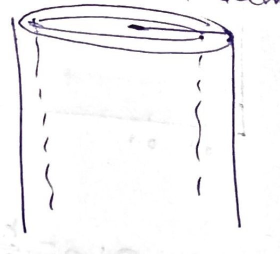
Heat Transfer Assignment 5.

$d = 40cm$



① $h = 125\ W/m^2K$

$\rho = 1000\ kg/m^3$

$c = 4200\ J/kg\text{-}K$

$T_\infty = 90°c$

$T_i = 10°c$

$$-E_{out} = E_{st}$$

$$hA(T - T_\infty) = -\rho V c \frac{dT}{dt}$$

$$\theta = T - T_\infty$$

$$\Rightarrow -\frac{hA\theta}{\rho V c} = \frac{d\theta}{dt}$$

$$\theta = \theta_i e^{-\frac{hA}{\rho Vc} t}$$

i) $$\frac{\theta}{\theta_i} = 0.99 = e^{-\frac{hA}{\rho Vc} t}$$

$$\cancel{t = \frac{\rho V c \ln \frac{1}{0.99}}{hA}}$$

$$\cancel{t = \frac{10^3 \times \pi R^2 L \, 4200}{125 \times 2\pi R L}}$$

$$\cancel{= \frac{10^5 \times 42 \times 20 \times 10^{-2}}{125 \times 2}}$$

$$\cancel{= 10^4 \times 42 / 125} = \cancel{0.33}\ \cancel{3.036 \times 10^3\ sec.}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho Vc} t}$$

$$\frac{0.99 T_\infty - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho Vc} t}$$

$$e^{-\frac{hA}{\rho Vc} t} = \frac{0.01 \cancel{T_\infty}\ 90}{80}$$

$$t = -\ln \frac{0.09}{8} \times \frac{10^3 \times \pi R^2 L\, 4200}{2\pi R L}$$

$$t = 15077.62\ sec$$

$$t = 251.29\ min$$

ii) $$Q = \int_0^\infty hA(T - T_\infty)\, dt$$

$$= \int_0^\infty hA\, \theta_i e^{-\frac{hA}{\rho Vc} t}\, dt$$

$$= \cancel{hA}\,\theta_i \frac{\left. e^{-\frac{hAt}{\rho Vc}} \right|_0^\infty}{-\frac{\cancel{hA}}{\rho Vc}}$$

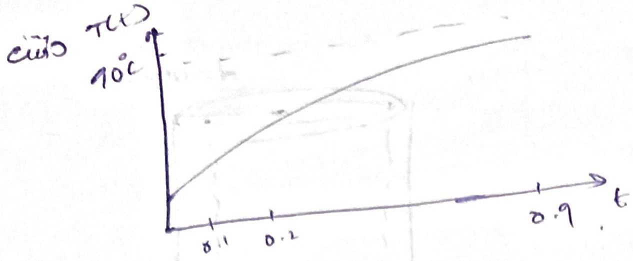
$$= \rho V c\, \theta_i$$

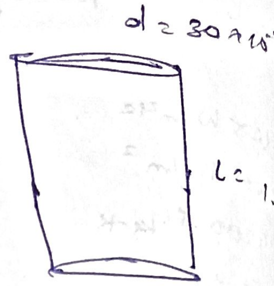
$$= 10^3 \times \pi \left(\frac{d}{2}\right)^2 L \times 4200 \times (-80)$$

$$= -\cancel{10^3} \times \pi\ 20^2 \times \cancel{10^{-4}} \times 70 \times 10^{-2} \times 4200 \times 8\cancel{0}$$

$$= 4\pi \times 7 \times 42 \times 10^3$$

$$= 3694.53 \times 10^3$$





(2)

$$\cancel{\theta = \theta_i e^{-\frac{hA}{\rho V c} t}} \qquad hA \overset{(T-T_\infty)}{\cancel{\frac{dT}{dt}}} = -\rho V c \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\lambda (T - T_\infty) \qquad \lambda = \frac{hA}{\rho V c} = \frac{8}{10^3} \times \frac{2\pi R L}{\pi R^2 L \times 4178}$$

$$= \frac{8 \times 2}{10^3 \times 15 \times 10^{-2} \times 4178}$$

$$\int_{t_0}^{t} dt = -\int_{T_0}^{T} \frac{dT}{\lambda (T - T_\infty)} \qquad = 2.55 \times 10^{-5}$$

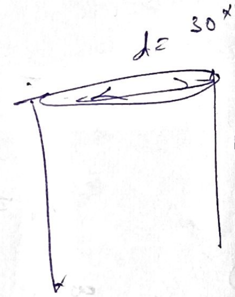
$$\cancel{t - t_0} = \quad \Delta t = -\frac{1}{\lambda} \ln \frac{(T - T_\infty)}{T_0 - T_\infty}$$

$$\Delta t = -\frac{1}{2.55 \times 10^{-5}} \ln (25 - 20)$$

$$=$$

$$\frac{dT}{dt} = -\lambda (T - T_\infty) = -\lambda \times 5$$

$$T - T_i = -5 \lambda t$$



(2) $\cancel{\Delta t} =$ $$\theta = \theta_i e^{-\frac{hA}{\rho V c} t}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{8 \times 2\pi R L}{10^3 \times \pi R^2 L \times 4178} t}$$

$$\frac{25 - 20}{37 - 20} = e^{-\frac{8 \times 2}{10^3 \times 15 \times 10^{-2} \times 4178} t}$$

$$\frac{5}{17} = e^{-2.55 \times 10^{-5} t}$$

$$\ln 17/5 = 2.55 \times 10^{-5} t$$

$t = \cancel{1.82 \times 10^4}$ sec 4.799×10^4

$t = \cancel{2.2 \text{ min}}$ 800 min = 13 hr 20 min

Time of death = ~~4.38~~ pm. 3.40 a.m

Q. 3

Using $\dfrac{T - T_s}{T_i - T_s} = \operatorname{erf}\left(\dfrac{x}{\sqrt{4\alpha t}}\right)$

$\cancel{57 - 180}$

$\dfrac{40 - 50}{35 - 50} = \operatorname{erf}\left(\dfrac{5 \times 10^{-2}}{2\sqrt{7 \times 10^{-7} t}}\right)$

$\dfrac{2}{3} = 0.66 = \operatorname{erf}\left(\sqrt{\dfrac{25 \times 10^{-4}}{4 \times 7 \times 10^{-7} t}}\right)$

$0.68 = \sqrt{\dfrac{25 \times 10^{-4}}{4 \times 7 \times 10^{-7} t}}$

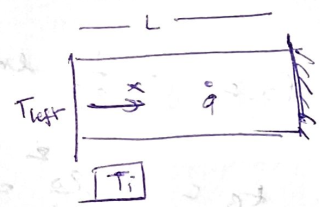
$t = \dfrac{25 \times 10^{3}}{4 \times 7 \times 0.68^2}$

$t = 1.93 \times 10^{3}$ sec

$t = 32.18$ min

Q. 5

$\dfrac{k}{\rho c} \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\dot{q}_0}{\rho c} = \dfrac{\partial T}{\partial t}$



& $\dot{q} = q_0 \sin\left(\dfrac{\pi x}{L}\right) e^{-t/\tau}$

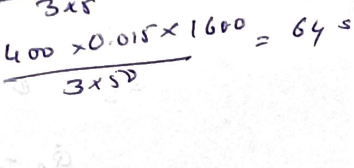
$k = 10$, $\alpha = 0.05$, $L = 1$, $T_i = 0$, $T_{left} = 100$, $\tau = 20$

comparison:
i) For low Nt, temp values diverges and attains unrealistics values, this happens because of high step value of (dt) which leads unstability & divergence → To be continu

Q 4

	D	c	ρ	k	h
			1600	170	5
sphere A	300	400		[1.7]	50
sphere B	30	1600	400		

$Bi = \dfrac{h(R_o/3)}{k}$ $\left(\because L_c = \dfrac{R_o}{3}\right)$



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$Bi_A = \dfrac{5 \times 0.15/3}{170} = 1.47 \times 10^{-3}$ $\quad Bi \ll 0.1$

$Bi_B = \dfrac{50 \times 0.015/3}{17} = 0.147$

Thermal time constant $\tau = \dfrac{\rho V c}{hA}$ $\Rightarrow$

$\tau_A = \dfrac{1600 \times 0.15 \times 400}{3 \times 5} = 6400$ s

$\tau_B = \dfrac{400 \times 0.015 \times 1600}{3 \times 50} = 64$ s

$\therefore$ B ~~slower~~ cools slower than A

as $\tau_B < \tau_A$

(b) As $Bi \ll 0.1$ for sphere A,

$$\frac{\theta}{\theta_i} = e^{-t/\tau} \Rightarrow t = -\tau_A \times \ln\frac{\theta}{\theta_i}$$

$$t = -6400 \ln\frac{415-320}{800-320}$$

$$= 10367.8 \text{ sec}$$

$$t = 2.88 \text{ hrs}$$

for sphere B, $Bi > 0.1$ $\therefore$ using exact solution

$$(Bi)_B = \frac{h r_o}{k} = \frac{50 \times 0.015}{1.7} = 0.441$$

$$\zeta_1 = 1.0992 \qquad C_1 = 1.1278$$

$$\theta^*(r^*=1, F_o) = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = \frac{415-320}{800-320} = 0.1979$$

$$\theta^*_o = \theta^* \zeta_1 r^* / \sin(\zeta_1 r^*) = \frac{0.1979 \times 1.099 \times 1}{\sin(1.099)}$$

$$= 0.2442$$

$$F_o = -\frac{1}{\zeta_1^2} \ln\frac{\theta^*_o}{C_1} = -\frac{1}{1.0992^2} \ln\left(\frac{0.2442}{1.1278}\right) = 1.266$$

$$t_B = F_o \frac{r_o^2}{\alpha} = F_o \frac{\rho c_p}{k} r_o^2 = 1.266 \times \frac{400 \times 1600}{1.7} \times 0.015^2$$

$$= 107 \text{ sec}$$

as fourier no. is > 0.2 therefore one term approx is accurate

(c) Sphere A (~~sphe~~ space isothermal behaviour)

$$E_{in} - E_{out} = \Delta E$$

$$-Q_A = \Delta E = E(t) - E(0)$$

$$Q_A = \rho c V [T - T_i] = 1600 \times 400 \times 4/3 \times \pi \times 0.015^3 (415 - 800)$$

$$Q_A = 3.48 \times 10^6 \text{ J}$$

sphere B

$$\frac{Q_B}{Q_0} = 1 - \frac{3\theta^*_o}{\zeta_1^3}\left[\sin\zeta_1 - \zeta_1\cos\zeta_1\right] = 1 - \frac{3 \times 0.2442}{1.0992^3} \times (\sin$$

$$= 0.784$$

$$Q_B = 0.784 Q_0 = 0.784 \times 1600 \times \frac{4}{3}\pi\, 0.015^3 \times (800-320) \times 400$$

$$Q_B = 3405 \text{ J}.$$

Q5

↳ case 1 : $q_0 = 1$

The plot depicts the evalution of temp distribution over time & space in 1D & T_i is uniformly initialised to $0°c$

~~Asin~~ $q_0 = 1$ is small and change is gradual

case II : $q_0 = 1000$

The key diff. in this case is amplitude, the larger T_0 which leads to more rapid changes exhibiliting more prononced varianion & higher peak temp.