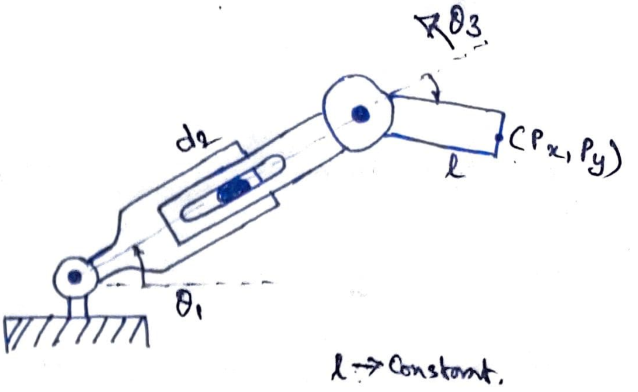
1.(a)



End effector position (P_x, P_y)

$$\left\{\begin{aligned} P_x &= d_2 \cos\theta_1 + l\cos(\theta_1+\theta_3) \\ P_y &= d_2 \sin\theta_1 + l\sin(\theta_1+\theta_3) \end{aligned}\right\} \text{—(A)}$$

Squaring and adding the both eq^n

$$P_x^2 + P_y^2 = d_2^2 + l^2 + 2d_2 l\left(\cos\theta_1\cos(\theta_1+\theta_3) + \sin\theta_1\sin(\theta_1+\theta_3)\right)$$
$$= d_2^2 + l^2 + 2d_2 l\cos\theta_3$$

$$\cos\theta_3 = \frac{P_x^2 + P_y^2 - (d_2^2 + l^2)}{2d_2 l}\ ; \quad \theta_3 = \cos^{-1}\left[\frac{P_x^2 + P_y^2 - (d_2^2 + l^2)}{2d_2 l}\right]$$

again, $P_x = d_2\cos\theta_1 + l\cos\theta_1\cos\theta_3 - l\sin\theta_1\sin\theta_3 = (d_2 + l\cos\theta_3)\cos\theta_1 - l\sin\theta_3\sin\theta_1$

$P_y = d_2\sin\theta_1 + l\sin\theta_1\cos\theta_3 + l\cos\theta_1\sin\theta_3 = (d_2 + l\cos\theta_3)\sin\theta_1 + l\sin\theta_3\cos\theta_1$

$$\Rightarrow \left\{\begin{aligned} P_x &= aC\theta_1 - bS\theta_1 \\ P_y &= aS\theta_1 + bC\theta_1 \end{aligned}\right\} \Longrightarrow \sin\theta_1 = \frac{P_y - \frac{b}{a}P_x}{(a - b^2/a)}$$

Where d_2 is assumed arbitrary as we have 2 eq^n with 3 variables. Infinite solution possible.

(b). Position (P_x, P_y) as well as orientation of endeffector is known, $\psi = (\theta_1 + \theta_3)$ w.r.t ground.

eq^n (A) simplifies into $\Longrightarrow$ $\left.\begin{aligned} P_x &= d_2 C\theta_1 + l\,C\psi \\ P_y &= d_2 S\theta_1 + l\,S\psi \end{aligned}\right\}$ — (B)

$$\left.\begin{aligned} d_2 C\theta_1 &= P_x - l\,C\psi \quad \text{—①} \\ d_2 S\theta_1 &= P_y - l\,S\psi \quad \text{—②} \end{aligned}\right\} \Rightarrow \quad \theta_1 = \tan^{-1}\left[\frac{P_y - lS\psi}{P_x - lC\psi}\right] \text{—(A1)}$$

Sq & add ①, ②

$$d_2 = \sqrt{P_x^2 + P_y^2 + l^2 - 2P_x l C\psi - 2P_y l S\psi} \quad \text{—— (A2)}$$

$$\theta_3 = \psi - \theta_1 \quad \text{—— (A 3)}$$

2.

- $\theta_1 = \tan^{-1}(P_y/P_x)$ —— (1)
- $P_z = d_2 + 1m$

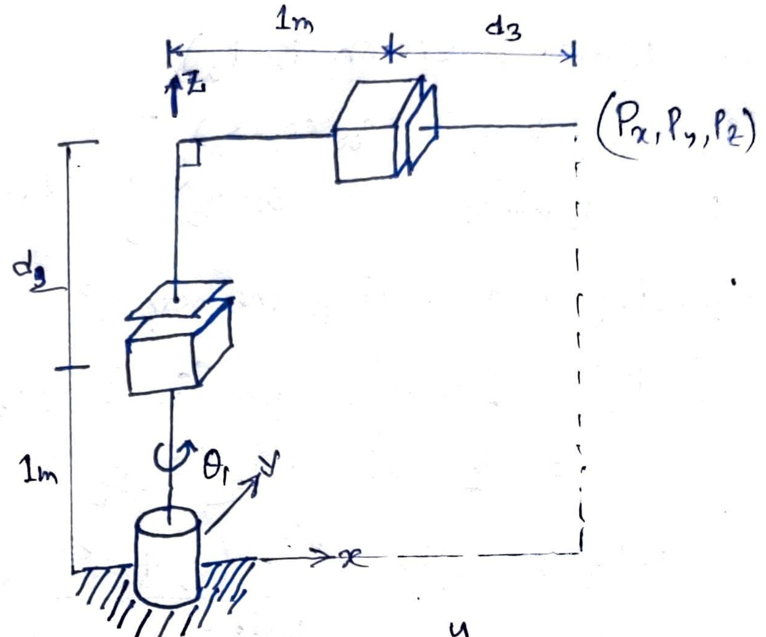
or, $d_2 = (P_z - 1)$ —— (2)

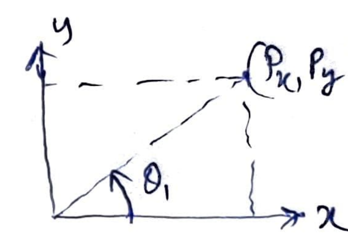
- $\cancel{P_z^2 = P_x^2 + P_y^2}$

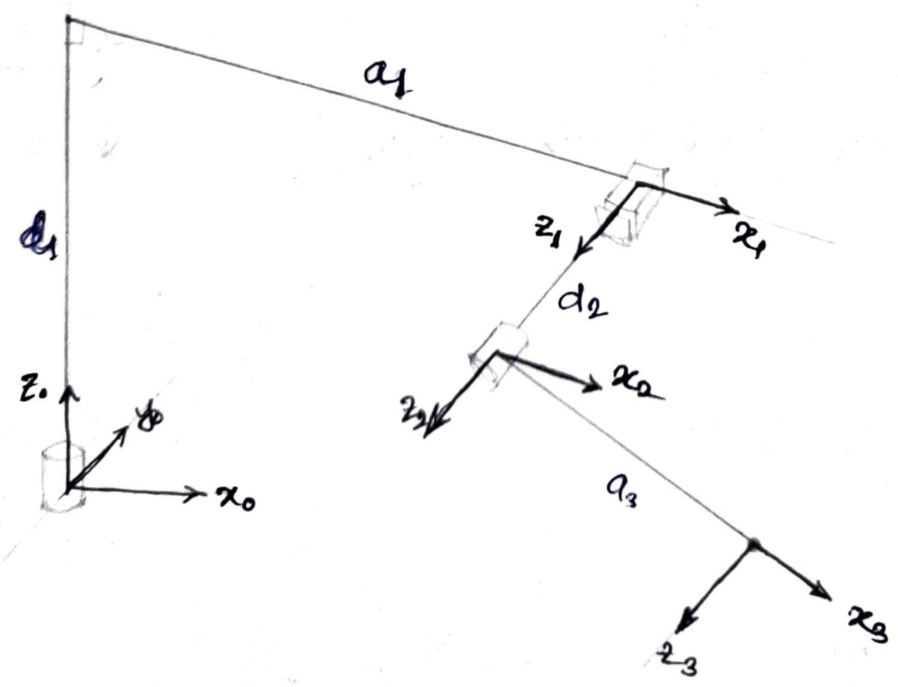
$P_z^2 + (1+d_3)^2 = (P_z^2 + P_y^2 + P_x^2)$

or $(1+d_3)^2 = P_x^2 + P_y^2$

$d_3 = \sqrt{(P_x^2 + P_y^2)} - 1$

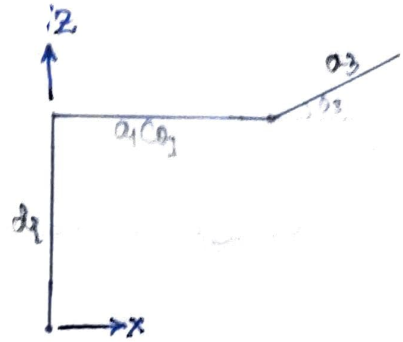






3(a) end effector position known (P_x, P_y, P_z)

[From Tutorial 3, we get]

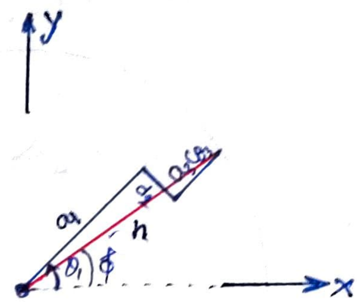


$$\begin{cases} P_x = a_1 C\theta_1 + d_2 S\theta_1 + a_3 C\theta_1 C\theta_3 & \text{---(1)} \\ P_y = a_1 S\theta_1 - d_2 C\theta_1 + a_3 S\theta_1 C\theta_3 & \text{---(2)} \\ P_z = d_1 + a_3 S\theta_3 & \text{---(3)} \end{cases}$$

From (3): $\theta_3 = \sin^{-1}\left[\dfrac{P_z - d_1}{a_3}\right] \Rightarrow \cos\theta_3 = \sqrt{\dfrac{a_3^2 - (P_z - d_1)^2}{a_3^2}}$

$$\theta_3 = \tan^{-1}\left[\frac{P_z - d_1}{\sqrt{a_3^2 - (P_z - d_1)^2}}\right]$$

recollecting (1) & (2)



$$P_x = (a_1 + a_3 C\theta_3) C\theta_1 + d_2 S\theta_1 \quad \text{---(4)}$$

$$P_y = (a_1 + a_3 C\theta_3) S\theta_1 - d_2 C\theta_1 \quad \text{---(5)}$$

$4 \times C\theta_1$ & $5 \times S\theta_1$ & addition:

$$P_x C\theta_1 + P_y S\theta_1 = (a_1 + a_3 C\theta_3)$$

or $h \cos(\theta_1 - \phi) = a_1 + a_3 C\theta_3$

$$\left\{ h = \sqrt{P_x^2 + P_y^2}, \quad \phi = \tan^{-1}\left(\frac{P_y}{P_x}\right) \right\}$$

or, $\cos(\theta_1 - \phi) = \dfrac{a_1 + a_3 C\theta_3}{h}$

$$\left\{ P_x = h C\phi, \quad P_y = h S\phi \right\}$$

$$\theta_1 = \phi + \cos^{-1}\left[\frac{a_1 + a_3 C\theta_3}{\sqrt{P_x^2 + P_y^2}}\right]$$

From eq^n (5): $d_2 = (a_1 + a_3 C\theta_3)\tan\theta_1 - P_y / C\theta_1$

(b) orientation of the last link of the manipulator (3-link) w.r.t. Base frame:

$$ {}^0_3R = \begin{bmatrix} -0.8 & 0.4242 & 0.4242 \\ 0.6 & 0.5656 & 0.5656 \\ 0 & 0.707 & -0.707 \end{bmatrix} $$

desired orientation of the end effector : ${}^0_6R = R_z(45^\circ)\, R_y(30^\circ)\, R_z(48^\circ)$

$$ = \begin{bmatrix} -0.0670 & -0.9330 & 0.3536 \\ 0.9330 & 0.0670 & 0.3536 \\ -0.3536 & 0.3536 & 0.8660 \end{bmatrix} $$

orientation of wrist w.r.t. end of 3rd Link of manipulator

$$ {}^3_6R = ({}^0_3R)^T\, {}^0_6R = \begin{bmatrix} 0.6134 & 0.7866 & -0.0707 \\ 0.2493 & -0.1079 & 0.9622 \\ 0.7493 & -0.6079 & -0.2623 \end{bmatrix} $$

$$ = \begin{bmatrix} C\theta_4 C\theta_5 C\theta_6 - S\theta_4 S\theta_6 & -C\theta_4 C\theta_5 S\theta_6 - S\theta_4 C\theta_6 & C\theta_4 S\theta_5 \\ S\theta_4 C\theta_5 C\theta_6 + C\theta_4 S\theta_6 & -S\theta_4 C\theta_5 S\theta_6 + C\theta_4 C\theta_6 & S\theta_4 S\theta_5 \\ -S\theta_5 C\theta_6 & S\theta_5 S\theta_6 & C\theta_5 \end{bmatrix} $$

$\theta_4 = \text{atan2}\,(r_{23}, r_{13}) = 94.2$

$\theta_5 = \text{atan2}\left(\sqrt{r_{13}^2 + r_{23}^2}\,, r_{33}\right) = 109.21$

$\theta_6 = \text{atan2}(r_{32}, -r_{31}) = -140.9$