Course Objectives

- To learn DOE techniques based on probability theory and statistical tools
- Understand proper methods for data collection, defining quality parameters, diagnostic tools, quality analysis, and interpretation for process/product improvement.
- Statistical DOE enables understanding of the relationship between multiple input variables/factors and the key responses, or product/process performance.

In-depth knowledge of these tools is quintessential while conducting scientific experiments

List of Topics*

Fundamental Concepts and Methods

- Quality philosophy and conceptual framework
- Statistical Methods and Probability Concepts for Data Characterization

Classical Design of Experiments

- Nature of variability, probability distributions
- Empirical models (regression, hypothesis testing, confidence intervals, applications)
- Two-level factorial designs (factor effects, ANOVA, residual analysis, interactions)
- General 2^k factorial designs
- Two-level fractional factorial designs

Response Surface Methodology

- First and second-order models and surfaces
- Central composite designs
- Multiple response analysis, Design rotatability, Box-Behnken design

Robust Design Method

- Quality loss function, signal, noise and control factors, product life cycle
- Matrix experiments using orthogonal arrays, analysis of means and variance, error prediction
- Steps in robust design: noise factors and testing, signal to noise ratio, degrees of freedom, selection of orthogonal arrays
- Conducting matrix experiments: randomization, confounding, result interpretation, verification
- Factor interactions, dynamic problems

Spring 2025

ME 794 Statistical Design of Experiments

Chapter 1

Fundamental Concepts

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Nature of Data

Question: Is Sachin Tendulkar a ‘good’ batsman?

How would you answer that?

- You need some data to decide ..
- In general, when we collect data, we are interested in,
  - How the process/product is behaving in terms of an output quality characteristic(s)
  - Perhaps, in terms of (a) average value of the characteristic, (b) the variation in individual measurements of the characteristic

Nature of Data

How would you collect the data?

In general, when we collect data, we have three choices,

- Observe the process ONCE, and use that observation as the absolute reflection of the process behavior

‘Fatal Error!’

- Observe ALL of the output of the process to get a true reflection of its behavior

Observing **entire population/universe**, i.e. all possible realizations of the process – a very large number. **Neither practical nor necessary**

- Observe PART OF the output of the process and use it to infer something about the process behavior

More ‘practical’! We are **‘sampling’** the process (population)

Sachin Tendulkar stats
Batsman

Career Batting Stats
Right-Handed Batsman

Format	M	Inn	NO	Runs	HS	Avg	BF	BF	SR	100s	50s
Test 1989–13	200	329	33	15921	248*	53.8	29437	379	-	51	68
ODI 1989–12	463	452	41	18426	200*	44.8	21367	147	86.2	49	96
T20I 2006	1	1	0	10	10	10.0	12	12	83.3	0	0
IPL 2008–13	78	78	11	2334	100*	34.8	1948	66	119.8	1	13

Nature of Data

We need an efficient and adequate method to collect and analyze the data

Critical Issues in Sampling

- How much do we sample?
- How do we sample?
- When do we sample?
- How/what can we say something about the process from the information contained in the sample?

We will keep exploring these answers throughout the course…

For now, it is important to understand the difference between a Sample vs. the Population

Sachin Tendulkar stats
Batsman

Career Batting Stats
Right-Handed Batsman

Format	M	Inn	NO	Runs	HS	Avg	BF	BF	SR	100s	50s
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Nature of Data

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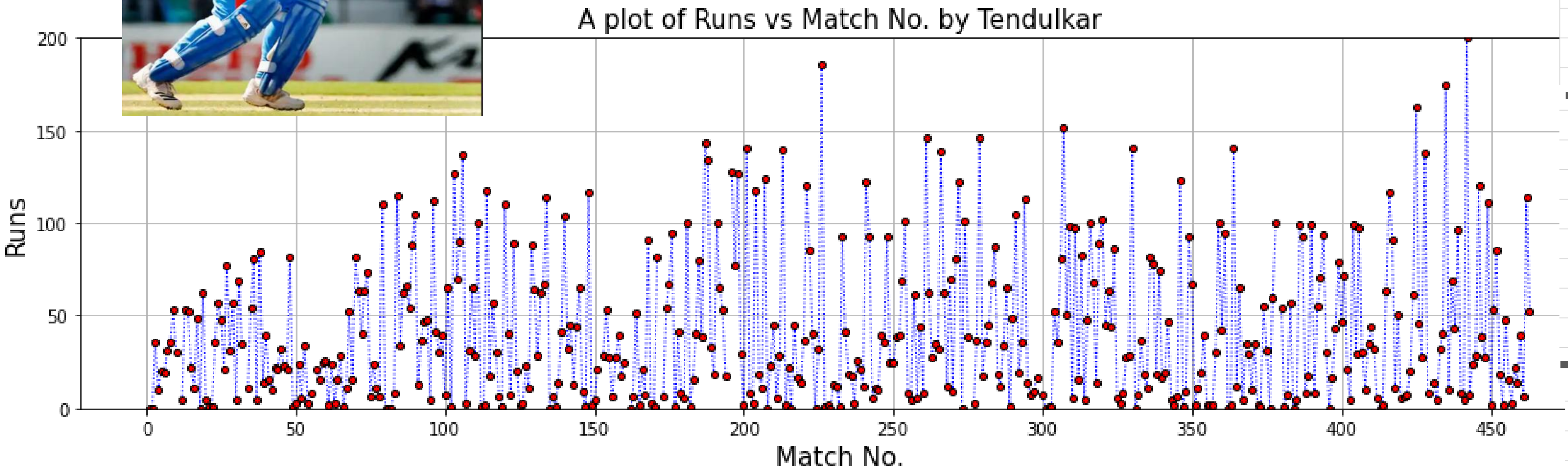
What do you notice?

Variability!



Match No.	Runs	Balls	Mins	4s	6s
463	52	48	93	5	1
462	114	147	205	12	1
461	6	19	25	1	-
460	39	30	48	5	-
459	14	15	34	2	-
458	22	23	33	3	-
457	3	12	21	-	-
456	15	24	25	2	-
455	48	63	87	5	-
454	2	6	9	-	-
453	18	14	21	2	-
285	18	16	20	3	-
284	87	67	120	13	1
283	68	79	110	10	-
282	45	60	101	6	-
281	36	43	71	6	-
280	17	42	49	3	-
279	146	132	186	17	-
278	37	35	68	5	-
10	30	29	-	1	2
9	53	41	-	7	1
8	36	22	-	3	2
7	31	26	31	3	-
6	19	35	38	1	1
5	20	25	-	1	-
4	10	12	-	-	-
3	36	39	51	5	-
2	0	2	2	-	-
1	0	2	-	-	-

ODI BATTING

Ref: http://www.cricketweb.net

Mat	Inns	NO	Runs	Avg	HS	BF	SR	50s	100s	0s	G 0s	4s	6s	ct	st
463	452	41	18426	44.83	200*	21367	86.23	96	49	19	1	2016	192	140	0

Sources of Variability

Taguchi suggests that variation in product and process function arises from three basic sources:

- **Outer Noise:** Sources of noise which influence performance as measured during field use under actual operating conditions, e.g., temp, humidity, supply voltage, vibration

- **Inner Noise:** Internal change in product characteristics such as drift from the nominal over time due to deterioration, e.g., mechanical wear, aging

- **Variational Noise:** Variation in the product parameters from one unit to another as a result of the manufacturing process, e.g., manufacturing imperfection

Example 1: Refrigerator

- **Outer Noise** **[Operating Conditions]**
  - The number of times the door is opened and closed
  - The amount of food kept and the initial temperature of the food
  - Variation in the ambient temperature
  - Supply voltage variation
- **Inner Noise** **[Deterioration]**
  - The leakage of Refrigerant
  - Mechanical Wear of Compressor parts
- **Variational Noise** **[Mfg/Use Imperfection]**
  - The tightness of door closure
  - The amount of refrigerant used

Example 2: Braking Distance of a Car

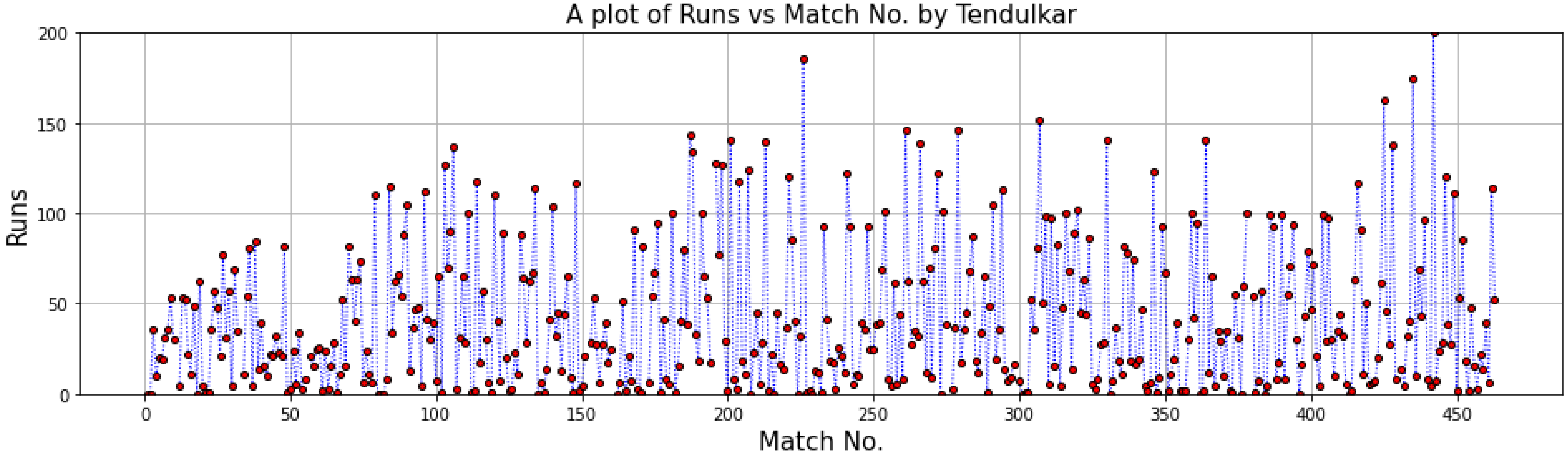
- **Outer Noise** **[Operating Conditions]**
  - Wet or dry road
  - Concrete or Asphalt pavement
  - Number of passengers in the car
- **Inner Noise** **[Deterioration]**
  - The leakage of brake fluid
  - Wear of brake drums and brake pads
- **Variational Noise** **[Mfg/Use Imperfection]**
  - Variation in friction coefficient of pads and drums
  - The amount of brake fluid

Data Characterization

Match No.	Runs	Balls
463	52	48
462	114	147
461	6	19
460	39	30
459	14	15
458	22	23
457	3	12
456	15	24
455	48	63
454	2	6
453	18	14
285	18	16
284	87	67
283	68	79
282	45	60
281	36	43
280	17	42
279	146	132
278	37	35
10	30	29
9	53	41
8	36	22
7	31	26
6	19	35
5	20	25
4	10	12
3	36	39
2	0	2
1	0	2

THREE Important Characteristics of Data

- Central Tendency
- Variability or Dispersion
- Shape of Frequency Distribution



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Central Tendency (Mean)

- Given a sample of n pieces of data $(y_1\ y_2, y_3, .., y_n)$ taken from a given population of size N, the arithmetic mean of the sample, denoted by $\bar{\boldsymbol{y}}$ is

$$\bar{y} = \sum_{k=1}^{n} \frac{y_i}{n}$$

- Population mean $\boldsymbol{\mu}$ given as,

$$\mu = \sum_{k=1}^{N} \frac{y_i}{N}$$

- **Note:** True mean ($\boldsymbol{\mu}$) of the population of size N could be different than sample mean $\bar{\boldsymbol{y}}$.

DIY

```
Can you show that as n -> N, the
sample mean -> population mean?
```

Dispersion/Variability

- **Range (R):** Difference between the largest value and the smallest value of the data

$$R = \max(y_i) - \min(y_i)$$

- **Variance (s^2):** Sample variance is given by

$$s^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n-1}$$

DIY

```
Why do we use (n-1) in the denominator for sample variance?
```

- Note, that the true variance (σ^2) of the population of size N could be different

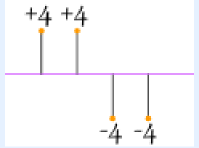
$$\sigma^2 = \sum_{i=1}^{N} \frac{(y_i - \mu)^2}{N}$$

- Notice (n-1) in the denominator for sample variance, while N for true variance
- Standard Deviation = Square root of Variance

Dispersion/Variability

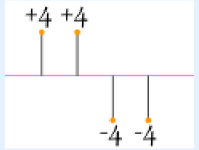
Why is variance defined as follows is a GOOD way to assess variability?

- What if we just add the deviations from the mean and take the average?

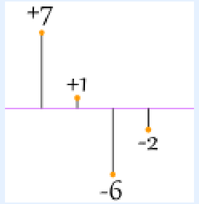


$$\frac{4 + 4 - 4 - 4}{4} = 0$$

- What if we just take absolute values of deviations from the mean?

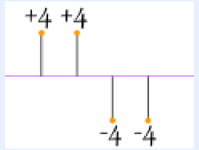


$$\frac{|4| + |4| + |-4| + |-4|}{4} = \frac{4 + 4 + 4 + 4}{4} = 4$$

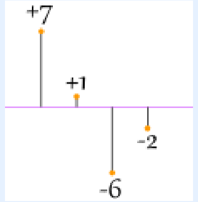


$$\frac{|7| + |1| + |-6| + |-2|}{4} = \frac{7 + 1 + 6 + 2}{4} = 4$$

- But, when we square..



$$\sqrt{\left(\frac{4^2 + 4^2 + (-4)^2 + (-4)^2}{4}\right)} = \sqrt{\left(\frac{64}{4}\right)} = 4$$



$$\sqrt{\left(\frac{7^2 + 1^2 + (-6)^2 + (-2)^2}{4}\right)} = \sqrt{\left(\frac{90}{4}\right)} = 4.74...$$

Ref: https://www.mathsisfun.com/data/standard-deviation.html

Median and Mode

- The median of a finite list of numbers is the "middle" number when those numbers are listed in order from smallest to greatest.
- Mode is the most frequent value in the data set

Examples:

- Test Scores out of 20: (0, 11, 15, 8, 18, 19, 7, 8, 9, 12)
- Test Scores out of 20: (0, 11, 15, 8, 18, 7, 8, 9, 12)
- Test Scores out of 20: (0, 11, 15, 8, 18, 19, 7, 8, 9, 15)

Data Characterization

- It has been shown that the mean (average) can be used to describe the data characteristics.
- However, it is possible to find two sets of data to have equal averages, but different degrees of scatter
- It has been a common mistake in many cases of applications to put all emphasis on the average but overlook the scatter of the data
- Such a mistake usually leads to unnecessary erroneous conclusions which could have been easily avoided if the scatter of the data had been considered.

Data Presentation

Table 1. Modified Table in Lee and Kim's Research (Adapted from Korean J Anesthesiol 2017; 70: 39-45)

Variable	Group	Baseline	After drug	1 min	3 min	5 min
SBP	C	135.1 ± 13.4	139.2 ± 17.1	186.0 ± 26.6*	160.1 ± 23.2*	140.7 ± 18.3
	D	135.4 ± 23.8	131.9 ± 13.5	165.2 ± 16.2*,†	127.9 ± 17.5†	108.4 ± 12.6†,‡
DBP	C	79.7 ± 9.8	79.4 ± 15.8	104.8 ± 14.9*	87.9 ± 15.5*	78.9 ± 11.6
	D	76.7 ± 8.3	78.4 ± 6.3	97.0 ± 14.5*	74.1 ± 8.3†	66.5 ± 7.2†,‡
MBP	C	100.3 ± 11.9	103.5 ± 16.8	137.2 ± 18.3*	116.9 ± 16.2*	103.9 ± 13.3
	D	97.7 ± 14.9	98.1 ± 8.7	123.4 ± 13.8*,†	95.4 ± 11.7†	83.4 ± 8.4†,‡

Values are expressed as mean ± SD. Group C: normal saline, Group D: dexmedetomidine. SBP: systolic blood pressure, DBP: diastolic blood pressu
MBP: mean blood pressure. HR: heart rate.

Table 2. Difference between a Regular Table and a Heat Map

Example of a regular table				Example of a heat map			
SBP	DBP	MBP	HR	SBP	DBP	MBP	HR
128	66	87	87	128	66	87	87
125	43	70	85	125	43	70	85
114	52	68	103	114	52	68	103
111	44	66	79	111	44	66	79
139	61	81	90	139	61	81	90
103	44	61	96	103	44	61	96
94	47	61	83	94	47	61	83

All numbers were created by the author. SBP: systolic blood pressure, DBP: diastolic blood pressure, MBP: mean blood pressure, HR: heart rate.

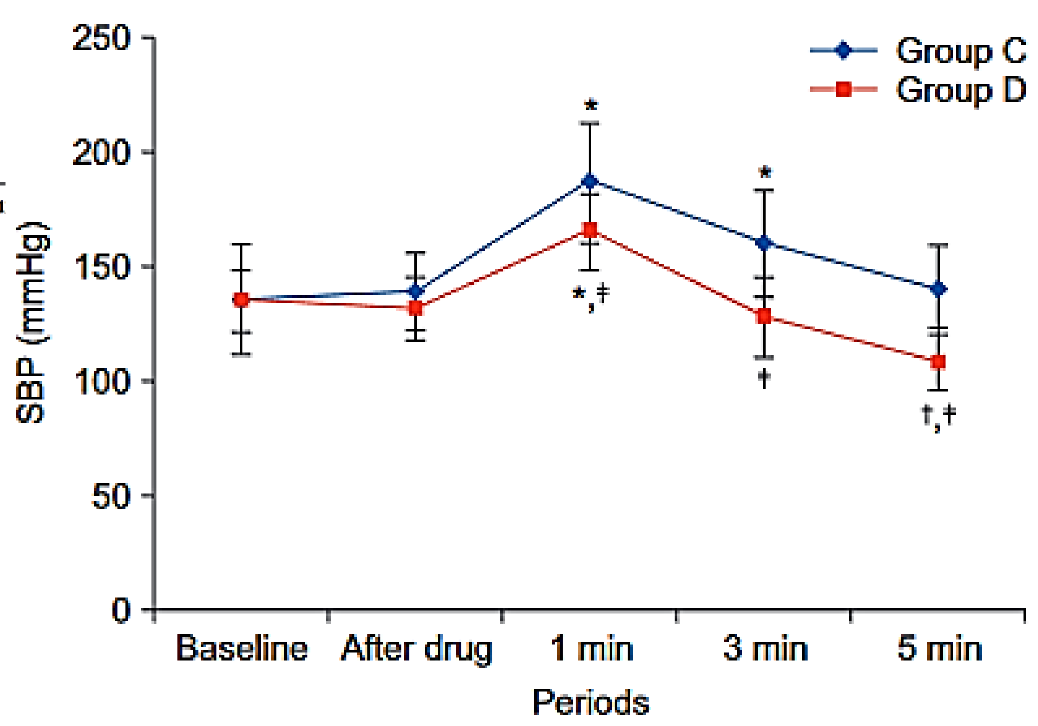
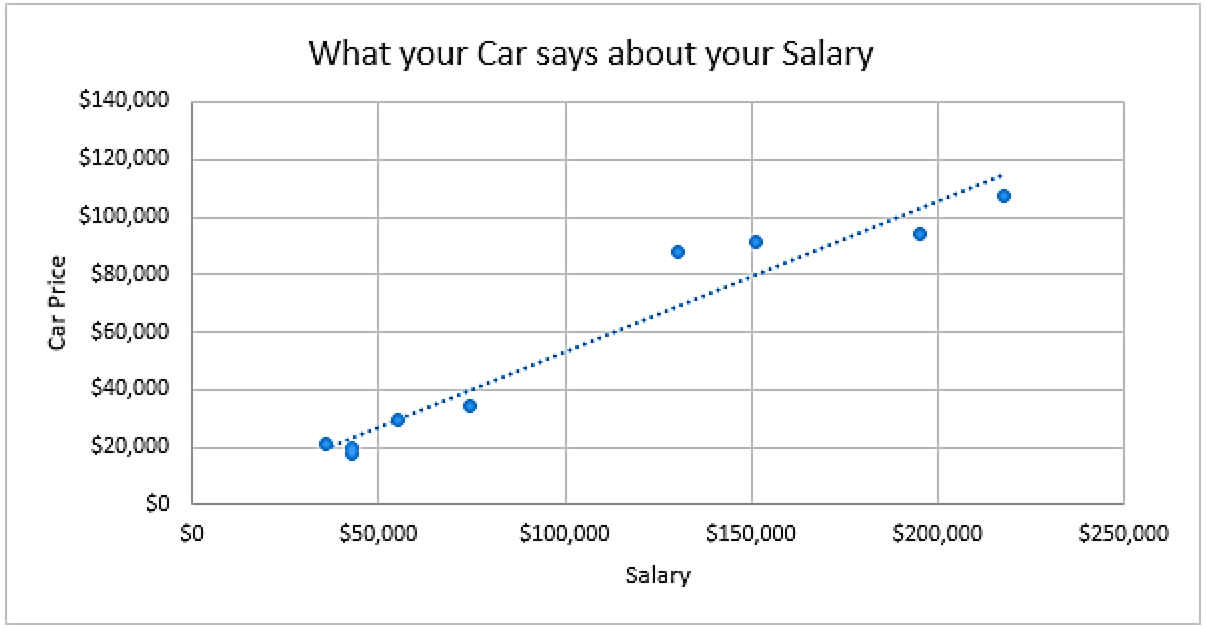


Fig. 1. Line graph with whiskers. Changes in systolic blood pressure (SBP) in the two groups. Group C: normal saline, Group D: dexmedetomidine.

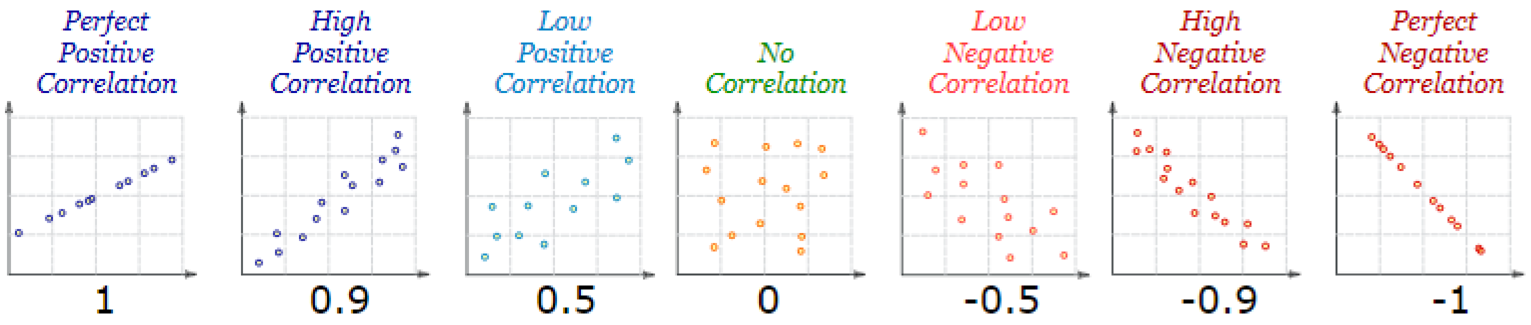
Reference: In, Junyong, and Sangseok Lee. "Statistical data presentation." *Korean journal of anesthesiology* 70.3 (2017): 267.

Data Presentation

Scatter Plot

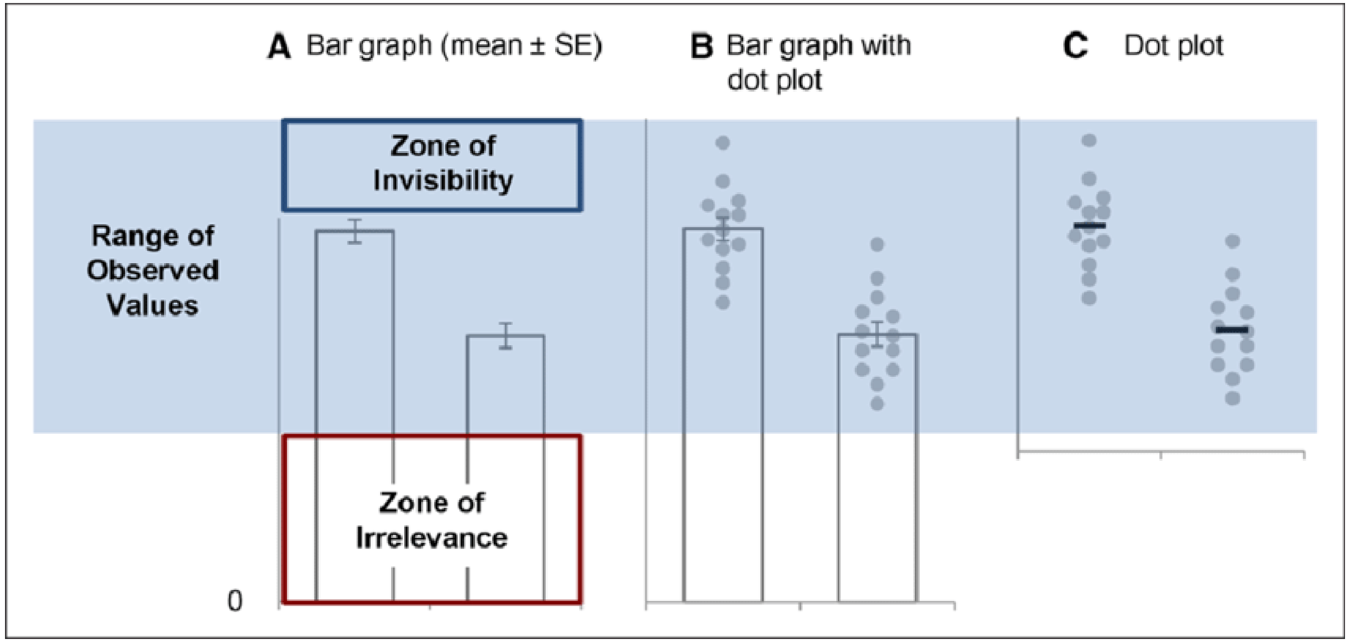


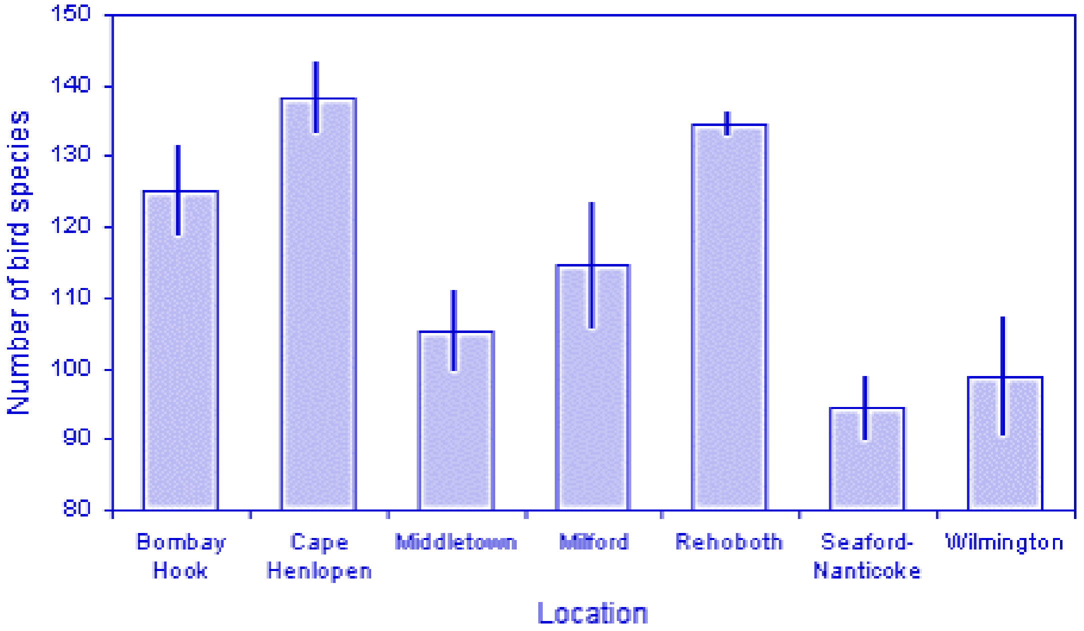
Scatter plots are used to investigate association between two variables



Data Presentation

Bar Graph



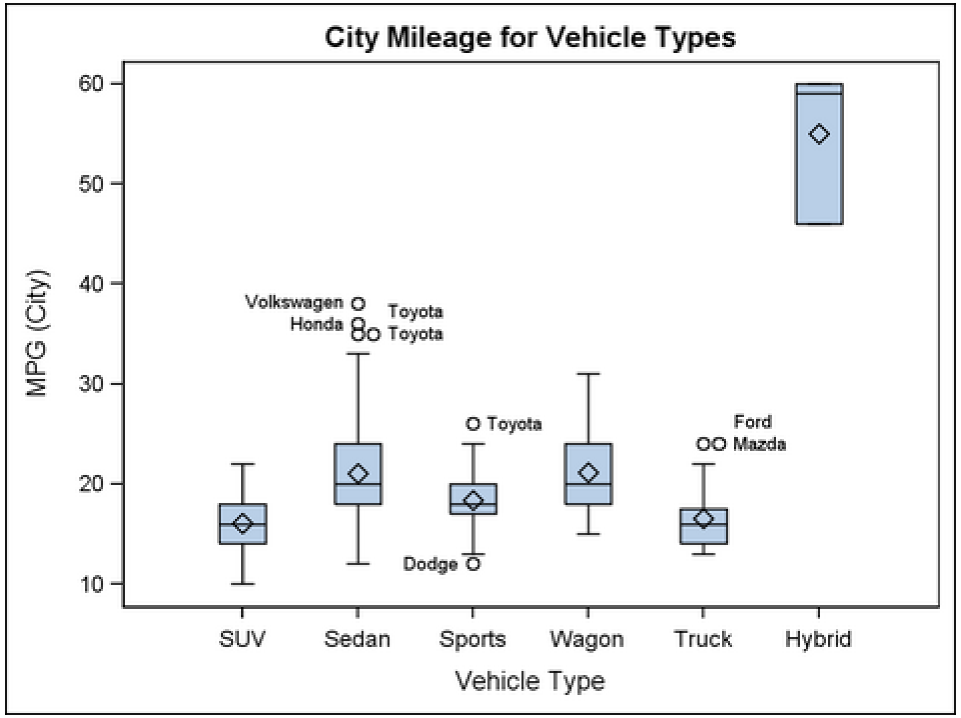


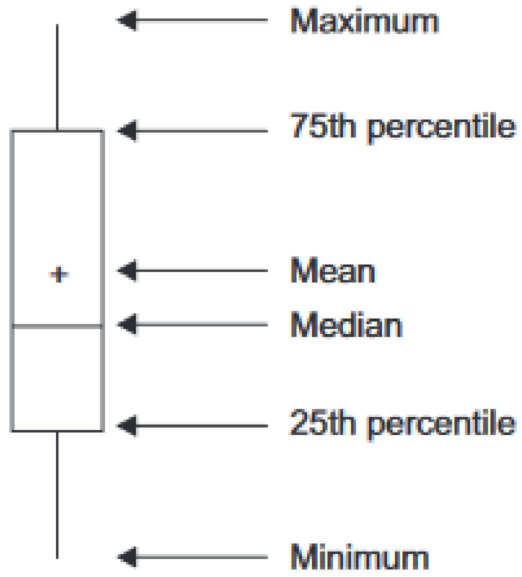
Bar graphs are used to indicate and compare values in discrete category or groups

Reference: In, Junyong, and Sangseok Lee. "Statistical data presentation." *Korean journal of anesthesiology* 70.3 (2017): 267.

Data Presentation

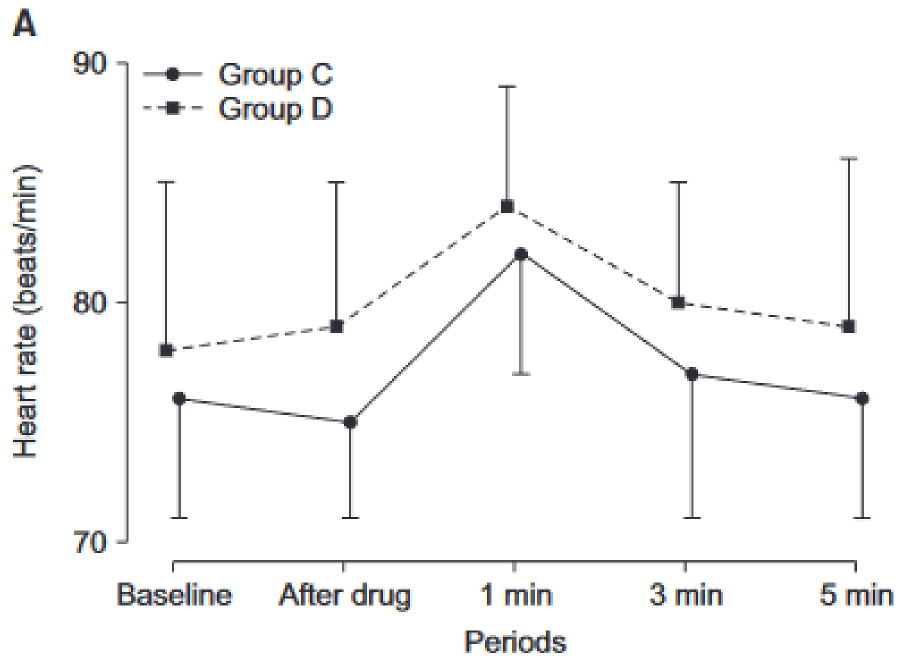
Box and Whisker Graph

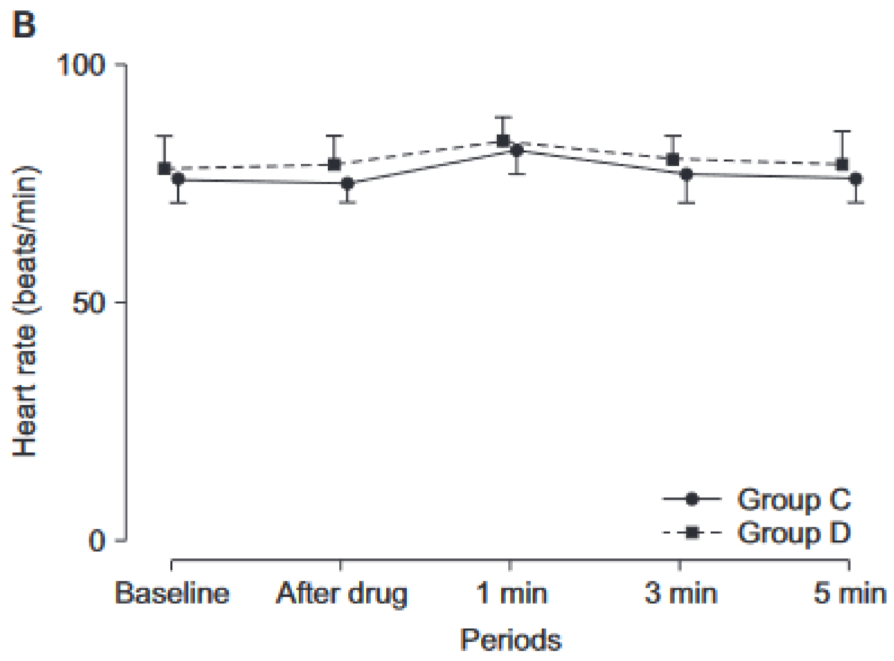




Reference: In, Junyong, and Sangseok Lee. "Statistical data presentation." *Korean journal of anesthesiology* 70.3 (2017): 267.

Data Presentation





Misleading Plot

Reference: In, Junyong, and Sangseok Lee. "Statistical data presentation." *Korean journal of anesthesiology* 70.3 (2017): 267.

Data Presentation

Table 3. Types of Charts depending on the Method of Analysis of the Data

Analysis	Subgroup	Number of variables	Type
Comparison	Among items	Two per items	Variable width column chart
		One per item	Bar/column chart
	Over time	Many periods	Circular area/line chart
		Few periods	Column/line chart
Relationship		Two	Scatter chart
		Three	Bubble chart
Distribution		Single	Column/line histogram
		Two	Scatter chart
		Three	Three-dimensional area chart
Comparison	Changing over time	Only relative differences matter	Stacked 100% column chart
		Relative and absolute differences matter	Stacked column chart
	Static	Simple share of total	Pie chart
		Accumulation	Waterfall chart
		Components of components	Stacked 100% column chart with subcomponents

Reference: In, Junyong, and Sangseok Lee. "Statistical data presentation." *Korean journal of anesthesiology* 70.3 (2017): 267.