1.

$$2\ddot{x} + \dot{x} = F(t)$$

Design a PD trajectory tracking controller to track a reference signal $x_d(t) = \sin t + \cos 2t$. The closed loop system should have a natural frequency less than 10 radians with a damping ratio greater than 0.707.

Set $F(t) = 2\left[K_p(x_d - x) + K_v(\dot{x}_d - \dot{x})\right] + 2\ddot{x}_d + \dot{x}$

Substituting,

$\ddot{e} + K_p e + K_v \dot{e} = 0$, $e = x_d - x$

choose K_p and K_v as needed.

2. Consider the coupled nonlinear system

$$\begin{aligned}\ddot{y}_1 + 3y_1y_2 + y_2^2 &= u_1 + y_2u_2, \quad &(1)\\ \ddot{y}_2 + (\cos y_1)\dot{y}_2 + 3(y_1 - y_2) &= u_2 - (\cos y_1)^2 y_2 u_1 \quad &(2)\end{aligned}$$

(a) Can these equations be written in the form Text

$$u_1 = f_1(\ddot{y}, \dot{y}, y); \qquad u_2 = f_2(\ddot{y}, \dot{y}, y)$$

solve (1) and (2) as linear eqns in u_1 and u_2

write your eqns in the form

$M(y)\ddot{y} + V(y, \dot{y}) + G(y) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

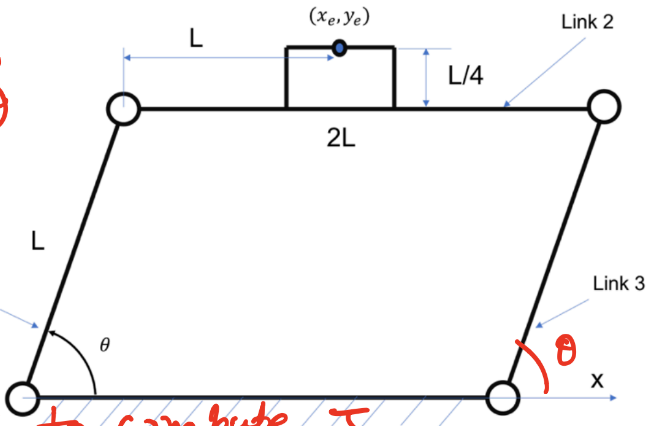
$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and design the controller.

3

Note $\omega_3 = \omega_1 = \dot{\theta}$

$\omega_2 = 0$

Write position co-ordinates of CoM of links and diff to compute J_{v_i}



The eqn. would be similar to that of a single DOF system. Design controller accordingly.

5

(a) For each link i, we have attached a frame $\{C_i\}$ to the center of mass (frame $\{2\}$ is same as frame $\{C_2\}$. Calculate matrices ${}^{0}_{C_1}T$ and ${}^{0}_{C_2}T$.

$${}^{0}v_{c_1} = \begin{bmatrix} -\frac{d_1}{2} s\theta_1 \\ \frac{d_1}{2} c\theta_1 \\ 0 \end{bmatrix} \dot{\theta}_1 \qquad {}^{0}v_{c_2} = \begin{bmatrix} -d_2 s\theta_1 \dot{\theta}_1 \\ d c\theta_1 \dot{\theta}_1 \\ \dot{d}_2 \end{bmatrix}$$

For this two-link manipulator, the mass matrix has the form

$$M(q) = m_1 J_{v1}^T J_{v1} + m_2 J_{v2}^T J_{v2} + J_{\omega 1}^T I_{c1} J_{\omega 1} + J_{\omega 2}^T I_{c1} J_{\omega 2}$$

$$= \underbrace{\begin{bmatrix} -\frac{d_1}{2} s\theta_1 & 0 \\ \frac{d_1}{2} c\theta_1 & 0 \\ 0 & 0 \end{bmatrix}}_{J_{c_1}} \dot{q}$$

where, J_{vi} is the Jacobian of the center of mass of link i, $J_{\omega i}$ is the angular velocity of link i, and I_{ci} is the inertia tensor of link i expressed in frame $\{C_i\}$.

(b) Calculate ${}^{0}J_{v1}$ and ${}^{0}J_{v2}$.

(c) Calculate ${}^{c1}J_{\omega 1}$ and ${}^{c2}J_{\omega 2}$. $\rightarrow \omega_1 = \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_1$

$$= \underbrace{\begin{bmatrix} -d s\theta_1 & 0 \\ d c\theta_1 & 0 \\ 0 & 1 \end{bmatrix}}_{J_{v_2}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d} \end{bmatrix}$$

(d) Calculate I_{c1} and I_{c2} in terms of the masses and dimensions of the links.

(e) Calculate the mass matrix $M(q)$.

(f) Calculate the other terms (gravity vector, Coriolis and centrifugal terms) and write out the equations of motion as:

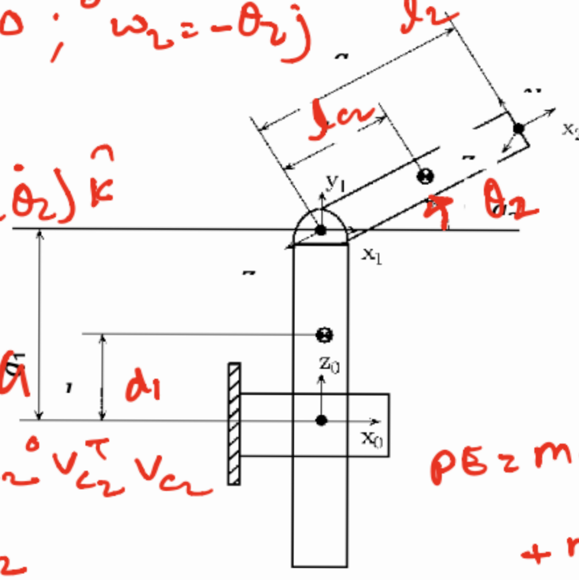
$$\tau_1 = f_1(\ddot{q}, \dot{q}, q); \qquad \tau_2 = f_2(\ddot{q}, \dot{q}, q)$$

g) $q_{1d} = c_1 t$, $q_{2d} = c_2 t$ $\quad \tau = M(q)\,[\text{PD control}_{\text{unit mass}}] + C\dot{q} + g(q)$

6

$^0v_{c1} = \dot{d}_1 \hat{k}$; $^0\omega_1 = 0$; $^0\omega_2 = -\dot{\theta}_2 \hat{j}$

$^0v_{c2} = -l_{c2}\, s\theta_2\, \dot{\theta}_2\, \hat{i} + (\dot{d}_1 + l_{c2}\, c\theta_2\, \dot{\theta}_2)\hat{k}$



$$K.E. = \frac{1}{2} m_1\, {}^0v_{c1}^T\, {}^0v_{c1} + \frac{1}{2} m_2\, {}^0v_{c2}^T\, {}^0v_{c2} + \frac{1}{2} I_{c2}\, {}^0\omega_2^T\, {}^0\omega_2$$

$$PE = m_1 g d_1 + m_2 g l_{c2} s\theta_2$$

use Lagrangian & Add friction term

a) Derive the dynamic equations governing the motion of this manipulator. Consider the coefficient of viscous friction at the prismatic joint to be c_1, and revolute joint to be frictionless. Note that the distance l_{C1} as link 1 moves!

b) Transform the equations in part (a) to obtain the equations of motion in task space

c) Design a task-space nonlinear decoupling PD trajectory controller with ω = 36 rad/s to follow some desired trajectory $[x_{\cdot}(t), y_{\cdot}(t)]^T$.

follow formula derived in class

Follow lecture notes to cancel non-linearities and get task-space eqns. in the form $\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$

end-effector position

design a_x, a_y as PD controllers for unit mass case.