

NCKU Programming Contest Training Course

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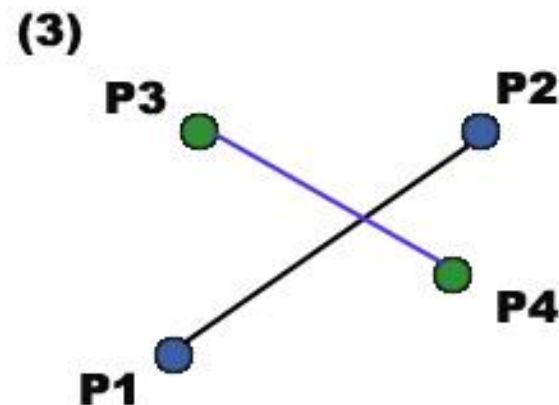
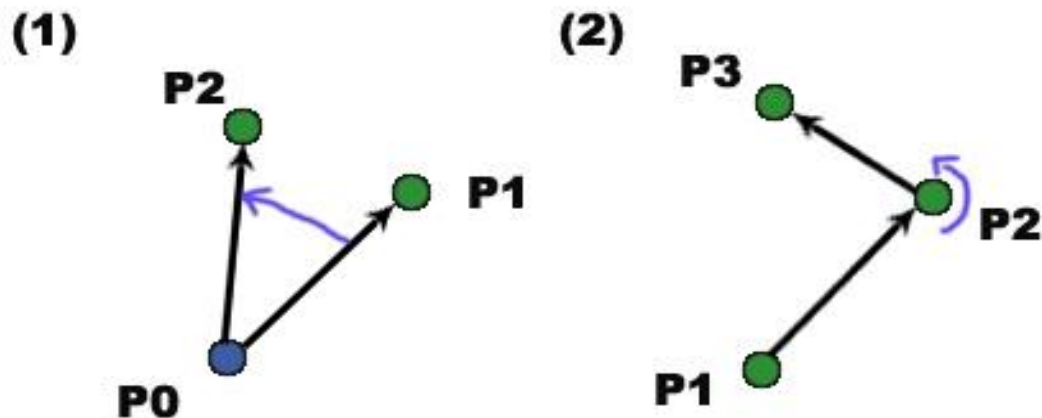
Outline

Line Intersection



Intersection

- Direction
 - Problem: Is vector $p_i \rightarrow p_j$ clockwise from vector $p_i \rightarrow p_k$
 - How to solve it efficiently?



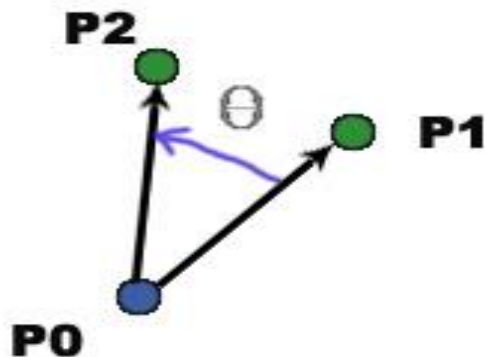
Cross Product

- Cross Product

$$p_1 \times p_2 = \det \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1 = |p_1| |p_2| \sin \theta$$

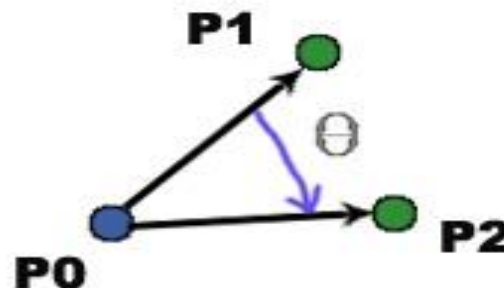
- $P_1 \times P_2 > 0$

$$\sin \theta > 0$$



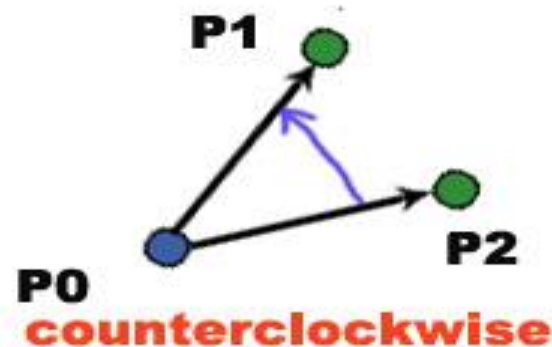
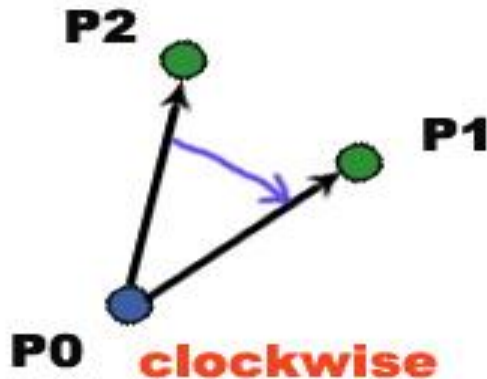
- $P_1 \times P_2 < 0$

$$\sin \theta < 0$$

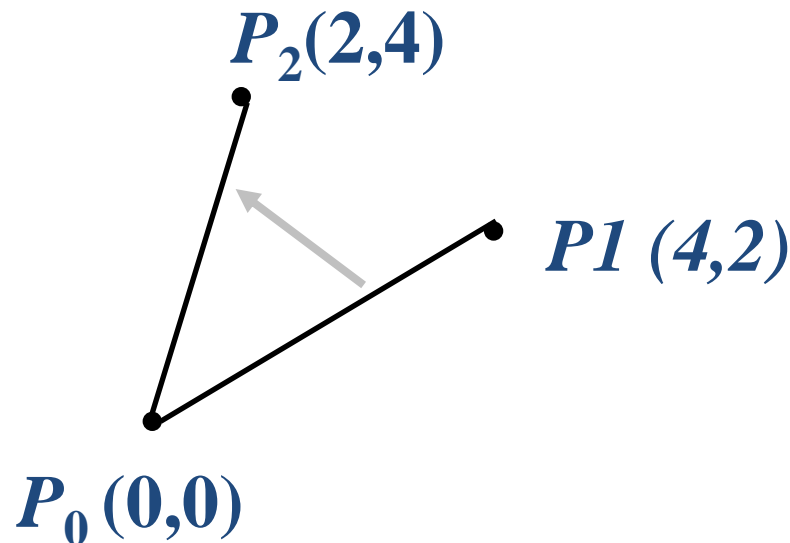


Cross Product

- The cross products of PP to PP can be represented as:
 - $(P_1 - P_0) \times (P_2 - P_0) = (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$
 - $(P_1 - P_0) \times (P_2 - P_0) > 0$
 - $(P_1 - P_0) \times (P_2 - P_0) < 0$



Cross Product



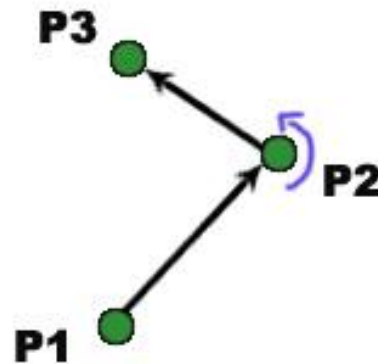
$$(P_1 - P_0) \times (P_2 - P_0) = (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0) = 4 \cdot 4 - 2 \cdot 2 = 12 > 0$$

$\vec{P_0P_2}$ is counterclockwise from $\vec{P_0P_1}$



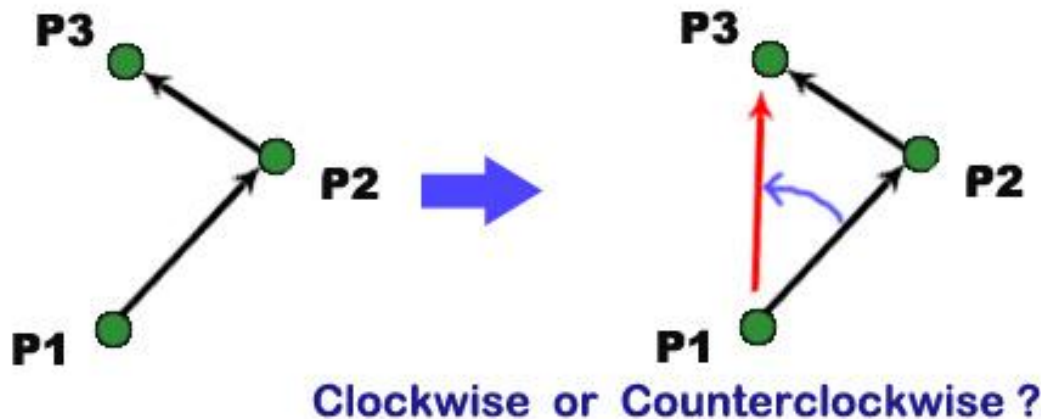
Cross Product

- If we traverse from p_1 to p_2 and then p_3 , should we make a turn left or turn right at point p_2 ?



Cross Product

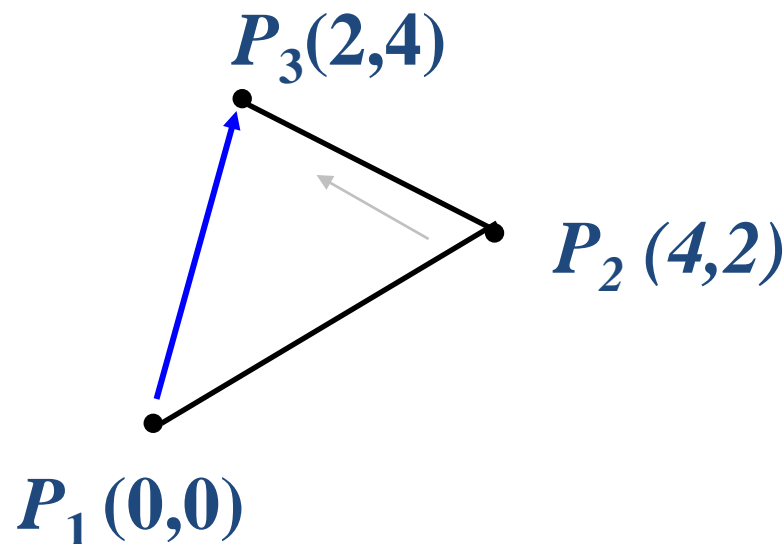
- Judge it by cross product



- Cross product d of p_1p_2 to p_1p_3 :
 - $d > 0$; Counterclockwise : left turn
 - $d < 0$; Clockwise : right turn



Cross Product



$$(P_2 - P_1) \times (P_3 - P_1) = (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$

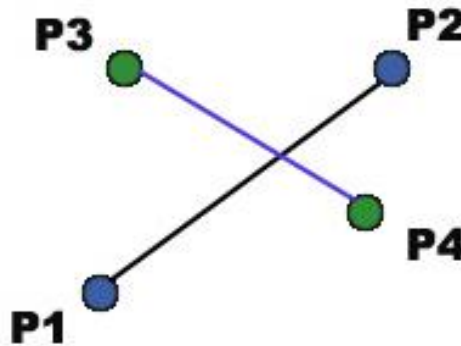
$$= 4 \cdot 4 - 2 \cdot 2 = 12 > 0$$

→ →
 P_1P_3 在 P_1P_2 逆時針方向
 : Left turn



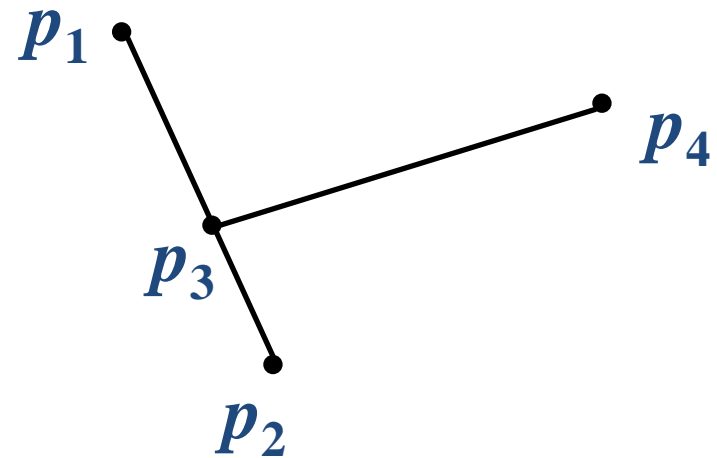
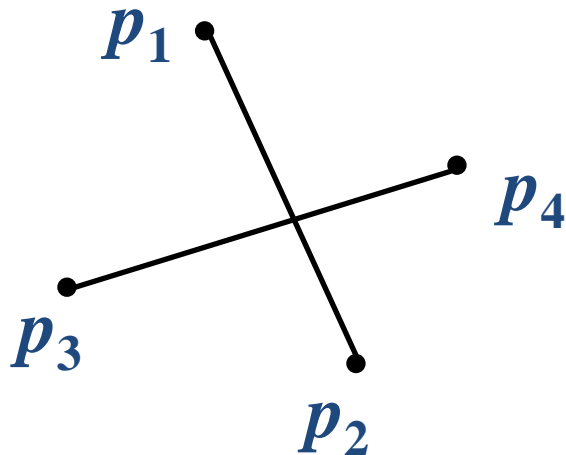
Cross Product

- Line/Segment intersection problem
 - Is the segment p_1p_2 crossing with segment p_3p_4 ?



Cross Product

- Two situation
 - (1) **Proper**: the ending point of the segment cannot be the intersected point
 - (2) **Non-proper**: intersected iff any intersection



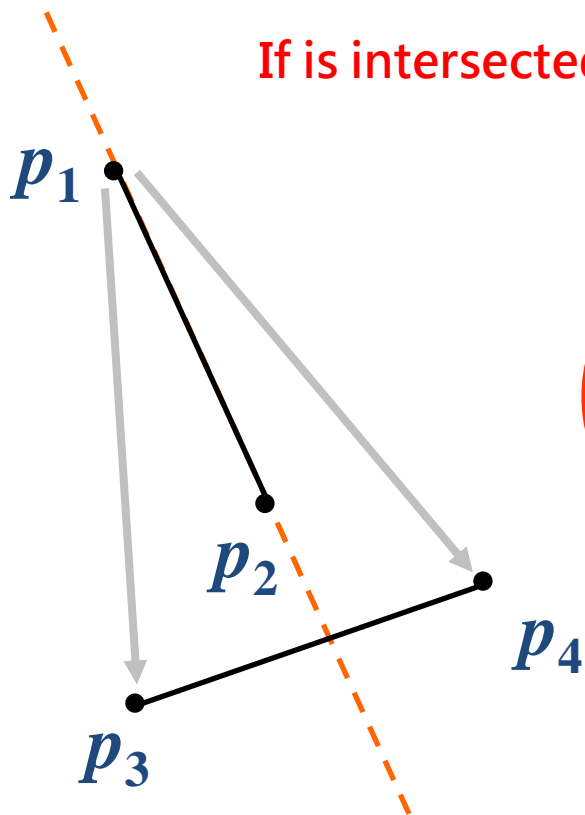
Cross Product

- Example



Cross Product

If is intersected, p_4 and p_3 will be opposite with respect to p_1p_2



$$\left(\overrightarrow{p_1p_2} \times \overrightarrow{p_1p_3} \right) \cdot \left(\overrightarrow{p_1p_2} \times \overrightarrow{p_1p_4} \right) < 0$$

< 0

> 0

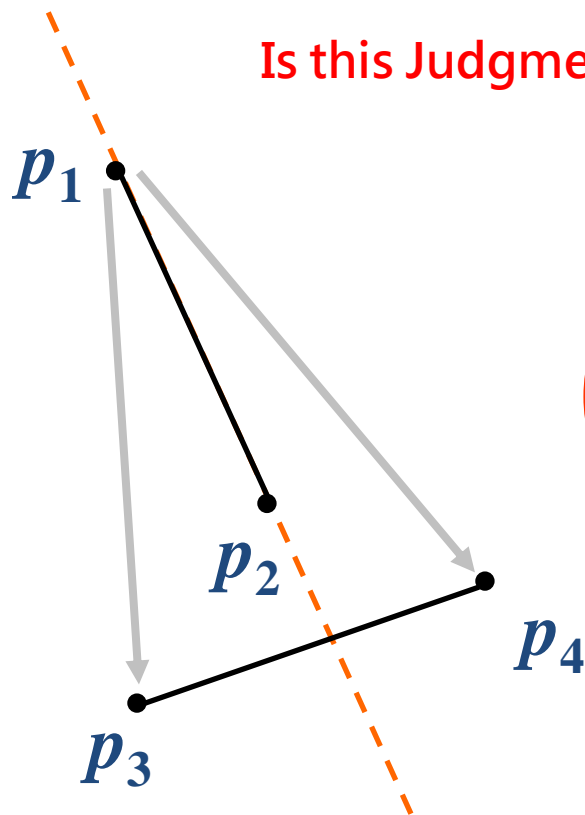
OR

> 0

< 0



Cross Product



Is this Judgment enough?

$$\left(\overrightarrow{p_1p_2} \times \overrightarrow{p_1p_3} \right) \cdot \left(\overrightarrow{p_1p_2} \times \overrightarrow{p_1p_4} \right) < 0$$

< 0

> 0

OR

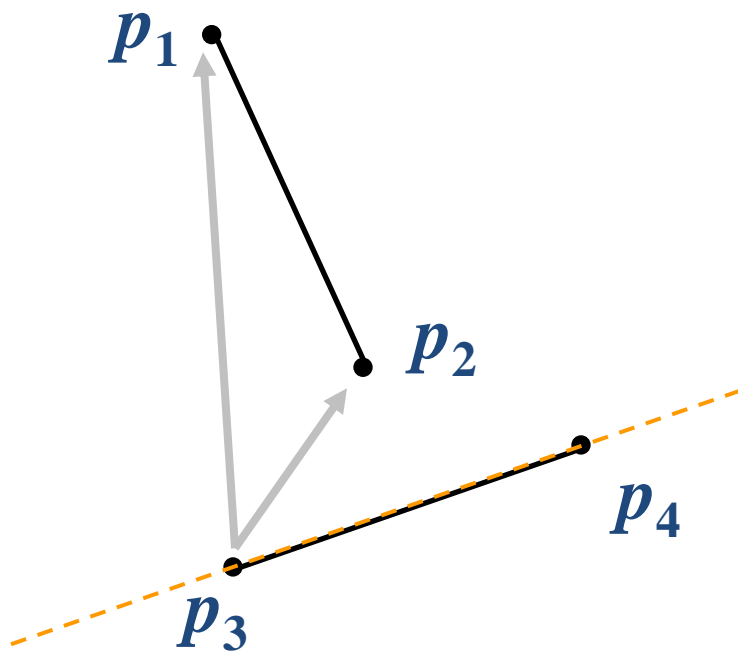
> 0

< 0



Cross Product

We should change the based line and judge it again...



$$(\overrightarrow{P_3P_2} \times \overrightarrow{P_3P_4}) \cdot (\overrightarrow{P_3P_1} \times \overrightarrow{P_3P_4}) > 0$$

False



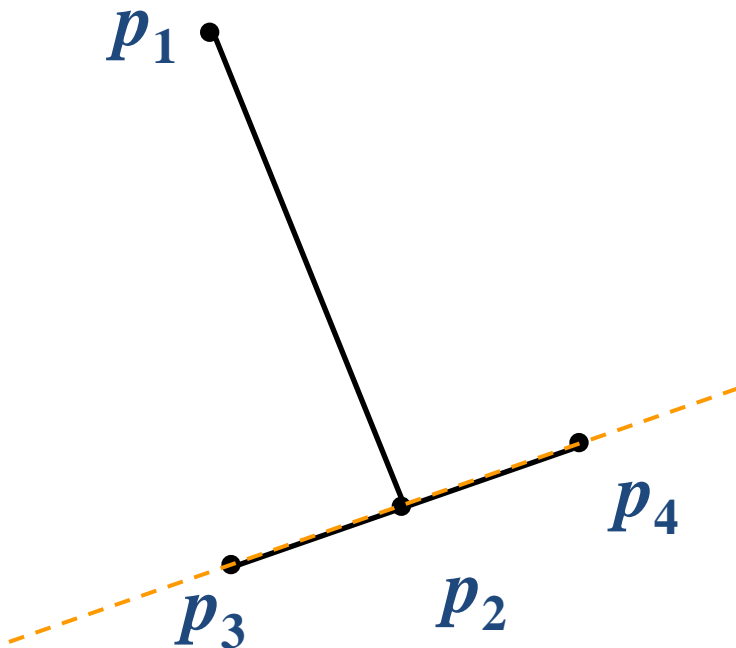
Cross Product

- Example



Cross Product

- Until now, we have solved the proper intersection problem...
- What about the non-proper (general) situation?



Cross product = 0

$$(\overrightarrow{P_3P_2} \times \overrightarrow{P_3P_4}) \cdot (\overrightarrow{P_3P_2} \times \overrightarrow{P_3P_4}) = 0$$



Cross Product

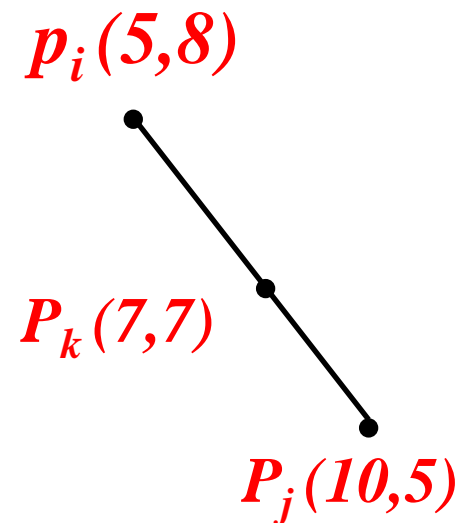
- Example



Cross Product

On-Segment(p_i, p_j, p_k)

- 1 if $\min(x_i, x_j) \leq x_k \leq \max(x_i, x_j)$
and $\min(y_i, y_j) \leq y_k \leq \max(y_i, y_j)$
- 2 then return TRUE
- 3 else return FALSE



$$X: 5 \leq 7 \leq 10$$

$$Y: 5 \leq 7 \leq 5 \quad \text{TRUE}$$



Cross Product

Segments-Intersect(p_1, p_2, p_3, p_4)

```

1   $d_1 \leftarrow (p_1 - p_3) \times (p_4 - p_3)$ 
2   $d_2 \leftarrow (p_2 - p_3) \times (p_4 - p_3)$ 
3   $d_3 \leftarrow (p_3 - p_1) \times (p_2 - p_1)$ 
4   $d_4 \leftarrow (p_4 - p_1) \times (p_2 - p_1)$ 
5  if  $d_1 d_2 < 0$  and  $d_3 d_4 < 0$  then return TRUE
6  if  $d_1 = 0$  and On-Segment( $p_3, p_4, p_1$ ) then return TRUE
7  if  $d_2 = 0$  and On-Segment( $p_3, p_4, p_2$ ) then return TRUE
8  if  $d_3 = 0$  and On-Segment( $p_1, p_2, p_3$ ) then return TRUE
9  if  $d_4 = 0$  and On-Segment( $p_1, p_2, p_4$ ) then return TRUE
10 return FALSE
  
```

Straddle each other

Judge if one of the ending point in a line is on the other line



Cross Product

- Example

POJ 1269

POJ 2653



Homework

PKU: 2318

PKU: 2826

UVa

PKU: 2398

PKU: 1039

152

PKU: 3304

10514

PKU: 1269

10709

PKU: 1556

191

PKU: 2653

273

PKU: 1066

378

PKU: 1410

527

PKU: 1696

866

PKU: 3347

10902

