Portfolio Optimization using MPT & CAPM

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Portfolio Optimization using MPT & CAPM

A variety of methods can be used to optimize the choice of stocks. In our report, we mainly use MPT & CAPM to predict three indicators to determine the merits of a portfolio: Sharpe Ratio, return, and risk.

Keywords: Modern Portfolio Theory (MPT), Capital Asset Pricing Model (CAPM),

Project Overview

Our project objective is to select the best asset distribution, out of the set of all portfolios being considered so as to compare the expected portfolio return, expected volatility, and the Sharpe Ratio when minimizing the variance (risk) and when maximizing the Sharpe Ratio. We compare the expected portfolio return, expected volatility, and the Sharpe Ratio when using different approaches: Modern Portfolio Theory (MPT) Vs. Capital Asset Pricing Model (CAPM). For MPT, we used Python, and for CAPM we used.

In the following sections, we will show the data and methodologies we used. We will also explain our detailed analysis upon each step in Excel and Python. After that, we will conclude the whole thing we did in the end.

Data and Methodologies

Data

1. **Technology**: AAPLConsumer
2. **Technology**: Adobe Inc. (ADBE)
3. **Consumer Cyclical**: Amazon (AMZN)
4. **Financial Services**: American Express Company (AXP)
5. **Energy**: Bristow Group Inc. (BRS)
6. **Utilities**: FE
7. **Healthcare**: Edwards Lifesciences Corporation (EW)
8. **Technology** GOOG
9. **Health Care**: JNJ **Consumer Discretionary**:WMT
10. **Financial Services**:Mastercard Incorporated (MA)
11. **Consumer Cyclical**: Netflix (NFLX)
12. **Technology**: NVIDIA Corporation (NVDA)
13. **Healthcare**: Regeneron Pharmaceuticals, Inc. (REGN)
14. **Basic Materials**: The Sherwin-Williams Company (SHW)
15. **Consumer Defensive**: Walmart Inc. (WMT)

To diversify our portfolio, we chose 15 stocks from different sectors. Historical adjusted prices were collected from Yahoo Finance, ranging from 12/29/2016 - 10/26/2018.

Methodologies

1.Modern Portfolio Theory (MPT)

We use this method in python, which is introduced by Harry Markowitz. It assumes investors are only concerned about efficient portfolios. Its key insight is that an asset's risk and return should not be assessed by itself, but by how it contributes to a portfolio's overall risk and return. It uses the variance of asset prices as a proxy for risk.[1]

2.Sharpe Ratio

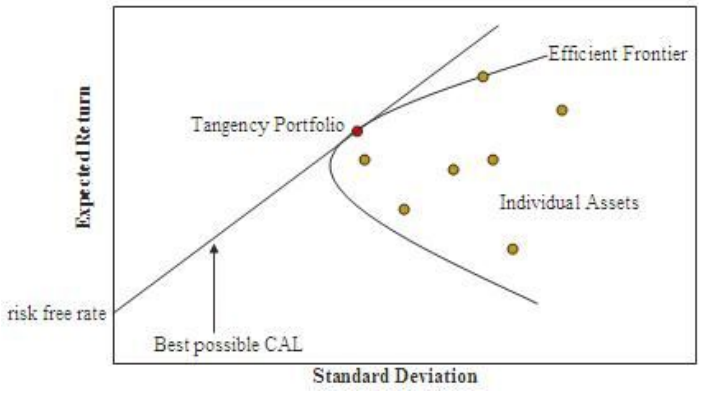
It an indicator uses standard deviation to measure a fund's risk-adjusted returns. The detailed way to calculate it is that the average return minus the risk-free return, then divided by the standard deviation of return on investment.

[1] Wigglesworth, Robin (11 April 2018). "How a volatility virus infected Wall Street". *The Financial Times*.

3.Monte Carlo Simulation

We implemented Monte Carlo simulation in python. Monte carlo simulation is used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models.[2] We use it to generate 1000,000 times of possible portfolios.

4.Efficient Frontier



We use this method in Python. As shown in the graph above, all the dots are portfolios generated by Monte Carlo simulation, but only the portfolios that lie on the curve are the most efficient ones, given a certain risk. This method focuses on the risk and return relationship for efficient portfolios. The straight line is called Capital Market Line, where it intersect with the efficient frontier is the Tangency Portfolio (with max sharpe ratio). By efficient frontier, we could also find the Min Variance Portfolio

5.Capital Asset Pricing Model (CAPM)

As an extension of Modern Portfolio Theory (MPT), it considers the relationship between expected return and risk for individual assets or securities. In Excel, we use CAPM to calculate the expected return of each stock with function presented below:

[2] https://www.investopedia.com/terms/m/montecarlosimulation.asp

$$Expected\ Return = r_f + \boldsymbol{\beta}(r_m - r_f)$$

$r_f = risk\ free\ rate$
$\boldsymbol{\beta} = Beta$
$r_m = return\ on\ the\ market$

The Beta in the function measures the volatility (systematic risk) of a portfolio in comparison to the entire market.

6.Solver

The Solver is a tool included in excel for doing optimizations. Our target is to found optimizations for three indicators:(1) Max Return, (2) Min Risk, (3) Max Sharpe Ratio.

7.Matplotlib & Tableau

We use Matplotlib & Tableau to do data visualizations.

Model-Python

Variables for the Excel Solver

There are five input variables in the Capital Asset Pricing Model. The market return and risk-free rate is generated by calculating the daily return of S&P500 Index and Treasury Bill, respectively. We have three objectives to optimize, and the common constraint for those three optimizations is that total weight equals to one. There are still specific constraints for each optimization. We

should make the standard deviation of the portfolio less than or equal to the standard deviation of each stock when maximizing the return of the portfolio, and the return of the portfolio larger than or equal to the return of each stock when minimizing the risk of the portfolio.

Input Variables: Expected return, Standard Deviation and Beta of each stock, Market Return (S&P500), and Risk-free rate (Treasury Bill).
Decision Variables: Stock weights.

Objectives: (1) Maximum return (2) Minimum risk (3) Maximum Sharpe Ratio.

Constraints: Total weight = 1

Other constraints for different objectives: (1) Maximum return: Std <= Std of each stock (2) Minimum risk: Return >= Return of each stock.

E(r), STDEV and Sharpe Ratio of each stock

For a given stock and a benchmark, the beta of the stock can be acquired by find this approximate formula , where is the return of stock, is the active return and is the return of the benchmark. The benchmark here is the market return.

	AAPL	ADBE	AMZN	AXP	BRS	FE	EW	GOOG
Beta	0.96974	1.14382	1.12396	1.44410	1.28212	0.69237	0.66123	0.95099

	JNJ	MA	NFLX	NVDA	REGN	SHW	WMT
Beta	0.56236	1.12910	0.95834	1.38815	1.14775	0.81311	0.53159

For the given beta, market return and risk-free rate, the expected return of each stock can be acquired by applying the formula . We calculate the standard deviation of each stock using the Function "= STDEV()" in Excel. The Sharpe Ratio can be obtained by applying the formula .

Individual Asset			
	E(r)	stdev	sharp
AAPL	7.069%	31.470%	22.462%
ADBE	7.789%	32.803%	23.745%
AMZN	7.710%	39.406%	19.565%
AXP	9.023%	37.396%	24.130%
BRS	8.354%	53.058%	15.744%
FE	5.943%	25.132%	23.646%
EW	5.802%	30.274%	19.164%
GOOG	7.003%	28.728%	24.376%
JNJ	5.401%	16.277%	33.184%
MA	7.732%	33.799%	22.875%
NFLX	7.019%	53.717%	13.066%
NVDA	8.793%	47.218%	18.622%
REGN	7.808%	47.808%	16.333%
SHW	6.434%	26.460%	24.317%
WMT	5.273%	19.897%	26.499%

Covariance Matrix

The covariance method is a risk management technique for calculating the value at risk of a portfolio of assets. The value at risk is a statistical risk management technique measuring the maximum loss that a portfolio is likely to face.

Our portfolio has multiple assets, so its volatility is calculated using a matrix, which is computed for all assets. The vector of the weights of the assets in the portfolio is multiplied by the transpose of the vector of the weights of the assets multiplied by the covariance matrix of all of the assets.

The covariance of a pair of stocks can be acquired by using the formula, where is the weight of each stock, is the variance of each stock, is the standard deviation of each stock, and is the correlation between two stocks. We use the Function "=MMULT()" and "=TRANSPOSE()" to acquire the covariance matrix of the portfolio with fifteen stocks.

Covariance Matrix															
	AAPL	ADBE	AMZN	AXP	BRS	FE	EW	GOOG	JNJ	MA	NFLX	NVDA	REGN	SHW	WMT
AAPL	0.099	0.0453	0.055	0.0519	0.0435	0.0218	0.0243	0.0482	0.017	0.0476	0.0449	0.0634	0.0468	0.0285	0.0177
ADBE	0.0453	0.1076	0.0586	0.0636	0.0499	0.0293	0.0318	0.0487	0.0239	0.0537	0.0508	0.0756	0.0552	0.0361	0.0222
AMZN	0.055	0.0586	0.1552	0.0603	0.0452	0.0228	0.0286	0.0605	0.0222	0.0582	0.0759	0.0687	0.0575	0.0373	0.0206
AXP	0.0519	0.0636	0.0603	0.1398	0.0726	0.0357	0.0321	0.0516	0.0275	0.0712	0.0488	0.0733	0.0623	0.0477	0.0267
BRS	0.0435	0.0499	0.0452	0.0726	0.2814	0.036	0.031	0.0418	0.0241	0.0552	0.0408	0.0693	0.0577	0.0348	0.0208
FE	0.0218	0.0293	0.0228	0.0357	0.036	0.0631	0.0174	0.0219	0.0184	0.028	0.0198	0.0319	0.0243	0.0188	0.0169
EW	0.0243	0.0318	0.0286	0.0321	0.031	0.0174	0.0916	0.0241	0.0183	0.0301	0.0314	0.0362	0.0389	0.0225	0.0157
GOOG	0.0482	0.0487	0.0605	0.0516	0.0418	0.0219	0.0241	0.0825	0.0195	0.0484	0.0443	0.0592	0.0473	0.0292	0.0181
JNJ	0.017	0.0239	0.0222	0.0275	0.0241	0.0184	0.0183	0.0195	0.0265	0.0229	0.0176	0.0246	0.0263	0.0173	0.015
MA	0.0476	0.0537	0.0582	0.0712	0.0552	0.028	0.0301	0.0484	0.0229	0.1142	0.0459	0.0674	0.0529	0.0375	0.0234
NFLX	0.0449	0.0508	0.0759	0.0488	0.0408	0.0198	0.0314	0.0443	0.0176	0.0459	0.2885	0.0719	0.0529	0.0338	0.0189
NVDA	0.0634	0.0756	0.0687	0.0733	0.0693	0.0319	0.0362	0.0592	0.0246	0.0674	0.0719	0.2229	0.0694	0.0413	0.0234
REGN	0.0468	0.0552	0.0575	0.0623	0.0577	0.0243	0.0389	0.0473	0.0263	0.0529	0.0529	0.0694	0.2285	0.0396	0.0259
SHW	0.0285	0.0361	0.0373	0.0477	0.0348	0.0188	0.0225	0.0292	0.0173	0.0375	0.0338	0.0413	0.0396	0.07	0.0194
WMT	0.0177	0.0222	0.0206	0.0267	0.0208	0.0169	0.0157	0.0181	0.015	0.0234	0.0189	0.0234	0.0259	0.0194	0.0396

Results from Solver

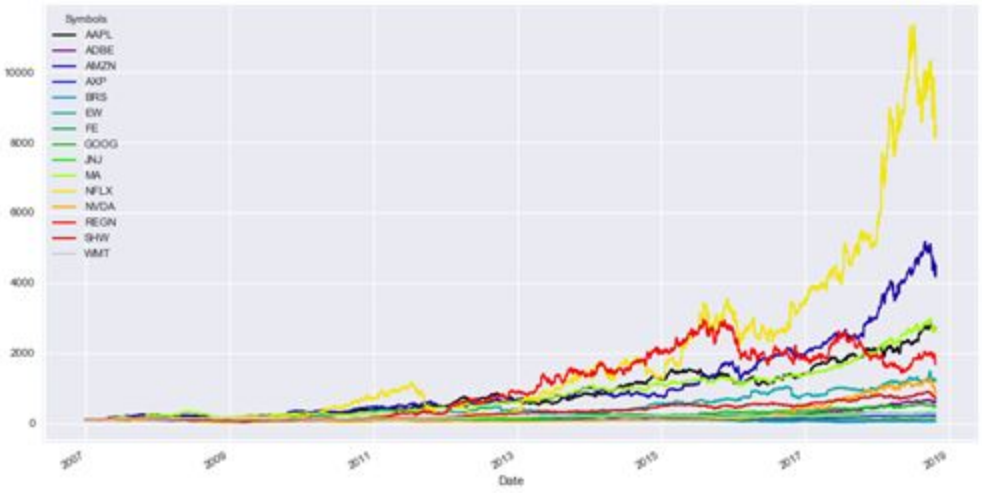
The rate of return on our portfolio is calculated as the weighted average rate of return on the multiple stocks within this portfolio. The variance of the portfolio is calculated as , where w is a column vector containing the weights of different stocks on this portfolio, is the covariance matrix, and is the transpose of the matrix .

The results indicate that portfolio with highest expected return is the most volatile, and portfolio with lowest expected return has the lowest volatility. If an investor is risk-averse, he might prefer the portfolio with 5.878% annual return and 0.1521 volatility. If an investor is a risk-tolerant, he might prefer the portfolio with 9.030% annual return and 0.3739 volatility.

Model-Python

With the same dataset, we analyzed the risks and return of different stocks.

Time-series Plot of the Data



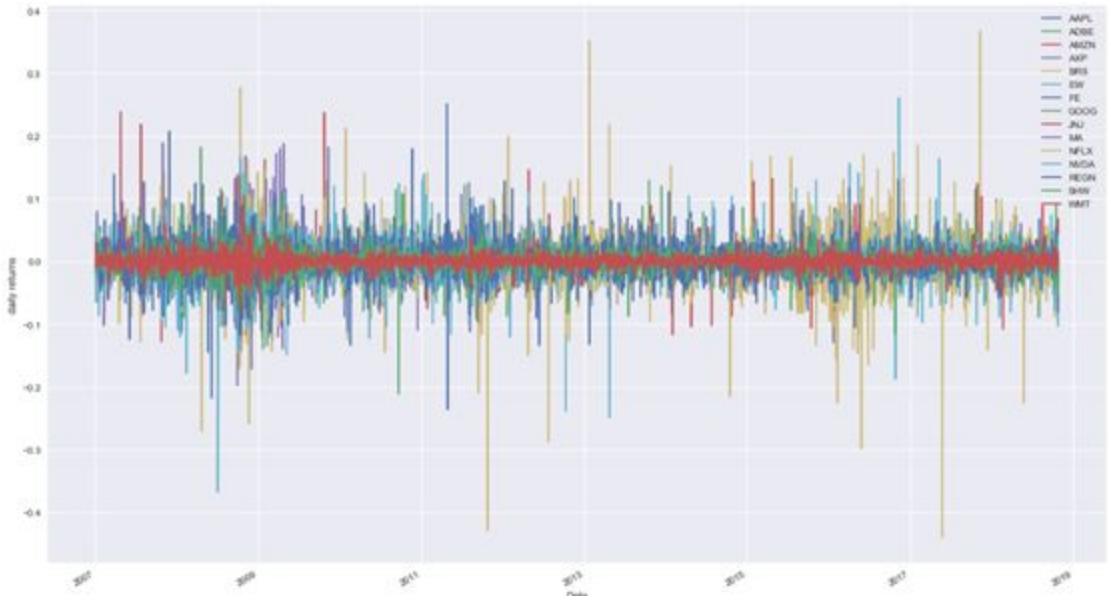
This time-series plot of shows the volatility of the different stocks. The yellow one is Netflix, and the blue one is AMAZON. They are increasing these years, which means that their risks are increasing these years. Also, the red line, Negeneron, is decreasing after 2015. Although the volatility is decreasing, its return is increasing. After adding it into the portfolio, the sharper ratio of the portfolio is higher, which is good for our result.

Daily log Return

Symbols / Date	AAPL	ADBE	AMZN	AXP	BRS	EW	FE	GOOG	JNJ	MA	NFLX	NVDA	REGN	SHW
2006-12-29	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2007-01-03	-0.012334	-0.029617	-0.019448	-0.002647	-0.037261	0.018952	0.002650	0.015322	0.005739	-0.021345	0.028590	-0.025449	-0.013039	-0.003308
2007-01-04	0.021953	0.022295	0.005155	-0.007317	-0.030961	0.001667	-0.001159	0.032963	0.012423	0.047896	-0.048508	-0.004723	0.018505	-0.005062
2007-01-05	-0.007147	-0.004912	-0.013718	-0.013272	-0.005950	-0.002502	-0.021423	0.008099	-0.009115	-0.000792	-0.021532	-0.064705	0.030261	-0.017439
2007-01-08	0.004926	-0.004194	-0.022935	0.009426	0.008025	-0.006073	-0.001862	-0.007437	-0.001652	0.000198	-0.040302	0.007400	0.016687	-0.004205

We use the daily log return to calculate the annual return

Individual Stocks' Volatility



This is the volatility of the individual stock. Some are higher, some are low, which diffuse the risk, good for our optimization.

Annualized Return

Symbols	Annual Return
AAPL	0.277590
ADBE	0.151354
AMZN	0.315646
AXP	0.059962
BRS	-0.086674
EW	0.209755
FE	0.004241
GOOG	0.130711
JNJ	0.091726
MA	0.276383
NFLX	0.372153
NVDA	0.182961
REGN	0.238486
SHW	0.164490
WMT	0.088228

There are 252 trading days in a year, we use the daily log return to get the annual return. It is calculated by the mean of daily return times the 252, There are 252 trading days in a year, that is why we use 252 here. The value of trading days is different every year, so we need to check it in a different year.

Covariance of Annualized Returns

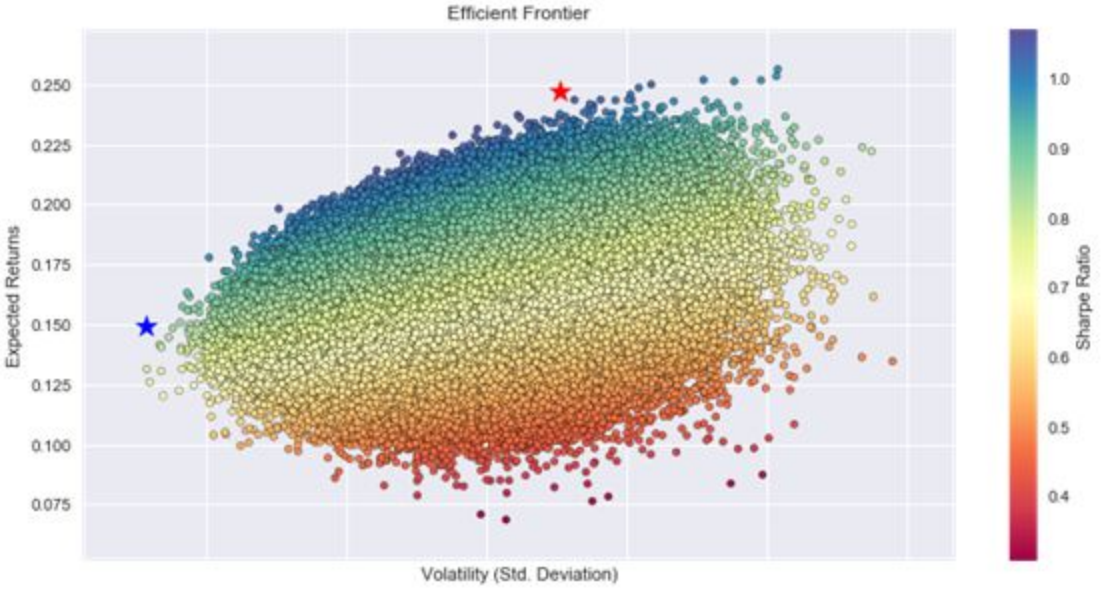
```
cov_annual = daily_ret.cov() * 252
cov_annual
```

Symbols	AAPL	ADBE	AMZN	AXP	BRS	EW	FE	GOOG	JNJ	MA	NFLX	NVDA	REGN	SHW	WM
Symbols															
AAPL	0.099431	0.045504	0.055081	0.052139	0.043469	0.024189	0.021638	0.048414	0.016935	0.047822	0.046042	0.063725	0.046705	0.028558	0.01771
ADBE	0.045504	0.107702	0.058823	0.063570	0.050128	0.032013	0.029146	0.048719	0.023825	0.053740	0.051037	0.075879	0.055317	0.036132	0.02210
AMZN	0.055081	0.058823	0.150438	0.060302	0.045269	0.028853	0.022697	0.060243	0.022169	0.058337	0.075665	0.069068	0.057385	0.037309	0.02072
AXP	0.052139	0.063570	0.060302	0.139052	0.073097	0.032202	0.035702	0.051736	0.027521	0.071425	0.048724	0.073565	0.062403	0.047752	0.02679
BRS	0.043469	0.050128	0.045269	0.073097	0.285086	0.031012	0.035697	0.041725	0.023977	0.055219	0.041180	0.069185	0.058290	0.035000	0.02082
EW	0.024189	0.032013	0.028853	0.032202	0.031012	0.093152	0.017273	0.024154	0.018277	0.030177	0.031825	0.036335	0.038833	0.022554	0.01579
FE	0.021638	0.029146	0.022697	0.035702	0.035697	0.017273	0.063028	0.021600	0.018206	0.027851	0.019585	0.031537	0.024283	0.018704	0.01676
GOOG	0.048414	0.048719	0.060243	0.051736	0.041725	0.024154	0.021600	0.081444	0.019414	0.048386	0.044372	0.059283	0.047197	0.029273	0.01797
JNJ	0.016935	0.023825	0.022169	0.027521	0.023977	0.018277	0.018206	0.019414	0.026368	0.022851	0.017577	0.024427	0.026191	0.017182	0.01494
MA	0.047822	0.053740	0.058337	0.071425	0.055219	0.030177	0.027851	0.048386	0.022851	0.112881	0.046268	0.067579	0.053123	0.037619	0.02340
NFLX	0.046042	0.051037	0.075665	0.048724	0.041180	0.031825	0.019585	0.044372	0.017577	0.046268	0.287272	0.072652	0.053110	0.033517	0.01890
NVDA	0.063725	0.075879	0.069068	0.073565	0.069185	0.036335	0.031537	0.059283	0.024427	0.067579	0.072652	0.224037	0.069388	0.041171	0.02322
REGN	0.046705	0.055317	0.057385	0.062403	0.058290	0.038833	0.024283	0.047197	0.026191	0.053123	0.053110	0.069388	0.224879	0.039670	0.02577
SHW	0.028558	0.036132	0.037309	0.047752	0.035000	0.022554	0.018704	0.029273	0.017182	0.037619	0.033517	0.041171	0.039670	0.069749	0.01925
WMT	0.017713	0.022101	0.020728	0.026794	0.020824	0.015794	0.016767	0.017977	0.014949	0.023400	0.018906	0.023228	0.025772	0.019257	0.03949

We used this covariance to calculate the stand deviation.

Portfolio Optimization using Efficient Frontier

We use Monte Carlo Simulation to run 1000,000 different randomly generated weights for the individual stocks and then calculate the expected return, expected volatility and Sharpe Ratio for each of the randomly generated portfolios.



The red star is the point which the sharper ratio of this portfolio is the biggest one among all that of the portfolios. Any Sharpe ratio greater than 1 is considered acceptable to good by investors. A ratio higher than 2 is rated as very good, and a ratio of 3 or higher is considered excellent. Although the biggest sharper ratio in our portfolios is almost 1, no so high, it is the best one in our portfolios.

The blue star is the point whose risk is the smallest one in our portfolios, we prefer the risk of our investment not too high, in order to ensure that we will not suffer too much lost.

Python Result

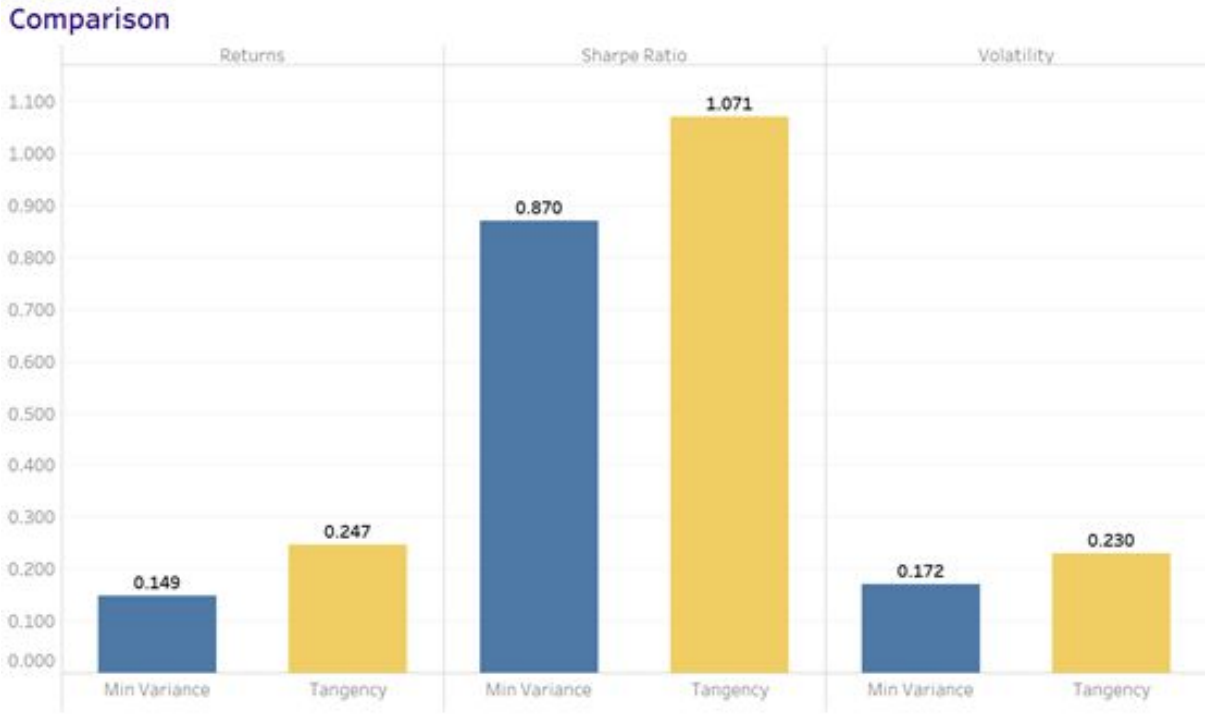
Min Variance Portfolio Weights/ Tangency Portfolio Weights

	Min Variance Portfolio
Returns	0.149296
Volatility	0.171509
Sharpe Ratio	0.870482
AAPL Weight	0.045764
ADBE Weight	0.009262
AMZN Weight	0.028537
AXP Weight	0.019046
BRS Weight	0.007946
FE Weight	0.126457
EW Weight	0.120894
GOOG Weight	0.039020
JNJ Weight	0.226423
MA Weight	0.124229
NFLX Weight	0.028474
NVDA Weight	0.022863
REGN Weight	0.010809
SHW Weight	0.056942
WMT Weight	0.133335

	Tangency Portfolio
Returns	0.246810
Volatility	0.230457
Sharpe Ratio	1.070959
AAPL Weight	0.169399
ADBE Weight	0.051639
AMZN Weight	0.100744
AXP Weight	0.026263
BRS Weight	0.005847
FE Weight	0.147941
EW Weight	0.005510
GOOG Weight	0.022043
JNJ Weight	0.020065
MA Weight	0.151688
NFLX Weight	0.164392
NVDA Weight	0.015969
REGN Weight	0.026410
SHW Weight	0.037747
WMT Weight	0.054342

Optimization Results

The volatility of the two results are almost the same, but the sharper ratio and return of the Yellow one are much higher (the yellow one is the stock with maximize sharpe ratio). We prefer sharper ratio bigger than 1, which make the investment sense. So we choose the portfolio with maximized shaper ratio as the optimization result.



Total Conclusion

We processed both MPT and CAPM so that we could do some comparisons between these two methods. By comparing the expected return, standard deviation and Sharpe Ratio for each portfolio based on Max Sharpe Ratio and Min Risk respectively, which lists below, We found MPT has higher expected return and Sharpe Ratio than CAPM.

Max Sharpe Ratio	Expected Return	Standard Deviation	Sharpe Ratio
MPT	24.7%	0.230	1.071
CAPM	5.9%	0.152	0.386

Min Risk	Expected Return	Standard Deviation	Sharpe Ratio
MPT	14.9%	0.172	0.870
CAPM	9.030%	0.3739	0.2415