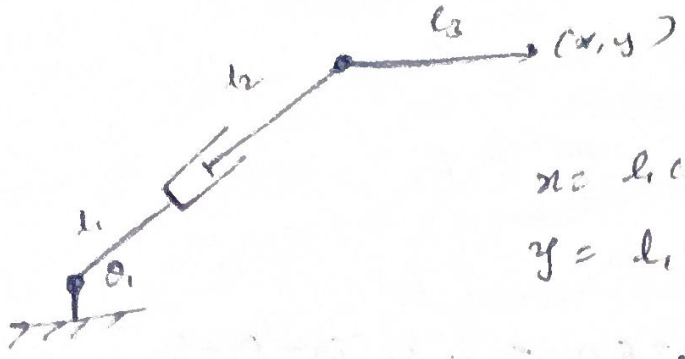
ME604 assignment #5

(1)



$$x = l_1 \cos\theta_1 + (l_2 + d_2)\cos(\theta_1 + \theta_3)$$
$$y = l_1 \sin\theta_1 + (l_2 + d_2)\sin(\theta_1 + \theta_3)$$

$$x' = x - l_3 \cos(\theta_1 + \theta_3)$$
$$y' = y - l_3 \sin(\theta_1 + \theta_3)$$

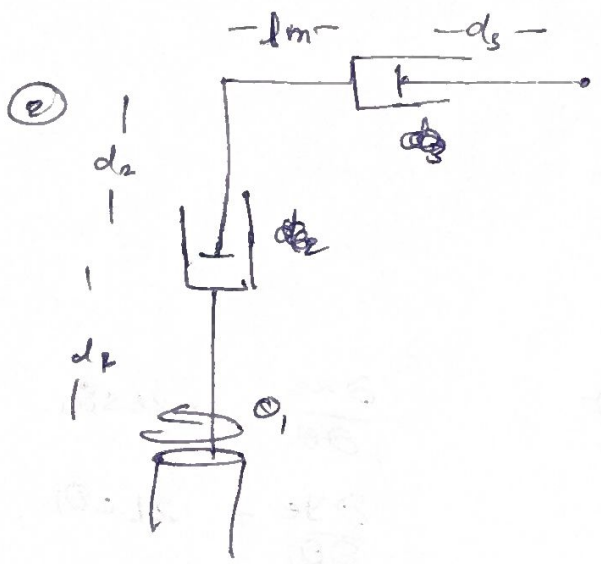
$$(x' - l_1\cos\theta_1)^2 + (y' - l_1\sin\theta_1)^2 = (l_2 + d_2)^2$$

$$\rightarrow \theta_1 = \tan^{-1}\left(\frac{y'}{x'}\right) \pm \cos^{-1}\left(\frac{x'^2 + y'^2 - (l_2+d_2)^2}{2l_1(l_2+d_2)}\right)$$

$$\rightarrow d_2 = \sqrt{(x' - l_1\cos\theta_1)^2 + (y' - l_1\sin\theta_1)^2} - l_2$$

$$\rightarrow \theta_3 = \tan^{-1}\left(\frac{y - l_1\sin\theta_1}{x - l_1\cos\theta_1}\right) - \theta_1$$

(2)

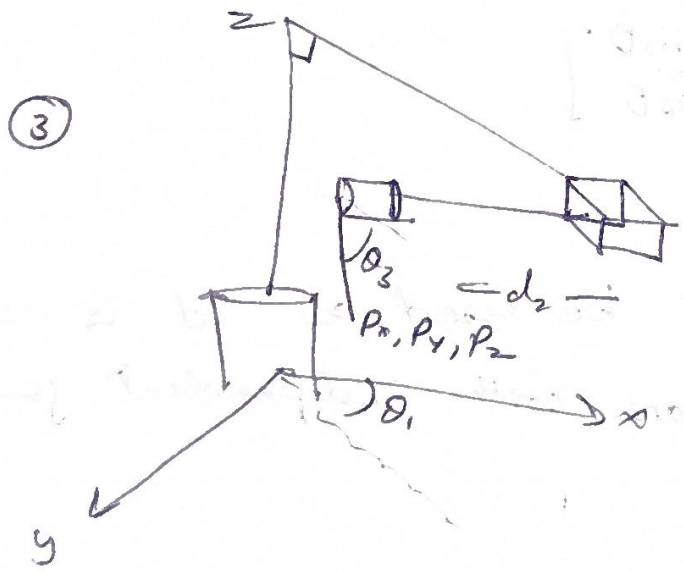


$$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

$$d_3 = \sqrt{P_x^2 + P_y^2} - l_1$$

$$d_2 = P_z - l_2$$

(3)



a) $$P_x = (l_1 + l_2\cos\theta_3)\cos\theta_1$$
$$P_y = (l_1 + d_2\cos\theta_3)\sin\theta_1$$
$$P_z = d_2 + l_2\sin\theta_3$$

$$\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

$$d_2 = P_z - l_2\sin\theta_3$$

$$\cos\theta_3 = \frac{P_x}{l_1 + l_2} \qquad \sin\theta_3 = \frac{P_z - d_2}{l_1 + l_2}$$

$$\theta_2 = \tan^{-1}\left(\frac{P_z - d_2}{P_x}\right)$$

b) ZYZ euler angles: $(45°, 30°, 45°)$

solve for $\theta_4, \theta_5, \theta_6$

$$R = R_z(45°)\, R_y(30°)\, R_z(45°)$$

$$\theta_4 = \tan^{-1}\left(\frac{r_{21}}{r_{11}}\right); \quad \theta_5 = \cos^{-1}(r_{33}); \quad \theta_6 = \tan^{-1}\left(\frac{r_{32}}{-r_{31}}\right)$$

(4) DH params:

	a_i	α_i	d_i	θ_i
1	305	−90	740	θ_1
2	1075	0	0	θ_2
3	250	−90	0	θ_3
4	0	90	1275	θ_4
5	240	90	0	θ_5
6	0	0	0	θ_6

$P_w = (P_{wx}, P_{wy}, P_{wz})$

$P_w = P_{ee} - a_6 R \hat{z}_T$ → const offset

↳ $P_{wx} = P_x - 240\, R_{13}$

$P_{wy} = P_y - 240\, R_{23}$

$P_{wz} = P_z - 240\, R_{33}$

$\theta_1 = \tan^{-1}\left(\frac{P_{wy}}{P_{wx}}\right)$

$$r = \sqrt{P_{wx}^2 + P_{wy}^2} \quad - q_1$$

$$z = P_{wz} - d_1$$

$L_1 = 107.5$, $L_2 = 250$

$$\cos\theta_3 = \frac{r^2 + s^2 - (L_1^2 + L_2^2)}{2 L_1 L_2} \quad ,$$

$$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right) - \tan^{-1}\left(\frac{L_2 \sin\theta_3}{L_1 + L_2 \cos\theta_3}\right)$$

$$\theta_4 = \tan^{-1}\left(\frac{R_{32}}{R_{31}}\right) \qquad R = R_{01}^{-1} R_{12}^{-1} R_{23}^{-1} R_{target}$$

$$\theta_5 = \cos^{-1}(R_{33})$$

$$\theta_6 = \tan^{-1}\left(\frac{R_{23}}{-R_{13}}\right)$$