NCKU Programming Contest Training Course
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NCKU Online Judge http://ncku.tk/

Outline

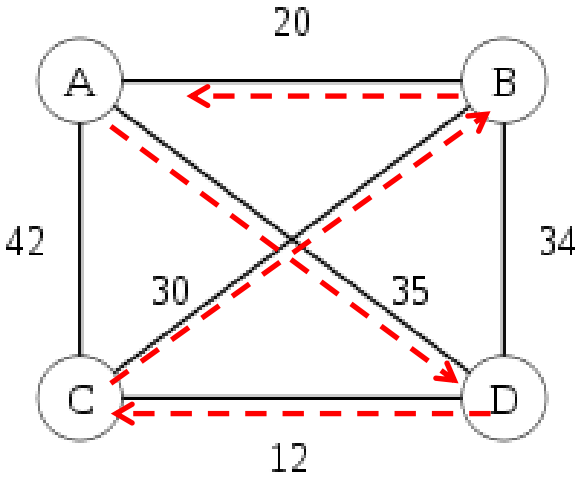
Traveling Salesman Problem

Chinese Postman Problem
Euler Path (Circuit)

- Background knowledge of Graph Theory
  - Basic terms (vertexes, edges…)
  - Degree of a vertex
  - Weighted Graph
  - Path, Cycle
  - Adjacent Matrix vs. Adjacent List

Traveling Salesman Problem

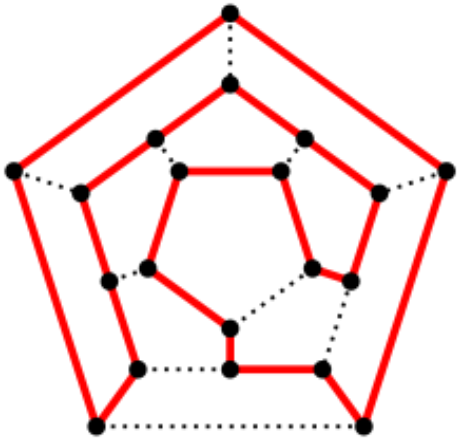
- Problem Definition
  - Given a list of **cities** and the **distances** between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?



Total Distance =
35+12+30+20 = 97

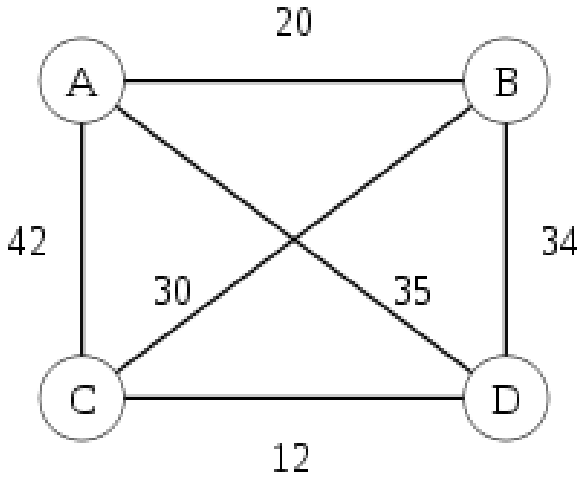
Traveling Salesman Problem

- Hamiltonian Cycle
  - **Hamiltonian path** (or traceable path) is a path in an undirected graph that visits each vertex exactly once. A **Hamiltonian cycle** is a Hamiltonian path that is a cycle.



Traveling Salesman Problem

- Any Idea?
  - Enumerate all paths?



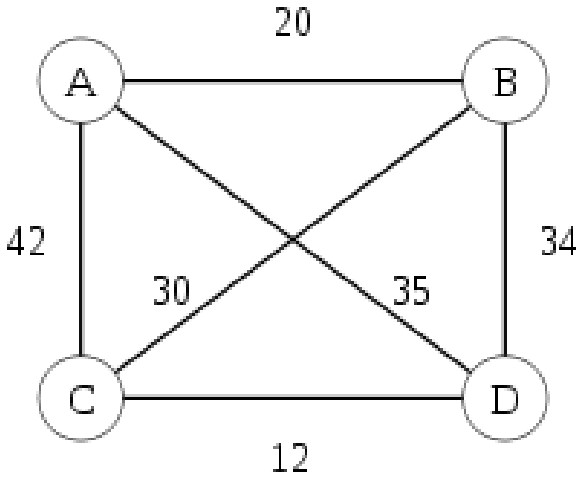
4 vertexes: Total 3! = 6 paths

A-B-C-D-A: 97
A-B-D-C-A: 108
A-C-B-D-A: 141
A-C-D-B-A: 108
A-D-B-C-A: 141
A-D-C-B-A: 97

N vertexes: Total (N-1)! paths

Traveling Salesman Problem

- Solving Clues
  - Part of the paths are repeatedly calculated!



Repeated path

```
A-B-C-D-A: 97
A-B-D-C-A: 108
A-C-B-D-A: 141
A-C-D-B-A: 108
A-D-B-C-A: 141
A-D-C-B-A: 97
```

Same subsets

Traveling Salesman Problem

- Advanced Dynamic Programming Technique – **Bitmask**
  - Similar to State Space Search
  - A "**bit sequence**" is used to encode a subset of vertexes.

  Ex: A graph with 6 nodes

  Subset: {1, 2, 4, 5} = 011011_2 (27_{10})

  Subset: {1, 3, 5, 6} = 110101_2 (53_{10})

  Subset: {1, 2, 3, 4} = 001111_2 (15_{10})

Traveling Salesman Problem

- DP Equation?
- Recursion?
- Top down or bottom up?

Traveling Salesman Problem

Example: **Simple TSP (NCKU Judge 173)**

Description:

The Traveling Salesman Problem is one of the most intensively studied problems in computational mathematics. These pages are devoted to the history, applications, and current research of this challenge of finding the shortest route visiting each member of a collection of locations and returning to your starting point.

Traveling Salesman Problem

input

```
2
0 1
2 0
```

output

```
3
```

Traveling Salesman Problem

- Code

Traveling Salesman Problem

- Challenge
  - **Nuts for nuts.. (NCKU OJ 148, Uva 10944)**
  - **Collecting Beepers (NCKU OJ 149, Uva 10496)**

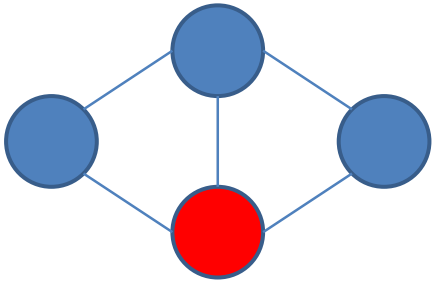
Outline

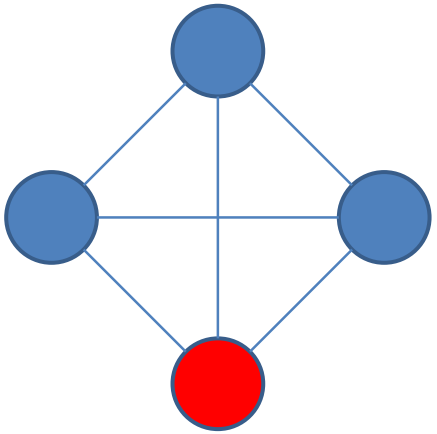
Traveling Salesman Problem

Chinese Postman Problem

Chinese Postman Problem

- Problem Definition
  - Given a (weighted) graph G(V, E). Start from a given vertex, finding the shortest route that cover each at least once and back to the origin vertex.



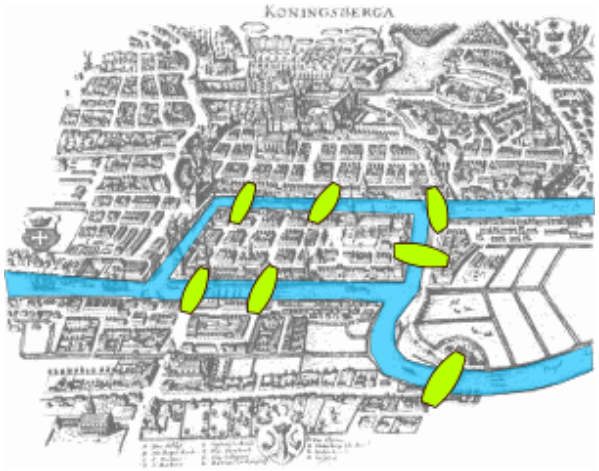


Chinese Postman Problem

- Euler Path (Circuit)
  - **Eulerian trail** (or **Eulerian path**) is a trail in a graph which visits every edge exactly once. Similarly, an **Eulerian circuit** or **Eulerian cycle** is an Eulerian trail which starts and ends on the same vertex.

Chinese Postman Problem

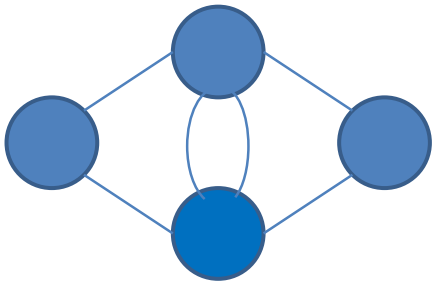
- How to determine whether a Euler Path (Circuit) can be found?



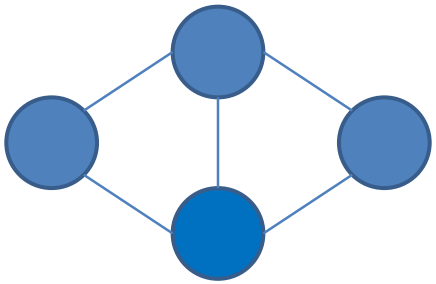
Seven Bridges of Königsberg

Chinese Postman Problem

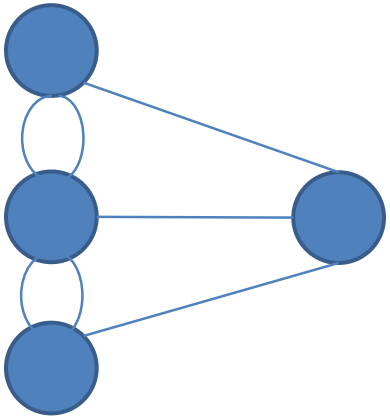
- Lemma
  - A Euler Path can be found in a graph if the number of vertexes with odd degree are less or equal than two. A Euler Circuit can be found in a graph if the number of vertexes with odd degree is equal to 0.



number of V_{odd} =



number of V_{odd} =



number of V_{odd} =

Chinese Postman Problem

- Example: UVa 10735 – Morning Walk

Description

Kamal is a *Motashota* guy. He has got a new job in Chittagong. So, he has moved to Chittagong from Dinajpur. He was getting fatter in Dinajpur as he had no work in his hand there. So, moving to Chittagong has turned to be a blessing for him. Every morning he takes a walk through the hilly roads of charming cityChittagong. He is enjoying this city very much. There are so many roads in Chittagong and every morning he takes different paths for his walking. But while choosing a path he makes sure he does not visit a road twice not even in his way back home. An intersection point of a road is not considered as the part of the road. In a sunny morning, he was thinking about how it would be if he could visit all the roads of the city in a single walk. Your task is to help Kamal in determining whether it is possible for him or not.

Chinese Postman Problem

input

```
2 2
1 0
0 1
2 1
0 1
```

output

```
Possible
Not Possible
```

Chinese Postman Problem

- Challenge
  - Colored Sticks (PKU 2513)

- Related Problem
  - Uva 10129
  - Uva 10441
  - Uva 10506
  - Uva 10596

Chinese Postman Problem

- Lets return the original problem..

(1) If we can found an Euler circuit in the given graph, the answer is obvious the summation of all edges' weight.

(2) If we can't…. ??