Summary of the paper "Asset Pricing Under Endogenous Expectations in An Artificial Stock Market"

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1 Introduction

This paper, authored by W. Brian Arthur, John H. Holland, Blake LeBaron, Richard G. Palmer, and Paul Tayler in 1996, was initially published in the book "The Economy as an Evolving Complex System II" and later republished in May 2018(By Google Scholar). The study focuses on the creation of an artificial stock market in which multiple agents trade a risky stock and a risk-free bond, aiming to optimize their portfolios. An intriguing aspect of this market is that agents' expectations of stock prices and dividends are endogenous, meaning that they do not share their beliefs with one another and may hold heterogeneous views. However, due to the endogeneity of expectations, agents are unaware of each other's expectations, making it challenging for them to deduce their own expectations accurately. To address this issue, the authors introduce a method called "Inductive Reasoning," enabling agents to form their expectations of dividends and prices in an inductive manner.

The authors conduct two experiments: the "slow rate of exploration experiments" and the "medium rates of exploration experiment". The distinction between these experiments lies in the speed at which agents update their beliefs and predictions regarding the market. In the "slow rate of exploration experiments" , the market eventually converges to a rational-expectations regime. Within this regime, the occurrence of bubbles, crashes, and autocorrelative behavior is impossible. Additionally, agents' holdings exhibit high homogeneity, and trading volume remains low. As a result, the efficient market theory is upheld in this regime. On the other hand, the "medium rates of exploration experiment" leads to a complex regime in which psychological behavior emerges.

This regime is characterized by the presence of bubbles, crashes, and technical trading in the market. Notably, agents' expectations in this regime are heterogeneous. This regime violates rational expectations theory clearly. In the end, the author conjecture that actual financial markets lie within the complex regime.

Subsequently, we provide a brief overview of how the authors implement inductive reasoning, outline the model of the asset market, describe the agent model, summarize the experiments and their corresponding results, and discuss possible explanations for the observed outcomes in the experiments.

2 Inductive Reasoning

If every agent in the market were to form their expectations of prices deductively, they would face difficulty since they would need to consider others' expectations of dividends and prices. The paper provides an example using the arbitrage pricing model to illustrate this idea. In arbitrage pricing mode, the equilibrium price should be:

$$p_t = \beta \Sigma_j w_{j,t}(E_j[d_{t+1}|I_t] + E_j[p_{t+1}|I_t]) \tag{1}$$

where: $\{d_t\}$ = a dividend sequence with stochastic payoff.
I_t = the shared information set of the market.
β = $\frac{1}{1+r}$, where r is paid by risk-free bond.
$w_{j,t}$ = $\frac{1}{\sigma^2_{j,t}}/\Sigma_k \frac{1}{\sigma^2_{k,t}}$, where $\sigma^2_{j,t} = var[d_{t+1} + p_{t+1}|I_t]$
j = j^{th} agent in the market.

Now, suppose agent i attempts rationally to deduce the expected price at *t+1*, based on the equation(1), we have:

$$E_i[p_{t+1}|I_t] = \beta E_i[\Sigma_j\{w_{j,t+1}(E_j[d_{t+2} + p_{t+2}|I_t])\}|I_t] \tag{2}$$

Thus, the agent i has to consider his expectation of others' expectations of dividends and price at period *t+2*. To eliminate the price expectation $E_j[p_{t+2}|I_t]$, he requires a further iteration, but it also requires his expectations of others' expectations of others' expectations of future dividends and prices and period *t+3*. Clearly, the repeated iteration goes to infinity, and it may lead to instability of the expectations. Suppose agent i believes that others believe future prices will increases, he may expect upward-moving prices. However, if he believes a reversion to lower values in likely, he may expect a reversion. Therefore, we can imagine swings and swift transitions in agents' beliefs. It becomes evident that deductive logic leads to indeterminable expectations, making it challenging to define perfect rationality in the market.

To address this issue, the authors propose a new approach, called inductive reasoning, for agents to form expectations. They suggest that agents act as statisticians who observe market data and derive expectational models through subjective reasoning. These models are considered hypotheses, and there is no objective method to verify them except by observing their performance in practice. Therefore, agents continuously create multiple "market hypotheses" about market prices and dividends and test several models simultaneously. Models that perform well in predicting market movements are retained and used for buying, selling, and decision-making, while poorly performing models are discarded. New models are periodically generated and tested for accuracy, replacing weaker models. Each inductively-rational agent generates models that compete within their mind, surviving or changing based on their predictive ability.

3 Model of the Asset Market and Agents

The authors make the following assumptions about the asset market:

- N heterogeneous agents optimize allocations between a risky stock and a risk-free bond.
- Each agent possess identical utility function, $u(c) = -e^{-\lambda c}$.
- Time is discrete.
- Bond is in infinite supply and pays r every period.

- Stock is issued in N units, and pays a dividend, d_t, which statisfies:
  (a) $d_t \sim \{d_t\}$, where $\{d_t\}$ is an exogenous stochastic process and is unknown to agents.
  (b) d_t follows AR(1) model. ie. $d_t = \bar{d} + \rho(d_{t-1} - \bar{d}) + \epsilon_t$
  (c) $\epsilon_t \overset{\text{iid}}{\sim} \mathrm{N}(0,\sigma_e^2)$
- $p_{t+1} + d_{t+1} \sim N(E_{i,t}[p_{t+1} + d_{t+1}], \sigma^2_{t,i,p+d})$,which implies that the demand function $x_{i,t} = \frac{E_{i,t}(p_{t+1}+d_{t+1})-p(1+r)}{\lambda\sigma^2_{i,t,p+d}}$
- Market clearing condition at t : $\Sigma_{i=1}^{N} X_{i,t} = N$

The authors make the following assumptions about the agents:

- Agents' Expectations are formed individually and inductively.
- Agents possess a multiplicity of linear forecasting models, which allows agents simulates non-linear forecasting models.
- Agents learn by discovering which of their hypotheses proved best.
- Agents develop new hypotheses from time to time by genetic algorithm.
- Agents form their expectations by a set of predictors, which consists of two component - market conditions and forecasting formula.
- At each period, agents possess M predictors, and they will use the most accurate ones, discard non-performing ones and create new ones.
- The expectations of agents are of the form : $E(p_{t+1} + d_{t+1}) = a(p_t + d_t) + b$

Under these assumptions, agents will first observe current state of the market, and then check whether his predictors match the state and find the most accurate ones. After that, agents will forecast price and dividend in the next period and calculate desired holdings and bid. Eventually, price and dividend are revealed and agents update the accuracy of their predictors.

4 Experiments and their results

The experiments have the following common settings:

- Each experiment runs 250,000 period with 25 times under different random seeds.
- There are 25 agents, each agent has 100 predictors, and each predictors have 12 market descriptors.
- $D_t \sim AR(1)$ with $\rho = 0.95$.

The settings are for "slow rate of exploration experiment":

- The genetic algorithm is invoked every 1,000 periods on average. ie. The new predictors will be created every 1,000 periods on average.
- The predictors are crossed over with probability 0.3. ie. The probability that two predictors will give birth to a new predictor is 0.3.
- The predictors' accuracy-updating parameter $\theta = \frac{1}{150}$. Briefly speaking, the higher the parameter is, the faster predictors will be updated.

And its corresponding results:

- The regime is called "rational-expectation regime" since it converges homogeneous rational expectations.
- Agents' holding is highly homogeneous.
- The trading volume in the market is low
- No bubbles, crashes and technical trading emerge.
- Efficient-Market theory upheld.

However, there are some differences between the equilibrium in this regime and theoretical rational expectation equilibrium:

- The equilibrium is not arrived deductively but inductively.
- The equilibrium is stochastic.

The setting for "medium rate of exploration experiment":

- The genetic algorithm is invoked every 250 periods on average.
- The predictors are crossed over with probability 0.1.
- The predictors' accuracy-updating parameter $\theta = \frac{1}{75}$.

And its corresponding results:

- The regime is called "complex regime" in which psychological behavior emerges.
- Bubbles, crashes and technical trading emerge.
- 300% larger in the volume of shares traded.
- Price volatility and trading volume show persistence, autocorrelation and cross-correlation.
- Each agents' expectations of prices and dividends are heterogeneous.
- Efficient-Market theory do not upheld.

5 Possible Explanation of the Results

The authors propose that agents with a high learning rate (speed of exploration) and non-rational expectation equilibrium (non-r.e.e) beliefs can develop mutually reinforcing beliefs. Specifically, in the presence of an upward market trend, certain agents' predictors occasionally generate forecasts aligned with the trend. This prompts these agents to engage in buying activities, resulting in price increases that validate their predictors. On top of that, The authors emphasize the importance of market information in facilitating the emergence of agents with mutually reinforcing forecasts. To test their hypothesis, they conducted an experiment where condition components of predictors are turned off, so agents have no idea about the market state. The experiment confirmed their conjecture, as the complex regime failed to appear, supporting the role of market information in driving mutual reinforcement among agents.