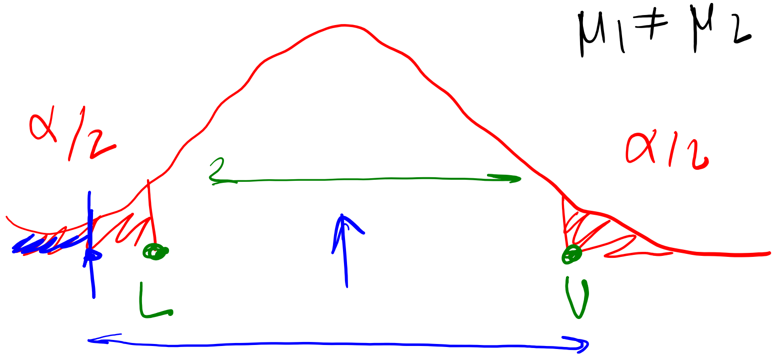
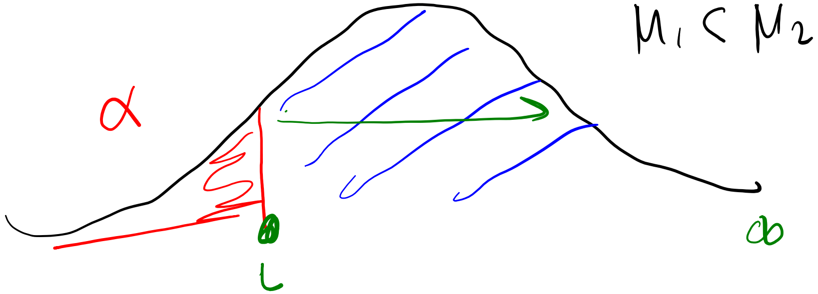
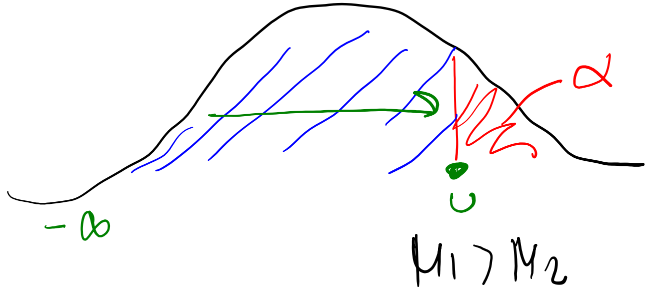
Recap

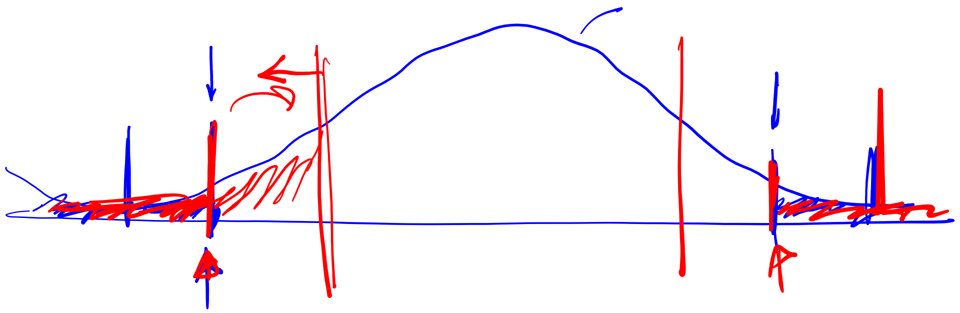






what is p value? $\Rightarrow$ minimum α at which H_0 is rejected.

any $\alpha < p \rightarrow H_0$ ✓



$$p = 2\,CDF(t_0)$$

Case 3: What happens when we have NO population data?

Example 1

An engineer is studying **the formulation of a Portland cement mortar**. He has **added a polymer latex emulsion** during mixing to determine if this impacts the curing time and tension bond strength of the mortar.

The experimenter prepared 10 samples of the original formulation and 10 samples of the modified formulation.

TABLE 2.1
Tension Bond Strength Data for the Portland Cement Formulation Experiment

j	Modified Mortar y_{1j}	Unmodified Mortar y_{2j}
1	16.85	16.62
2	16.40	16.75
3	17.21	17.37
4	16.35	17.12
5	16.52	16.98
6	17.04	16.87
7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

Question: Does adding polymer latex emulsion change the strength?

Case 3: What happens when we have NO population data?

Example 2

Who is a better ODI batsman, Virat or Babar? (Based on the runs scored in an inning)

Batsman	One sample each of 10 ODI innings	Sample Mean	Sample Std. Dev
Virat	00, 53, 34, 31, 00, 54, 96, 20, 10, 19	**31.7**	29.6
Babar	12, 09, 91, 79, 51, 45, 41, 46, 29, 33	**43.6**	26.0

What is the hypothesis test?

What is the statistical (mathematical) model based on the hypothesis?

Example 1: Mortar Formula

■ **TABLE 2.1**
Tension Bond Strength Data for the Portland Cement Formulation Experiment

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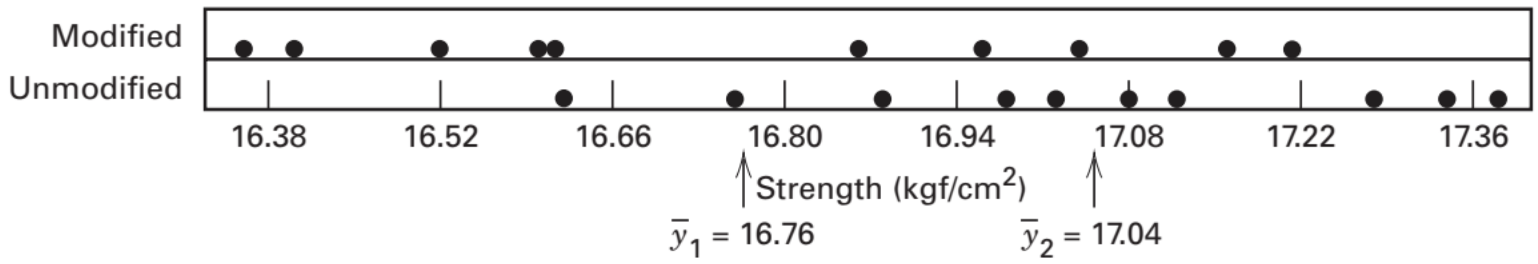
7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

Ref: Design and Analysis of Experiments, 8th Ed.

Each of the observations in the Portland cement experiment described above would be called a **run**. Notice that the individual runs differ, so there is fluctuation, or **noise**, in the observed bond strengths. This noise is usually called **experimental error** or simply **error**. It is a **statistical error**, meaning that it arises from variation that is uncontrolled and generally unavoidable. The presence of error or noise implies that the response variable, tension bond strength, is a **random variable**. A random variable may be either **discrete** or **continuous**. If the set of all possible values of the random variable is either finite or countably infinite, then the random variable is discrete, whereas if the set of all possible values of the random variable is an interval, then the random variable is continuous.



■ **FIGURE 2.1** **Dot diagram for the tension bond strength data in Table 2.1**

Example 1: Mortar Formula

Let $y_{11}, y_{12}, y_{13}, .. y_{1n1}$ be n_1 observations from the first factor level (Modified Mortar)

and $y_{21}, y_{22}, y_{23}, .. y_{2n1}$ be n_2 observations from the second factor level (UNmodified Mortar)

What is the hypothesis test?

A simple statistical model to describe the data is

$$y_{ij} = \mu_i + \epsilon_{ij} \begin{cases} i = 1, 2 \\ j = 1, 2, \ldots, n_i \end{cases}$$

where y_{ij} is the jth observation from factor level i, μ_i is the mean of the response at the ith factor level, and ϵ_{ij} is a normal random variable associated with the ijth observation.

■ **TABLE 2.1**

Tension Bond Strength Data for the Portland Cement Formulation Experiment

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7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

Ref: Design and Analysis of Experiments, 8th Ed.

Example 1: Mortar Formula

We **assume** that the random error components ϵ_{1j} and ϵ_{2j} are normally distributed with means 0 and variances σ_1^2 and σ_2^2

Which would follow that the y_{1j} and y_{2j} are normally distributed with means μ_1 *and* μ_2 and variances σ_1^2 and σ_2^2

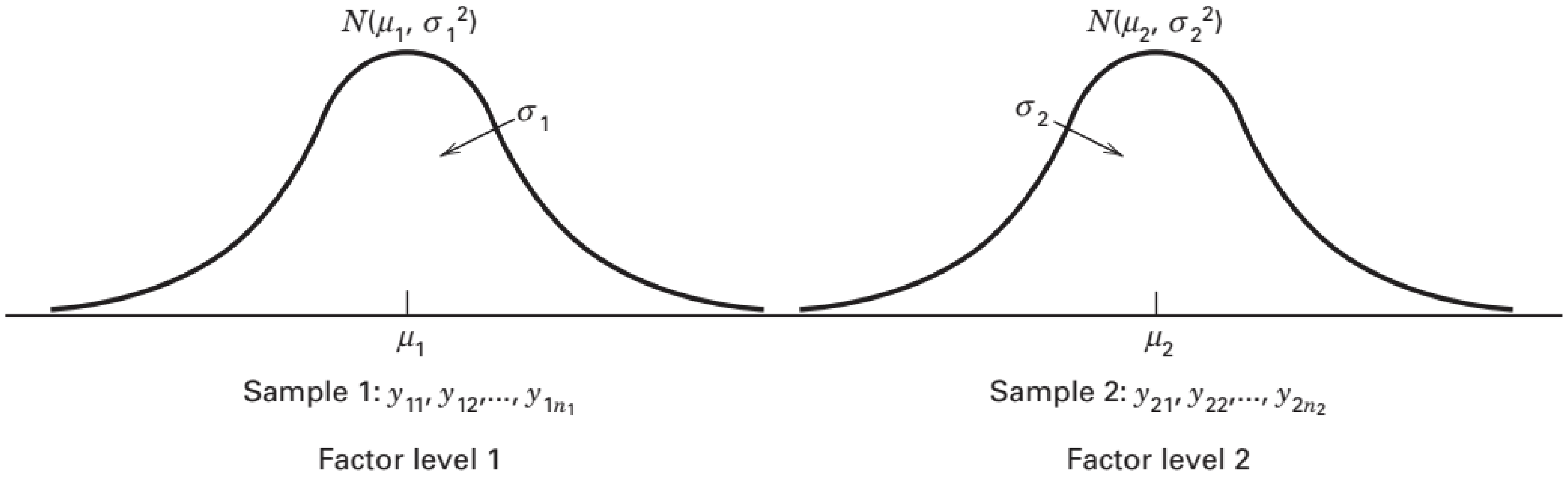

TABLE 2.1
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7	16.96	17.34
8	17.15	17.02
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10	16.57	17.27

Example 1: Mortar Formula

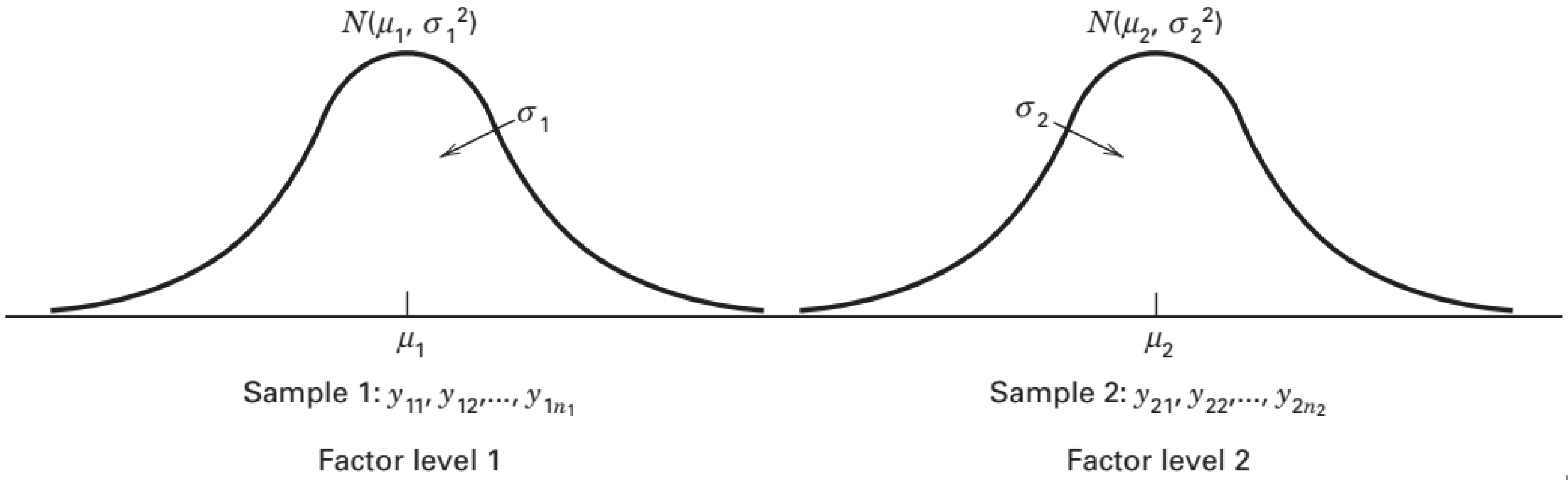
Now the question is whether μ_1 & μ_2 are statistically different

Hypothesis Testing

$$H_0: \mu_1 = \mu_2$$ Null Hypothesis

$$H_1: \mu_1 \neq \mu_2$$ Alternate Hypothesis (two-sided)

$\mu_1 < \mu_2$ or if $\mu_1 > \mu_2$.



■ **TABLE 2.1**
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10	16.57	17.27

Two-Sample t-Test

Suppose that we could assume that the variances of tension bond strengths were identical for both mortar formulations. $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Then the **appropriate test statistic** to use for comparing two treatment means in the completely randomized design is $\frac{SS}{V} = S^2$

$\Sigma\, y_{1j} - \bar{y}_1 = 0$

$\Sigma\, y_{2j} - \bar{y}_2 = 0$

$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$

Pooled Var $= \frac{SS}{V}$

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$= \frac{SS}{V}$

$= \frac{SS_1 + SS_2}{n_1 + n_2 - 2}$

$= \frac{S_1^2 (n_1 - 1) + S_2^2 (n_2 - 1)}{n_1 + n_2 - 2}$

Where

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

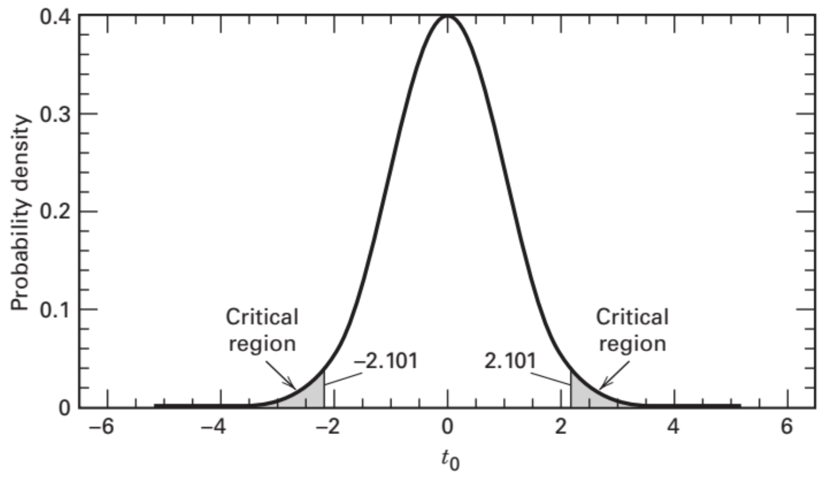
S_p^2 is an estimate of the common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$

■ TABLE 2.1
Tension Bond Strength Data for the Portland Cement Formulation Experiment

j	Modified Mortar y_{1j} (S_1^2, $\bar{y}_1$)	Unmodified Mortar y_{2j} (S_2^2, $\bar{y}_2$)
1	16.85	16.62
2	16.40	16.75
3	17.21	17.37
4	16.35	17.12
5	16.52	16.98
6	17.04	16.87
7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

Two-Sample t-Test Procedure

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$



1) To determine whether to reject $H_0: \mu_1 = \mu_2$, we would compare t_0 to the t-distribution with $(n_1 + n_2 - 2)$ degrees of freedom.

2) If $t_0 > t_{\frac{\alpha}{2}, n_1+n_2-2}$ OR $t_0 < -t_{\frac{\alpha}{2}, n_1+n_2-2}$, then we will reject $H_0: \mu_1 = \mu_2$

t-Test

Justification of Two-Sample t-Test

If we were sampling from two independent normal distributions, then the distribution of $\overline{y_1} - \overline{y_2}$ will be a normal distribution with mean $\mu_1 - \mu_2$ and variance $\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

If σ^2 were known, and if $H_0: \mu_1 = \mu_2$ were true, then the Z_0 distribution would be a normal distribution with mean 0 and variance 1

$$Z_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

But since we do NOT know σ^2, we use S_p^2

and the normal distribution changes to t-distribution with $(n_1 + n_2 - 2)$ degrees of freedom.

Two-Sample t-Test

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{1/n_1 + 1/n_2}} = \frac{16.76 - 17.04}{\sqrt{0.081}\sqrt{\frac{2}{10}}} = -2.21$$

In this example

Modified Mortar	Unmodified Mortar
$\bar{y}_1 = 16.76$ kgf/cm^2	$\bar{y}_2 = 17.04$ kgf/cm^2
$S_1^2 = 0.100$	$S_2^2 = 0.061$
$S_1 = 0.316$	$S_2 = 0.248$
$n_1 = 10$	$n_2 = 10$

$$S_p^2 = \frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1+n_2-2} = \frac{0.1\times 9 + 0.061\times 9}{18}$$

$$= 0.161/2 = 0.0805 \rightarrow S_p =$$

■ **TABLE 2.1**
Tension Bond Strength Data for the Portland Cement Formulation Experiment

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7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

t-Test

Two-Sample t-Test

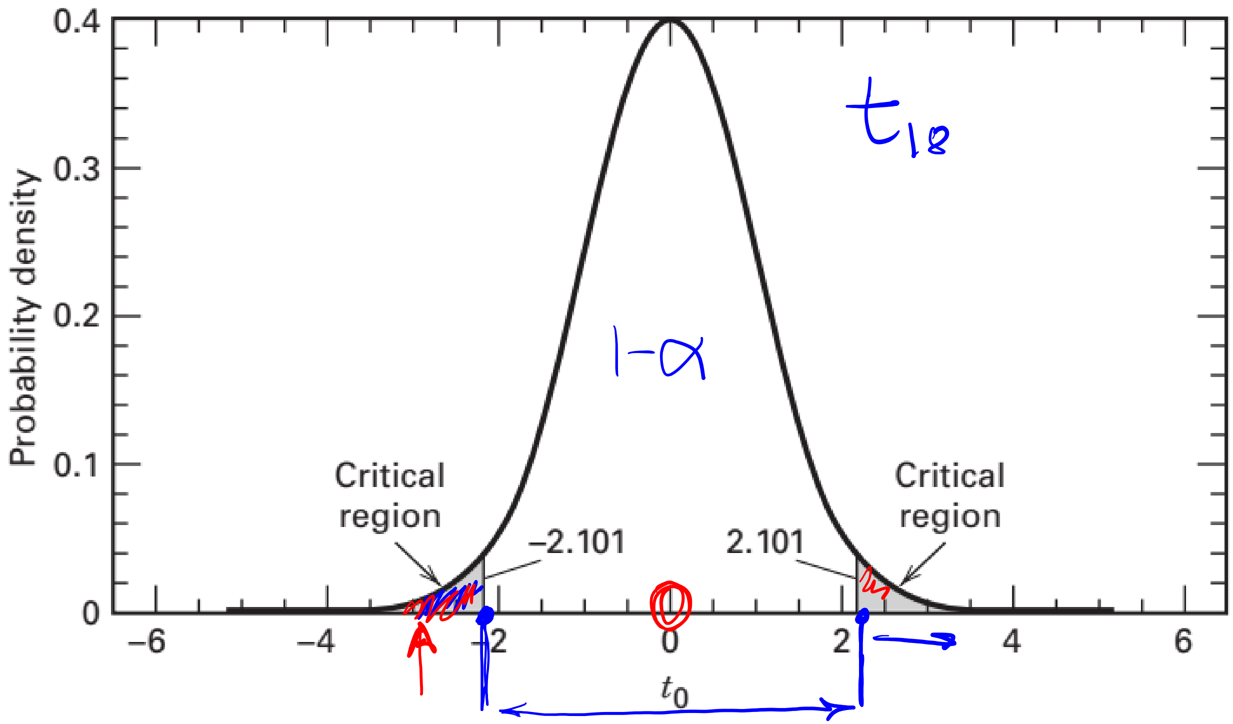
$\alpha = 5\%$

In this example

Modified Mortar	Unmodified Mortar
$\bar{y}_1 = 16.76 \text{ kgf/cm}^2$	$\bar{y}_2 = 17.04 \text{ kgf/cm}^2$
$S_1^2 = 0.100$	$S_2^2 = 0.061$
$S_1 = 0.316$	$S_2 = 0.248$
$n_1 = 10$	$n_2 = 10$

your $t_0 = -2.21$

H_0 is Rejected.



■ **FIGURE 2.10** The t distribution with 18 degrees of freedom with the critical region $\pm t_{0.025,18} = \pm 2.101$

Furthermore, $n_1 + n_2 - 2 = 10 + 10 - 2 = 18$, and if we choose $\alpha = 0.05$, then we would reject $H_0 : \mu_1 = \mu_2$ if the numerical value of the test statistic $t_0 > t_{0.025,18} = 2.101$, or if $t_0 < -t_{0.025,18} = -2.101$. These boundaries of the critical region are shown on the reference distribution (t with 18 degrees of freedom) in Figure 2.10.

t-Test Calculations

Two-Sample t-Test

In this example

Modified Mortar	Unmodified Mortar
$\bar{y}_1 = 16.76\ \text{kgf/cm}^2$	$\bar{y}_2 = 17.04\ \text{kgf/cm}^2$
$S_1^2 = 0.100$	$S_2^2 = 0.061$
$S_1 = 0.316$	$S_2 = 0.248$
$n_1 = 10$	$n_2 = 10$

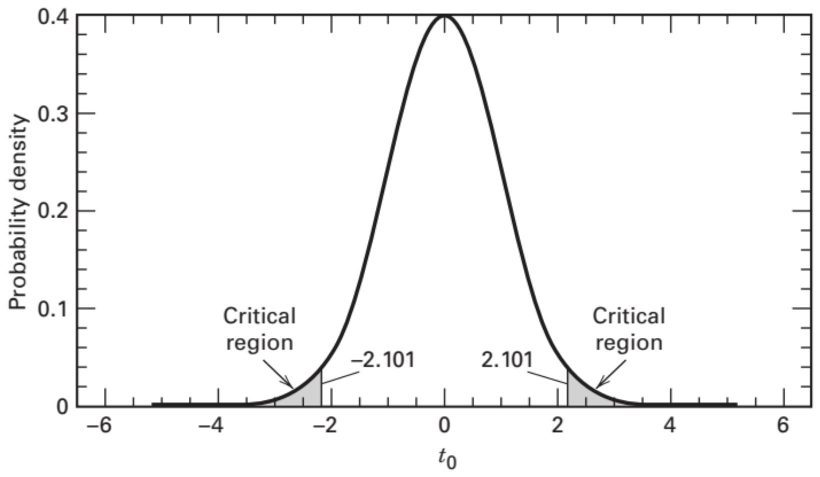


FIGURE 2.10 The t distribution with 18 degrees of freedom with the critical region $\pm t_{0.025,18} = \pm 2.101$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$= \frac{9(0.100) + 9(0.061)}{10 + 10 - 2} = 0.081$$

$$S_p = 0.284$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{16.76 - 17.04}{0.284\sqrt{\frac{1}{10} + \frac{1}{10}}}$$

$$= \frac{-0.28}{0.127} = -2.20$$

We Reject $H_0 : \mu_1 = \mu_2$ at Significance level of 0.05

P-Value

Two-Sample t-Test

In this example, we concluded that we Reject $H_0: \mu_1 = \mu_2$ at significance level of α = 0.05

Do you see any problem/limitation of this?

For example, what will be the conclusion if the significance level is 0.04 or 0.03 or 0.01?

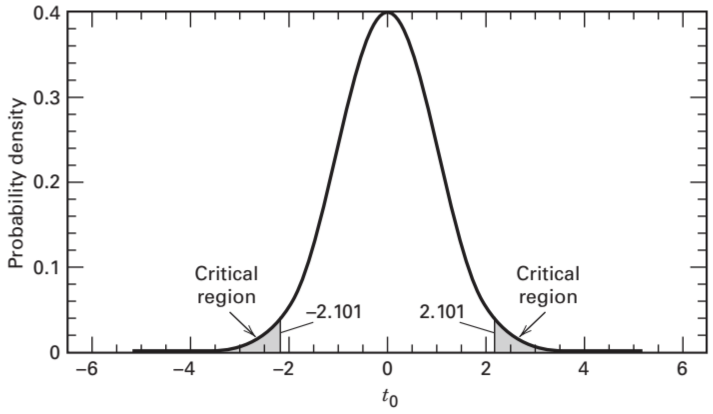
We do not know whether the test-statistic t0 lies just barely in the rejection region OR very far into the rejection region

Thus, we can specify P-value, which is the minimum significance value which will

Result in rejection of the null hypothesis

For example, in the mortar experiments, the null hypothesis will be rejected for

any level of significance > 0.0411



■ **FIGURE 2.10** The t distribution with 18 degrees of freedom with the critical region $\pm t_{0.025,18} = \pm 2.101$

Concept of Confidence Interval

- Given a random sample of '*n*' observations from some process of interest and an estimate of the process mean, it is of interest to make some statement about the *"goodness" of that sample mean*, as an estimate of μ, i.e., the degree of belief or confidence that can be placed on it.
- One way of approaching this problem is through *the concept of the confidence interval.*

- **Remember:** Distribution of sample means is a normal distribution (CLT)
- *That means,* for random samples of size 'n' drawn from a population, we expect that 95% of all sample means will be within an interval of $\mu \pm 1.96$ standard deviations of the distribution of the sample mean, i.e., $\mu \pm \frac{1.96\sigma_x}{\sqrt{n}}$

 https://www.mathsisfun.com/data/standard-normal-distribution-table.html

Concept of Confidence Interval

In other words, $\bar{y} \pm \frac{1.96\sigma_y}{\sqrt{n}}$ is called a 95% confidence interval for the true mean μ

$\mu \pm 1.96\,\sigma_{\bar{y}}$

$\bar{y}$

In general,

$\boldsymbol{\bar{y} \pm \left(z_{1-\frac{\alpha}{2}}\right)\frac{\sigma_y}{\sqrt{n}}}$ **is a 100*(1-α)% confidence interval for the true mean μ**

μ

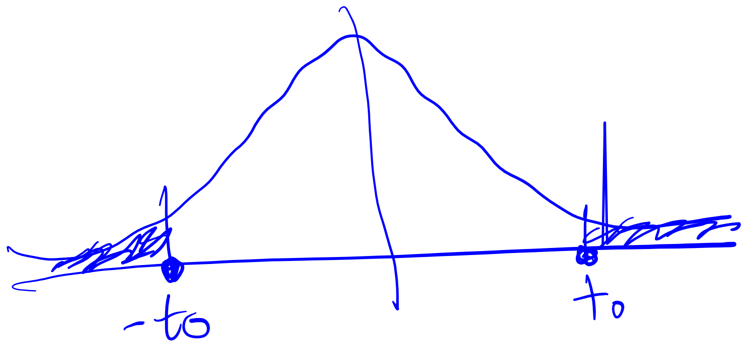
$\bar{y} \pm 1.96\,\sigma_{\bar{y}}$

95%

When sample size is small and σ_y is UNKNOWN,

the confidence interval is given by $\boldsymbol{\bar{y} \pm \left(t_{v,1-\frac{\alpha}{2}}\right)\frac{s}{\sqrt{n}}}$

Where v = n-1 is the degree of freedom



$t_{v,\alpha/2} = -t_{v,1-\alpha/2}$

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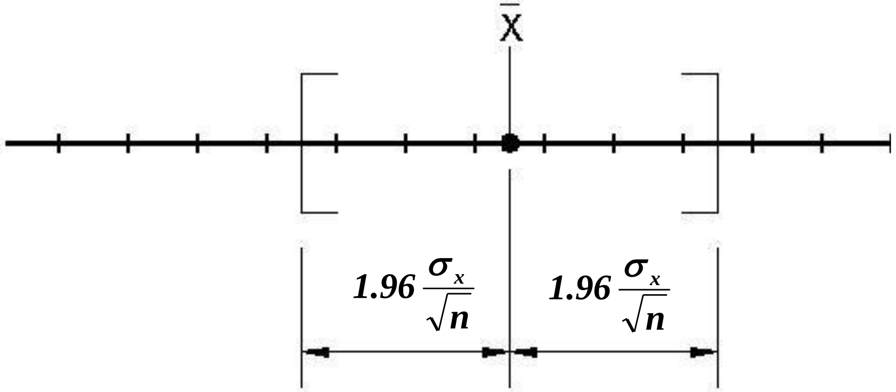
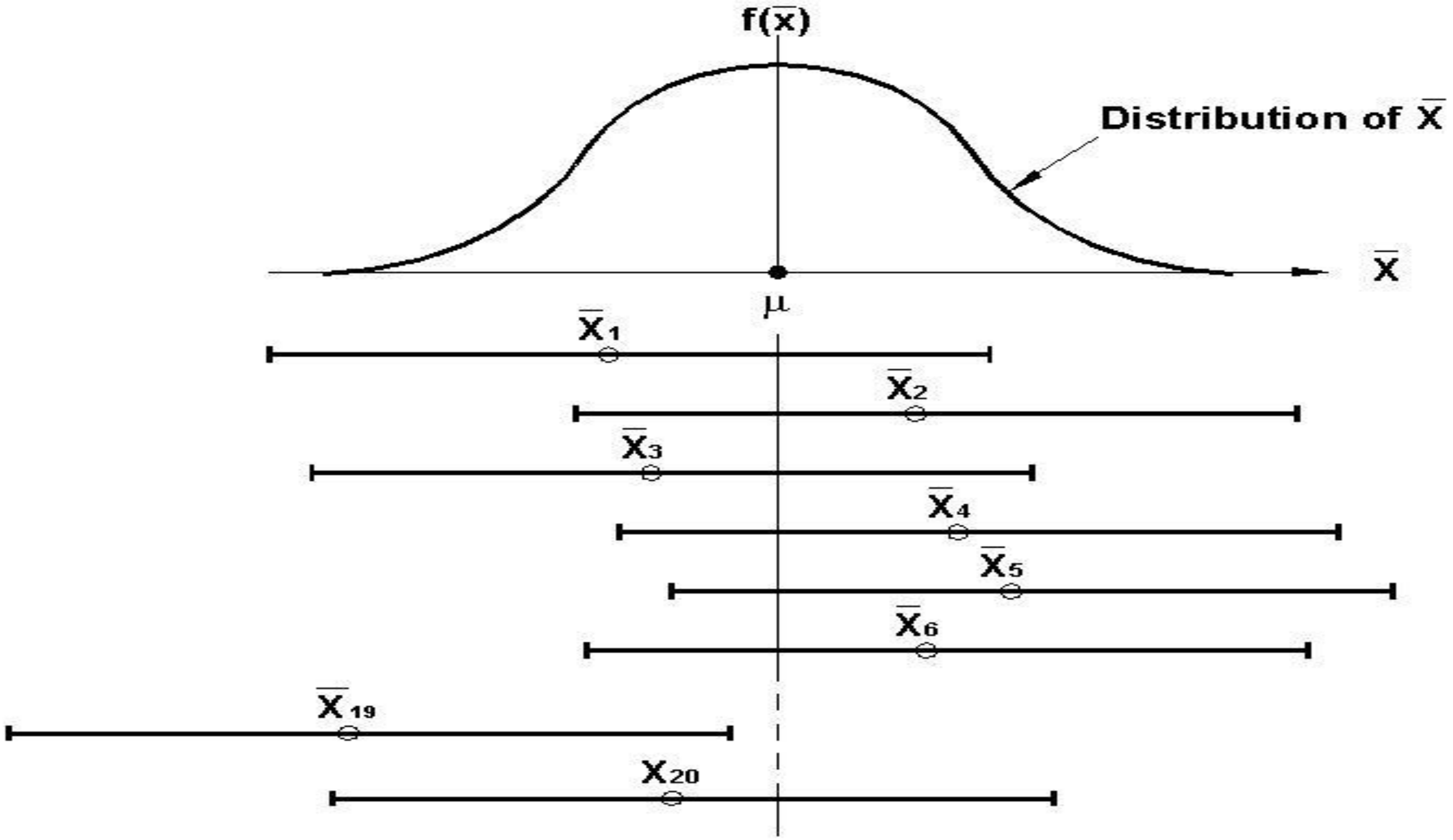
Confidence Interval





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Confidence Interval Approach

To define a confidence interval, suppose that θ is an unknown parameter. To obtain an interval estimate of θ, we need to find two statistics L and U such that the probability statement

$$P(L \leqslant \theta \leqslant U) = 1 - \alpha \tag{2.27}$$

is true. The interval

$$L \leqslant \theta \leqslant U \tag{2.28}$$

is called a **100(1 − α) percent confidence interval** for the parameter θ. The interpretation of this interval is that if, in repeated random samplings, a large number of such intervals are constructed, 100(1 − α) percent of them will contain the true value of θ. The statistics L and U are called the **lower** and **upper confidence limits**, respectively, and 1 − α is called the **confidence coefficient**. If $\alpha = 0.05$, Equation 2.28 is called a 95 percent confidence interval for θ. Note that confidence intervals have a frequency interpretation; that is, we do not know if the statement is true for this specific sample, but we do know that the *method* used to produce the confidence interval yields correct statements 100(1 − α) percent of the time.

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$$-t_{\alpha/2} = t_{1-\alpha/2}$$

Suppose that we wish to find a $100(1-\alpha)$ percent confidence interval on the true difference in means $\mu_1 - \mu_2$ for the Portland cement problem. The interval can be derived in the following way. The statistic

$$\frac{\bar{y}_1 - \bar{y}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

is distributed as $t_{n_1+n_2-2}$. Thus,

$$P\left(-t_{\alpha/2,n_1+n_2-2} \le \frac{\bar{y}_1 - \bar{y}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \le t_{\alpha/2,n_1+n_2-2}\right) = 1 - \alpha$$

or

$$P\left(\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right) = 1 - \alpha$$

$$\Delta y = y_1 - y_2$$

$$\overline{\Delta y} = \bar{y}_1 - \bar{y}_2$$

$$\frac{\overline{\Delta y} - \Delta\mu}{S_p\sqrt{1/n_1 + 1/n_2}}$$

Comparing Equations 2.29 and 2.27, we see that

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

is a $100(1-\alpha)$ percent confidence interval for $\mu_1 - \mu_2$.

The actual 95 percent confidence interval estimate for the difference in mean tension bond strength for the formulations of Portland cement mortar is found by substituting in Equation 2.30 as follows:

$$16.76 - 17.04 - (2.101)0.284\sqrt{\tfrac{1}{10} + \tfrac{1}{10}} \leq \mu_1 - \mu_2$$
$$\leq 16.76 - 17.04 + (2.101)0.284\sqrt{\tfrac{1}{10} + \tfrac{1}{10}}$$
$$-0.28 - 0.27 \leq \mu_1 - \mu_2 \leq -0.28 + 0.27$$
$$-0.55 \leq \mu_1 - \mu_2 \leq -0.01$$

95% $\alpha = 5\%$

$\mu_1 \neq \mu_2$

$\mu_1 - \mu_2 \neq 0$

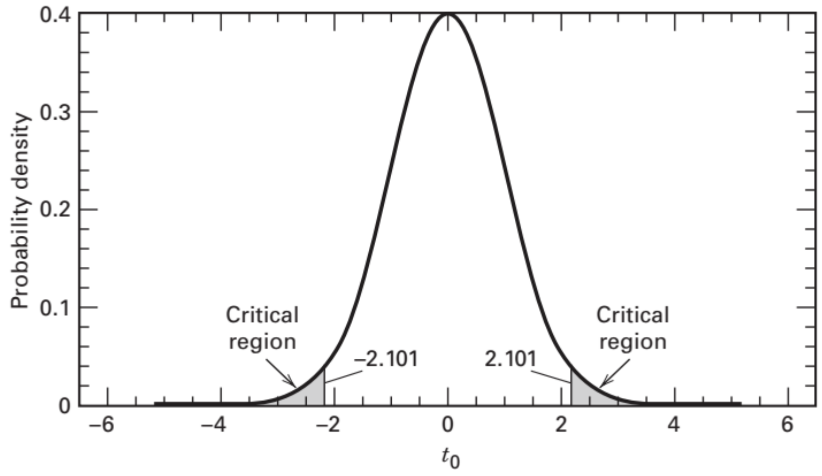
Note that because $\mu_1 - \mu_2 = 0$ is *not* included in this interval, the data do not support the hypothesis that $\mu_1 = \mu_2$ at the 5 percent level of significance (recall that the P-value for the two-sample t-test was 0.042, just slightly less than 0.05).

~~H_0~~

Recap: Comparison when we do NOT know σ

Two-Sample t-Test Procedure (Two-Sided)

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$



1) To determine whether to reject $H_0: \mu_1 = \mu_2$, we would compare t_0 to the t-distribution with $(n_1 + n_2 - 2)$ degrees of freedom.

2) If $t_0 > t_{\frac{\alpha}{2}, n_1+n_2-2}$ OR $t_0 < -t_{\frac{\alpha}{2}, n_1+n_2-2}$, then we will reject $H_0: \mu_1 = \mu_2$

Recap: Comparison when we do NOT know σ

Two-Sample t-Test Procedure (Two-Sided) using Confidence Interval

$$P\left(-t_{\alpha/2,n_1+n_2-2} \leq \frac{\bar{y}_1 - \bar{y}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \leq t_{\alpha/2,n_1+n_2-2}\right) = 1 - \alpha$$

or

$$P\left(\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \right.$$

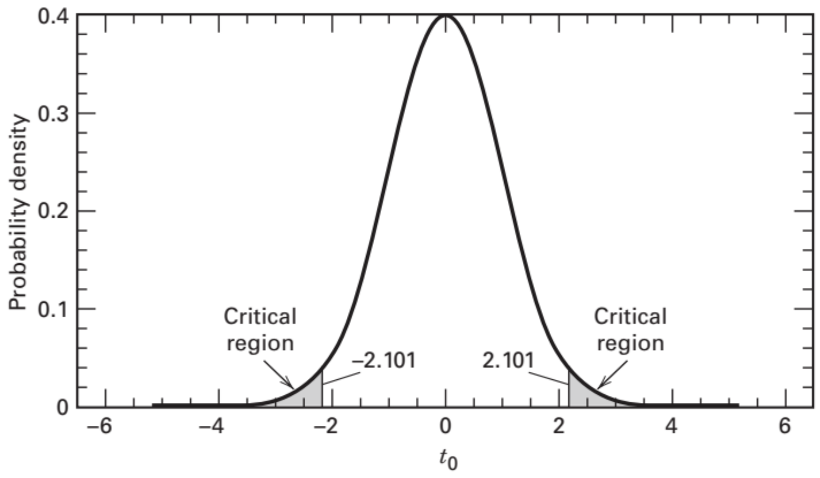
$$\left. \leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right) = 1 - \alpha$$

Comparing Equations 2.29 and 2.27, we see that

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2$$

$$\leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

is a $100(1 - \alpha)$ percent confidence interval for $\mu_1 - \mu_2$.

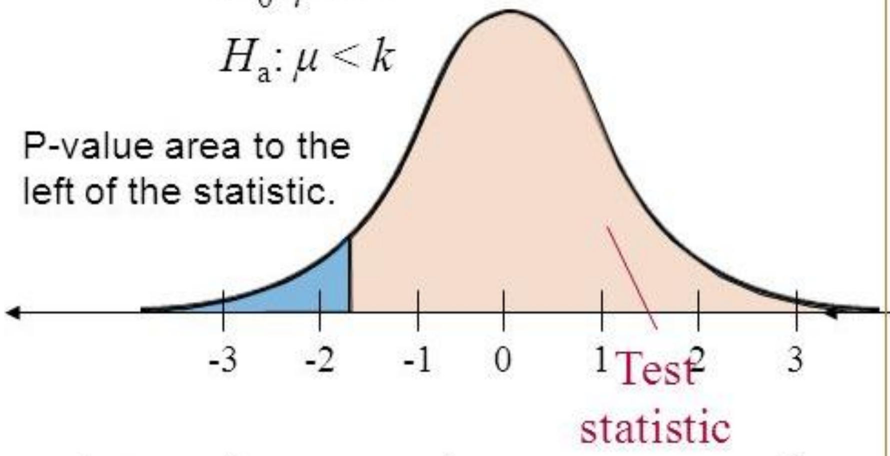


One-sided Tests

Left Tailed Test: The alternative hypothesis H_a contains the less-than inequality symbol (<).

$H_0\text{: } \mu \geq k$

$H_a\text{: } \mu < k$



A water faucet manufacturer announces that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

$H_0\text{: } \mu \geq 2.5$

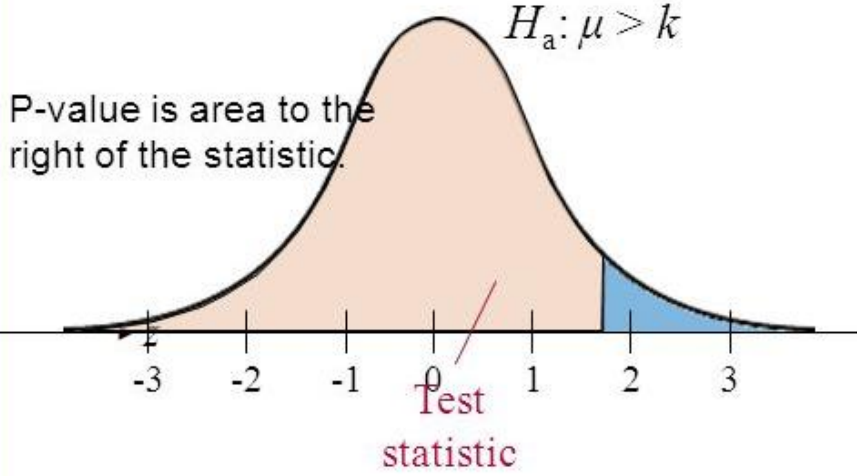
$H_a\text{: } \mu < 2.5$

Larson/Farber 4th ed.

Right Tailed Test: The alternative hypothesis H_a contains the less-than inequality symbol (>).

$H_0\text{: } \mu \leq k$

$H_a\text{: } \mu > k$



A cereal company says: Mean weight of box is more than 20 oz.

$H_0\text{: } \mu \leq 20$

$H_a\text{: } \mu > 20$

8

Comparative Experiments

Example 2

Who is a better ODI batsman, Virat or Babar? (Based on the runs sc

Batsman	One sample each of 10 ODI innings	Sample Mean	Sample Std. Dev
Virat	00, 53, 34, 31, 00, 54, 96, 20, 10, 19	**31.7**	29.6
Babar	12, 09, 91, 79, 51, 45, 41, 46, 29, 33	**43.6**	26.0

What is the hypothesis test?

What is the statistical (mathematical) model based on the hypothesis?

What's the statistical conclusion?

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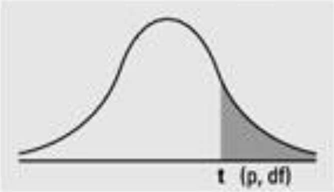
Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216

Example 3

Given that 9 bearings made by a certain process have an average diameter of 0.305 cm and the sample standard deviation of 0.003 cm, construct a 99 % confidence interval for the true mean diameter of bearings made by the process. What is the width of the confidence interval?

Degrees of freedom (ν)	Amount of area in one tail (α)							
	0.0005	0.001	0.005	0.010	0.025	0.050	0.100	0.200
1	636.6192	318.3088	63.65674	31.82052	12.70620	6.313752	3.077684	1.376382
2	31.59905	22.32712	9.924843	6.964557	4.302653	2.919986	1.885618	1.060660
3	12.92398	10.21453	5.840909	4.540703	3.182446	2.353363	1.637744	0.978472
4	8.610302	7.173182	4.604095	3.746947	2.776445	2.131847	1.533206	0.940965
5	6.868827	5.893430	4.032143	3.364930	2.570582	2.015048	1.475884	0.919544
6	5.958816	5.207626	3.707428	3.142668	2.446912	1.943180	1.439756	0.905703
7	5.407883	4.785290	3.499483	2.997952	2.364624	1.894579	1.414924	0.896030
8	5.041305	4.500791	3.355387	2.896459	2.306004	1.859548	1.396815	0.888890
9	4.780913	4.296806	3.249836	2.821438	2.262157	1.833113	1.383029	0.883404
10	4.586894	4.143700	3.169273	2.763769	2.228139	1.812461	1.372184	0.879058
11	4.436979	4.024701	3.105807	2.718079	2.200985	1.795885	1.363430	0.875530
12	4.317791	3.929633	3.054540	2.680998	2.178813	1.782288	1.356217	0.872609
13	4.220832	3.851982	3.012276	2.650309	2.160369	1.770933	1.350171	0.870152
14	4.140454	3.787390	2.976843	2.624494	2.144787	1.761310	1.345030	0.868055
15	4.072765	3.732834	2.946713	2.602480	2.131450	1.753050	1.340606	0.866245
16	4.014996	3.686155	2.920782	2.583487	2.119905	1.745884	1.336757	0.864667
17	3.965126	3.645767	2.898231	2.566934	2.109816	1.739607	1.333379	0.863279

https://www.mathsisfun.com/data/standard-normal-distribution-table.html

Choice of Sample Size

- Selection of appropriate sample size 'n' is critical in any experimental design
- In the previous example, have a look at the length of 100*(1-α)% confidence interval for difference in means $(\mu_1 - \mu_2)$
- It was determined by

$$t_{\alpha/2,\, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2$$

$$\leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

is a $100(1-\alpha)$ percent confidence interval for $\mu_1 - \mu_2$.

- What is the effect of sample size on this width?
  - Say n1 = n2 = n, and α = 0.05, Sp could be anything (we don't have control over it)
  - So essentially, the width is a function of

$$t_{\alpha/2,\, 2n-2}\, S_p \sqrt{\frac{2}{n}}$$