第3讲　电路的等效变换

1 2*b*法求解电路

2 电阻等效变换

2.1 串并联

2.2 平衡电桥

2.3 Y-Δ变换

2.4 含受控源二端网络的入端电阻

3 电源等效变换

3. 1 理想独立源等效变换

3. 2 实际独立源等效变换

对等效的理解

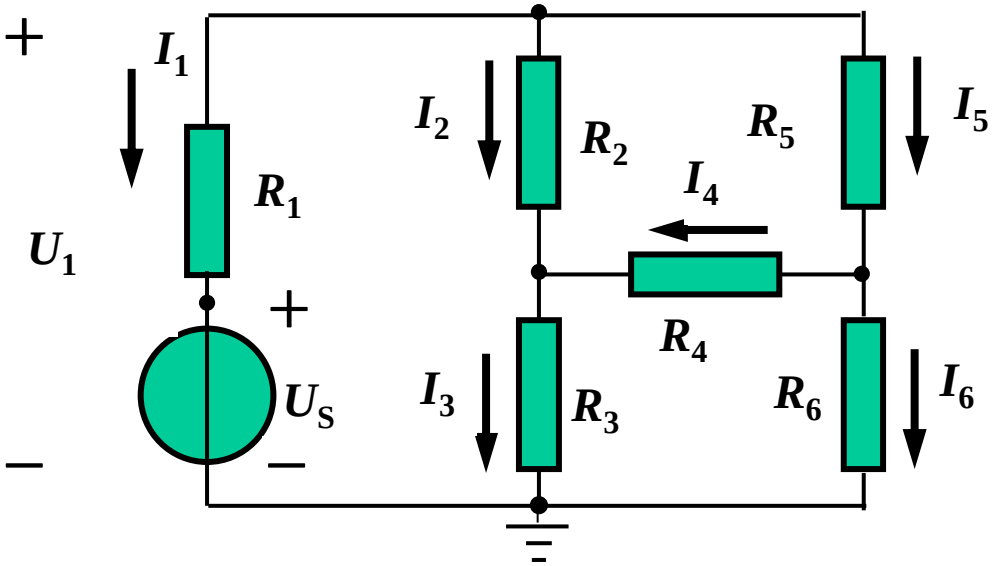
1 2*b*法求解电路

所有支路电压与电流采用关联参考方向。求电流I_1～I_6。

n=4

b=6

l=7



12个变量待求解
(每个支路的电压电流)

元件约束有6个方程

$U_1=R_1I_1+U_s$

$U_2=R_2I_2$

$U_3=R_3I_3$

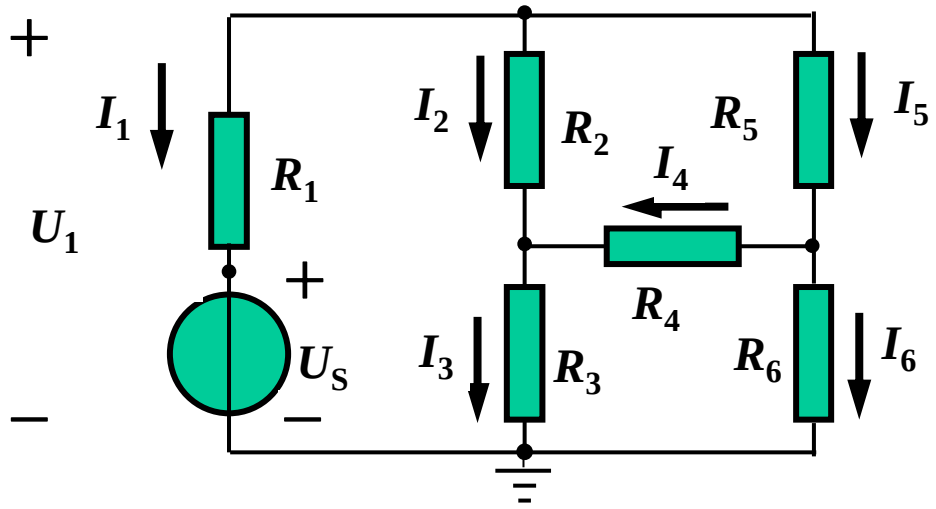
$U_4=R_4I_4$

$U_5=R_5I_5$

$U_6=R_6I_6$

$n=4$ $b=6$ $l=7$

原则上需要12个独立方程



n节点b支路电路

$2b$个未知量

$2b$个方程

b个独立元件约束

n-1个独立KCL

b-n+1个独立KVL

$2b$法

$$I_1+I_2+I_5=0$$
$$-I_2+I_3-I_4=0$$
$$I_4-I_5+I_6=0$$

$$U_1-U_2-U_3=0$$
$$U_2-U_4-U_5=0$$
$$U_3+U_4-U_6=0$$

$$U_1=R_1I_1+U_s$$
$$U_2=R_2I_2$$
$$U_3=R_3I_3$$
$$U_4=R_4I_4$$
$$U_5=R_5I_5$$
$$U_6=R_6I_6$$

理由？教材附录B

元件约束6个方程

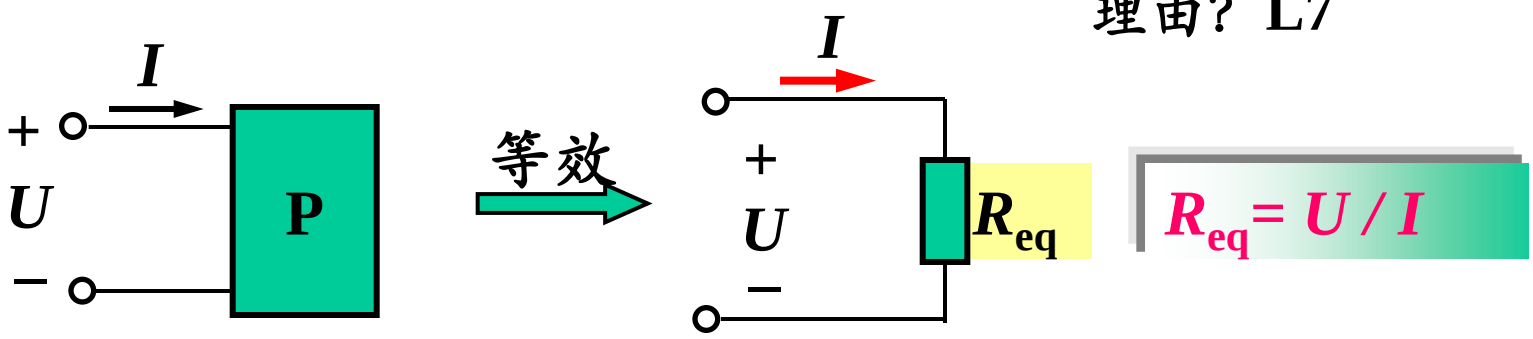
2.1 串并联

二端网络：与外部只有两个接线端相连的网络。

无独立源二端网络：网络内部没有独立源的二端网络。

一个无独立源二端电阻网络可以用端口的入端电阻来等效。

理由？L7

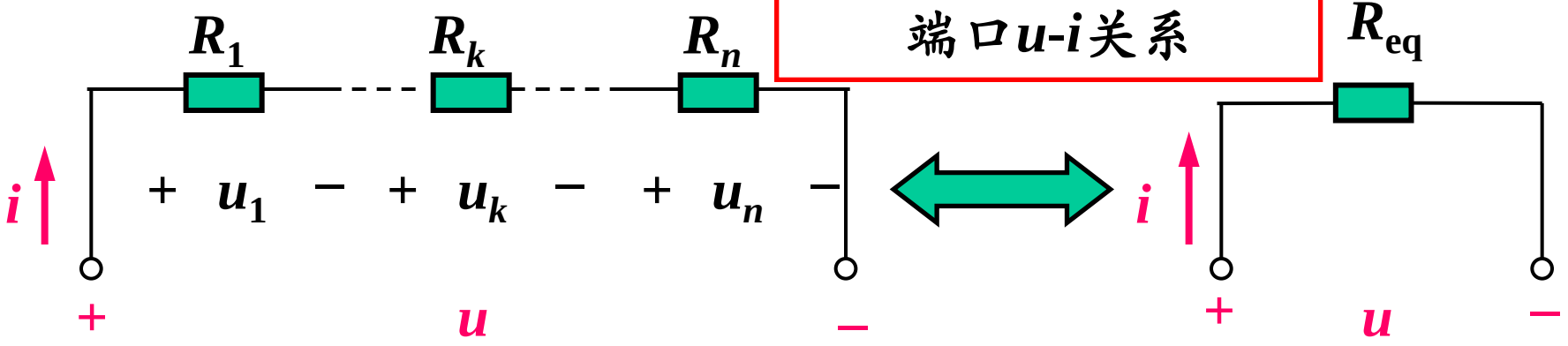


两个电路等效：两个电路*u-i*关系的形式和参数均一样

I、 电阻元件串联（无分叉地首尾相连）

等效的相对性

等效电阻 R_{eq}



KVL $u= u_1+ u_2 +\ldots+u_k+\ldots+u_n$

欧姆定律 $u_k = R_k\, i \quad (k=1, 2, \ldots, n)$

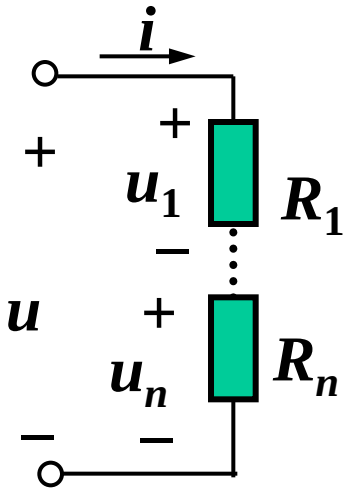
$u= (R_1+ R_2 +\ldots+R_k+\ldots+ R_n)\, i$ $\qquad u= R_{eq} i$

$$R_{eq}= R_1+ R_2 +\ldots+R_n$$

串联电阻元件的分压

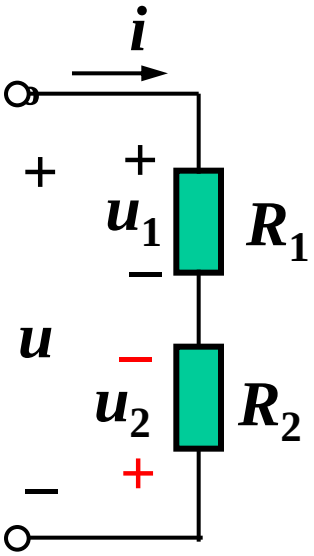
$$\frac{u_k}{u}=\frac{R_k i}{R_{\text{eq}} i}=\frac{R_k}{R_{\text{eq}}}$$

$$u_k=\frac{R_k}{R_{\text{eq}}}u$$



电阻越大，压降越大

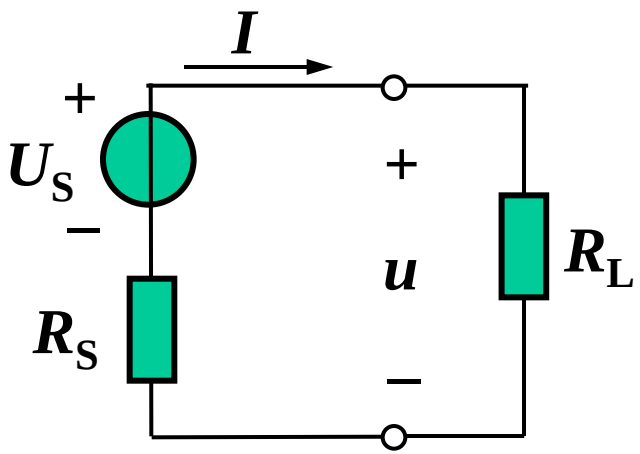
例



$$u_1=\frac{R_1}{R_1+R_2}u$$

$$u_2=\ominus\frac{R_2}{R_1+R_2}u$$

（参考方向！）



$$u = \frac{R_L}{R_L + R_S} U_S$$

U_S：电压形式表示的信号源

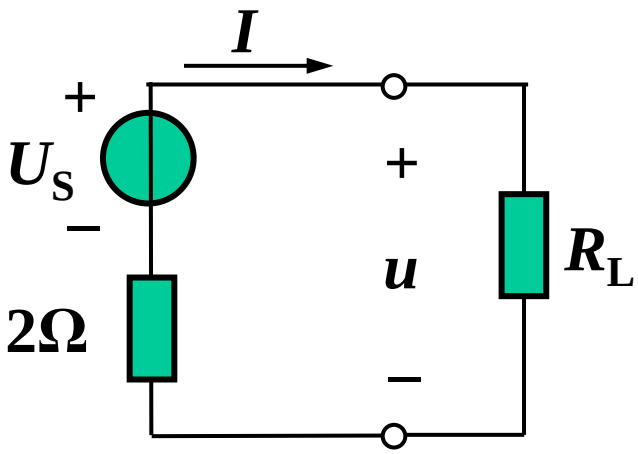
负载电阻R_L相对越大，负载上得到的信号越大

电压源内阻R_S相对越小，为负载提供信号的能力越强

单选题 1分

设置

R_L=_____Ω时，其上获得最大电压



A 2

B 1

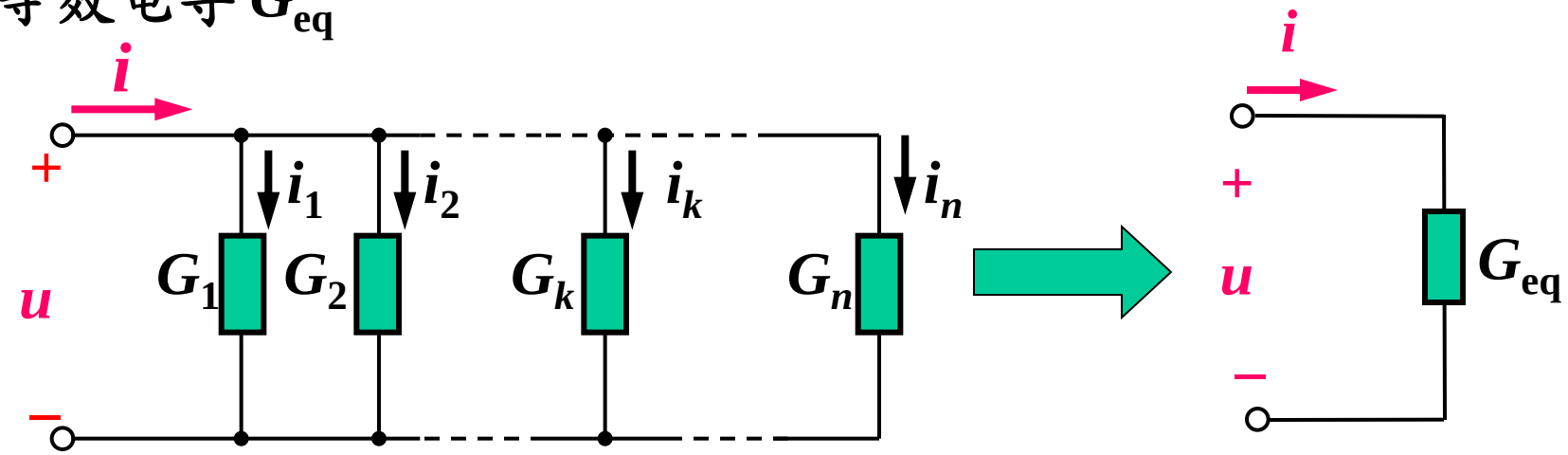
C 0

D ∞

提交

II、 并联电阻元件（元件共用两个接线端）

等效电导 G_{eq}



KCL $i = i_1 + i_2 + \ldots + i_k + \ldots + i_n$

$$i_k = G_k u$$

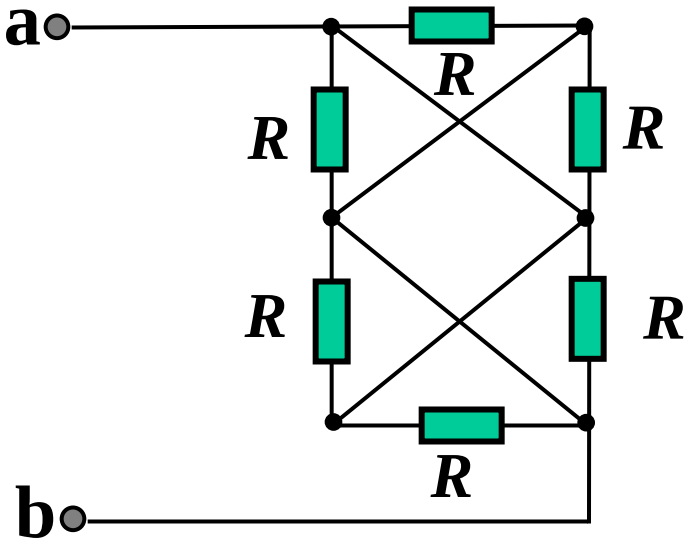
$$= uG_1 + uG_2 + \ldots + uG_n = u(G_1 + G_2 + \ldots + G_n) = u\,G_{eq}$$

$$G_{eq} = G_1 + G_2 + \ldots + G_n$$

单选题 1分

设置

R_{ab} = _____.



A 3*R*

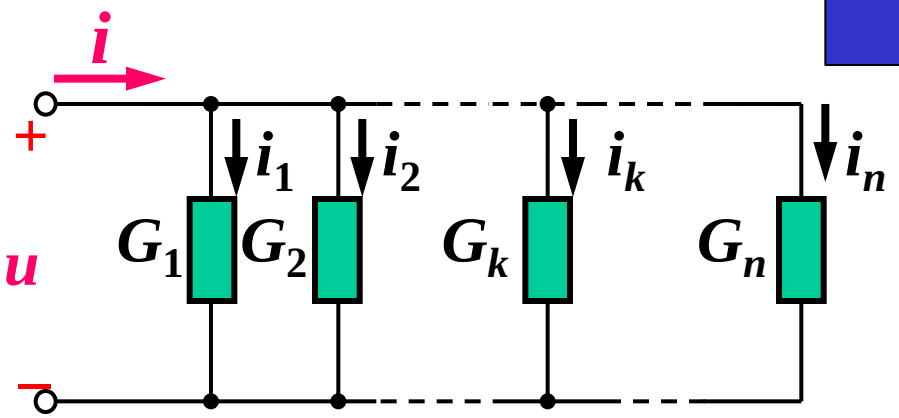
B 6*R*

C *R*/3

D *R*/6

提交

并联电阻器的分流

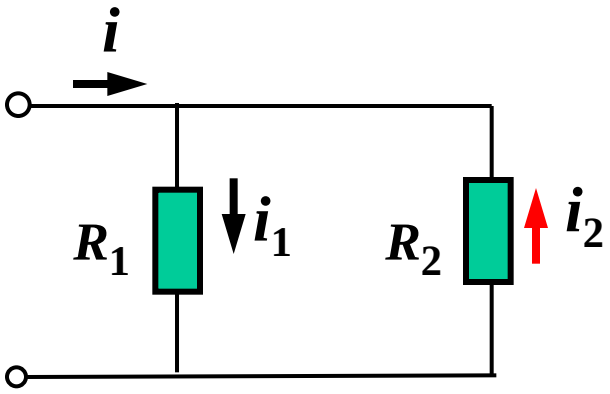


$$\frac{i_k}{i}=\frac{G_k u}{G_{eq} u}=\frac{G_k}{G_{eq}}$$

$$i_k=\frac{G_k}{G_{eq}} i$$

电导越大（电阻越小），电流越大。

例



$$i_1=\frac{1/R_1}{1/R_1+1/R_2} i=\frac{R_2}{R_1+R_2} i$$

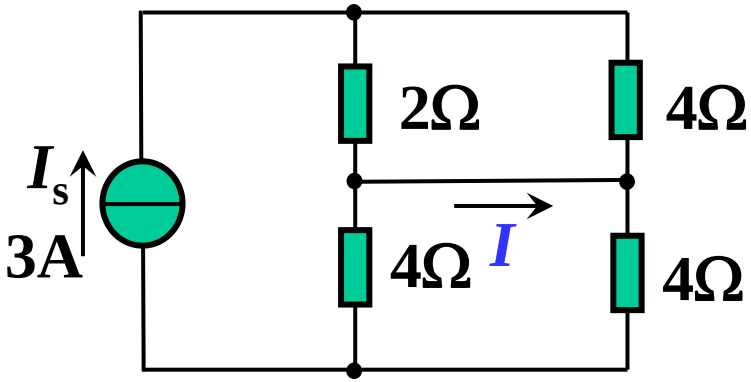
$$i_2=\frac{-1/R_2}{1/R_1+1/R_2} i=-\frac{R_1}{R_1+R_2} i$$

参考方向!

单选题 1分

设置

I= _____A



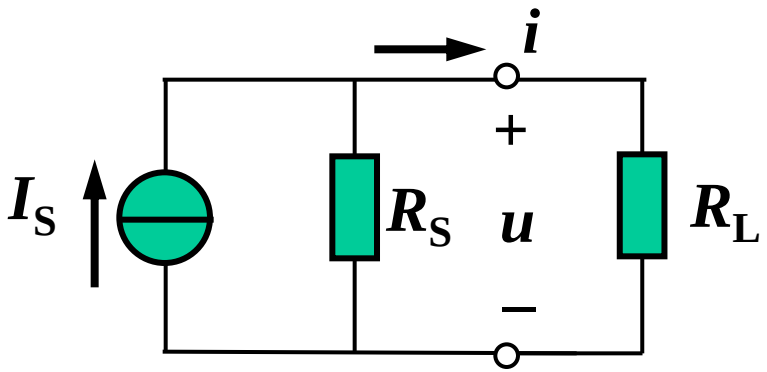
A 0

B 0.5

C −0.5

D 1

提交



$$i = \frac{R_S}{R_L + R_S} I_S$$

I_S：电流形式表示的信号源

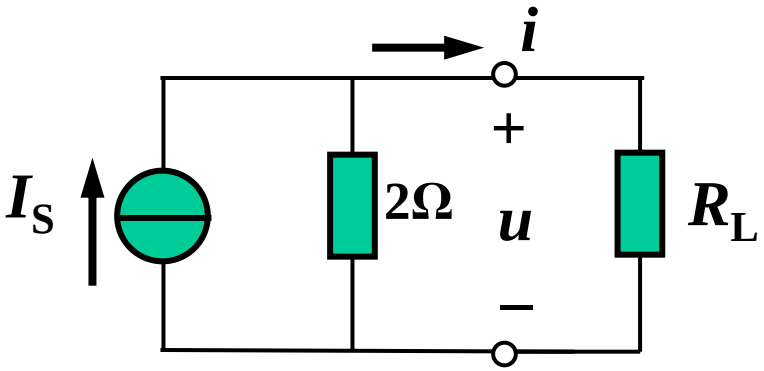
负载电阻R_L相对越小，负载上得到的信号越大

电流源内阻R_S相对越大，为负载提供信号的能力越强

单选题 1分

设置

R_L=___Ω时，其上流过最大电流



A 0

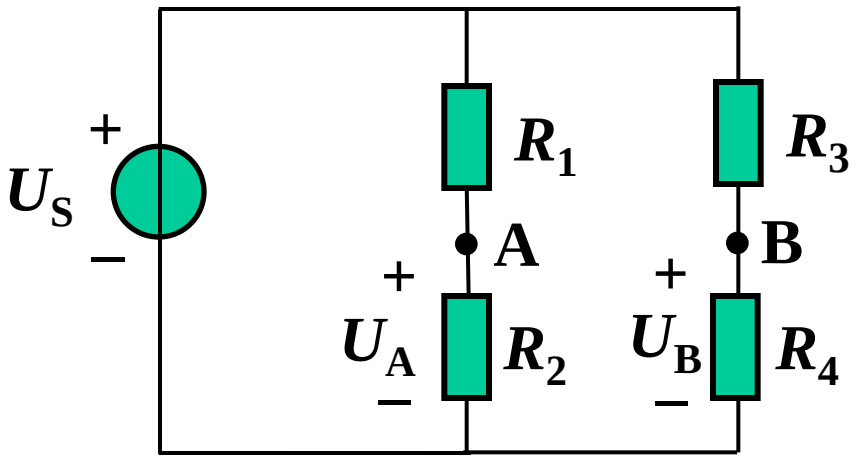
B 2

C 4

D ∞

提交

2.2 平衡电桥



$$U_A = \frac{R_2}{R_1 + R_2} U_S \qquad U_B = \frac{R_4}{R_3 + R_4} U_S$$

如果

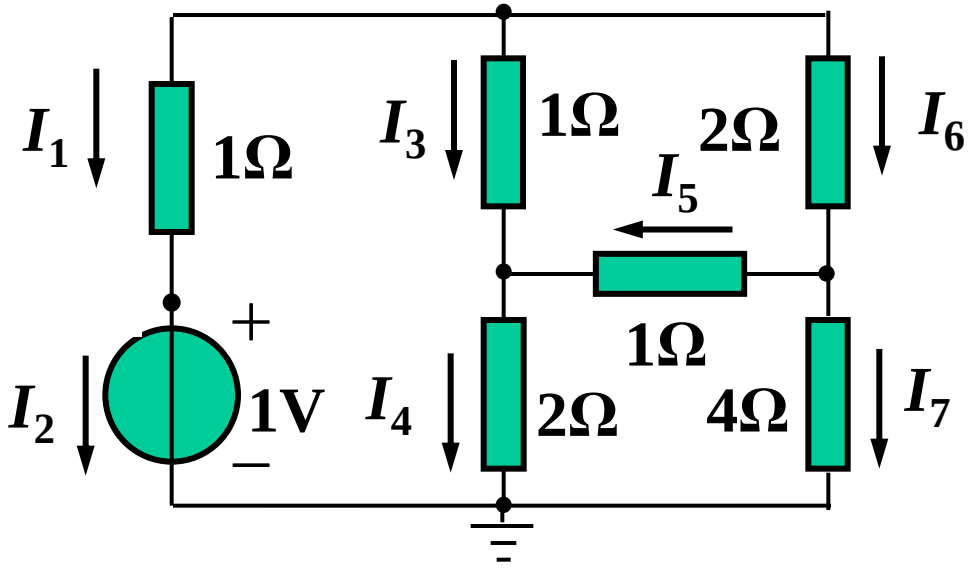
$$R_1 R_4 = R_2 R_3$$

$$U_A = \frac{R_2}{R_1 + R_2} U_S = \frac{R_2}{\dfrac{R_2 R_3}{R_4} + R_2} U_S = \frac{R_4}{R_3 + R_4} U_S = U_B$$

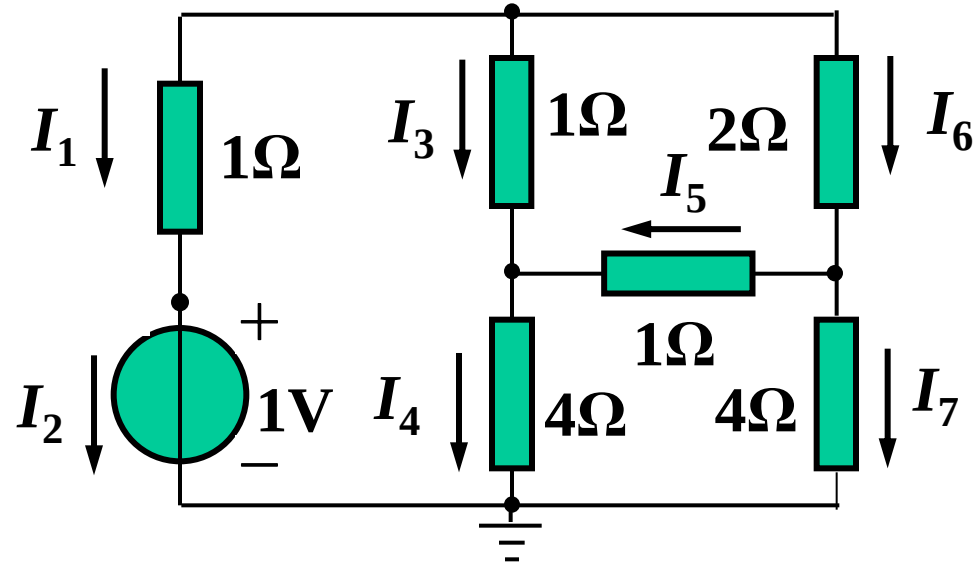
A-B为“等电位点”

A-B间(开路)电压为零

等电位点间接任意电阻(含开短路)不影响电路的电压电流分布(L7解释)

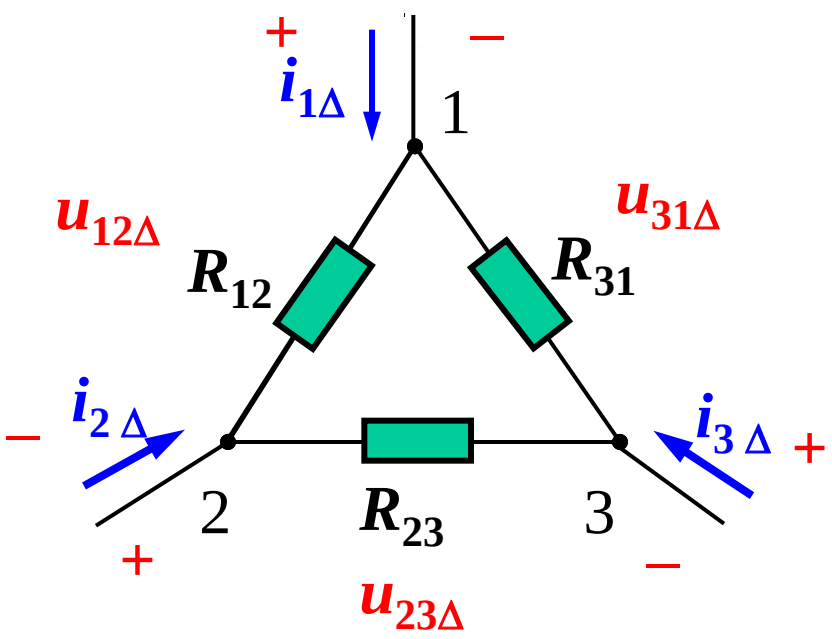

I_1
1Ω
I_3
1Ω
2Ω
I_6
I_5
1Ω
+
I_2
1V
−
I_4
2Ω
4Ω
I_7

V

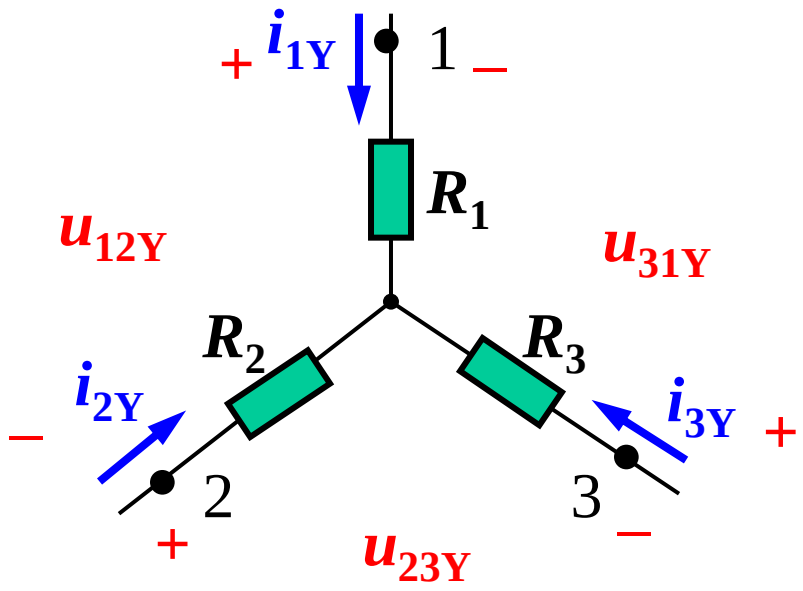

I_1
1Ω
I_3
1Ω
2Ω
I_6
I_5
1Ω
+
I_2
1V
−
I_4
4Ω
4Ω
I_7

?

2.3 Y—Δ变换



Δ 网络

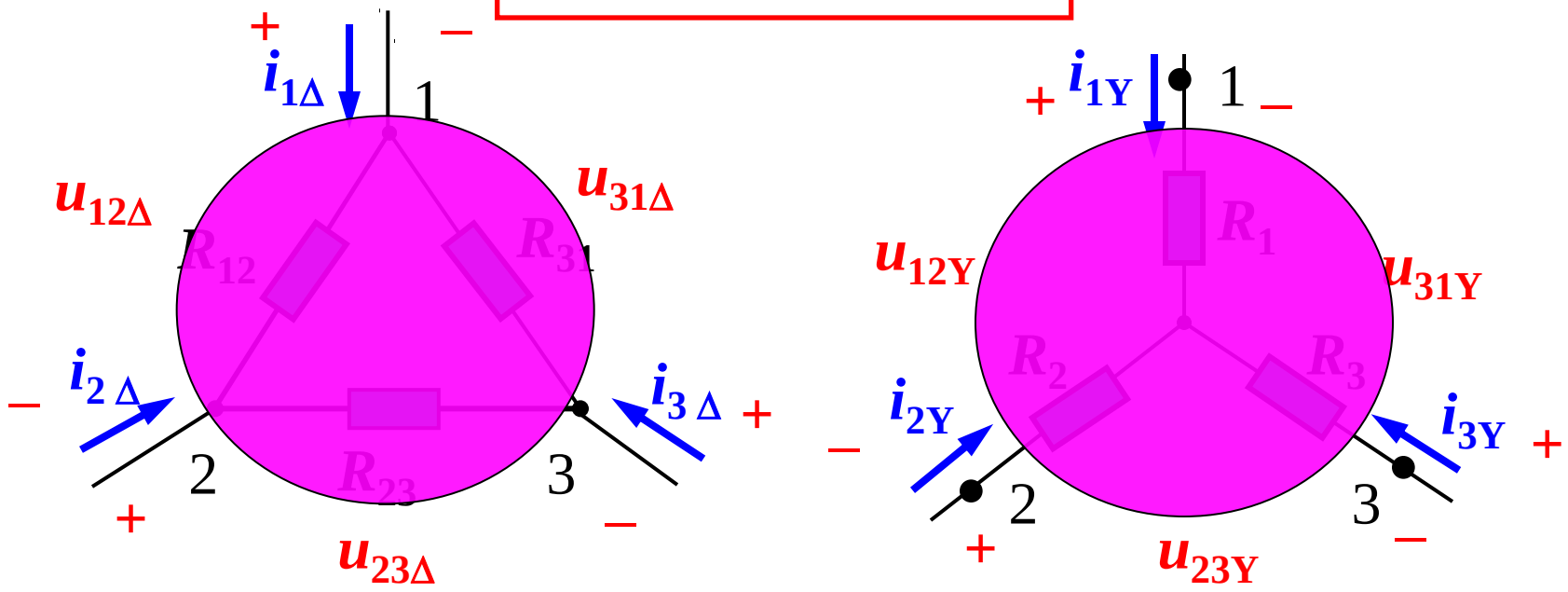


Y 网络

在怎样的条件下，上面的Δ和Y网络对外等效?

Δ—Y 等效条件:

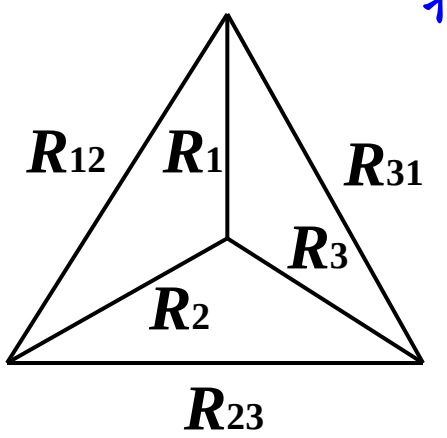
等效电路具有相同的端口u-i关系

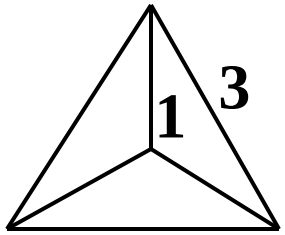


$(i_{1\Delta},\ i_{2\Delta},\ i_{3\Delta},\ u_{12\Delta},\ u_{23\Delta},\ u_{31\Delta})$之间满足的关系 =

$(i_{1Y}, i_{2Y}, i_{3Y}, u_{12Y}, u_{23Y}, u_{31Y})$之间满足的关系

特别地： Δ或Y的三个电阻具有相同阻值





$$R_\Delta = 3R_Y$$

Y → Δ

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

Δ→ Y

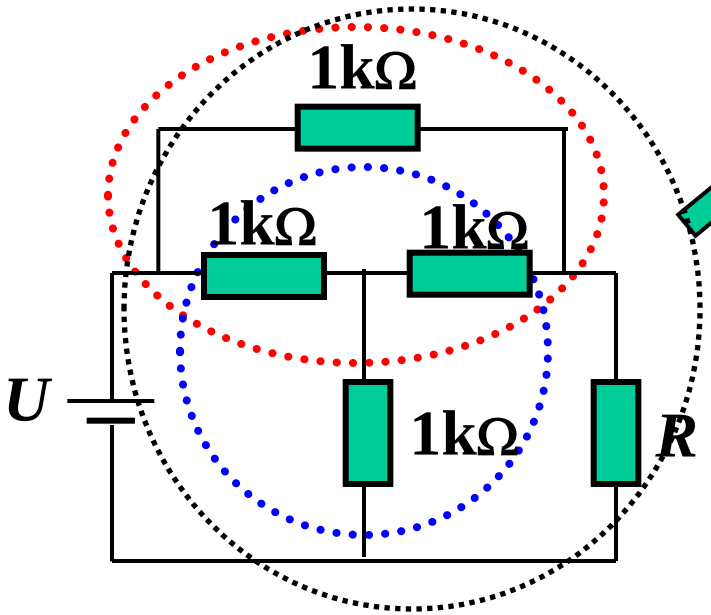
$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}}$$

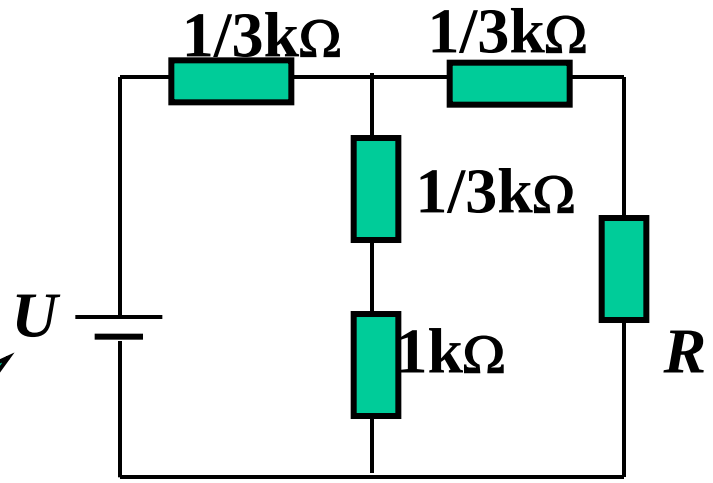
$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

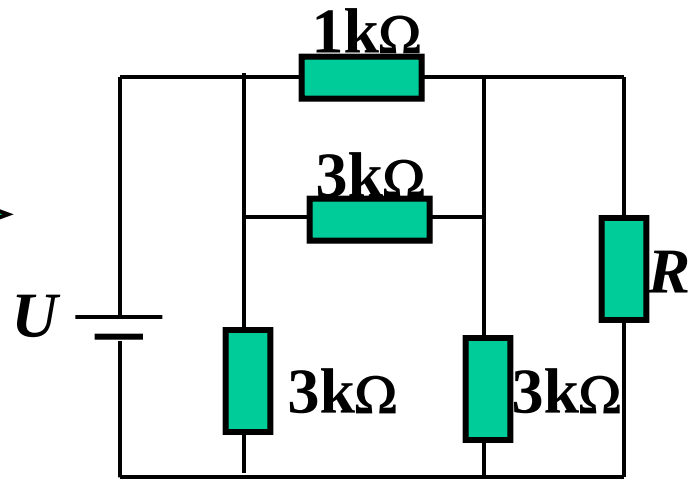
等效的相对性

例： 求*R*上电压



电桥



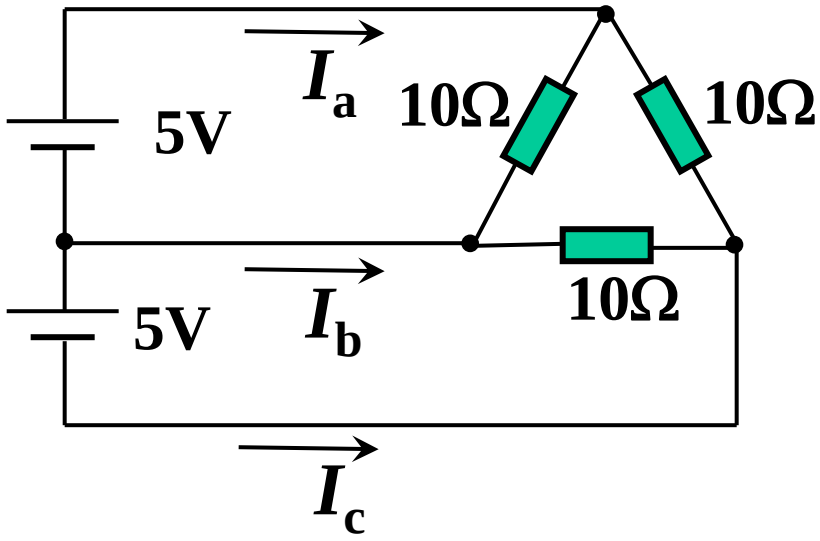


单选题 1分

设置

I_a= _____A

（最先答对的3位同学有红包）



A 0

B 0.5

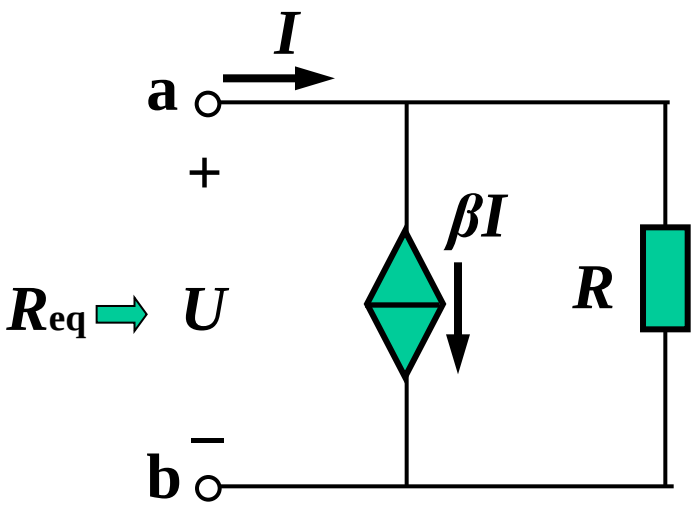
C −0.5

D 1.5

提交

2.4 含受控源二端网络的入端电阻

求入端等效电阻 ⇒ 求端口上的电压电流关系 ⇒ 加压求流或加流求压



$$I=\beta I+\frac{U}{R}\Longrightarrow U=R(1-\beta)I$$

$$R_{eq}=U/I=(1-\beta)R$$

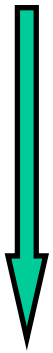
思考：有没有含受控源二端网络加压求流无法求出R_{eq}的情况？

总结：如何求二端网络的入端电阻

串并联

平衡电桥

Δ—Y变换



电阻二端网络求解顺序

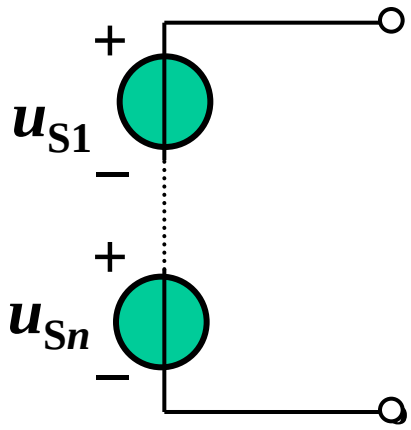
加压求流/

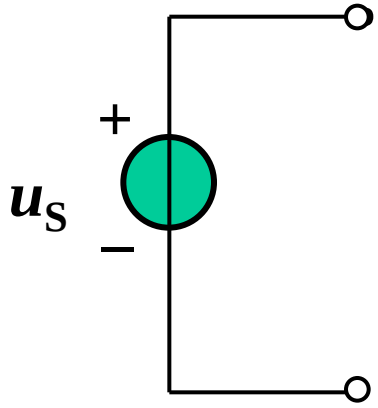
加流求压

含受控源二端网络入端电阻

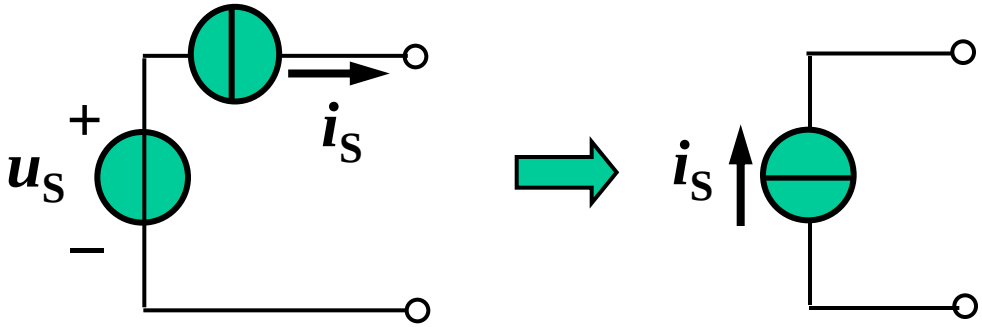
3.1 理想独立源的串并联

I、 理想独立源的串联





$$u_S=\sum u_{Sk}$$

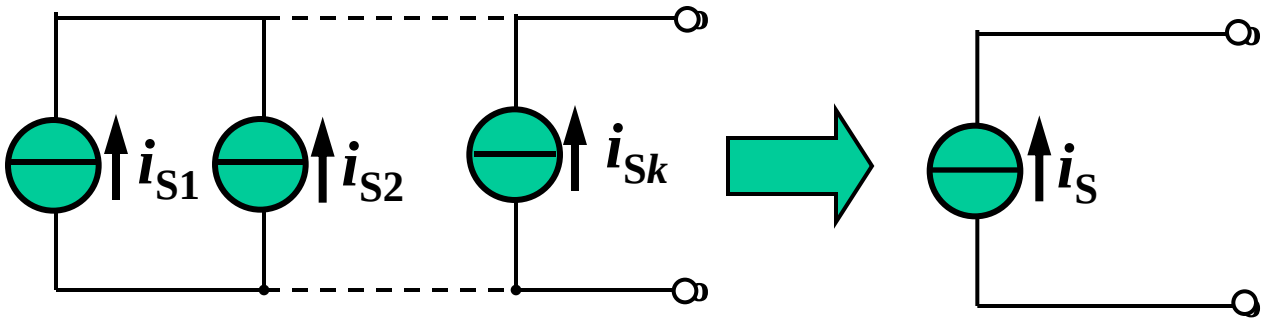


和电流源串联

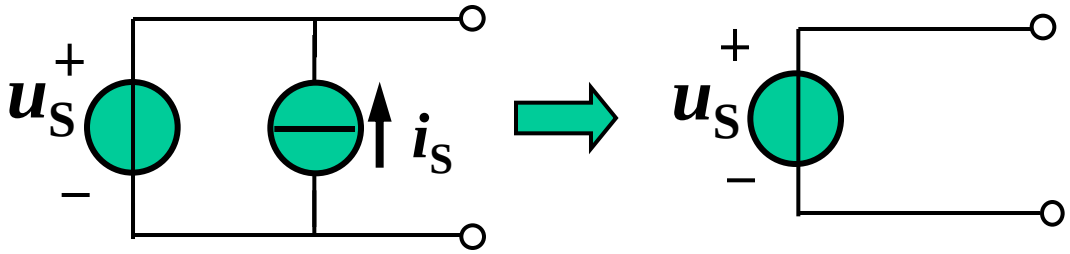
的电压源有什么用?

此处可以有弹幕

II、 理想独立源的并联



$$i_S = \sum i_{Sk}, \quad i_S = i_{S1} + i_{S2} + \cdots + i_{Sn}$$

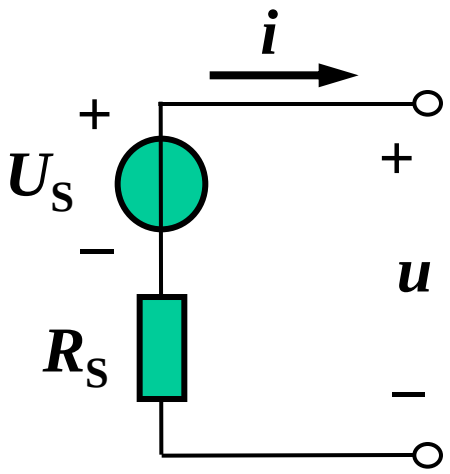


和电压源并联

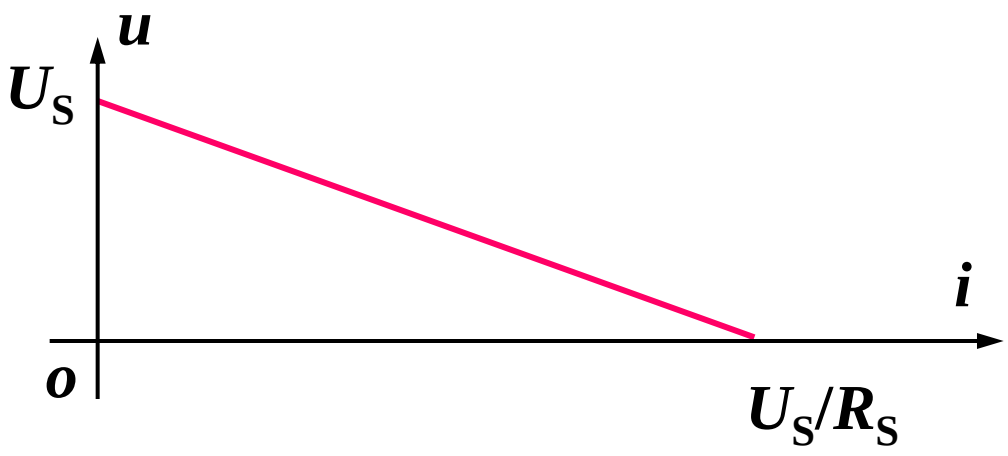
的电流源有什么用?

3.2 实际独立源的等效变换

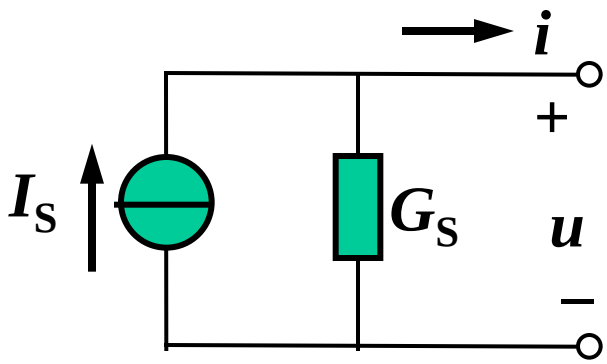
I、实际独立电压源



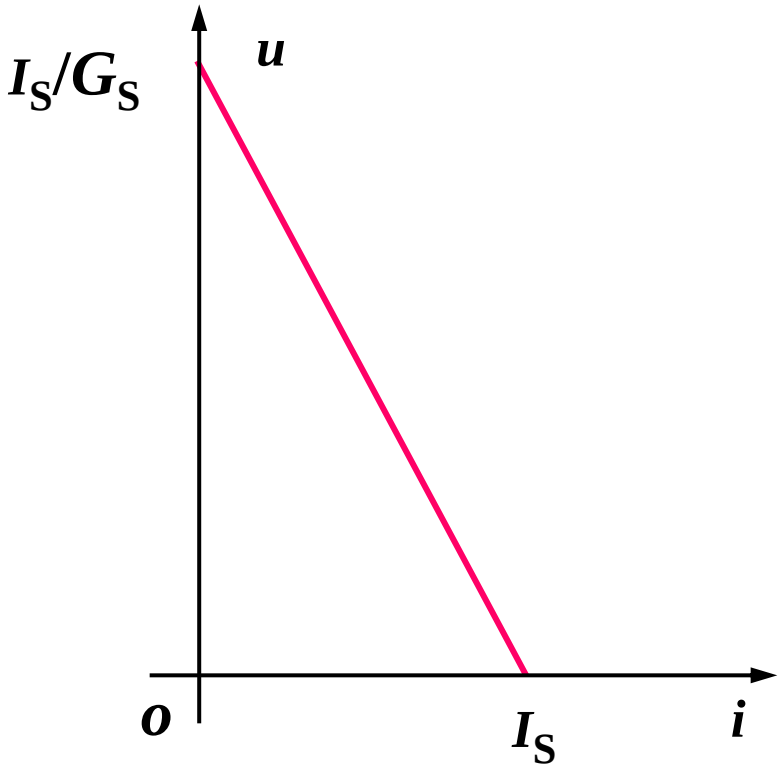
$$u = U_S - R_S\, i$$



II、 实际独立电流源

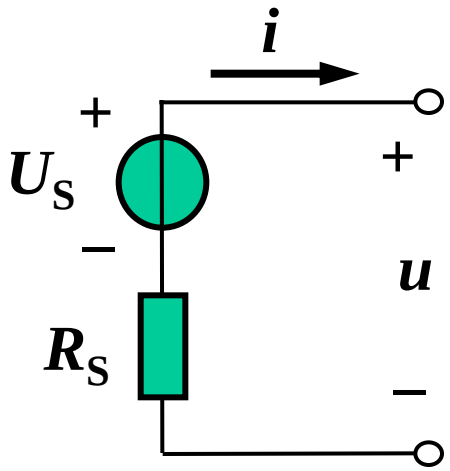


$$i = I_S - G_S u$$

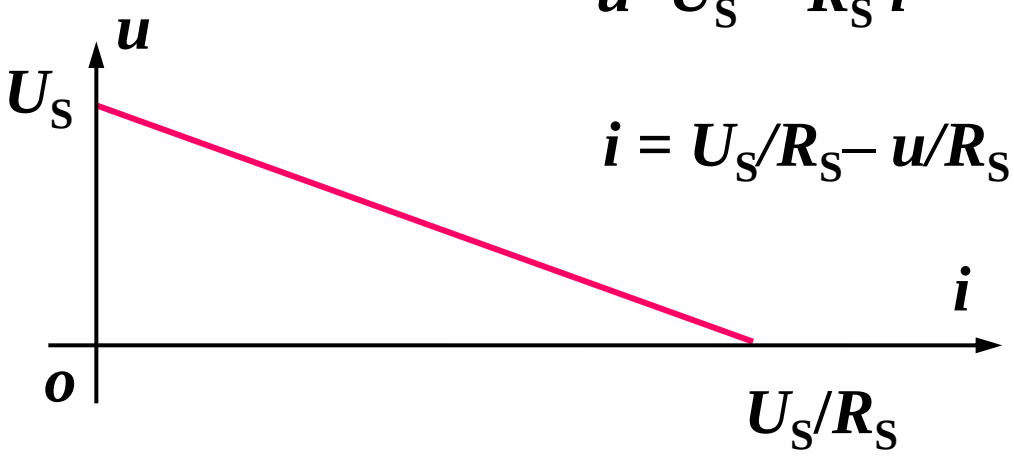


III、电源等效变换

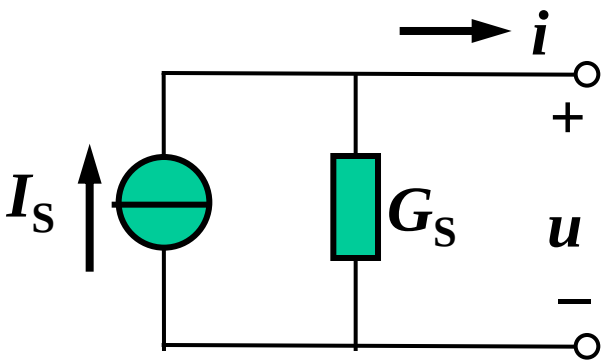
二者如何等效?

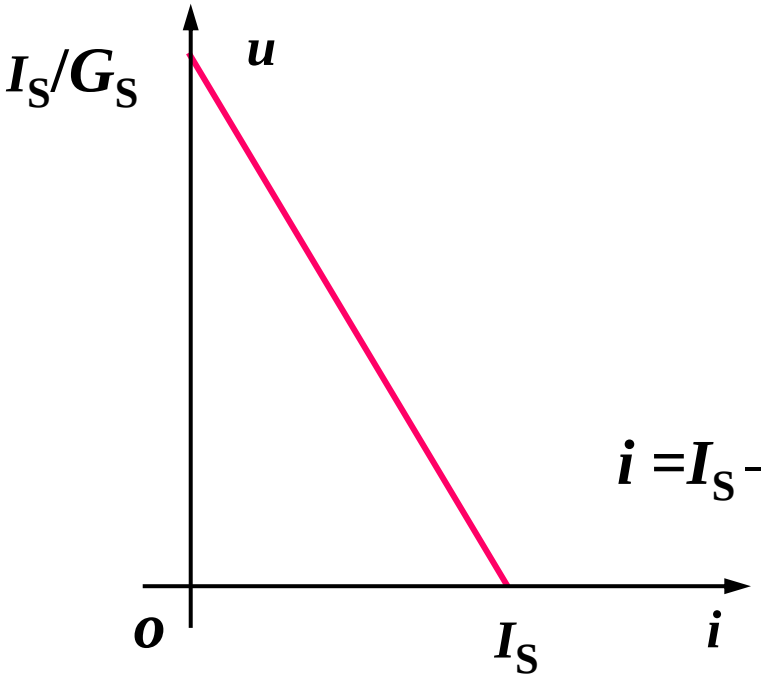


$u=U_S - R_S\, i$



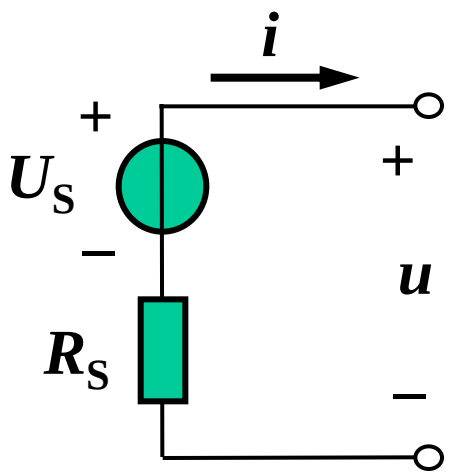
$i = U_S/R_S - u/R_S$



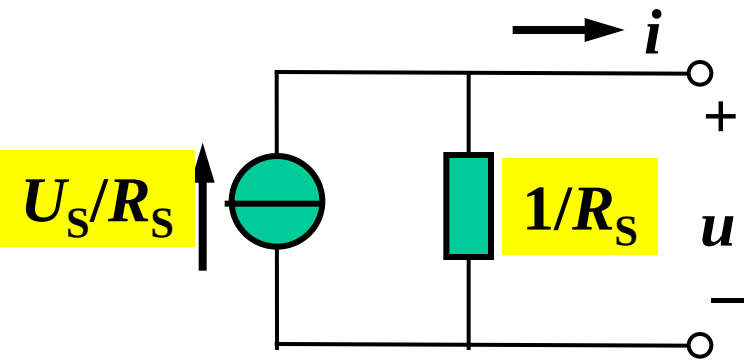


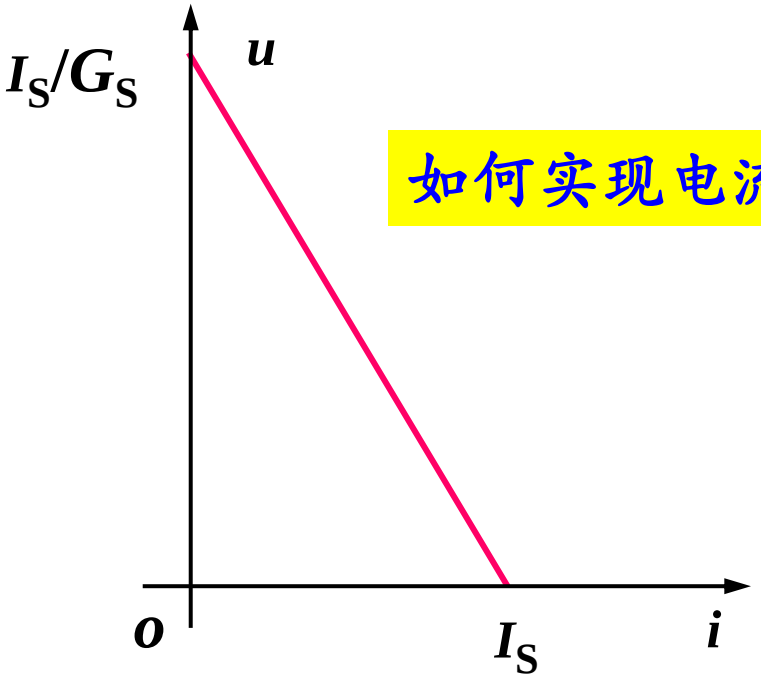
$i = I_S - G_S u$

$I_S=U_S/R_S$ ， $G_S=1/R_S$



此处可以有弹幕



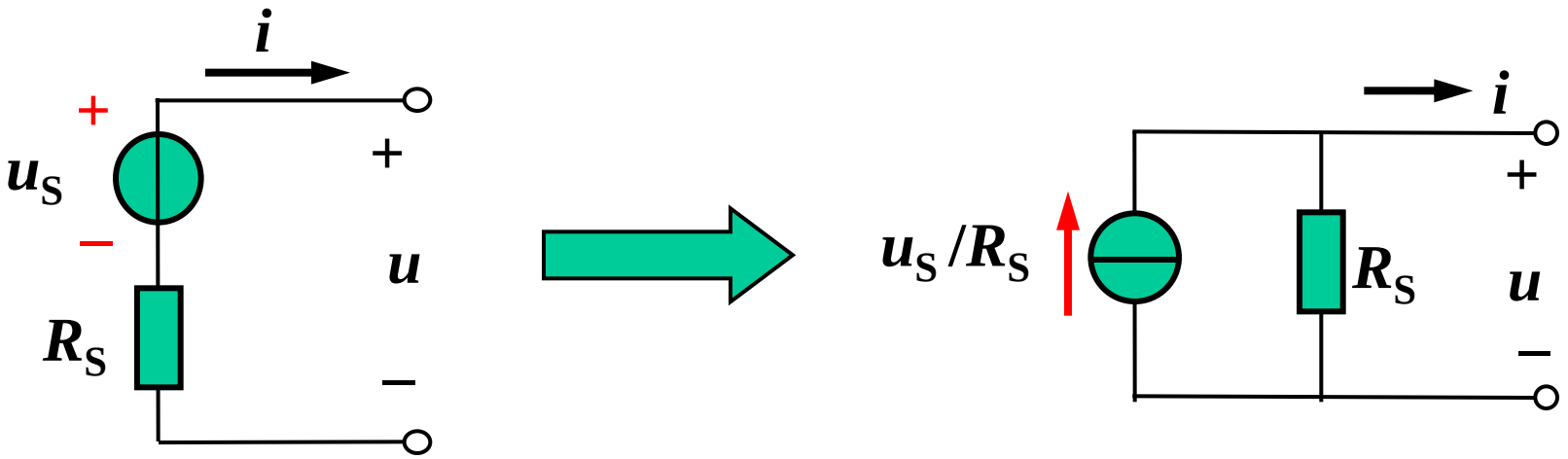


如何实现电流源?

$I_S=U_S/R_S$ ， $G_S=1/R_S$

从电压源变换为电流源

等效的相对性

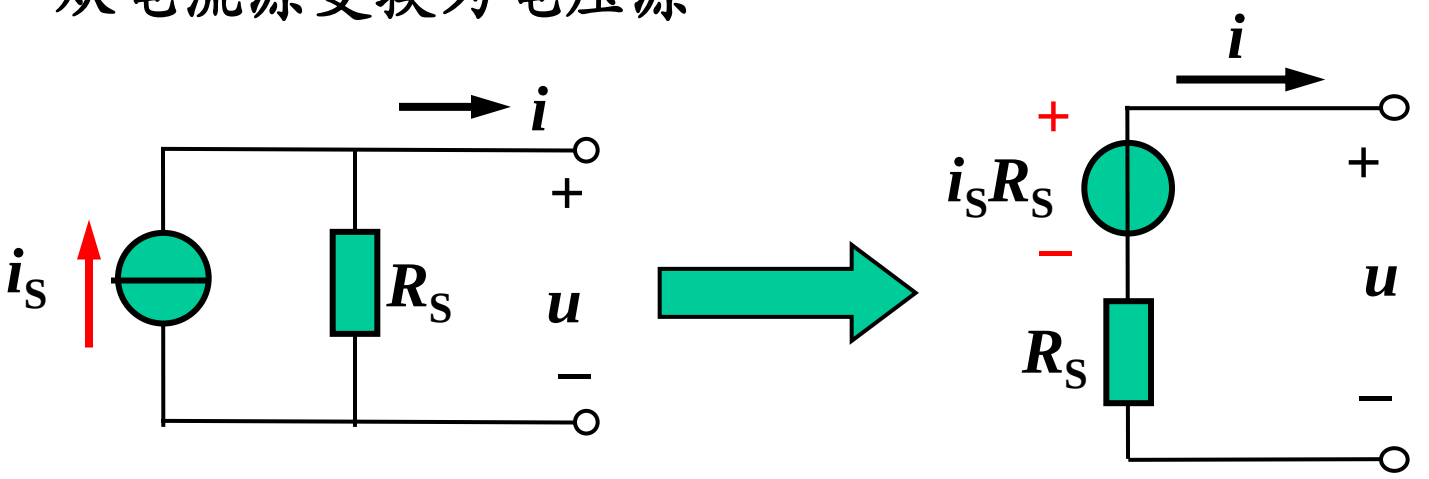


$$u = u_S - iR_S$$

$$i = \frac{u_S}{R_S} - \frac{u}{R_S}$$

注意参考方向

从电流源变换为电压源



$$i = i_S - \frac{u}{R_S}$$

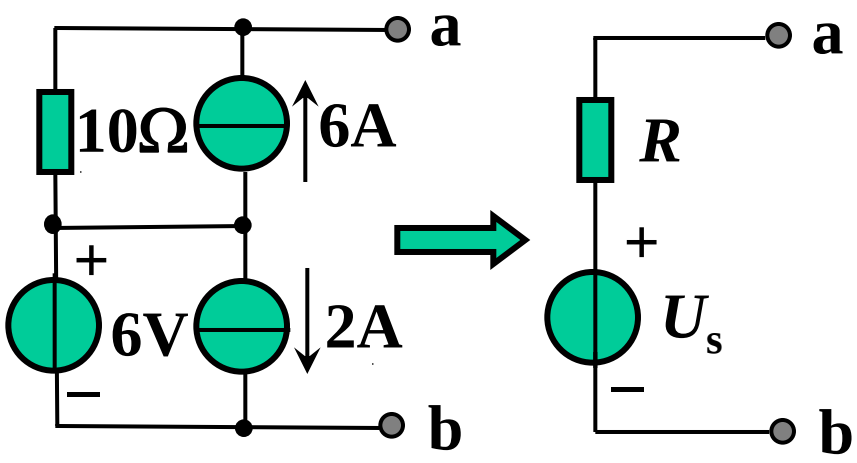
$$u = i_S R_S - i R_S$$

注意参考方向

单选题 1分

设置

U_s= _____V



A 6

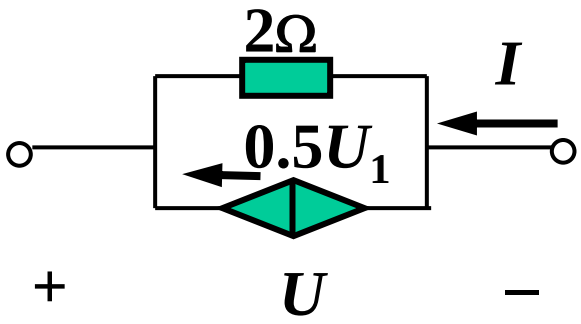
B 60

C −60

D 66

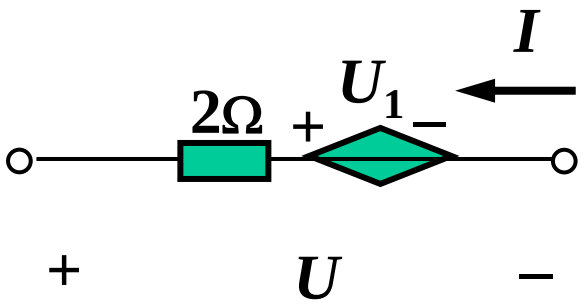
提交

讨论：受控源的等效变换

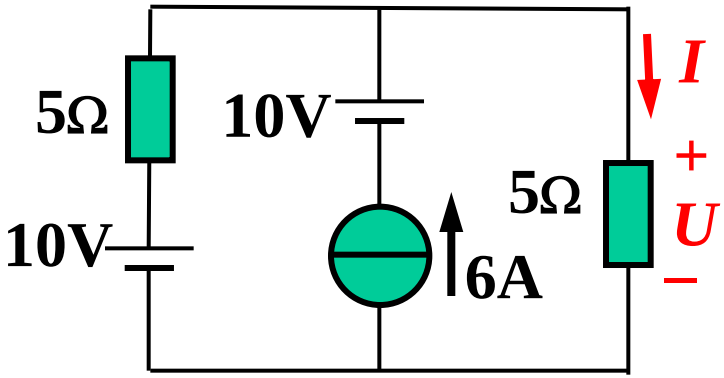


成立吗?

什么条件下成立?



例　求电压U。



$U=4\times5=20\text{V}$