

# Week1

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## 1 Examples

### 1.1 Duopoly

$$x_1(p_1, p_2) = \begin{cases} 0 & \text{if } p_2 < p_1 \\ x(p_1) & \text{if } p_1 < p_2 \\ x(p_2)/2 & \text{if } p_1 = p_2 \end{cases}$$

$\max(p_1 x(p_1, p_2))$  for  $p_1$

### 1.2 Auctions

values known to only agents and no sharing of information

places bids

highest bid wins

model:

$1, 2, 3, \dots, N$

$v_1, v_2, v_3, \dots, v_N \rightarrow \text{values}$

$b_1, b_2, b_3, \dots, b_N \rightarrow \text{bids}$

max bidden value wins

$\rightarrow b_1^* = \max(b_1, b_2, \dots, b_N)$

but has to pay second max bid

$\hat{b}^* \rightarrow$  second best bid

### 1.3 NCG

$N = \{1, 2, \dots, n\} \rightarrow$  set of players

$S = \{S_{i,i \in N}\} \rightarrow$  set of actions

$U = \{U_{i,i \in N}\} \rightarrow$  set of utilities

$U_i(a_i, a_{-i}) \forall a_i \in S_i, a_{-i} \in S_{-i} = S_1 \times S_2 \times \dots \times S_{i+1} \dots$

For 2 players:  $N = 2$

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ x(p_1) & a_1 = 2 \& a_{-1} \in S_{-i} \\ x(p_2)/2 & a_2 = 2 \end{cases}$$

For  $N$  players:  $N = N$

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ -\frac{n_2(a_{-i})+1}{N} & a_1 = 2 \& a_{-1} \in S_{-i} \end{cases}$$

#### 1.4 Hotelling Game