Exploring high dimensional asset dependence through Vine Copulas

Stellenbosch University/ Eighty20

Copulas

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Introduction

- In the past session, you have encountered a vast array of financial models
  - Basic ARIMA models for the mean equation
  - GARCH extensions to deal with heteroscedasticity
  - Multivariate GARCH models that deal with dependence modeling
- Theoretical problem arises when we talk about **dependence**
  - Capturing co-movement between financial asset returns with linear correlation has been the staple approach in modern finance since the birth of Harry Markowitz's portfolio theory
  - But linear correlation is only appropriate when the dependence structure (or joint distribution) follow a normal distribution

Goal

- To introduce to you an extension in the field of risk management
- Grasp basic concepts and generators within the field of copulas
  - Learn to walk, before we can run
  - Revisit your statistics
- Understand the field of copulas to such an extent that you might go on to do a PhD in this field ;-)

Fields where copulas are applied

- Quantitative finance
  - Stress-tests and robustness checks
  - "Downside/crisis/panic regimes" where extreme downside events are important
  - Pool of asset evaluation
  - Latest development: Vine Copulas
  - Hot research page here
- Civil engineering
- Warranty data analysis
- Medicine

Introduction to copulas

- Copula stems from the latin verb copulare; bond or tie.
  - Regulated financial institutions are under pressure to build robust internal models to account for risk exposure
  - Fundamental ideology of these internal models rely on joint dependency among whole basket of mixed instruments
  - This issue can be addresed through the copula instrument
  - It functions as a linking mechanism between uniform marginals through the inverse cdf
- Copula theory was first developed by Sklar in 1959 Nelsen (2007).

Introduction to copulas (Sklar)

- Sklar's theorem forms the basis for copula models as:
  - It does not require identical marginal distributions and allows for n-dimensional expansion
- Let X be a random variable with marginal cumulative distribution function:
  - $F_X(x) = \mathcal{P}(X \leq x)$
  - Probability that random variable X takes on a value less or equal to point of evaluation
  - $F_X(x) \sim U(0,1)$

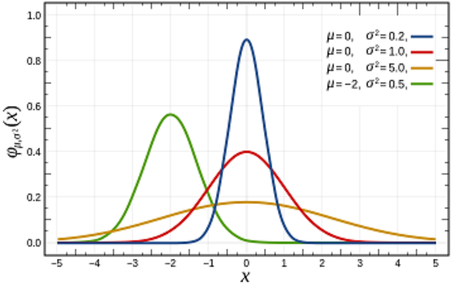
Introduction to copulas (Sklar cont.)

- If we now denote the inverse CDF (Quantile funtion) as F_x^{-1}
  - $U \sim U(0,1)$ then $F_x^{-1}(U) \sim F(X)$
- This allows a simple way for us to simulate observations from the F_X provided the inverse cdf, F_X^{-1} is easy to calculate
- Think, median is $F_X^{-1}(0.5)$

Lets have a look visually

Transformations

- PDF
- CDF
- CDF^{-1}



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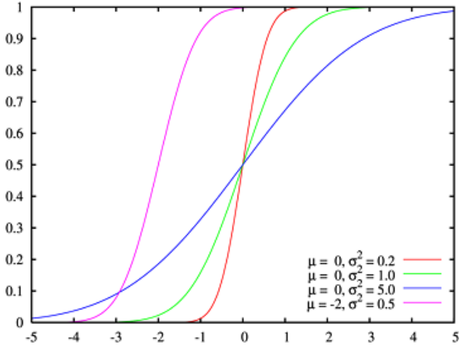

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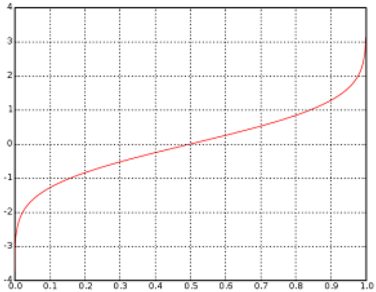


Definitions and basic properties

- Define the uniform distribution on an interval $(0,1)$ by $U(0,1)$, i.e the probability of a random variable U satisfying $P(U \leq u) = u$ for $u \in (0,1)$
  - This is the quantile function ($Q = F^{-1}$) Probability transformation implies that if X has a distribution function F, then $F(X) \sim U(0,1)$ iff F is continous

Definitions and basic properties (cont)

Definition (Copula): A d-dimensional copula is the distrubutiom function $\mathcal{C}$ of a random vector U whose components U_k are uniformly distributed

$$C(u_1, \dots, u_d) = P(U_1 \leq u_1, \dots, U_d \leq u_d), (u_1, \dots, u_d) \in (0,1)^d \tag{1}$$

Thus Sklar's theorem states:

$$\begin{aligned} C(F_1(x_1), \dots, F_d(x_d)) &= P(U_1 \leq F_1(x_1), \dots, U_d \leq F_d(x_d)) \\ &= P(F_1^{-1}(U_1) \leq x_1, \dots, F_d^{-1}(U_d) \leq x_d) \\ &= F(x_1, \dots, x_d) \end{aligned} \tag{2}$$

Joint distribution function:

- This represents the joint distribution function function F can be expressed in terms of a copula C and the marginal distristributions $(F_1, \ldots, F_d)$. Modeling them seperately
- **Easy Def:** A Copula is a function that couples the joint distribution function to its univariate marginal distribution
- Dependence or correlation coefficient dependent on marginal distributions. This one to one mapping of correlation and dependece only works in case of elliptical joint distribution
- For copulas, we use Kendall's Tau - non-linear concordance measure

Kendall's Tau

- Let (X_1, Y_1) and (X_2, Y_2) be i.i.d random vectors, each with joint distribution function H
- Tau is then defined as the probability of concordance minus the probability of discordance

$$\tau = \tau_{X,Y} = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2) - (Y_1 - Y_2) < 0) \tag{3}$$

- Tau is the difference between the probability that the observed data are in the same order versus the probability that the observed data are no in the same order

Vine-Copulas

- A vine is a graphical tool for labeling constraints in high-dimensional probability distributions
- Regular Vines from part of what is know as pair copula construction
- Trees are constructed between copulas based on what is know as maximum spanning degree (Or concordance measure)
- Under suitable differentiability conditions, any multivariate density $F_{1\ldots n}$ on n variables may be represented in closed form as a product of univariate densities and (conditional) copula densities on any R-vine V

Vine copulas kurowicka2006

The R-vine copula density is uniquely identified according to

Theorom 4.2 of @kurowicka2006:

$$c(F_1(x_1), \cdots, F_d(x_d)) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)} \left(F(x_{j(e)} | \mathbf{x}_{D(e)}) \right) \tag{4}$$

- Introduction to VineCopula
- Website for the research here

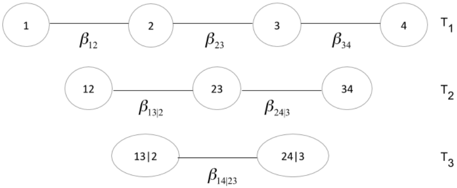
Different structures

Vine copula specifications are base upon graph theory and more so Vines.

- This gives a lot of scope for the structure of the arrangement of assets
- R-vine, D-Vine, C-Vine

D-Vine, C-Vine and R-Vine

Each of the structures provide their own insight into the dynamics of the market



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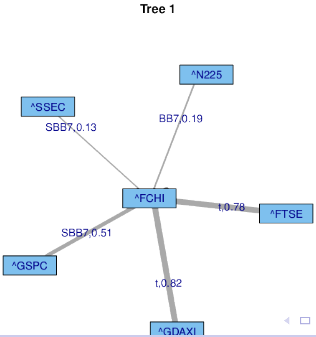
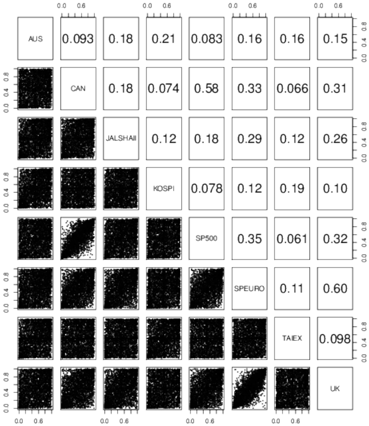


Applications

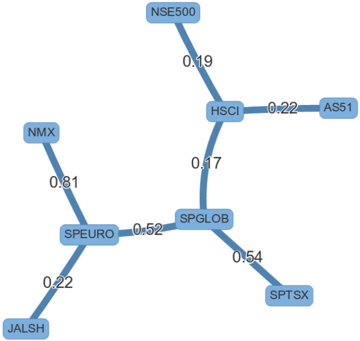
- C-vine offers a unique opportunity from centeralized market player evalution
  - CAPM
- Value at risk estimation of large portfolios bottom up
- Modeling complex dependence measures

A look into energy market dependence using Vine Copula



Energ market vines

- Pre - GFC
- GFC
- Post - GFC



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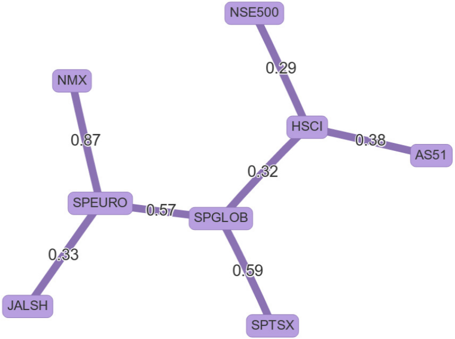

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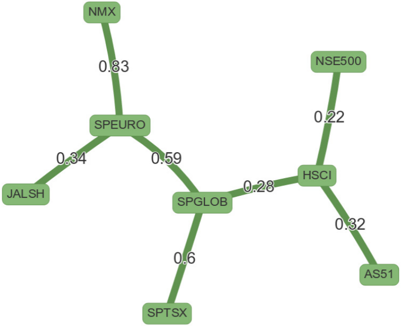
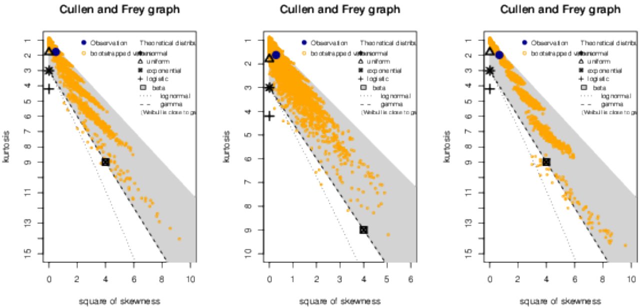


Quantifying dynamic dependence



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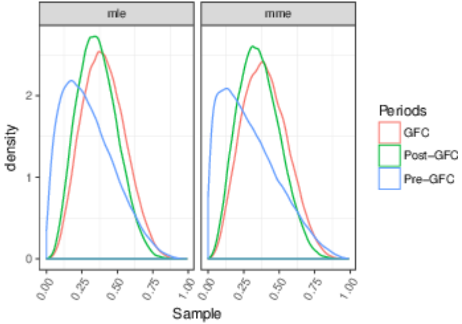

Final results

	hypothesis	fit Type	estimate	statistic	p.value	conf.low	conf.high	alternative
1	Pre-GFC/GFC	MLE	-0.11	1296773158.00	1.00	-0.11	Inf	greater
2	Pre-GFC/GFC	MME	-0.11	1360073661.00	1.00	-0.11	Inf	greater
3	Pre-GFC/Post-GFC	MLE	-0.07	1718227541.00	1.00	-0.07	Inf	greater
4	Pre-GFC/Post-GFC	MME	-0.07	1727208393.00	1.00	-0.07	Inf	greater

Table : Mann-Whitey location test results

Conclusion

- Copulas act as a unique tool in to model non-conforming marginals that weren't possible before
- Vine Copulas have the ability to model complex relationships talks to their flexibility in their structuring
- Informs on how assets are dependent - whether its tail dependence or general symmetric driving co-dependency
- Opens the doors to practitioners (such as risk managers), to be better equiped in dealing with modern day finance

Contact information

- hanjo.oden@gmail.com
- https://github.com/HanjoStudy/R_finance_20170323

References

Nelsen, Roger B. 2007. *An Introduction to Copulas*. Springer Science & Business Media.