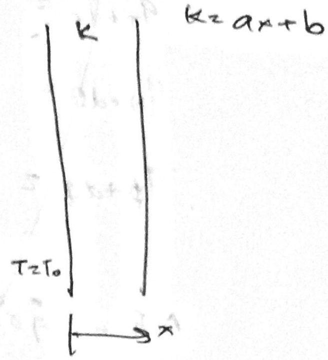
ME 346 -
Assingment 1

1

a) let $q'' =$ const $= P$



$$P = -k \frac{dT}{dx}$$

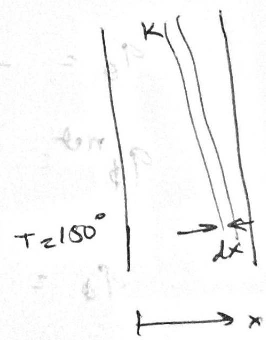
$$P = -(ax+b)\frac{dT}{dx}$$

$$\boxed{\frac{dT}{dx} = \frac{-P}{ax+b}}$$

b) $$\int_{T_0}^{T} dT = \int_0^x -\frac{P}{ax+b}$$

$$\boxed{T = T_0 + -P \ln \frac{ax+b}{b}}$$

2



a) $$dq = -\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right)dx$$

$$\dot{q}_{gen} = \dot{q}\, A dx$$
↳ per unit vol.

$$\dot{q} \cancel{A dx} = -k\cancel{A}\frac{d^2T}{dx^2}\cancel{dx}$$

as $T = a + bx^2$

$$\dot{q} = -45 \times (2b)$$

$$= +9 \times 10^4 \ W/m^3$$

b) $$q''_x = -k\frac{dT}{dx} = -k(2bx)$$

$q''(x=0) = 0$

$q''(x=0.1) = -45(2 \times (-1000) \times 0.1)$

$= 9000 \ W/m^2$

③

$$q_{r+dr} - q_r = \frac{\partial}{\partial r} q_r \, dr$$

$$q_{\theta+d\theta} - q_\theta = \frac{\partial}{\partial \theta} q_\theta \, d\theta$$

$$q_{\phi+d\phi} - q_\phi = \frac{\partial}{\partial \phi} q_\phi \, d\phi$$

$$dq^{net} = dq_r + dq_\theta + d\phi$$

$$dq_r = -kA \frac{\partial T}{\partial r} = -k \, r^2 \sin\theta \, d\theta \, d\phi \, \frac{\partial T}{\partial r}$$

$$dq_\theta = -kA \frac{\partial T}{\partial \theta} = -k \, r \sin\theta \, dr \, d\phi \, \frac{\partial T}{\partial \theta}$$

$$dq_\phi = -kA \frac{\partial T}{\partial \phi} = -k \, r \, dr \, d\theta \, \frac{\partial T}{\partial \phi}$$

$$\Delta E_{gen} = \dot{q} \, r^2 \sin\theta \, dr \, d\theta \, d\phi \, dt$$

$$dq_{net} + \Delta E_{gen} = \rho c \, r^2 \sin\theta \, d\theta \, dr \, d\phi$$

$$\dot{q} r^2 \sin\theta \, dr \, d\theta \, d\phi + k \, dr \, d\theta \, d\phi \left[\frac{\partial}{\partial r} r^2 \sin\theta \frac{\partial T}{\partial r} + \frac{\partial}{\partial \theta} r \sin\theta \frac{\partial T}{\partial \theta} + \frac{\partial}{\partial \phi} r \frac{\partial T}{\partial \phi} \right]$$

$$\dot{q} + k \left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \sin\theta \frac{\partial T}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial^2 T}{\partial \phi^2} \right] = \rho c \frac{\partial T}{\partial t}$$

④

$$\frac{\partial}{\partial r} k r^2 \frac{\partial T}{\partial r} = 0 \quad \text{—①}$$

$$\frac{T(r) - T(r_1)}{r - r_1} = \frac{T(r_2) - T(r_1)}{r_2 - r_1}$$

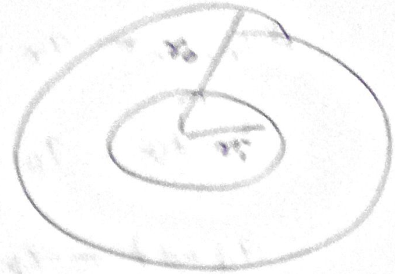
$$T(r) = T(r_1) + \frac{T(r_2) - T(r_1)}{r_2 - r_1} \times (r - r_1)$$

$$\frac{\partial}{\partial r} k r^2 \left(\frac{T(r_2) - T(r_1)}{r_2 - r_1} \right) = 0 \Rightarrow k r^2 = C_1$$

$$k = \frac{C_1}{r^2}$$

(5)

$T(r) = C_1 \ln\left(\frac{r}{r_0}\right) + C_2$



a)

$$\frac{\partial T}{\partial r} = \frac{C_1}{r}$$

$$q = \frac{\partial}{\partial r} kA \frac{\partial T}{\partial r} = 0$$

$$kA \frac{\partial T}{\partial r} = C_i'$$

$$\int dT = \int \frac{C_i'}{A} \, dr$$

$$kA \frac{C_1}{r} = C_i'$$

$A \sim r$ (A for cylinder $= 2\pi r L$)

$\therefore$ Cylinder

b) $q' = -kA \frac{dT}{dr} = -k \cdot 2\pi \cancel{r} L \frac{C_1}{\cancel{r}} = \text{const.}$

heat rate

$q'' = -k \frac{dT}{dr} = -k \frac{C_1}{r}$

flux.