ME604: Introduction to Robotics

Spring 2025

Quiz 2

- **Duration: 45 minutes**
- **Write legibly and show all your work.**
- **You may refer to up to 1 A4 sized sheet of handwritten material during the exam.**

Similar to Assignment 4 P2. See solution.

1. [3 marks] Given the Euler angle transformation $R = R_z(\psi)R_x(\theta)R_z(\phi)$. Derive the expression for ω such that $dR/dt = S(\omega)R$. You need not multiply matrices to provide the expanded expression.

2. [3 marks] Consider the two-link RP manipulator in Fig 1a. Determine, through direct differentiation, the 3X2 Jacobian matrix that relates the velocity of the center of mass of link 2 (origin of frame {2}) to the joint rates $\dot{\theta}_1$ and $\dot{d}_2$. write the position vector, p = [L1c1 L1s1 d2]. Differentiate to find velocity expression, and write in matrix form to get the Jacobian.

3. Consider the 6 DOF manipulator shown in Fig 1b.

   a. [3 marks] Assign DH frames for the manipulator Done in Q1. No need to make table of DH parameters
   b. [2 marks] At the instant shown, what will the angular velocity of the end-effector (expressed in ground frame) be if all revolute joints turn at unit rate? See A4. P4
   c. [1 mark] Assume that at the instant shown, joint 2 rotates at 1 rad/s. Is it possible to choose other joint rates such that the resultant velocity of point P is 1 m/s in horizontal direction? If yes, suggest one such set of joint rates along with a suitable value for b_3; if not, justify your answer. See A4. P3, P4
   d. [6 marks] Derive the expression (as a function of joint variables) for the 3X3 Jacobian relating the velocity of the center of the wrist 'C' to the joint rates corresponding to the first 3 joints.
   e. [BONUS: 2 marks] Identify any one singular configuration based upon the expression obtained in part (d) above, and comment on how the robot arm loses 'mobility' in that configuration.

Note that rectangles denote revolute joints with axes lying in the plane of the figure (axis of joint being parallel to the long edge of the rectangle), while circles with dots indicate revolute joints with axes normal to the plane of the figure. The dashpot indicates a prismatic joint.

2d. This is longer. You need write the three transformation matrices from frame 1 to frame 0, and from frame 2 to frame 0.
From the transformation matrices, find z1 and z2 (col 3) in terms of frame {0} unit vectors, and locations of origins o1 and o2. z0 is just [0 0 1].
Write the tranformation matrix from frame 3 to frame 0 (you only need the last col) to find location of o3.
Write the Jacobian using the formula derived
[z0X(o3 - o0) z1X(o3 - o1) z2]
Note that joint 3 is prismatic.
Note that joint 4 doesn't change position of C!

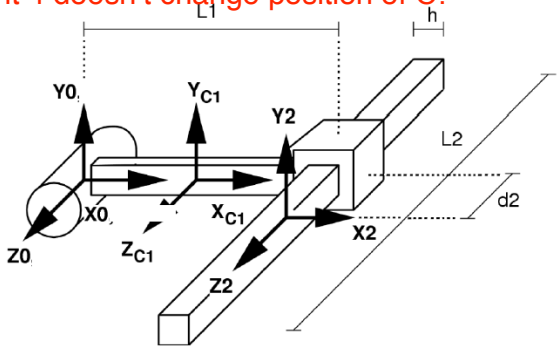

Figure 1(a)

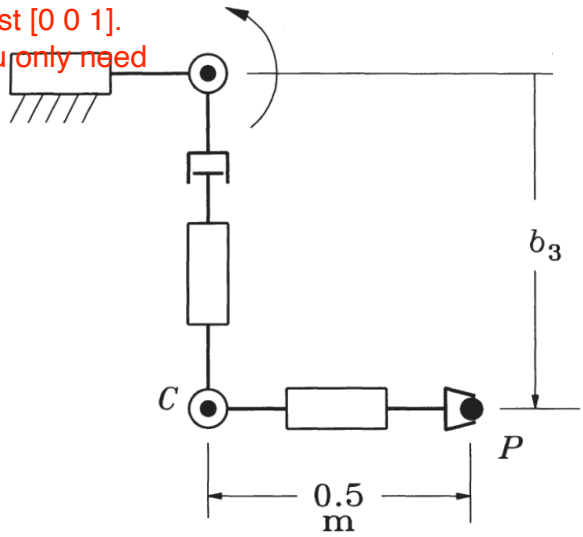

Figure1(b)