

# Mathematical Backdoors in Symmetric Encryption Systems<sup>\*</sup>

## Proposal for a Backdoored AES-like Block Cipher

Arnaud Bannier and Eric Filiol

Operational Cryptology and Virology Lab, ESIEA,  
38 rue des Drs Calmette et Guérin, 53000 Laval, France,  
{bannier, filioli}@esiea.fr

**Keywords:** Cryptography, Encryption Algorithms, Backdoor, Trapdoor, Cryptanalysis, Block Cipher, AES.

**Abstract.** Recent years have shown that more than ever governments and intelligence agencies try to control and bypass the cryptographic means used for the protection of data. Backdooring encryption algorithms is considered as the best way to enforce cryptographic control. Until now, only implementation backdoors (at the protocol/implementation/management level) are generally considered. In this paper we propose to address the most critical issue of backdoors: mathematical backdoors or by-design backdoors, which are put directly at the mathematical design of the encryption algorithm. While the algorithm may be totally public, proving that there is a backdoor, identifying it and exploiting it, may be an intractable problem. We intend to explain that it is probably possible to design and put such backdoors. Considering a particular family (among all the possible ones), we present BEA-1, a block cipher algorithm which is similar to the AES and which contains a mathematical backdoor enabling an operational and effective cryptanalysis. The BEA-1 algorithm (80-bit block size, 120-bit key, 11 rounds) is designed to resist to linear and differential cryptanalyses. A challenge will be proposed to the cryptography community soon. Its aim is to assess whether our backdoor is easily detectable and exploitable or not.

## 1 Introduction

Despite the fact that in the late 90s/early 2000s, citizens have partially obtained the freedom for using cryptography, the recent years have shown that more than ever, governments and intelligence agencies still try to control and bypass the cryptographic means used for the protection of data and of private life. Snowden's leaks were a first upheaval. A tremendous number of secret projects (from NSA, GCHQ) have been revealed to the public opinion which proves this situation.

While the need for the security of everyday life activities (for companies, for citizens) requires more and more cryptography, recent bothering initiatives by political decision-makers ask for an even stronger control over cryptography not to say preparing the simple prohibition or ban of cryptographic application such as telegram. At the same time, the EU as well as a number of security agencies (such as French ANSSI, German BSI) confirmed that it was nonsense and militate for the mandatory use of end-to-end encryption.

The recurring approaches and attempts consist in making the implementation of backdoors mandatory. The simplest and naive approach consists in enforcing key escrowing at the operators'

---

<sup>\*</sup> This work has been presented at the FORmal Methods for Security Engineering (ForSE) conference 2017

level. But point-to-point encryption solutions (which are not equal to end-to-end encryption) like Telegram or Proton mail enable to prevent it. A number of different backdoor techniques are regularly mentioned or proposed.

The most critical aspect in implementation backdoors lies on the fact that hackers or analysts may find them more or less easily and worse may exploit them. This is the reason why it is likely that IT operators or developers are very reluctant to accept backdoors until now. In case of leak, they will inevitably lose users' confidence and favor the development of trusted services abroad. In fact, the backdoor issue arises due to the fact that only implementation backdoors (at the protocol/implementation/management level) are generally considered.

In this paper we address the most critical issue of backdoors: mathematical or by-design backdoors. In other words, the backdoor is put directly at the mathematical design of the encryption algorithm. While the algorithm may be totally public, proving that there is a backdoor, identifying it and exploiting it, may be an intractable problem, unless you know the backdoor. To some extent, the RSA's Dual\_EC\_DRBG standard case falls within this category [14]. Other non-public examples are known within the military cryptanalysis community, and partially revealed to the public thanks to the 1995 Hans Buehler case [15]. This kind of backdoor is the most difficult one to address and there is quite no public work on that topic. It is generally the technical realm of a few among the most eminent intelligence agencies (namely NSA, GCHQ, SVR/GRU) which moreover have the ability and power to step in and to influence the international standardization processes.

We intend to explain that it is probably possible to design and put such backdoors. Considering a particular case of mathematical backdoors (among all the possible ones) based on our previous work [2], we present a block cipher algorithm which is similar to the AES and which contains a mathematical backdoor enabling an operational and effective cryptanalysis (in other words in a limited time on a modern desktop computer and with a limited number of plaintext/ciphertext pairs). This block cipher algorithm (80-bit block, 120-bit key size, 11 rounds) is designed to resist to linear and differential cryptanalyses.

This paper is organized as follows. In Section 2 we explore the concept of backdoors and trapdoors and we identify two main categories, each containing itself subcategories depending on the nature of the cipher (stream or block ciphers). This observation is backed by the personal experience of the second author as a military cryptanalyst. We also present the state-of-the-art, history and previous work regarding backdoors, mostly in symmetric cryptography. In Section 3, we present our backdoored block cipher algorithm BEA-1 (standing for *Backdoored Encryption Algorithm 1*), based on our work [2]. This is a particular family of trapdoors using a suitable partition of the plaintext and ciphertext spaces. In Section 4, we discuss the cryptographic security of this cipher, with respect to linear and differential cryptanalyses. We also propose a cryptographic challenge to the cryptography community, regarding the backdoor identification and exploitation. We suppose that this backdoor is likely to be detected. Such a challenge should enable to prove or disprove this claim. Lastly we conclude in Section 5 and explore future work.

## 2 The Concept of Backdoor

### 2.1 Definition and Classification Proposal

Trapdoors are a two-face, key concept in modern cryptography. It is primarily related to the concept of “*trapdoor function*” — a function that is easy to compute in one direction, yet difficult to compute in the opposite direction without special information, called the “*trapdoor*”. This first “face” relates most of the time to asymmetric cryptography algorithms. It is a necessary condition to get reversibility between the sender/receiver (encryption) or the signer/verifier

(digital signature). The trapdoor mechanism is always fully public and detailed. The security and the core principle is based on the existence of a secret information (the private key) which is essentially part of the trapdoor. In other words, the private key can be seen as *the* trapdoor.

The second “face” of the concept of trapdoor relates to the more subtle and perverse concept of “mathematical backdoor” and is a key issue in symmetric cryptography (even if the issue of backdoors may be extended to asymmetric cryptography; see for example the case of the DUAL\_EC\_DRBG [14], or the case of trapdoored primes addresses recently in [8]).

In this case, the aim is to insert hidden mathematical weaknesses which enable one who knows them to break the cipher. If possible, these weaknesses should be independent of the secret key. Somehow, it consists to create a hidden asymmetry to the detriment of the legitimate users of the communication and to the benefit of the eavesdropper. In this context, the existence of a backdoor is a strongly undesirable property.

In the rest of the present section, we will oppose the term of trapdoor (desirable property) to that of backdoor (undesirable property). While the term of trapdoor has been already used in the very few literature covering this second face of this problem, we suggest however to use the term of backdoor to describe the issue of hidden mathematical weaknesses. This would avoid ambiguity and maybe would favor the research work around a topic which is nowadays mostly addressed by governmental entities in the context of cryptography control and regulations.

Inserting backdoors in encryption algorithms underlies quite systematically the choice of cryptographic standards (DES, AES. . .). The reason is that the testing, validation and selection process is always conducted by governmental entities (NIST or equivalent) with the technical support of secret entities (NSA or equivalent). So an interesting and critical research area is: “how easy and feasible is it to design and to insert backdoors (at the mathematical level) in encryption algorithms?”. In this paper, we intend to address one very particular case of this question. It is important to keep in mind that a backdoor may be itself defined in the following two ways.

- As a “natural weakness” known — but non disclosed — only by the tester/validator/final decision-maker (e.g. the NSA as it could have been the case for the AES challenge). The best historic example is that of the differential cryptanalysis. Following Biham and Shamir’s seminal work in 1991 [3], NSA acknowledged that it was aware of that cryptanalysis years ago [13]. Most of experts estimate that it was nearly 20 years ahead. However a number of non public, commercial block ciphers in the early 90s may be weak with respect to differential cryptanalysis.
- As an intended design weakness put by the author of the algorithm. To the authors knowledge, there is no known cases for public algorithms yet.

As far as symmetric cryptography is concerned, there are two major families of cipher systems for which the issue of backdoor must be considered differently.

- *Stream ciphers*. Their design complexity is rather low since they mostly rely on algebraic primitives (LFSRs and Boolean functions which have intensely been studied in the open literature). Until the late 70s, backdoors relied on the fact that quite all algorithms were proprietary and hence secret. It was then easy to hide non primitive polynomials, weak combining Boolean functions. The Hans Buehler case in 1995 [15] shed light on that particular case.
- *Block ciphers*. This class of encryption algorithms is rather recent (end of the 70s for the public part). They exhibit so a huge combinatorial complexity that it is reasonable to think to backdoors. As described in [6] for a  $k$ -bit secret key and a  $m$ -bit input/output block cipher there are  $((2^m)!)^{2^k}$  possible such block ciphers. For such an algorithm, the number of possible internal states is so huge that we are condemned to have only a local view of the system, that

is, the round function or the basic cryptographic primitives. We cannot be sure that there is no degeneration effect at a higher level. This point has been addressed in [6] when considering linear cryptanalysis. Therefore, it seems reasonable to think that this combinatorial richness of block cipher may be used to hide backdoors.

Since block ciphers are the most widely used encryption algorithms nowadays by the general public and the industry, we will focus on them in the rest of the paper. Backdoors in stream ciphers have quite never been exposed to the public.

## 2.2 Previous Work

One of the first trapdoor cipher was proposed in 1997 by Rijmen and Preneel [11]. The S-boxes are selected randomly and then modified to be weak to the linear cryptanalysis. They are finally applied to a Feistel cipher such as CAST or LOKI91. But because of the big size of the S-boxes, the linear table of such an S-box cannot be computed. However the knowledge of the trapdoor gives a good linear approximation of the S-boxes which is then used in a linear cryptanalysis. As an example, the authors created a 64-bit block cipher based on CAST cipher, and four  $8 \times 32$  S-boxes. If the parameters of the trapdoors are known, a probabilistic algorithm allows to recover the key easily. Such a family of trapdoor ciphers leads to recover only a part of the key, and the authors claim that the trapdoor is undetectable. But in [16], Wu and al. discovered a way to recover the trapdoor if the attacker knows its global design but not the parameters. They also showed that there exists no parameter allowing to hide the trapdoor. Nevertheless, it is worthwhile to mention that in practice, if a real cipher containing a trapdoor is given, the presence of the trapdoor will certainly not be revealed.

In [10], a DES-like trapdoor cipher exploiting a weakness induced by the round functions is presented. The group generated by the round functions acts imprimitively on the message space to allow the design of the trapdoor. In other words, this group preserves a partition of the message space between input and output of the round function. Such a construction leads to the design of a trapdoor cipher composed of 32 rounds and using a 80 bits key. The knowledge of the trapdoor allows to recover the key using  $2^{41}$  operations and  $2^{32}$  plaintexts. Even if the mathematical material to build the trapdoor is given, no general algorithm is detailed to construct such S-boxes. Furthermore, as the author says, S-boxes using these principles are incomplete (half of the cipher text bits are independent of half of the plaintext bits). Finally, the security against the differential attack is said *not as high as one might expect*.

More recently in [1], the authors created non-surjective S-boxes embedding a parity check to create a trapdoor cipher. The message space is thus divided into cosets and leads to create an attack on this DES-like cipher in less than  $2^{23}$  operations. The security of the whole algorithm, particularly against linear and differential cryptanalyses is not given and the authors admit that their attack is dependent on the first and last permutation of the cipher. Finally, the non-surjective S-boxes may lead to detect easily the trapdoor by simply calculating the image of each input vector. This problem is naturally avoided in a Substitution-Permutation Network (SPN) in which S-boxes are bijective by definition.

In a slightly different context, Caranti and al. answer to Patterson's question by the affirmative in [5], by proving that the imprimitivity of the group generated by round functions is actually related to the cosets of a linear subspace. They also give some conditions to create such a primitive group to design a secure cipher that cannot contain such trapdoor, and finally show that the AES respects these conditions. They add in [4] an algorithm to verify this last condition simply and show that AES and Serpent S-boxes verify this property.

### 3 Description of BEA-1

The algorithm BEA-1 (standing for *Backdoored Encryption Algorithm version 1*) is based on our research work on partition-based trapdoors [2]. This section is intended to describe this cipher precisely. The cipher operates on 80-bit data blocks using a 120-bit master key. Our algorithm is directly inspired by Rijndael [6], the block cipher designed by Joan Daemen and Vincent Rijmen which is now known as the AES (the encryption standard proposed by the USA, under the auspices of NIST and NSA [9]). Consequently, our cipher is a Substitution-Permutation Network.

The encryption consists in applying eleven times a simple keyed operation called *round function* to the data block. A different 80-bit round key is used for each iteration of the round function. Since the last round is slightly different and uses two round keys, the encryption requires twelve 80-bit round keys. These round keys are derived from the 120-bit master key using an algorithm called *key schedule* (depicted in Figure 1).

The round function in Figure 2 is made up of three distinct stages: a *key addition*, a *substitution layer* and a *diffusion layer*. The key addition is just a bitwise XOR between the data block and the round key. The substitution layer consists in the parallel evaluation of four different S-boxes and is the only part of the cipher which is not linear or affine. Following the design principles of the AES, the diffusion layer comes in two parts: the **ShiftRows** and the **MixColumns** operations.

The decryption is straightforward from the encryption since all the components are bijective. Thus, to decrypt, we just have to apply the inverse operations in the reverse order. Remark that the key addition and the **ShiftRows** are involutions, therefore the same operations are used in the decryption process. **In contrast to the AES**, the algorithm works with **bundles of 10 bits instead of 8 bits**. Let  $\mathbb{F}_2$  denote the Galois Field of order 2. Any  $10n$ -bit block  $x$  is seen as  $n$ -tuple of 10-bit bundles  $(x_0, \dots, x_{n-1})$ , and thus as an element of  $(\mathbb{F}_2^{10})^n$ . The hexadecimal notation is used to denote any 10-bit bundle. For example, **37A stands for 1101111010 in  $\mathbb{F}_2^{10}$** .

The S-boxes  **$S_0, S_1, S_2$  and  $S_3$**  are four permutations of  $\mathbb{F}_2^{10}$  given in Appendix. The linear map  $M : (\mathbb{F}_2^{10})^4 \rightarrow (\mathbb{F}_2^{10})^4$  processes four 10-bit bundles. Because of the linearity of this map,  $M$  is only defined on the standard basis of  $(\mathbb{F}_2^{10})^4$ . For convenience, its inverse  $M^{-1}$  is also given in Appendix.

A pseudo-code for the key schedule is given in Algorithm 1. To provide an overview of its structure, the first step is represented in Figure 1. This representation also emphasizes the similarities between our key schedule and the AES one. The pseudo-code for the encryption and decryption functions are respectively given in Algorithms 2 and 3. The notation  $[a \parallel b]$  denotes the concatenation of the vectors  $a$  and  $b$ . Again, an overview of the round function is given in Figure 2.

## 4 Cryptographic Security Analysis of BEA-1

### 4.1 Differential and Linear Cryptanalyses

In [6], Daemen and Rijmen introduced the differential and the linear branch numbers of a linear transformation. With an exhaustive search, it can be checked that the differential and linear branch numbers of  $M$  are both equal to 5, which is the maximum. This implies that any 2-round trail has at least 5 active S-boxes. Thus, a 10-round trail involves at least 25 active S-boxes.

Note that all the S-boxes are (at most) differentially 40-uniform and linearly 128-uniform. Therefore, the probability of any 10-round differential trail is upper bounded by  $(\frac{40}{1024})^{25} \approx 2^{-116.9}$  and the absolute bias of a 10-round linear trail is upper bounded by  $(\frac{128}{512})^{25} = 2^{-50}$ .

**Algorithm 1 - ExpandKey**

Input. The 120-bit master key  $K = (K_0, \dots, K_{11}) \in (\mathbb{F}_2^{10})^{12}$ .

Output. The twelve 80-bit round keys  $k^0, \dots, k^{11} \in (\mathbb{F}_2^{10})^8$ .

```

1   $(k_0, \dots, k_{11}) \leftarrow (K_0, \dots, K_{11})$ 
2  For i from 0 to 6 do
3       $x \leftarrow M(k_{12i+8}, \dots, k_{12i+11})$ 
4       $x \leftarrow (S_j(x_j))_{0 \leq j \leq 3}$ 
5       $x \leftarrow (x_0 \oplus (3^i \bmod 2^{10}), x_1, x_2, x_3)$ 
6       $(k_{12i+12}, \dots, k_{12i+15}) \leftarrow (k_{12i+0}, \dots, k_{12i+3}) \oplus x$ 
7       $(k_{12i+16}, \dots, k_{12i+19}) \leftarrow (k_{12i+4}, \dots, k_{12i+7}) \oplus (k_{12i+12}, \dots, k_{12i+15})$ 
8       $(k_{12i+20}, \dots, k_{12i+23}) \leftarrow (k_{12i+8}, \dots, k_{12i+11}) \oplus (k_{12i+16}, \dots, k_{12i+19})$ 
9  For r from 0 to 11 do
10      $k^r \leftarrow (k_{8r+i})_{0 \leq i \leq 7}$ 

```

**Algorithm 2 - Encrypt**

Input. The 120-bit master key  $K \in (\mathbb{F}_2^{10})^{12}$  and the 80-bit plaintext block  $p \in (\mathbb{F}_2^{10})^8$ .

Output. The 80-bit ciphertext block  $c \in (\mathbb{F}_2^{10})^8$ .

```

1   $k^0, \dots, k^{11} \leftarrow \text{ExpandKey}(K)$ 
2   $x \leftarrow p$ 
3  For r from 0 to 9 do
4       $x \leftarrow x \oplus k^r$  AddRoundKey
5       $x \leftarrow (S_{i \bmod 4}(x_i))_{0 \leq i \leq 7}$  SubBundles
6       $x \leftarrow (x_0, x_5, x_2, x_7, x_4, x_1, x_6, x_3)$  ShiftRows
7       $x \leftarrow [M(x_0, x_1, x_2, x_3) \parallel M(x_4, x_5, x_6, x_7)]$  MixColumns
8       $x \leftarrow x \oplus k^{10}$  AddRoundKey
9       $x \leftarrow (S_{i \bmod 4}(x_i))_{0 \leq i \leq 7}$  SubBundles
10      $x \leftarrow (x_0, x_5, x_2, x_7, x_4, x_1, x_6, x_3)$  ShiftRows
11      $x \leftarrow x \oplus k^{11}$  AddRoundKey
12  $c \leftarrow x$ 

```

**Algorithm 3 - Decrypt**

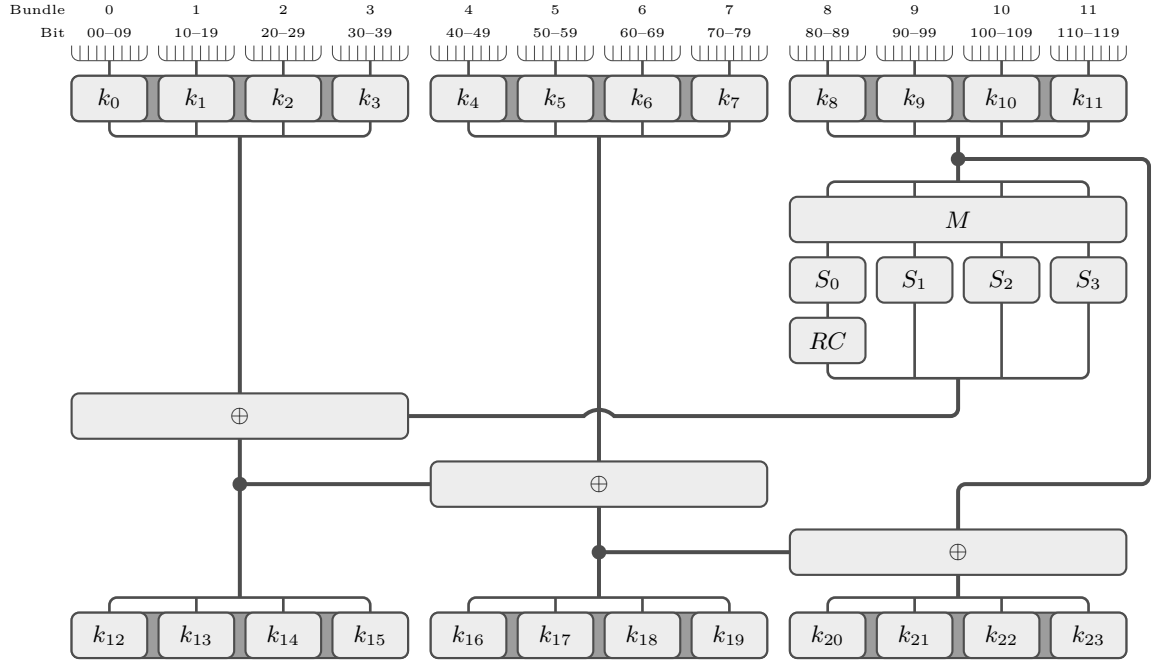
Input. The 120-bit master key  $K \in (\mathbb{F}_2^{10})^{12}$  and the 80-bit ciphertext block  $c \in (\mathbb{F}_2^{10})^8$ .

Output. The 80-bit plaintext block  $p \in (\mathbb{F}_2^{10})^8$ .

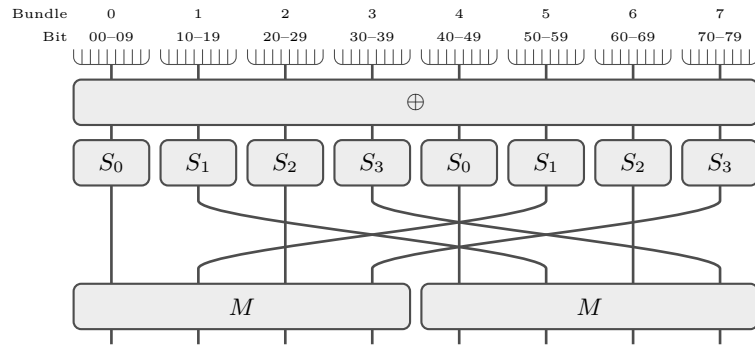
```

1   $k^0, \dots, k^{11} \leftarrow \text{ExpandKey}(K)$ 
2   $x \leftarrow c$ 
3   $x \leftarrow x \oplus k^{11}$  AddRoundKey
4   $x \leftarrow (x_0, x_5, x_2, x_7, x_4, x_1, x_6, x_3)$  InvShiftRows
5   $x \leftarrow (S_{i \bmod 4}^{-1}(x_i))_{0 \leq i \leq 7}$  InvSubBundles
6   $x \leftarrow x \oplus k^{10}$  AddRoundKey
7  For r from 9 to 0 do
8       $x \leftarrow [M^{-1}(x_0, x_1, x_2, x_3) \parallel M^{-1}(x_4, x_5, x_6, x_7)]$  InvMixColumns
9       $x \leftarrow (x_0, x_5, x_2, x_7, x_4, x_1, x_6, x_3)$  InvShiftRows
10      $x \leftarrow (S_{i \bmod 4}^{-1}(x_i))_{0 \leq i \leq 7}$  InvSubBundles
11      $x \leftarrow x \oplus k^r$  AddRoundKey
12  $p \leftarrow x$ 

```



**Fig. 1.** A diagrammatic representation of the key schedule **ExpandKey**



**Fig. 2.** A diagrammatic representation of the round function

Consequently, a differential cryptanalysis of the 10-round version of our cipher would require at least  $2^{117}$  chosen plaintext/ciphertext pairs and a linear cryptanalysis would require  $2^{100}$  known plaintext/ciphertext pairs.

Even if this is a rough approximation since it does not take into account the inter-column diffusion provided by the **ShiftRows** operation, it suffices to prove the cipher practical resistance against classical differential and linear cryptanalyses. In fact, there is only  $2^{80}$  different plaintext/ciphertext pairs for a fixed master key.

## 4.2 Statistical Analysis of BEA-1

Any cryptographic algorithm must behave as a random generator or at least must exhibit enough randomness properties. Therefore, its outputs for different classes of inputs must pass all the reference statistical testings. The most widely used is the NIST's Statistical Test Suite (STS) [12].

We have performed the statistical analysis for BEA-1 with respect to all the tests which are implemented in STS. Our encryption has passed all the tests successfully. This result is of rather high importance since

- STS is the tool recommended by the US government to evaluate statistical properties of any secure encryption algorithm. It is explicitly mandatory to consider it in the industry.
- The presence of our backdoor remains statistically undetectable which proves that if statistical properties are a necessary condition for cryptographic security it is absolutely not a sufficient property. It may be bypassed by considering statistical simulation techniques [7]. Algebraic or combinatorial weaknesses moreover remains out of reach from statistical analysis.

## 4.3 Cryptographic Challenge

We propose a cryptographic challenge whose aims is twofold:

- identifying and explaining what our backdoor consists in,
- exploiting this backdoor in the most efficient way (in terms of computing time, memory requirements, the number of required plaintext/ciphertext pairs).

We have run our own full cryptanalysis implementation several times. Each time, we retrieve the 120-bit key successfully.

This challenge will be officially launched right after the presentation of the present paper, on the [arxiv.org](https://arxiv.org). To take part, participants must send the following data to both authors (prior to any publication):

- the description of the backdoor,
- the description of the attack to exploit the backdoor successfully.
- the relevant source codes. They will help us to sort the different proposals with respect to, first the number of required plaintext/ciphertext pairs, second the computing time on our reference computer.

Incentive (non monetary) awards will be awarded to the three best attacks. Our attack as the reference solution will be presented in at the RusCrypto 2017 conference around end of March 2017. Consequently, the challenge holds until this date. Moreover the best attack will be considered for publication in the Journal in Computer Virology and Hacking Techniques.



## 5 Conclusion and Future Work

In this paper, we have proposed an AES-like encryption algorithm which contains a backdoor at its design level. This algorithm, named BEA-1, exhibits many of the desirable properties that any secure algorithm should. However, it is absolutely unsuitable for actually protection information. Indeed, we manage to break it with a rather limited amount of resources successfully.

While it is a humble, first step in a larger research work, it illustrates the issue of using foreign encryption algorithms which may contains such hidden weaknesses. The very final aim of our work is to prove that it is feasible to embed such undetectable intended weaknesses. It is consequently a critical issue to have a broader work conducted in this research area and we hope that other people will also consider it as such.

The next step will be to consider more sophisticated combinatorial structures.

## References

1. Vesela Angelova and Yuri Borissov. Plaintext recovery in des-like cryptosystems based on s-boxes with embedded parity check. *Serdica Journal of Computing*, 7(3):257p–270p, 2013.
2. Arnaud Bannier, Nicolas Bodin, and Eric Filiol. Partition-based trapdoor ciphers. Cryptology ePrint Archive, Report 2016/493, 2016. <http://eprint.iacr.org/2016/493>.
3. Eli Biham and Adi Shamir. *Differential cryptanalysis of the data encryption standard*, volume 28. Springer, 1993.
4. A Caranti, Francesca Dalla Volta, and Massimiliano Sala. On some block ciphers and imprimitive groups. *Applicable algebra in engineering, communication and computing*, 20(5-6):339–350, 2009.
5. A Caranti, F Dalla Volta, Massimiliano Sala, and Francesca Villani. Imprimitive permutations groups generated by the round functions of key-alternating block ciphers and truncated differential cryptanalysis. *arXiv preprint math/0606022*, 2006.
6. Joan Daemen and Vincent Rijmen. *The design of Rijndael*. Springer Verlag, 2002.
7. Eric Filiol and Sébastien Josse. A statistical model for undecidable viral detection. *Journal in Computer Virology*, 3(2):65–74, 2007.
8. Joshua Fried, Pierrick Gaudry, Nadia Heninger, and Emmanuel Thomé. A kilobit hidden snfs discrete logarithm computation. Cryptology ePrint Archive, Report 2016/961, 2016. <http://eprint.iacr.org/2016/961>.
9. NIST. Advanced encryption standard archive, 1998–2000. <http://csrc.nist.gov/archive/aes/>.
10. Kenneth G Paterson. Imprimitive permutation groups and trapdoors in iterated block ciphers. In *Fast Software Encryption*, pages 201–214. Springer, 1999.
11. Vincent Rijmen and Bart Preneel. A family of trapdoor ciphers. In *Fast Software Encryption*, pages 139–148. Springer, 1997.
12. Andrew Rukhin, Juan Soto, James Nechvatal, Elaine Barker, Stefan Leigh, Mark Levenson, David Banks, Alan Heckert, James Dray, San Vo, Andrew Rukhin, Juan Soto, Miles Smid, Stefan Leigh, Mark Vangel, Alan Heckert, James Dray, and Lawrence E Bassham Iii. A statistical test suite for random and pseudorandom number generators for cryptographic applications, 2001.
13. Bruce Schneier. The nsa’s cryptographic capabilities, 1998–2000. <http://csrc.nist.gov/archive/aes/>.
14. Dan Shumow and Niels Ferguson. On the possibility of a back door in the nist sp800-90 dual ec prng. In *Proc. Crypto*, volume 7, 2007.
15. Res Strehle. *Verschlüsselt: der Fall Hans Bühler*. Werd, 1994.
16. Hongjun Wu, Feng Bao, Robert H Deng, and Qin-Zhong Ye. Cryptanalysis of rijmen-preneel trap-door ciphers. In *Advances in Cryptology—Asiacrypt’98*, pages 126–132. Springer, 1998.

## Appendix

The present appendix contains the different tables for the S-boxes (Figures 4 and 5), the linear map  $M$  and its inverse  $M^{-1}$  (Figure 3). They can be copied and pasted for a practical implementation of the encryption algorithm.

$x$	$\mapsto$	$M(x)$	$x$	$\mapsto$	$M^{-1}(x)$
(001,000,000,000)	$\mapsto$	(112,1BC,36C,0C5)	(001,000,000,000)	$\mapsto$	(10B,221,09D,398)
(002,000,000,000)	$\mapsto$	(344,394,342,165)	(002,000,000,000)	$\mapsto$	(1AE,1E9,2CB,245)
(004,000,000,000)	$\mapsto$	(23F,15B,0C7,0A7)	(004,000,000,000)	$\mapsto$	(1AB,11E,05F,3A4)
(008,000,000,000)	$\mapsto$	(215,11F,1E0,2E7)	(008,000,000,000)	$\mapsto$	(08D,04D,016,34C)
(010,000,000,000)	$\mapsto$	(2D9,10A,0C4,095)	(010,000,000,000)	$\mapsto$	(0AD,337,3C5,2D4)
(020,000,000,000)	$\mapsto$	(231,120,322,016)	(020,000,000,000)	$\mapsto$	(322,3FD,3D5,0E5)
(040,000,000,000)	$\mapsto$	(3C6,010,0EC,261)	(040,000,000,000)	$\mapsto$	(002,246,2E2,380)
(080,000,000,000)	$\mapsto$	(32C,199,2C5,07A)	(080,000,000,000)	$\mapsto$	(1E9,3FE,238,329)
(100,000,000,000)	$\mapsto$	(35C,13E,212,110)	(100,000,000,000)	$\mapsto$	(0F5,1BD,210,210)
(200,000,000,000)	$\mapsto$	(13E,20F,253,0BC)	(200,000,000,000)	$\mapsto$	(2D8,209,353,243)
(000,001,000,000)	$\mapsto$	(237,252,004,0F8)	(000,001,000,000)	$\mapsto$	(07D,2BB,037,3C8)
(000,002,000,000)	$\mapsto$	(0CC,32A,01A,2DB)	(000,002,000,000)	$\mapsto$	(055,128,25A,17F)
(000,004,000,000)	$\mapsto$	(13B,2FA,328,38C)	(000,004,000,000)	$\mapsto$	(0EB,2FD,3C3,176)
(000,008,000,000)	$\mapsto$	(022,37D,08D,3D4)	(000,008,000,000)	$\mapsto$	(3D1,236,09D,2F1)
(000,010,000,000)	$\mapsto$	(1F4,1C5,1FF,31D)	(000,010,000,000)	$\mapsto$	(06D,1BE,3EB,0BE)
(000,020,000,000)	$\mapsto$	(39A,062,38C,2EB)	(000,020,000,000)	$\mapsto$	(3D9,069,21B,11B)
(000,040,000,000)	$\mapsto$	(006,131,32E,12B)	(000,040,000,000)	$\mapsto$	(3AA,29E,239,1C0)
(000,080,000,000)	$\mapsto$	(15E,0BF,1E2,04F)	(000,080,000,000)	$\mapsto$	(0BD,1B1,18E,2AB)
(000,100,000,000)	$\mapsto$	(17E,011,198,3C5)	(000,100,000,000)	$\mapsto$	(2D7,1F4,378,157)
(000,200,000,000)	$\mapsto$	(0E6,0ED,314,289)	(000,200,000,000)	$\mapsto$	(395,295,38D,129)
(000,000,001,000)	$\mapsto$	(075,380,371,2E9)	(000,000,001,000)	$\mapsto$	(15E,23B,378,376)
(000,000,002,000)	$\mapsto$	(38B,1A6,221,260)	(000,000,002,000)	$\mapsto$	(0D0,34D,18C,354)
(000,000,004,000)	$\mapsto$	(019,08E,280,1A7)	(000,000,004,000)	$\mapsto$	(084,128,167,20B)
(000,000,008,000)	$\mapsto$	(0DC,0B1,061,3DE)	(000,000,008,000)	$\mapsto$	(1C7,3F1,063,33C)
(000,000,010,000)	$\mapsto$	(189,2AB,1A6,39D)	(000,000,010,000)	$\mapsto$	(141,222,031,28A)
(000,000,020,000)	$\mapsto$	(1B2,0A7,178,208)	(000,000,020,000)	$\mapsto$	(009,1D9,3CC,131)
(000,000,040,000)	$\mapsto$	(269,2CC,27E,1CD)	(000,000,040,000)	$\mapsto$	(169,1A1,02D,39B)
(000,000,080,000)	$\mapsto$	(09A,1DD,336,34B)	(000,000,080,000)	$\mapsto$	(0C8,111,34B,38E)
(000,000,100,000)	$\mapsto$	(2D5,29F,072,04D)	(000,000,100,000)	$\mapsto$	(263,36C,361,369)
(000,000,200,000)	$\mapsto$	(009,175,254,3ED)	(000,000,200,000)	$\mapsto$	(0A6,050,36D,016)
(000,000,000,001)	$\mapsto$	(28D,172,3EA,24E)	(000,000,000,001)	$\mapsto$	(015,371,2DC,0E2)
(000,000,000,002)	$\mapsto$	(058,044,3A0,281)	(000,000,000,002)	$\mapsto$	(04A,1EC,1B6,3B4)
(000,000,000,004)	$\mapsto$	(22D,1C8,221,18B)	(000,000,000,004)	$\mapsto$	(2BE,1DD,223,1FA)
(000,000,000,008)	$\mapsto$	(370,1D0,3CD,07F)	(000,000,000,008)	$\mapsto$	(322,319,244,300)
(000,000,000,010)	$\mapsto$	(256,130,382,067)	(000,000,000,010)	$\mapsto$	(19A,0E6,364,0F2)
(000,000,000,020)	$\mapsto$	(37F,282,3A4,3D8)	(000,000,000,020)	$\mapsto$	(13C,355,058,07F)
(000,000,000,040)	$\mapsto$	(165,3BA,19B,0F7)	(000,000,000,040)	$\mapsto$	(211,2D9,1B2,362)
(000,000,000,080)	$\mapsto$	(1C7,259,17E,0BE)	(000,000,000,080)	$\mapsto$	(14F,3D2,0E2,1C7)
(000,000,000,100)	$\mapsto$	(38E,3D2,2CD,21C)	(000,000,000,100)	$\mapsto$	(005,38F,215,2DF)
(000,000,000,200)	$\mapsto$	(099,176,3BC,031)	(000,000,000,200)	$\mapsto$	(03D,208,27E,249)

**Fig. 3.** Specification of  $M$  and  $M^{-1}$

$S_0$	..0	..1	..2	..3	..4	..5	..6	..7	..8	..9	..A	..B	..C	..D	..E	..F
00.	08A	026	0A0	1E1	183	3DB	1A4	083	110	350	085	2E5	3B4	195	359	2E6
01.	33A	26B	209	217	1CE	2E3	0C0	136	129	0C8	3D6	054	040	3F2	09F	322
02.	11B	07F	139	07D	2CF	02A	268	227	246	1C5	12B	3B6	16C	20D	1E7	35B
03.	313	0CD	11E	1E6	117	355	182	0E6	094	1B9	19C	28C	2B9	336	0AF	19D
04.	2BC	1A9	31B	02E	282	2AE	272	2E9	3AA	1DD	013	2D3	30F	35A	159	1BB
05.	11C	12A	248	3C7	28B	191	025	173	018	38D	1A1	185	007	156	378	312
06.	0C9	143	05D	3FA	038	3DE	081	0F9	2D1	3FB	1C7	3E0	1DC	16A	2D8	23F
07.	030	1EB	3AF	311	36D	3BD	3C9	348	261	1AF	071	3EE	3BA	3AB	1B8	3CA
08.	22B	118	279	0F6	3FF	122	1B2	360	1D6	1B6	3D4	3BB	3B3	0EA	097	308
09.	3A9	086	0AE	15A	253	058	0BB	3D5	01D	1A3	23E	053	35D	277	384	0E2
0A.	233	2B8	2AF	0D0	1B1	105	0B3	215	2A2	27F	2DB	17E	12C	3A2	18E	2AC
0B.	321	09C	294	04C	036	2F1	3D2	18D	188	349	128	069	198	2F4	3DC	370
0C.	138	324	23C	1FD	082	247	005	0A3	0F0	273	152	17B	1A0	1C8	04E	34C
0D.	12F	0CC	075	10E	290	021	1AE	211	3E6	17A	276	289	3B5	123	01F	048
0E.	201	08F	29A	002	179	32E	120	1AC	1E3	109	079	37C	297	096	12D	323
0F.	165	04C	18B	0AB	1FF	230	25B	3D3	111	07E	21C	1BE	187	30E	34A	318
10.	269	343	29F	395	1AD	1D2	023	2DE	1B3	35E	2D7	044	206	3F1	310	0A7
11.	287	3C3	2A5	213	3E4	3DA	0FD	140	38E	2C2	154	254	15F	02C	1FB	1ED
12.	1C6	051	062	090	214	14B	190	15D	0A1	186	032	0B9	1DA	239	3D1	383
13.	331	06D	02D	009	2FC	3AD	2AA	363	1EF	38F	39A	2DC	3BF	106	39B	31F
14.	03E	0DE	18C	067	0CF	155	2CE	240	05E	0E8	0C4	149	08C	3E5	2A1	150
15.	1D1	228	3DF	0E0	3F6	193	19B	27D	2B0	35C	0E3	171	180	022	00E	358
16.	161	0EE	365	15B	0C3	2D0	3E1	06C	119	283	0F1	3B9	212	226	076	382
17.	38C	1D3	15C	082	22C	314	056	216	364	3DD	1E9	020	176	389	2F2	073
18.	06F	27E	027	14E	177	26D	1BA	0EC	25A	194	3C6	2F9	221	0E1	3F4	0B7
19.	14F	293	144	0FB	2F0	3ED	0F4	1CC	0C6	065	028	315	3E2	2DD	274	0FA
1A.	0D3	041	080	2C5	072	08D	339	2A3	1F1	1DF	2F5	267	015	0B1	275	21B
1B.	091	03F	259	18F	1C3	27B	319	153	0D2	0BD	2D0	064	000	379	2F6	2A9
1C.	142	0F5	3EF	03B	3F8	344	3BC	265	0E7	334	238	08E	347	174	18C	162
1D.	112	1D0	01C	292	2C0	0E9	2B6	301	0C1	3D0	369	1C0	1E4	1F7	08A	2FA
1E.	3CB	34D	2BE	28F	09A	39D	232	262	333	2F8	397	2C4	06E	27A	317	017
1F.	327	26C	325	167	05B	36C	362	004	3F7	0F7	20B	22D	222	2D2	0CA	196
20.	33F	3B2	17D	302	146	170	367	18A	1DE	0B5	099	3BE	2C1	0BC	2A0	01B
21.	11D	010	342	169	366	2EC	088	361	291	131	2FF	199	1CA	3B0	00D	24F
22.	2B7	063	3EB	281	0A5	070	1C8	07B	270	2CC	398	32B	1C1	396	278	39E
23.	160	0FF	1A2	0D4	024	24B	178	1BD	326	2EF	28D	392	21F	24A	10B	042
24.	141	256	229	218	0EB	260	145	050	035	0E5	300	3AE	1E2	34E	223	20A
25.	164	02F	0C5	210	1A6	258	3F5	32D	1B4	2EA	1C4	3D0	381	371	2D9	101
26.	3C8	3F3	1F2	10F	0D1	1BF	2D6	320	390	25E	249	341	33B	203	087	23A
27.	09E	095	2C8	3A6	0F2	263	108	307	3E8	3C4	2BB	14C	36E	13E	2C9	376
28.	014	00F	0DA	133	163	05C	0AA	1E5	019	37D	043	1FC	184	07A	3FE	03D
29.	0FE	25F	26E	3B7	135	2E8	3B1	1B7	012	2CA	0C2	113	001	271	1D8	01A
2A.	16F	1C9	0AD	236	299	3CF	3EC	24E	3F0	1D4	3CC	2BF	2C3	338	1B5	25C
2B.	181	052	243	1F3	11F	2EE	332	32C	034	3A8	2B4	34F	031	305	006	124
2C.	13F	13C	19E	3A0	17C	2E2	3CE	345	3CD	0EF	205	31A	23D	06B	059	19F
2D.	1E0	3D8	3F9	103	337	0DB	14D	353	127	0CE	385	114	107	3D7	057	288
2E.	04F	2B2	2CB	039	234	2B5	2E1	32A	2FB	115	116	37B	3A5	092	373	17F
2F.	21E	2AB	37F	2FD	2ED	2BA	1EA	125	208	16E	33C	0A9	2F3	3C2	3C1	21D
30.	11A	0A4	3EA	047	157	25D	1D9	10A	16D	20E	098	2B1	340	22E	241	078
31.	1F5	0ED	31E	298	347	30C	1CF	05F	351	0E4	335	046	151	24C	1EE	235
32.	12E	2A6	1A5	061	3A1	29C	011	066	093	03A	38A	1F8	1F0	084	134	356
33.	225	20C	3D9	2E4	0A8	0BE	1FE	0FC	0C7	377	2F7	07C	074	045	1E8	05A
34.	36B	36F	37E	375	04D	1FA	257	13B	089	220	399	008	158	2D5	068	280
35.	357	0DD	0BF	1B0	2A7	23B	255	3FC	00A	330	244	200	016	008	126	0A6
36.	09B	37A	284	2D4	0F3	28E	237	31D	0DF	368	386	060	374	31C	033	26A
37.	100	394	1F9	04B	391	39F	30B	00C	077	2EB	3E3	231	29B	049	202	224
38.	132	2DA	2A8	286	06A	189	130	13D	1EC	29D	104	387	32F	316	207	137
39.	0DC	02B	1D7	21A	354	39C	0B6	329	285	3A4	0D9	245	2B3	0D6	33E	252
3A.	0CB	1DB	172	296	192	04A	244	250	1F6	2AD	2C6	346	09D	388	328	3A3
3B.	2C7	3E7	29E	3C0	0D5	22A	1F4	168	3FD	242	102	3C5	0F8	251	264	2DF
3C.	27C	029	003	3B8	10C	380	10D	295	303	197	1CD	219	13A	306	166	304
3D.	175	19A	0D8	28A	0A2	26F	3B8	1C2	148	30A	0B8	24D	1A7	121	15E	372
3E.	0B4	266	22F	2FE	0B0	055	01E	3AC	14A	2E0	34B	1D5	3E9	393	2E7	037
3F.	20F	0D7	1A8	1AB	16B	36A	352	204	2BD	08B	147	1AA	35F	03C	309	33D
$S_1$	..0	..1	..2	..3	..4	..5	..6	..7	..8	..9	..A	..B	..C	..D	..E	..F
00.	021	09B	37A	3AB	0DF	016	1FE	004	07C	3BE	141	397	300	185	00C	1A7
01.	2FA	3AA	235	0B9	003	3CF	14A	18F	356	363	173	2E4	168	0CF	373	379
02.	2CA	326	16B	393	283	2E0	2B9	3E9	12F	247	3D8	07B	288	146	30F	267
03.	15C	01F	22C	0F8	10F	35D	367	343	1EC	047	008	062	2CF	3D6	36B	148
04.	0B4	2E3	25E	234	0D2	1F8	184	2FF	2EB	2BB	3A1	34F	312	10B	2EA	04D
05.	1B1	2FE	084	229	216	337	0D4	08D	21F	035	164	32A	1AA	182	24B	1BF
06.	245	257	01E	34E	375	197	292	1DD	14D	190	27E	13D	137	3A3	228	392
07.	010	34C	389	114	3B9	28B	325	210	1E7	30B	388	1A1	094	088	038	1C2
08.	305	38E	112	0AA	01B	260	3C1	104	30E	3D4	0EF	079	347	382	22E	09D
09.	1E6	087	278	20D	25B	060	215	2C6	3E0	055	3F9	179	252	1B5	105	368
0A.	029	1E9	2C4	2C5	037	233	204	133	3BD	20B	37D	1AE	03D	116	1B2	2F3
0B.	266	333	08F	050	1B9	328	26F	1EA	1A9	0E6	291	2ED	05E	162	1EE	362
0C.	15B	351	20F	17D	088	2D5	259	271	14F	2F5	011	3E7	14B	391	248	0B2
0D.	119	3CD	160	23E	06A	0D0	3C3	01C	171	3D3	349	061	16F	0FB	1DF	342
0E.	082	068	218	2E9	3B3	225	2F9	230	020	223	151	0C5	2A9	0FE	096	045
0F.	0F2	0DA	03A	015	049	370	14C	255	369	193	38A	20E	0B1	3A6	039	387
10.	24C	030	315	3CA	0A1	0C6	02C	203	107	115	3FE	244	26C	264	106	109
11.	123	090	36F	28F	1A3	19D	0BE	317	19B	25C	117	0ED	395	0BF	37E	3E4
12.	04C	3FB	103	2E6	3C8	11E	3D1	279	316	38C	277	286	0B1	074	213	1F7
13.	3C5	095	2FC	09F	2B5	332	05C	31F	324	09E	2DD	3FC	19F	111	2A7	2B0
14.	091	329	106	10E	012	273	2EC	341	0B0	174	2DB	1C7	102	2D3	2EE	1B0
15.	03F	2D4	364	131	0A6	275	00A	386	052	3DC	339	11A	211	02A	27F	0DD
16.	318	27B	17B	2D7	1E4	285	0AC	269	3F4	1EF	093	3BB	307	08E	3B0	0EB
17.	209	2CB	0BB	3A5	129	1CA	027	028	3E6	064	221	125				

$S_2$	..0	..1	..2	..3	..4	..5	..6	..7	..8	..9	..A	..B	..C	..D	..E	..F
00.	12E	38B	18E	131	039	10D	2DE	246	286	2BE	315	384	21D	142	06D	0CA
01.	2A2	2CE	264	085	374	3BB	3B9	187	3E6	3BC	207	002	392	1B5	0BA	318
02.	39C	2EE	1DA	125	019	063	27E	126	19A	082	305	0E3	206	0A0	009	3C6
03.	100	3F3	2AD	199	102	108	1DB	2F0	310	245	0A8	116	022	3C1	028	332
04.	1E1	2E7	0DA	2B7	0CB	07C	2A0	240	150	165	258	2C8	0C4	334	36B	2D1
05.	1E0	138	39A	0FF	1A7	10C	353	19B	171	038	3BD	000	3A2	1B8	282	2EB
06.	1D4	3D4	20F	23C	0D7	154	012	0DF	3A8	237	09E	155	2E2	189	2F2	136
07.	1B4	381	273	123	052	12C	158	033	2D2	3D3	23B	3B8	2F7	160	341	124
08.	337	1E6	3BE	327	1D3	045	2E4	107	1C2	263	2A4	2CF	244	196	36A	16D
09.	0A1	2C3	004	049	209	3A3	221	361	01E	1B0	05D	319	21B	249	2B2	399
0A.	198	26A	080	1B1	340	28A	33C	316	0FC	37F	1A8	134	17F	3DF	34F	3E5
0B.	2D9	32A	34A	1D1	09D	3FB	0BE	3EA	383	036	3B6	222	22E	2B6	3A6	0FA
0C.	1C1	0B2	113	3E8	129	34C	153	333	07E	01F	01D	213	299	0F8	130	1B9
0D.	182	0A2	1A1	3D5	119	10F	24C	020	097	3F0	280	112	04C	14D	1EB	307
0E.	386	0AE	322	2FE	0C5	3D7	1AF	345	05B	3F5	110	1C8	03C	1C5	35A	0C0
0F.	3F1	15B	338	1CB	0F4	2B4	00F	3A1	242	03D	29D	1B3	003	114	3FA	313
10.	35F	217	261	2C1	15D	28F	390	1C9	1DD	3C7	14F	11D	066	04D	03B	0E9
11.	2BA	2FD	347	191	04A	0BB	194	148	256	360	326	257	1AE	396	09B	2CD
12.	1E7	3CD	1FF	269	040	3E7	0BA	216	0C9	33B	3D9	1BC	2B1	325	11B	16F
13.	053	22A	186	180	27D	11F	2A9	13E	3E1	0D4	24E	1D2	2FC	3C9	1FB	31A
14.	3DE	1D7	025	372	339	2C7	2ED	25F	0A7	098	2EF	247	0E8	2D3	105	09F
15.	2CC	36D	31F	24B	1D8	241	068	211	2AF	0AA	355	35C	026	2BD	238	0EF
16.	35B	233	05A	1BE	291	368	137	035	298	140	26B	1E4	379	07F	3EB	164
17.	20B	12D	375	1BF	12F	1AA	18B	268	3F4	364	0F7	1CC	0B9	3C5	060	19D
18.	22B	17C	11C	0B1	23A	3B4	05F	2F5	219	224	0E5	042	06F	39D	21B	023
19.	1DE	177	190	395	274	359	0E2	2E9	397	0F1	010	099	17D	08E	314	317
1A.	0DC	03F	1AC	1A6	132	152	195	3AD	3E9	3C2	1BB	0F0	0CD	074	178	174
1B.	184	3E0	389	2FB	1A9	0B7	250	27B	06C	13B	0FB	296	297	30F	350	14E
1C.	007	10E	19C	055	351	034	175	103	272	02D	2C0	21C	047	20D	0EA	29B
1D.	13F	1DF	162	376	0BF	1CA	3EC	2B9	3FE	388	133	0A9	33A	304	1FE	059
1E.	13D	0BD	294	02B	127	1E8	275	07B	14C	018	031	1C6	0A3	0EC	27C	087
1F.	3BD	380	284	1FA	1F5	00A	3E2	02E	228	285	34B	311	075	2F1	1C4	094
20.	3FF	202	27F	2F9	30D	135	33F	301	3D8	2C6	3D2	309	057	073	1F1	289
21.	3B5	3CE	111	0B4	20E	1BA	1F7	24A	394	157	366	336	39B	017	25C	3C4
22.	1EC	2BC	144	1E9	193	16A	33D	344	295	079	027	2D4	38A	17A	292	0AC
23.	0F2	3E6	1EF	0BB	106	071	2DA	3F7	084	037	2AB	330	0B0	2DB	07A	22D
24.	00C	149	0AF	290	2E0	122	283	32E	3AE	3C3	1D9	2E5	37B	0BC	265	32D
25.	089	2CB	115	081	18A	255	05C	1A3	287	0D0	276	32C	0C3	30B	226	1C0
26.	2F3	0A5	121	2AA	21A	091	208	3EE	230	320	385	28E	21E	3F9	11E	05E
27.	159	281	0C8	37C	0DD	188	04F	26E	33E	2F8	3A0	3B3	3C8	227	1A2	3AA
28.	302	36E	3BF	19E	212	13C	24D	0B3	141	3EF	1CE	262	145	362	346	176
29.	1E3	14B	349	3DD	093	3F8	070	0D3	1EA	3BA	248	146	201	243	1F6	205
2A.	1CD	20A	2F6	00E	267	26D	2A7	1FC	2D0	0D1	38E	006	30A	3E3	2C5	28B
2B.	2D5	3A7	1D5	3A4	101	2D7	34E	2B5	072	26C	090	1F8	1F9	3AF	1F0	0C2
2C.	2C2	21A	06A	0AB	1EE	109	16B	15E	161	38C	156	271	279	369	342	1D6
2D.	01A	016	352	173	34D	354	181	185	1A5	23F	16C	030	215	1C3	2EA	0CF
2E.	0DE	078	18D	01B	117	393	3F2	39E	37D	1BD	24F	18C	29A	0A4	08D	187
2F.	015	065	0C1	251	00D	348	014	21F	001	008	2CA	321	1B2	1B6	043	147
30.	223	0B6	054	1AB	0FD	373	31E	323	20C	151	10B	288	2DF	041	349	2E3
31.	32F	0ED	277	179	278	3F6	23E	252	077	04A	120	200	308	300	312	04B
32.	1ED	048	30C	183	0D2	39F	3B7	0AD	3FD	204	050	1C7	197	3BF	046	04E
33.	1F3	343	051	1F2	169	266	25A	26F	0F3	2B0	095	17B	31B	0F6	05E	2DC
34.	225	36C	377	253	058	0E1	021	31C	3D6	1AD	167	2AC	06B	23D	398	032
35.	35D	2FA	00B	391	239	0F5	335	02C	083	143	02A	29E	36F	214	104	14A
36.	0E4	096	19F	2E1	1FD	30E	28D	07D	11A	0EE	0EB	370	358	1DC	163	056
37.	367	03A	2D6	363	229	3DA	08B	1E5	270	2E8	2FF	168	10A	2AE	170	28C
38.	2A3	08C	1CF	076	3D1	32B	2EC	2A5	2C9	2A6	29C	3ED	09C	0CC	2A8	203
39.	0FE	293	29F	2F4	0E7	232	0CE	3AB	13A	011	3D8	220	0D8	1F4	22F	236
3A.	062	1A4	27A	128	329	324	067	365	024	2B8	3E4	0D6	3AC	3A5	172	306
3B.	16E	0DB	3C0	25B	088	2D8	303	380	259	2A1	1E2	0B5	02F	029	356	2BF
3C.	2C4	03E	2BB	3B1	17E	3CC	01C	25E	2B3	069	0A6	15C	0D5	3DC	118	2E6
3D.	2DD	235	1BF	371	064	260	0C7	0E0	0C6	1D0	254	0D9	37A	387	3CB	234
3E.	3D0	15F	3B2	08F	15A	013	331	328	06E	25D	0F9	092	166	378	31D	139
3F.	005	09A	12B	061	231	1A0	3CF	3CA	382	192	086	357	22C	12A	3FC	37E

$S_3$	..0	..1	..2	..3	..4	..5	..6	..7	..8	..9	..A	..B	..C	..D	..E	..F
00.	200	084	1B5	30A	25A	151	174	3F9	113	3B4	35B	291	332	170	021	31E
01.	00E	2FC	023	0B0	3A9	259	2BC	378	031	050	0D0	1FF	26C	0D5	214	23E
02.	1AB	0AB	3AC	036	0E2	2F6	07A	0EA	2CB	0FE	24E	280	138	073	219	37A
03.	2E2	27C	032	162	285	13C	0B6	1ED	0B3	2F5	2C6	34B	335	1EF	26E	073
04.	273	17E	30F	2E7	14B	3BC	1CE	039	315	01A	144	1C4	20A	17B	362	10D
05.	235	1D9	2F9	0A4	052	0E3	0BD	061	02C	140	0E1	156	10E	250	288	1BE
06.	07C	2B8	05D	242	192	0A8	3B0	0DB	129	2AF	063	3AF	3D1	0C8	0A6	029
07.	2B9	3B8	0D2	078	2A2	06E	2CF	3CF	0EF	0E7	019	1F1	07E	1BB	2C7	251
08.	36A	2CA	076	216	2E5	0E6	1DD	2FE	390	277	1D2	394	2C5	022	05A	396
09.	0F4	265	0FD	150	057	111	2EC	29C	3DF	11F	13A	158	388	1D3	3C8	386
0A.	38B	279	064	1A4	028	22F	1D5	352	2C8	257	3C4	355	104	322	2C1	382
0B.	1DF	1A9	137	3DC	015	096	2AA	2A4	3F6	1A3	3DA	086	2E8	343	233	11A
0C.	0A5	38D	328	348	292	132	3F4	059	31C	1AC	1C6	3BF	1C2	36D	1D8	0ED
0D.	191	3D3	3D4	3DE	0E8	373	034	23C	224	3C5	11E	393	00B	308	2DA	00F
0E.	209	230	19D	184	1B8	339	360	2D7	011	305	17A	324	344	128	3F0	073
0F.	317	0B4	0BD	18E	035	0C9	345	0D3	37D	3CA	284	3EF	0D0	197	36B	06B
10.	08B	10B	18A	218	046	32A	2CC	0A5	254	3FC	066	246	24D	232	0A2	145
11.	2DB	199	37F	1E1	392	3F3	1C8	1CD	136	2D0	325	27B	068	1F5	077	22D
12.	12F	2F4	0B2	2E9	3CC	296	2EA	116	30D	276	02D	266	09C	25E	157	195
13.	3A1	3F2	3D7	130	258	227	0D4	26B	027	1EA	379	329	179	2D5	0C4	09F
14.	39E	09E	1FD	15B	126	2B3	15E	012	21A	372	356	154	042	017	217	19B
15.	1F8	261	3ED	14A	1FB	110	037	1B7	079	045	3C9	0EE	2B6	107	3CB	302
16.	19E	21D	1E5	205	25F	3BD	196	198	337	069	32E	0DF	3EE	201	0BC	3C2
17.	01D	37B	3C0	0A3	22E	123	2A9	0DE	2A7	2FF	3A5	05B	38F			