1.4.2

◀ $\begin{bmatrix} 3 & -3 & 3 \\ 2 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -3 \\ 2 & 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -3 & 4 \\ 2 & 0 & 3 \\ -3 & -1 & -1 \end{bmatrix},$

1.4.5

1. $\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}^2.$

◀ $\begin{bmatrix} 4 & & & \\ & 4 & & \\ & & 4 & \\ & & & 4 \end{bmatrix}.$ ▶

2. $\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}^k.$

◀ 由于 $\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}^2 = 4I_4$, 故 $\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}^{2m} = 4^m I_4$, $\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}^{2m+1} = 4^m I_4 \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix} = 4^m \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}.$ ▶

1.4.6

1. 计算 $A = I_2 + J_2, B = I_2 + J_2^T, C = J_2 - J_2^T$.

◀ $A = \begin{bmatrix} 1 & 1 \\ & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}.$ ▶

4. 计算 $AB + BC, B(A + C), (A + C)B$, 三者是否相等?

◀ $AB + BC = \begin{bmatrix} 2 & 2 \\ & 2 \end{bmatrix}, B(A + C) = \begin{bmatrix} 1 & 2 \\ & 3 \end{bmatrix}, (A + C)B = \begin{bmatrix} 3 & 2 \\ & 1 \end{bmatrix}$. 不相等. ▶

6. 计算 $(AB)^2$ 与 A^2B^2, 二者是否相等?

◀ $(AB)^2 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}, A^2B^2 = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$. 不相等. ▶

8. 计算 $\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix}$.

◀ $\begin{bmatrix}5 & 6\end{bmatrix}$. ▶

9. 计算 $J_4\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, J_4^T\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, J_4^2\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, J_4^4$.

◀ $\begin{bmatrix}2\\3\\4\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\\3\end{bmatrix}, \begin{bmatrix}3\\4\\0\\0\end{bmatrix}, 0$. ▶

1.4.10

1. p, q, r 取何值时, 有 $AB = BA$?

◀ 只需考虑 $\begin{bmatrix}p & 0\\q & r\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}p & 0\\q & r\end{bmatrix}$. 解得 $q = 0, p = r$. ▶

2. z 取何值时, 有 $BC = CB$?

◀ $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}0 & z\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & z\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ 等价于 $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & z\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & z\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$, 或 $z\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = z\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$. 这恒成立. ▶

3. p, q, r, z 取何值时, 有 $ABC = CAB$?

◀ 与上面类似地, 提出 C 的因子 z, 原式等价于 $z\begin{bmatrix}p & 0\\q & r\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = z\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}p & 0\\q & r\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$. 解得 $z = 0$ 或 $q = 0, p = r$. ▶

1.4.13

◀ 直接写出矩阵元素进行验证. ▶

1.4.18

◀ $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$, $(AA^T)^T = (A^T)^T A^T = AA^T$, $(A^T A)^T = A^T (A^T)^T = A^T A$, $(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$. ▶

1.4.21

◀ 对任意的列向量 $x, 0 = x^T A^2 x = x^T A^T A x = (Ax)^T (Ax)$, 故列向量 $Ax = 0$. 这说明 $A = 0$. ▶

1. 5. 1

1. $$\begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}.$$

◀ $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$. ▶

2. $$\begin{bmatrix} 1 & & \\ -1 & 1 & \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \end{bmatrix}.$$

◀ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{bmatrix}$. ▶

5. $$\begin{bmatrix} 1 & & & & \\ -1 & 1 & & & \\ -1 & & 1 & & \\ -1 & & & 1 & \\ -1 & & & & 1 \end{bmatrix}\begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ 1 & 2 & 1 & & \\ 1 & 3 & 3 & 1 & \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}.$$

◀ $\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & 2 & 1 & & \\ & 3 & 3 & 1 & \\ & 4 & 6 & 4 & 1 \end{bmatrix}$. ▶

1. 5. 2. 1

◀ $\begin{bmatrix} 1 & 1 & \\ & 1 & 1 \\ & & 1 \end{bmatrix}^{-1} = (I_3 + \begin{bmatrix} & 1 & \\ & & 1 \\ & & \end{bmatrix})^{-1} = I_3 - \begin{bmatrix} & 1 & \\ & & 1 \\ & & \end{bmatrix} + \begin{bmatrix} & 1 & \\ & & 1 \\ & & \end{bmatrix}^2 = \begin{bmatrix} 1 & -1 & 1 \\ & 1 & -1 \\ & & 1 \end{bmatrix}$. ▶

1. 5. 2. 2

2. $$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix}.$$

◀ $\begin{bmatrix} 2 & -1 & -1 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$. ▶

1.5.2.4

◀ $\frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$. ▶

1.5.3

◀ 用第 3 行消掉前两行的第 3 个元素, $\begin{bmatrix} 1 & 2 & -1 \\ 3 & a & -2 \\ 5 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 6 & 0 & 0 \\ 13 & a-4 & 0 \\ 5 & -2 & 1 \end{bmatrix}$. $a = 4$. ▶

1.5.4

◀ $Ax = e_1$ 的解为 $x_1 = \begin{bmatrix} 1 \\ -\frac{2}{3} \\ -\frac{1}{9} \end{bmatrix}$, $Ax = e_2$ 的解为 $x_2 = \begin{bmatrix} 0 \\ \frac{1}{3} \\ -\frac{5}{18} \end{bmatrix}$, $Ax = e_3$ 的解为 $x_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{6} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & & \\ -\frac{2}{3} & \frac{1}{3} & \\ -\frac{1}{9} & -\frac{5}{18} & \frac{1}{6} \end{bmatrix}$. ▶

1.5.5

1. $\begin{bmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix}$.

◀ $$\begin{bmatrix} 2 & -1 & & & 1 & & & \\ -1 & 2 & -1 & & & 1 & & \\ & -1 & 2 & -1 & & & 1 & \\ & & -1 & 2 & & & & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & -1 & 2 & -1 & & & 1 & \\ & & -1 & 2 & & & & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & \frac{4}{3} & -1 & \frac{1}{3} & \frac{2}{3} & 1 & \\ & & -1 & 2 & & & & 1 \end{bmatrix} \to$$

$$\begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & \frac{4}{3} & -1 & \frac{1}{3} & \frac{2}{3} & 1 & \\ & & & \frac{5}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & \frac{4}{3} & -1 & \frac{1}{3} & \frac{2}{3} & 1 & \\ & & & \frac{5}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & 1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & \frac{4}{3} & -1 & \frac{1}{3} & \frac{2}{3} & 1 & \\ & & & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix} \to$$

$$\begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & \frac{4}{3} & & \frac{8}{15} & \frac{16}{15} & \frac{8}{5} & \frac{4}{5} \\ & & & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & \frac{3}{2} & -1 & & \frac{1}{2} & 1 & & \\ & & 1 & & \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{3}{5} \\ & & & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix} \to \begin{bmatrix} 2 & -1 & & & 1 & & & \\ & 1 & & & \frac{3}{5} & \frac{6}{5} & \frac{4}{5} & \frac{2}{5} \\ & & 1 & & \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{3}{5} \\ & & & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix} \to$$

$\begin{bmatrix} 1 & & & & \frac{4}{5} & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} \\ & 1 & & & \frac{3}{5} & \frac{6}{5} & \frac{4}{5} & \frac{2}{5} \\ & & 1 & & \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{3}{5} \\ & & & 1 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix}$. 逆矩阵是 $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} & \frac{2}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{6}{5} & \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} \end{bmatrix}$. ▶

2. $\begin{bmatrix} 1 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix}$.

◀ $\begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$. ▶

4. $\begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdots & & \\ 1 & & & \end{bmatrix}$.

◀ $\begin{bmatrix} & & & 1 \\ & & 1 & \\ & \cdots & & \\ 1 & & & \end{bmatrix}$. ▶

7. $\begin{bmatrix} 1 & -a & & \\ & 1 & -b & \\ & & 1 & -c \\ & & & 1 \end{bmatrix}$.

◀ $\begin{bmatrix} 1 & a & ab & abc \\ & 1 & b & bc \\ & & 1 & c \\ & & & 1 \end{bmatrix}$. ▶

8. $\begin{bmatrix} 1 & a & & \\ & 1 & \ddots & \\ & & \ddots & a \\ & & & 1 \end{bmatrix}_{n\times n}$.

◀ $\begin{bmatrix} 1 & -a & (-a)^2 & \cdots & (-a)^{n-1} \\ & 1 & -a & \ddots & \vdots \\ & & 1 & \ddots & (-a)^2 \\ & & & \ddots & -a \\ & & & & 1 \end{bmatrix}$. ▶

本次作业以计算题居多，绝大部分错误都是计算出错，希望各位同学下次能细心运算。证明题完成情况良好。

部分同学的作业只给出答案而未写计算过程。即使计算繁琐，也请简单写明关键步骤。

做错较多的题目是 1.5.1.1、1.5.5.1、1.5.5.2 和 1.5.5.8，主要表现为对矩阵的乘法和逆矩阵的求法不够熟练，导致计算结果错误。下面给出这几道题目的计算过程。

练习 1.5.1.1 部分同学将结果算成 $\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$，请参考以下运算过程。

$$\begin{aligned}\begin{bmatrix}&&1\\&1&\\1&&\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\begin{bmatrix}&&1\\&1&\\1&&\end{bmatrix}&=\begin{bmatrix}7&8&9\\4&5&6\\1&2&3\end{bmatrix}\begin{bmatrix}&&1\\&1&\\1&&\end{bmatrix}\\&=\begin{bmatrix}9&8&7\\6&5&4\\3&2&1\end{bmatrix}\end{aligned}$$

练习 1.5.5.1 这道题目运算复杂，有的同学将分母算成了 7；该题需要一步一步地细心计算，提高自己对矩阵行变换的熟练度。

$$\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\-1&2&-1&&&1&&\\&-1&2&-1&&&1&\\&&-1&2&&&&1\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&-1&2&-1&&&1&\\&&-1&2&&&&1\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&&\frac{4}{3}&-1&\frac{1}{3}&\frac{2}{3}&1&\\&&-1&2&&&&1\end{array}\right]\longrightarrow$$

$$\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&&\frac{4}{3}&-1&\frac{1}{3}&\frac{2}{3}&1&\\&&&\frac{5}{4}&\frac{1}{4}&\frac{2}{4}&\frac{3}{4}&1\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&&\frac{4}{3}&-1&\frac{1}{3}&\frac{2}{3}&1&\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&&\frac{4}{3}&&\frac{8}{15}&\frac{16}{15}&\frac{24}{15}&\frac{4}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow$$

$$\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&\frac{3}{2}&-1&&\frac{1}{2}&1&&\\&&1&&\frac{2}{5}&\frac{4}{5}&\frac{6}{5}&\frac{3}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&\frac{8}{5}&&&\\&\frac{3}{2}&&&\frac{1}{2}&1&\frac{6}{5}&\frac{3}{5}\\&&1&&\frac{2}{5}&\frac{4}{5}&\frac{6}{5}&\frac{3}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}2&-1&&&1&&&\\&1&&&\frac{3}{5}&\frac{6}{5}&\frac{4}{5}&\frac{2}{5}\\&&1&&\frac{2}{5}&\frac{4}{5}&\frac{6}{5}&\frac{3}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow$$

$$\left[\begin{array}{cccc:cccc}2&&&&\frac{8}{5}&\frac{6}{5}&\frac{4}{5}&\frac{2}{5}\\&1&&&\frac{3}{5}&\frac{6}{5}&\frac{4}{5}&\frac{2}{5}\\&&1&&\frac{2}{5}&\frac{4}{5}&\frac{6}{5}&\frac{3}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]\longrightarrow\left[\begin{array}{cccc:cccc}1&&&&\frac{4}{5}&\frac{3}{5}&\frac{2}{5}&\frac{1}{5}\\&1&&&\frac{3}{5}&\frac{6}{5}&\frac{4}{5}&\frac{2}{5}\\&&1&&\frac{2}{5}&\frac{4}{5}&\frac{6}{5}&\frac{3}{5}\\&&&1&\frac{1}{5}&\frac{2}{5}&\frac{3}{5}&\frac{4}{5}\end{array}\right]$$

练习 1.5.5.2 该题与上一题只差了一个数字，结果却截然不同。同学们在运算时要注意它与上一题的差别，避免错误。写过程时可像下面这样，写出关键的中间步骤，不要直接写答案。

$$
\left[\begin{array}{cccc|cccc}
1 & -1 & & & 1 & & & \\
-1 & 2 & -1 & & & 1 & & \\
 & -1 & 2 & -1 & & & 1 & \\
 & & -1 & 2 & & & & 1
\end{array}\right]
\longrightarrow
\left[\begin{array}{cccc|cccc}
1 & -1 & & & 1 & & & \\
 & 1 & -1 & & 1 & 1 & & \\
 & & 1 & -1 & 1 & 1 & 1 & \\
 & & & 1 & 1 & 1 & 1 & 1
\end{array}\right]
\longrightarrow
\left[\begin{array}{cccc|cccc}
1 & & & & 4 & 3 & 2 & 1 \\
 & 1 & & & 3 & 3 & 2 & 1 \\
 & & 1 & & 2 & 2 & 2 & 1 \\
 & & & 1 & 1 & 1 & 1 & 1
\end{array}\right]
$$

大致思路：该题的矩阵很特殊。先把第一行加到第二行，再把第二行加到第三行，再把第三行加到第四行；接着把第四行加到第三行，再把第三行加到第二行，再把第二行加到第一行。

练习 1.5.5.8 部分同学将答案右上角的 $(-a)^{(n-1)}$ 写为 $(-a)^{(n)}$ 或 $a^{(n-1)}$，参考下述解答。

$$
\left[\begin{array}{ccccc|ccccc}
1 & a & & & & 1 & & & & \\
 & \ddots & \ddots & & & & \ddots & & & \\
 & & 1 & a & & & & 1 & & \\
 & & & 1 & a & & & & 1 & \\
 & & & & 1 & & & & & 1
\end{array}\right]
\longrightarrow
$$

$$
\left[\begin{array}{ccccc|ccccc}
1 & a & & & & 1 & & & & \\
 & \ddots & \ddots & & & & \ddots & & & \\
 & & 1 & a & & & & 1 & & \\
 & & & 1 & & & & & 1 & -a \\
 & & & & 1 & & & & & 1
\end{array}\right]
\longrightarrow
$$

$$
\left[\begin{array}{ccccc|ccccc}
1 & a & & & & 1 & & & & \\
 & \ddots & \ddots & & & & \ddots & & & \\
 & & 1 & & & & & 1 & -a & (-a)^2 \\
 & & & 1 & & & & & 1 & -a \\
 & & & & 1 & & & & & 1
\end{array}\right]
\longrightarrow
$$

$$
\left[\begin{array}{cccccc|cccccc}
1 & a & & & & & 1 & a & & & & \\
 & 1 & 0 & & & & & 1 & \cdots & (-a)^{(n-4)} & (-a)^{(n-3)} & (-a)^{(n-2)} \\
 & & \ddots & & & & & & \ddots & \vdots & \vdots & \vdots \\
 & & & 1 & & & & & & 1 & -a & (-a)^2 \\
 & & & & 1 & & & & & & 1 & -a \\
 & & & & & 1 & & & & & & 1
\end{array}\right]
\longrightarrow
$$

$$
\left[\begin{array}{cccccc|cccccc}
1 & & & & & & 1 & a & \cdots & (-a)^{(n-3)} & (-a)^{(n-2)} & (-a)^{(n-1)} \\
 & 1 & & & & & & 1 & \cdots & (-a)^{(n-4)} & (-a)^{(n-3)} & (-a)^{(n-2)} \\
 & & \ddots & & & & & & \ddots & \vdots & \vdots & \vdots \\
 & & & 1 & & & & & & 1 & -a & (-a)^2 \\
 & & & & 1 & & & & & & 1 & -a \\
 & & & & & 1 & & & & & & 1
\end{array}\right]
$$