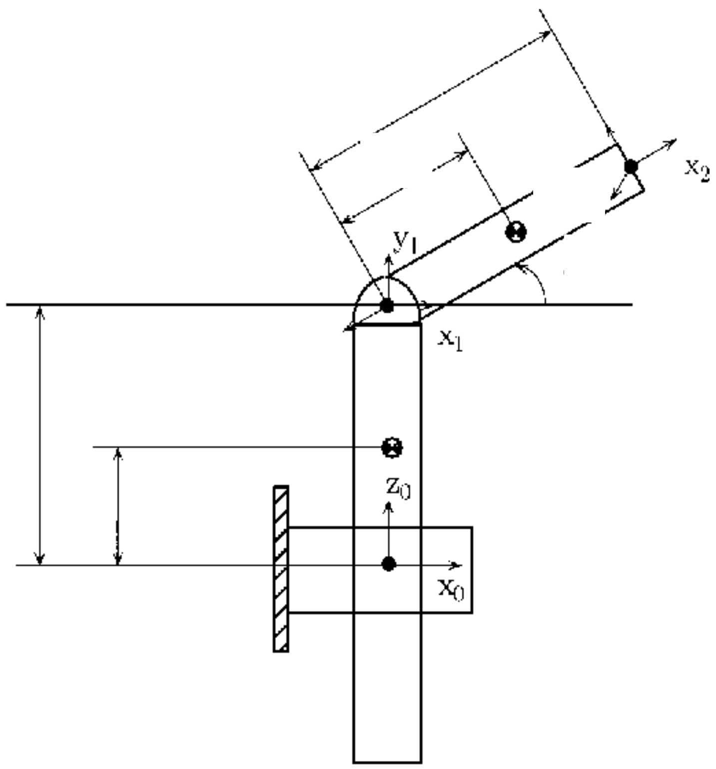
ME604: Introduction to Robotics

Spring 2025

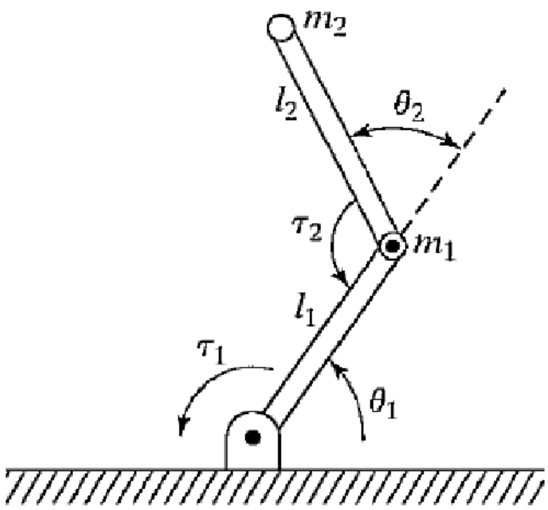
Assignment 6

1. Consider the planar 2-link manipulator shown below.

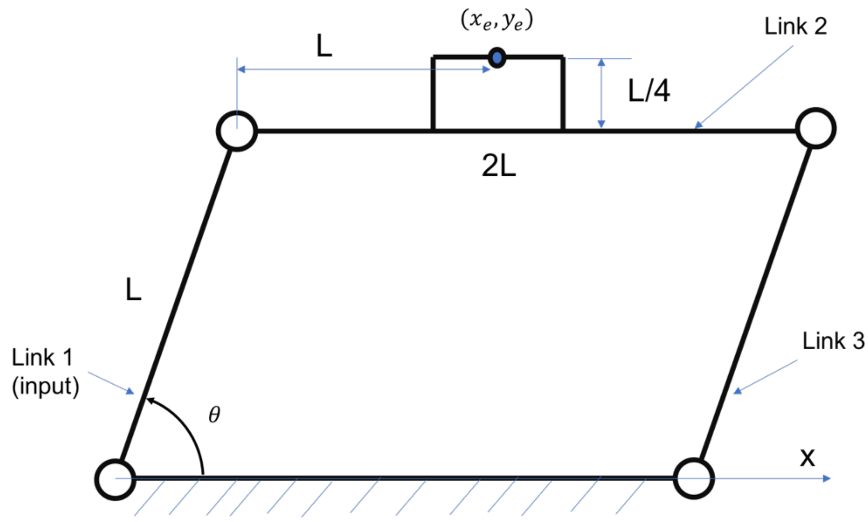


   Derive the dynamic equations governing the motion of this manipulator. Consider the joints to be frictionless. Note that the distance l_{c1} as link 1 moves!

2. Consider the planar 2R manipulator shown below. The links are modeled as point masses with the mass concentrated at the ends. Hence, the moment of inertia of the links about the axes passing through the center of mass can be neglected. Each of the links is driven with identical geared motors with gear ratio r, rotor inertia J_m and damping B_m. You may assume that the torque generated by the motors is given by KV/R where V is the applied voltage, and K & R are constants. Derive the dynamic equations governing the motion of this manipulator in terms of the motor angles. Assume $l_1 = l_2$ and $m_1 = m_2$.



3. Consider the parallelogram-based single degree of freedom robot mechanism shown below. The mechanical structure is such that the opposing links of the manipulator remain parallel to each other at all times.



a. Determine the position and velocity of the end-effector as functions of the input joint angle θ and joint velocity $\dot{\theta}$.
b. Write the 2X1 Jacobians relating the velocities of center of mass of links 1, 2 and 3 to the input joint velocity.
c. Write the expression for kinetic energy of the manipulator. Assume all links to be uniform rigid bars of same density.
d. Derive the equation of motion for the manipulator in the form $\tau = f(\ddot{\theta}, \dot{\theta}, \theta)$, where, τ is the torque applied to the input link.
e. What are the maximum and minimum values of the inertia? At what values of θ do they occur?

4. Consider a rigid body undergoing a pure rotation with no external forces acting on it. The kinetic energy is then given as

$$K = \frac{1}{2}(I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2)$$

with respect to a coordinate frame located at the center of mass and whose coordinate axes are principal axes. Take as generalized coordinates the Euler angles ϕ, θ, ψ and show that the Euler-Lagrange equations of motion of the rotating body are

$$\begin{aligned} I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y\omega_z &= 0 \\ I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z\omega_x &= 0 \\ I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x\omega_y &= 0. \end{aligned}$$