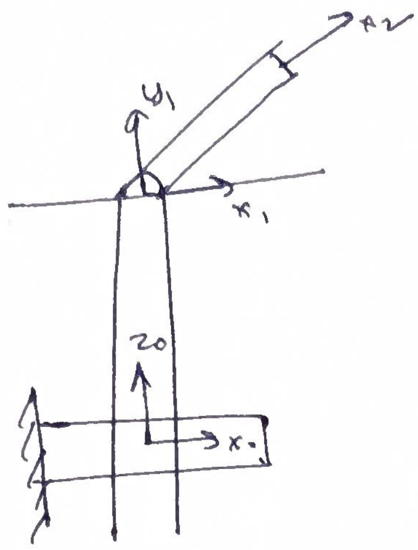
Assignment # A 6

(1)



$x_1 = \frac{l_1}{2}\cos\theta_1$, $y_1 = \frac{l_1}{2}\sin\theta_1$

$x_2 = l_1\cos\theta_1 + \frac{l_2}{2}\cos(\theta_1+\theta_2)$

$y_2 = l_1\sin\theta_1 + \frac{l_2}{2}\sin(\theta_1+\theta_2)$

$v_{c_1}^2 = \left(\frac{l_1}{2}\dot\theta_1\right)^2$

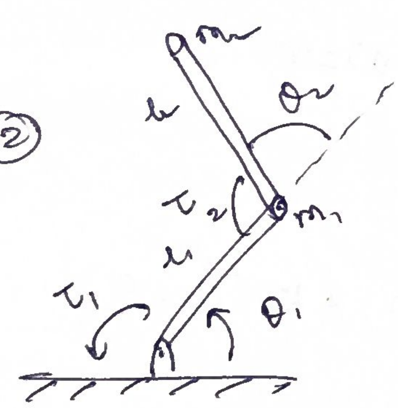
$v_{c_2}^2 = \left(l_1\dot\theta_1 + \frac{l_2}{2}(\dot\theta_1+\dot\theta_2)\right)^2$

$K.E. = \frac{1}{2}m_1 v_{c_1}^2 + \frac{1}{2}I_1\dot\theta_1^2 + \frac{1}{2}m_2 v_{c_2}^2 + \frac{1}{2}I_2(\dot\theta_1+\dot\theta_2)^2$

$U = m_1 g y_1 + m_2 g y_2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta_i}\right) - \frac{\partial L}{\partial\theta_i} = 0$; $L = T - U$

(2)



Link
$K.E. = \frac{1}{2}m_1(l_1\dot\theta_1)^2 + \frac{1}{2}I_1\dot\theta_1^2 +$
$\frac{1}{2}m_2\left[(l_1\dot\theta_1 + l_2\dot\theta_2)^2\right] + \frac{1}{2}I_2(\dot\theta_1+\dot\theta_2)^2$

motor
$K.E. = \frac{1}{2}J_m\dot\phi_1^2 + \frac{1}{2}J_m\dot\phi_2^2$

$\theta_1 = \frac{\phi_1}{\gamma}$, $\theta_2 = \frac{\phi_2}{\gamma}$

Total KE = links KE + motor KE

potential Energy = $U = m_1 g y_1 + m g y_2$

$y_1 = l_1\sin\theta_1$ $y_2 = l_1 s\theta_1 + l_2 s(\theta_1+\theta_2)$

eqn's of motion

$$L = T - U$$

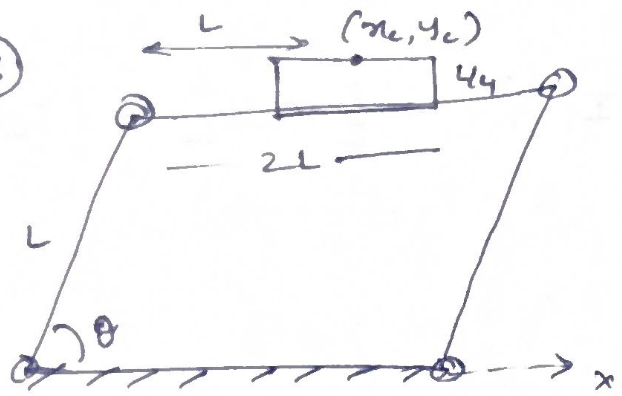
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta} = \tau_i \quad \text{—(1)}$$

Torques in terms of voltage

$$\tau_i = \frac{k V_i}{R}$$

substituting in (1) to get $\tau(V)$

(3)



$$e_i = \theta_{d_i} - \theta_i$$

$$\dot{e}_i = \dot{\theta}_{d_i} - \dot{\theta}_i$$

$$\tau_i = k_p e_i + k_d \dot{e}_i$$

for $i = 1, 2$

dynamics of ~~2R~~ 2R

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau$$

PD control:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = k_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$$

$$e = \theta_d - \theta \quad , \quad \dot{e} = \dot{\theta}_d - \dot{\theta}$$

$$\Rightarrow \quad M(\theta)(\ddot{\theta}_d - \ddot{e}) + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = k_p e + k_d \dot{e}$$

$$M(\theta)\ddot{e} = M(\theta)\ddot{\theta}_d - (C(\theta, \dot{\theta})\dot{\theta} + G(\theta)) - k_p e + k_d \dot{e}$$

$$M(\theta) \sim M_0$$

$\ddot{e}$ ~~$+ M_0$~~

$$\ddot{e} + M_0^{-1} K_d \dot{e} + M_0^{-1} K_p e = M_0^{-1} \left(M(\theta) \ddot{\theta}_d - c(\theta, \dot{\theta}) \dot{\theta} - G(\theta) \right)$$

$$\ddot{e} + K_d' \dot{e} + K_p' e = 0$$

$$K_d = 2 \xi \sqrt{K_p}$$

for critical damping $\xi = 1$

$$K_d = 2\sqrt{K_p}$$

b) a) $x = f_1(\theta)$, $y = f_2(\theta)$

$$V_{EE} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J \dot{\theta}$$

b) $V_{com,i} = J_i \dot{\theta}$

$$J_1 = \begin{bmatrix} J_{11} \\ J_{12} \end{bmatrix}, \quad J_2 = \begin{bmatrix} J_{21} \\ J_{22} \end{bmatrix}, \quad J_3 = \begin{bmatrix} J_{31} \\ J_{32} \end{bmatrix}$$

c) $\Rightarrow KE = \frac{1}{2} \sum_{i=1}^{3} m_i (J_i \dot{\theta})^T (J_i \dot{\theta})$

$$+ \frac{1}{2} \sum_{i=1}^{3} I_i \dot{\theta}^2$$

$$\Rightarrow KE = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

d) equat^n of motions

$$\tau = f(\ddot{\theta}, \dot{\theta}, \theta)$$

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{\theta}}\right) - \frac{\partial KE}{\partial \theta} = \tau$$

$$M(\theta) \ddot{\theta} + c(\theta, \dot{\theta}) \dot{\theta} + G(\theta) = \tau$$

(4)

$$KE = \frac{1}{2}\left(I_{xx}\,\omega_x^2 + I_{yy}\,\omega_y^2 + I_{zz}\,\omega_z^2 \right)$$

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\phi}\, s\theta\, s\psi + \dot{\theta}\, c\phi \\ \dot{\phi}\, s\theta\, c\psi - \dot{\theta}\, s\psi \\ \dot{\phi}\, c\theta + \dot{\psi} \end{bmatrix}$$

$$L = K - U$$

$$L = \frac{1}{2}\left(I_{xx}\,\omega_x^2 + I_{yy}\,\omega_y^2 + I_{zz}\,\omega_z^2 \right)$$

$$= \frac{1}{2} I_{xx} \left(\dot{\phi}\, s\theta\, s\psi + \dot{\theta}\, c\psi \right)^2$$

$$+ \frac{1}{2} I_{yy} \left(\dot{\phi}\, s\theta\, c\psi - \dot{\theta}\, s\psi \right)^2$$

$$+ \frac{1}{2} I_{zz} \left(\dot{\phi}\, c\theta + \dot{\psi} \right)^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$$

x:

$$\frac{d}{dt}\left(I_{xx}\,\omega_x \right) - \left(I_{zz} - I_{yy} \right) \omega_y \omega_z = 0$$

$$\Rightarrow \quad I_{xx}\,\dot{\omega}_x \mp \left(I_{zz} - I_{yy} \right) \omega_z \omega_z = 0$$

similary for y, z:

$$\Rightarrow \quad I_{yy}\,\dot{\omega}_y \mp \left(I_{xx} - I_{zz} \right) \omega_z \omega_x = 0$$

$$\Rightarrow \quad I_{zz}\,\dot{\omega}_z \mp \left(I_{yy} - I_{xx} \right) \omega_x \omega_y = 0$$