Assignment-1

1. All coordinate frames 1 to 6 are expressed in frame 0, thus they constitute unit vectors $\hat{}$ ${}^{0}_{n}R$

$$\therefore\ {}^{0}_{1}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\quad {}^{0}_{2}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},\quad {}^{0}_{3}R = \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{pmatrix}$$

$$\cancel{0}\,{}^{0}_{4}R = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix},\quad {}^{0}_{5}R = \begin{pmatrix} 0 & -1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix},\quad {}^{0}_{6}R = \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$${}^{\cancel{0}}_{2}R = {}^{1}_{0}R\,{}^{0}_{2}R = {}^{0}_{1}R^{T}\,{}^{2}_{0}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * {}^{2}_{0}R = {}^{2}_{0}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

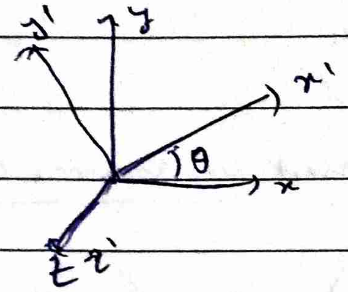
$${}^{2}_{3}R = {}^{0}_{2}R^{T}\,{}^{0}_{3}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{-1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$${}^{3}_{4}R = {}^{0}_{3}R^{T}\,{}^{0}_{4}R = \cancel{0} \begin{pmatrix} 0 & 0 & -1 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$${}^{4}_{5}R = {}^{0}_{4}R^{T}\,{}^{0}_{5}R = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$${}^{\cancel{0}\,5}_{6}R = {}^{\cancel{0}}_{5}R^{T}\,{}^{0}_{6}R = \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ -1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{2} + \frac{1}{2\sqrt{2}}\right) & \frac{1}{2} & \left(\frac{-1}{2\sqrt{2}} + \frac{1}{2}\right) \\ \frac{-1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \left(\frac{-1}{2\sqrt{2}} + \frac{1}{2}\right) & \frac{-1}{2} & \frac{1}{2} + \frac{1}{2\sqrt{2}} \end{pmatrix}$$

2. (a)



~~$R_x(\theta) = x'$~~ xyz : frame 1

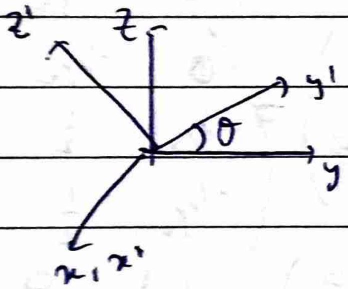
$x'y'z'$: frame 2

$$R_z(\theta) = [\,{}^1(x') \quad {}^1(y') \quad {}^1(z')\,]$$

$$\begin{aligned} {}^2x &= x'\cos\theta - y'\sin\theta \\ {}^2y &= x'\sin\theta + y'\cos\theta \\ {}^2z &= z \end{aligned}$$

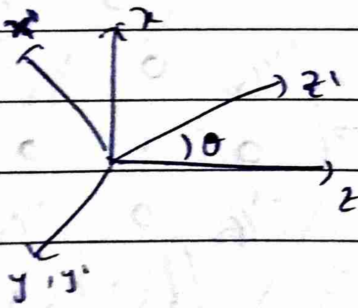
$$\Rightarrow \cancel{R_z(\theta) =} \quad \begin{aligned} {}^1(x') &= x\cos\theta + y\sin\theta \\ {}^1(y') &= (-\sin\theta)x + \cos\theta\, y. \\ {}^1(z') &= z \end{aligned}$$

$$\rightarrow R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\begin{aligned} y' &= (\cos\theta)y + (\sin\theta)z \\ z' &= (-\sin\theta)y + (\cos\theta)z \\ x' &= x \end{aligned}$$

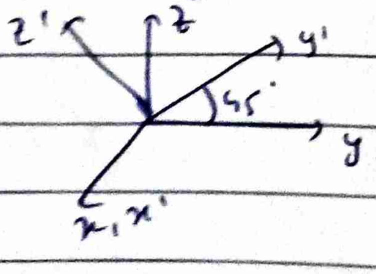
$$\Rightarrow R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$



$$\begin{aligned} z' &= (\cos\theta)z + (\sin\theta)x \\ x' &= (-\sin\theta)\cancel{x}z + (\cos\theta)\cancel{z}x \\ y' &= y \end{aligned}$$

$$\Rightarrow R_y(\theta) = \cancel{\sin\theta} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

b) I) Rotⁿ by 45° abt x-axis:



$$\Rightarrow {}^A_B R = R_x(-45^\circ) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

II) Rotⁿ abt y-axis by 90° - given by $R_y(90°)$

$${}^{A}_{B}R \Leftarrow R_y(90°)\,{}^{A}_{B}R \qquad R_y(90°)\,{}^{A}_{B}R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ 0 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{-1}{\sqrt2} & \frac{1}{\sqrt2} \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ -1 & 0 & 0 \end{pmatrix}$$

III) Rotⁿ about z by 45° ⇒ given by $R_z(45°)$

$${}^{A}_{B}R \Leftarrow R_z(45°)\cdot{}^{A}_{B}R = \begin{pmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ -1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$

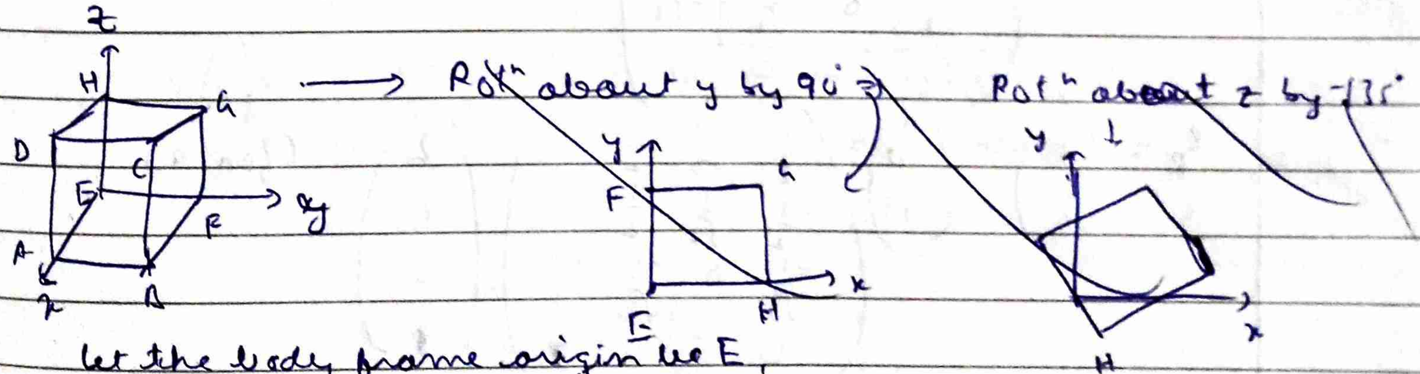
IV) Rotⁿ about x-axis by -45° ⇒ given by $R_x(-45°)$

$${}^{A}_{B}R \Leftarrow R_x(-45°)\,{}^{A}_{B}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt2} & +\frac{1}{\sqrt2} \\ 0 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ -\frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} \end{pmatrix}$$

V) Rotⁿ about z-axis by 45°: given by $R_z(45°)$

$${}^{A}_{B}R \Leftarrow R_z(45°)\,{}^{A}_{B}R = \begin{pmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} & 0 \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ -\frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt2} & \frac{1}{2} \\ -\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} \end{pmatrix}$$

3.



Let the body frame origin be E, defined by axes EA, EF, EH; initially it is aligned with world frame. In the final orientaⁿ,

$$\hat{EF} = -\cos(45°)\,\hat{i} + \sin(45°)\,\hat{j} = \frac{-1}{\sqrt2}\hat{i} + \frac{1}{\sqrt2}\hat{j} + 0\hat{k}$$

$$\hat{EH} = -\cos(45°)\,\hat{i} - \sin(45°)\,\hat{j} = \frac{-1}{\sqrt2}\hat{i} - \frac{1}{\sqrt2}\hat{j} + 0\hat{k}$$

$$\hat{EA} = \hat{k}$$

In the initial frame, $\hat{FA} = \hat{i}$, $\hat{EF} = \hat{j}$, $\hat{EH} = \hat{k}$ → aligned with world frame

$$\Rightarrow {}^{A}_{B}R = \begin{pmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 1 & 0 & 0 \end{pmatrix}$$ → Rotates box to final orientaⁿ

4. a) Rotⁿ by R, followed by translaⁿ by $[1,1,1]^T = v$

Standard homogeneous transformaⁿ:

$$\begin{pmatrix} {}^{A}p \\ 1 \end{pmatrix} = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{AB} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{pmatrix} {}^{B}p \\ 1 \end{pmatrix}$$

$$\Rightarrow T = \left(\begin{array}{ccc|c} \frac{1}{2} & -1/\sqrt{2} & -1/2 & 1 \\ -1/2 & -1/\sqrt{2} & 1/2 & 1 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

b) Translaⁿ by $[1,1,1]^T$, followed by rotⁿ by R

→ Now the vector ${}^{A}v$ ~~P_{AB}~~ also gets rotated by R.

∴ The translaⁿ vector is ~~$R P_{AB}$~~ Rv

$$= \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & -1/2 \\ -1/2 & -1/\sqrt{2} & \frac{1}{2} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -1/\sqrt{2} \\ -\sqrt{2} \end{pmatrix} = {}^{A}P_{AB}$$

$$\Rightarrow T = \left(\begin{array}{ccc|c} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} & \frac{-1}{\sqrt{2}} \\ -1/2 & -1/\sqrt{2} & 1/2 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & \frac{-1}{\sqrt{2}} & -\sqrt{2} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

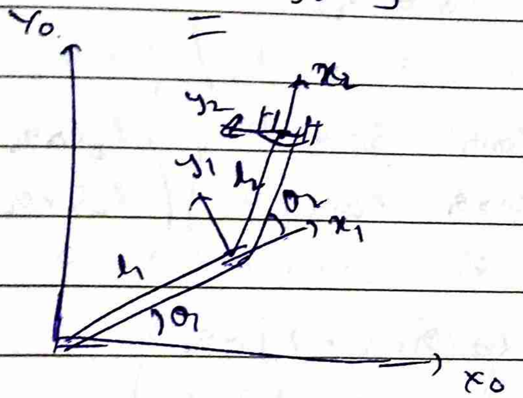
$${}^{B}p = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} {}^{A}p \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} & 1 \\ \frac{-1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} & 1 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$ (for a)

$$\Rightarrow {}^{A}p = \begin{bmatrix} \left(2 - \frac{1}{\sqrt{2}}\right) \\ \left(\frac{-1}{\sqrt{2}}\right) \\ -\sqrt{2}+1 \end{bmatrix}$$

Eq (6), $\begin{pmatrix} ^{A}P \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{-1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

$\Rightarrow {}^{A}P = \begin{bmatrix} (1-\sqrt{2}) \\ (-1-\sqrt{2}) \\ -2\sqrt{2} \end{bmatrix}$

5. (a) I)



a) ${}^{0}_{1}T = \begin{pmatrix} {}^{0}_{1}R & {}^{0}P_{01} \\ 0 & 1 \end{pmatrix}$; ${}^{0}_{1}R = R_z(\theta_1) = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

${}^{0}\vec{P}_{01} = \begin{pmatrix} l_1\cos\theta_1 \\ l_1\sin\theta_1 \\ 0 \end{pmatrix}$

$\Rightarrow {}^{0}_{1}T = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

${}^{1}_{2}T = \begin{pmatrix} {}^{1}_{2}R & {}^{1}P_{12} \\ 0 & 1 \end{pmatrix}$; ${}^{1}_{2}R = R_z(\theta_2) = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\cancel{0p}\ {}^{1}\vec{P}_{12} = \begin{pmatrix} l_2\cos\theta_2 \\ l_2\sin\theta_2 \\ 0 \end{pmatrix}$.

$\Rightarrow {}^{1}_{2}T = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_1 & 0 & l_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

b) $${}^0_2T = {}^0_1T\,{}^1_2T = \begin{pmatrix} {}^0_1R & {}^0P_{01} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} {}^1_2R & {}^1P_{12} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} {}^0_1R\,{}^1_2R & {}^0_1R\,{}^1P_{12} + {}^0P_{01} \\ 0 & 1 \end{pmatrix}$$

$$= {}^0_1R\,{}^1_2R = R_z(\theta_1+\theta_2)$$

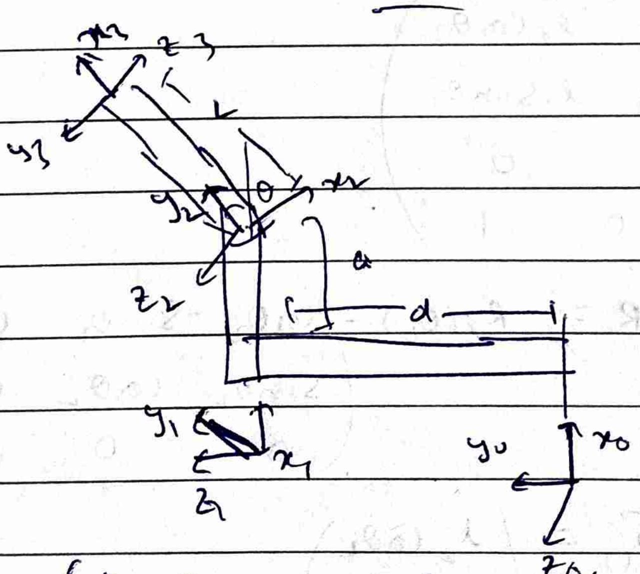
$$= \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^0_1R\,{}^1P_{12} + {}^0P_{01} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} l_2\cos\theta_2 \\ l_2\sin\theta_2 \\ 0 \end{pmatrix} + \begin{pmatrix} l_1\cos\theta_1 \\ l_1\sin\theta_1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 \end{pmatrix}$$

$$\Rightarrow {}^2_0T = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

II) a)



$${}^0_1T = \begin{pmatrix} {}^0_1R & {}^0P_{01} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T = \begin{pmatrix} {}^1_2R & {}^1P_{12} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta & 0 & a \\ 0 & 0 & -1 & 0 \\ -\cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T = \begin{pmatrix} {}^2_3R & {}^2P_{23} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

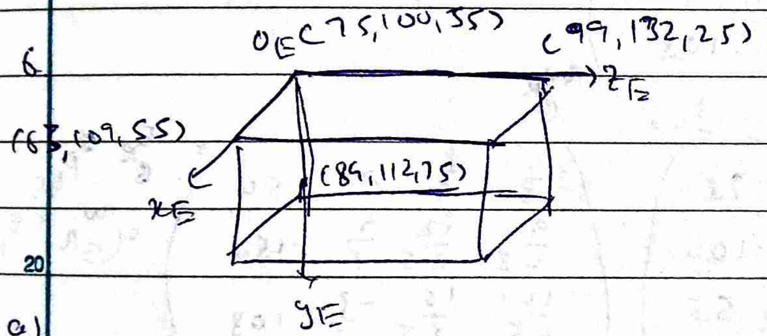
b) $${}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T = \begin{pmatrix} {}^0_3R & {}^0P_{03} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & a+L\cos\theta \\ \sin\theta & \cos\theta & 0 & d+L\sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0P_{03} = d\hat{y}_0 + a\hat{x}_0 + L[\cos\theta\,\hat{x}_0 + \sin\theta\,\hat{y}_0]$$

$$= (a+L\cos\theta)\hat{x}_0 + (d+L\sin\theta)\hat{y}_0$$

$$=$$

6.



a)

$${}^W\vec{P}_{WE} = (75,100,55)^T$$

$$\vec{O_E x_E} = (-12,9,0)^T \equiv {}^Wx_E = \left(-\frac{4}{5}, \frac{3}{5}, 0\right)^T$$

$$\vec{O_E y_E} = \cancel{24}(9,12,20)^T \equiv {}^Wy_E = \left(\frac{9}{25}, \frac{12}{25}, \frac{20}{25}\right)^T$$

$$\vec{O_E z_E} = (24,32,-30)^T \equiv {}^Wz_E = \left(\frac{12}{25}, \frac{16}{25}, -\frac{3}{5}\right)^T$$

$$\Rightarrow {}^W_ER = \cancel{\begin{matrix} -4/5 & 9/25 \\ 3/5 & 12/25 \\ 0 & 20/25 \end{matrix}} \quad \cancel{\begin{matrix} -4/5 \\ 9/25 \\ 12/25 \end{matrix}} \quad \begin{pmatrix} -4/5 & 9/25 & 12/25 \\ 3/5 & 12/25 & 16/25 \\ 0 & 4/5 & -3/5 \end{pmatrix}$$

$$\Rightarrow {}^W_ET = \begin{pmatrix} {}^W_ER & {}^WP_{WE} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{4}{5} & 9/25 & 12/25 & 75 \\ 3/5 & 12/25 & 16/25 & 100 \\ 0 & 4/5 & -3/5 & 55 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore \cancel{{}^{E}_{W}T} =$

b) $${}^{B}_{E}T = \begin{pmatrix} -4/5 & 9/25 & 12/25 & 68 \\ 3/5 & 12/25 & 16/25 & 174 \\ 0 & 4/5 & -3/5 & -60 \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{B}\begin{pmatrix} {}^{B}_{E}R & {}^{B}p_{BE} \\ 0 & 1 \end{pmatrix}$$

${}^{W}_{B}T = ?$; $= {}^{W}_{E}T\ {}^{E}_{B}T$; thus we need ${}^{E}_{B}T = ({}^{B}_{E}T)^{-1}$

$$({}^{B}_{E}T)^{-1} = {}^{E}_{B}T = \begin{pmatrix} ({}^{B}_{E}R)^{T} & {}^{E}p_{EB} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} ({}^{B}_{E}R)^{T} & ({}^{B}_{E}R)^{T}(-{}^{B}p_{BE}) \\ 0 & 1 \end{pmatrix}$$

Also, ${}^{B}_{E}R = {}^{W}_{E}R$ $$= \begin{pmatrix} ({}^{B}_{E}R)^{T} & -({}^{B}_{E}R)^{T}\ {}^{B}p_{BE} \\ 0 & 1 \end{pmatrix}$$

Now. $$({}^{B}_{E}R)^{T} = \begin{pmatrix} -4/5 & 3/5 & 0 \\ 9/25 & 12/25 & 4/5 \\ 12/25 & 16/25 & -3/5 \end{pmatrix} = {}^{E}_{B}R$$

$$\begin{pmatrix} -4/5 & 3/5 & 0 \\ 9/25 & 12/25 & 4/5 \\ 12/25 & 16/25 & -3/5 \end{pmatrix} \begin{pmatrix} -68 \\ -174 \\ 60 \end{pmatrix} = \begin{pmatrix} \cancel{-93.52}\ -50 \\ \cancel{-108}\ -156 \\ 108 \end{pmatrix} = {}^{E}p_{EB}$$

$$\therefore ({}^{B}_{E}T)^{-1} = \begin{pmatrix} -4/5 & 3/5 & 0 & -50 \\ 9/25 & 12/25 & 4/5 & -156 \\ 12/25 & 16/25 & -3/5 & 108 \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^{E}_{B}T$$

${}^{W}_{B}R$

$$\therefore {}^{W}_{B}T = \begin{pmatrix} -4/5 & 9/25 & 12/25 & 75 \\ 3/5 & 12/25 & 16/25 & 100 \\ 0 & 4/5 & -3/5 & 55 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -4/5 & 3/5 & 0 & -50 \\ 9/25 & 12/25 & 4/5 & -156 \\ 12/25 & 16/25 & -3/5 & 108 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

${}^{E}_{B}R\ {}^{B}p_{BE} = ({}^{W}_{E}R)^{T}\ {}^{B}p_{BE}$

$$= \begin{pmatrix} 1 & 0 & 0 & \cancel{25}\,143 \\ 0 & 1 & 0 & 274 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ $\underline{\text{Ans}}$

$$= \begin{pmatrix} {}^{W}_{E}R\ {}^{E}_{B}R & {}^{W}_{B}R\ {}^{E}p_{EB} + {}^{W}p_{WE} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} I & {}^{R}p_{BE} + {}^{W}p_{WE} \\ 0 & 1 \end{pmatrix} ; \quad \therefore \underline{{}^{W}_{E}R = {}^{B}_{E}R}$$

7. ${}^{F_1}\vec{OP} = (1,-1,1)^T$; ${}^1_2R = \begin{pmatrix} 1 & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & 1 & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & 1 \end{pmatrix} \times \frac{1}{3}$

a) ${}^{F_2}\vec{SP} = (-1,0,0)^T$

$${}^{F_1}\vec{SP} = {}^{F_1}\vec{OP} + {}^1_2R\,{}^{F_2}\vec{SP}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 \\ 1+\sqrt{3} \\ 1-\sqrt{3} \end{pmatrix} = \begin{bmatrix} 2/3 \\ -\frac{4}{3} - \frac{1}{\sqrt{3}} \\ 2/3 + 1/\sqrt{3} \end{bmatrix}$$

b) Plane ABC: $x+y+z=1 \equiv {}^{F_1}x + {}^{F_1}y + {}^{F_1}z = 1$.

~~In frame F_1~~, $\Rightarrow \underbrace{{}^{F_1}(x,y,z)^T}_{\vec{r}} \, \underbrace{{}^{F_1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{n}} = 1 \equiv {}^{F_1}\vec{r}\cdot{}^{F_1}\vec{n} = 1$

In frame F_1, express $\vec{r}$, $\vec{n}$ in F_2, dot product will still be 1

$\Rightarrow {}^{F_2}\vec{r}\cdot{}^{F_2}\vec{n} = 1$

${}^2\vec{x}\cdot{}^2\vec{n} = 1$; ${}^2\vec{x} = ?$ ${}^2\vec{n} = ?$

$$\begin{pmatrix} {}^2_1R & {}^2P_{21} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} {}^1x \\ 1 \end{pmatrix} = \begin{pmatrix} {}^2x \\ 1 \end{pmatrix} ; \begin{pmatrix} {}^2_1R & {}^2P_{21} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} {}^1n \\ 1 \end{pmatrix} = \begin{pmatrix} {}^2n \\ 1 \end{pmatrix}$$

$${}^2_1R = {}^1_2R^T = \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 \end{bmatrix}$$

$${}^1P_{12} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} ; \quad {}^2P_{21} = -{}^2_1R\,{}^1P_{12}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} -1+2\sqrt{3} \\ -1 \\ -2\sqrt{3}-1 \end{bmatrix}$$

$$\therefore \begin{pmatrix} {}^2n \\ 1 \end{pmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} & (-1+2\sqrt{3}) \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} & -1 \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 & (-1-2\sqrt{3}) \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 2+2\sqrt{3} \\ 2 \\ 2-2\sqrt{3} \\ 1 \end{pmatrix} \Rightarrow {}^2n = \begin{pmatrix} 2+2\sqrt{3} \\ 2 \\ 2-2\sqrt{3} \end{pmatrix}$$

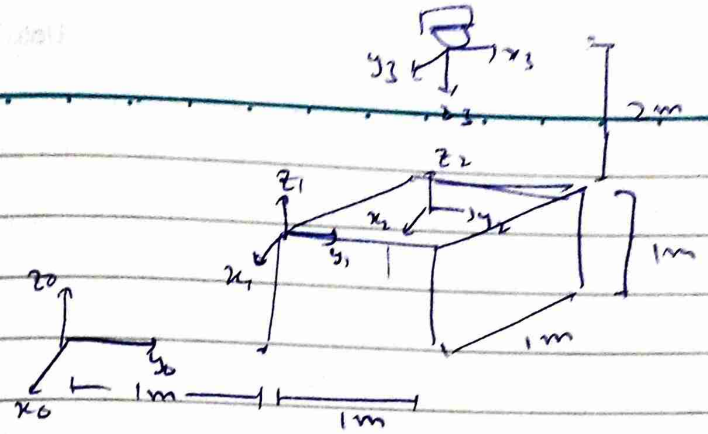
Also, for a general vector ${}^1v = (x\ y\ z)^T$,

$$\begin{pmatrix} {}^2v \\ 1 \end{pmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1+\sqrt{3} & 1-\sqrt{3} & -1+2\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} & -1 \\ 1+\sqrt{3} & 1-\sqrt{3} & 1 & -1-2\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\Rightarrow$ Thus, eqⁿ of plane in F_2-coordinates

$$= {}^2(x\ y\ z)^T \cdot \frac{1}{3}\begin{pmatrix} 2+2\sqrt{3} \\ 2 \\ 2-2\sqrt{3} \end{pmatrix} = 1$$ [express $(x\ y\ z)^T$ in F_2-coordinates]

$$\equiv \underline{(2+2\sqrt{3})x + 2y + (2-2\sqrt{3})z = 3}$$

8. 

a) $${}^0_1T = \begin{pmatrix} {}^0_1R & {}^0P_{01} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_2T = \begin{pmatrix} {}^0_2R & {}^0P_{02} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_3T = \begin{pmatrix} {}^0_3R & {}^0P_{03} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) ∵ the block is rotated by 45°, followed by disp of ${}^2(0.5\ 0\ 0)^T$;

∴ the transform^n from this new alignment (say '4') to the given block ~~base~~ frame (2) = ${}^2_4T = \begin{pmatrix} {}^2_4R & {}^2P_{24} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_z(45°) & {}^2P_{24} \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Also, $${}^0_4T = {}^0_2T\,{}^2_4T = \begin{pmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 1.5 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$