ME604 Assignment #2

Date :

(1) $R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$ $R_y(\psi) = \begin{bmatrix} c\psi & 0 & s\psi \\ 0 & 1 & 0 \\ -s\psi & 0 & c\psi \end{bmatrix}$ $R_z(\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R = R_y(\psi)\, R_z(\theta)\, R_x(\phi)$

(2) $R = \left[R_z(\theta)\, R_x(\psi)\, R_z(\theta)\, R_x(\phi) \right]$

$= \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$

(3) ~~$R_z(45^\circ)$~~ Rotated by 45° along z_0

$B' = R_z(45^\circ) \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt2 \\ 1 \end{bmatrix}$

Translation by 2 units in y_0

$B'' = B' + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2+\sqrt2 \\ 1 \end{bmatrix}$

Rotated by 90° along x_0

$B''' = R_x(90^\circ) \cdot B'' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2+\sqrt2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -1 \\ 2+\sqrt2 \end{bmatrix}$

Rotation about O_1A by 45°

$R_x(45^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt2 & -1/\sqrt2 \\ 0 & 1/\sqrt2 & 1/\sqrt2 \end{bmatrix}$.

$B'''' = R_x(45^\circ) \cdot B''' = \begin{bmatrix} 0 \\ \frac{-1}{\sqrt2} - \sqrt2 - 1 \\ \frac{-1}{\sqrt2} + \sqrt2 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-\sqrt2 - 3}{\sqrt2} \\ \frac{-\sqrt2 + 1}{\sqrt2} \end{bmatrix}$

$\therefore \left(0,\ \frac{-3-\sqrt2}{\sqrt2},\ \frac{1-\sqrt2}{\sqrt2} \right)$

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(4) $R_{y_0}(90^\circ) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$R_{z_1}(45^\circ) = \begin{bmatrix} \cancel{\cos}\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{\sqrt2}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt2 \end{bmatrix}$

$R = R_{y_0}(90^\circ)\, R_{z_1}(45^\circ) = \frac{1}{\sqrt2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt2 \end{bmatrix}$

$= \frac{1}{\sqrt2}\begin{bmatrix} 0 & 0 & \sqrt2 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$

axis angle representation:

$\theta = \cos^{-1}\left(\frac{Tr(R) - 1}{2}\right) = \cos^{-1}\left(\frac{\frac{1}{\sqrt2} - 1}{2}\right) = \cancel{\cos^{-1} 0}$

$= \cancel{90^\circ}\ \cos^{-1}\left(\frac{1 - \sqrt2}{2\sqrt2}\right)$

$= 98.4^\circ$

rotation axis:

$r_x = \frac{R_{32} - R_{23}}{2\sin\theta} = \frac{0 - 1}{2(0.985)} = -0.508$

$r_y = \frac{R_{13} - R_{31}}{2\sin\theta} = 0.866$

$r_z = \frac{R_{21} - R_{12}}{2\sin\theta} = 0.508$

(5) $k = \frac{1}{\sqrt3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$R(k,\theta) = I + \sin\theta\, [k]_\times + (1 - \cos\theta)\, kk^T$

$\theta = \pi/2$

$[k]_\times = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix}$

$kk^T = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \frac{1}{\sqrt3}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$R(k,\theta) = I + (1)\cdot\frac{1}{\sqrt2}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} + (1 - 0)\cdot\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

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$$R(k,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -0.577 & 0.577 \\ 0.577 & 0 & -0.577 \\ -0.577 & 0.577 & 0 \end{bmatrix} + \begin{bmatrix} 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \\ 0.333 & 0.333 & 0.333 \end{bmatrix}$$

$$= \begin{bmatrix} 1.333 & -0.244 & 0.910 \\ 0.91 & 1.333 & -0.244 \\ -0.244 & 0.91 & 1.333 \end{bmatrix}$$

Verificat^n from formula:

$$v' = R(k,\theta)\,v = v\cos\theta + (k\times v)\sin\theta + (1-\cos\theta)(k^T v)\,k$$

$$v' = (k\times v) + (k^T v)\,k$$

(6) → possible sets of Euler angles

ZXY, ZYZ, XYX, XZX, YXY, YZY

→ NO, ZZX is not valid Euler angle sequence because it violates non ~~of~~ redundancy condition as euler angles must have:

i) No 2 consecutive rots be about same axis

ii) Middle rot is different to ensure 3D transformat^n

→ Rotation Matrix of ZYZ

i) Rotate by α about z axis $= R_z(\alpha)$

ii) Rotate by θ about y axis $= R_y(\theta)$

iii) -u- by γ -u- z axis $= R_z(\gamma)$

$$R = (R_z(\alpha)\,\overset{I}{\cancel{R_y(0)}}\,R_z(\gamma) = R_z(\alpha)\,R_z(\gamma)$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\gamma) & -\sin(\alpha+\gamma) & 0 \\ \sin(\alpha+\gamma) & \cos(\alpha+\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for $\left(\alpha = \frac{\pi}{2}, \gamma = \frac{\pi}{2}\right)$ ~~RO~~ $R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ direction to x_2 axis

$\alpha = 90^\circ, \gamma = 90^\circ$

$$x_2 = \begin{bmatrix} \cos\alpha\cos\gamma - \sin\alpha\sin\gamma \\ \sin\alpha\cos\gamma + \cos\alpha\sin\gamma \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

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