ME604: Introduction to Robotics

Spring 2025

Assignment 1

1. The direction cosines of the i, j, k unit vectors corresponding six reference frames, RF1 to RF6 are (expressed in the reference frame RF0) give below:

$$RF1: \begin{Bmatrix}1\\0\\0\end{Bmatrix}, \begin{Bmatrix}0\\1\\0\end{Bmatrix}, \begin{Bmatrix}0\\0\\1\end{Bmatrix} \quad RF2: \begin{Bmatrix}1\\0\\0\end{Bmatrix}, \begin{Bmatrix}0\\ \frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{Bmatrix}, \begin{Bmatrix}0\\ \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{Bmatrix}$$

$$RF3: \begin{Bmatrix}0\\0\\-1\end{Bmatrix}, \begin{Bmatrix}\frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\end{Bmatrix}, \begin{Bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\end{Bmatrix} \quad RF4: \begin{Bmatrix}0\\0\\-1\end{Bmatrix}, \begin{Bmatrix}-1\\0\\0\end{Bmatrix}, \begin{Bmatrix}0\\1\\0\end{Bmatrix}$$

$$RF5: \begin{Bmatrix}0\\ \frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{Bmatrix}, \begin{Bmatrix}-1\\0\\0\end{Bmatrix}, \begin{Bmatrix}0\\ \frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{Bmatrix} \quad RF6: \begin{Bmatrix}\frac{1}{2}\\ \frac{-1}{2}\\ \frac{-1}{\sqrt{2}}\end{Bmatrix}, \begin{Bmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\\ 0\end{Bmatrix}, \begin{Bmatrix}\frac{-1}{2}\\ \frac{1}{2}\\ \frac{-1}{\sqrt{2}}\end{Bmatrix}$$

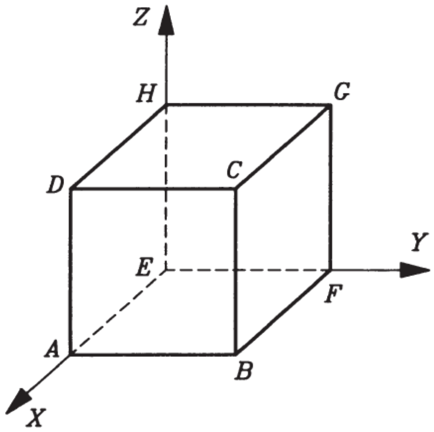
Obtain the rotation matrices ${}^{0}_{n}R$, ${}^{n-1}_{\ \ n}R$, for n = 1 to 6.

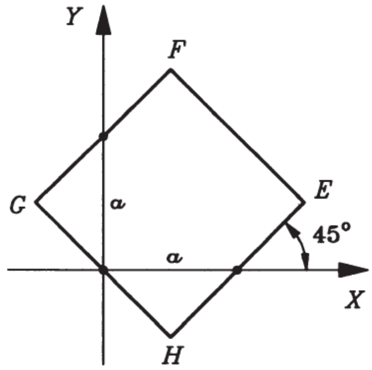
2. a. Compute the rotation matrices $R_X(\theta), R_Y(\theta)$ and $R_Z(\theta)$ that rotate a vector by an angle θ about the x, y and z axis, respectively.

   b. Consider a rigid body with a reference frame {B} affixed to it. Initially, the body is placed with frame {B} aligned with the world frame {A}. The body undergoes the following sequence of transformations (all with respect to the axes of the world frame):
      I. Rotation by -45 degrees about x-axis.
      II. Rotation by 90 degrees about y-axis.
      III. Rotation by 45 degrees about z-axis.
      IV. Rotation by -45 degrees about x-axis
      V. Rotation by 45 degrees about z-axis

Sketch frame {B} and compute the rotation matrix ${}^{A}_{B}R$ after each step.

3. Shown below (left) is a cube that is to be displaced in an assembly operation to a configuration in which face EFGH lies in the XY plane as shown in the figure (right). What is the rotation matrix that relates the final configuration (right) of the cube to the initial one? Justify your answer.





4. Consider a rigid body with a reference frame {B} affixed to it. Initially, the body is placed with frame {B} aligned with the world frame {A}. Consider the following cases

   a. The body is rotated using the rotation operator R given below, and subsequently translated by 1 unit along vector $[1 \quad 1 \quad 1]$ of frame {A}.
   b. The body is first body is first translated by 1 unit along vector $[1 \quad 1 \quad 1]$ of frame {A}, and subsequently rotated using the rotation operator R, which represents rotation about some axis in {A}, given below.

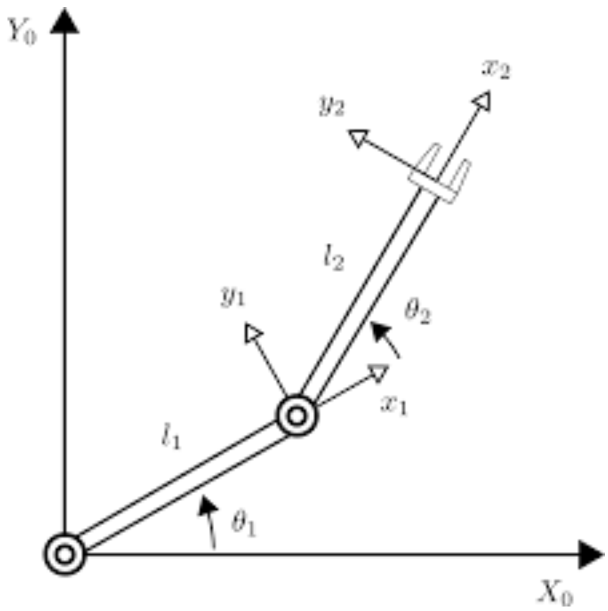
$$R = \begin{bmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

Compute the homogenous transformation matrices ${}^{A}_{B}T$ for both cases, and obtain the co-ordinates of point P in {A} if its co-ordinates in frame {B} are

$$P_B = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

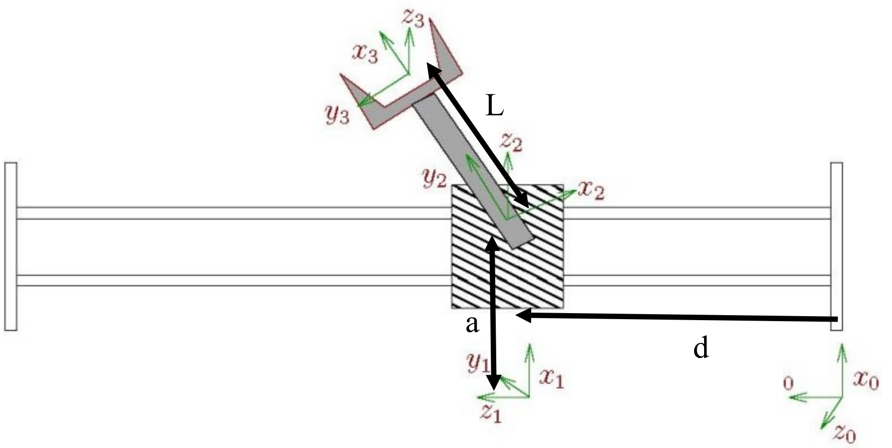
5. For each of the robots shown below
   a. Write the transformation matrices ${}^{n-1}_{n}T$.
   b. Compute the transformation matrix from the end-effector frame to the world frame {0}.

I.



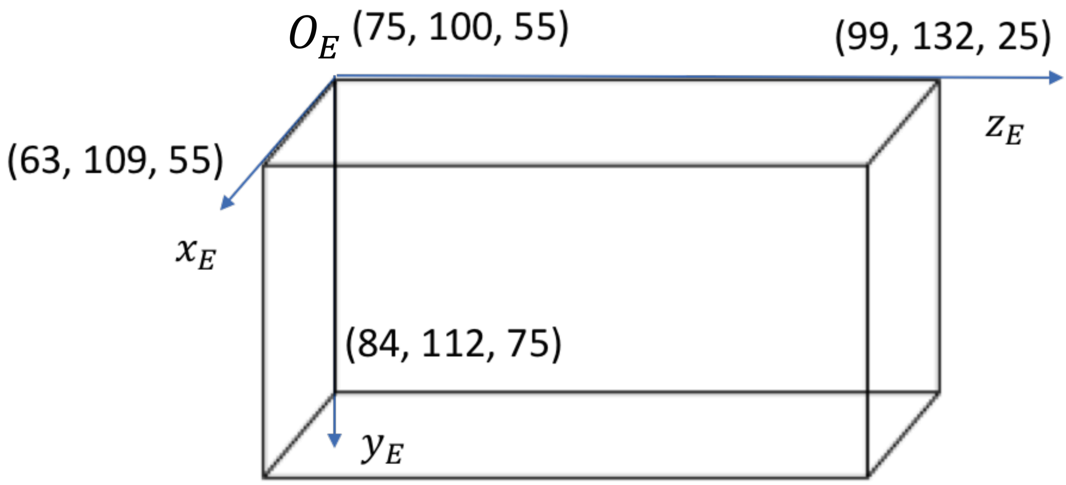
Assume that frame {0} being attached to a fixed base and frame {i} is attached to the i-th body. The circles represent joints that allow rotation about the axis passing through the center and coming out of the plane of paper.

II.



The block (marked with diagonal lines) can slide along a fixed frame, while the shaded link rotates about axis z_2. Assume that angle between axis y_2 and x_1 is θ.

6. The end-effector of a robot holds a cuboidal object such that a corner of the cuboid is located at the origin of the end-effector frame. Further, the cuboid is oriented such that the longest, intermediate and shortest edges are aligned along the z, y and x axis of the end-effector frame, as shown in the figure below. A camera is used to determine the co-ordinates of the corners of the cuboid in the world frame.



a) Determine the homogeneous transformation matrix relating the end-effector frame to the world frame at this instant.

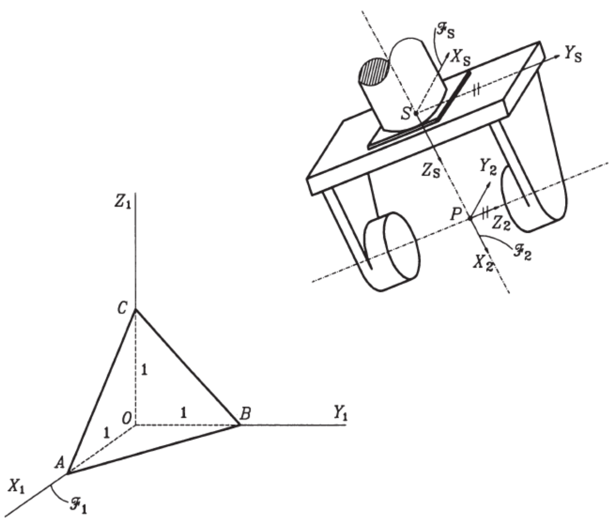
b) If the homogeneous transformation matrix relating the robot base frame to the end-effector frame is given by,

$$ {}^{B}_{E}T = \begin{bmatrix} -4/5 & 9/25 & 12/25 & 68 \\ 3/5 & 12/25 & 16/25 & 174 \\ 0 & 4/5 & -3/5 & -60 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

find the homogeneous transformation relating the base frame to the world frame.

7. The axes X_1, Y_1, Z_1 of a frame F_1 are attached to the base of a robotic manipulator, whereas axes X_2, Y_2, Z_2 of a second frame F_2 are attached to its end-effector, as shown below. Origin P of F_2 has F_1- coordinates (1, -1, 1). Furthermore, the orientation of the end effector with respect to the base is defined by a rotation matrix

$$ {}^{1}_{2}R = \frac{1}{3}\begin{bmatrix} 1 & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & 1 & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & 1 \end{bmatrix}. $$



a. If distance between P and S be 1 unit, find the F_1- and F_2- coordinates of point S.
b. The end-effector is approaching plane ABC shown in the figure. What is the equation of the plane in F_2- coordinates?
Hint: Equation of a plane is of the form $ax + by + cz + d = 0$.

8. Consider the figure below. The cube measuring 20 cm on a side is placed at the center of the table as shown. The camera is situated directly above the center of the cube, 2 m above the tabletop. Coordinate frames $x_0y_0z_0, x_1y_1z_1, x_2y_2z_2$ and $x_3y_3z_3$ are attached to the robot base, table top, center of the cube and the camera, respectively. Note that $x_2y_2z_2$ is attached to the center of the bottom face of the cube.

   a) Find the homogenous transformation relating each of the frames to the base frame.

   b) Suppose the cube is rotated by 45° about z_1 and subsequently moved by 0.5m along x_0. Compute the new homogeneous transformation relating the block frame to the base frame.