Mechanics of Machining Processes

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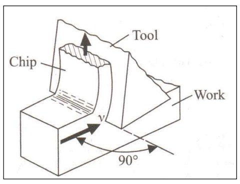
Outline

- Mechanics of Chip formation
- Merchant's Analysis – Circle diagram
- Cutting Forces, Power, Energy
- Stress, Strain, Specific cutting energy

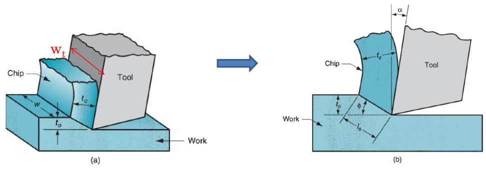
Orthogonal machining



Characteristics

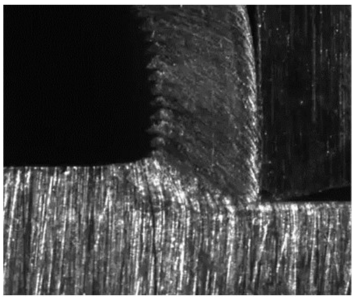
- Cutting edge is *perpendicular* to cutting velocity
- Plain Strain 2 D planar phenomena
- Simple analysis,; gives consistent and good results close to actual

Orthogonal Machining Characteristics

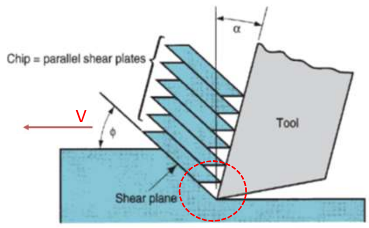


- Width of cutting tool (w_t) > width of chip (w)
- Width of chip (w) >> uncut chip thickness t_0 (about >10 times)
- No Spread of the chip along the tool width (plane strain condition)
- The tool is considered to be perfectly sharp. No 'ploughing' (rubbing).

Machining Process video

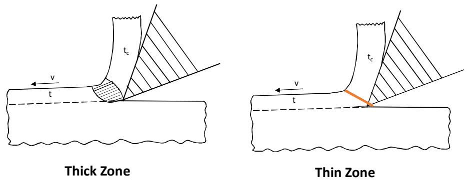


Piispanen's Sliding Card Analogy



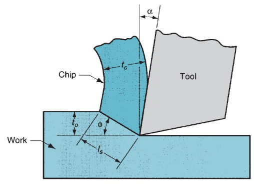
Displacement of a Pack of Cards by the Cutting Tool along a Shear Plane at an angle of ϕ with the Cutting Velocity V

Shear zone deformation



- Shear deformation along a zone – Tool tip to Free surface
  - *Thick* Zone : Wedge shaped
  - *Thin* Zone: Shear Plane
- Material does not deform until the shear plane is reached
  - Primary shear zone – Shear plane
  - Secondary shear zone – Chip Tool interface
- For simple analysis, a single shear plane (Thin zone) is assumed

Basic Terminology



- t : Umcut chip thickness (= depth of cut)
- tc : Chip thickness (tc > t)
- Chip thickness ratio (rc) = t / tc (< 1)
- Shear Angle : ϕ
- Cutting Velocity : V
- Length of Shear Plane: Ls
- Shear strength of work material : τ_S

Merchant's Analysis -Forces in machining

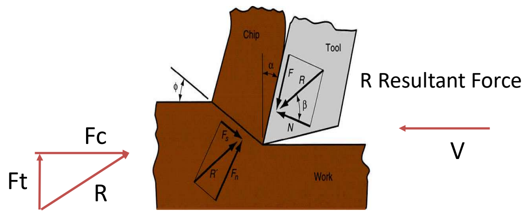
M. E. Merchant

Objective

Prediction of Cutting Forces in machining

- Assumptions
  - Orthogonal machining
  - 2D Planar deformation, No spread of material
  - Cutting edge of tool is sharp
    - No tool wear, point contact
    - Continuous Chip flow, No Built up edge,
  - Constant Cutting velocity V
  - Chip is in stable equilibrium under forces
  - Shear strength of work material τ_s is *constant.*

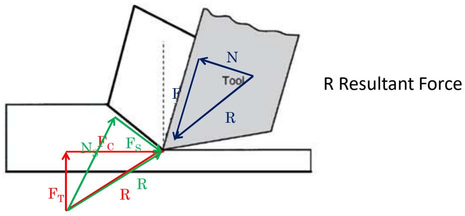
Forces in Orthogonal machining



Resultant Cutting Force R is resolved into 3 Pairs

Fc , Ft – Along and Perpendicular to Velocity V
Fs , Fn – Along and Perpendicular to Shear Plane
F , N – Along and Perpendicular to Tool Rake face

Forces in orthogonal machining

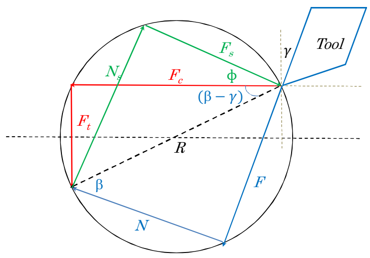


F_C Horizontal cutting force along V
F_T Vertical thrust force holds tool in position

F Friction resistance to the chip flow
N ormal force to the hip flow direction

F_S hear force, resistance to the shear of the metal
N_S Normal force, to shear plane

Merchant Circle



$$F_s = R\,Cos(\phi+\beta-\gamma)$$
$$F_c = R\,Cos(\beta-\gamma)$$
$$F = R\,Sin\,\beta$$
$$F_c = \frac{F_s\,Cos(\beta-\gamma)}{Cos(\phi+\beta-\gamma)}$$
$$F_s = \frac{F_c\,Cos(\phi+\beta-\gamma)}{Cos(\beta-\gamma)}$$
$$F_s = \tau A_s = \tau\frac{wt_0}{Sin\phi}$$

Estimation of Cutting Forces

Shear Force on Shear Plane

$$F_s = \tau_s A_s$$
$$= \tau_s \frac{w\,t}{Sin\phi}$$

From Merchant's Circle , Resultant Force R

$$R = \frac{\tau_s.w.t}{\sin\emptyset\, Cos(\phi+\beta-\gamma)} = \frac{Fs}{Cos\ (\varphi+\beta-\gamma)}$$

Other components of resultant force R can be calculated

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Force Relationships

Shear Plane decomposition of Resultant Force:

$$F_s = F_c \cos\emptyset - F_t\, sin\emptyset$$
$$N_s = F_t \cos\emptyset + F_c\, sin\emptyset = F\tan(\emptyset+\beta-\gamma)$$

Tool Chip Interface decomposition of resultant force:

$$F = F_c\, sin\gamma + F_t \cos\gamma$$
$$N = F_c \cos\gamma - F_t\, sin\gamma$$

Tool-chip interface friction coefficient

$$\mu = \tan\beta = \frac{F}{N} = \frac{F_t + F_c \mathrm{ta}}{F_c - F_t \tan\gamma}$$

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Cutting Force and Power

Cutting Force

$$F_c = RCos(\beta-\gamma)$$
$$= \frac{\tau_s.w.t\cos(\beta-\gamma)}{\sin\emptyset\, Cos(\phi+\beta-\gamma)}$$

Power in cutting P

$$P = F_c\ .V$$
$$P = \frac{\tau_s.w.t\cos(\beta-\gamma).V}{\sin\emptyset\, Cos(\phi+\beta-\gamma)}$$

How to choose shear angle ϕ ?

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Merchant's Theory

- Nature always tries to take the path of least resistance -
  - *Minimize* Cutting Energy
- During machining process, *shear angle ϕ assumes a value such that least amount of energy is consumed*
  - Cutting force F_c should be *minimum;* V is Constant
- Assumption
  - Friction between Chip-Tool is *independent* of Shear angle ϕ
  - Shear yield stress τ of the work material is *constant*

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Optimum Shear Angle ϕ

Merchant's Theory

Shear angle ϕ assumes a value to *Minimize* P

$$P = \frac{\tau_s.w.t\cos(\beta-\gamma).V}{\sin\emptyset Cos(\phi+\beta-\gamma)} \qquad \frac{dp}{d\emptyset} = 0$$

$$\frac{d\ (\mathrm{Sin}\,\emptyset\, Cos(\phi+\beta-\gamma))}{d\emptyset} = 0$$

$$\mathrm{Cos}\,\emptyset.Cos(\phi+\beta-\gamma)\ -\ \mathrm{Sin}\,\emptyset.Sin(\phi+\beta-\gamma) = 0$$

$$Cos(2\phi+\beta-\gamma) = 0 = Cos\frac{\pi}{2}$$

$$2\phi+\beta-\gamma = \frac{\pi}{2} \qquad \boxed{\phi = \frac{\pi}{4} - \frac{1}{2}(\beta-\gamma)}$$

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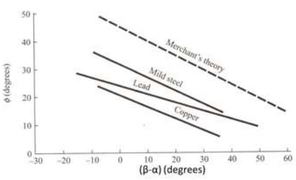
Experimental Verification of shear angle



Better correlation for soft materials e.g., plastic, but not for metals.

Limitations in Merchants analysis – Assu,ptions

- Shear stress τ is constant and is independent of normal stress,
- Simplified friction conditions on chip-tool interface

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Modified Merchant theory

- With modified Merchant theory:

$$\tau_S = \tau_{S0} + K\sigma$$

$$\sigma = \frac{N_s}{wt/Sin\phi}$$

$$\tau_S = \tau_{S0} + K\frac{N_s}{wt/Sin\phi}$$

$$2\emptyset + \beta - \gamma = C_m$$

$$C_m = cot^{-1}K$$

Machining constant: C_m (degree)

Work material (hot rolled steel)		C_m (degrees)
AISI	1010	69.8
AISI	1020	69.6
AISI	1045	78.0
AISI	2340	76.2
AISI	3140	70.6
AISI	4340	74.5
Stainless	303	92
Stainless	304	82

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Lee and Shaffer Shear model

Based on the Theory of *Plasticity*
Slip Line Filed Model of zone of deformation

<u>Assumptions</u>

- Material is Ideal Rigid Plastic
- Shear plane has the maximum shear stress.
- Effect of strain and strain rate on flow stress is neglected
- Effect of Temperature rise during deformation is negligible.

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Slip Line Field for Deformation zone

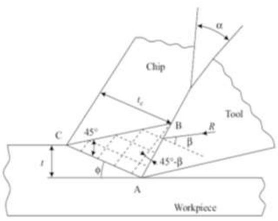

Fig. 2.12 Slip-line field for orthogonal cutting.

AC – Shear Plane - Maximum Shear Stress
CB – Free Surface – at 45 degrees to AC
CAB – Zone of Deformation
R – Resultant Cutting Force
β - Chip Tool interface Friction Angle

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Shear Angle relation

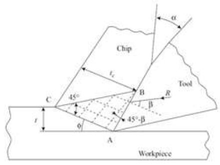

Fig. 2.12 Slip-line field for orthogonal cutting.

Angle CAB = 45 + β

Φ +Angle CAB = 90 + γ

Thus Shear Angle Φ is

Φ = 45 – (β – γ)

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Limitations of Lee and Shaffer model

Shear angle relation

$$\Phi = \frac{\pi}{4} - (\beta - \gamma)$$

Impracticable cases
- γ = 0, β = 45, - Very High friction, BUE
- Negative Rake angle γ with High friction angle β

<u>Limitations of the model</u>

- Not good correlation with experimental results for many work materials
- Considers material to be Ideally Rigid Plastic
- Effect of Strain rate and temperature neglected

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Shear Angle relations

Ernst & Merchant equation : $2\emptyset + \beta - \gamma = \frac{\pi}{2}$

Merchant's Second solution : $2\emptyset + \beta - \gamma = C_m$

Lee & Shaffer equation : $\emptyset + \beta - \gamma = \frac{\pi}{4}$

Stabler equation : $\emptyset + \beta - \frac{\gamma}{2} = \frac{\pi}{4}$

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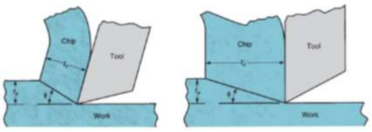
Effect of Shear angle variation



Shear Force Fs = τ. Area of Shear plane

$= \tau \cdot \frac{wt}{\sin \varphi}$

Increase of shear angle φ causes

- Smaller shear zone area
- Smaller chip size
- Less energy consumption
- Lower Temperatures and forces

Machining process becomes *more efficient*

Controlling shear angle

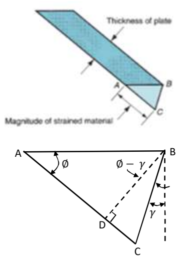
Merchant's Shear angle relation

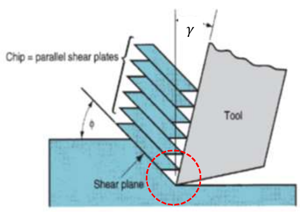
$\Phi = \frac{\pi}{4} - \frac{1}{2}(\beta - \gamma)$

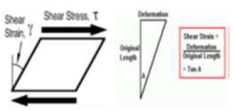
To increase shear angle Φ

- Increase tool rake angle γ
  - Use of Positive rake angle tools
- Reduce friction angle β
  - Use of cutting fluid / lubricant

Shear strain in machining





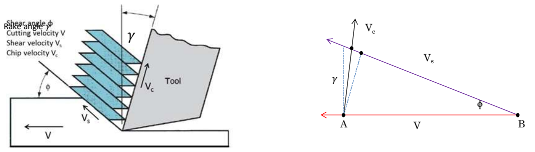


Shear strain (γ) = $\frac{AC}{BD} = \frac{AD+DC}{BD}$

Shear Strain (Y) = $Cot\emptyset + \tan(\emptyset - \gamma)$

Typical value of shear strain vary 2 – 5

Velocity and Strain Rate



V - Cutting velocity, V_s - Shear velocity along shear plane, V_c - Chip velocity along the rake surface

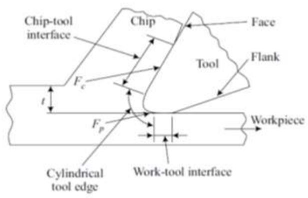
$$\frac{V}{\cos(\emptyset - \gamma)} = \frac{V_S}{\cos \gamma} = \frac{V_C}{\sin \varphi}$$

Vc = V.rc   rc – chip thickness ratio = $\frac{\sin \emptyset}{\cos(\emptyset - \gamma)}$

Shear strain rate (γ') = V_S/d; where d is shear plane thickness (0.001 – 0.01 mm)

- Typical shear strain rates, $\gamma = 10^4 \sim 10^6 s^{-1}$

Tool Work Interaction



- Tool has a Finite Nose radius
- Cuttingl Force Fc comprises of
  - Cutting Force component Fct and
  - Ploughing force Fp– Rubbing of Tool Nose

Fc = Fct + Fp

Specific Cutting Energy

Power consumption during machining

P = Cutting Force x Cutting velocity
= $F_c V$

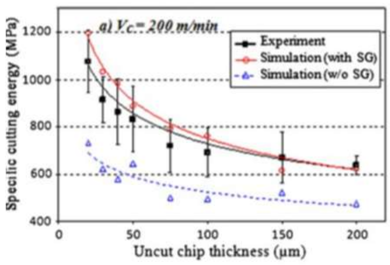
Material Removal Rate MRR = w.t.V

Specific Cutting energy = P/MRR
= F_c/wt
= (Fct +Fp)/w.t
= Constant + $\frac{Fp}{w.t}$

Specific Cutting Energy $\propto \frac{1}{t}$

Size Effect

Specific Cutting Energy *varies Inversely* with Uncut chip thickness t.



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Specific Cutting Energy U

Specific Cutting Energy U = Power in Cutting / MRR

U = $\frac{Fc\ V}{w\ t\ V}$

U varies with uncut chip thickness t

$U_e = U_0 (t)^{-0.4}$

U in J / mm^3, w and t in mm

F_c = 1000 w t U_e

F_c in N

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Specific Cutting Energy U_o

Material	Hardness BHN	Hardness R_C	U_0 (J/mm^3)
Steel	85–200		1.4
		35–40	1.6
		40–50	1.9
		50–55	2.4
		55–58	4.0
Stainless steel	135–275		1.4
		30–45	1.6
Cast iron	110–190		0.8
	190–320		1.6
Al alloys	30–150		0.35
Copper		$80R_B$	1.2
Copper alloys		$10–80R_B$	0.8
		$80–100R_B$	1.2

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Estimation of Power Consumption

Unit Power – Power consumed per Unit MRR

Machinability Data from extensive experimentation

Material	Unit Power (kW/cm^3/min)
Cast irons	0.044–0.08
Steels	
Soft	0.05–0.066
$0 < R_c < 45$	0.065–0.09
$50 < R_c < 60$	0.09–0.2
Stainless steels	0.055–0.09
Magnesium alloys	0.007–0.009
Titanium	0.053–0.066
Aluminum alloys	0.012–0.022

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