# FORECASTING OF THE INFLATION RATES IN UGANDA: A COMPARISON OF ARIMA, SARIMA AND VECM MODELS

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# Acknowledgement

I would like to thank everyone who directly or indirectly helped me finish my thesis, the help ranged from motivation to advice. A special thank you to my parents, sisters and my friend Catherine for being there for me whenever I needed them.

To my supervisor Niklas Karlsson who always created time for me, listened to my ideas and provided continuous advice. I also appreciate my examiner Sune Karlsson for accepting to grade my work.

Further appreciation goes to the faculty members that provided me with their contribution to the finishing my thesis and finally thanks to the Swedish institute for giving me this opportunity to do my masters.

# Abstract

Structural changes in the economy are a big factor in development of new models with different variables and parameter. This study considered the possible existence of seasonality in Uganda's inflation developed three models; the ARIMA model which does not consider seasonality, the SARIMA model which considers the component of seasonality in the series, the VECM model which is multivariate model with three variables; the Uganda inflation rates, Uganda exchange rates and the world coffee prices. The study used monthly data from April 1998 to September 2015. The in-sample forecast accuracy and the one-step ahead out-of-sample forecast accuracy of the three models for was compared using RMSE, MAE and the Diebold-Mariano test.

The research results imply that Uganda's inflation rate has an aspect of seasonality since the SARIMA model performed better than the ARIMA model when it comes to both out-of-sample and in-sample forecast performance based on the RMSE and MAE respectively. VECM model performed worse than the ARIMA and SARIMA models under both the in-sample and out-of-sample performance since it had the maximum RMSE and MAE.

The Diebold-Marino test concludes that the SARIMA and ARIMA models have equal forecast accuracy for both in-sample and out-of-sample forecast. There is also a difference in both the in-sample and out-of-sample forecast accuracy of the VECM as compared to both ARIMA and SARIMA models.

**KEYWORDS:** Uganda Inflation, ARIMA model, SARIMA model, VECM model, Forecast comparison

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# 1 INTRODUCTION

Inflation is a rise in the average price of goods over time. The inflation is based on prices of goods that are commonly consumed by the public. These could be classified under different commodity baskets for example food and non-alcoholic beverages, alcoholic beverages and tobacco, clothing and footwear, housing and household services, health, transport, restaurants and hotels among others. The inflation rate is measured using the Consumer Price Index (CPI) which is the current price of a collection of goods and services in terms of the same period prices in the previous year. (Begg et al. (1994);Begg et al. (2014)).

Inflation affects economic decisions because of its volatility and therefore attracts the attention of economists for instance by doing forecasts of future inflation rates. Both business entities and consumers make economic decisions based on the forecasted inflation therefore affected by inflation uncertainty. Long term plans become hard to achieve so short term investments are chosen over more profitable long term investments because the aspect of profit in the long run is made to seem impossible and unclear; the will to invest and save could possibly decline and companies direct their resources in order to avoid the related risks(Devereux, 1989).

There is possible transfer of wealth between the debtors and the creditors when the inflation rate differs from its projection since repayment of the loans is done with money of different value. This calls for the need to adjust for inflation by both investors and governments when it comes to all future planning which can lead to reduced inflation uncertainty and a reduction in financial costs.

Uganda's past inflation since the 1980's can be grouped into three main episodes; a high inflation period in the 1980s, a relatively stable period from 1995-2007 and a more volatile moderate inflation episode in the past 7 years. A new government got into power in 1987 and began an economic recovery program that resulted into a reduction in the excess money supply. Macro-economic imbalances were the major drivers of the inflation in the 1990's. This inflation period was characterized by shortages of key consumer goods that resulted into black market sales of goods with government controlled prices (Aron et al., 2015).

Inflation rates are greatly influenced by the political environment. They have been observed to increase every after five years particularly towards presidential campaigns/elections. In the

year 2005 before the 2006 general elections, the inflation rate between March and July was 10.0%-10.6%, this was dealt with well by the Bank of Uganda that made sure the inflation rate dropped back to a single digit during and shortly after the elections. Bank of Uganda records also show that inflation was in the range of 10.9% -11.2% for the months from May, 2008 to December, 2009.

Despite the rate dropping to 0.2% in the October of 2010 right before the elections, the rates soared again right after the elections on food and fuel prices in March 2011 during which the inflation rate was recorded at 11.2%, this inflation rate never declined but even rose to higher figures for example 30.5%, 29.0% and 27.0% in the months of October, November and December, respectively (Bariyo, 2011). The inflation rates published by Central Bank of Uganda indicate that from April, 2010 to April, 2011 the inflation rate was a single digit between 5.7%-9.6%. This developed the need to do this study and find out more about the possible seasonality of Uganda's inflation rates throughout the years.

Aron et al. (2015) suggests that structural changes in the economy make the already existing models for inflation in Uganda outdated. They pointed out models used in past studies like that of Kasekende (1990), Barungi (1997) and Abuka and Wandera (2001) among others which emphasised how inflation rates were influenced by domestic money and foreign factors during the period of economic stabilisation. This thesis will consider world coffee prices as the only external factor that could affect Uganda's inflation. The Exchange rate of Uganda will also be considered as a domestic factor for multivariate model development. A few studies have used at least one of these predictors of inflation but not in the exact combination that will be considered here.

The main objective of the study is to compare both in-sample and out-of-sample forecasting performance between two univariate models (ARIMA, SARIMA) and VECM as the only multivariate model used to model the inflation rates in Uganda. In the process, we will be able to answer the questions: Does Uganda's inflation exhibit any seasonality? Does a seasonal adjusted model have superior forecasting accuracy? And finally, do the multivariate models have superior forecasting accuracy than the univariate ones when it comes to forecasting the inflation rates in Uganda?

The monthly data used in this study is Uganda's inflation rates, the exchange rates of Uganda and the world coffee prices from June 1998 to September 2015 which were obtained from the Bank of Uganda and the World Bank websites. The portion of the data from June 1998 to April

2011 will be used to build the different models whereas the remaining data (2011:05 to 2015:09) used to access the out of sample forecast performance of the models. After using the models to obtain the forecasts and their intervals, Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and the Diebold-Mariano test will be used to measure and study the equality in the accuracy of the developed models.

The second section presents the relevant past literature on forecasting inflation. This is followed by the third section that includes the methodology used in the thesis; the different models, tests as well as their corresponding assumptions and null hypothesis. The data used, its source and some descriptive statistics is put together in section four which also includes the analysis results. Finally, the concluding discussion about the obtained results and recommendation for future studies is in the final and fifth section.

# 2 LITERATURE REVIEW

## 2.1 Inflation in Uganda

Coffee is a major contributor in Uganda's given that it accounts for more than 90% of the export earnings and about 56% of the revenue. This makes the international price of coffee one of the critical external factors that affect the economy of Uganda for example through its effect on the terms of trade and the creation of fiscal and balance payments problem. This was said by Barungi (1997) who used the Engle Granger 2-step procedure to develop an Error Correction Model after testing for cointegration. Her aim was to determine the role played by the monetary base and real exchange rates in determining Uganda's inflationary process and it turned out that only the monetary expansion (monetary value of M2) is the main source of price variation in the short run.

Inflation can be measured by the consumer price index (CPI) for food and non-food prices. In Uganda, the domestic Money supply has a small influence on non-food inflation but does not affect food and fuel inflation. After the East African Community Regional integration, there has been an international influence on the inflation dynamics of Uganda because of the food prices in Kenya. Factors like trade balances, current account balances, fiscal balance, terms of trade and trade openness also have an impact on Uganda's inflation. On the other hand, no evidence proves a link between mobile money and Uganda's inflation, (Aron et al., 2015).

Several other studies that have evidence that suggests the relationship between inflation and other factors; the long run solution for the log CPI depends on log money supply (M2) and the log of nominal effective exchange rate (Kihangire and Mugyenyi, 2005). Kabundi (2012) used monthly data from January 1999 to October 2011 to study the factors that explain the dynamics of inflation in Uganda. He included both external and domestic factors in a single equation Error Correction Model (ECM) then concluded that the main determinants of inflation in Uganda are monetary aggregate, world food prices, domestic supply and demand effects. He also suggests that money growth, world food prices and energy prices have a short term impact on inflation.

## 2.2 Forecasting inflation and model comparison

Unemployment turned out to be a less important predictor for USA inflation as compared to housing starts, capacity utilization and trade sales. This was a conclusion by Stock and Watson (1999) who used 189 different predictors in a modified bivariate Philips curve to forecast inflation. In a later study by Goodhart and Hofmann (2000) and Stock and Watson (2001), that asset prices were slightly successful in Predicting US inflation despite the fact that the consideration of different horizons made the forecast results not consistent. The atheoretical autoregressive model generally out performs the bivariate Phillips curve suggested by Stock and Watson (1999), this was established by Atkeson and Ohanian (2001) and also supported by Cecchetti et al. (2000) who said that the autoregressive model is the most robust across different forecasting horizons. Similar studies conducted for European countries identified price and labour variables as useful predictors for most countries (Banerjee et al. (2005), Arratibel et al. (2009) ). This tells us that each economy has its own major predictors of inflation and therefore different models that best forecast the inflation.

Fannoh et al. (2014) used ARIMA $(0,1,0)$ $(2,0,0)_{12}$ to appropriately model Liberia's monthly inflation rates. The residuals showed no evidence of ARCH effect and serial correlation after the ARCH-LM and Ljung-Box test respectively. They used monthly inflation data from January to December 2006 to build the ARIMA model based on the Box-Jenkins methodology considering seasonality. There has been a good number of studies based on the Seasonal Autoregressive Integrated Moving Average (SARIMA) as proposed by Box and Jenkins (1976). Several studies suggest that the SARIMA model has a forecasting advantage over other time series models; Schulze and Prinz (2009) alleged that the SARIMA model was better than the Holt Winters exponential smoothing approach to forecasting transhipment in German.

The ARCH and GARCH models are known standard tools when it comes to the uncertainty/ volatility of inflation rates. Few studies have considered the use of GARCH or Multivariate GARCH models in forecasting the inflation rate in Uganda but still some cases in other countries can be discussed. Uwilingiyimana et al. (2015) used ARIMA and GARCH models to forecast Kenya's inflation based on historical monthly data from 2000 to 2014. Their empirical research used ordinary least square estimation concluded that although ARIMA (1,1,12) provided better estimates than the GARCH (1,2) model, the combination of ARIMA (1,1,12)-GARCH (1,2) out-performed both of them. A similar study was done by Abdelaal et al. (2012) who developed a short term forecasting GARCH model to explore the volatility feature of

Egyptian inflation rate using 175 monthly observations from January 1996 to July 2010. The number of observations used here are quite few in case of out-of-sample forecasting.

Gikungu et al. (2015) modelled the inflation rates of Kenya and specified a SARIMA $(0,1,0)(0,0,1)_4$. Akaike information criterion, prediction tests including RMSE, MAE and MAPE were used in the model selection. The Jarque-Bera normality test provided evidence in support of normally distributed residuals. The PACF and ACF also showed white noise and homoscedastic residuals. The 8 quarter out-of sample forecasts for 2014 and 2015 provided fluctuating results with an increasing trend towards the end of 2015.

For a country like Turkey with a high dominant fisheries industry that has had a steady increase in production throughout the years, there is a high possibility of this industry to have an effect on the inflation of that country. SAYGI and EMİROĞLU (2014) found out that Fisheries export explains 66% of the Turkish inflation. He carried out a study to explain the factors that affect the inflation rate using multiple regression model and considered 27 years of time series data.

Fritzer et al. (2002) studied the Australian inflation and concluded that the univariate models outperform multivariate models. Junttila (2001), also agrees with the fact that ARIMA models perform better in terms of forecasting than other time series models. There is evidence from the literature that the causality of inflation with respect to a certain micro economic variable depends on the structure of the analysed economy.

Aron et al. (2015) points out the demonstration by Aron and Muellbauer (2013) that multivariate approaches perform better than the univariate models in differences as earlier concluded by Stock and Watson (1999, 2001). This superiority does not only hold for emerging markets like Uganda but also well-established ones like USA. A stable cointegration between inflation rate and its chief components would improve forecasts.

# 3 METHODOLOGY

In this section, the models to be compared in this research work are discussed. The ARIMA, SARIMA and VECM models are introduced. The section also discusses how to obtain forecasts from the models, the different tests that will be used and how the forecast accuracy will be used to compare the models.

## 3.1 ARIMA

The dynamic structure of the data could require high order *AR* or *MA* models; this turns out to be bulky because of the number of parameters. Box et al. (2011) propose an autoregressive moving-average (*ARMA*) model that reduces the number of parameters by combining both the *AR* and *MA* models. The representation *ARMA*(*p,q*) is used to imply a generalization of AR(*p*) with some minor modification to include the MA(*q*) component. Both p and q are integers and the orders of the *AR* and the *MA* component respectively. Allowing the *AR* polynomial to be stationary produces an extension of the *ARMA* (*p,q*) model known as autoregressive integrated moving average (*ARIMA*(*p,d,q*)) model with d as the number of times a series is differenced to make it stationary (Tsay, 2005).

If $\Delta$ is the difference operator, the time series process $\{y_t\}$ can be described as an *ARIMA*(*p,d,q*) if its d:th difference $\Delta^d y_t$ is *ARMA*(*p,q*).

*ARMA*(*p,q*) is represented as:

$$y_t = \emptyset_0 + \sum_{i=1}^{p} \emptyset_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i};$$

where $\{\varepsilon_t\}$ is a white noise series with zero mean and variance $\sigma^2$ denoted as $WN(0,\sigma^2)$.

The lag operator is defined as $L^k y_t = y_{t-k}$ so $\Delta^d y_t = (1-L)^d y_t$. Hence we define both the autoregressive-*AR*(*p*) and moving average-*MA*(*q*) operators using corresponding order standard autoregressive and moving average polynomials $\emptyset$ *and* $\theta$ in L as follows:

$$\emptyset(L) = 1 - \emptyset_1 L - \emptyset_2 L^2 - \cdots - \emptyset_{p-1} L^{p-1} - \emptyset_p L^p$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_{q-1} L^{q-1} - \theta_q L^q$$

We note that $\emptyset(L) \neq 0 \ for \ |\emptyset| < 1$.

Therefore we will generally write the *ARIMA*(*p,d,q*) model as:

$$\emptyset(L)(1-L)^d y_t = \theta(L)\varepsilon_t; \{\varepsilon_t\} \sim WN(0,\sigma^2)$$

The time series process $\{y_t\}$ is stationary if and only if $d = 0$; this reduces the $ARIMA(p,d,q)$ to $ARMA(p,q)$.

## 3.1.1 ARIMA model selection

Akaike Information Criterion (AIC or AICc) or the Bayesian Information Criterion (BIC) that were developed by Akaike (1974) and Schwarz (1978) will be used to select the final model. The AICc has a small sample size correction for the AIC and also converges to AIC in large samples Hurvich and Tsai (1989). AIC and BIC are penalty statistic function used to measure goodness of fit of an estimated statistical model. Several competing models are developed and ranked according to the AIC, AICc or BIC and the one with the lowest information criterion value is chosen as the best.

The information criteria idea is based on the extent to which the fitted values of the model approximate the true values. The penalty aspect discourages over fitting of the models so penalty increases with the number of estimated parameters. The AIC, AICc and BIC are computed as follows:

$$AIC = 2k - 2\log(L) = 2k - n log\left(\frac{RSS}{n}\right)$$

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

$$BIC = log(\sigma_e^2) + \frac{k}{n} log(n)$$

Where

$k$-number of parameters in the statistical model

$RSS$-residual sum of squares for the estimated model

$n$- the number of observations

$\sigma_e^2$-the variance of the residuals

### 3.1.2 Forecasting using ARIMA model

The next step of model building is forecasting using the model that passes the diagnostic tests for example serial correlation and normality. Forecasting involves obtaining information about unobserved outcomes of events.

Considering a difference stationary series $\{y_t\}$ of order $d$, forecasts of an $ARIMA(p,d,q)$ are obtained the same way as those of an $ARMA(p,q)$. Denoting the forecast origin and available information by t and $Y_t$ respectively, the h multistep(s) ahead forecast can be computed recursively as below:

$$\hat{y}_t(h) = E(y_{t+h}|Y_t) = \emptyset_0 + \sum_{i=1}^{p} \emptyset_i \hat{y}_t(h-i) - \sum_{i=1}^{q} \theta_i \hat{\varepsilon}_t(h-i)$$

where: $\hat{\varepsilon}_{t+h-1} = \begin{cases} \varepsilon_{t+h-i} \ if \ h-i \leq 0 \\ 0 \ if \ h-i > 0 \end{cases}$

And the error associated to this forecast is $\quad \hat{\varepsilon}_t(h) = y_{t+h} - \hat{y}_t(h)$

The h-step ahead forecast error variance FEV(h) for $\{y_t\}$ can be obtained after expressing the $ARMA(p,q)$ model weighted sum of disturbances-$\varepsilon_t$

$$y_t = \varepsilon_t + \pi_1 \varepsilon_{t-1} + \cdots + \pi_k \varepsilon_{t-k}$$

where the weights $\pi$ are functions of model parameters $\emptyset \ and \ \theta$. So the h-step ahead forecast error variance $FEV(h) = (1 + \pi_1^2 + \pi_2^2 + \cdots + \pi_{h-1}^2)\sigma^2$.

This gives us the 95% h-step(s) ahead forecast confidence interval under the normality assumption, as:

$$\hat{y}_{t+h} \pm 1.96\sqrt{FEV(h)}$$

## 3.2 SARIMA model

The Box-Jenkins ARIMA model is generalised into a Seasonal Autoregressive Integrated Moving Average (SARIMA) model that accounts for both seasonal and non-seasonal characterized data. The SARIMA model is derived from the ARIMA model described above and also uses information on past observations and past errors of the series.

Since the ARIMA model is inefficient for those series with both seasonal and non-seasonal behaviour for example in terms of wrong order selection, the SARIMA model is preferred when any seasonal behaviour is suspected in the series. The SARIMA model also sometimes referred to as the Multiplicative Seasonal Autoregressive Integrated Moving Average model, is denoted as *ARIMA(p,d,q) (P,D,Q)*s.

The corresponding lag form of the model is:

$$\emptyset(L)\varphi(L^S)(1-L)^d(1-L^S)^D y_t = \theta(L)\vartheta(L^S)\varepsilon_t$$

This model includes the following AR and MA characteristic polynomials in L of order *p* and *q* respectively:

$$\emptyset(L) = 1 - \emptyset_1 L - \emptyset_2 L^2 - \cdots - \emptyset_{p-1}L^{p-1} - \emptyset_p L^p$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_{q-1}L^{q-1} - \theta_q L^q$$

Also Seasonal polynomial functions of order *P* and *Q* respectively as represented below:

$$\varphi(L^S) = 1 - \varphi_1 L^S - \varphi_2 L^{2S} - \cdots - \emptyset_{P-1}L^{(P-1)S} - \varphi_P L^{PS}$$

$$\vartheta(L^S) = 1 - \vartheta_1 L^S - \vartheta_2 L^{2S} - \cdots - \vartheta_{Q-1}L^{(Q-1)S} - \vartheta_Q L^{QS}$$

Where: $\{y_t\}$ - the observable time series

$\{\varepsilon_t\}$ -white noise series

*p,d,q* – order of non-seasonal AR, differencing and non-seasonal MA respectively

*P,D,Q*- order of seasonal AR, differencing and seasonal MA respectively

L-lag operator $L^k y_t = y_{t-k}$

*S*-seasonal order for example *S=12* for monthly data

## 3.2.1 SARIMA model Selection

The first step towards the development of the SARIMA model is to examine whether the series satisfy the stationarity condition; this implies time invariant mean, variance as well as co-variance. The HEGY test which will be explained later on in section 3.6.1 is used to check for stationarity of the series.

Looking at the patterns of the ACF and PACF is helpful in determining the orders *p,q,P and Q*. These use the information about internal correlation between time series observations at different times apart to provide an idea about the seasonal and non-seasonal lags. Both the ACF and PACF have spikes and cut off at lag *k* and lag *ks* at the non-seasonal and seasonal levels respectively. The order of the model is given by the number of significant spikes. **Table 1** summarizes the behavior of ACF and PACF; it was adopted from AIDOO (2011) who also adopted it from Shumway and Stoffer (2006).

*Table 1 Behaviour of ACF and PACF for seasonal and Non-seasonal ARMA(p,q)*

|  |  | **AR(p)** | **MA(q)** | **ARMA(p,q)** |
|---|---|---|---|---|
| **Non-seasonal ARMA(p,q)** | **ACF** | tails off at lag k k=1,2,3,... | cuts off after lag q | Tails off |
|  | **PACF** | cuts off after lag p | Tails off at lags k k=1,2,3,... | Tails off |
|  |  | **AR(P)s** | **MA(Q)s** | **ARMA(P,Q)s** |
| **pure-seasonal ARMA(p,q)** | **ACF** | tails off at lag ks k=1,2,3,... | cuts off after lag Qs | Tails off at ks |
|  | **PACF** | cuts off after lag Ps | Tails off at lags ks k=1,2,3,... | Tails off at ks |

The ACF and PACF could result into several different models whose parameters are estimated using the Maximum Likelihood method. The model with the minimum value of AIC and BIC selection criterion as defined in earlier **section 3.1.1** is chosen as the most appropriate model. The last step in model selection is residual diagnostic checking and if the model passes these diagnostic checks, then it can be used to forecast.

### 3.2.2 Forecasting using the SARIMA model

Finally, simple SARIMA model like $SARIMA(0,1,1)$ $(1,0,1)_{12}$ will be used to demonstrate how forecasts are obtained from the selected SARIMA model. Cryer and Chan (2008) demonstrated these steps below:

$$y_t - y_{t-1} = \emptyset(y_{t-12} - y_{t-13}) + \varepsilon_t - \theta\varepsilon_{t-1} - \vartheta\varepsilon_{t-12} + \theta\vartheta\varepsilon_{t-13} \tag{3.2a}$$

Equation 3.2a gives the corresponding one step ahead forecast and the two steps ahead forecast as elaborated in equations 3.2b and 3.2c respectively.

$$\hat{y}_{t+1} = y_t + \emptyset(y_{t-11} - y_{t-12}) - \theta\varepsilon_t - \vartheta\varepsilon_{t-11} + \theta\vartheta\varepsilon_{t-12} \tag{3.2b}$$

$$\hat{y}_{t+2} = \hat{y}_{t+1} + \emptyset(y_{t-10} - y_{t-11}) - \vartheta\varepsilon_{t-10} + \theta\vartheta\varepsilon_{t-11} \tag{3.2c}$$

This pattern goes on, the residual terms $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{13}$ will be included in the first thirteen forecasts after which the AR part of the model takes over and produces the $l > 13$ steps ahead forecasts in equation 3.2d below.

$$\hat{y}_{t+l} = \hat{y}_{t+l-1} + \emptyset y_{t+l-12} - \emptyset y_{t+l-13} \tag{3.2d}$$

### 3.3 VECM model

The VAR model can be used to study the relationship between inflation and other factors that are suspected to affect it. This is an extension of the univariate auto regression to a vector of time series variables so a $VAR(p)$ is obtained when the number of lags in each univariate equation is $p$. An N-variate and $p$th order vector autoregressive model with time series $Y_t = \{y_{1t}, y_{2t}, \dots, y_{Nt}\}$ is an Nx1 matrix and can be written as;

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t$$

where the time $t = 1, 2, \dots, n$

$\quad A_i - (N \times N)$ Coefficient matrix for $i = 1, \dots, p$

$\quad \varepsilon_t$- iid N-dimensional multivariate normal distribution $N(0, \Sigma)$. So it is an Nx1 matrix

The VAR $(p)$ model defined above assumes that the mean and variance of the error term are time invariant.

The VAR($p$) model defined above can be transformed into a vector error correction model below:

$$\Delta Y_t = \pi Y_{t-p} + \theta_1 \Delta Y_{t-1} + \cdots + \theta_{p-1} \Delta Y_{t-p+1} + u_t$$

Where: $\theta_i = -(I - A_1 - \ldots - A_i), i = 1, \ldots, p-1$

$\pi = \alpha \beta^T = -(I - A_1 - \ldots - A_p).$

The VECM is appropriate when the variables are co-integrated. Taking the cointegration rank as r, the dimensions of $\alpha$ and $\beta$ is $N \times r$. $\alpha$ is the laoding matrix and $\beta$ contains the coefficients of the co-integration relationship.

## 3.5 Forecast accuracy comparison

The main purpose of estimating the time series models in this research is to use the selected model in prediction of the future value of inflation that can be for decision making. The model with the minimum forecast errors compared to the others is said to be better suggesting that it also has higher accuracy. The research will examine the accuracy of each selected model for both in-sample and out-of-sample forecast. When it comes to comparison between in-sample and out-of-sample model comparison, the one with fewer out-of-sample forecast errors is picked over its counterpart.

The out-of-sample forecast accuracy of the estimated models is obtained by using the remaining 25% of the data after using the 75% to develop the model. Forecast accuracy measures such as the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) will be used in this research and the best model is taken to be the one with a minimum MAE or RMSE.

$$MAE = \frac{1}{T} \sum_{t=1}^{T} |e_t|$$

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (e_t)^2}$$

Where the difference between the actual observation $y_t$ and the forecasted value $\hat{y}_t$ is the forecast error $e_t = \hat{y}_t - y_t$. T is the sample size.

Another test that will be used to compare the models is the Diebold-Mariano (DM) test that was suggested by Diebold and Mariano (1995). It checks for the existence of significant differences between the forecasting accuracy of two models. The DM test has the null hypothesis of no difference between the forecast accuracy of the two models.

Suppose we have two competing forecasts $y_{(i)t+h/t}$ from two models i=1,2, the corresponding forecast errors can be computed as $\varepsilon_{(i)t+h/t} = y_{t+h} - y_{(i)t+h/t}$. The h-steps forecasts are computed for $t = t_1, \dots, T$ producing a series of forecast errors $\{\varepsilon_{(i)t+h/t}\}_{t_1}^{T}$ which will also be serially correlated because of the overlapping data used to compute the forecasts.

The accuracy of each forecast is measured using a loss function $L(y_{t+h}, y_{(i)t+h/t}) = L(\varepsilon_{(i)t+h/t})$ which is in most cases taken as the squared errors or Absolute errors. The Diebold-Mariano test with the null hypothesis of equal forecast accuracy between the models has the following loss function and test statistic:

$$loss\ funtion\ d_t = L(\varepsilon_{(1)t+h/t}) - L(\varepsilon_{(2)t+h/t})$$

$$null\ hypothesis\ H_0 : E[L(\varepsilon_{(1)t+h/t})] = E[L(\varepsilon_{(2)t+h/t})]$$

$$alternative\ hypothesis\ H_0 : E[L(\varepsilon_{(1)t+h/t})] \neq E[L(\varepsilon_{(2)t+h/t})]$$

$$DM\ test\ statistic : S = \frac{\bar{d}}{(\widehat{avar}(\bar{d}))^{1/2}} = \frac{\bar{d}}{(\widehat{LRV}_{\bar{d}}/T)^{1/2}}$$

where $\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$ and $LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j$, $\gamma_j = cov(d_t, d_{t-j})$

$\widehat{LRV}_{\bar{d}}$ is a consistent estimate of the long asymptotic variance of $\sqrt{T}\bar{d}$, it is also used in the test statistic because of the serially correlated sample of loss differentials for $h > 1$. The DM test statistic S approximates a standard normal distribution $S \sim N(0,1)$ hence we reject the null hypothesis at 5% level of confidence if $|S| > 1.96$.

## 3.6 Other Tests to be used

## 3.6.1 Seasonal Unit Root test

As one of the objectives of this research, seasonal integration will be examined and later a SARIMA model for the seasonally adjusted data will be constructed. Data can be seasonal in nature but if it is not seasonal, then the dynamics of the estimated model could be distorted when the seasonally adjusted data is used (Wallis, 1974). This research will consider the Hylleberg-Engle-Granger-Yoo (HEGY) test suggested by Hylleberg et al. (1990) to test for the presence of seasonal unit root in the observable series.

The HEGY test determines the class of seasonal processes (deterministic, stationary or integrated) that generates seasonality. A series $y_t$ is considered an integrated seasonal process if it has a seasonal unit root as well as a peak at any seasonal frequency in its spectrum other than the zero frequency and we can say it is integrated of order d at frequency $\theta$ implying $y_t \sim I_\theta(d)$. As illustrated by Ronderos (2015), the method used to perform the test on monthly data is as follows;

The equation below is estimated by OLS.

$$\Phi(L)_{y_{8,t}} = \mu_t + \beta_1 y_{1,t-1} + \beta_2 y_{2,t-1} + \beta_3 y_{3,t-1} + \beta_4 y_{3,t-2} + \beta_5 y_{4,t-1} + \beta_6 y_{4,t-2} + \beta_7 y_{5,t-1}$$
$$+ \beta_8 y_{5,t-2} + \beta_9 y_{6,t-1} + \beta_{10} y_{6,t-2} + \beta_{11} y_{7,t-1} + \beta_{12} y_{7,t-2} + e_t$$

where
$$y_{8,t} = (1 - L^{12})y_t$$

$$y_{1,t} = (1 + L)(1 + L^2)(1 + L^4 + L^8)y_t$$

$$y_{2,t} = -(1 - L)(1 + L^2)(1 + L^4 + L^8)y_t$$

$$y_{3,t} = -(1 - L^2)(1 + L^4 + L^8)y_t$$

$$y_{4,t} = -(1 - L^4)(1 - \sqrt{3}L + L^2)(1 + L^2 + L^4)y_t$$

$$y_{5,t} = -(1 - L^4)(1 + \sqrt{3}L + L^2)(1 + L^2 + L^4)y_t$$

$$y_{6,t} = -(1 - L^4)(1 - L^2 + L^4)(1 - L + L^2)y_t$$

$$y_{7,t} = -(1 - L^4)(1 - L^2 + L^4)(1 + L + L^2)y_t$$

If $\beta_1 = 0, \beta_2 = 0$ then the null hypothesis of presence of non-seasonal unit root cannot be rejected. Also if $\beta_3 = \beta_4 = 0$, the null hypothesis of presence of non-seasonal unit root cannot be rejected and these can be tested using a joint F-test which has a nonstandard distribution. Finally, considering the rest of the coefficients $\beta_i = \beta_{i+1} = 0$ for $i = 5,7,9,11$, then the null hypothesis of the presence of seasonal unit root cannot be rejected and these hypotheses can also be jointly tested using F-test which has a nonstandard distribution.

### 3.6.2 Stationarity Test

A stationary series behaves differently from a nonstationary series. A problem like spurious regression can result from modelling a nonstationary time series. A stationary process is also defined as one with no unit root; its mean and variance are constant. Differencing a series can make it stationary. Differencing once leads to the 1$^{st}$ differences values of the series, then the second time differencing results into the 2$^{nd}$ differenced values of the series and so on. Most economic series do not need to be differenced more than twice to become stationary.

Data that does not need any differencing is taken to be integrated of order 0 whereas the data that becomes stationary after the 1$^{st}$ differencing is said to be integrated of order 1. The research will employ both the Augmented Dickey Fuller test suggested by Dickey and Fuller (1979) and the Phillips-Perron test recommended by Phillips and Perron (1988) to test for stationarity of the variables.

The regression model estimated which these tests are based on is:

$$\Delta y_t = \beta_0 + \gamma t + \theta y_{t-1} + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \cdots + \beta_k \Delta y_{t-k}$$

$$where: \Delta y_t = y_t - y_{t-1}, \beta_0 \ is \ a \ constant, \ \gamma \ is \ the \ coefficient \ of \ trend$$

$k$ is the number of lags to include and normally it is about 3 lags. The number of lags under the ADF test can be computed using:

$$lag_{max} = \left[12 \left(\frac{T}{100}\right)^{1/4}\right] \text{ with T as the sample size.}$$

A series that need differencing has the coefficient $\theta$ of zero and $\theta < 0$ if it is already stationary. This gives the null hypothesis $\theta = 0$ and alternative hypothesis $\theta < 0$.

The t-statistic is given by: $t_{\theta=0} = \frac{\hat{\theta}}{SE(\hat{\theta})}$ if no trend is considered and it is compared to the relevant critical values of the Dickey-Fuller distribution.

On the other hand, the Phillips-Perron test also builds on the same principles as the Augmented Dickey Fuller test but it ignores any serial correlation in the regression used for the test and adds a correction to the t-test statistic and produces the following test statistic:

$$C = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{\frac{1}{2}} t_{\theta=0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right)\left(\frac{T.SE(\hat{\theta})}{\hat{\sigma}^2}\right)$$

where $\hat{\sigma}^2 \ and \ \hat{\lambda}^2$ are the consistent estimates of the variance parameters

$$\sigma^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E[u_t^2]$$

$$\lambda^2 = \lim_{T\to\infty} \sum_{t=1}^{T} E\left[T^{-1}\left(\sum_{t=1}^{T} u_t\right)^2\right]$$

The $Z_t$ statistic approximates the same asymptotic distribution as the ADF $t_{\theta=0}$ statistic. The Phillips-Perron test is robust to general forms of heteroscedasticity in the error term $u_t$. This study will choose the results of the Phillips-Perron test over those of the ADF test in case of any contradicting results.

### 3.6.3 Test for co-integration

The Johansen and Juselius test of cointegration will be used in the study to test for any co-integration relationship between the series. The test is divided into two variations; the first one is the maximum Eigen value test with the null hypothesis of r cointegrating relations against the alternative hypothesis of r+1 cointegration relations for $r = 0,1,2,...,n-1$. The maximum eigen value test statistic is;

$$LR_{max}(r/n+1) = -Tlog(1-\hat{\lambda})$$

where $\lambda \ and \ T$ represent the Maximum Eigen value and sample size respectively.

The second option of the Johansen and Juselius cointegration test is known as the trace test and it has the null hypothesis of r cointegrating relations with the test statistic as;

$$LR_{tr}(r/n) = -T \sum_{i=r+1}^{n} log(1-\hat{\lambda}_i)$$

Where n is the number of variables in the system and $r = 0,1,2,...,n-1$. The results of the trace test should be preferred in cases where the two tests produce different results (Johansen and Juselius, 1990).

# 4 DATA AND EMPIRICAL RESULTS

This section describes the properties of the inflation rate series and the other two independent series which are later used in the building of the models under consideration. At the later stage, a comparison of the obtained forecasts by the models is done. This study utilises the E-views, R-software and the already developed r packages and codes.

## 4.1 Data

The research considered data on three variables namely; inflation rates, the exchange rates and the world coffee prices. 207 Monthly data observations of these three variables from June-1998 to September-2015 was employed. The out-of-sample forecasts were done by considering the first 155 observations from June-1998 to April-2011 for model development and the other 52 observations to compute the out of sample model forecast accuracy. Data on the two domestic variables (inflation rates and the exchange rates) was obtained from the Bank of Uganda website and the external variable (the world coffee prices) was obtained from the World Bank website. The only transformations done were to take the tenth logarithm of Uganda's exchange rates and the world coffee prices were computed as the average of the two traded coffee types, Robusta and Arabica coffee.

## 4.2 Descriptive statistics

**Table 2** presents the descriptive statistics of the five variables. Based on the Jarque-Bera test of normality, none of the variables is normally distributed. The Ugandan inflation rates exhibited negative inflation rates right after the currency reforms that occurred in the 1980's. The standard deviations of inflation rates, and the world coffee prices suggest some the volatility of these variables.

*Table 2 Descriptive statistics of the variables*

|                      | INFLATION | Log(EXCHANGE_RATE) | COFFEE |
|----------------------|-----------|--------------------|--------|
| Mean                 | 6.911     | 3.296              | 2.228  |
| Median               | 5.986     | 3.268              | 2.221  |
| Maximum              | 30.477    | 3.564              | 4.602  |
| Minimum              | -5.361    | 3.092              | 0.878  |
| Std. Dev.            | 6.184     | 0.092              | 0.907  |
| Skewness             | 1.141     | 0.903              | 0.417  |
| Kurtosis             | 5.334     | 3.569              | 2.545  |
| Jarque-Bera(P-value) | <0.0005   | <0.0005            | 0.02   |
| Observations         | 207       | 207                | 207    |

The original variables without any differencing adjustments are not stationary. The stationary test in **Table 3** and the time series plots of all the variables in **Figure 1** give a good picture of this. The Phillips-Perron test contradicted with the ADF test for inflation but the Phillips-Perron test results were considered, the exchange rates have an increasing trend for the considered time period.

*Table 3 P-values of stationary tests for original data variables*

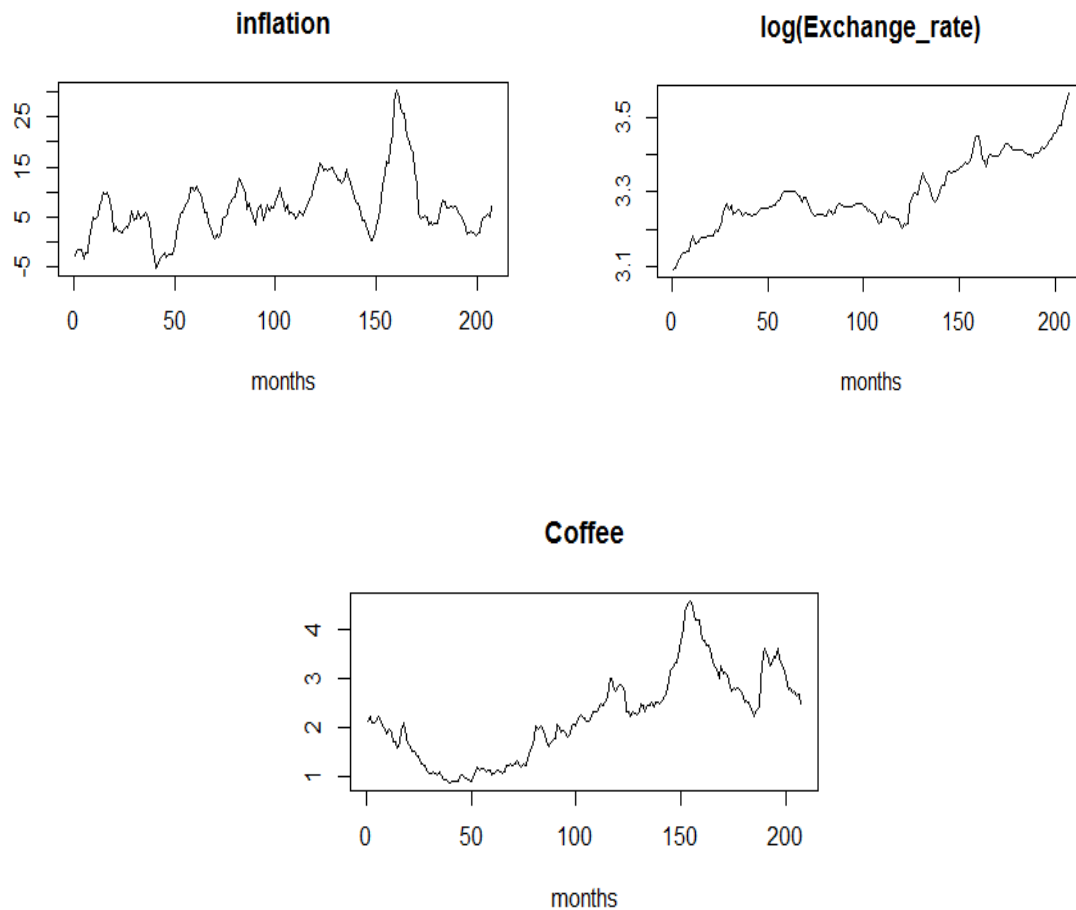|  | **Inflation** | **log(Exchange rates)** | **Coffee** |
|---|---|---|---|
| Phillips-Perron | 0.16 | 0.76 | 0.48 |
| ADF | <0.01 | 0.77 | 0.28 |



*Figure 1 Time series plots of original data variables*

Since the Phillips-Perron test in **Table 3** above suggests that all the series are unit root stationary (*P-value>0.05*), taking the first difference of each series as shown by the stationarity test in **Table 4** and time series plot **Figures 2** makes it stationary.
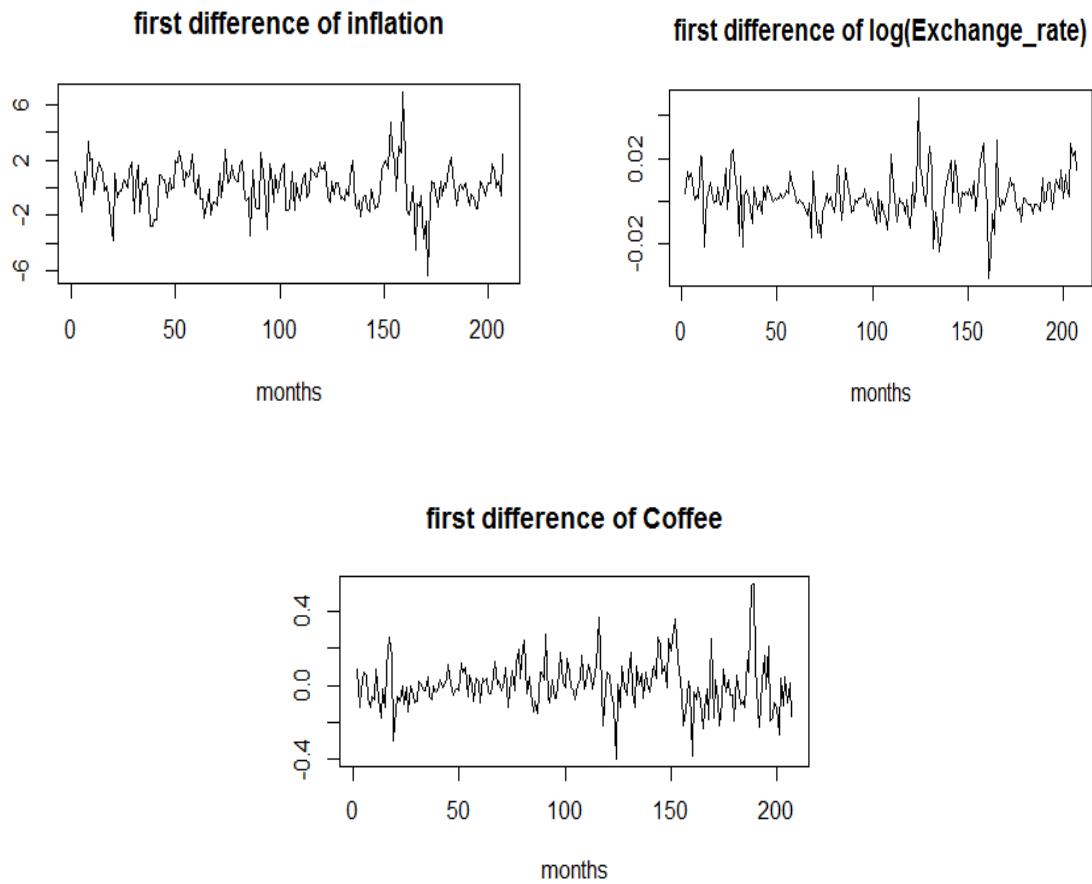






*Figure 2 Time series plots of differenced data variables*

*Table 4 P-values of stationary tests for differenced data variables*

|                 | Inflation | log(Exchange rates) | Coffee |
|-----------------|-----------|---------------------|--------|
| Phillips-Perron | 0.01      | 0.01                | 0.01   |
| ADF             | <0.01     | <0.01               | <0.01  |

## 4.3 Univariate models

## 4.3.1 ARIMA model

This section follows the ARIMA model methodology presented in section 3 to identify the best model.

**ARIMA Model identification**

Since we already know from **Tables 3 and 4** that the inflation rates are not originally stationary but become stationary after the first difference, we can go on and identify an ARIMA model using this information. **Figure 3** shows the acf and pacf plots of the first differenced inflation rates ignoring any possibility of seasonality in the data. Based on the ARIMA model identification in section 3, the PACF plot has a significant spike at the first lag while the ACF plot has 3 significant spikes at the low lags. From this observation, the AR part of the model could have a lag of one while the MA part can possibly have lags 1 or 2. Several models were suggested and their AIC and BIC computed.

The significant spike at lag 12 for both ACF and PACF plots could be as a result of possible seasonality in the series. Models with AR or MA part of lags more than 12 were also included to assess the impact on the 12th significant lags. The models were restricted and lags between 1 and 12 were dropped. An example of the notation for the restricted ARIMA models is *ARIMA*(1,1,[1,12,13]) implying that for the MA part of the ARIMA considered the first lag, the 12th lag and the 13th lag. These models, their information criterion and residual diagnostics are presented in **Table 5.**

Based on the information criterion described in section 3, the AIC and BIC chose *ARIMA(1,1,[1,12])* as the model that fit the data well best. The model with the least BIC also satisfies the requirement of no serial correlation in the residuals and it was considered for forecasting. The suggested models were *ARIMA(1,1,[1,12]), ARIMA(1,1,[1,12,13])* and *ARIMA(2,1,[1,12])* since they have the least values of information criteria. All the three models satisfy both the two requirements of no serial correlation and no ARCH process in the residuals, they were all considered for further analysis and forecasting since the chosen model is not necessarily the one with the best forecast.
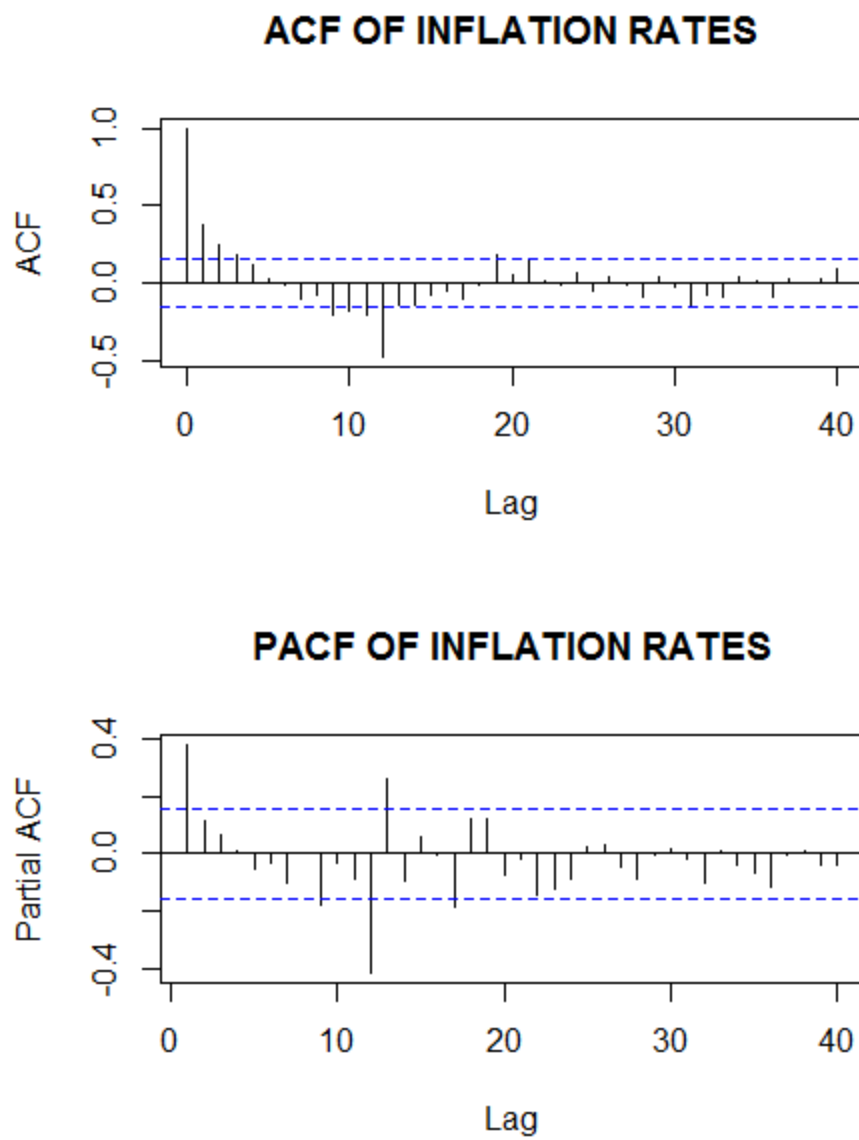
## ACF OF INFLATION RATES



## PACF OF INFLATION RATES



*Figure 3 ACF and PACF plots of differenced inflation rates*

*Table 5 AIC or BIC values of suggested ARIMA models*

| | Information criteria | | Residual diagnostics(p-values) | |
| | | | ARCH-LM | |
| Model | AIC | BIC | test | Ljung-box test(40 lags) |
| --- | --- | --- | --- | --- |
| ARIMA(1,1,0) | 530.557 | 536.644 | 0.093 | 0.000 |
| ARIMA(1,1,1) | 529.641 | 538.772 | 0.029 | 0.000 |
| ARIMA(1,1,2) | 531.641 | 543.815 | 0.028 | 0.000 |
| ARIMA(2,1,0) | 530.315 | 539.445 | 0.048 | 0.000 |
| ARIMA(2,1,1) | 531.641 | 543.815 | 0.028 | 0.000 |
| ARIMA(2,1,2) | 531.024 | 546.241 | 0.045 | 0.000 |
| ARIMA(5,1,1) | 536.715 | 558.019 | 0.031 | 0.000 |
| ARIMA(6,1,1) | 526.674 | 551.022 | 0.133 | 0.000 |
| ARIMA(10,1,1) | 529.079 | 565.600 | 0.192 | 0.001 |
| ARIMA(11,1,1) | 519.812 | 559.293 | 0.075 | 0.000 |
| ARIMA([1,12],1,0) | 491.114 | 530.595 | 0.895 | 0.108 |
| ARIMA([1,12,13],1,0) | 486.557 | 529.074 | 0.633 | 0.159 |
| ARIMA([1,12,13],1,1) | 488.495 | 534.049 | 0.637 | 0.160 |
| ARIMA([1,12,13],1,2) | 490.199 | 538.791 | 0.620 | 0.191 |
| ARIMA([1,12,13,14],1,0) | 486.593 | 532.146 | 0.635 | 0.128 |
| ARIMA([1,12,13,14],1,2) | 490.486 | 542.114 | 0.612 | 0.132 |
| ARIMA([1,12,13,14],1,1) | 488.591 | 537.182 | 0.633 | 0.128 |
| ARIMA(1,1,[1,12]) | **472.610** | **515.127** | 0.523 | 0.813 |
| ARIMA(0,1,[1,12]) | 481.470 | 520.950 | 0.829 | 0.237 |
| ARIMA(1,1,[1,12,13]) | **473.736** | **519.290** | 0.540 | 0.872 |
| ARIMA(1,1,[1,12,13,14]) | 475.718 | 524.309 | 0.525 | 0.869 |
| ARIMA(2,1,[1,12]) | **473.982** | **519.537** | 0.546 | 0.849 |
| ARIMA(2,1,[1,12,13,14]) | 476.896 | 528.524 | 0.600 | 0.814 |
| ARIMA(2,1,[1,12,13]) | 475.693 | 524.285 | 0.520 | 0.876 |

## ARIMA Model estimation and evaluation

**Tables 6** gives the estimated parameters of the models followed by the residual diagnostics for the four models. For All the three models their residuals have no serial correlation based on the Ljung-Box test (40 lags) results at the bottom of **Table 6** and the residual plots in **Figure 4** also support the no serial correlation test result. Still from **table 6**, the ARCH-LM test result show that all the models have no ARCH effect in their residuals. We went on with forecasting since the assumption of white noise was fulfilled by all the suggested models.
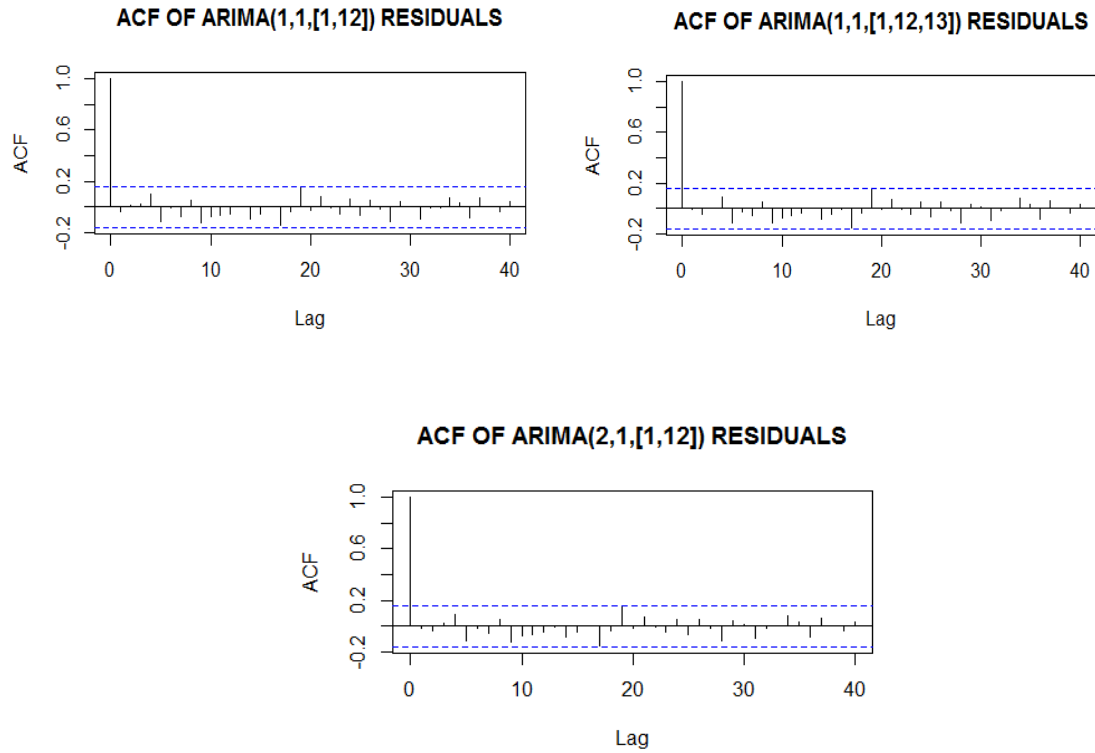
*Figure 4 ACF plots of ARIMA(1,1,[1,12]), ARIMA(1,1,[1,12,13]) and ARIMA(2,1,[1,12]) residuals*

*Table 6 Parameter estimates of selected ARIMA models*

| Parameter | ARIMA(1,1,[1,12]) | ARIMA(1,1,[1,12,13]) | ARIMA(2,1,[1,12]) |
|---|---|---|---|
| AR(1) | 0.3478 | 0.5505 | 0.3055 |
| *std.error* | *0.1013* | *0.2005* | *0.1099* |
| AR(2) | | | 0.0691 |
| *std.error* | | | *0.0868* |
| MA(1) | -0.0001 | -0.2286 | 0.0244 |
| *std.error* | *0.0824* | *0.2334* | *0.0828* |
| MA(12) | -0.7434 | -0.075 | -0.7489 |
| *std.error* | *0.0737* | *0.076* | *0.0754* |
| MA(13) | | 0.1934 | |
| *std.error* | | *0.1879* | |
| | **Residual diagnostics P-values** | | |
| ARCH-LM test | 0.523 | 0.540 | 0.546 |
| Ljung-Box Test | 0.813 | 0.872 | 0.849 |

## 4.3.2 SARIMA model

The easiest way to check whether there is seasonality in the inflation rates is to plot the monthly averages and look at the pattern of the averages. **Figure 5** is a plot of the monthly averages of the inflation rates and it is evident that the inflation rates are on average high during certain months of the year and low during the others implying a sign of possible seasonality. December shows the least inflation rates on average whereas the months of March, April and May have the highest average inflation rates. The HEGY test of seasonality was also carried out using Eviews and the results are shown in **Table 7.** The results of the HEGY test imply existence non-seasonal unit root but no seasonal unit root so we can reject the null hypothesis of seasonality in Uganda's inflation rates data.
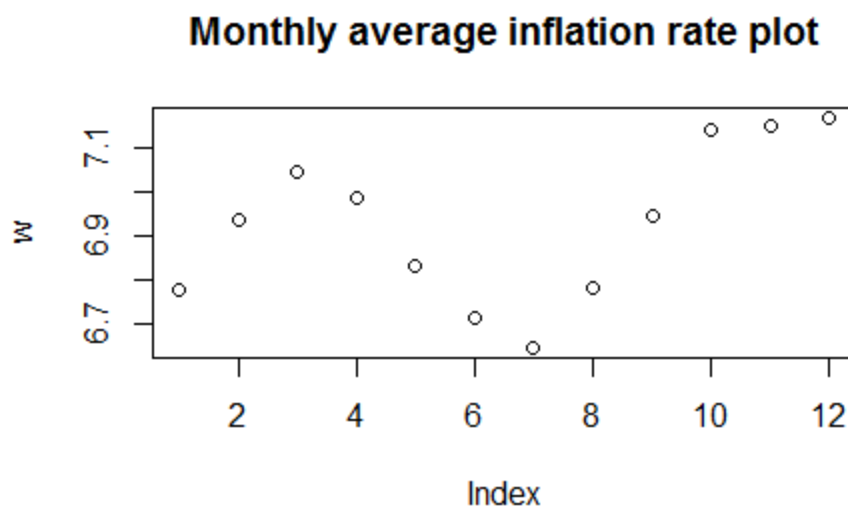


*Figure 5Monthly average inflation rates plot, starting from the month of June*

*Table 7 HEGY test of seasonality results*

| Null-hypothesis | P-value |
| --- | --- |
| Non-seasonal unit root | 0.0642 |
| Seasonal unit root(12 months) | 0.0000 |

**SARIMA Model identification**

Identification of the SARIMA model was based on the ACF and PACF plots in **Figure 3** which were also used to identify ARIMA models. The non-seasonal terms of the model are obtained by examining the early lags of the plots. From the early lags of the ACF plot, the MA part of the model can have lags 1 or 2 based on the spikes while the early lags of the PACF suggest a non-seasonal AR part with a possible lag of 1. On the other hand, the Seasonal terms of the SARIMA model are determined by examining the significant spikes around multiples of 12 like 12, 24 and 36 since we have monthly data. The ACF plot in **Figure 3** suggest possible lags of 1, 2 or 3 for the seasonal MA part of the SARIMA model. The seasonal AR part for the SARIMA model can have lags of 1 or 2 based on the significant spikes around the $12^{th}$ spike in the PACF plot.

From the above observation based on the ACF and PACF plots in **Figure 3,** 30 possible SARIMA models were tried and the results of their AIC and BIC statistics presented in **Table 8.** Both the AIC and BIC suggested *SARIMA(1,1,0)(0,0,1)$_{12}$* followed by *SARIMA(1,1,0)(1,0,1)$_{12}$* and *SARIMA(1,1,1)(0,0,1)$_{12}$* and all the three models were considered for further analysis and comparison since they also satisfied the requirement of no serial correlation in the residuals.

*Table 8 AIC and BIC of possible SARIMA models*

| SARIMA model | AIC | BIC | Ljung-Box Test p-value(40 lags) |
|---|---|---|---|
| $(1,1,1)(0,0,0)_{12}$ | 529.641 | 538.752 | 0.000 |
| $(1,1,1)(1,0,1)12$ | 473.162 | 488.347 | 0.862 |
| $(1,1,1)(1,0,2)12$ | 473.716 | 491.937 | 0.920 |
| $(1,1,1)(1,0,0)12$ | 486.509 | 498.657 | 0.163 |
| $(1,1,1)(0,0,1)12$ | **471.832** | **483.980** | 0.864 |
| $(1,1,1)(2,0,0)12$ | 482.323 | 497.507 | 0.236 |
| $(1,1,1)(0,0,2)12$ | 473.231 | 488.416 | 0.861 |
| $(1,1,1)(2,0,1)12$ | 474.924 | 493.146 | 0.880 |
| $(1,1,1)(2,0,2)12$ | 475.717 | 496.975 | 0.920 |
| $(1,1,2)(0,0,0)12$ | 531.641 | 543.789 | 0.000 |
| $(1,1,2)(1,0,1)12$ | 474.956 | 538.752 | 0.877 |
| $(1,1,2)(1,0,2)12$ | 475.537 | 496.796 | 0.928 |
| $(1,1,2)(1,0,0)12$ | 488.120 | 503.385 | 0.191 |
| $(1,1,2)(0,0,1)12$ | 473.683 | 488.867 | 0.872 |
| $(1,1,2)(2,0,0)12$ | 484.025 | 502.247 | 0.268 |
| $(1,1,2)(0,0,2)12$ | 475.030 | 493.252 | 0.875 |
| $(1,1,2)(2,0,1)12$ | 476.714 | 497.972 | 0.894 |
| $(2,1,1)(0,0,0)12$ | 531.641 | 543.789 | 0.000 |
| $(2,1,1)(1,0,1)12$ | 474.022 | 492.243 | 0.846 |
| $(2,1,1)(1,0,2)12$ | 474.343 | 495.601 | 0.921 |
| $(2,1,1)(1,0,0)12$ | 487.470 | 502.655 | 0.158 |
| $(2,1,1)(0,0,1)12$ | 473.731 | 488.916 | 0.870 |
| $(2,1,1)(2,0,0)12$ | 484.185 | 502.406 | 0.250 |
| $(2,1,1)(0,0,2)12$ | 474.143 | 492.365 | 0.839 |
| $(2,1,1)(2,0,1)12$ | 476.793 | 501.233 | 0.889 |
| $(2,1,1)(2,0,2)12$ | 477.631 | 501.926 | 0.927 |
| $(1,1,0)(1,0,0)12$ | 484.589 | 493.700 | 0.1645 |
| $(1,1,0)(1,0,1)12$ | **471.532** | **483.679** | 0.851 |
| $(1,1,0)(0,0,1)12$ | **470.610** | **479.721** | 0.813 |

The parameter estimates of *SARIMA(1,1,0)(0,0,1)$_{12}$* , *SARIMA(1,1,0)(1,0,1)$_{12}$* and *SARIMA(1,1,1)(0,0,1)$_{12}$* and their residual diagnostics are presented in **Table 9 and 10** respectively. The Ljung-Box test in **Table 10** provides support for no serial correlation in the residuals for all the three models. The conclusion of the Ljung-Box test in tables is also supported ACF plots of the residuals in **figure 6. Table 10** also provides evidence of no ARCH

process in the model residuals. Since all the four models satisfy these two requirements, they were all used in the forecasting stage and comparison of forecasts.

*Table 9 Parameter estimates of selected SARIMA models*

| Parameter | $(1,1,1)(0,0,1)_{12}$ | $(1,1,0)(1,0,1)_{12}$ | $(1,1,0)(0,0,1)_{12}$ |
|---|---|---|---|
| AR(1) | 0.5511 | 0.3362 | 0.3477 |
| *std.error* | *0.2082* | *0.0774* | *0.0761* |
| MA(1) | -0.2339 | | |
| *std.error* | *0.2441* | | |
| SAR(1) | | -0.1312 | |
| *std.error* | | *0.1251* | |
| SMA(1) | -0.7457 | -0.665 | -0.7434 |
| *std.error* | *0.0743* | *0.1131* | *0.0728* |

*Table 10 Residual Diagnostics P-values of selected SARIMA models*

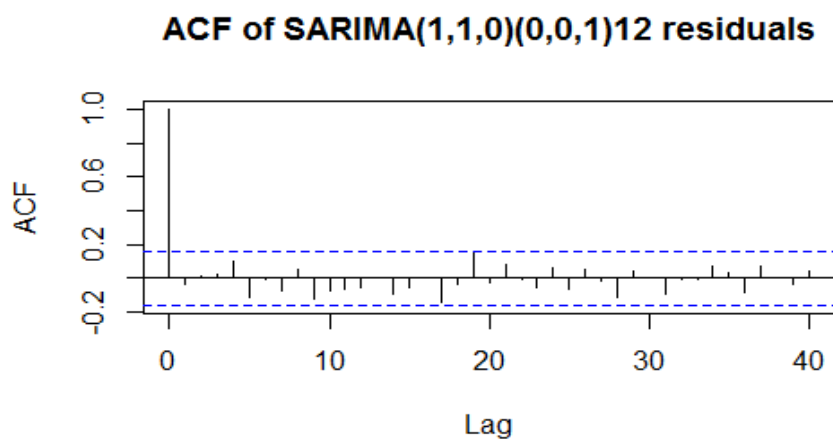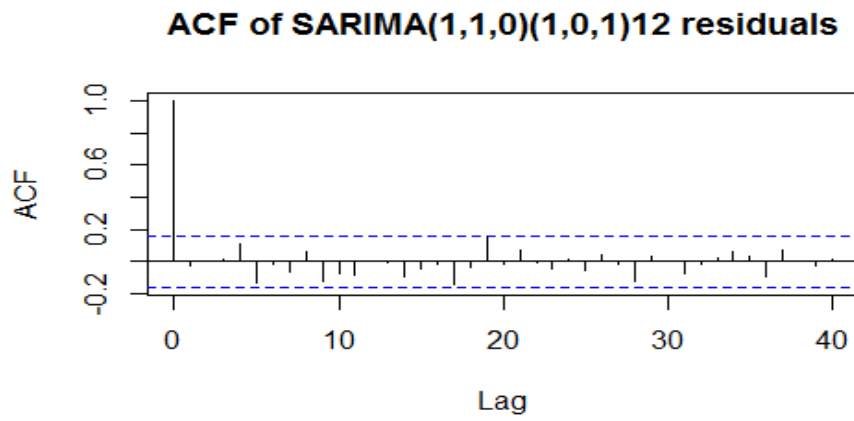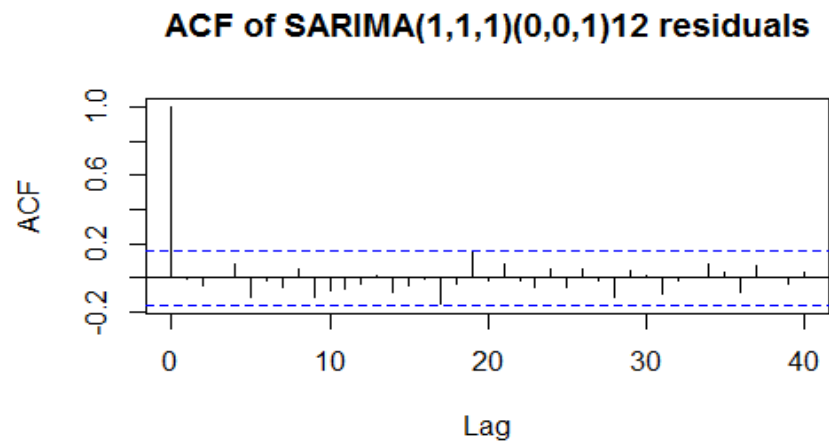| | $(1,1,1)(0,0,1)_{12}$ | $(1,1,0)(1,0,1)_{12}$ | $(1,1,0)(0,0,1)_{12}$ |
|---|---|---|---|
| Log-likelihood | -231.920 | -231.770 | -232.300 |
| ARCH-LM test | 0.509 | 0.528 | 0.523 |
| Ljung-Box Test | 0.863 | 0.850 | 0.813 |

*Figure 6 ACF plots of the SARIMA model residuals*

## 4.4 VECM model

This section follows the VECM model methodology presented in section 3. The model identification and results, the selection criterion statistic and residual diagnostics are presented below. The VECM model is an extension of VAR that assumes a co-integration relationship between the dependent variable and the independent variable(s), therefore there should be at least one co-integration relationship for the VECM model to be appropriate. Before the cointegration test was done, the number of lags for the unrestricted VAR model using stationary (differenced once) data were determined using AIC information criteria and BIC information criteria (**see Table 11**). The lags suggested by both the AIC and AIC were used to compute the cointegration tests and the results are presented in **Table 12.** The Eigen value Johansen's test of cointegration suggests no cointegration relationships while the trace version of the Johansen's cointegration test suggests at most one cointegration relationships as pointed out in section 3.6.3, the results of the trace test were chosen over those of the Eigen value Johannsen's test of cointegration.

*Table 11 Unrestricted VAR model lag selection*

| Information criteria | AIC | BIC | HQ |
|---|---|---|---|
| Number of lags | 2 | 2 | 2 |

*Table 12 Johansens's Trace test of cointegration results*

| | Trace test | | Maximum Eigen value test | |
|---|---|---|---|---|
| Number of relations | Test statistic | 5% critical values | Test statistic | 5% critical values |
| r<=2 | 4.43 | 9.24 | 4.43 | 9.24 |
| r<=1 | **18.05** | 19.96 | 13.63 | 15.67 |
| r=0 | 38.05 | 53.12 | **20.00** | 22.00 |

**VECM Model identification**

Considering one cointegration relationship obtained in **Table 12**. The 2 lags of the VAR model correspond to 1 lag of the VECM model after transformation so as shown in **Table 13**, this is used as the starting point for lags in the VECM model, lags are increased by one until there is no serial correlation in the model residuals. In Table 13, Serial correlation disappears at lag 2

and still does not exist at a higher lag of 3. The parameter estimates of the selected VECM (2) model are in **Table 14** with their standard errors. The residual diagnostics in Table **15** show that the model residuals have no serial correlation and no ARCH process. This VECM model is considered for forecast accuracy comparison.

*Table 13: VECM information criterion and serial correlation test*

| Number of lags | AIC | BIC | Ljung-Box test P-value |
|---|---|---|---|
| 1 | -2025.774 | -1974.26 | 0.0493 |
| 2 | -2008.91 | -1930.29 | 0.0673 |
| 3 | -1985.34 | -1879.74 | 0.0916 |

*Table 14: VECM(2) parameter estimation and the cointegration relationship matrix*

| Cointegrating vector(estimated by ML) | | | |
|---|---|---|---|
| | Inflation | Exchange Rate | Coffee |
| r1 | 1 | -58.361 | -4.205 |
| | **Parameter estimates** | | |
| | **Inflation** | **Exchange Rate** | **Coffee** |
| ECT1 | -0.0828 | 0.0003 | -0.0062 |
| std.error | 0.0226* | 0.0002 | 0.0018* |
| Intercept | -15.7893 | 0.088 | -1.1756 |
| std.error | 4.3172* | 0.0315 | 0.3402* |
| inflation-1 | 0.3603 | -0.0008 | 0.0101 |
| std.error | 0.0799* | 0.0006 | 0.0063 |
| Exchange rate-1 | 5.5998 | 0.2786 | 1.2675 |
| std.error | 11.5033 | 0.0839* | 0.9065 |
| coffee-1 | 0.2109 | -0.0167 | 0.2298 |
| std.error | 1.0826 | 0.0079* | 0.0853* |
| inflation-2 | 0.1889 | 0.0006 | -0.0039 |
| std.error | 0.0826* | 0.0006 | 0.0065 |
| Exchange rate-2 | -10.8596 | 0.0328 | -1.2401 |
| std.error | 11.3642 | 0.0829 | 0.8955 |
| coffee-2 | -2.0414 | 0.0078 | -0.1124 |
| std.error | 1.0853 | 0.0079 | 0.0855 |

*Table 15: VECM(2) residual diagnostics*

| Test | Portmanteau test | ARCH LM |
|---|---|---|
| p-value | 0.067 | 0.5449 |

## 4.5 Forecast comparison of the models

The major objective of this research is to compare the forecast ability of ARIMA, SARIMA and VECM models when it comes to forecasting inflation rates of Uganda. Comparison of the different models would lead to choosing that one model that provides the better forecasts than the rest hence leading in better decisions by policy makers based on the forecasted inflation rates. This comparison of the suggested models is based on the methodology of the model with the minimum MAE and RMSE elaborated in section 3. The in-sample and out-of-sample MAE plus RMSE of the different models estimated earlier in this section presented in **Table 16.**

First for the in-sample forecast accuracy; the MAE and RMSE suggested *ARIMA(1,1,[1,12,13])* as the best ARIMA model. There was a contradiction when it came to the SARIMA models as MAE *suggested SARIMA(1,1,0)(1,0,1)$_{12}$* as the best SARIMA model which was different from *SARIMA(1,1,1)(0,0,1)$_{12}$* suggested by the RMSE. This contradiction makes the choice of a better model hard based on the in-sample and also insufficient to disagree a study by Junttila (2001) who also concluded that ARIMA models perform better than other time series models. The *VECM(2)* model was out performed by both *SARIMA(1,1,0)(1,0,1)$_{12}$* suggested by MAE and *ARIMA(1,1,[1,12,13])* suggested by the RMSE.

Secondly, for the out-of-sample forecast accuracy still in **Table 16***; ARIMA(2,1,[1,12])* was suggested as the best ARIMA model by both the MAE and RMSE. There was still contradiction as in the in-sample forecast accuracy measure since both MAE and RMSE differently suggest *SARIMA(1,1,1)(0,0,1)$_{12}$* and *SARIMA(1,1,0)(1,0,1)$_{12}$* respectively as the best SARIMA models. Also as in the in-sample forecast accuracy measure, The best univariate models (*ARIMA(2,1,[1,12])* and *SARIMA(1,1,1)(0,0,1)$_{12}$* ) out performed *VECM(2)*.

The Diebold and Marino test was used to examine whether there was any difference in the forecast accuracy (two tailed test with null of models with equal forecast accuracy) between *ARIMA(1,1,[1,12,13]), ARIMA(2,1,[1,12]), SARIMA(1,1,0)(1,0,1)$_{12}$* and the *VECM(2)* model for the in sample forecasts. Based on the results of the Diebold and Marino test in **Table 17**, at 5% significant level, we reject the null hypothesis that forecasts accuracy of *SARIMA(1,1,0)(1,0,1)$_{12}$* is equal to the forecast accuracy of *VECM(2)* and conclude that the *VECM(2)* does not have equal forecast accuracy as the *SARIMA(1,1,0)(1,0,1)$_{12}$*. *ARIMA(1,1,[1,12,13])* was also compared to the *VECM(2)* model and the Diebold and Mariano test does not provide enough evidence to conclude that the forecast accuracy of the two models

is equal. Still from the results of **Table 17,** *VECM(2)* forecast accuracy was not equal to the forecast accuracy of both *SARIMA(1,1,0)(1,0,1)$_{12}$* and *ARIMA(2,1,[1,12])* based on the out-of-sample forecast.

As shown in **Table 18** Another comparison of accuracy using the Diebold Mariano test was carried out to test for the null of equality of forecast accuracy between the best ARIMA and SARIMA models suggested by MAE and RMSE for both in-sample and out-of-sample forecasts. For the in sample forecast, the results suggest that the forecast accuracy of *SARIMA(1,1,0)(1,0,1)$_{12}$* and *ARIMA(1,1,[1,12,13])* is equal. On the other hand, for the out-of-sample forecasts, *SARIMA(1,1,1)(0,0,1)$_{12}$* and *ARIMA(2,1,[1,12])* also have equal forecast accuracy.

**Figures 7, 8, 9 and 10** give a visualised presentation of the Diebold and Mariano test. It is good to note how close the *VECM(2)* out of sample forecasts are to the observed data despite the fact that both RMSE and MAE suggested it as the model with the worst forecast accuracy. It is also hard to clearly observe distinction in the relative forecast performance of the models from the figures especially between the ARIMA and SARIMA models.

*Table 16: One month ahead forecast accuracy of the different models*

| | In-sample | | Out-of-sample | |
|---|---|---|---|---|
| **Model** | **MAE** | **RMSE** | **MAE** | **RMSE** |
| *ARIMA(1,1,[1,12])* | 0.8409 | 1.0561 | 0.7829 | 1.2441 |
| *ARIMA(1,1,[1,12,13])* | 0.8377 | **1.0520** | 0.7706 | 1.2375 |
| *ARIMA(2,1,[1,12])* | 0.8386 | 1.0531 | **0.7696** | 1.2349 |
| *SARIMA(1,1,1)(0,0,1)$_{12}$* | 0.8393 | 1.0530 | 0.7793 | 1.2423 |
| *SARIMA(1,1,0)(1,0,1)$_{12}$* | **0.8362** | 1.0539 | 0.7898 | **1.2191** |
| *SARIMA(1,1,0)(0,0,1)$_{12}$* | 0.8409 | 1.0561 | 0.7828 | 1.2441 |
| *VECM(2)* | 2.3717 | 2.9859 | 1.1663 | 1.5803 |

*Table 17: Diebold-Marino equal accuracy test results of SARIMA and ARIMA models Verses VECM model (forecast horizon=1)*

| | Models | DM-statistic | P-value |
|---|---|---|---|
| | SARIMA(1,1,0)(1,0,1)$_{12}$ Vs VECM(2) | -3.2368 | 0.001 |
| In-Sample | ARIMA(1,1,[1,12,13]) Vs VECM(2) | -3.4217 | 0.001 |
| | SARIMA(1,1,0)(1,0,1)$_{12}$ Vs VECM(2) | -2.710 | 0.009 |
| Out-of-sample | ARIMA(2,1,[1,12]) Vs VECM(2) | -2.612 | 0.012 |

*Table 18: Diebold-Marino equal accuracy test results of SARIMA against ARIMA models (forecast horizon =1)*

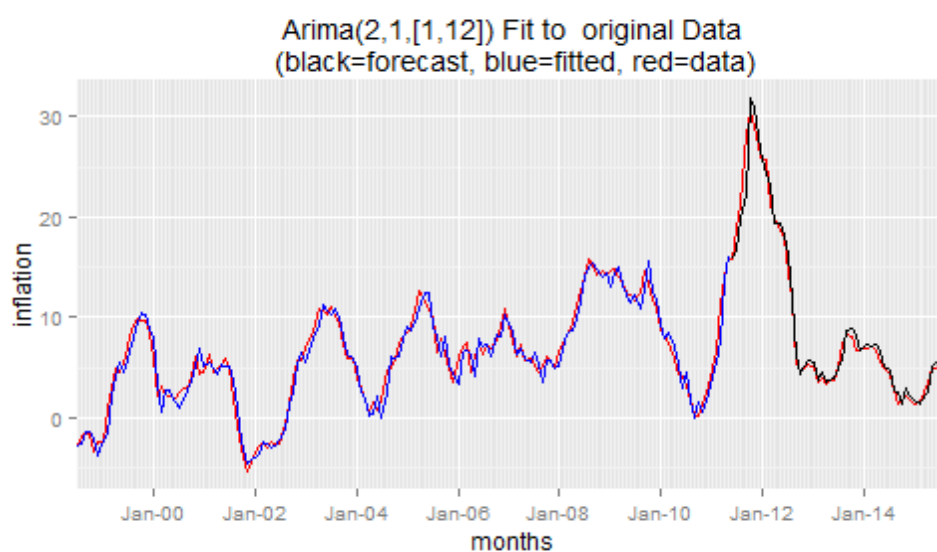| | Models | DM-statistic | P-value |
|---|---|---|---|
| In-Sample | SARIMA(1,1,0)(1,0,1)$_{12}$ Vs ARIMA(1,1,[1,12,13]) | -0.218 | 0.828 |
| Out-of-sample | SARIMA(1,1,0)(1,0,1)$_{12}$ Vs ARIMA(2,1,[1,12]) | -0.795 | 0.430 |



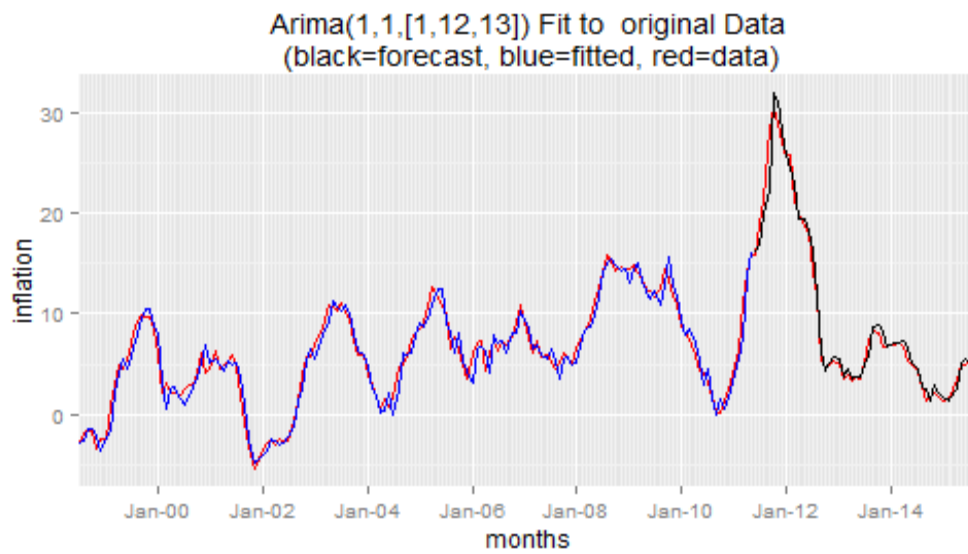*Figure 7:plot of ARIMA(1,1,[1,12]) fit to the original data and out of sample forecasts*

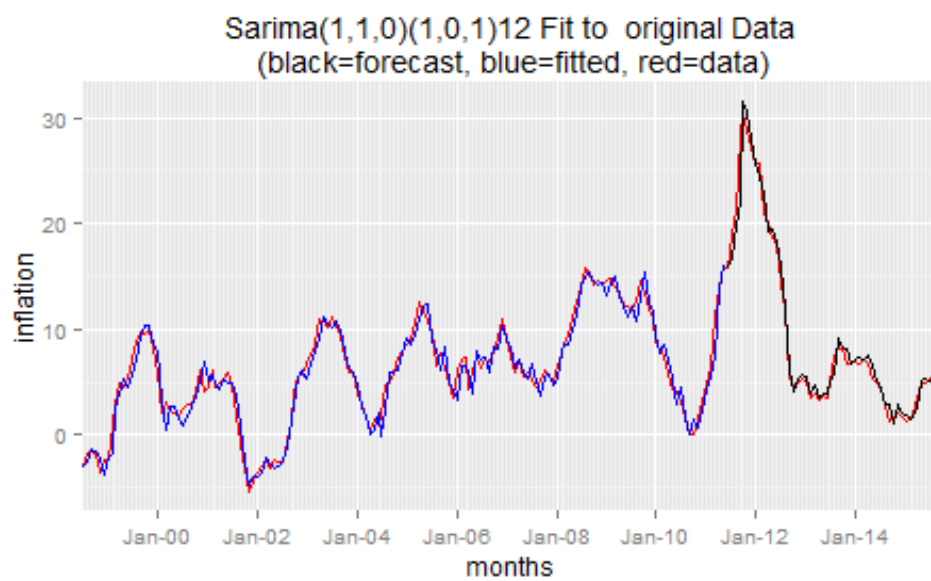*Figure 8:plot of ARIMA(1,1,[1,12,13]) fit to the original data and out of sample forecasts*



*Figure 9: Plot of SARIMA(1,1,0)(1,0,1)12  fit to the original data and out of sample forecasts*
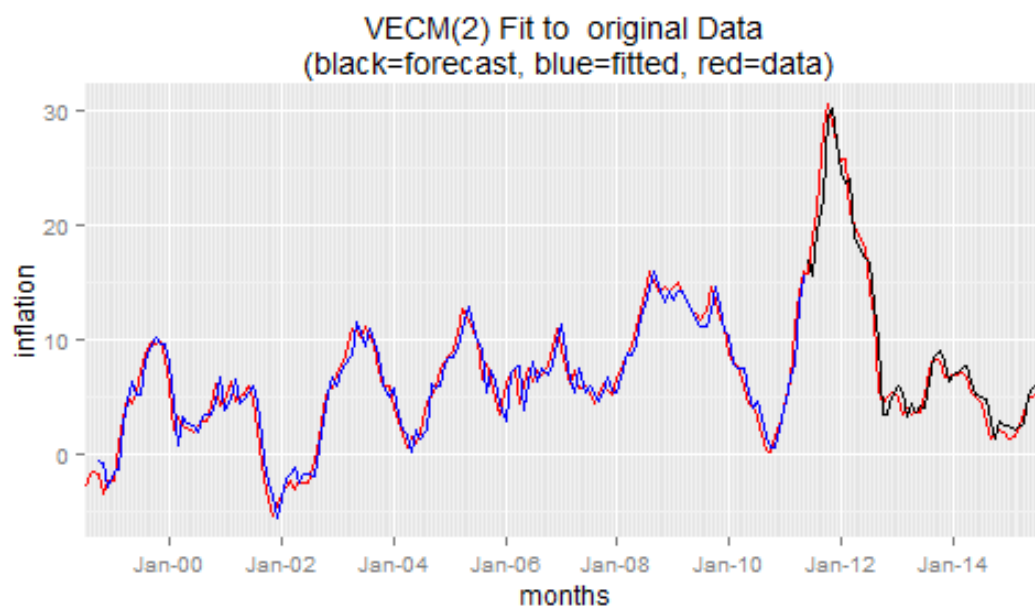
*Figure 10: Plot of VECM(2) fit to the original data and out of sample forecasts*

# 5 CONCLUSION

The research results imply that Uganda's inflation rate has an aspect of seasonality since the SARIMA model performed better than the ARIMA model when it comes to both out-of-sample and in-sample forecast performance based on the RMSE and MAE respectively. Based on the Diebold-Mariano test for both in-sample and out-of-sample forecast accuracy, the SARIMA models perform significantly equal to ARIMA models.

VECM model performed worse than the ARIMA and SARIMA models under both the in-sample and out-of-sample performance since it had the maximum RMSE and MAE, the VECM model turned out to be have less forecasting accuracy than SARIMA and ARIMA models for out-of-sample and in-sample forecasts respectively. These findings are similar to a study done by Fritzer et al. (2002) on the Australian inflation.

The two univariate models (SARIMA and ARIMA) performed better than the multivariate model (VECM) for both in-sample and out-of-sample performance. The Diebold-Marino test which was applied to both the in-sample and out-of-sample forecasts of the SARIMA and VECM as well as ARIMA and VECM model suggested that there was significant difference between the forecast accuracy of VECM and the two univariate models. This study can conclude that there is a difference in forecast accuracy of the VECM and the univariate models (SARIMA and ARIMA) based on the RMSE, MAE and Diebold-Mariano test.

**Recommendation:** Future studies can consider more variations in the models used in this study or even other models for example the ARCH-GARCH models, SETAR, VARMA. It would also be a good idea to look at how these models behave under different forecast horizons. For the sake of the multivariate models, different predictors other than exchange rate and world coffee prices can be chosen.

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