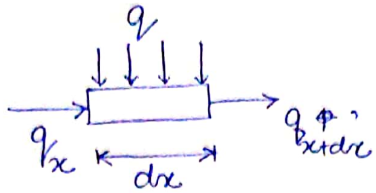
Let us take a small differential volume, C.V.



We know, $q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$

$$\therefore \quad q_x \cancel{\phi} t + q'' dx \cancel{\phi} = q_{x+dx} \cancel{\phi} t.$$

$$\Rightarrow \cancel{q_x t} + q'' dx = \cancel{q_x t} + \frac{\partial q_x}{\partial x} dx\, t$$

$$\Rightarrow q'' dx = \frac{\partial q_x}{\partial x} dx\, t.$$

$$\Rightarrow \frac{q''}{t} = \frac{d}{dx}\left(-k \frac{dT}{dx}\right)$$

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{q''}{kt} = 0.$$

$$\Rightarrow \frac{dT}{dx} = -\frac{q''}{kt} x + C_1$$

$$T(x) = -\frac{q'' x^2}{2kt} + C_1 x + C_2.$$

At $x = 0$, $T = T_0$.

$$\Rightarrow C_2 = T_0.$$

and at $x = L \Rightarrow T = T_0 \Rightarrow \cancel{T_0} = -\frac{q'' L^2}{2kt} + C_1 L + \cancel{T_0}$

$$\Rightarrow C_1 = \frac{q'' L}{2kt}.$$

$$\therefore \boxed{T(x) = \frac{q''\cancel{L}}{2kt}(Lx - x^2) + T_0.}$$

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At $x = 0$,

~~Q/kH~~ ~~q~~

$$Q = -kA\frac{dT}{dx}$$

$$= -k(wt)\left[-\cancel{\frac{q''x}{kt}}^{\,0} + \frac{q''L}{2kt}\right]$$

$$= -\frac{q''L\omega}{2}$$

(Assumed heat input,
⇒ heat output at $x = 0$).

At $x = L$

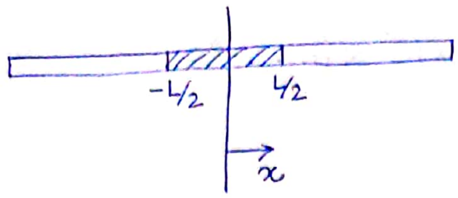
$$Q = -k\omega t\left[-\frac{q''L}{kt} + \frac{q''L}{2kt}\right]$$

$$= \frac{q''L\omega}{2}$$

(Assumed heat output,
sign shows output ~~at~~ at $x = L$).

∴ net heat output $= q''L\omega$.

3. using symmetry.



For $0 < x < \frac{L}{2}$

$$\frac{d^2T}{dx^2} + \frac{q_{gen}}{k} = 0.$$

$$\Rightarrow \frac{dT}{dx} = -\frac{q_{gen}\, x}{k} + C_1.$$

$$\Rightarrow T(x) = -\frac{\dot{q}_{gen}\, x^2}{2k} + C_1 x + C_2 \rightarrow (1).$$

Boundary condⁿ,

$$\left.\frac{dT}{dx}\right|_{x=0} = 0 \Rightarrow C_1 = 0.$$

$$T\left(x = \frac{L}{2}\right) = T_b \Rightarrow -\frac{q_{gen} L^2}{8K} + C_2 = T_b.$$

$$\Rightarrow C_2 = T_b + \frac{\dot{q}_{gen} L^2}{8k} \rightarrow (2).$$

For ∞ long fin,

$$Q = \sqrt{hpKA_c}\,(T_b - T_\infty).$$

$$\Rightarrow \dot{q}_{gen} A_c (L/2) = \sqrt{hpKA_c}\,(T_b - T_\infty)$$

plugging values.

$$T_b = \overset{20}{\cancel{T_\infty}} + \frac{7.5\times10^6 \times (\pi/4)(0.005^2)(30\times10^{-3})}{2\times\sqrt{10\times\pi\times0.005\times25\times(\pi/4)\,0.005^2}}$$

$$= 271.55°C.$$

Using T_b in (2),

$$C_2 = 271.55 + \frac{7.5 \times 10^6 \times (30 \times 10^{-3})^2}{8 \times 25}$$

$$= 305.3\ ^\circ C$$

Using C_2 in (1)

$$T(x) = -\frac{\dot{q}_{gen}\, x^2}{2K} + 305.3$$

$$T(x=0) = T_0 = 305.3^\circ C$$