ME604: Introduction to Robotics

Spring 2025

Assignment 7: Practice problems

1. Consider a single degree of freedom system described by the equation

$$2\ddot{x} + \dot{x} = F(t)$$

Design a PD trajectory tracking controller to track a reference signal $x_d(t) = \sin t + \cos 2t$. The closed loop system should have a natural frequency less than 10 radians with a damping ratio greater than 0.707.

2. Consider the coupled nonlinear system

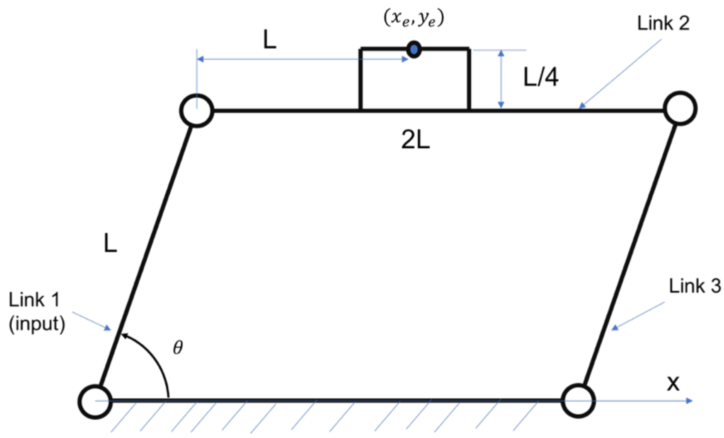
$$\ddot{y}_1 + 3y_1y_2 + y_2^2 = u_1 + y_2u_2,$$
$$\ddot{y}_2 + (\cos y_1)\dot{y}_2 + 3(y_1 - y_2) = u_2 - (\cos y_1)^2 y_2 u_1$$

(a) Can these equations be written in the form

$$u_1 = f_1(\ddot{y}, \dot{y}, y); \qquad u_2 = f_2(\ddot{y}, \dot{y}, y)$$

(b) Find an inverse dynamics control so that the closed loop system is linear and decoupled, with each subsystem having natural frequency 10 radians, and damping ratio 0.5.

3. Consider the parallelogram-based single degree of freedom robot mechanism shown below. The mechanical structure is such that the opposing links of the manipulator remain parallel to each other at all times.

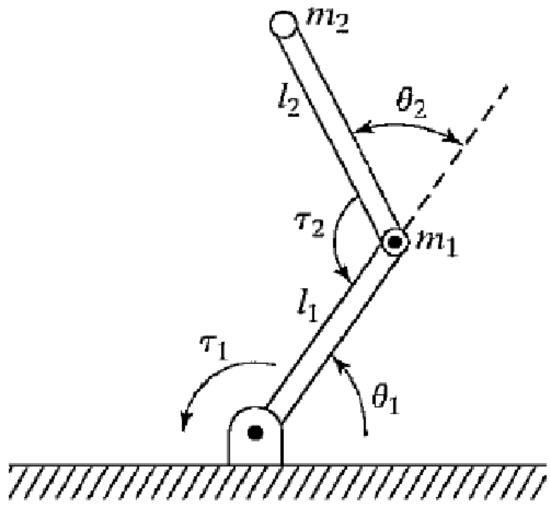


Derive the equation of motion for the manipulator in the form $\tau = f(\ddot{\theta}, \dot{\theta}, \theta)$, where, τ is the torque applied to the input link and design a critically damped set point PD controller with ω = 4 rad/s, to move the input link to a desired angle θ_d.

4. Consider the planar 2R manipulator shown below. The links are modeled as point masses with the mass concentrated at the ends. Hence, the moment of inertia of the links about the axes passing through the center of mass can be neglected.

   Each of the links is driven with identical geared motors with gear ratio r, rotor inertia J_0 and damping B_0. You may assume that the torque generated by the motors is given by KV/R where V is the applied voltage, and K & R are constants.

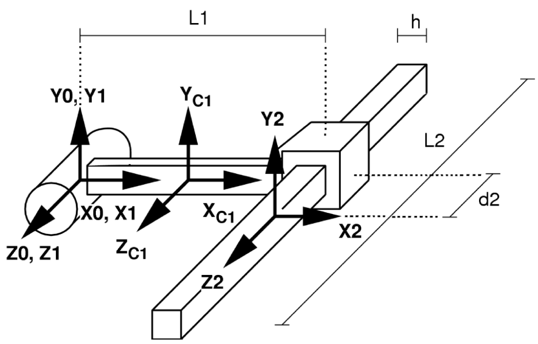
   a) Derive the dynamic equations governing the motion of this manipulator in terms of the motor angles. Assume $l_1 = l_2$ and $m_1 = m_2$. (Done in earlier assignment.)
   b) Find a set of joint trajectories (as functions of time) that will move the robot end-effector along a circle of radius $3l_1/2$ units at a speed of 10π rad/s.
   c) Design an independent joint PD trajectory tracking controller to follow the



   joint trajectories computed above. The controller should be critically damped with ω = 36 rad/s for the closed loop system. For controller design, you may neglect any terms that are divided by r in your equations.

   d) What is the steady state error if a disturbance torque τ_d. acts on joint 1.

5. Consider the robot manipulator shown below. Links of this manipulator are modeled as bars of uniform density, having square cross-sections of thickness h, lengths L_1 and L_2 and total masses of m_1 and m_2, with center of mass as shown. Assume that the joints to be massless.



(a) For each link i, we have attached a frame $\{C_i\}$ to the center of mass (frame $\{2\}$ is same as frame $\{C_2\}$. Calculate matrices ${}^{0}_{C_1}T$ and ${}^{0}_{C_2}T$.

For this two-link manipulator, the inertia matrix has the form

$$D(q) = m_1 J_{v1}^T J_{v1} + m_2 J_{v2}^T J_{v2} + J_{\omega 1}^T I_{c1} J_{\omega 1} + J_{\omega 2}^T I_{c1} J_{\omega 2}$$

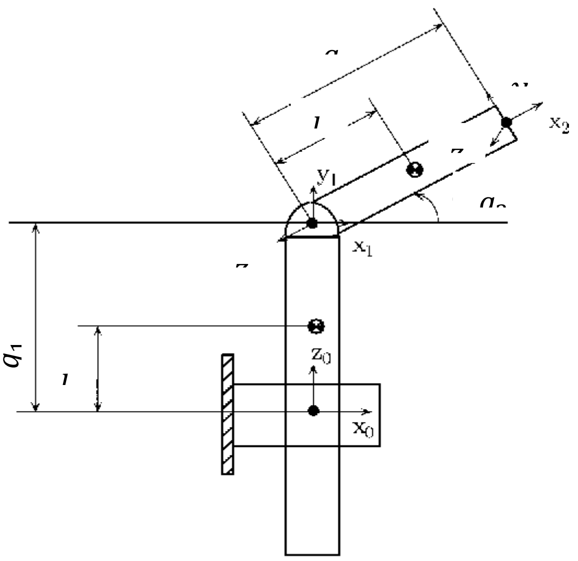
where, J_{vi} is the Jacobian of the center of mass of link i, $J_{\omega i}$ is the angular velocity of link i, and I_{ci} is the inertia tensor of link i expressed in frame $\{C_i\}$.

(b) Calculate ${}^{0}J_{v1}$ and ${}^{0}J_{v2}$.
(c) Calculate ${}^{c1}J_{\omega 1}$ and ${}^{c2}J_{\omega 2}$.
(d) Calculate I_{c1} and I_{c2} in terms of the masses and dimensions of the links.
(e) Calculate the inertia matrix $D(q)$.
(f) Calculate the other terms (gravity vector, Coriolis and centrifugal terms) and write out the equations of motion as:

$$\tau_1 = f_1(\ddot{q}, \dot{q}, q); \qquad \tau_2 = f_2(\ddot{q}, \dot{q}, q)$$

(g) Design an inverse dynamics based decoupling controller for the robot to follow an helical path with pitch p at some constant speed.

6. Consider the planar 2-link manipulator shown below.



a) Transform the dynamic equations derived in the previous assignment to obtain the equations of motion in task space.
b) Design a task-space nonlinear decoupling PD trajectory controller with ω = 36 rad/s to follow some desired trajectory $[x_{.}(t), y_{.}(t)]$.