第20讲　三相电路

本节课需要用计算器

1　三相电源与三相电路　　相线关系

2　对称三相电路的分析　　抽单相

3　不对称三相电路分析简介

4　三相电路的功率　　三相功率的计算

1 三相电源与三相电路

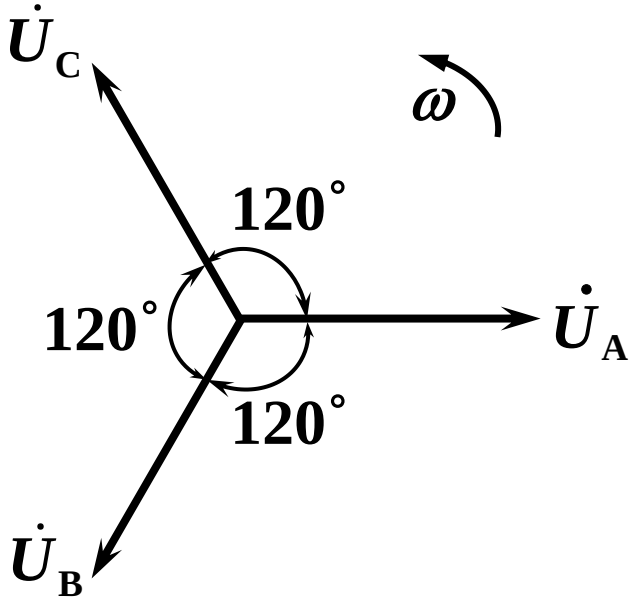
课前预习

(1) 对称三相电源

$$\dot{U}_A = U\angle 0^o$$

$$\dot{U}_B = U\angle -120^o$$

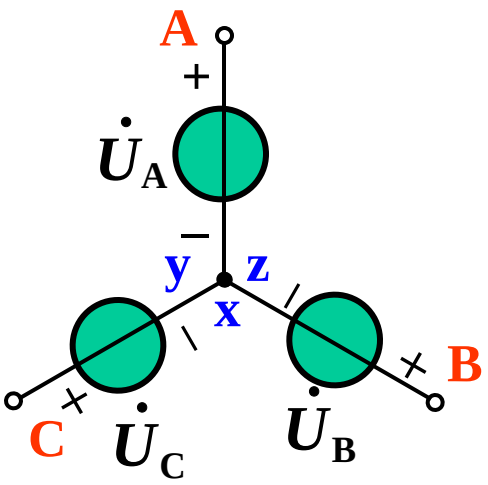
$$\dot{U}_C = U\angle -240^\circ = U\angle 120^o$$

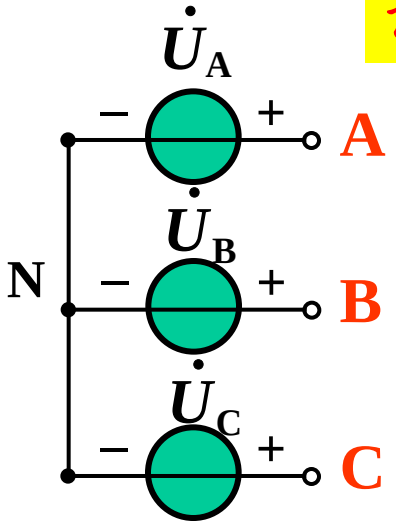


（2）对称三相电源连接

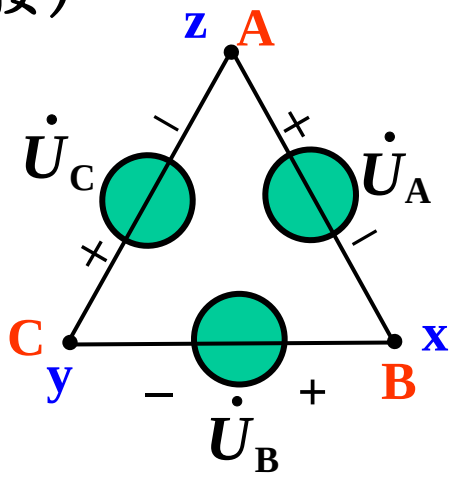
课前预习

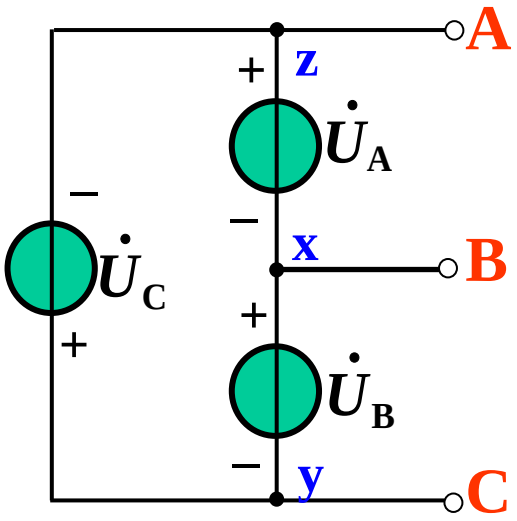
星形联接(Y接)



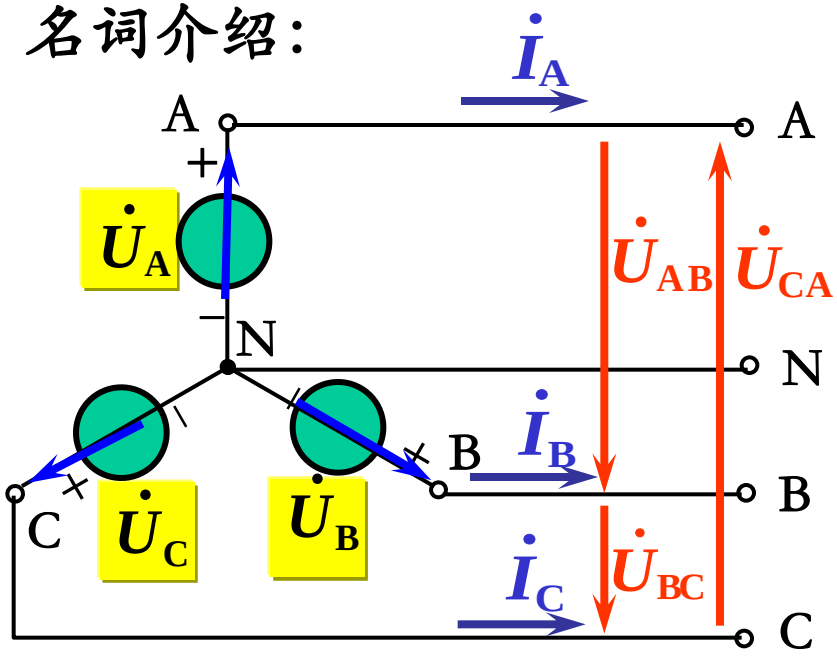


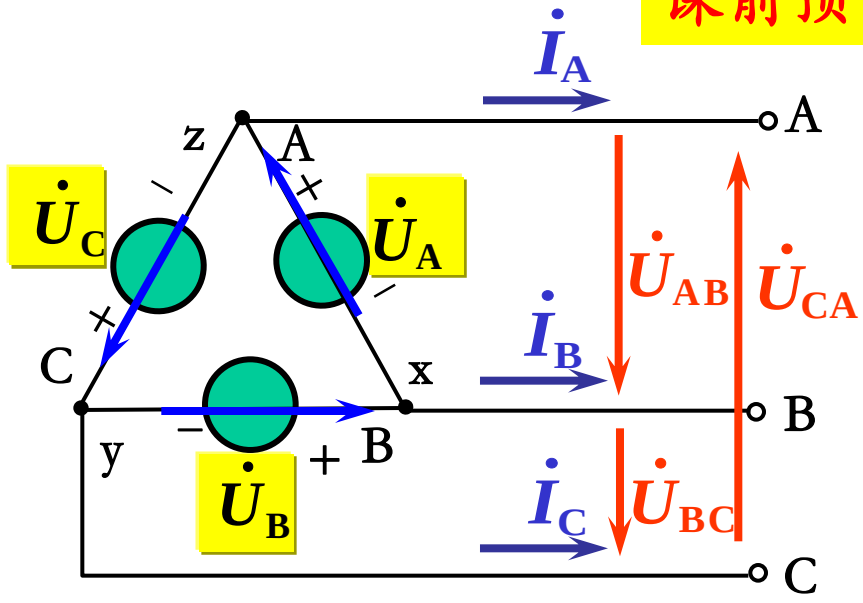
三角形联接(Δ 接)





名词介绍：





(1) 端线(火线)　(2) 中线　(3) 三相三线制与三相四线制。

(4) 线电流 $\dot{I}_A$，$\dot{I}_B$，$\dot{I}_C$　(5) 线电压 $\dot{U}_{AB}$，$\dot{U}_{BC}$，$\dot{U}_{CA}$

(6) 相电流　(7) 相电压 $\dot{U}_A$，$\dot{U}_B$，$\dot{U}_C$

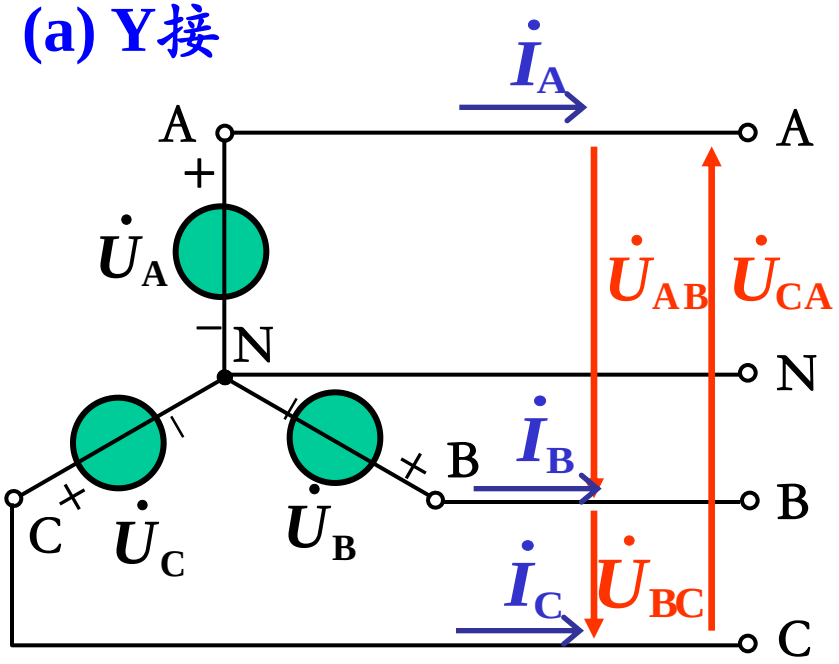
(3) 对称三相电源的相线关系

设 $\dot{U}_{AN}=\dot{U}_A=U\angle 0^o$

$\dot{U}_{BN}=\dot{U}_B=U\angle -120^o$

$\dot{U}_{CN}=\dot{U}_C=U\angle 120^o$

(a) Y接



线电压对称

$$U_L=\sqrt{3}U_P$$

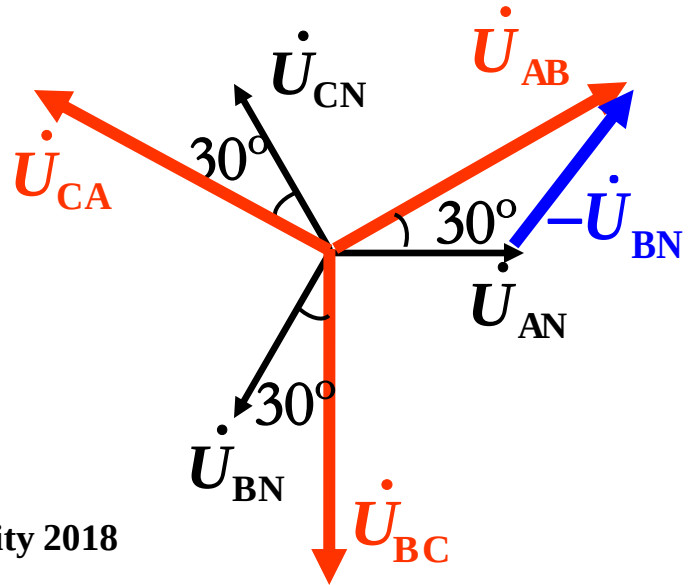
线电压相位领先对应相电压30°

线电流=相电流

$$\dot{U}_{AB}=\dot{U}_{AN}-\dot{U}_{BN}=\sqrt{3}U\angle 30^o$$

$$\dot{U}_{BC}=\dot{U}_{BN}-\dot{U}_{CN}=\sqrt{3}U\angle -90^o$$

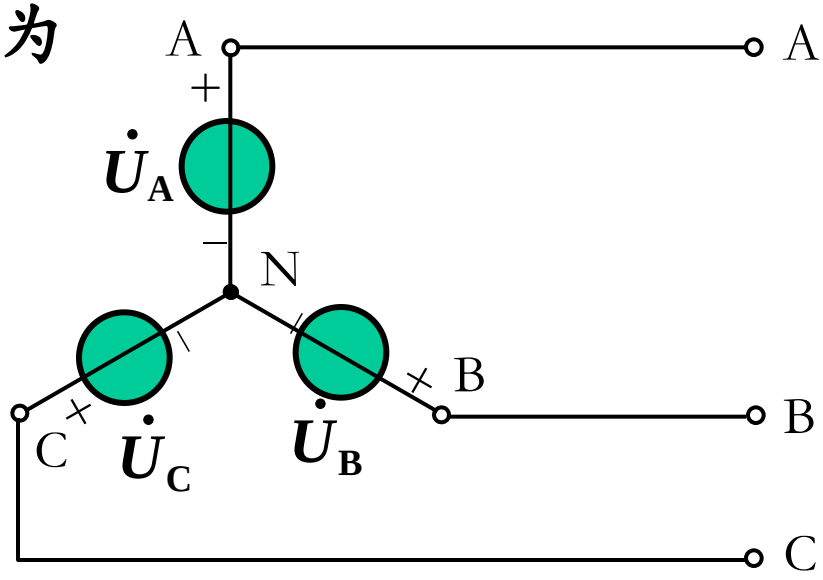
$$\dot{U}_{CA}=\dot{U}_{CN}-\dot{U}_{AN}=\sqrt{3}U\angle 150^o$$



单选题 1分

设置

$\dot{U}_A$ 和$\dot{U}_{CB}$之间的相位关系为



A $\dot{U}_A$ 领先$\dot{U}_{CB}$ 90°

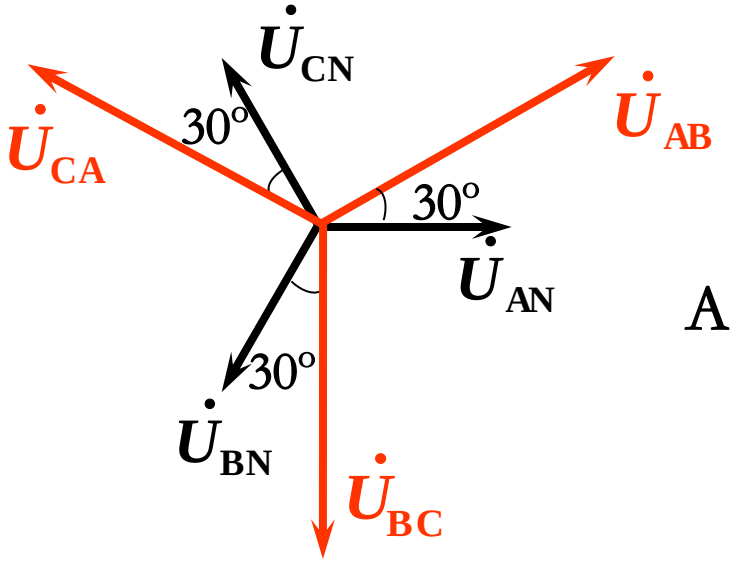
B $\dot{U}_A$ 落后$\dot{U}_{CB}$ 90°

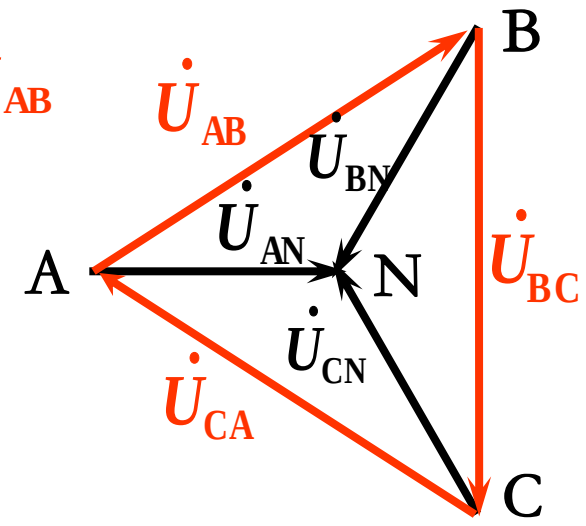
C $\dot{U}_A$ 领先$\dot{U}_{CB}$ 30°

D $\dot{U}_A$ 落后$\dot{U}_{CB}$ 30°

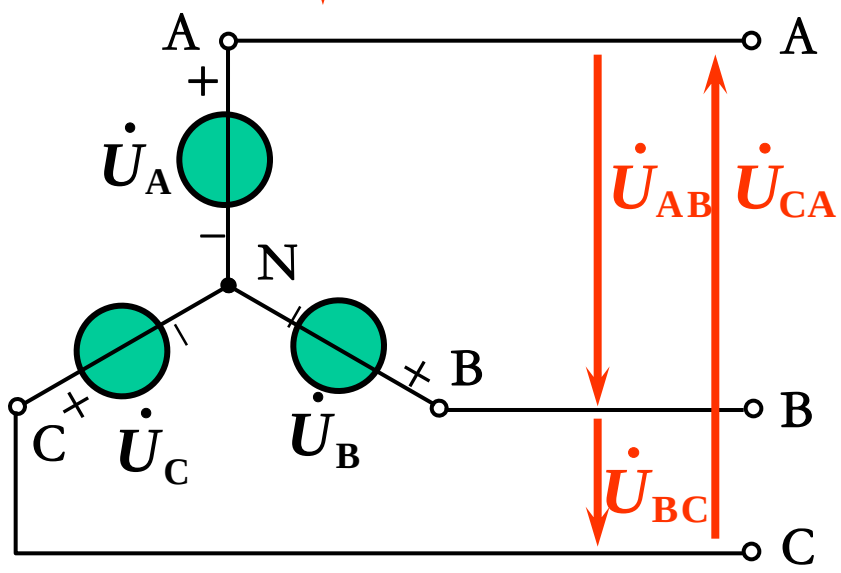
提交

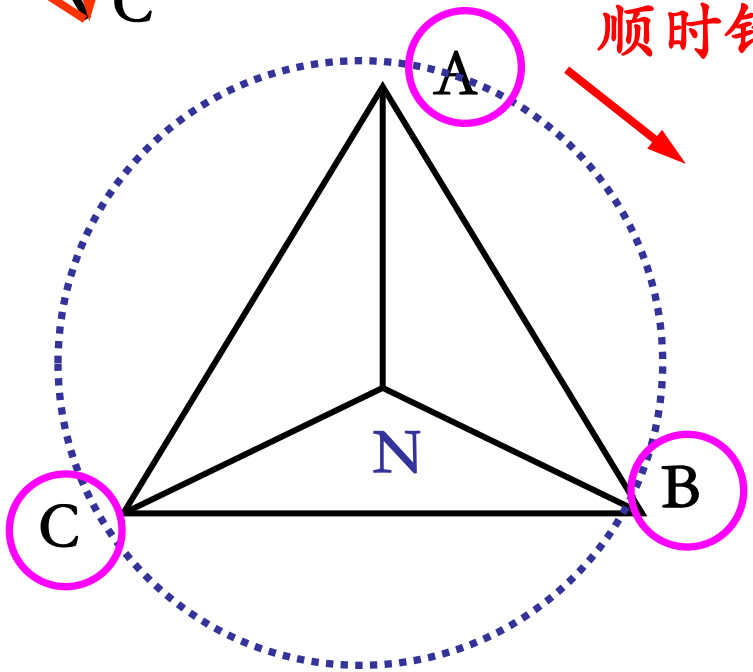
相量图　以N为参考点，图中各点的电位由该点与N点的距离决定



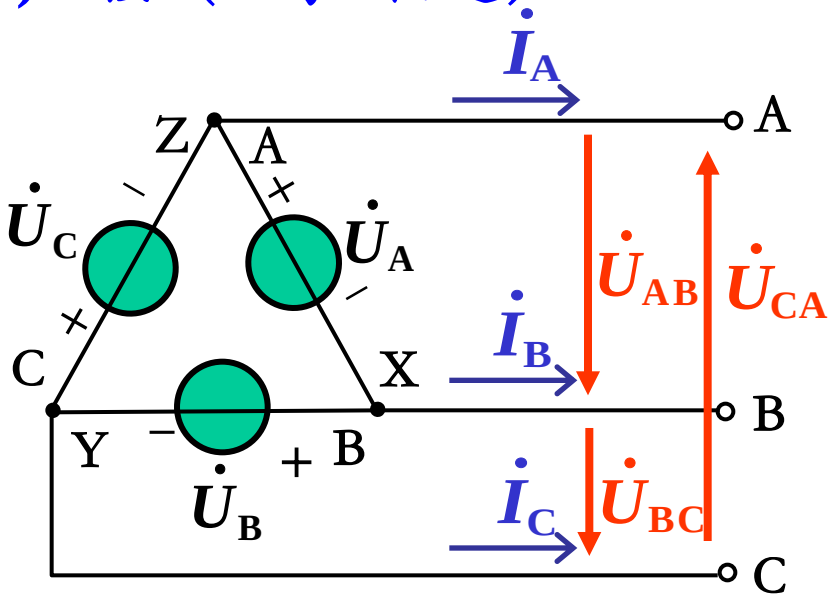


位形图：用图形方式表示电位的相量图





(b) Δ接（X与B相连）



设 $\dot{U}_A = U\angle 0^o$

$\dot{U}_B = U\angle -120^o$

$\dot{U}_C = U\angle 120^o$

$\dot{U}_{AB} = \dot{U}_A = U\angle 0^o$

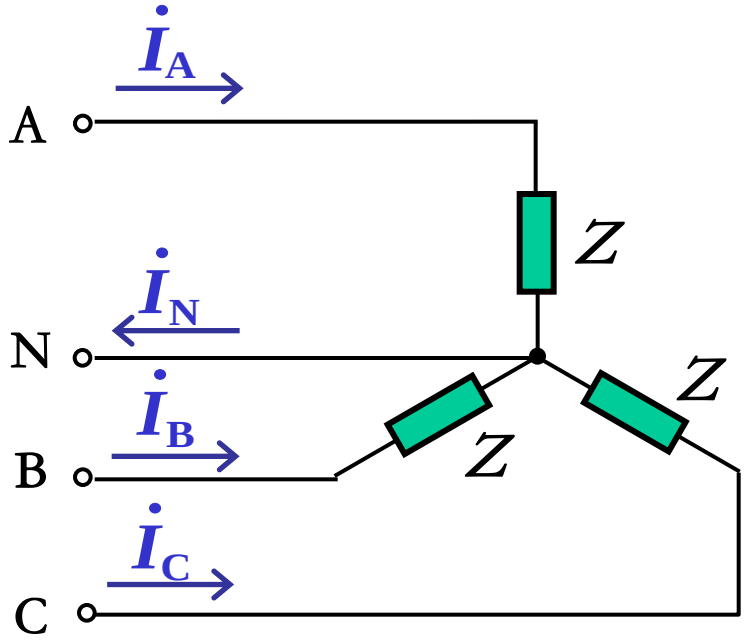
$\dot{U}_{BC} = \dot{U}_B = U\angle -120^o$

$\dot{U}_{CA} = \dot{U}_C = U\angle 120^o$

线电压＝相电压

(4) 对称三相负载的相线关系

(a) Y接



设 $\dot{U}_{AN}=\dot{U}_A=U\angle 0^o$

$\dot{U}_{BN}=\dot{U}_B=U\angle -120^o$

$\dot{U}_{CN}=\dot{U}_C=U\angle 120^o$

对Y接法的对称电源讨论得出的结论对Y接法的对称负载一样成立。

$$\dot{U}_{AB}=\dot{U}_{AN}-\dot{U}_{BN}=\sqrt{3}U\angle 30^o$$

$$\dot{U}_{BC}=\dot{U}_{BN}-\dot{U}_{CN}=\sqrt{3}U\angle -90^o$$

$$\dot{U}_{CA}=\dot{U}_{CN}-\dot{U}_{AN}=\sqrt{3}U\angle 150^o$$

线电流=相电流

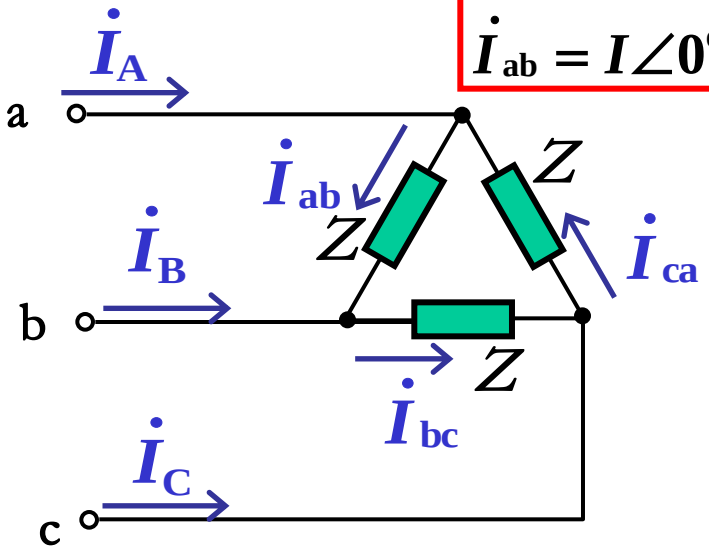
线电压对称

$$U_L=\sqrt{3}U_P$$

线电压领先对应相电压30°

(b) Δ接

$\dot{I}_{ab} = I\angle 0^o$　　$\dot{I}_{bc} = I\angle -120^o$　　$\dot{I}_{ca} = I\angle 120^o$



$$\dot{I}_{ab} = \frac{\dot{U}_{ab}}{Z}$$

$$\dot{I}_{bc} = \frac{\dot{U}_{bc}}{Z}$$

$$\dot{I}_{ca} = \frac{\dot{U}_{ca}}{Z}$$

线电压(即相电压）对称

相电流对称

线电流

$$\dot{I}_A = \dot{I}_{ab} - \dot{I}_{ca} = \sqrt{3}\,\dot{I}_{ab}\angle -30^o$$

$$\dot{I}_B = \dot{I}_{bc} - \dot{I}_{ab} = \sqrt{3}\,\dot{I}_{bc}\angle -30^o$$

$$\dot{I}_C = \dot{I}_{ca} - \dot{I}_{bc} = \sqrt{3}\,\dot{I}_{ca}\angle -30^o$$

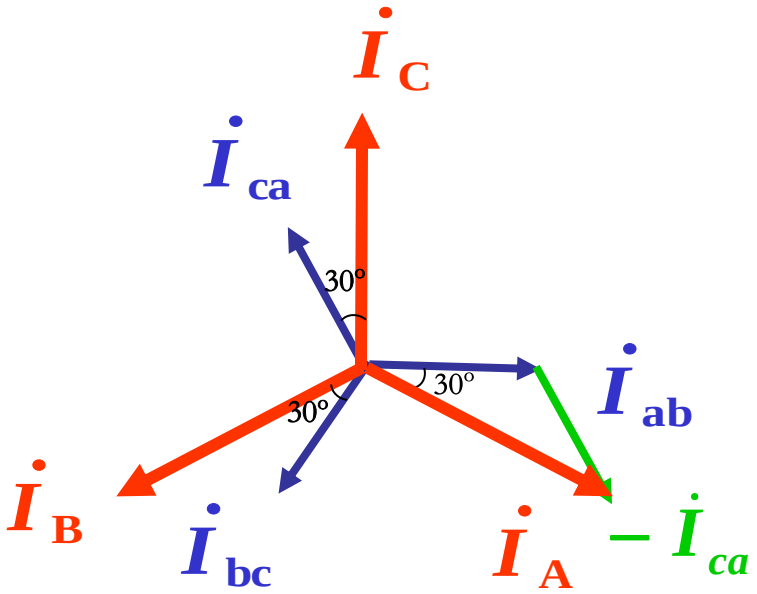
线电流也对称

线电流对称

$$I_L = \sqrt{3} I_P$$

线电流落后对应相电流30°

线电压＝相电压



(5) 三相电路

三相制电力系统：由三个频率相同、相位互差120°的正弦交流电源供电的系统。

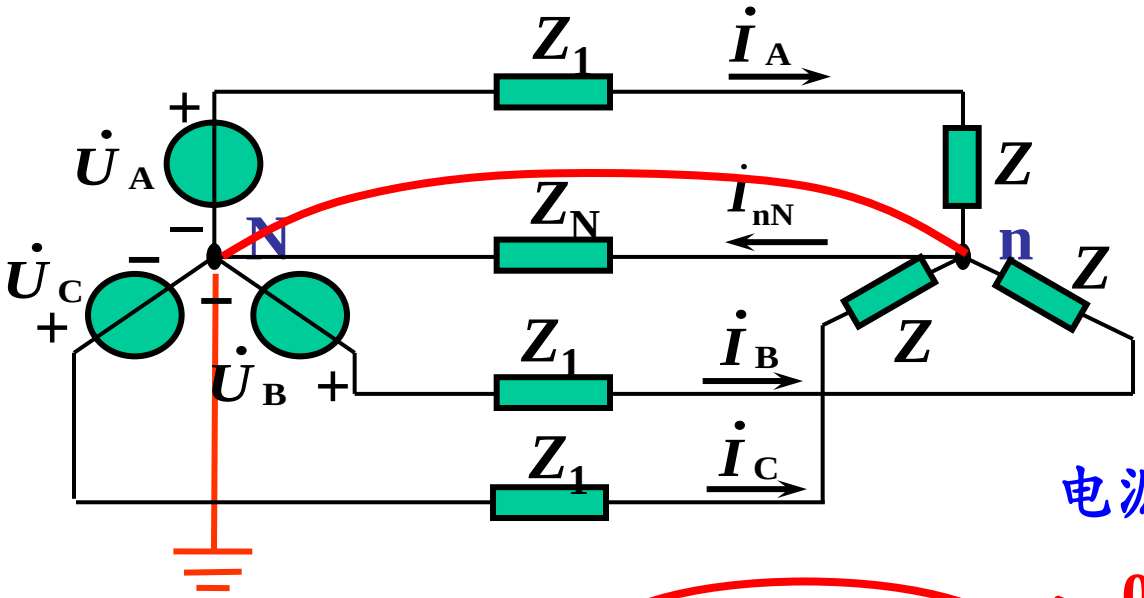
三相制优点：见补充材料

对称三相电路：由对称三相电源和对称三相负载联接而成

对称三相负载：三相负载阻抗模相等，阻抗角相同

不对称
- 电源不对称 —— 程度小(由系统保证)
- 负载不对称 —— 情况很多

2 对称三相电路的分析



$$设\dot{U}_A = U\angle 0^\circ$$

$$\dot{U}_B = U\angle -120^o$$

$$\dot{U}_C = U\angle +120^o$$

电源中点与负载中点等电位

$$\dot{U}_{nN} = 0$$

$$(\frac{3}{Z+Z_1}+\frac{1}{Z_N})\dot{U}_{nN} = \frac{\dot{U}_A+\dot{U}_B+\dot{U}_C}{Z+Z_1}$$

$\dot{U}_A+\dot{U}_B+\dot{U}_C \to 0$

三个单相可单独计算

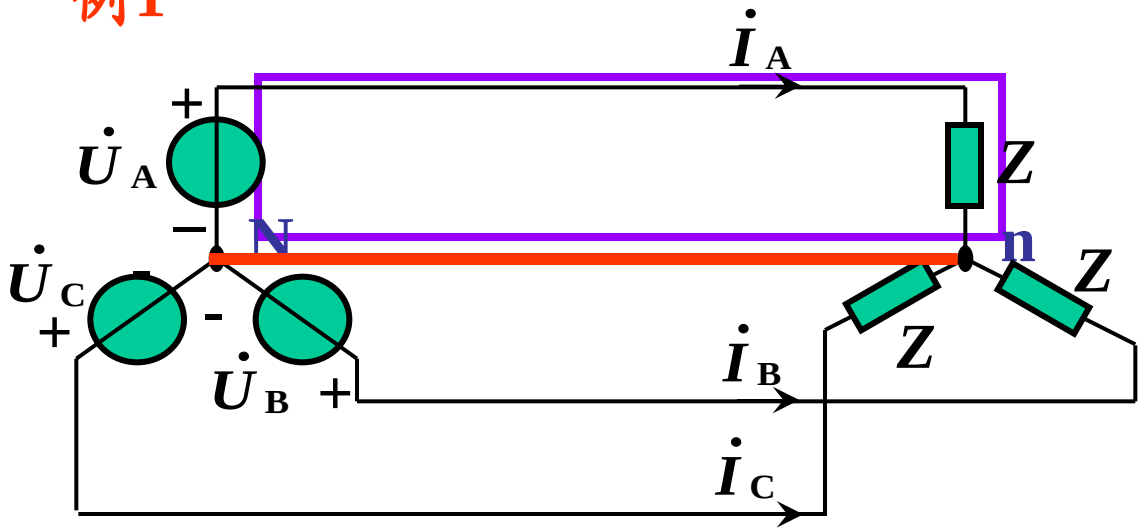
$$\dot{I}_A = \frac{\dot{U}_A}{Z+Z_1} \qquad \dot{I}_B = \frac{\dot{U}_B}{Z+Z_1} \qquad \dot{I}_C = \frac{\dot{U}_C}{Z+Z_1}$$

$\dot{I}_A$, $\dot{I}_B$, $\dot{I}_C$对称

可只抽A相计算

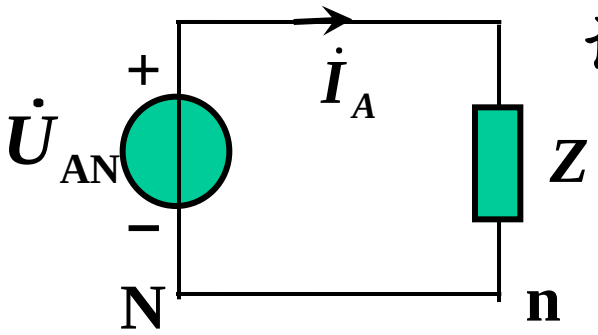
抽单相

例1



已知对称三相电源的线电压为380V，对称负载$Z=100\angle 30°\Omega$求线电流。

解：连接中线Nn，取A相为例计算



设 $\dot{U}_{AB}=380\angle 30^{o}\,\mathrm{V}$ 则 $\dot{U}_{AN}=220\angle 0^{o}\,\mathrm{V}$

$$\dot{I}_A=\frac{\dot{U}_{AN}}{Z}=\frac{220\angle 0^{o}}{100\angle 30^{o}}=2.2\angle -30^{o}\,\mathrm{A}$$

由对称性，得 $\dot{I}_B=2.2\angle -150^{o}\,\mathrm{A}$

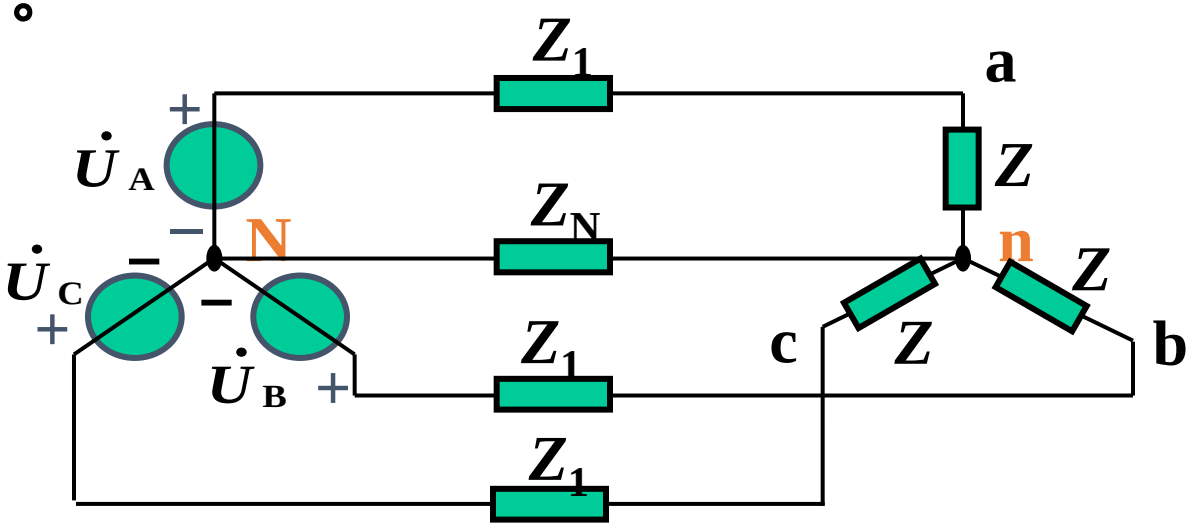
$\dot{I}_C=2.2\angle 90^{o}\,\mathrm{A}$

单选题 1分

设置

图示对称三相电路中，各相电源电压有效值为220V，$Z=Z_1=10\Omega$，$Z_N=50\Omega$

则 $U_{cn}=$ _____ 。



A 183.33V

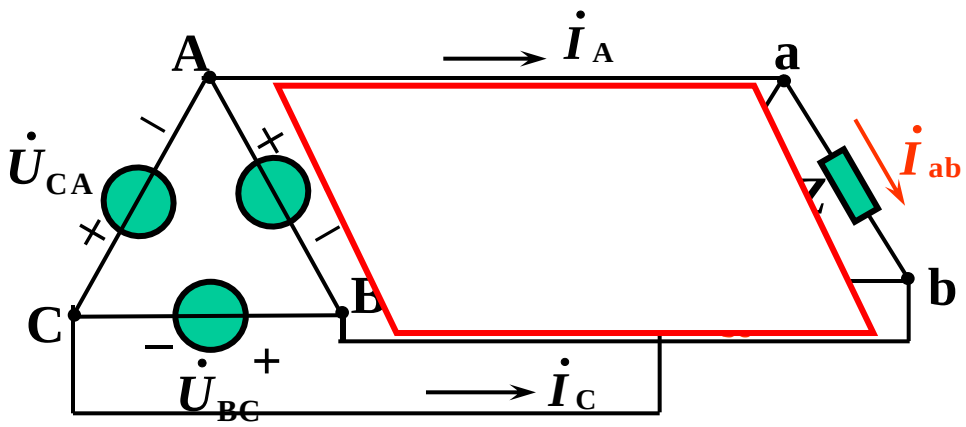
B 220V

C 3.14V

D 110V

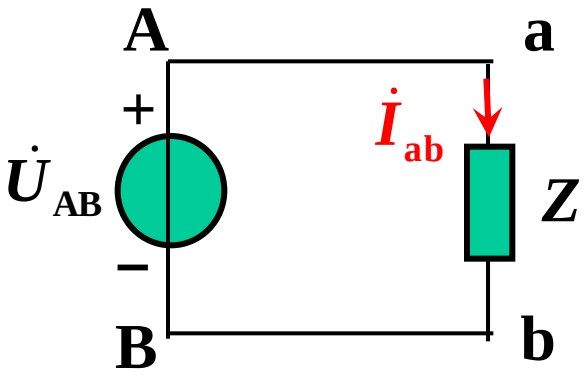
提交

例2



已知对称三相电源的线电压为380V，对称负载$Z=100\angle 30°\Omega$求线电流。

解1　取A相求相电流 $\dot{I}_{ab}$



设　$\dot{U}_{AB}=380\angle 0^{o}\,\mathrm{V}$

$$\dot{I}_{ab}=\frac{\dot{U}_{AB}}{Z}=\frac{380\angle 0^{o}}{100\angle 30^{o}}=3.8\angle -30^{o}\,\mathrm{A}$$

$$\dot{I}_{A}=\sqrt{3}\times 3.8\angle -30^{o}-30^{o}=6.58\angle -60^{o}\,\mathrm{A}$$

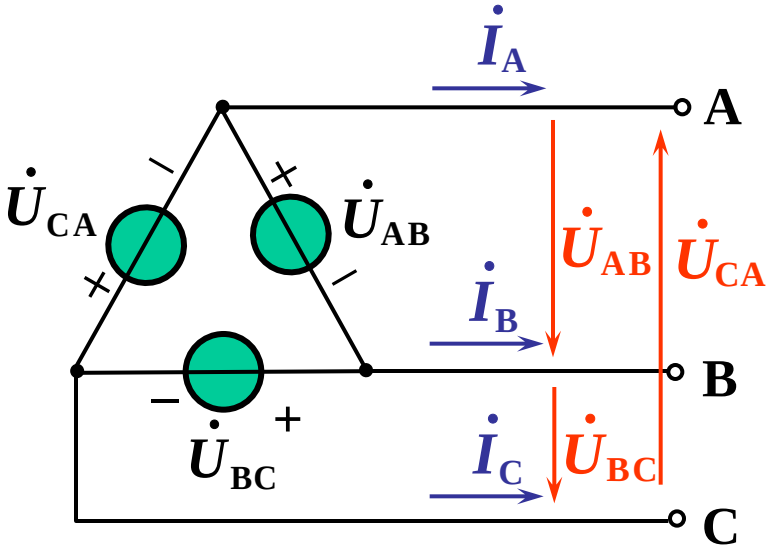
由对称性，得

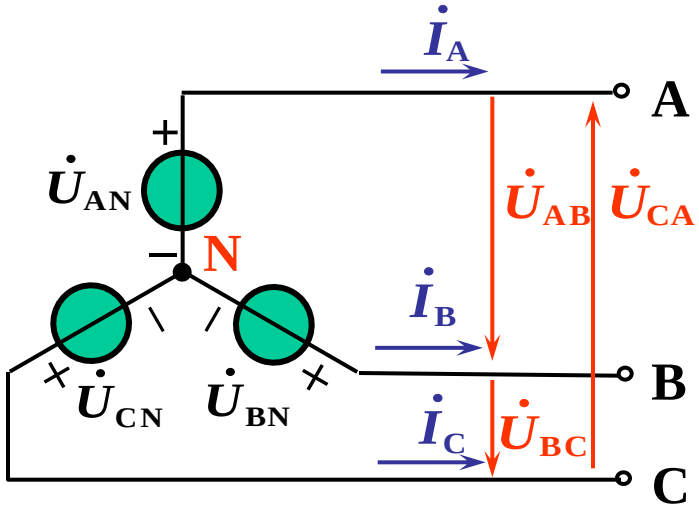
$$\dot{I}_{B}=6.58\angle -180^{o}=-6.58\mathrm{A}$$

$$\dot{I}_{C}=6.58\angle 60^{o}\,\mathrm{A}$$

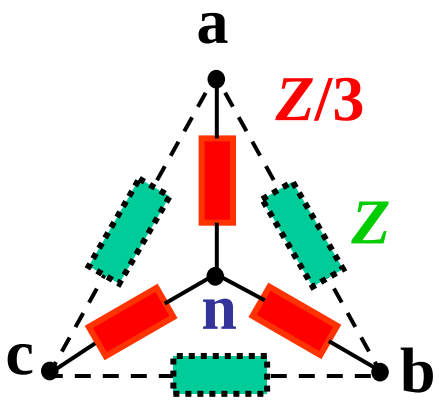
解2　　化为Y - Y

将Δ接电源用Y接电源替代，保证其线电压相等





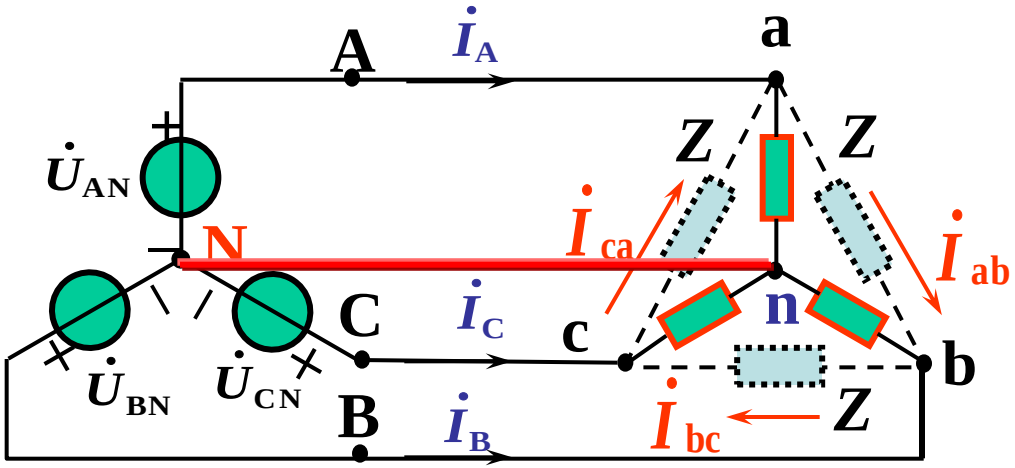
将负载Δ -Y变换



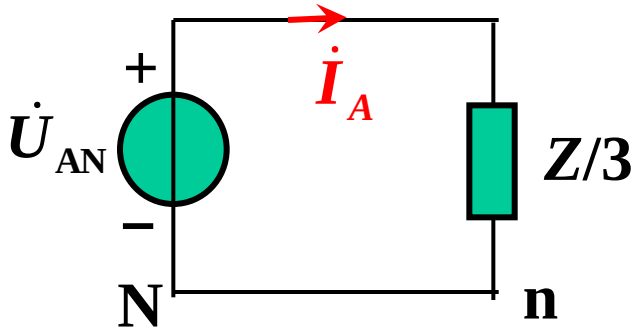
$$\dot{U}_{AN}=\frac{1}{\sqrt{3}}\dot{U}_{AB}\angle-30^{o}$$

$$\dot{U}_{BN}=\frac{1}{\sqrt{3}}\dot{U}_{BC}\angle-30^{o}$$

$$\dot{U}_{CN}=\frac{1}{\sqrt{3}}\dot{U}_{CA}\angle-30^{o}$$



解： 连接中线Nn，取A相为例计算



设 $\dot{U}_{AB} = 380\angle 0^{o}\,\mathrm{V}$

则 $\dot{U}_{AN} = 220\angle -30^{o}\,\mathrm{V}$

$$\dot{I}_A = \frac{\dot{U}_{AN}}{Z/3} = \frac{220\angle -30^{o}}{100\angle 30^{o}/3} = 6.6\angle -60^{o}\,\mathrm{A}$$

由对称性，得 $\dot{I}_B = 6.6\angle -180^{o} = -6.6\mathrm{A}$

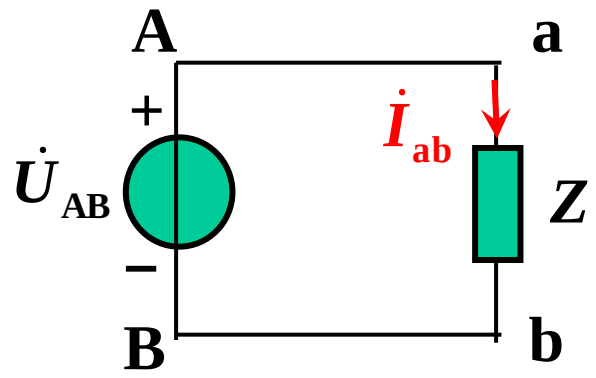
$$\dot{I}_C = 6.6\angle 60^{o}\,\mathrm{A}$$

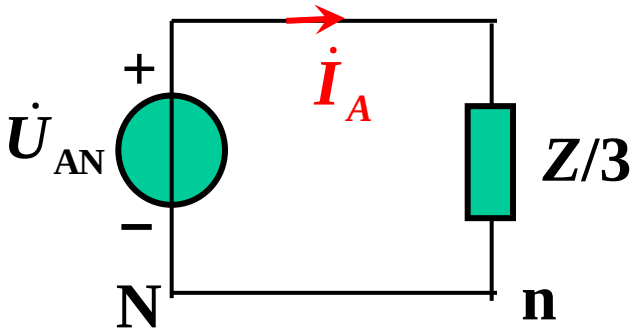
投票 最多可选1项

设置

前面两种方法，

你觉得哪种好?





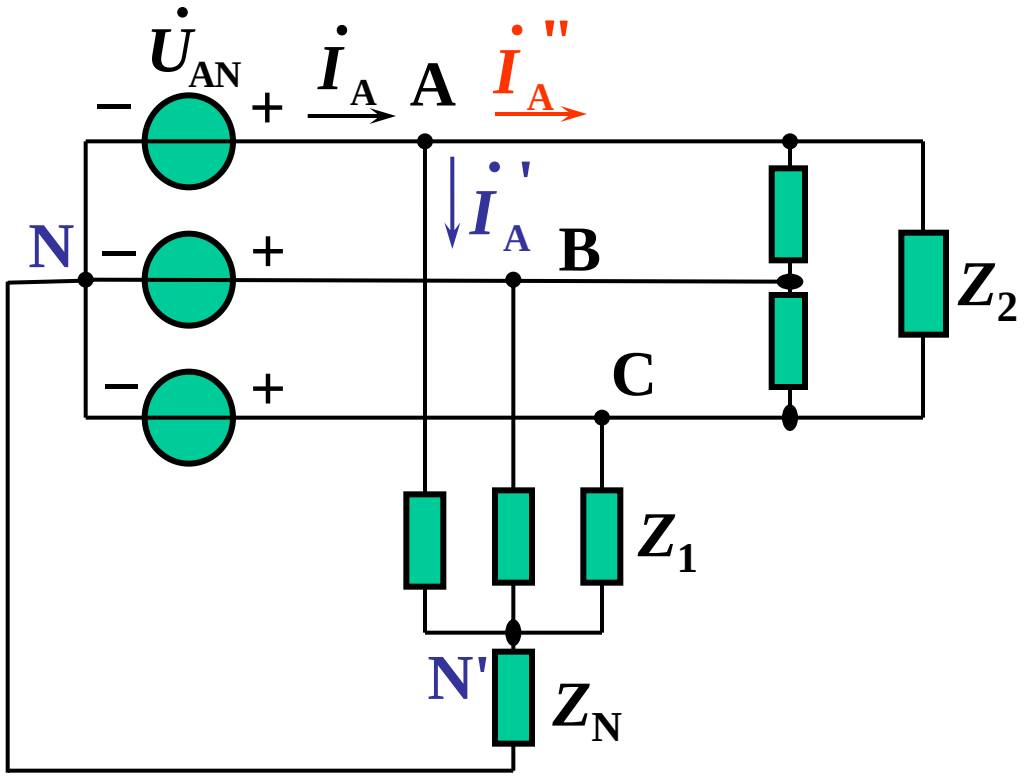
A 解法1

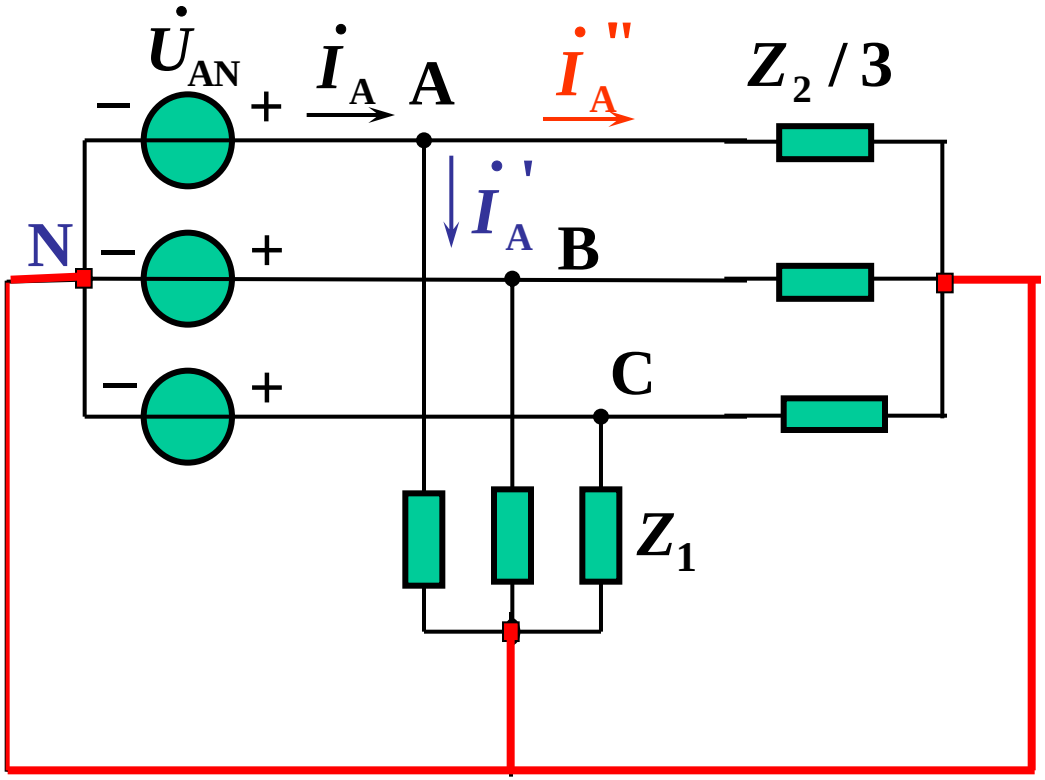
B 解法2

提交

例3 如图对称三相电路，电源线电压为380V，$|Z_1|=10\Omega$，$\cos\varphi_1=0.6$(滞后)，$Z_2=-j50\Omega$，$Z_N=1+j2\Omega$。

求 $\dot{I}_A$



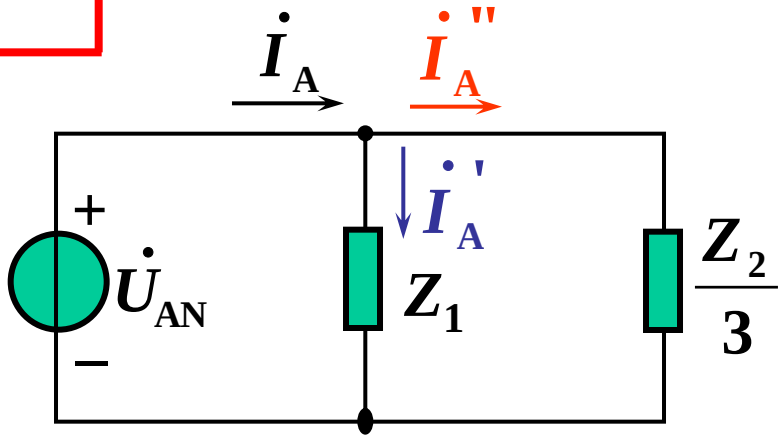


负载进行Δ—Y变换

短路中线阻抗Z_n

连接电源和负载中点

画A相计算电路



设 $\dot{U}_{AB} = 380\angle 30^\circ\ \text{V}$

$\dot{U}_{AN} = 220\angle 0^\circ \text{V}$

$\dot{I}_A = 13.9\angle -18.4^\circ \text{A}$

课后计算

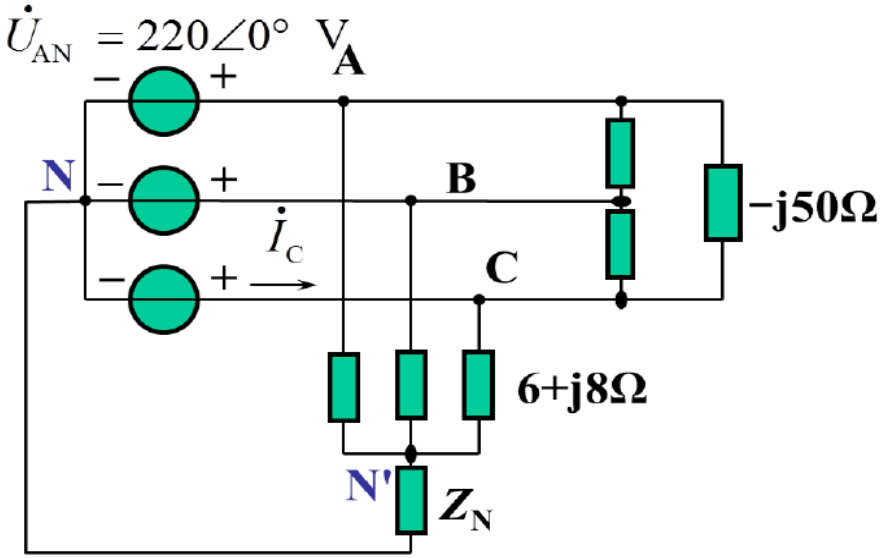
单选题 1分

设置

对称三相电路中，$Z_N=1\Omega$，

其他参数如图所示，

则 $\dot{I}_C$ =



A $6.5\angle 101.6^\circ$

B $6.5\angle -101.6^\circ$

C $13.9\angle 101.6^\circ$

D $13.9\angle -138.4^\circ$

$\dot{I}_A = 13.9\angle -18.4^o\,A$

提交

对称三相电路的一般计算方法

(1) 将所有三相电源、负载都化为等值Y连接;

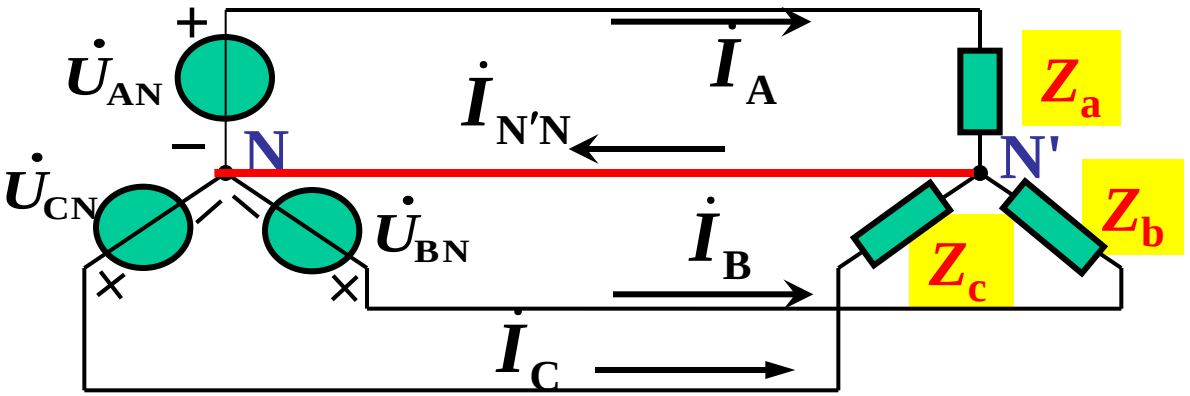
(2) 连接各负载和电源中点，中线上若有阻抗则不计;

(3) 画出A相计算电路，求出A相的电压、电流;

(4) 根据Δ 接、Y接时线量、相量之间的关系和对称性，求出原电路其他相的电流、电压。

3 不对称三相电路分析简介

（1） 有中线



(1) 负载上的相电压仍为对称三相电压；

$$\dot{I}_A = \frac{\dot{U}_{AN}}{Z_a} \qquad \dot{I}_B = \frac{\dot{U}_{BN}}{Z_b} \qquad \dot{I}_C = \frac{\dot{U}_{CN}}{Z_c}$$

各相可分别计算

(2) 由于三相负载不对称，则三相电流不对称；

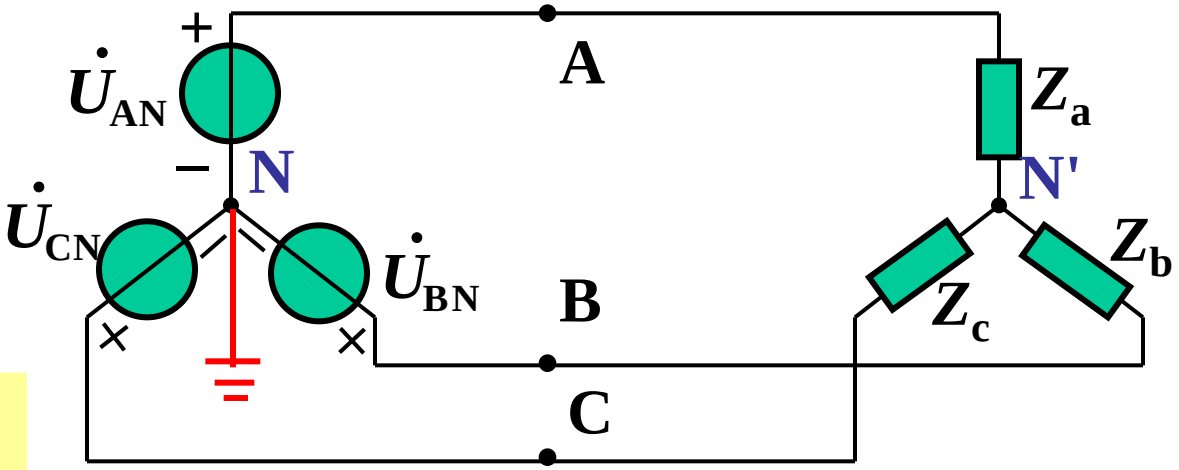
不能抽单相

$$\dot{I}_{N'N} = \dot{I}_A + \dot{I}_B + \dot{I}_C = \frac{\dot{U}_{AN}}{Z_a} + \frac{\dot{U}_{BN}}{Z_b} + \frac{\dot{U}_{CN}}{Z_c} \neq 0$$

(3) 中线电流一般不为零。

三相负载Z_a、Z_b、Z_c不相同。

（2）无中线



无法分别计算各相

节点电压法

$$\dot{U}_{N'N}=\frac{\dot{U}_{AN}/Z_a+\dot{U}_{BN}/Z_b+\dot{U}_{CN}/Z_c}{1/Z_a+1/Z_b+1/Z_c}\neq 0$$

负载各相电压：

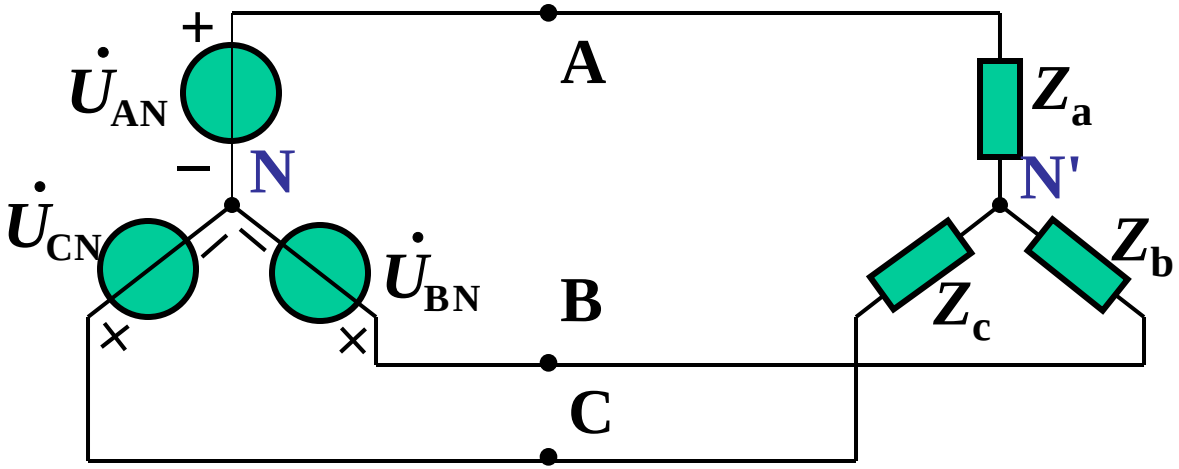
$$\left.\begin{aligned}\dot{U}_{AN'}&=\dot{U}_{AN}-\dot{U}_{N'N}\\ \dot{U}_{BN'}&=\dot{U}_{BN}-\dot{U}_{N'N}\\ \dot{U}_{CN'}&=\dot{U}_{CN}-\dot{U}_{N'N}\end{aligned}\right\}$$

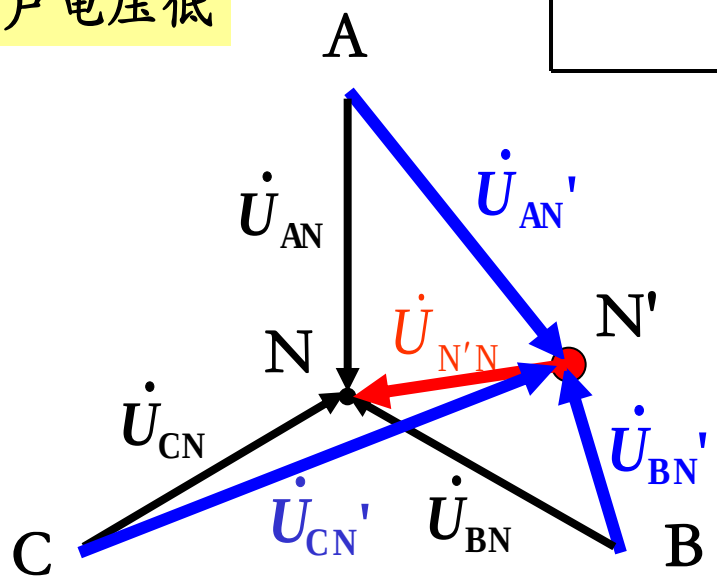
相电压不对称

线(相)电流也不对称

有的用户电压高

有的用户电压低





$$\dot{U}_{N'N} \neq 0$$

$$\dot{U}_{AN'} = \dot{U}_{AN} - \dot{U}_{N'N}$$

$$\dot{U}_{BN'} = \dot{U}_{BN} - \dot{U}_{N'N}$$

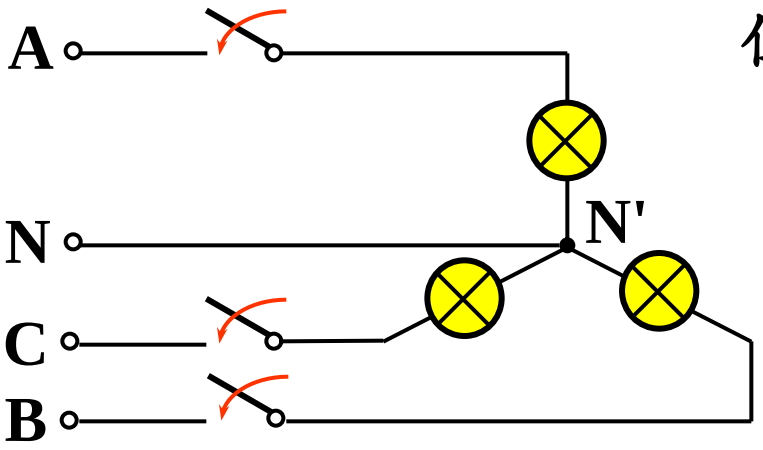
$$\dot{U}_{CN'} = \dot{U}_{CN} - \dot{U}_{N'N}$$

负载中点与电源中点不重合，这个现象称为中点位移。

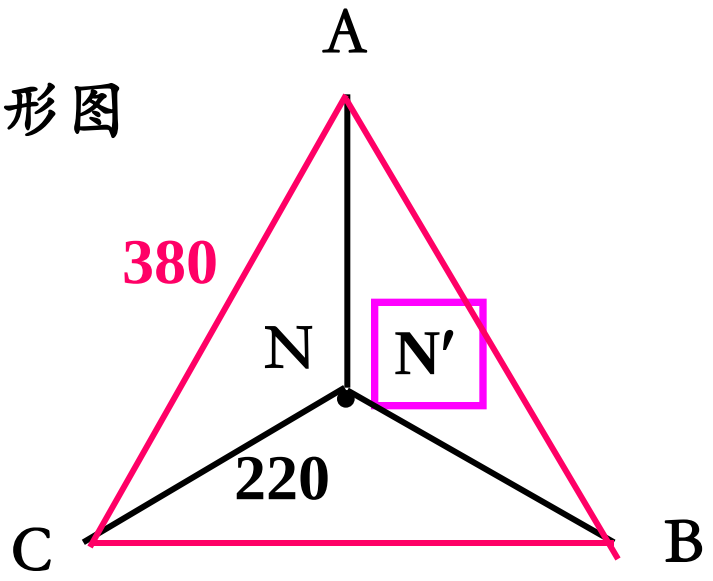
$\dot{U}_{N'N}$ 称为中点位移电压

例1　照明电路。

(1) 正常情况下，三相四线制，中线阻抗为零。

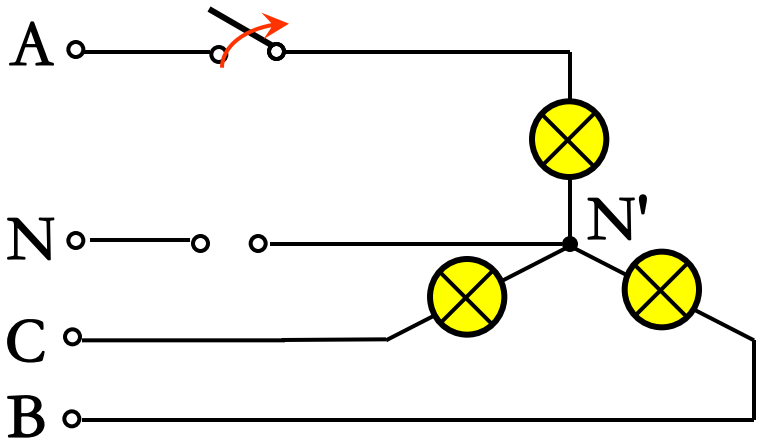


位形图

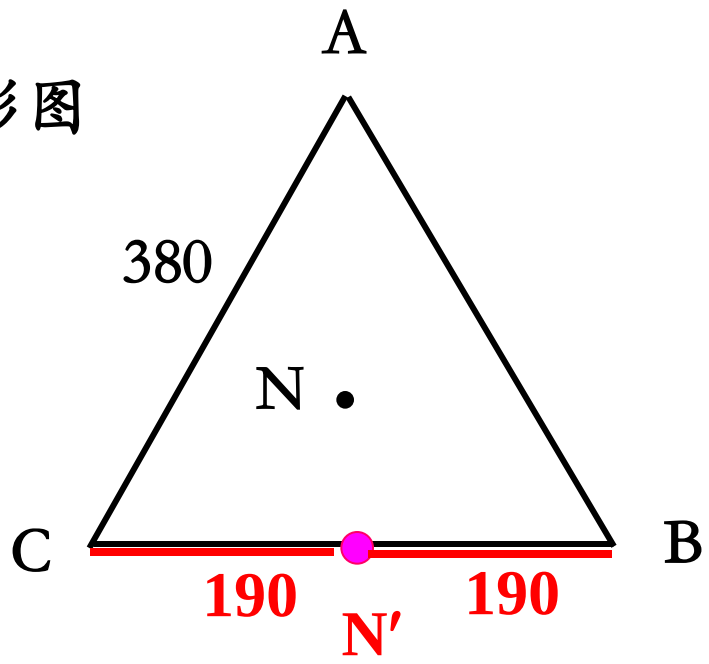


N’在哪里?

(2) 假设中线断了(三相三线制), A相电灯没有接入电路(三相不对称)



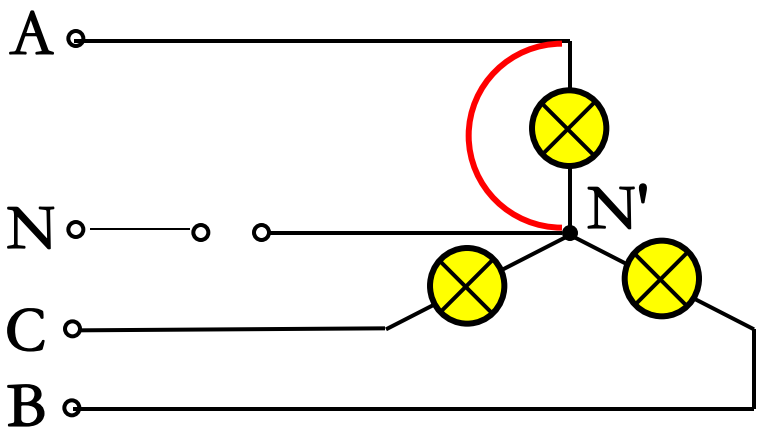
位形图



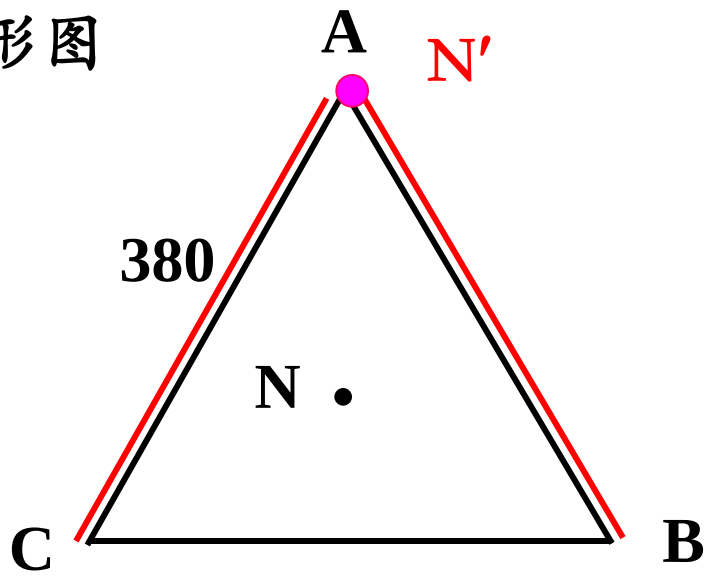
灯泡未在额定电压下工作，灯光昏暗。

N’在哪里?

(3) 中线断了且A相短路

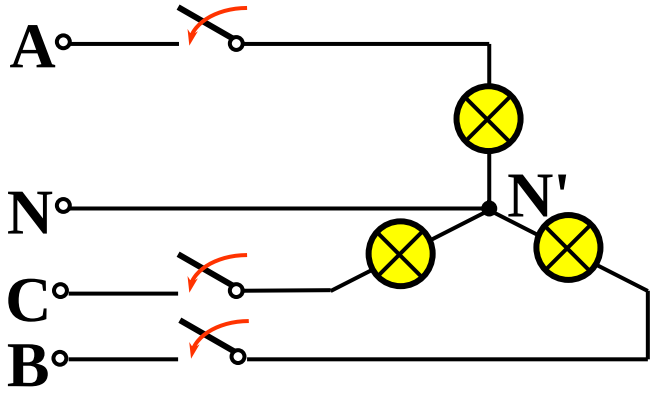


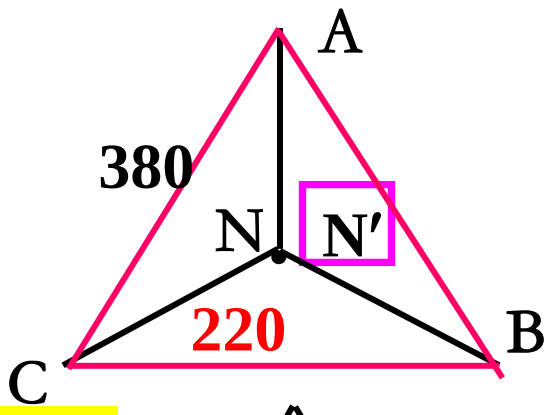
位形图



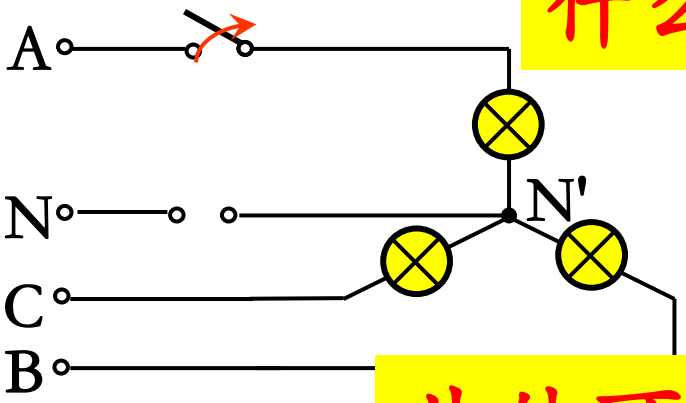
灯泡电压超过额定工作电压，烧坏了。

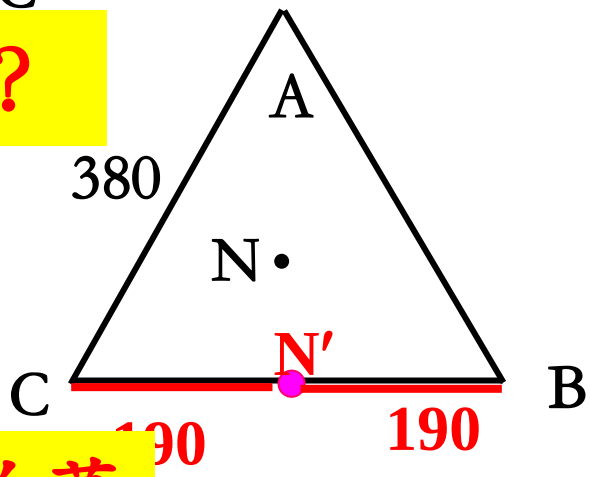
N’在哪里?



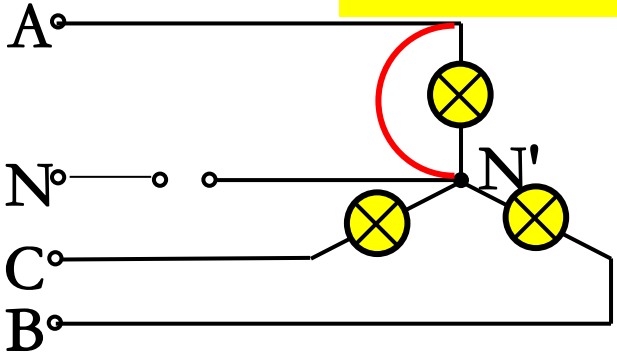


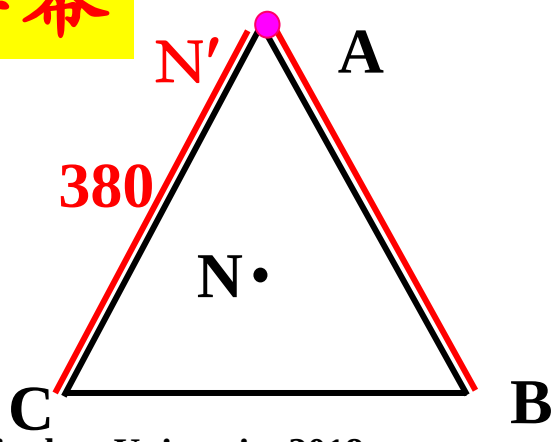
什么结论?





此处可以有弹幕

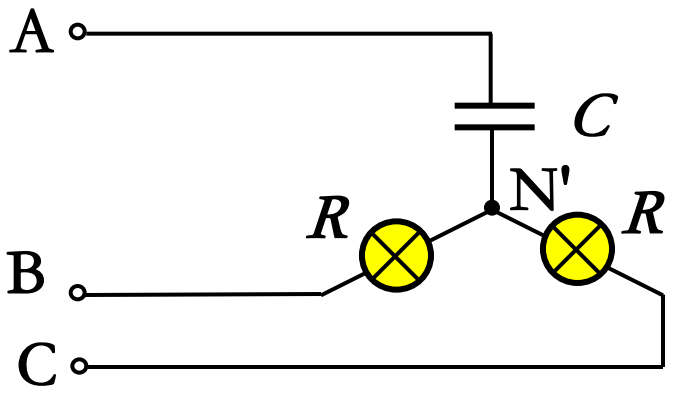




例2　已知$1/(\omega C)=R$，

三相电源对称。

求：灯泡承受的电压。



解

$$\text{设}\dot{U}_{AN}=U\angle 0^{o}\text{V},\ \dot{U}_{BN}=U\angle -120^{o}\text{V},\ \dot{U}_{CN}=U\angle 120^{o}\text{V}$$

$$\dot{U}_{N'N}=\frac{\mathrm{j}\omega C\dot{U}_{AN}+\dot{U}_{BN}/R+\dot{U}_{CN}/R}{\mathrm{j}\omega C+1/R+1/R}=\frac{\mathrm{j}\dot{U}_{AN}+\dot{U}_{BN}+\dot{U}_{CN}}{2+\mathrm{j}1}\rightarrow -\dot{U}_{AN}$$

$$=\frac{(-1+\mathrm{j})\dot{U}_{AN}}{2+\mathrm{j}1}=0.632\angle 108.4^{o}\,\dot{U}_{AN}=0.632U\angle 108.4^{o}\text{V}$$

$$\dot{U}_{BN'}=\dot{U}_{BN}-\dot{U}_{N'N}=U\angle -120^{o}-0.632U\angle 108.4^{o}=1.5U\angle -101.5^{o}\text{V}$$

$$\dot{U}_{CN'}=\dot{U}_{CN}-\dot{U}_{N'N}=U\angle 120^{o}-0.632U\angle 108.4^{o}=0.4U\angle 138.4^{o}\text{V}$$

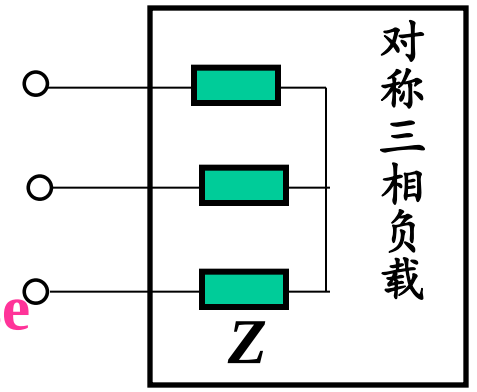
假设灯泡承载1.5U电压不会坏

这个电路什么实际应用功能？

此处可以有弹幕

4 三相电路的功率

(1) 对称三相电路的平均功率



一相负载的功率 $P_p=U_pI_p\cos\varphi_p$

phase

三相总功率 $P_3=3P_p=3U_pI_p\cos\varphi_p$

$Z=|Z|\angle\varphi_p$

Y接： $U_L=\sqrt{3}U_p$， $I_L=I_p$

line

$$P_3=3\cdot\frac{1}{\sqrt{3}}U_LI_L\cos\varphi_p=\sqrt{3}U_LI_L\cos\varphi_p$$

Δ接： $U_L=U_p$， $I_L=\sqrt{3}I_p$

$$P_3=3U_L\cdot\frac{1}{\sqrt{3}}I_L\cos\varphi_p=\sqrt{3}U_LI_L\cos\varphi_p$$

注意： φ_p 为相电压与相电流的相位差角(Y接负载单相阻抗角)。

（2） 对称三相电路的无功功率

$$Q_3 = 3U_P I_P \sin\varphi_P = \sqrt{3}U_L I_L \sin\varphi_P$$

（3）对称三相电路的复功率

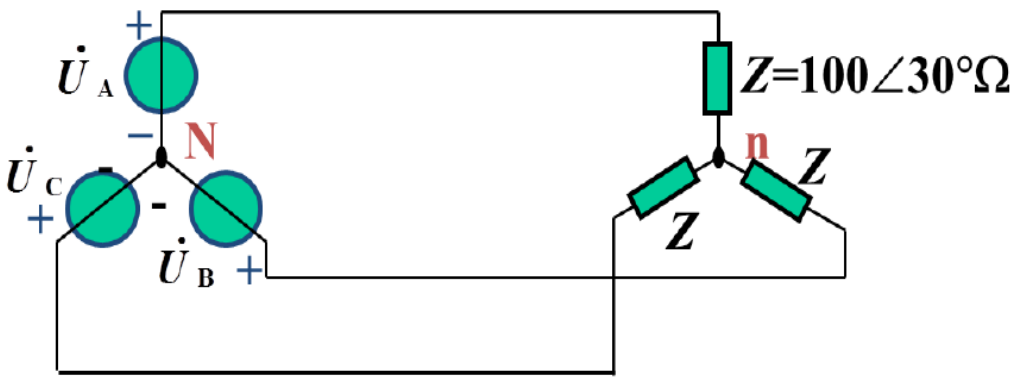
$$\bar{S}_3 = 3\dot{U}_P \dot{I}_P^*$$

（4） 对称三相电路的瞬时功率

$$p = p_A + p_B + p_C = 3U_p I_p \cos\varphi_p = P$$

设置

对称三相电路如图所示，已知$\dot{U}_A = 220\angle 30^\circ$则三相负载吸收的有功功率为



A 1257W

B 419W

C 346W

D 726W

提交

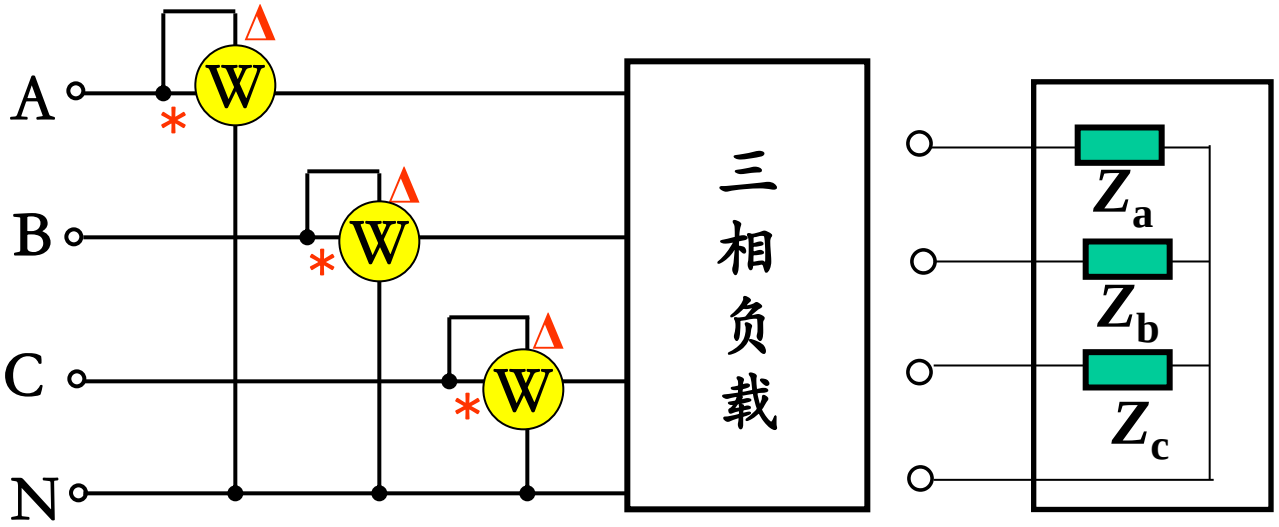
(5) 三相电路功率的测量（可以不对称）

(a) 三表法：

$$P_{总} = P_A + P_B + P_C$$

适用于

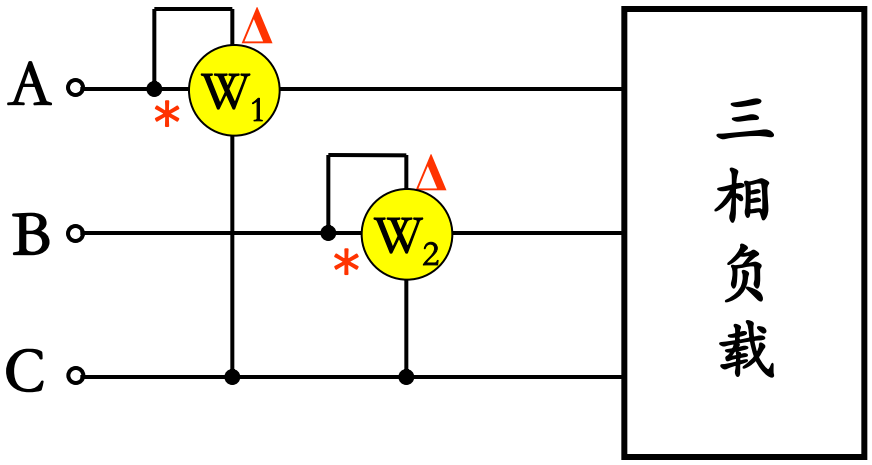
三相四线制



若负载对称，则需一块表，读数乘以 3。

如果是三相三线制怎么办?

(b) 二表法:



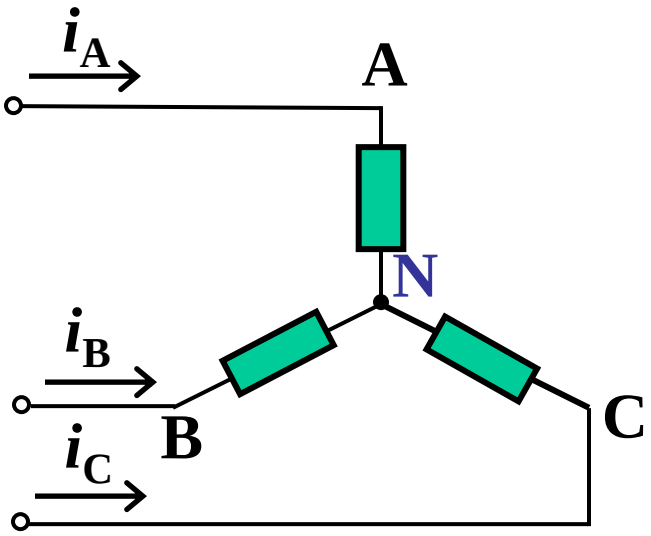
若W_1的读数为P_1，W_2的读数为P_2，则 $P_{总}=P_1+P_2$

即为三相总功率。

证明：(设负载为Y接)

$$p_{总}=u_{AN}i_A+u_{BN}i_B+u_{CN}i_C$$

$$i_C=-(i_A+i_B)$$



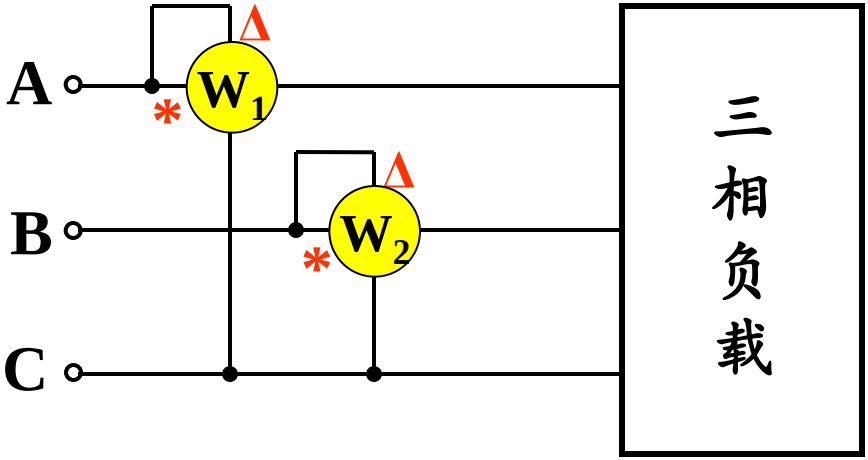
$$p_{总}=(u_{AN}-u_{CN})i_A+(u_{BN}-u_{CN})i_B$$

$$=u_{AC}i_A+u_{BC}i_B$$

$$P_{总}=U_{AC}I_A\cos\varphi_1+U_{BC}I_B\cos\varphi_2$$

φ_1：u_{AC} 领先i_A的相位角，

φ_2：u_{BC} 领先i_B的相位角。



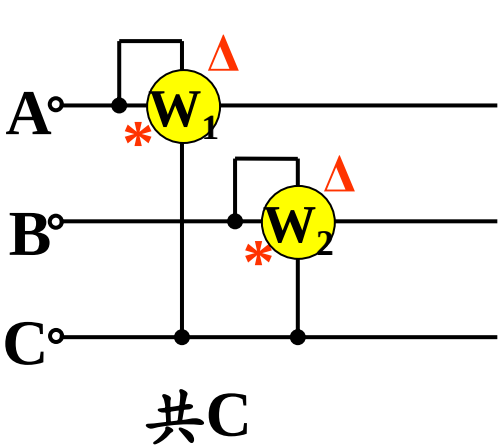
注意：

1. 只有在 $i_A+i_B+i_C=0$ 这个条件下，才能用二表法。二表法不能用于不对称三相四线制，但对称三相四线制可用。

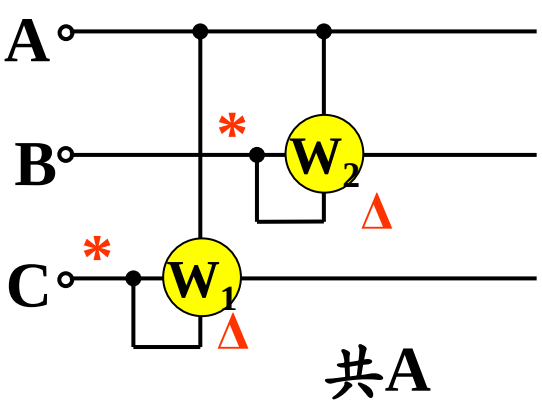
2. 两块表读数的代数和为三相总功率，单块表的单独读数无意义。

3. 按正确极性接线时，二表中可能有一个表的读数为负。

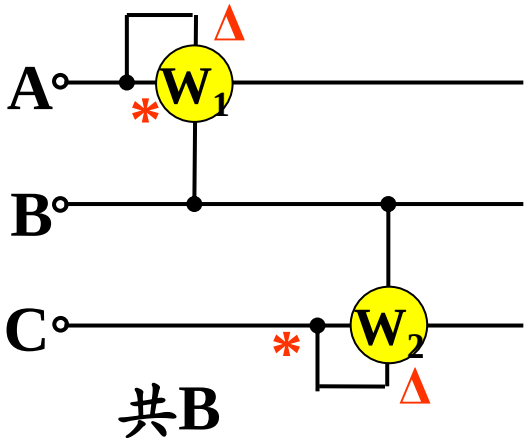
4. 两表法测三相功率的接线方式有三种。



共C



共A

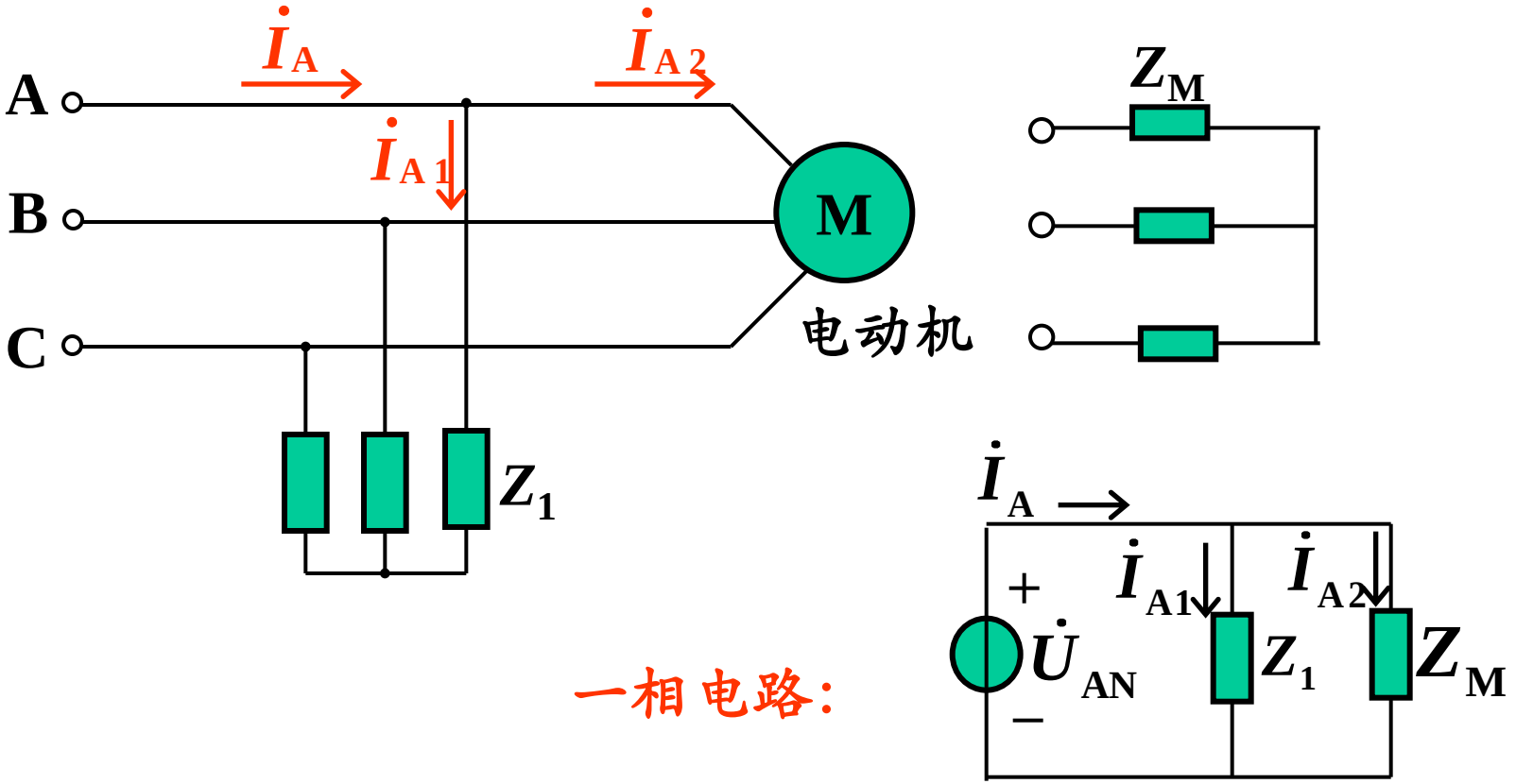


共B

例:U_L =380V, Z_1=30+j40Ω, 电动机 P_M =1700W, cosφ =0.8(滞后)。

求: (1) 线电流和电源发出总功率;

(2) 用两表法测电动机负载的功率，画接线图，求两表读数。



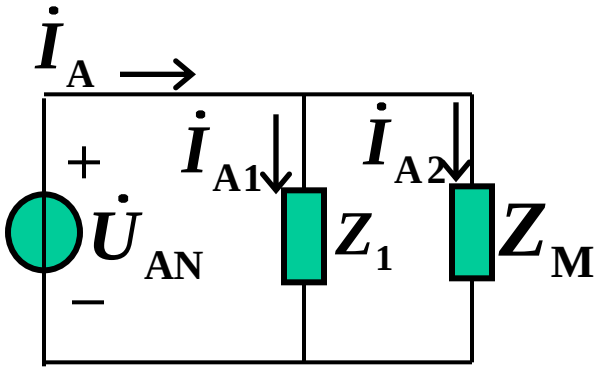
一相电路:

U_L =380V, Z_1=30+j40Ω, 电动机 P_M =1700W, cosφ =0.8(滞后)。

解

(1) 设 $\dot{U}_{AN} = 220\angle 0^\circ$ V

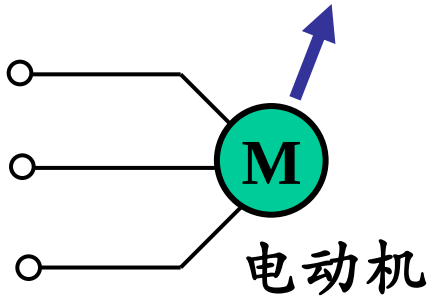
$$\dot{I}_{A1} = \frac{\dot{U}_{AN}}{Z_1} = \frac{220\angle 0^\circ}{30 + j40} = 4.41\angle -53.1^\circ \text{ A}$$



电动机负载:

$$P_M = \sqrt{3} U_L I_{A2} \cos\varphi = 1700\text{W}$$

$$I_{A2} = \frac{P_M}{\sqrt{3} U_L \cos\varphi} = \frac{P_M}{\sqrt{3} \times 380 \times 0.8} = 3.23\text{A}$$



$\cos\varphi = 0.8$(滞后), $\varphi = 36.9^\circ$ → Y接模型单相阻抗角

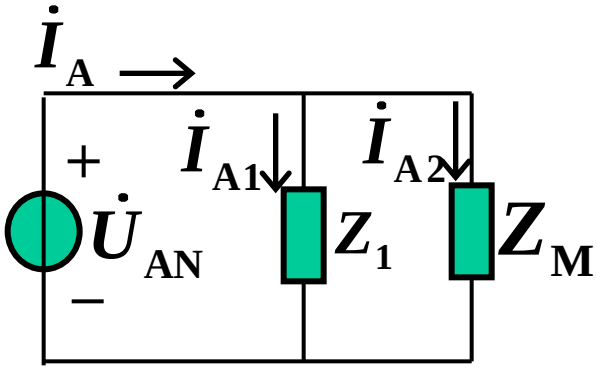
↓

A相电压电流相位差

$$\dot{I}_{A2} = 3.23\angle -36.9^\circ \text{ A}$$

$\dot{U}_{AN} = 220\angle 0^\circ \text{ V}$ $\dot{I}_{A1} = 4.41\angle -53.1^\circ \text{ A}$

$\dot{I}_{A2} = 3.23\angle -36.9^\circ \text{ A}$



总电流：

$$\dot{I}_A = \dot{I}_{A1} + \dot{I}_{A2}$$
$$= 4.41\angle -53.1^\circ + 3.23\angle -36.9^\circ = 7.56\angle -46.2^\circ \text{ A}$$

$$P_{总} = \sqrt{3} U_L I_A \cos\varphi_{P总} \qquad \varphi_{P总} = \psi_{\dot{U}_{AN}} - \psi_{\dot{I}_A} = 46.2^\circ$$

$$= \sqrt{3} \times 380 \times 7.56 \times \cos 46.2^\circ = 3.44\text{kW}$$

(2) 两表的接法如图。

$\dot{I}_{A2} = 3.23\angle -36.9^{\circ}\,\mathrm{A}$

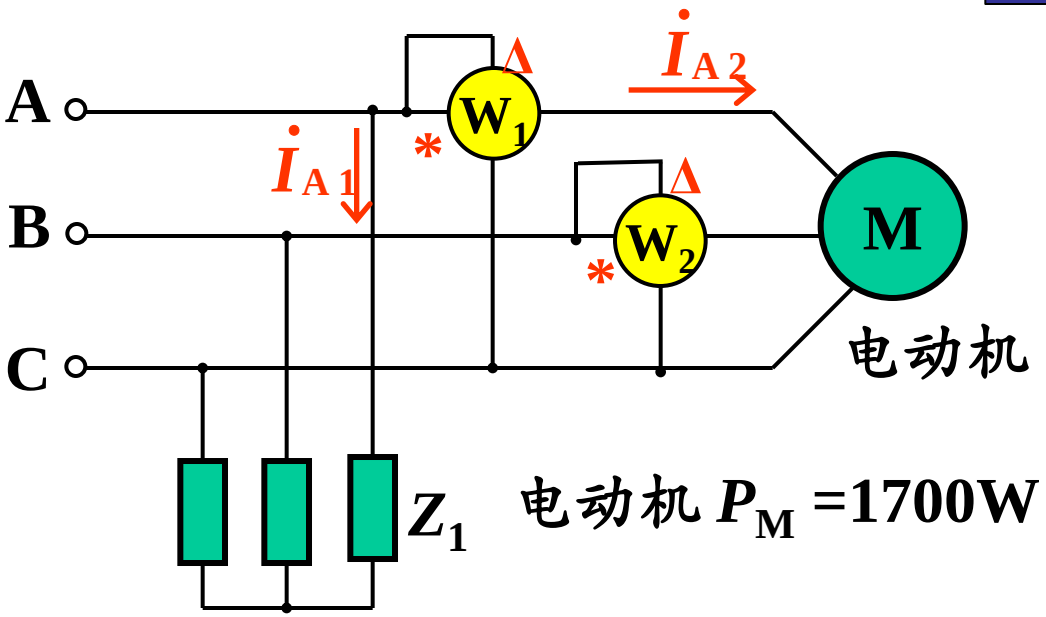
$\dot{I}_{B2} = 3.23\angle -156.9^{\circ}\,\mathrm{A}$

$\dot{U}_{AB} = 380\angle 30^{\circ}\,\mathrm{V}$

$\dot{U}_{AC} = -\dot{U}_{CA} = -380\angle 150^{\circ}\,\mathrm{V}$

$= 380\angle -30^{\circ}\,\mathrm{V}$

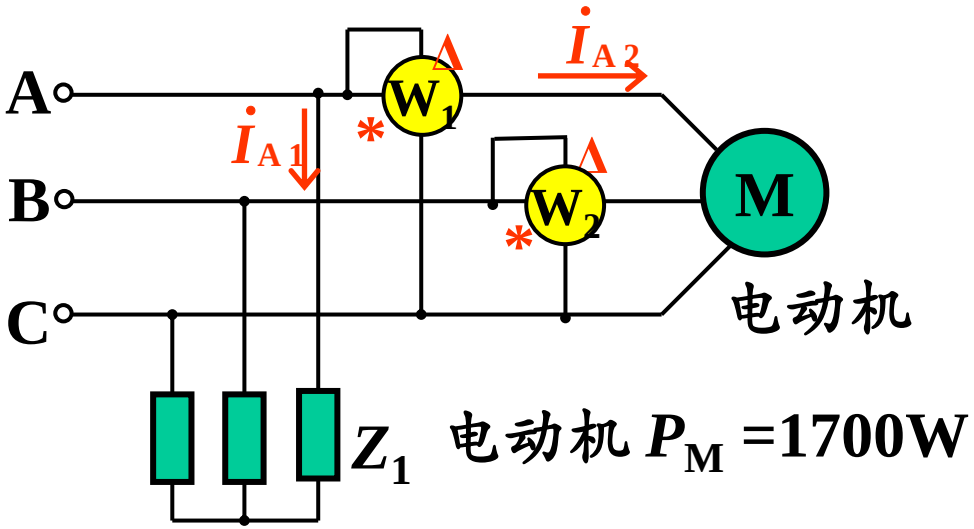
$\dot{U}_{BC} = 380\angle -90^{\circ}\,\mathrm{V}$



单选题 1分

设置

表W_1的读数为：



$$\dot{I}_{A2} = 3.23\angle -36.9^{\circ}\,\text{A}$$

$$\dot{U}_{AC} = -\dot{U}_{CA} = 380\angle -30^{\circ}\,\text{V}$$

A 1219W

B 1227.4W

C 481.5W

D -1219W

提交

(2) 两表的接法如图。

$\dot{I}_{A2} = 3.23\angle -36.9^{\circ}\ \mathrm{A}$

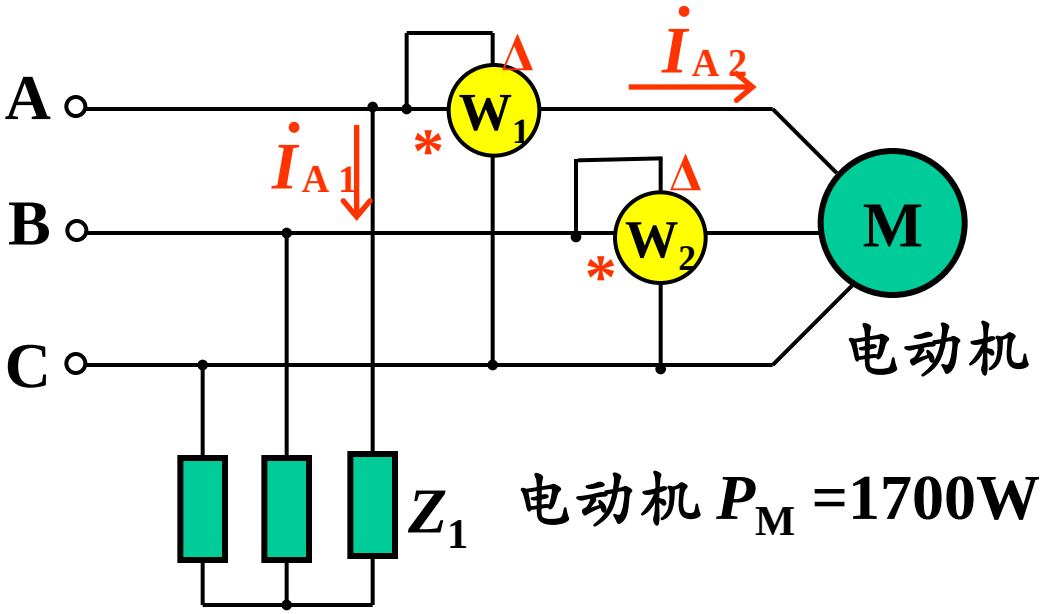
$\dot{I}_{B2} = 3.23\angle -156.9^{\circ}\ \mathrm{A}$

$\dot{U}_{AB} = 380\angle 30^{\circ}\ \mathrm{V}$

$\dot{U}_{AC} = -\dot{U}_{CA} = -380\angle 150^{\circ}\ \mathrm{V}$

$= 380\angle -30^{\circ}\ \mathrm{V}$

$\dot{U}_{BC} = 380\angle -90^{\circ}\ \mathrm{V}$



表W_1的读数：$P_1 = U_{AC} I_{A2} \cos\varphi_1 = 380\times 3.23\cos(-30^{\circ} + 36.9^{\circ})$

$= 380\times 3.23\cos(6.9^{\circ}) = 1219\mathrm{W}$

1700W

表W_2的读数：$P_2 = U_{BC} I_{B2} \cos\varphi_2 = 380\times 3.23\cos(-90^{\circ} + 156.9^{\circ})$

$= 380\times 3.23\cos(66.9^{\circ}) = 481\mathrm{W}$

总结

- 相线关系是重中之重
  - 线值是对应相值的 $\sqrt{3}$ 倍
  - 线电压领先对应相电压30度
  - 线电流落后对应相电流30度
- 对称三相抽单(A)相
- 不对称三相电路节点法
- 三表法和两表法均可适用于不对称三相电路