Spring Semester **February 9, 2020**

Quiz 1

IE 616: Decision Analysis and Game Theory

Closed book exam. No clarification during exam. Make proper assumptions if required and state them correctly. Total Marks 45. Duration 1:30 hours

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Exercise 1 *Show that any Strongly Dominant Strategy Equilibrium is a Nash Equilibrium.*

Marks 5

Exercise 2 *State true or false for the following statements.* **Marks 1 each**

(i) A Pure Strategy Nash Equilibrium provides insurance for a player against unilateral deviations by any other player.

(ii) Every Nash equilibrium is a dominant strategy equilibrium.

(iii) Suppose a strategic form game $(N; (S_i); (u_i))$ has a pure strategy Nash equilibrium $(s_1^*, \ldots, s_n^*)$. Then $u_i(s_1^*, \ldots, s_n^*) \geq \underline{v}_i$, where $\underline{v}_i$ is the max-min value.

(iv) A two-player game is symmetric if the two players have the same strategy set $S_1 = S_2$ and the payoff functions satisfy $u_1(s_1, s_2) = u_2(s_2, s_1)$ for each $s_1, s_2 \in S_1$. Then the set of equilibria of a two-player symmetric game is a symmetric set: if (s_1, s_2) is an NE, then (s_2, s_1) is also an NE.

Exercise 3 *Consider a two player game with strategy set of each player as [0,1] and the utility function gives as:*

$$u_1(x, y) = 3xy - 2x - 2y + 2, \forall x \in [0, 1], \forall y \in [0, 1]$$
$$u_2(x, y) = -4xy + 2x + y, \forall x \in [0, 1], \forall y \in [0, 1]$$

Find the maxmin strategies of both players. **Marks 10**

Exercise 4 *Suppose a strategic form game $< N, (S_i), (u_i) >$ has a mixed strategy Nash Equilibrium $(\sigma_1^*, \cdots, \sigma_n^*)$.Then, show that*

$$u_i(\sigma_1^*, \cdots, \sigma_n^*) \geq \bar{v}_i \geq \underline{v}_i \quad \forall\, i \in N$$

where $\bar{v}_i$ is the min-max value in mixed strategies of player i and $\underline{v}_i$ is the max-min value in mixed strategies of player i. **Marks 8**

Exercise 5 *For a given instance of Prisoners' dilemma,*

	NC	C
NC	$-4, -4$	$-2, -x$
C	$-x, -2$	$-x, -x$

Find the values of x for which (in pure strategies): **Marks 4+2+2**

1. *the profile (C, C) is a strongly dominant strategy equilibrium.*

2. *the profile (C, C) is a not even a weakly dominant strategy equilibrium.*

3. *the profile (C, C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.*

In each case, say whether it is possible to find such an x. Justify your answer in each case.

Exercise 6 *Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and $m(= n - k)$ support candidate B. Each citizen decides to vote or not to vote. A citizen who does not vote, receives the payoff of 2 if the candidate she supports wins, 1 if there is a tie, and 0 if her candidate loses. The citizen that votes have to pay some extra cost for voting; thus a citizen who votes receives the payoffs $2 - c$, $1 - c$,and $-c$ respectively in the three cases (her candidate wins, ties, her candidate loses), where $0 < c < 1$.* **Marks 4+3+3**

1. *For $k = m = 1$, write this as a strategic form game.*

2. *For $k = m > 1$, check the following:*

   (a) *Is the action profile in which everyone votes a Nash equilibrium?*

   (b) *Is there any Nash equilibrium in which the candidates tie and not everyone votes?*

OR

The Mumbai traffic police department has come up with a new rule to stop people from honking excessively. According to this rule, if the sound intensity at a signal goes above 80Db, the signal will turn red and reset its timer to the full time. Say there are two travellers, waiting at the signal, refer them as Player 1 and Player 2.

Both the players can opt for honk or stay quiet as per their will. They can honk respectively with intensities h_1 and h_2.

Player 1 gets guilty pleasure worth of a_1 units if he/she causes the traffic jam. He/she gets time saving worth of a_2 if the traffic runs smoothly.

Player 2 is in hurry and has no interest in causing a traffic jam. He/she loses on time worth of a_3 if he/she gets caught into a traffic jam. If the traffic runs smoothly, Player 2 gets a reward a_4. **Marks 4+3+3**

1. *Formulate this as a strategic form game.*

2. *If $h_1 + h_2 < 80$, what are the Nash equilibria?*

3. *If $h_1 + h_2 \geq 80$ but $h_1 < 80$ and $a_1 > a_2$. What is the equilibrium of this game ?*