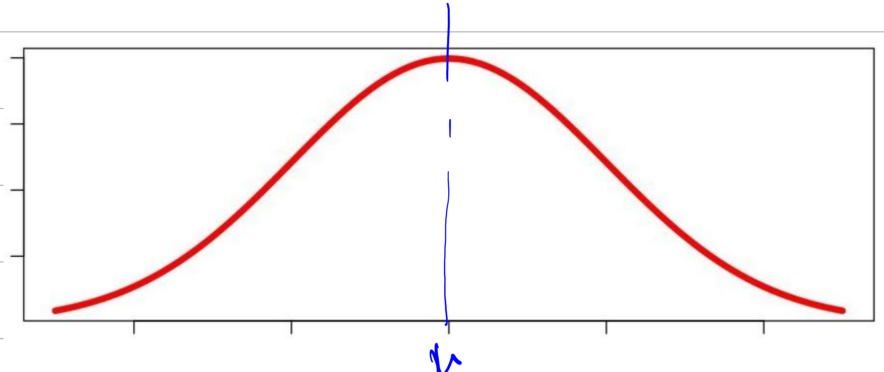
Recap

- What is a **variable** (say, y)?
- What is the **probability density function** ‘f(y)’ of the variable ‘y’ ?
- What is the **function g(y)** defined over the variable ‘y’?
- What is **‘sample’ vs ‘population**’ of ‘y’?
- What is the **‘expected value’** or expectation E[]? Say E[y] or E[g(y)]?

- **Given the PDF** of a variable ‘y’, how do we figure out **mean, median, mode, variance**, …

Normal or Gaussian PDF

DIY



$$f(y) = \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-b}{a}\right)^2\right) \quad y \in [-\infty, \infty]$$

DIY

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right)$$

- What is mean?

$\mu = b$

- What is variance and std. deviation?

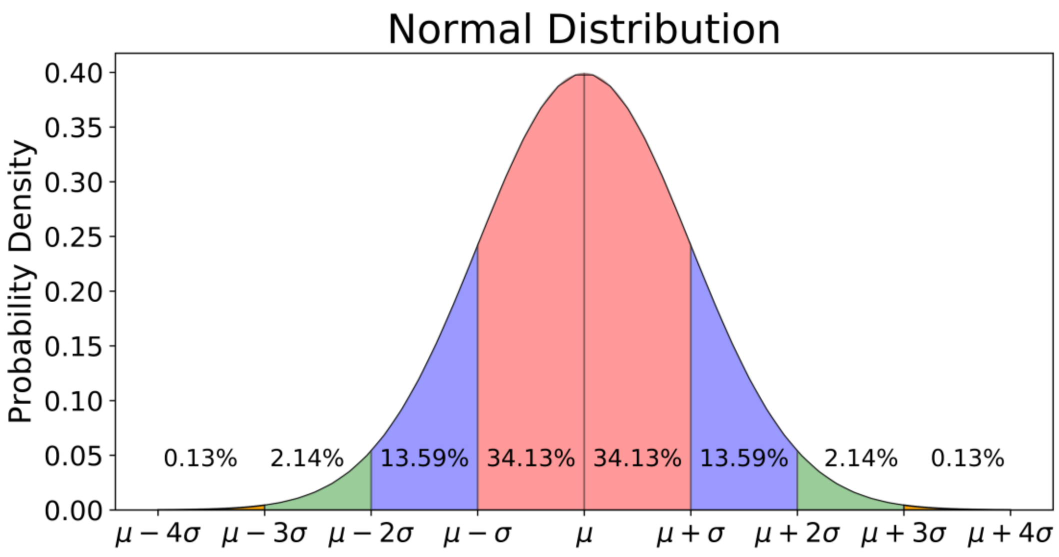
$\sigma^2 = a^2 \quad , \quad \sigma = a$

- What are median and mode?

mean = median = mode = μ = b

Normal PDF

- The 'Normal' distribution, does an excellent job of approximating the relative frequencies of many natural and "man-made" phenomena, e.g. dimension of machined parts, the strength of steel samples, etc.
- It is a bell-shaped curve, symmetric about the objects, which fell off quite rapidly beyond a distance of about one standard deviation from the mean.



NOTE: Although the PDF is defined from –∞ to +∞, most of the density is distributed over a narrow range near the mean (μ)

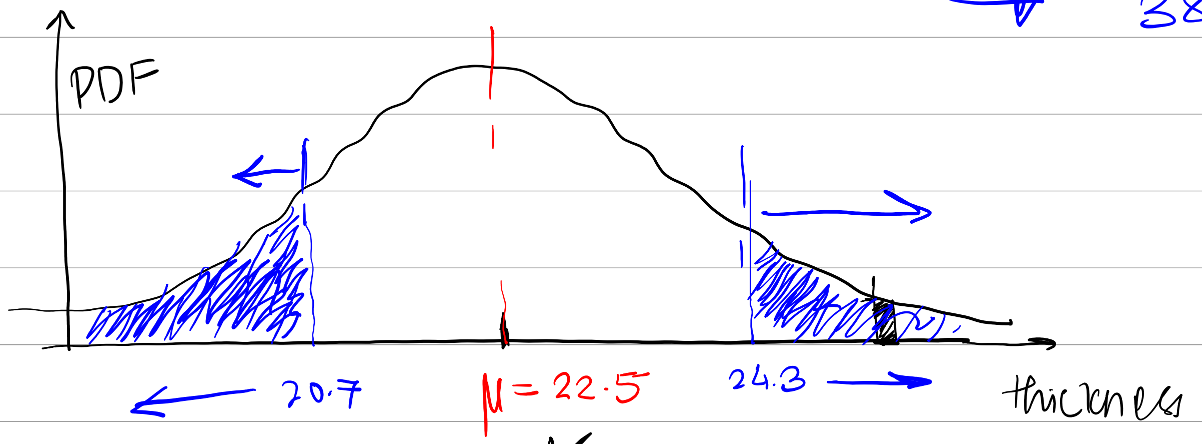
- **68.26% of the observations fall between $\mu - \sigma$ and $\mu + \sigma$**
- **95.46% of the observations fall between $\mu - 2\sigma$ and $\mu + 2\sigma$**
- **99.73% of the observations fall between $\mu - 3\sigma$ and $\mu + 3\sigma$**

Normal PDF Example

The heat shield plates for the space shuttle must have a closely measured thickness in order to withstand the rigors of heat from re-entry.

After testing 400 of them, the engineer found the thickness was normally distributed with a mean of 22.5 mm. It was also found that 382 plates were within 22.5 ± 2.50 mm.

If the defective plates deviate more than 1.80 mm from the mean, find the number of plates to be rejected during the testing



→ 382/400 are within 22.5 ± 2.50 mm

How many out of 400 beyond (20.7, 24.3)

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right)$$

~95% plates are within 22.5 ± 2.50 mm

I also know that in ND, 95% population lies bet $\mu \pm 2\sigma$

$\Rightarrow 2\sigma = 2.50$ mm $\Rightarrow \sigma = 1.25$

Normal PDF Example

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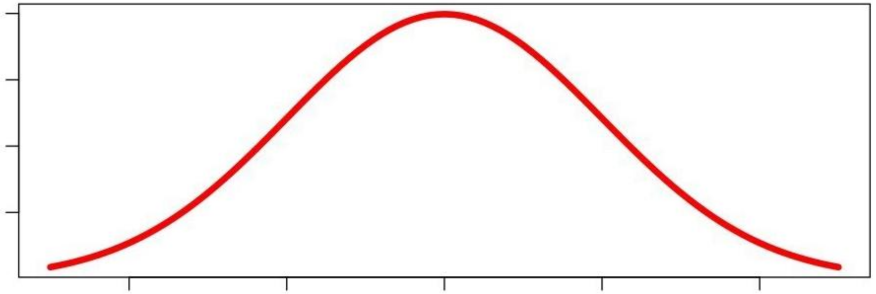
If the defective plates deviate more than 1.80 mm from the mean, find the number of plates to be rejected during the testing

Once we know μ & σ we have $f(y)$

fraction of plates rejected $= \int_{-\infty}^{20.7} f(y)\,dy + \int_{24.3}^{\infty} f(y)\,dy$

Standard Normal PDF



'General Form'

$$f(y) = \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-b}{a}\right)^2\right), \qquad -\infty \le y \le \infty$$

Substitute, $z = \frac{y-\mu}{\sigma}$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right), \qquad -\infty \le z \le \infty$$

$$\mu = 0, \sigma = 1$$

Standard Normal Probabilities

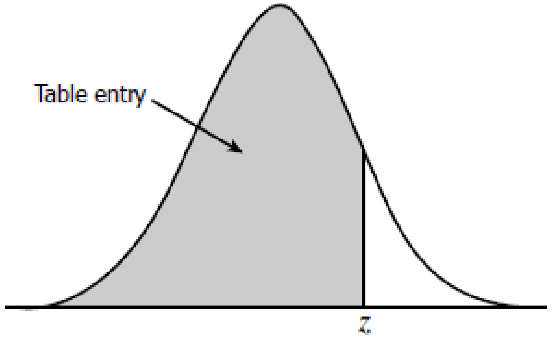


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177

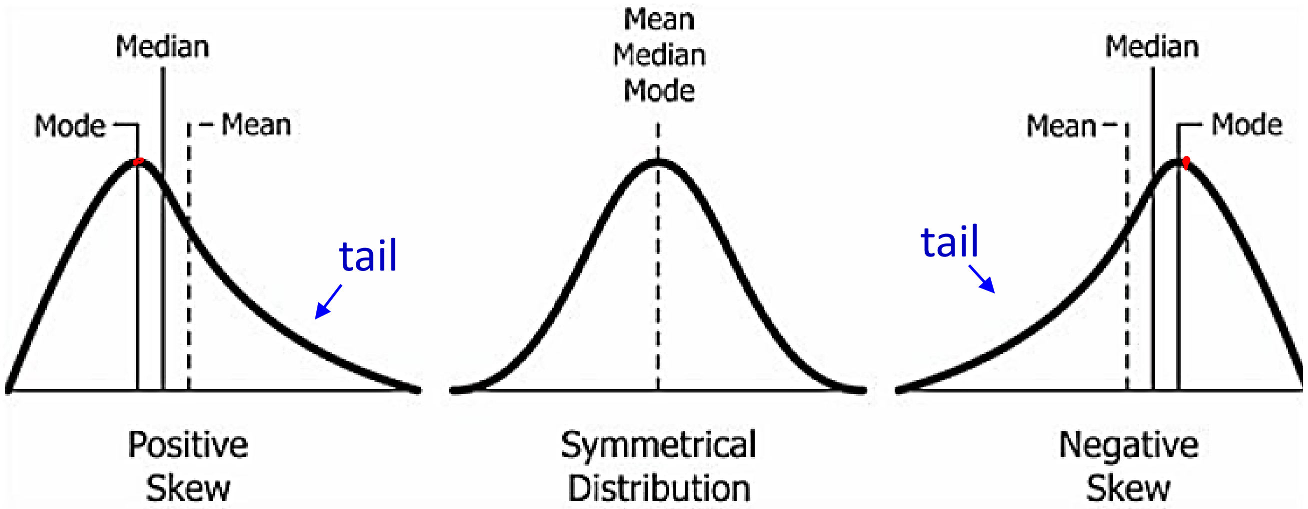
Other Statistical Parameters

Name	Definition	Symbol
mean	$E[X]$	μ
variance	$E[(X-\mu)^2]$	σ^2
standard deviation	$\sqrt{\sigma^2}$	σ
skewness	$E[(X-\mu)^3]/\sigma^3$	γ_1
kurtosis	$E[(X-\mu)^4]/\sigma^4 - 3$	γ_2

Skewness

skewness	$E[(X-\mu)^3]/\sigma^3$	γ_1

- Skewness can tell us about symmetry: . It measures the lack of symmetry in data distribution.
- It is the degree of distortion from the symmetrical bell curve or the normal distribution
- It differentiates extreme values in one versus the other tail
- A symmetrical distribution will have a skewness of 0

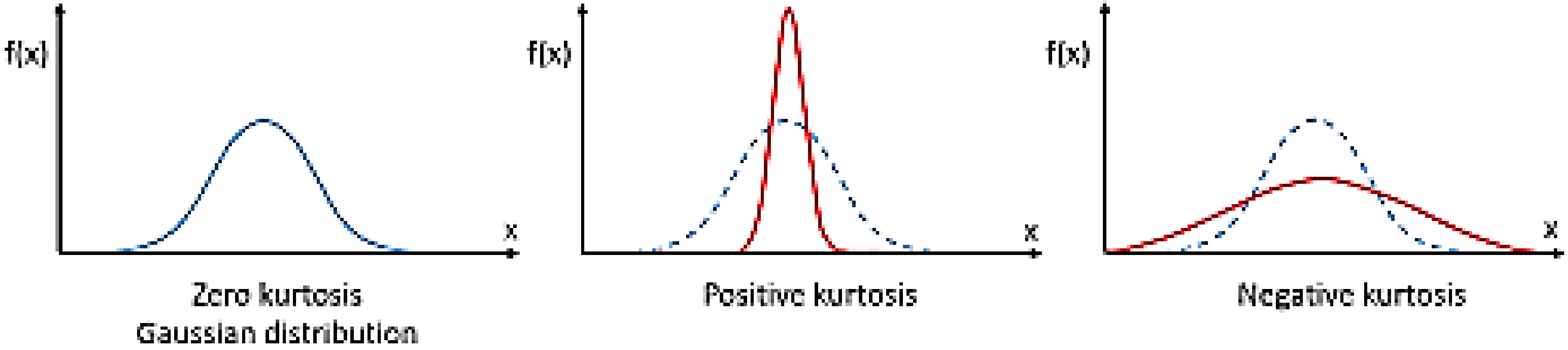


- If the skewness is between -0.5 and 0.5, the data are fairly symmetrical.
- If the skewness is between -1 and -0.5 (negatively skewed) or between 0.5 and 1 (positively skewed), the data are moderately skewed.
- If the skewness is less than -1 (negatively skewed) or greater than 1 (positively skewed), the data are highly skewed.

Kurtosis

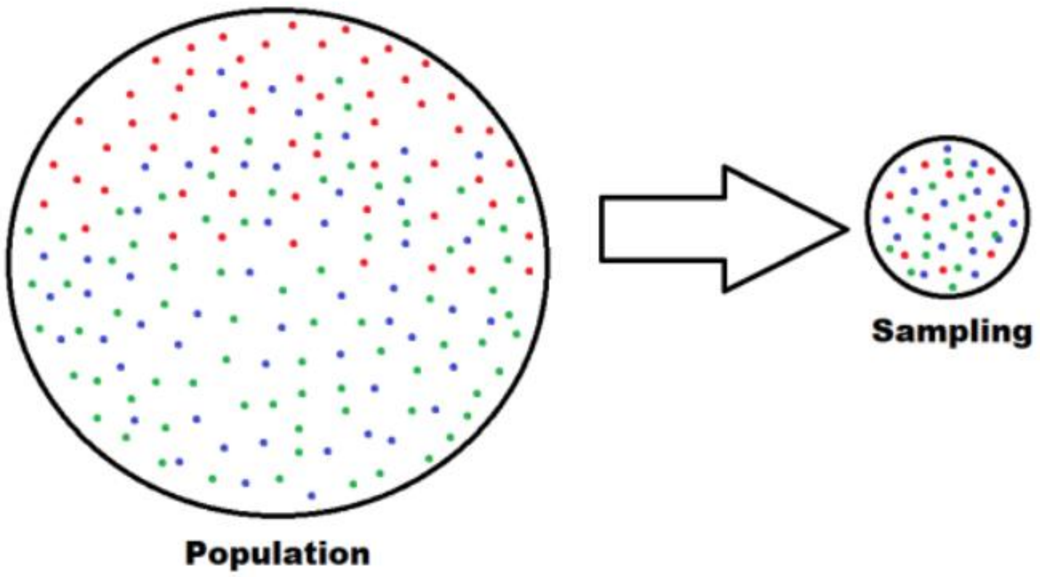
kurtosis	$E[(X-\mu)^4]/\sigma^4 - 3$	γ_2

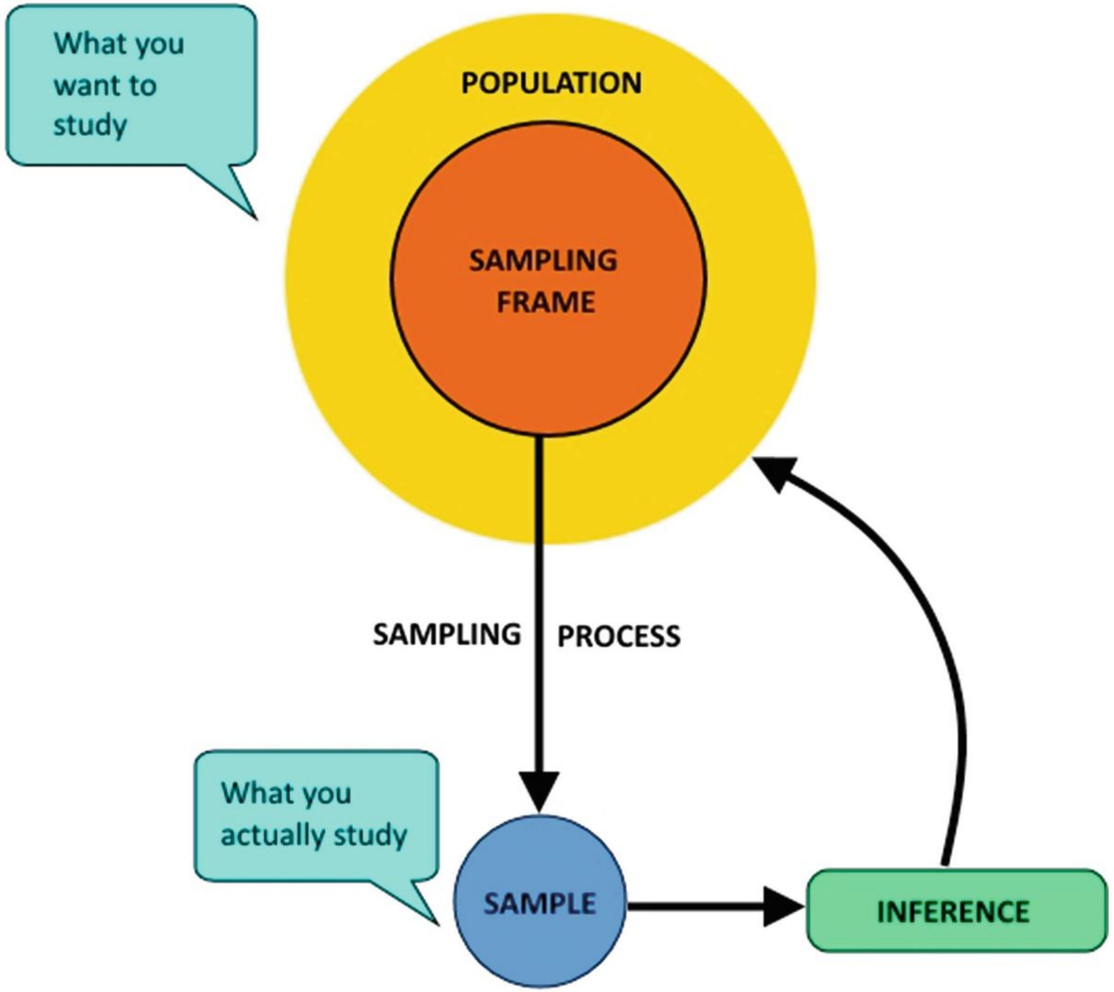
- Kurtosis is all about the tails of the distribution — the peakedness or flatness.
- It is used to describe the extreme values in one versus the other tail.
- It is actually the measure of outliers present in the distribution.
- High kurtosis data has heavy tails or outliers.
- Low kurtosis data has light tails or lack of outliers.



What is Sampling?

https://images.app.goo.gl/cvN1oMakejPsdhva7

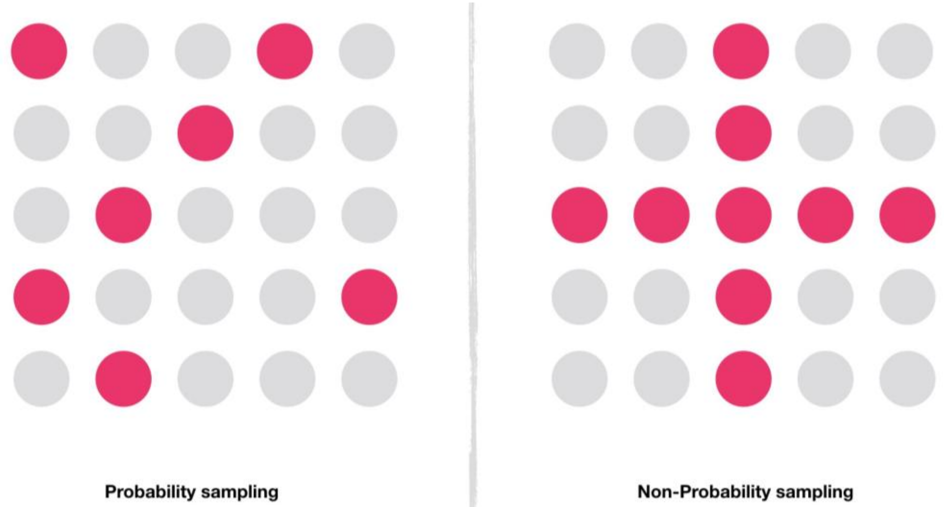




Sampling Methods

- **Probability Sampling**
  - When *each entity* of the population has *a finite, non-zero probability* of being into the sample
  - Sampling procedure involves random sampling and without bias
- **Non-probability Sampling**
  - Some units of the population have zero chance of selection
  - OR probability of selection cannot be determined accurately

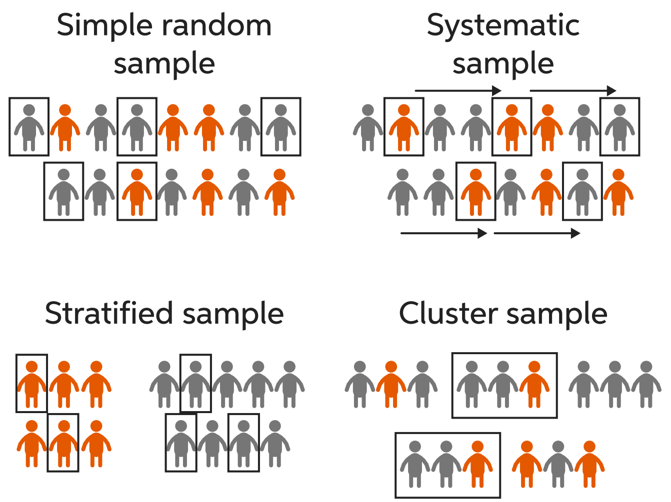
Probability sampling	Non-probability sampling
The samples are randomly selected.	Samples are selected on the basis of the researcher's subjective judgment.
Everyone in the population has an equal chance of getting selected.	Not everyone has an equal chance to participate.
Researchers use this technique when they want to keep a tab on sampling bias.	Sampling bias is not a concern for the researcher.
Useful in an environment having a diverse population.	Useful in an environment that shares similar traits.
Used when the researcher wants to create accurate samples.	This method does not help in representing the population accurately.
Finding the correct audience is not simple.	Finding an audience is very simple.



https://www.questionpro.com/blog/probability-sampling/

Probability Sampling

- **Simple Random Sampling**
  - Each subject/unit selected at random, independent from each other
  - Typically done when the population is large
- **Systematic Sampling**
  - Arrange the population in some order, and pick a unit at regular intervals from the list
  - When population is logically homogenous
  - E.g. You ask every 10^{th} customer entering a shop about his purchase habits
- **Stratified Sampling**
  - Population divided into groups/stratas based on some characteristics
  - Then population is sampled randomly within each strata
  - E.g. If 38% of the population is college-educated, then 38% of the sample is randomly selected from the college-educated subset of the population
- **Cluster Sampling**
  - Random sample is drawn from a cluster of data, rather than individual samples
  - E.g. An NGO wants to create a sample of girls across five neighboring towns to provide education. Using single-stage sampling, the NGO randomly selects towns (clusters) to form a sample and extend help to the girls deprived of education in those towns.



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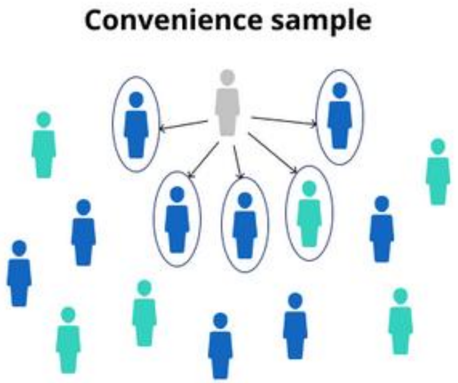
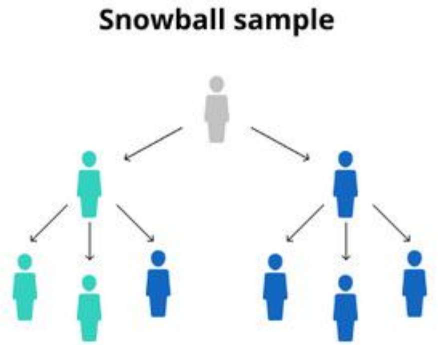
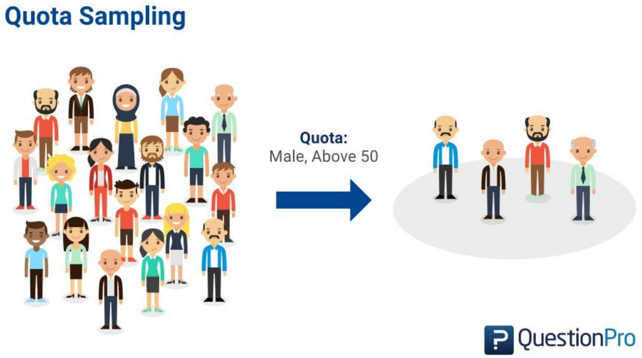
Non-Probability Sampling

- **Convenience Sampling**
  - Each subject/unit is selected on the basis of convenience, availability, reach, etc.
  - Typically during preliminary research
- **Snowball Sampling**
  - One unit refers you to the next unit
  - Costs of sampling are lower





- **Quota Sampling**
  - Population divided into mutually exclusive subgroups and non-random set of observations chosen from each subgroup



Distribution of Sample Means

CEP2022_Notebook (1.6)

- We say that the sample mean ($\bar{y}$) gives us *an estimate* of the population mean μ
- But, since the sample is only a small subset of the entire population, *$\bar{y}$ is an uncertain estimate of μ*

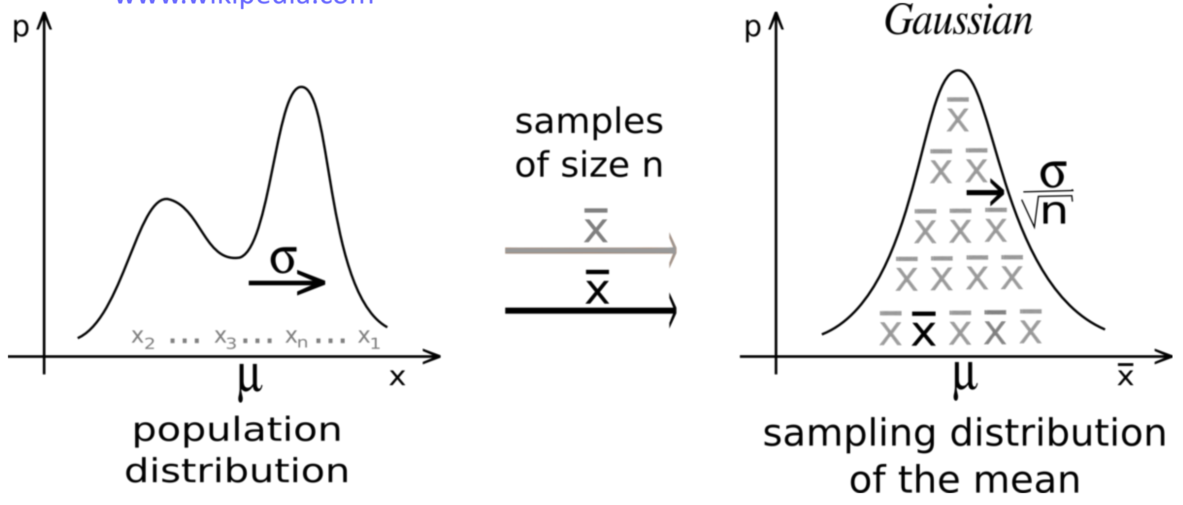
- What if, we sample the population *several times*, each time calculating the sample mean $\bar{y}$
- Let's say we do it 'k' times, we get sample means as $\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4, \ldots.., \bar{y}_k$
- How would these sample means behave?
  - How close are they from *μ?*
  - What's the *mean of sample means*?
  - What's the *standard deviation of sample means*?
  - More importantly, *what's their frequency distribution*?

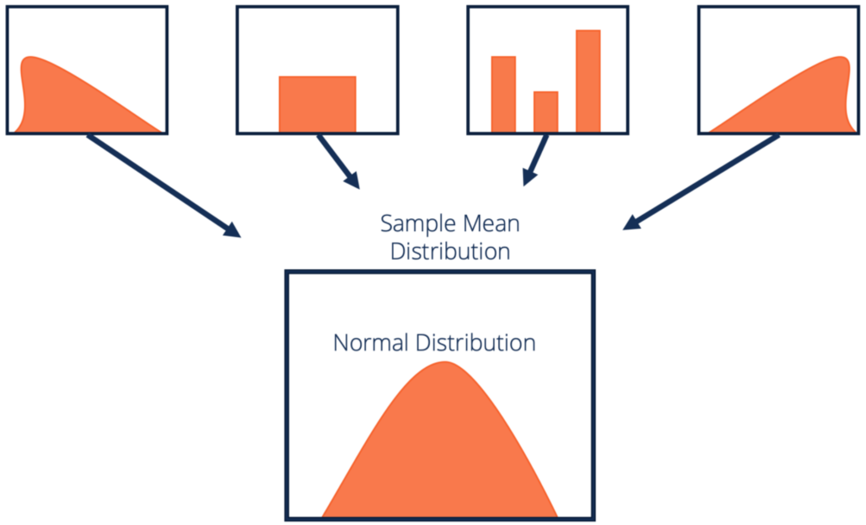
Central Limit Theorem

"The *distribution of sample means* $(\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4, \dots, \bar{y}_k)$ follows *a normal distribution*, even when the original variable y is NOT normally distributed."

www.moriah.com

www.wikipedia.com





- What is the mean of distribution of sample means?

- What is the variance of distribution of sample means?

Central Limit Theorem

DIY: Read

"In non-mathematical language, the "CLT" says that whatever the PDF of a variable is, if we randomly sample a "large" number (say k) of independent values from that random variable, the sum or mean of those k values, if collected repeatedly, will have a Normal distribution.

It takes some extra thought to understand what is going on here. The process I am describing here takes a sample of (independent) outcomes, e.g., the weights of all of the rats chosen for an experiment, and calculates the mean weight (or sum of weights). Then we consider the less practical process of repeating the whole experiment many, many times (taking a new sample of rats each time). If we would do this, the CLT says that a histogram of all of these mean weights across all of these experiments would show a Gaussian shape, even if the histogram of the individual weights of any one experiment were not following a Gaussian distribution.

By the way, the distribution of the means across many experiments is usually called the sampling distribution of the mean."

- Seltman, Howard J. "Experimental design and analysis." (2012)