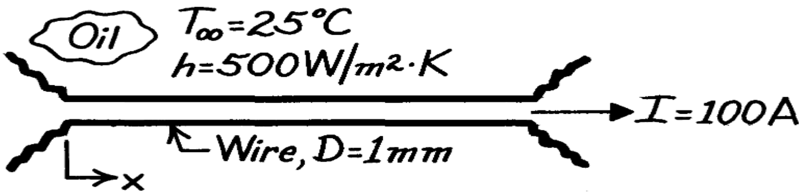
Solution 1

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of its steady-state value.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.

PROPERTIES: Wire (given): $\rho = 8000$ kg/m^3, $c_p = 500$ J/kg·K, k = 20 W/m·K, $R'_e = 0.01$ Ω/m.

ANALYSIS: Since

$$\mathrm{Bi} = \frac{h\left(r_o/2\right)}{k} = \frac{500\ \mathrm{W/m^2\cdot K}\left(2.5\times10^{-4}\mathrm{m}\right)}{20\ \mathrm{W/m\cdot K}} = 0.006 < 0.1$$

the lumped capacitance method can be used. The problem has been analyzed in Example 1.4, and without radiation the steady-state temperature is given by

$$\pi\, Dh\left(T - T_\infty\right) = I^2 R'_e.$$

Hence

$$T = T_\infty + \frac{I^2 R'_e}{\pi\, Dh} = 25^\circ\mathrm{C} + \frac{(100\mathrm{A})^2\, 0.01\Omega/\mathrm{m}}{\pi(0.001\ \mathrm{m})500\ \mathrm{W/m^2\cdot K}} = 88.7^\circ\mathrm{C}.$$ <

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.4)

$$\frac{dT}{dt} = \frac{I^2 R'_e}{\rho c_p\left(\pi D^2/4\right)} - \frac{4h}{\rho\, c_p D}\left(T - T_\infty\right).$$

With $T = T_i = 25$°C at t = 0, the solution is

$$\frac{T - T_\infty - \left(I^2 R'_e/\pi\, Dh\right)}{T_i - T_\infty - \left(I^2 R'_e/\pi\, Dh\right)} = \exp\left(-\frac{4h}{\rho\, c_p D}t\right).$$

Substituting numerical values, find

$$\frac{87.7 - 25 - 63.7}{25 - 25 - 63.7} = \exp\left(-\frac{4\times500\ \mathrm{W/m^2\cdot K}}{8000\ \mathrm{kg/m^3}\times500\ \mathrm{J/kg\cdot K}\times0.001\ \mathrm{m}}t\right)$$

$$t = 8.31\mathrm{s}.$$ <

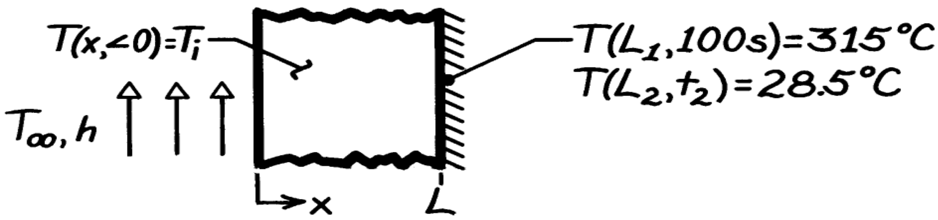
COMMENTS: The time to reach steady state increases with increasing ρ, c_p and D and with decreasing h.

Solution 2

KNOWN: One-dimensional wall, initially at a uniform temperature, T_i, is suddenly exposed to a convection process (T_∞, h). For wall #1, the time (t_1 = 100s) required to reach a specified temperature at x = L is prescribed, $T(L_1, t_1) = 315°C$.

FIND: For wall #2 of different thickness and thermal conditions, the time, t_2, required for $T(L_2, t_2) = 28°C$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: The properties, thickness and thermal conditions for the two walls are:

Wall	L(m)	$\alpha(m^2/s)$	k(W/m·K)	$T_i(°C)$	$T_\infty(°C)$	$h(W/m^2·K)$
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

The dimensionless functional dependence for the one-dimensional, transient temperature distribution, Eq. 5.38, is

$$\theta^* = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = f\left(x^*,\ Bi,\ Fo\right)$$

where

$$x^* = x/L \qquad Bi = hL/k \qquad Fo = \alpha t/L^2.$$

If the parameters x*, Bi, and Fo are the same for both walls, then $\theta_1^* = \theta_2^*$. Evaluate these parameters:

Wall	x*	Bi	Fo	θ*
1	1	0.40	0.150	0.85
2	1	0.40	$1.563\times10^{-4}\ t_2$	0.85

where

$$\theta_1^* = \frac{315-400}{300-400} = 0.85 \qquad \theta_2^* = \frac{28.5-20}{30-20} = 0.85.$$

It follows that

$$Fo_2 = Fo_1 \qquad 1.563\times10^{-4} t_2 = 0.150$$

$$t_2 = 960s.$$

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