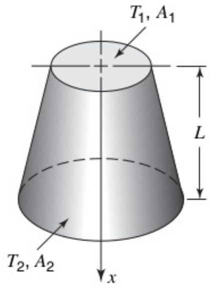
Assignment-2

1. Heat is transferred by conduction (assumed to be one-dimensional) along the axial direction through the truncated conical section shown in the figure. The two base surfaces are maintained at constant temperatures: T_1 at the top, and T_2 , at the bottom, where $T_1 > T_2$: Evaluate the heat transfer rate, q_x , when
(a) The thermal conductivity is constant. [2 marks]
(b) The thermal conductivity varies with temperature according to $k= k_0 - \alpha T$, where α is a constant. [3 marks]



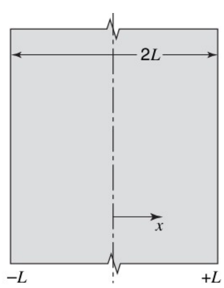
2. Radioactive waste (k = 20 W/mK) is stored in a cylindrical stainless steel (k =15 W/mK) container with inner and outer diameters of 1.0 and 1.2 m, respectively. Thermal energy is generated uniformly within the waste material at a volumetric rate of $2x10^5$ W/m^3 : The outer container surface is exposed to water at 25°C, with a surface coefficient of 1000 W/m^{2}K: The ends of the cylindrical assembly are insulated so that all heat transfer occurs in the radial direction. For this situation determine
(a) The steady-state temperatures at the inner and outer surfaces of the stainless steel. [4 marks]
(b) The steady-state temperature at the center of the waste material. [2 marks]

3. Consider a hollow sphere of inner radius r_i and outer radius r_o maintained at constant inner surface temperature T_i and outer surface temperature T_o.
(a) Under steady-state operating conditions with no heat generation, derive an expression for thermal resistance offered for conduction heat transfer. [3 marks]
(b) Now let's keep the inner radius fixed at r_i and vary the outer radius r_o. Include the convective heat transfer from the outer surface of the sphere to ambient air at temperature T_∞ with convective heat transfer coefficient of h_o. Derive an expression for critical outer radius r_c at which heat transfer from T_i to T_∞ is maximum. [3 marks]
(c) For the critical radius derived in (b), are the thermal resistances $R_{conduction}$ and $R_{convection}$, equal? What can be concluded from this? [3 marks]
(d) Now consider the same sphere under steady-state operating conditions with volumetric heat generation of Q W/m^3. Start with the diffusion equation and derive an expression for temperature gradient inside the sphere. For the boundary condition given below, show that $q''(r_i) = 0$. What does it mean physically? [3 marks]

$$q''(r_0)=\frac{Q}{3}\left(r_0-\frac{r_i^3}{r_0^2}\right)$$

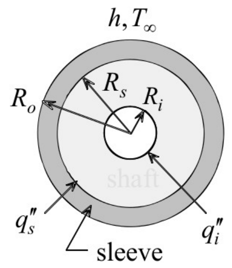
4. Derive an expression for temperature distribution in a flat plate with constant thermal conductivity k and temperature T_L at both surfaces as shown in the figure. The energy-generation term varies linearly with temperature as shown, where β is a constant. Assume one-dimensional steady-state heat conduction. Neglect other modes of heat transfer.

$$\dot{q}=\dot{q}_L[1+\beta(T-T_L)]$$



[6 marks]

5. A hollow shaft of outer radius R_s and inner radius R_i rotates inside a sleeve of inner radius Rs and outer radius R_o . Frictional heat is generated at the interface at a flux q''_s . At the inner shaft surface heat is added at a flux q''_i . The sleeve is cooled by convection with a heat transfer coefficient h. The ambient temperature is T_∞. Determine the steady state one-dimensional temperature distribution in the sleeve. [6 marks]



6. A very thin electric element is wedged between two plates of conductivities k_1 and k_2 . The element dissipates uniform heat flux q_0''. The thickness of one plate is L_1 and, that of the other is L_2 . One plate is insulated while the other exchanges heat by convection. The ambient temperature is T_∞ and the heat transfer coefficient is h . Determine the temperature of the insulated surface for one-dimensional steady state conduction. [5 marks]