Week1

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1 Examples

1.1 Duopoly

$$x_1(p_1, p_2) = \begin{cases} 0 & if p_2 < p_1 \\ x(p_1) & if p_1 < p_2 \\ x(p_2)/2 & if p_1 = p_2 \end{cases}$$

$max(p_1 x(p_1, p_2))$ for p_1

1.2 Auctions

values known to only agents and no sharing of information
places bids
highest bid wins

model:
$1, 2, 3, \dots N$
$v_1, v_2, v_3, \dots v_N \rightarrow values$
$b_1, b_2, b_3, \dots b_N \rightarrow bids$

max bidden value wins
$\rightarrow b_1^* = max(b_1, b_2, \dots b_N)$

but has to pay second max bid
$\hat{b^*} \rightarrow$ second best bid

1.3 NCG

$N = \{1, 2, \dots, n\} \rightarrow$ set of players
$S = \{S_{i, i \in N}\} \rightarrow$ set of actions
$U = \{U_{i, i \in N}\} \rightarrow$ set of utilities
$U_i(a_i, a_{-i}) \forall a_i \in S_i, a_{-i} \in S_{-i} = S_1 \times S_2 \times \dots S_{i+1} \dots$

For 2 players: N= 2

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ x(p_1) & a_1 = 2 \& a_{-1} \in S_{-i} \\ x(p_2)/2 & a_2 = 2 \end{cases}$$

For N players: N= N

$$U_1(a_1, a_{-1}) = \begin{cases} -C & a_1 = 1 \& a_{-1} \in S_{-i} \\ -\frac{n_2(a_{-i})+1}{N} & a_1 = 2 \& a_{-1} \in S_{-i} \end{cases}$$

1.4 Hotelling Game

$N = \{H_1, H_2\}$
$S_1 = S_2$

People are lazy, will try to go to the nearest hotel

$$U_1(a_1, a_{-1}) = \begin{cases} \frac{a_1 + a_{-1}}{2} & a_1 < a_{-1} \\ 0 & a_1 = a_{-1} \\ L - \frac{a_1 + a_{-1}}{2} & a_1 > a_{-1} \end{cases}$$

- Order of decisions
- Best Response

BR $\rightarrow$ BEST Response
$BR_i(a_{-i}) = arg \max_{a_i} U_i(a_i, a_{-i})$

Situations where players dont know each others actions
Best strategy is to anticiapate ie assume $a_{-i} = \tilde{a}$
Then, play $BR_i = BR_i(\tilde{a})$

1.5 Free Riding

- Simultaneous move games (assumption)

eg. N people in area, decide whether to participate in a cleaniong activity or not

- n_p participates
- Every person who participates gets a negative utility of $-v$
- Everybody in area gets positive reward of $g(n_p)$

Num of people participated,
$n_p(a_{-i}) = \sum_{j!=i}^{N} 1_{a_j=1} = \sum_{j!=i} a_j$

Utility:
$U_1(a_i, a_{-i}) = -a_i.v + g(n_p(a_{-i}) + a_i)$

2 Week2

2.1

Prove that all Dominant Eqm are Nash Eqm

2.2 Nash Equilibrium

2.3 Domination

2.3.1 Weak Dominantion

2.3.2 Strong Dominantion

2.3.3 Very Weak Dominantion

2.3.4 Strongly Dominant strategy

2.3.5 Weakly Dominant strategy

2.3.6 Very Weakly Dominant strategy

2.3.7 Strongest Dominant strategy Eqn