Travelling Salesman Problem

by Hung Nguyen

Problem statement

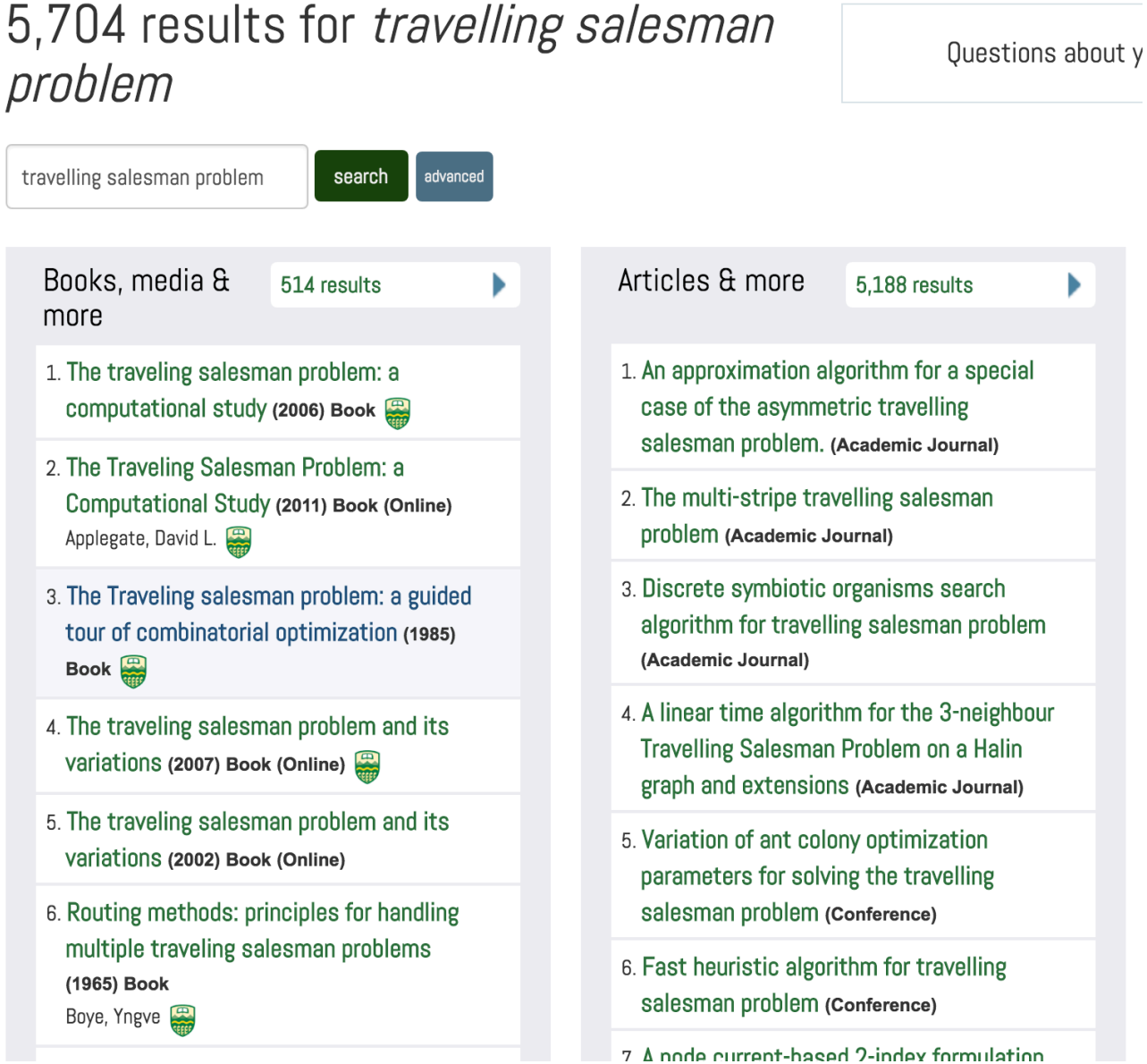
“Given a list of cities and the distances between each pair of cities, find the shortest possible route that visits each city and returns to the origin city”

- Graph problem
- Find the shortest path that visits all the nodes and come back to the origin node.

Applications

- Vehicle routing
  - UPS delivery
  - School bus routing

- Let's go micro…
  - Computing Wiring
  - PCB production



Exact algorithm

Brute force…

Time complexity: $O(n!)$

Try all paths – n! permutation of edges.

Not practical!

Exact algorithm (cont.)

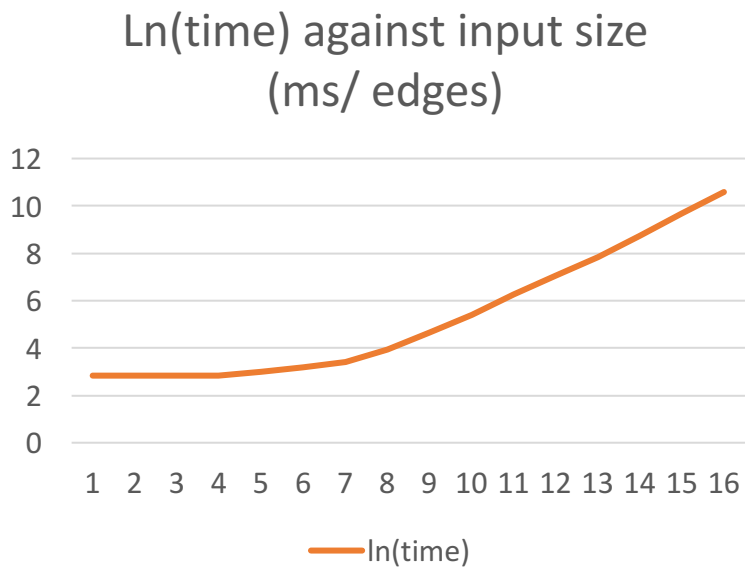
DP solution: Held-Karp algorithm

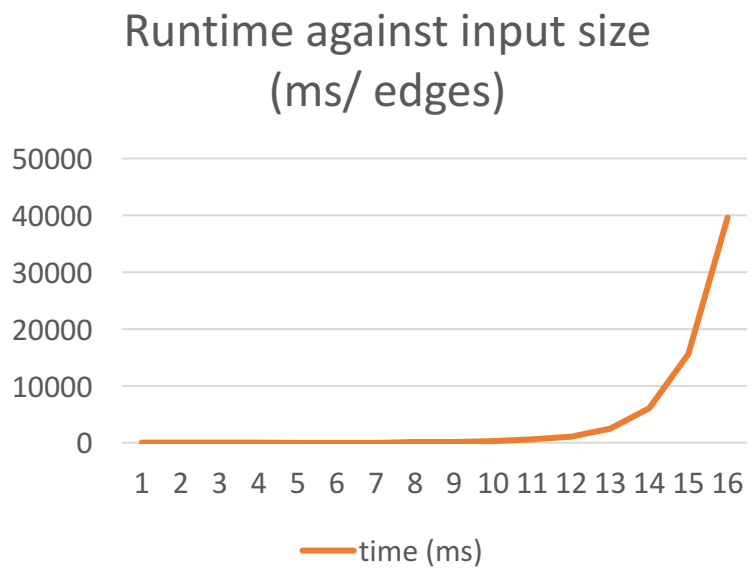
Time complexity: $O(2^n * n^2)$

Implementation: https://github.com/CarlEkerot/held-karp

Also, not practical!

…

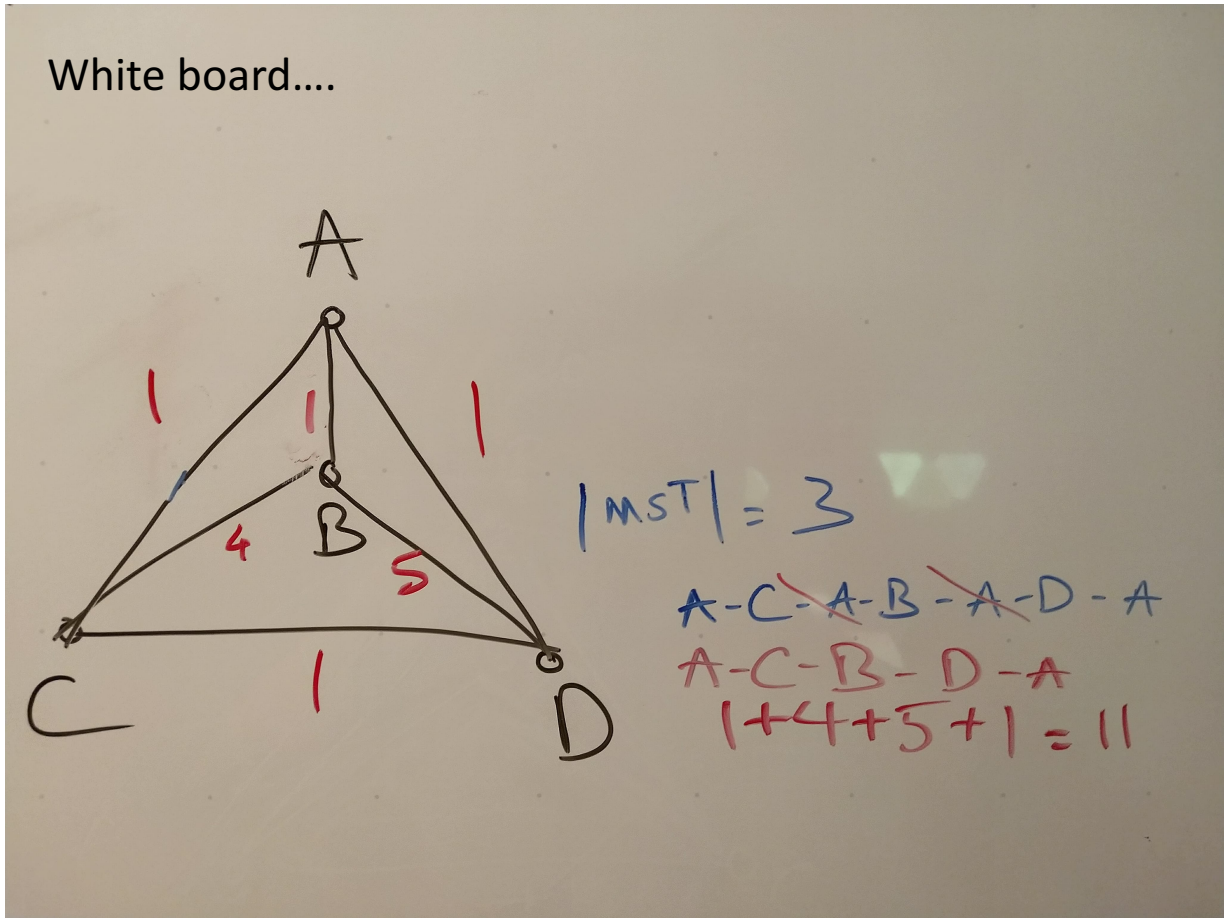




Heuristic – Approximation

- Idea: find some thing ”cheaper” but “good enough”.
  - Walk on the MST edges to complete the cycle (Here we duplicate the nodes, the result is Hamiltonian cycle)
  - We ”collapse” nodes on the path
- How good is the solution?
  - Minimum Spanning Tree: the cost of the optimal path cannot be smaller than the size of MST (by MST definition) $C \leq 2 \times MST \leq 2\ OPT$
  - ➢ 2-approximation: we can guarantee the found path is at most twice as long as the optimal path.
- Time complexity: $O(n^2 \log n)$
- Let’s try out some code!
- There is a caveat: only works with metric TSP (triangle inequality holds)
  - Why?

Triangle inequality



The upper-bound does not hold on cost

i.e. $\cancel{C \leq 2 \times MST}$

The reason is when we reduce the path, the cost might increase, i.e. $\text{Path}(A - C - B) < Path(A - B)$

Sources

- MST: Code Archieve
- Fleury: https://www.geeksforgeeks.org/fleurys-algorithm-for-printing-eulerian-path/
- Held-karp: https://github.com/CarlEkerot/held-karp
- This presentation: https://github.com/Hank-TNguyen/CMPUT-403-demo

Thx!

UVa problem

- https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&page=show_problem&problem=52
  - UVa_116 – Unidirectional TSP