

NCKU Programming Contest Training Course

Course 9

2013/03/13

Sheng-Chi You(rabbit125)

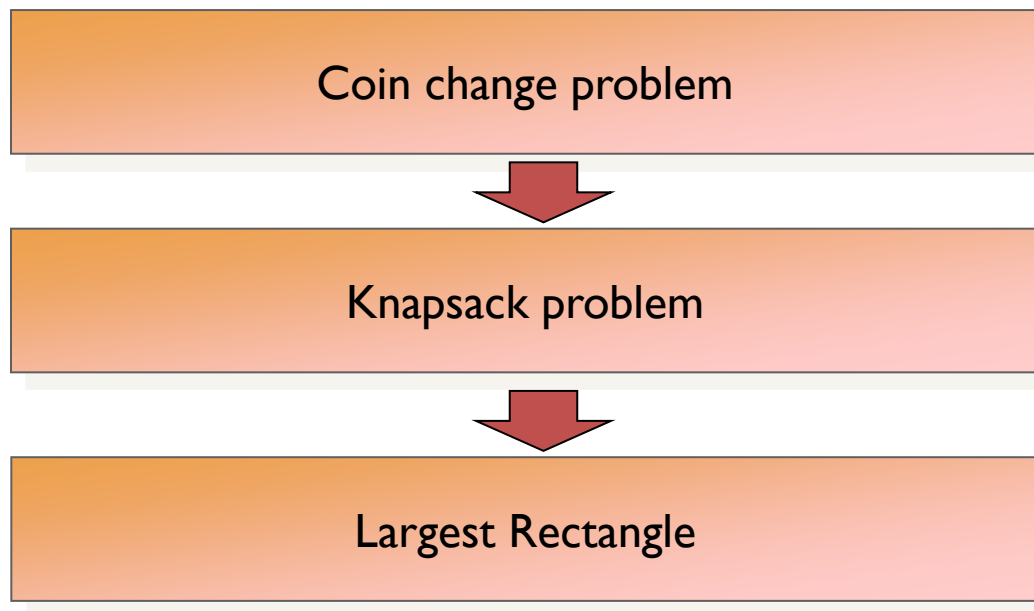
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http://myweb.ncku.edu.tw/~f74986133/Course_9.rar

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Tainan, Taiwan



Outline



Coin change problem

- Coin change problem
 - 0/1 Coin change problem
 - Unbounded Coin change problem
 - Limited Coin change problem
 - ...



Coin Change

類型

- 硬幣限制各一個(0/1 背包變型)
 - 是否湊得某個價位 / 湊得某價位的方法數
- 硬幣無限
 - 是否湊得某個價位 / 湊得某價位的方法數 / 湊得某個價位的最少硬幣用量
 - 湊得某個價位的硬幣用量 (錢用量不多時)
- 硬幣有限
 - 是否湊得某個價位 / 湊得某個價位的最少硬幣用量

相同觀念

- 按照題意設計
- 注意設定紀錄維度, 意義, 以及初始化數值



Coin Change

- 硬幣**限制各一個**，是否湊得某價位
- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	0	0	0	0	0	0	0



Coin Change

- 硬幣限制各一個，是否湊得某價位
- $Dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	0	0	0	0	0	0	0



Coin Change

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- Money value $v[i]=2,5$
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dp	1	0	0	0	0	0	0	0	0



Coin Change

- 硬幣限制各一個，是否湊得某價位
- $dp[0] = 1$;
- Money value $v[i] = 2, 5$
- $\text{if } (dp[j - v[i]] == \text{true}) dp[j] = dp[j - v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	0	0



Coin Change

- 硬幣限制各一個，是否湊得某價位
- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	0	0



Coin Change

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- $dp[0]=1$;
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- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	1	0



Coin Change

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- $Dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	1	0	1	0



Coin Change

- 硬幣**無限**，是否湊得某個價位
- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	0	0	0	0	0	0	0



Coin Change

- 硬幣無限，是否湊得某個價位
- $Dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	0	0



Coin Change

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	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	0	0



Coin Change

- 硬幣無限，是否湊得某個價位
- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	0	0	0	0



Coin Change

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- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	0	1	0	1



Coin Change

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- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	1	1	0	1



Coin Change

- 硬幣無限，是否湊得某個價位
- $dp[0]=1$;
- Money value $v[i]=2,5$
- $if (dp[j-v[i]] == true) dp[j]=dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	1	1	1	1



Coin Change

- 硬幣無限，湊得某個價位有幾種？
- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	0	0	0	0	0	0	0



Coin Change

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- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	0	0	0	0	0



Coin Change

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- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	0	0	0	0



Coin Change

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- $Dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	0	1	0	1	0	1



Coin Change

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- $dp[0]=1$;
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- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	1	1	0	1	0	1



Coin Change

- 硬幣無限，湊得某個價位有幾種？
- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	1	1	1	1	0	1



Coin Change

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- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	1	1	1	2	0	1

|+|



Coin Change

- 硬幣無限，湊得某個價位有幾種？
- $dp[0]=1$;
- Money value $v[i]=2,3$
- $if (dp[j-v[i]] == true) dp[j] += dp[j-v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	1	1	1	2	1	1



Coin Change

- 硬幣無限，湊得某個價位有幾種？
- $dp[0] = 1$;
- Money value $v[i] = 2, 3$
- $\text{if } (dp[j - v[i]] == \text{true}) dp[j] += dp[j - v[i]]$;

	0	1	2	3	4	5	6	7	8
dp	1	0	1	1	1	1	2	1	2

|+|



Coin Change

- 硬幣無限，湊得某個價位的最少硬幣用量
- $dp[0]=0$;
- Money value $v[i]=2,3$
- $dp[j]=\min (dp[j], dp[j-v[i]]+1);$

	0	1	2	3	4	5	6	7	8
dp	0	INF	INF	INF	INF	INF	INF	INF	INF



Coin Change

- 硬幣無限，湊得某個價位的最少硬幣用量
- $dp[0]=0$;
- Money value $v[i]=2,3$
- $dp[j]=\min (dp[j], dp[j-v[i]]+1);$
- **Greedy** : Money value 大的優先(找錢問題)

	0	1	2	3	4	5	6	7	8
dp	0	INF	INF	INF	INF	INF	INF	INF	INF



Practice

- 基礎：UVA 674
- 進階：UVA 10306



Coin Change

- 硬幣**有限**，是否**湊得某個價位的最少硬幣用量**
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次??

	0	1	2	3	4	5	6	7	8
dp	0	INF	INF	INF	INF	INF	INF	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次??

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	2	INF	INF	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次??

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	1	INF	INF	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次??

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	1	INF	1	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次??

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	1	INF	1	INF	2



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- 硬幣無限的方法跑 m 次?? \rightarrow *It is wrong!*

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	1	INF	1	INF	2



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- for $j = \max$ to $v[i]$

run $m[i]$ 次

	0	1	2	3	4	5	6	7	8
dp	0	INF	INF	INF	INF	INF	INF	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- for $j = \max$ to $v[i]$

run $m[i]$ 次

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	2	INF	INF	INF	INF



Coin Change

- 硬幣有限，是否湊得某個價位的最少硬幣用量
- Money value $v[i] = \{2, 4\}$
- Number of $v[i] \rightarrow m[] = \{2, 1\}$
- for $j = \max$ to $v[i]$

run $m[i]$ 次

	0	1	2	3	4	5	6	7	8
dp	0	INF	1	INF	2	INF	2	INF	3

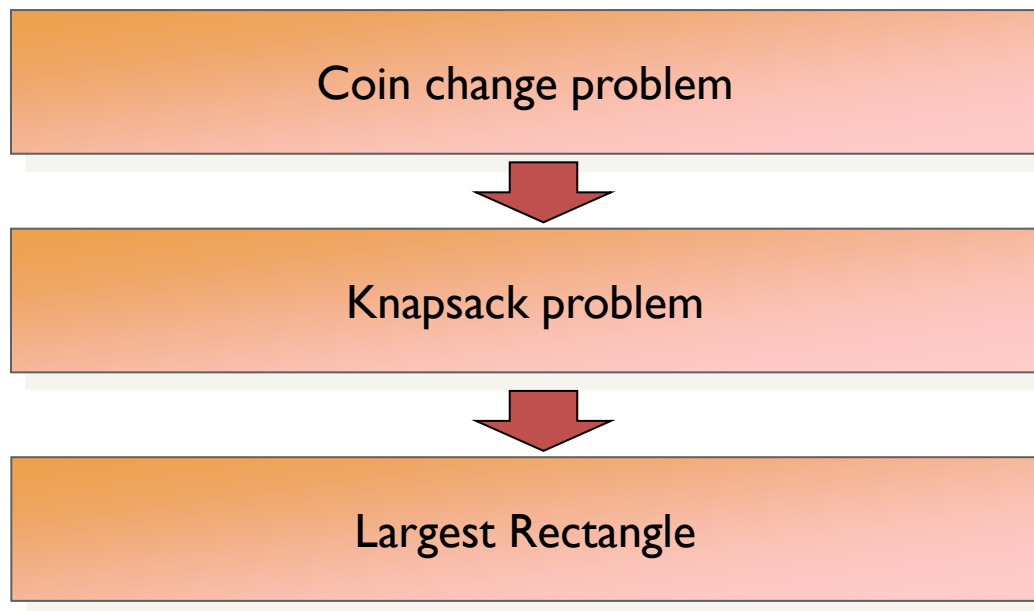


Practice

- UVA 166



Outline



Knapsack problem

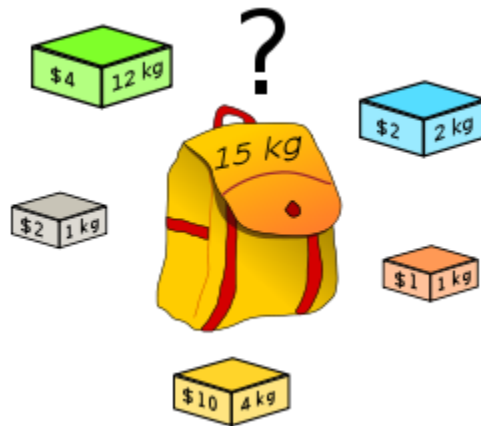
- Knapsack problem
 - 0/1 Knapsack problem
 - Unbounded Knapsack problem
 - ...



0/1 Knapsack problem

★ Problem Description:

- Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than a given limit and the total value is as large as possible.



• Brute force

- Each item has 2 status: put in the bag or not
- If there are N items, it will cost $O(2^N)$ to check all the possibilities.



0/1 Knapsack problem

- ★ Status representation and transfer function:
 - $dp[n][m]$ store the maximum value that we put some of first n items in the bag and weight m
 - $dp[n][m] = \max(\underbrace{dp[n-1][m]}_{\text{不取第}n\text{個物品}}, \underbrace{dp[n-1][m-w[n]] + v[n]}_{\text{取第}n\text{個物品放入背包}})$
- Top-down DP can be written as follow:

```

int dp[N+1][W+1], v[N], w[N];           // top-down, N items with maximum total weight W
bool isfind[N][W];
int knapsack(int n, int m){
    if (m < 0) return -INF;               // basic constrain
    if (n == 0) return 0;
    if (isfind[n][m]) return dp[n][m];   // isfind before
    dp[n][m] = max(knapsack(n-1, m),
                   knapsack(n-1, m-w[n]) + v[n]); // recursively call
    isfind[n][m] = true;                 // record isfind
    return dp[n][m];
}
  
```



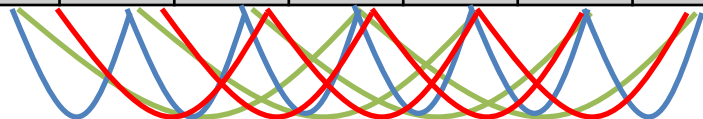
0/1 Knapsack problem

★ Bottom-up DP can be written as follow:

```

int c[W+1], v[N], w[N];                                // bottom-up
int knapsack( int n, int w){
    memset(c, 0, sizeof(c));                            // Initialize basic constrain
    for (int i = 0; i < n; i++)
        for (int j = W; j - w[i] >= 0; j--)              // Back to the front
            c[j] = max( c[j], c[j - w[i]] + v[i] );      // Update lookup table
    return c[w];
}
  
```

c[0]	c[1]	c[2]	c[3]	c[4]	c[5]	c[6]
0	20	30	30	30	30	60



item	w	v
0	3	10
1	1	20
2	2	30

W
6



Dynamic programming

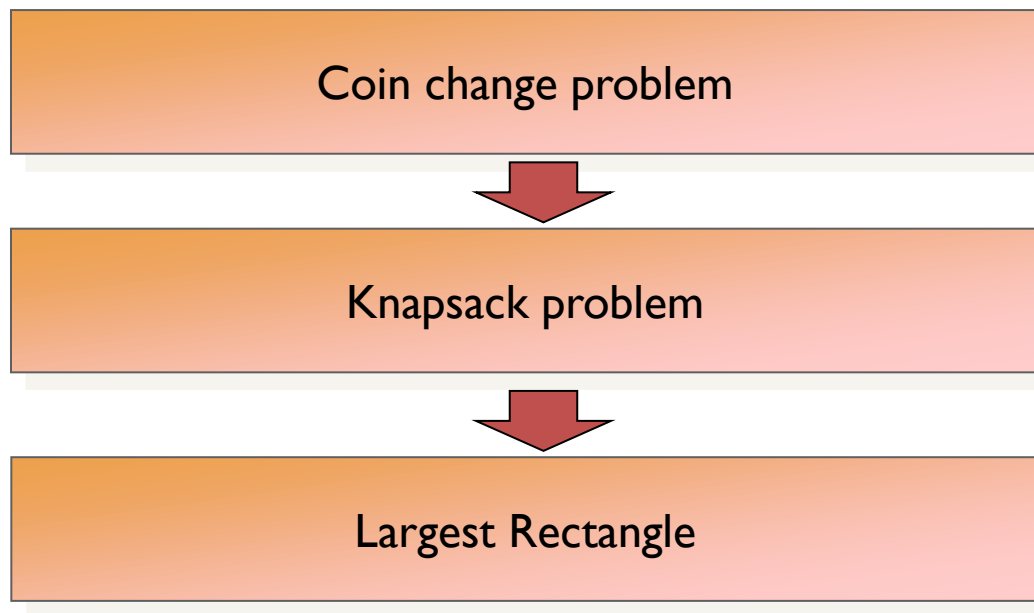


Practice

- ZeroJudge2 d155



Outline



Largest Rectangle

- Largest Rectangle
 - Maximum Sub-array Sum problem 1D (Array)
 - Maximum Sub-array Sum problem 2D (Rectangle)
 - Max size of Rectangle expansion
 - ...



- Maximum Sub-Array Sum Problem
 - Maximum subarray summation problem is to find a subarray which contains a set of continuous elements in which the summation is maximum.
 - The elements in the subarray must be **continuous**.
 - This problem can be extended to **multiple dimension**.
 - Two general method to solve this problem
 - Brute force method
 - DP based method



MSS

- Maximum Sub-Array Sum Problem

- Example for one dimension

- Given an array $a[10] = \{1, 2, -6, 3, -2, 4, -1, 3, 2, -4\}$
- Subarray can be $\{1, 2, -6\}$ with summation $= 1+2-6 = -3$, can be also be $\{-1, 3, 2, -4\}$ with summation $= -0$, and so on.

- Example for two dimension

- Given an array:

Subarray with summation value
 $= 14$

1	2	3	4
2	6	-4	3
4	3	-3	3
5	2	-1	-1



MSS (1D)

- Maximum Sub-Array Sum Problem (1D)
- Rule 1
 - Order?
- Rule 2
 - Category



MSS (1D)

- Rule 3
 - Define a *max_sum* that represents the optimal value and define a variable *sum* that represents a temporary summation.
 - Given an array $a[1 \dots N]$.

```
int maximum_subarray()
{
    int max_sum = 0, sum = 0;
    for (int i=0; i<N; ++i)
    {
        sum += a[i];
        if (sum < 0) sum = 0;
        if (sum > max_sum) max_sum = sum;
    }
    return max_sum;
}
```

// 隨時計算總和
// 零總比負數好
// 隨時紀錄最大值



MSS (1D)

- Rule 4
 - Program

```
int maximum_subarray()
{
    int max_sum = 0, sum = 0;
    for (int i=0; i<N; ++i)
    {
        sum += a[i];
        if (sum < 0) sum = 0;
        if (sum > max_sum) max_sum = sum;
    }
    return max_sum;
}
```

// 隨時計算總和
// 零總比負數好
// 隨時紀錄最大值

- Rule 5
 - Trace



MSS (2D)

- Maximum Sub-Array Sum Problem (2D)
 - UVA 108

0	-2	-7	0
9	2	-6	2
-4	1	-4	1
-1	8	0	-2



DP method

- Exercise
 - UVA 10684 (1D MSS problem)
 - UVA 108 (2D MSS problem)
- Review
 - Time complexity $O(?)$
 - Space complexity $O(?)$
 - Compare with the brute force method.



References

- 演算法筆記
 - <http://www.csie.ntnu.edu.tw/~u91029/KnapsackProblem.html#a4>
 - <http://acm.nudt.edu.cn/~twcourse/MoneyChangingProblem.html>
 - <http://www.csie.ntnu.edu.tw/~u91029/LargestEmptyRectangle.html>



Homework 9

- ZeroJudge2
 - d207, d208, d197, d167, ,d155, d156, d146, d267, d171
- PKU
 - 1050
- Uva
 - 10285
- NCKUOJ
 - 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 132



Notice

- 例行月賽! 記得先報名!!
- 3/26(二) [CPE](#) 18:30~9:30
- 3/27(三) [ITSA](#) 18:00~21:00
- 3/28(四) [PTC](#) 19:00~22:00

- 3/30~4/7 Practice!!!! Practice!!!! Practice!!!!



Homework 9+

- **PKU (total 124 problems)**

1837, 1836, 1260, 2533, 3176, 3034, 1925, 2948, 3280, 1054, 1191, 2250, 1159,
1018, 1050, 1083, 1088, 1125, 1143, 1157, 1163, 1178, 1179, 1185, 1208, 1276,
1322, 1414, 1456, 1458, 1609, 1644, 1664, 1690, 1699, 1740, 1742, 1887, 1926,
1936, 1952, 1953, 1958, 1959, 1962, 1975, 1989, 2018, 2029, 2033, 2063, 2081,
2082, 2181, 2184, 2192, 2231, 2279, 2329, 2336, 2346, 2353, 2355, 2356, 2385,
2392, 2424, 1019, 1037, 1080, 1112, 1141, 1170, 1192, 1239, 1655, 1695, 1707,
1733, 1737, 1837, 1850, 1920, 1934, 1937, 1964, 2039, 2138, 2151, 2161, 2178,
1015, 1635, 1636, 1671, 1682, 1692, 1704, 1717, 1722, 1726, 1732, 1770, 1821,
1853, 1949, 2019, 2127, 2176, 2228, 2287, 2342, 2374, 2378, 2384, 2411, 1579,
1080, 3356, 2533, 1631, 1157, 1014, 1160



Homework 9+

- **UVA (total ? problems)**

103, 108, 111, 116, 147, 164, 166, 231, 348, 357, 437, 473, 481, 497, 507, 531, 562, 590, 607, 620, 624, 674, 709, 711, 714, 787, 825, 836, 882, 907, 909, 910, 926, 944, 986, 988, 990, 991, 10003, 10029, 10032, 10036, 10037, 10051, 10066, 10069, 10074, 10081, 10100, 10111, 10118, 10128, 10130, 10131, 10149, 10151, 10154, 10157, 10159, 10163, 10166, 10169, 10185, 10192, 10201, 10207, 10247, 10259, 10261, 10271, 10280, 10285, 10296, 10304, 10306, 10313, 10340, 10358, 10400, 10401, 10404, 10405, 10453, 10482, 10496, 10534, 10549, 10558, 10559, 10564, 10593, 10599, 10604, 10605, 10616, 10617, 10618, 10625, 10626, 10635, 10643, 10645, 10648, 10650, 10651, 10654, 10663, 10664, 10665, 10667, 10681, 10684, 10688, 10690, 10700, 10702, 10712, 10721, 10722, 10723, 10739, 10755, 10759, 10817, 10827, 10891, 10910, 10911, 10912, 10913, 10917, 10918, 10943, 10953, 10970, 11002, 11003, 11008, 11022, 11026, 11052, 11081, 11087, 11125, 11126, 11133, 11137, 11149, 11151, 11153, 11158, 11162, 11171, 11176, 11238, 11258, 11259, 11284, 11307, 11311, 11312, 11331, 11341, 11370, 11372, 11391, 11394, 11400, 11404, 11420, 11421, 11427, 11432, 11438, 11441, 11450, 11471, 11472, 11485, 11500



Homework 9+

- **ZJ2 (total 9 problems)**

d013, d018, d023, d025, d034, d039, d061, d078, d079, d083



Thank You For Attention!

