Betweenness

For an undirected graph with n nodes (vertices), the betweenness of node k is computed as follows:

1. For each unordered pair of nodes (i, j), compute all shortest paths between them. The shortest path is the path with the fewest steps between node i and node j.
2. For each unordered pair of nodes (i, j), determine the fraction of shortest paths that pass through the node in question (here, node k).
3. Sum this fraction over all unordered pairs of nodes (i, j).
4. Say $s_k(i,j)$ is the proportion of shortest paths between node i and node j containing node k. Then,

$$\text{betweenness}(k) = \sum_{(i<j):i\neq j,k\notin\{i,j\}} s_k(i,j).$$

Betweenness

As in degree, betweenness measures network connectivity of one node relative to others. Betweenness may be normalized, dividing the earlier expression by the number of unordered pairs of nodes not including k, which for an undirected graph is $(n-1)(n-2)/2$. For illustration, consider an undirected star graph. This is a graph with a center node that is connected to all other nodes, while none of the other nodes (the leaves in the star graph) are connected among themselves. Each shortest path between two star points goes through the center point; the betweenness of the center point is $(n-1)(n-2)/2$ (or 1, if normalized). The leaves in a star graph are not contained in shortest paths, and have betweenness 0.

Betweenness measures how much influence a node has over connections between other nodes. It measures total graph connectivity, rather than counting next door neighbors.

Betweenness

Comment: In directed graphs, the degree of a node is the number of links that point to or from the node. In directed graphs, betweenness$(k) = \sum_{(i,j):i\neq j,k\notin\{i,j\}} s_k(i,j)$, summing over all ordered pairs of nodes as the shortest path from node i to node j may be different from the shortest path from node j to node i. Normalization is achieved by dividing the sum by the number of ordered pairs of nodes not including k, which is $(n-1)(n-2)$.

Below, we illustrate betweenness for the nodes in the undirected network among the 16 families in Medevial Florence. Take Peruzzi, as example, with betweenness 2. It arises from the connections between Bisheri and Castellani (two shortest paths involving three nodes, with one going through Peruzzi), Bisheri and Barbadori (two shortest paths involving four nodes, with one going through Peruzzi), Guadagni and Castellani (two shortest paths involving four nodes, with one path going through Perruzi), and Lamberteschi and Castellani (two shortest paths involving five nodes with one going through Peruzzi). Hence betweenness for Peruzzi is $(1/2) + (1/2) + (1/2) + (1/2) = 2$.