

# Exploring high dimensional asset dependence through Vine Copulas

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Copulas

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# Introduction

- In the past session, you have encountered a vast array of financial models
  - Basic ARIMA models for the mean equation
  - GARCH extensions to deal with heteroscedasticity
  - Multivariate GARCH models that deal with dependence modeling
- Theoretical problem arises when we talk about **dependence**
  - Capturing co-movement between financial asset returns with linear correlation has been the staple approach in modern finance since the birth of Harry Markowitz's portfolio theory
  - But linear correlation is only appropriate when the dependence structure (or joint distribution) follow a normal distribution

# Goal

- To introduce to you an extension in the field of risk management
- Grasp basic concepts and generators within the field of copulas
  - Learn to walk, before we can run
  - Revisit your statistics
- Understand the field of copulas to such an extent that you might go on to do a PhD in this field ;-)

# Fields where copulas are applied

- Quantitative finance
  - Stress-tests and robustness checks
  - “Downside/crisis/panic regimes” where extreme downside events are important
  - Pool of asset evaluation
  - Latest development: Vine Copulas
  - Hot research page [here](#)
- Civil engineering
- Warranty data analysis
- Medicine

# Introduction to copulas

- Copula stems from the latin verb copulare; bond or tie.
  - Regulated financial institutions are under pressure to build robust internal models to account for risk exposure
  - Fundamental ideology of these internal models rely on joint dependency among whole basket of mixed instruments
  - This issue can be addressed through the copula instrument
  - It functions as a linking mechanism between uniform marginals through the inverse cdf
- Copula theory was first developed by Sklar in 1959 Nelsen (2007).

# Introduction to copulas (Sklar)

- Sklar's theorem forms the basis for copula models as:
  - It does not require identical marginal distributions and allows for n-dimensional expansion
- Let  $X$  be a random variable with marginal cumulative distribution function:
  - $F_X(x) = \mathcal{P}(X \leq x)$
  - Probability that random variable  $X$  takes on a value less or equal to point of evaluation
  - $F_X(x) \sim U(0, 1)$

# Introduction to copulas (Sklar cont.)

- If we now denote the inverse CDF (Quantile function) as  $F_X^{-1}$ 
  - $U \sim U(0,1)$  then  $F_X^{-1}(U) \sim F(X)$
- This allows a simple way for us to simulate observations from the  $F_X$  provided the inverse cdf,  $F_X^{-1}$  is easy to calculate
- Think, median is  $F_X^{-1}(0.5)$

Lets have a look visually

# Definitions and basic properties

- Define the uniform distribution on an interval  $(0, 1)$  by  $U(0, 1)$ , i.e the probability of a random variable  $U$  satisfying  $P(U \leq u) = u$  for  $u \in (0, 1)$ 
  - This is the quantile function ( $Q = F^{-1}$ ) Probability transformation implies that if  $X$  has a distribution function  $F$ , then  $F(X) \sim U(0, 1)$  iff  $F$  is continuous



## Definitions and basic properties (cont)

**Definition (Copula):** A d-dimensional copula is the distribution function  $\mathcal{C}$  of a random vector  $U$  whose components  $U_k$  are

uniformly distributed  
$$\mathcal{C}(u_1, \dots, u_d) = P(U_1 \leq u_1, \dots, U_d \leq u_d), (u_1, \dots, u_d) \in (0, 1)^d$$

Thus Sklar's theorem states:

$$\begin{aligned}\mathcal{C}(F_1(x_1), \dots, F_d(x_d)) &= P(U_1 \leq F_1(x_1), \dots, U_d \leq F_d(x_d)) \\ &= P(F_1^{-1}(U_1) \leq x_1, \dots, F_d^{-1}(U_d) \leq x_d) \\ &= F(x_1, \dots, x_d)\end{aligned}\tag{2}$$

# Joint distribution function:

- This represents the joint distribution function  $F$  can be expressed in terms of a copula  $C$  and the marginal distributions  $(F_1, \dots, F_d)$ . Modeling them separately
- **Easy Def:** A Copula is a function that couples the joint distribution function to its univariate marginal distribution
- Dependence or correlation coefficient dependent on marginal distributions. This one to one mapping of correlation and dependence only works in case of elliptical joint distribution
- For copulas, we use Kendall's Tau - non-linear concordance measure

# Kendall's Tau

- Let  $(X_1, Y_1)$  and  $(X_2, Y_2)$  be i.i.d random vectors, each with joint distribution function  $H$
- Tau is then defined as the probability of concordance minus the probability of discordance
$$\tau = \tau_{X,Y} = P((X_1 - X_2)(Y_1 - Y_2) > 0) - P((X_1 - X_2) - (Y_1 - Y_2) < 0)$$
- Tau is the difference between the probability that the observed data are in the same order versus the probability that the observed data are not in the same order

# Vine-Copulas

- A vine is a graphical tool for labeling constraints in high-dimensional probability distributions
- Regular Vines from part of what is known as pair copula construction
- Trees are constructed between copulas based on what is known as maximum spanning degree (Or concordance measure)
- Under suitable differentiability conditions, any multivariate density  $F_{1,...,n}$  on  $n$  variables may be represented in closed form as a product of univariate densities and (conditional) copula densities on any R-vine  $V$

# Vine copulas kurowicka2006

The R-vine copula density is uniquely identified according to

Theorem 4.2 of @kurowicka2006:

$$c(F_1(x_1), \dots, F_d(x_d)) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e), k(e) | D(e)} (F(x_{j(e)} | \mathbf{x}_{D(e)})) \quad (4)$$

- Introduction to **VineCopula**
- Website for the research **here**

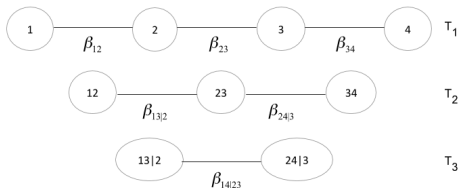
# Different structures

Vine copula specifications are based upon graph theory and more so Vines.

- This gives a lot of scope for the structure of the arrangement of assets
- R-vine, D-Vine, C-Vine

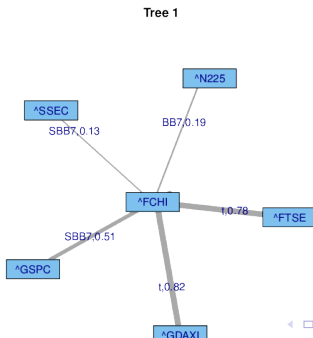
# D-Vine, C-Vine and R-Vine

Each of the structures provide their own insight into the dynamics  
of the market



# D-Vine, C-Vine and R-Vine

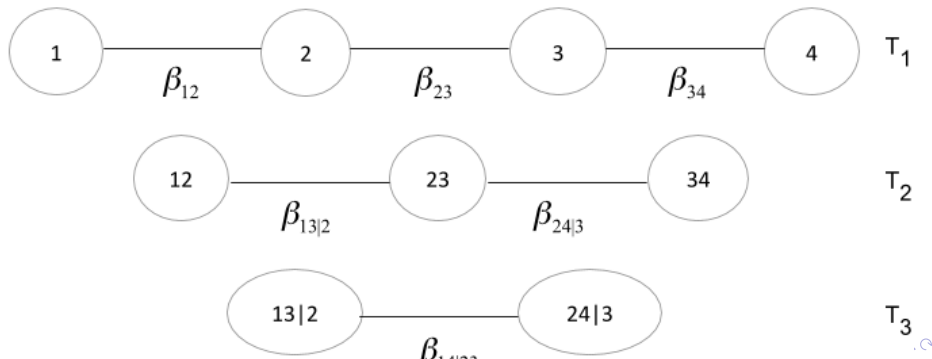
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# D-Vine, C-Vine and R-Vine

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of the market



# Applications

- C-vine offers a unique opportunity from centralized market player evaluation
  - CAPM
- Value at risk estimation of large portfolios bottom up
- Modeling complex dependence measures

# End

Questions?

# Contact information

- [hanjo.oden@gmail.com](mailto:hanjo.oden@gmail.com)
- <https://github.com/HanjoStudy>

# References

Nelsen, Roger B. 2007. *An Introduction to Copulas*. Springer Science & Business Media.