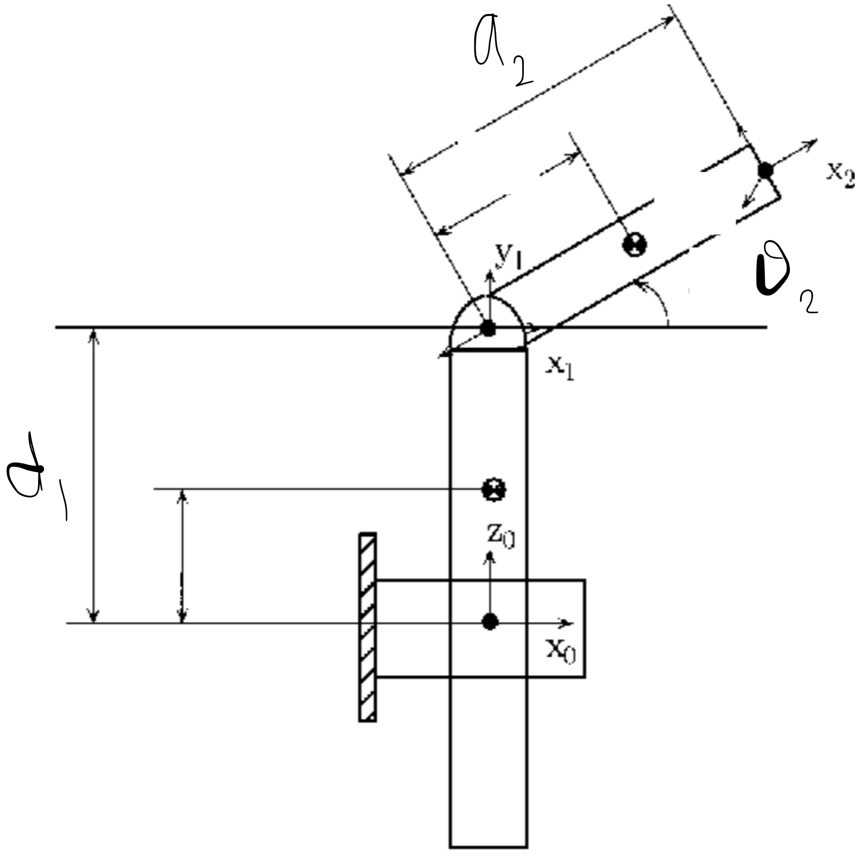
1.



a.

Co-ordinates of origin of frame {2} o2 (in fixed frame x0y0): $\begin{bmatrix} a_2 c_2 \\ 0 \\ d_1 + a_2 s_2 \end{bmatrix}$

Taking partial derivatives wrt to joint variables (d_1, θ_2)

$$J_v = \begin{bmatrix} 0 & -a_2 s_2 \\ 0 & 0 \\ 1 & a_2 c_2 \end{bmatrix}$$

Alternatively,

$$J_v = \left[\, z_0 \quad {}^0z_1 \times ({}^0o_2 - {}^0o_1) \,\right]$$

$$\begin{bmatrix} a_2 c_2 \\ 0 \\ a_2 s_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Note that these vectors can be obtained using the DH parameter table and corresponding transformation matrices.
Singularity: s2 = 0 (endeffector cant be moved in x0 direction at that instant)

Q2> Given $R = R_x(\psi) R_y(\theta) R_z(\phi)$

Find ω, such that $\frac{dR}{dt} = S(\omega) R$

We already know properties

$$\frac{d R_x(\theta)}{dt} = \dot{\theta} S(\hat{i}) R_x$$

Applying the same with the chain Rule on R

$$\frac{dR}{dt} = \left(\frac{d R_x(\psi)}{dt}\right) R_y(\theta) R_z(\phi) + R_x(\psi)\left(\frac{d R_y(\theta)}{dt}\right) R_z(\phi) + R_x(\psi) R_y(\theta)\left(\frac{d R_z(\phi)}{dt}\right)$$

$$= \dot{\psi} S(\hat{i}) R_x R_y R_z + R_x(\dot{\theta} S(\hat{j}) R_y) R_z + R_x R_y(\dot{\phi} S(\hat{k}) R_z)$$

$$= S(\dot{\psi}\hat{i}) \underbrace{R_x R_y R_z}_{R} + R_x S(\dot{\theta}\hat{j}) R_x^T \underbrace{R_x R_y R_z}_{R} + R_x R_y S(\dot{\phi}\hat{k}) R_y^T R_x^T \underbrace{R_x R_y R_z}_{R}$$

Using

$$R S(a) R^T = S(Ra)$$

$$= S(\dot{\psi}\hat{i}) R + S(R_x \dot{\theta}\hat{j}) R + S(R_x R_y \dot{\phi}\hat{k}) R.$$

$$= \left[\begin{bmatrix} \dot{\psi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}}\begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix}\right] R$$

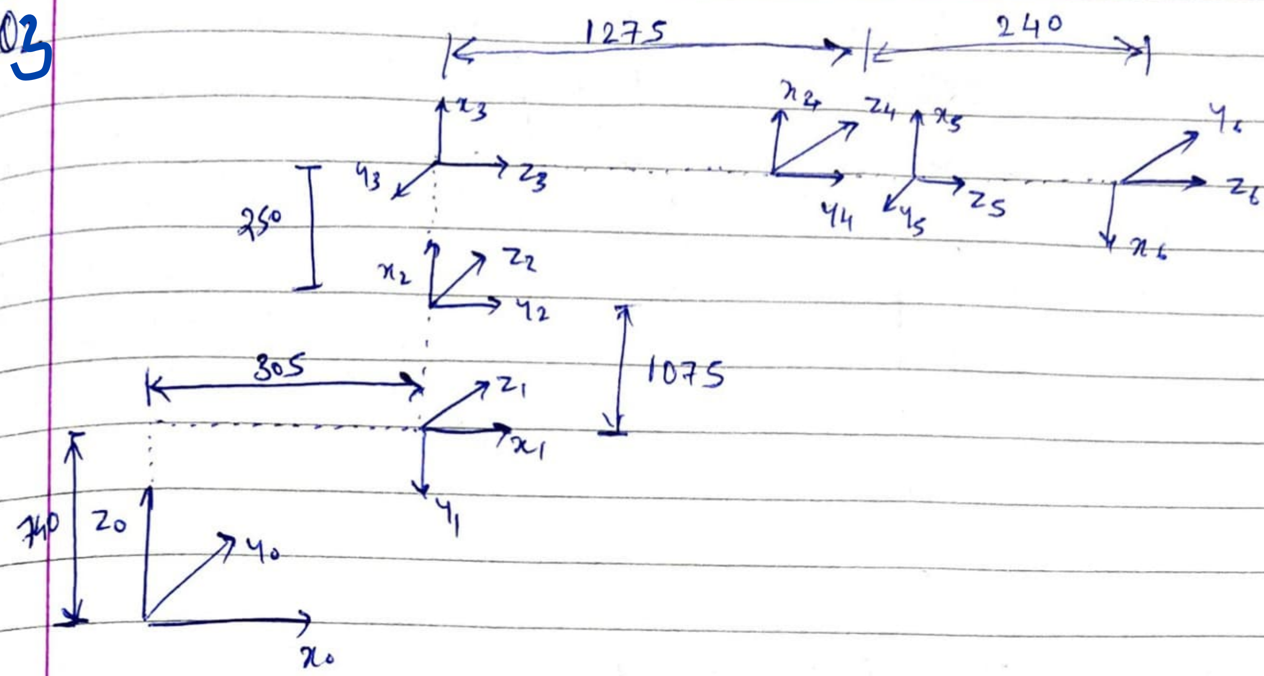
$$\begin{bmatrix} c\theta & 0 & s\theta \\ s\theta s\psi & c\psi & -s\psi c\theta \\ -s\theta c\theta\ \ s\psi & & c\psi c\theta \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix}$$

$$= \left[\begin{bmatrix} \dot\psi \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \dot\theta\, c\psi \\ \dot\theta\, s\psi \end{bmatrix} + \begin{bmatrix} \dot\phi\, s\theta \\ -\dot\phi\, s\psi c\theta \\ \dot\phi\, c\psi c\theta \end{bmatrix} \right] R$$

$$\underbrace{\left\{ (\dot\psi + \dot\phi s\theta)\,\hat{i} + (\dot\theta c\psi - \dot\phi s\psi c\theta)\,\hat{j} + (\dot\theta s\psi + \dot\phi c\psi c\theta)\,\hat{k} \right\}}_{\omega}$$

Page No.:
Date:

Q3



We have to find the Jacobian, of an industrial manipulator at the given instant.

$$J_i = \begin{bmatrix} Z_{i-1} \times (P_n - P_{i-1}) \\ Z_{i-1} \end{bmatrix}_{6\times n} ; \quad i = 1, 2, \ldots n$$

Here we have $n = 6$
hence we will have $[J]_{6\times 6}$

$$J = \begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} Z_0 \times (P_6 - P_0) \\ Z_0 \end{bmatrix}_{6\times 1} \begin{matrix} \rightarrow J_v \\ \rightarrow J_\omega \end{matrix}$$

$$Z_0 \times (P_6 - P_0) = S(Z_0)(P_6 - P_0)$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1820 \\ 0 \\ 2065 \end{bmatrix}$$

Page No.:
Date:

$$J_v = \begin{bmatrix} 0 \\ 1820 \\ 0 \end{bmatrix} \quad J_\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 0 \\ 1820 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} Z_1 \times (P_6 - P_1) \\ Z_1 \end{bmatrix} \begin{matrix} \to J_{v_2} \\ \to J_{\omega_2} \end{matrix}$$

$$J_{v_2} = {}^0R_1 \, S(Z_1)(P_6 - P_1)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1515 \\ -1325 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} +1325 \\ 1515 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1325 \\ 0 \\ -1515 \end{bmatrix}$$

$$J_{\omega_2} = {}^0R_1 \, Z_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} 1325 \\ 0 \\ -1515 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Page No.:
Date:

$$J_3 = \begin{bmatrix} z_2 \times (P_6 - P_2) \\ z_2 \end{bmatrix} \begin{matrix} \rightarrow J_{v_3} \\ \rightarrow J_{\omega_3} \end{matrix}$$

$$J_{v_3} = {}^0R_2\, S(z_2)(P_6 - P_2)$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 250 \\ 1515 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 250 \\ 0 \\ -1515 \end{bmatrix}$$

$$J_{\omega_3} = {}^0R_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 250 \\ 0 \\ -1515 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} z_3 \times (P_6 - P_3) \\ z_3 \end{bmatrix} \begin{matrix} \rightarrow J_{v_4} \\ \rightarrow J_{\omega_4} \end{matrix}$$

$$J_{v_4} = {}^0R_3\, S(z_3)(P_6 - P_3)$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1515 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{\omega_3} = {}^0R_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$J_5 = \begin{bmatrix} Z_4 \times (P_6 - P_4) \\ Z_4 \end{bmatrix} \begin{matrix} \rightarrow J_{v_5} \\ \rightarrow J_{w_5} \end{matrix}$$

$$J_{v_5} = {}^0R_4\, S(Z_4)(P_6 - P_4)$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 240 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -240 \end{bmatrix}$$

$$J_{w_5} = {}^0_4R\, Z_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad ; \quad J_5 = \begin{bmatrix} 0 \\ 0 \\ -240 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$J_6 = \begin{bmatrix} Z_5 \times (P_6 - P_5) \\ Z_5 \end{bmatrix} \begin{matrix} \rightarrow J_{v_6} \\ \rightarrow J_{w_6} \end{matrix}$$

$$J_{v_6} = {}^0_5R\, S(Z_5)(P_6 - P_5)$$

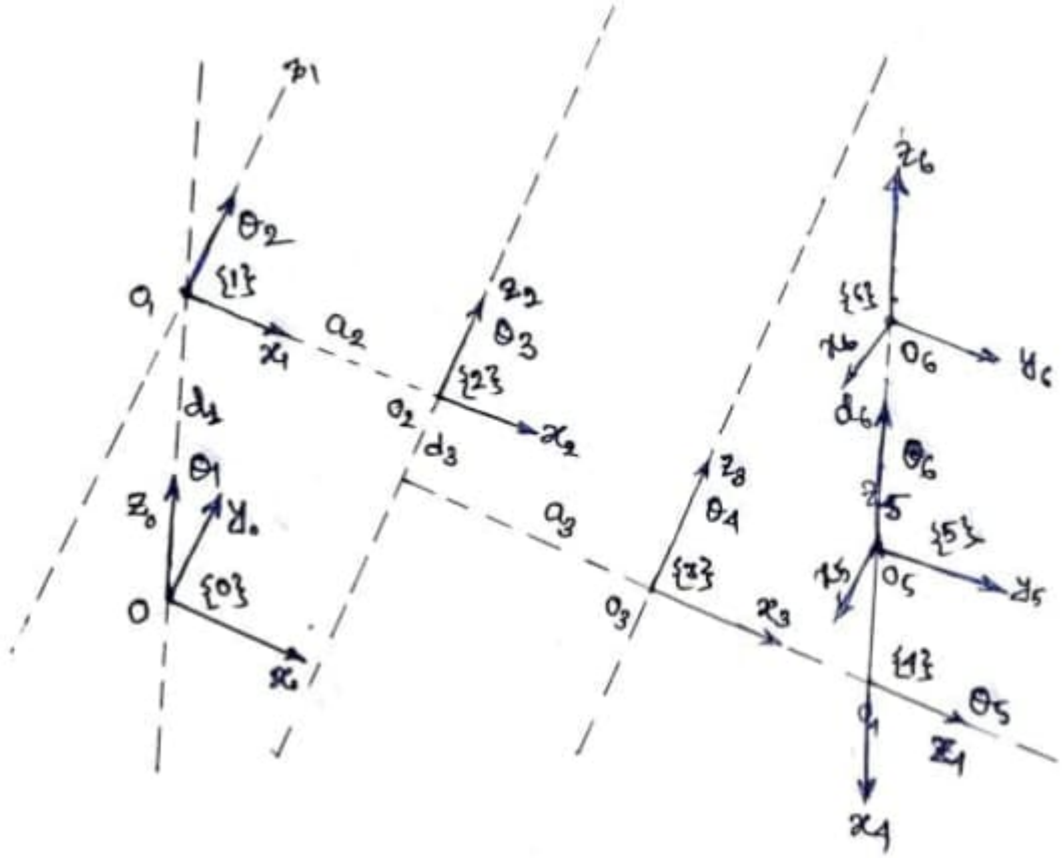
$$J_{v_6} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 240 \end{bmatrix}$$

$$J_{v_6} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad ; \quad J_{w_6} = {}^0_5R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$J_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 1325 & 250 & 0 & 0 & 0 \\ 1820 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1515 & -1515 & 0 & -240 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. PUMA manipulator:



	θ_i	d_i	a_i	α_i
$({}^0_1T)$	θ_1	13	0	$-\pi/2$
$({}^1_2T)$	θ_2	0	8	0
$({}^2_3T)$	θ_3	-4	8	0
$({}^3_4T)$	θ_4	0	0	$\pi/2$
$({}^4_5T)$	θ_5	0	0	$\pi/2$
$({}^5_6T)$	θ_6	1	0	0

{1} → shoulder rotation, z_1
{2} → Elbowrotation, z_2
{3} → Wrist rotation; z_3
{5} → Flange rotation, z_5

$o_3 = o_4 = o_5$ (same point)
so, link length & joint distances will be zero.

- $d_1 = 13$ in, $d_3 = 4$ in, $d_6 = 1$ in
- $a_2 = a_3 = 8$ in

(b) all joint velocities(angular) are = 1 $\quad \dot{q} = [1\,1\,1\,1\,1\,1]^T$

$$J_\omega = \begin{bmatrix} z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$ (all expressed w.r.t world frame)

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

angular velocity of the flange, $\omega = J_\omega \dot{q} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ rad/s

Now, flange holds a screw driver of 6 in long, so the joint distance of the {6}th frame will be $(1+6) = 7$ in

$${}^0O_6 = \begin{bmatrix} 8+8 \\ -4 \\ 13+1+6 \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ 20 \end{bmatrix};\ {}^0O_1 = \begin{bmatrix} 0 \\ 0 \\ 13 \end{bmatrix};\ {}^0O_2 = \begin{bmatrix} 8 \\ 0 \\ 13 \end{bmatrix};\ {}^0O_3 = {}^0O_4 = {}^0O_5 = \begin{bmatrix} 16 \\ -4 \\ 13 \end{bmatrix}$$

$$({}^0O_6 - {}^0O_1) = \begin{bmatrix} 16 \\ -4 \\ 7 \end{bmatrix};\ ({}^0O_6 - {}^0O_2) = \begin{bmatrix} 8 \\ -4 \\ 7 \end{bmatrix};\ ({}^0O_6 - {}^0O_3) = ({}^0O_6 - {}^0O_4) = ({}^0O_6 - {}^0O_5) = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$$

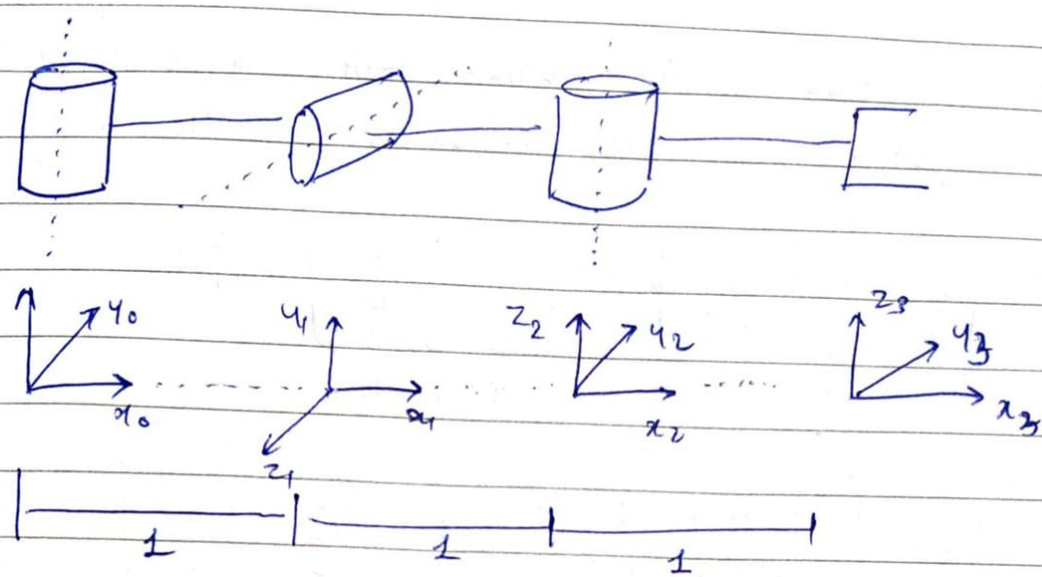
* Linear velocity Jacobian:

$$J_v = \begin{bmatrix} z_0 \times {}^0O_6 & {}^0z_1 \times ({}^0O_6 - {}^0O_1) & {}^0z_2 \times ({}^0O_6 - {}^0O_2) & {}^0z_3 \times ({}^0O_6 - {}^0O_3) & {}^0z_4 \times ({}^0O_6 - {}^0O_4) & {}^0z_5 \times ({}^0O_6 - {}^0O_5) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 7 & 7 & 7 & 0 & 0 \\ 16 & 0 & 0 & 0 & -7 & 0 \\ 0 & -16 & -8 & 0 & 0 & 0 \end{bmatrix}$$

Velocity of the tip, $v = J_v \dot{q} = \begin{bmatrix} 25 \\ 9 \\ -24 \end{bmatrix}$ in/s

Q5



DH Parameters:

(a)

Joint	a	α	d	θ
1	1	90°	0	θ_1^*
2	1	-90°	0	θ_2^*
3	1	0	0	θ_3^*

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ ← General Homogeneous transformation Matrix

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & C\theta_1 \\ S\theta_1 & 0 & -C\theta_1 & S\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} C\theta_2 & 0 & -S\theta_2 & C\theta_2 \\ S\theta_2 & 0 & C\theta_2 & S\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $^0_3T = A_1 A_2 A_3$

$$= \begin{bmatrix} C\theta_1 C\theta_2 C\theta_3 - S\theta_1 S\theta_3 & -C\theta_3 S\theta_1 - C\theta_1 C\theta_2 S\theta_3 & -C\theta_1 S\theta_2 & \cdots \\ C\theta_1 S\theta_3 + C\theta_2 C\theta_3 S\theta_1 & C\theta_1 C\theta_3 - C\theta_2 S\theta_1 S\theta_3 & -S\theta_1 S\theta_2 & \cdots \\ C\theta_3 S\theta_2 & -S\theta_2 S\theta_3 & C\theta_2 & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}$$

$$\begin{matrix} C\theta_1 + C\theta_1 C\theta_2 - S\theta_1 S\theta_3 + C\theta_1 C\theta_2 C\theta_3 \\ S\theta_1 + C\theta_2 S\theta_1 + C\theta_1 S\theta_3 + C\theta_2 C\theta_3 S\theta_1 \\ S\theta_2 + C\theta_3 S\theta_2 \\ 1 \end{matrix} \Bigg]$$

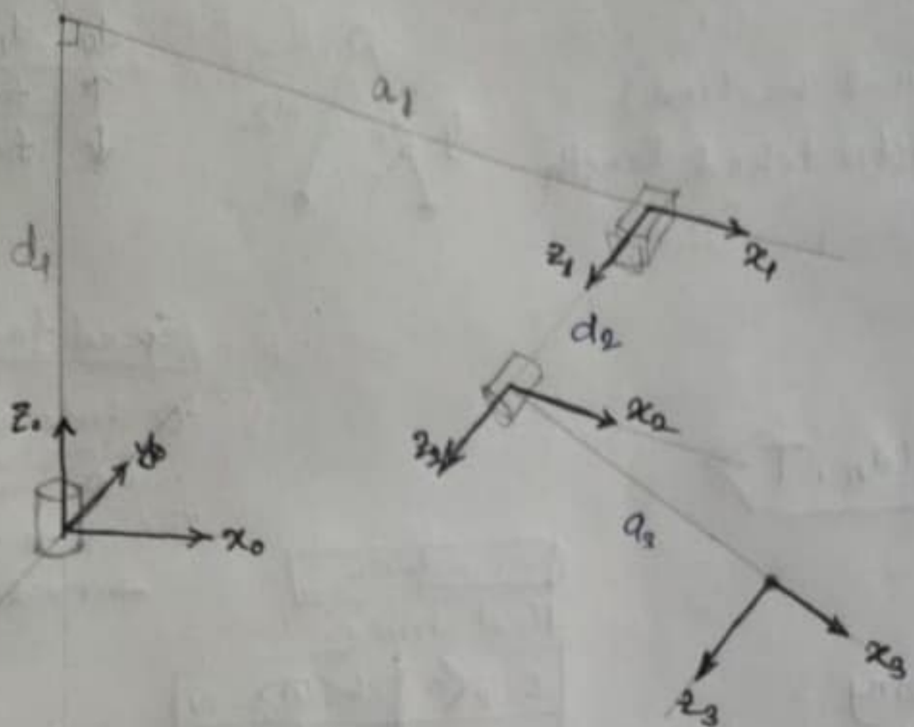
(C) To find the basic Jacobian
use the explicit form:
Take derivative of the end effector position (last column of T^0_3)
for J_V
and use Z-axes of each frame (3^{rd} column of each ^0_iT matrix)
for J_ω

$$J_0 = \begin{bmatrix} \dfrac{\partial\, ^0P_3}{\partial\theta_1} & \dfrac{\partial\, ^0P_3}{\partial\theta_2} & \dfrac{\partial\, ^0P_3}{\partial\theta_3} \\ ^0Z_1 & ^0Z_2 & ^0Z_3 \end{bmatrix}$$

$$
J_0 = \left[\begin{array}{cc}
-S\theta_1 - C\theta_2 S\theta_1 - C\theta_1 S\theta_3 - C\theta_2 C\theta_3 S\theta_1 & -C\theta_1 S\theta_2 - C\theta_1 C\theta_3 S\theta_2 \quad \cdots \\
C\theta_1 + C\theta_1 C\theta_2 - S\theta_1 S\theta_3 + C\theta_1 C\theta_2 C\theta_3 & -C\theta_3 S\theta_1 S\theta_2 - S\theta_1 S\theta_2 \quad \cdots \\
0 & C\theta_2 + C\theta_2 C\theta_3 \quad \cdots \\
S\theta_1 & -C\theta_1 S\theta_2 \quad \cdots \\
-C\theta_1 & -S\theta_1 S\theta_2 \quad \cdots \\
0 & C\theta_2 \quad \cdots
\end{array}\right.
$$

$$
\left.\begin{array}{c}
-C\theta_3 S\theta_1 - C\theta_1 C\theta_2 S\theta_3 \\
C\theta_1 C\theta_3 - C\theta_2 S\theta_1 S\theta_3 \\
-S\theta_2 S\theta_3 \\
-C\theta_1 S\theta_2 \\
-S\theta_1 S\theta_2 \\
C\theta_2
\end{array}\right]
$$

6



θ_i	d_i	a_i	α_i
θ_1	d_1	a_1	$\pi/2$
0	d_2	0	0
θ_3	0	a_3	0

$d_1, a_1, a_3 \rightarrow$ fixed length
$d_2 \rightarrow$ variable

(b)
$$^0_1T = \begin{array}{cccc} & & {}^0z_1 & {}^0o_1 \\ \end{array} \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad ^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad ^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0_2T = \begin{array}{cccc} & & {}^0z_2 & {}^0o_2 \\ \end{array} \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 + d_2s_1 \\ s_1 & 0 & -c_1 & a_1s_1 - d_2c_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad ^0_3T = \begin{array}{cccc} & & {}^0z_3 & {}^0o_3 \\ \end{array} \begin{bmatrix} c_1c_3 & -c_1s_3 & s_1 & a_1c_1 + d_2s_1 + a_3c_1c_3 \\ c_3s_1 & -s_1s_3 & -c_1 & a_1s_1 - d_2c_1 + a_3s_1c_3 \\ s_3 & c_3 & 0 & a_3s_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $({}^0O_3 - {}^0O_2) = \begin{bmatrix} a_3 c_1 c_3 \\ a_3 s_1 c_3 \\ a_3 s_3 \end{bmatrix}$

$$J_\omega = \begin{bmatrix} z_0 & 0 & z_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & s_1 \\ 0 & 0 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_v = \begin{bmatrix} {}^0z_0 \times {}^0O_3 & {}^0z_1 & {}^0z_2 \times ({}^0O_3 - {}^0O_2) \end{bmatrix} = \begin{bmatrix} d_2c_1 - a_1s_1 - a_3s_1c_3 & s_1 & -a_3s_3c_1 \\ d_2s_1 + a_1c_1 + a_3c_1c_3 & -c_1 & -a_3s_1s_3 \\ 0 & 0 & a_3c_3 \end{bmatrix}$$

Basic Jacobian, $J = \begin{bmatrix} J_v \\ \hline J_\omega \end{bmatrix}$

(c) Jacobian expressed in frame {1}:

We have the relationship ${}^0v = {}^0J(q)\dot{q}$

the velocity vector represented in frame {1}, ${}^1v = \underbrace{{}^1_0R\,{}^0J(q)}\,\dot{q}$

${}^1J(q) = {}^1J_v$

$${}^1J(q) = {}^1J_v = {}^0_1R^T\,{}^0J_v.$$

$$= \begin{bmatrix} c_1 & s_1 & 0 \\ 0 & 0 & 1 \\ s_1 & -c_1 & 0 \end{bmatrix} \begin{bmatrix} d_2c_1 - a_1s_1 - a_3s_1c_3 & s_1 & -a_3s_3c_1 \\ d_2s_1 + a_1c_1 + a_3c_1c_3 & -c_1 & -a_3s_1s_3 \\ 0 & 0 & a_3c_3 \end{bmatrix} = \begin{bmatrix} d_2 & 0 & -a_3s_3 \\ 0 & 0 & a_3c_3 \\ -a_1 - a_3c_3 & 1 & 0 \end{bmatrix}$$

(d) Singularity: when the Jacobian matrix loses rank.

if $\cos\theta_3 = 0$ i.e. $\theta_3 = \pm\pi/2$; the rank of $J_v = 2$, singularity apears in z_0 direction.