Date :

ME 604

Assignment #1

(1) coordinates frames 1 to 6 are in from 0

$${}^{0}_{1}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}_{2}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$${}^{0}_{3}R = \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{pmatrix}$$

$${}^{0}_{4}R = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

$${}^{0}_{5}R = \begin{pmatrix} 0 & -1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$$

$${}^{0}_{6}R = \begin{pmatrix} 1/2 & -1/\sqrt{2} & -1/\sqrt{2} \\ -1/2 & -1/\sqrt{2} & 1/2 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$$

$${}^{1}_{2}R = {}^{1}_{0}R \, {}^{0}_{2}R = {}^{0}_{1}R^{T} \, {}^{0}_{2}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{2}_{0}R = {}^{0}_{2}R^{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

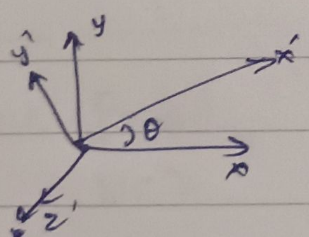
$${}^{2}_{3}R = {}^{0}_{2}R^{T} \, {}^{0}_{3}R = \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & 1/2 \\ -1/\sqrt{2} & 1/2 & 1/2 \end{pmatrix}$$

$${}^{3}_{4}R = {}^{0}_{3}R^{T} \, {}^{0}_{4}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$${}^{2}_{5}R = {}^{0}_{4}R^{T} \, {}^{0}_{5}R = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$${}^{5}_{6}R = {}^{0}_{5}R^{T} \, {}^{0}_{6}R = \frac{1}{2}\begin{pmatrix} 1+\frac{1}{\sqrt{2}} & 1 & \frac{-1}{\sqrt{2}}+1 \\ -1 & \sqrt{2} & 1 \\ \frac{-1}{\sqrt{2}}+1 & -1 & 1+\frac{1}{\sqrt{2}} \end{pmatrix}$$

(2) (a)



$${}^{1}(x') = x\cos\theta + y\sin\theta$$

$${}^{1}(y') = -x\sin\theta + y\cos\theta$$

$${}^{1}z' = z$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b)

$$y' = \cos\theta \, y + \sin\theta \, z$$

$$z' = -\sin\theta \, y + \cos\theta \, z$$

$$x' = x$$

$$R_{x'}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{pmatrix}$$

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$z' = \cos\theta\, z + \sin\theta\, x$ $\qquad R_y(\theta) = \begin{pmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{pmatrix}$

$x' = -\sin\theta\, z + \cos\theta\, x$

$y' = y$

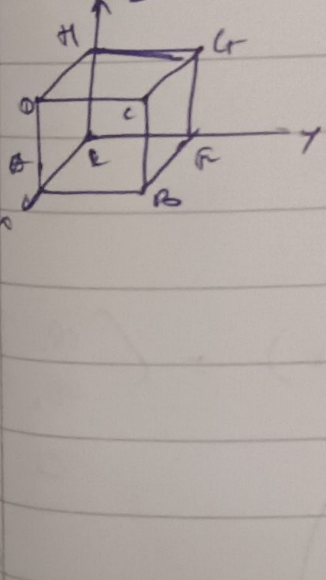
2) $\;{}^{A}_{B}R = R_x(-45^\circ) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

II) $\;R_y(90^\circ)\,{}^{A}_{B}R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{pmatrix}$

IV) $\;R_x(-45^\circ)\,{}^{A}_{B}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$

III) $\;R_z(45^\circ)\,{}^{A}_{B}R = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$

(V) $\;R_z(45^\circ)\,{}^{A}_{B}R = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$



final oreo :-

$\hat{EP} = -\cos 45^\circ\, \hat{i} + \sin 45^\circ\, \hat{j} = \frac{1}{\sqrt{2}}(-\hat{i} + \hat{j} + 0\hat{k})$

$\hat{EH} = \frac{1}{\sqrt{2}}(-\hat{i} - \hat{j} + 0\hat{k})$

$\hat{EA} = \hat{k}$

initial oreo :-

$\hat{EA} = \hat{i}, \quad \hat{EP} = \hat{j}, \quad \hat{EH} = \hat{k}$

$${}^{A}_{B}R = \begin{pmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} {}^{A}_{B}R & {}^{A}r_{AB} \\ 0 & 1 \end{pmatrix}$$

a) R, then Translatⁿ by $[1,1,1]^T$

$$T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & -\frac{1}{2} & 1 \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 1 \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

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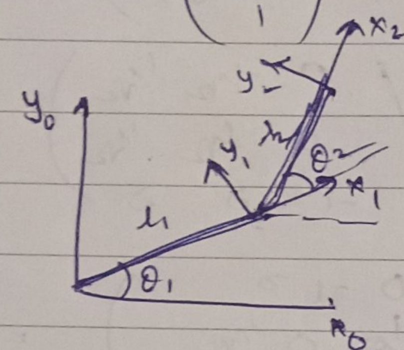
b) Translat^n by $[1\,1\,1]^T$ then R

$$Rv = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt2} & \frac{1}{2} \\ -\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} \end{pmatrix} = {}^{A}P_{AB}$$

$$T = \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt2} & \frac{-1}{2} & \frac{-1}{\sqrt2} \\ \frac{-1}{2} & \frac{-1}{\sqrt2} & \frac{1}{2} & \frac{-1}{\sqrt2} \\ -\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} {}^{A}P \\ 1 \end{pmatrix} = T \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} = {}^{A}P = \begin{pmatrix} 2-\frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} \\ \frac{1}{\sqrt2}+1 \end{pmatrix}$$

(5) a)



a) ${}^{0}_{1}T = \begin{pmatrix} {}^{0}_{1}R & {}^{0}P_{0B} \\ 0 & 1 \end{pmatrix}$

$${}^{0}_{1}R = R_z(\theta_1) = \begin{pmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}_{1}T = \begin{pmatrix} c\theta_1 & -s\theta_1 & 0 & l_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{1}_{2}T = \begin{pmatrix} {}^{1}_{2}R & {}^{1}P_{12} \\ 0 & 1 \end{pmatrix} \; ; \; {}^{1}_{2}R = R_z(\theta_2) = \begin{pmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{1}_{2}T = \begin{pmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ s\theta_2 & c\theta_1 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $${}^{0}_{2}T = {}^{0}_{1}T\,{}^{1}_{2}T = \begin{pmatrix} {}^{0}_{1}R & {}^{0}P_{01} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} {}^{1}_{2}R & {}^{1}P_{12} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^{0}_{1}R\,{}^{1}_{2}R & {}^{0}_{1}R\,{}^{1}P_{12} + \\ 0 & 1 \end{pmatrix}$$

$${}^{0}_{2}R = R_z(\theta_1+\theta_2) = \begin{pmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$${}^{0}_{1}R\,{}^{1}P_{12} + {}^{0}P_{01} = \begin{pmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} l_2 c\theta_2 \\ l_2 s\theta_2 \\ 0 \end{pmatrix} + \begin{pmatrix} l_1 c\theta_1 \\ l_1 s\theta_1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} l_2 c(\theta_1+\theta_2) + l_1 \cos\theta_1 \\ l_2 s(\theta_1+\theta_2) + l_1 s\theta_1 \end{pmatrix}$$

$${}^{2}_{0}T_1 = \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 & l_2 c(\theta_1+\theta_2) + l_1 c\theta_1 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 & l_2 s(\theta_1+\theta_2) + l_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

II)

$${}^{0}_{1}T = \begin{pmatrix} {}^{0}_{1}R & {}^{0}P_{01} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{1}_{2}T = \begin{pmatrix} {}^{1}_{2}R & {}^{1}P_{12} \\ 0 & 1 \end{pmatrix} = \begin{bmatrix} s\theta & c\theta & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -c\theta & s\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{2}_{3}T = \begin{pmatrix} {}^{2}_{3}R & {}^{2}P_{23} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $${}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T$$

$$= \begin{pmatrix} {}^{0}_{2}R & {}^{0}P_{03} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} c\theta & -s\theta & 0 & a+Lc\theta \\ s\theta & c\theta & 0 & d+Ls\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^{0}P_{03} = d\,\hat{y}_0 + a\hat{x}_0 + L[\cos\theta\,\hat{x}_0 + \sin\theta\,\hat{y}_0)$$

$$= (a+L\cos\theta)\,\hat{x}_0 + (d+L\sin\theta)\,\hat{y}_0$$

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6) $^{w}p_{wE} = (75, 100, 53)^T$

$^{0}_{E}x_E = (-12, 9, 0)^T \;=\; ^{w}x_E = \left(\frac{-4}{5}, \frac{3}{5}, 0\right)^T$

$^{0}_{E}y_E = (9, 12, 20)^T \;=\; ^{w}y_E = \left(\frac{9}{25}, \frac{12}{25}, \frac{20}{25}\right)^T$

$^{0}_{E}z_E = (24, 32, -30)^T \;=\; ^{w}z_E = \left(\frac{12}{25}, \frac{16}{25}, \frac{-3}{5}\right)^T$

$$^{w}_{E}R = \begin{pmatrix} -4/5 & 9/25 & 12/25 \\ 3/5 & 12/25 & 16/25 \\ 0 & 4/5 & -3/5 \end{pmatrix}$$

$$^{w}_{E}T = \left(\begin{array}{c|c} ^{w}_{E}R & ^{w}p_{wE} \\ \hline 0 & 1 \end{array}\right) = \left[\begin{array}{c|c} - & \\ \hline 0 & 1 \end{array}\right]$$

$$^{B}_{E}T_1 = \begin{pmatrix} -4/5 & 9/25 & 12/25 & 68 \\ 3/5 & 12/25 & 16/25 & 174 \\ 0 & -4/5 & -3/5 & -60 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} ^{B}_{E}R & ^{B}p_{BE} \\ 0 & 1 \end{pmatrix}$$

$$^{w}_{B}T = \begin{pmatrix} ^{w}_{E}T & ^{E}_{B}T \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 143 \\ 0 & 1 & 0 & 274 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7) a) $^{F_1}\vec{OP} = (1, -1, 1)^T$

$^{F_2}\vec{SP} = (-1, 0, 0)^T$

$^{F_1}\vec{SP} = {}^{F_1}OP_1 + {}^{1}_{2}R \, {}^{F_2}\vec{SP}$

$$= \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & 1 & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \tan\sqrt{3} \\ \tan \end{pmatrix}$$

$$= \begin{bmatrix} 2/3 \\ -\frac{4}{3} - \frac{1}{\sqrt{3}} \\ \frac{2}{3} + \frac{1}{\sqrt{3}} \end{bmatrix}$$

b) $^{F_1}[x + y + z] = 1$

$$\begin{bmatrix} ^{2}_{1}R & ^{2}p_{21} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \\ 1 \end{bmatrix} = \begin{pmatrix} 2 \\ 9 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} \cancel{^{2}_{1}R} & \cancel{^{1}p_{21}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cancel{1} \end{bmatrix}$$

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$${}^{2}_{1}R = \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{2} & 1 & 1+\sqrt{3} \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 \end{bmatrix}$$

$${}^{1}P_{12} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \; ; \quad {}^{2}P_{21} = -{}^{2}_{1}R \; {}^{1}P_{12}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \\ 1+\sqrt{2} & 1-\sqrt{3} & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} -1+2\sqrt{3} \\ -1 \\ -2\sqrt{2}-1 \end{pmatrix}$$

$$\begin{bmatrix} \cancel{y} \\ x \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} {}^{2}R_{1} & {}^{1}P_{12} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

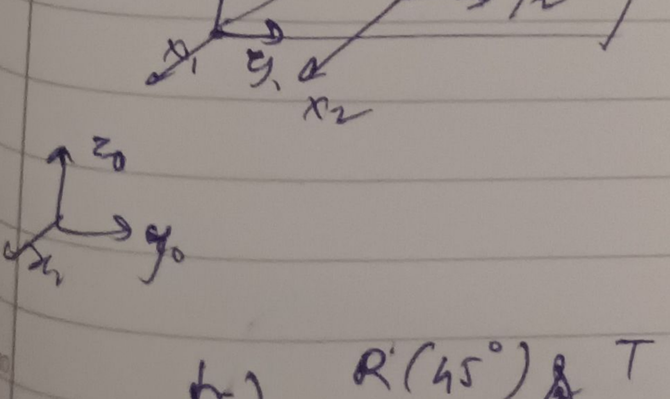
$$= \frac{1}{3}\begin{bmatrix} 2+2\sqrt{3} \\ 2 \\ 2-2\sqrt{3} \end{bmatrix}$$

eq^n in F_2

$${}^{2}[x\ y\ z]^{T} \cdot \frac{2}{3}\begin{bmatrix} \cancel{2}+\cancel{2}\sqrt{3} \\ \cancel{2} \\ \cancel{2}-\cancel{2}\sqrt{3} \end{bmatrix} = 1$$

$$\frac{2}{3}\left[(1+\sqrt{3})x + y + (1-\sqrt{3})z \right] = 1$$

8)



a) $${}^{0}_{1}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $${}^{0}_{2}T = \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c) $${}^{0}_{3}T = \begin{pmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $R(45^\circ)$ & $T \to [0.5, 0, 0]$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{4}T = {}^{0}_{2}T \; {}^{2}_{4}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 1.05 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{pmatrix}$$