IE 616: Decision Analysis and Game Theory
Closed book exam. No clarification during exam. Make proper assumptions if required and state them correctly. Total Marks 35. Duration 1:15 hours

---

Question 1. Manu (Player 1), and Nanda (Player 2), are business partners. Each of the partners has to determine the amount of effort they will put into the business, which is denoted by e_i, $i = 1, 2$. These are any non-negative real numbers. The cost of effort e_i for Player i is ce_i, where $c > 0$ is equal for both players. The success of the business depends on the amount of effort put in by the players; the businesss profit is denoted by $r(e_1, e_2) = e_1^{\alpha_1} e_2^{\alpha_2}$, where $\alpha_1, \alpha_2 \in (0, 1)$ are fixed constants known by Manu and Nanda, and the profit is shared equally between the two partners. Each players utility is given by the difference between the share of the profit received by that player and the cost of the effort they put into the business. Answer the following questions:

1. Describe this situation as a strategic-form game. Note that the set of strategies of each player is a continuum.   **Marks 3**

2. Find the best response of player 1, against e_2 of player 2 (for any given e_2)   **Marks 3**

3. Find the Nash equilibria of the game.   **Marks 3**

4. Comment on the result   **Marks 1**

Question 2. Construct an example of a two player game with 2 possible action's where   **Marks 2**

1. all the strategy pairs are Nash equilibrium

2. there is no Pure strategy Nash equilibrium

Question 3. Brew n Bites cafe and Kresit canteen, both located inside IIT Bombay, are offering samosas either at Rs 5, Rs 10 or at Rs 12 per piece. It is expected that 6000 samosa per month are eaten by the IIT students, who can choose one of the cafe randomly with equal probability. The ousiders consume 4000 samosa per week and prefer the cafe with lowest price. In case the two shops offers at the same rate, the outsiders get split evenly. What price (among 5, 10, 12 Rs) should the two cafe offer for samosa in order to maximize their utility? Do we have pure NE? Do we have any other Mixed strategy NE ?
     Marks 8

Question 4. Consider a two player game shown below. Find the mixed strategy NE of this game.

Player 2

		X	Y	Z
	A	(3, 1)	(0, 1)	(0, 0)
Player 1	B	(0, 1)	(4, 1)	(0, 0)
	C	(1, 1)	(1, 1)	(5, 0)

Table 1: Pay-off matrix

     Marks 5

Question 5. **First Price Auction.** Assume two bidders with valuations v_1 and v_2 for an object. Their bids are in terms of positive integers and so are v_1, v_2. Further these values are common knowledge. The bidder with higher bid wins the auction and pays the amount that he had bid. If both bid the same amount, one of them gets the object with equal probability $1/2$. For this game, compute a pure strategy Nash equilibrium.   **Marks 10**