第11讲　二阶动态电路

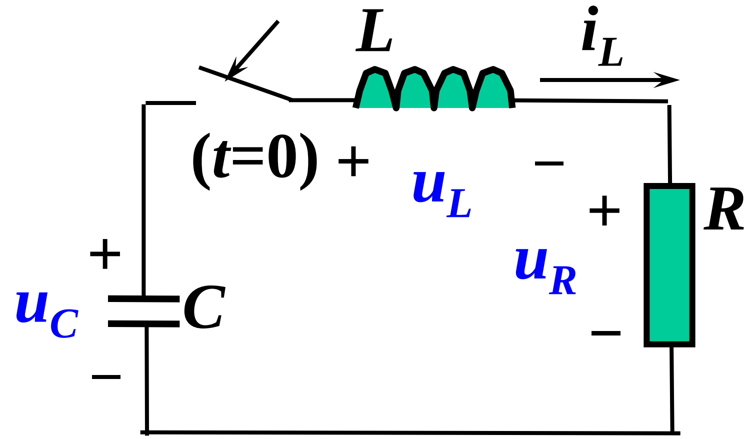
1 RLC串联二阶电路

2 RLC并联二阶电路

3 二阶电路的直觉解法

4 二阶电路的应用

1 RLC串联二阶电路



零输入RLC串联

(1) 列方程

$$\begin{cases} u_C = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} + Ri_L \\ i_L = -C\dfrac{\mathrm{d}u_C}{\mathrm{d}t} \end{cases}$$

i_L 代入上式

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$

u_C 代入下式

$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}i_L}{\mathrm{d}t} + \frac{1}{LC}i_L = 0$$

衰减系数 α

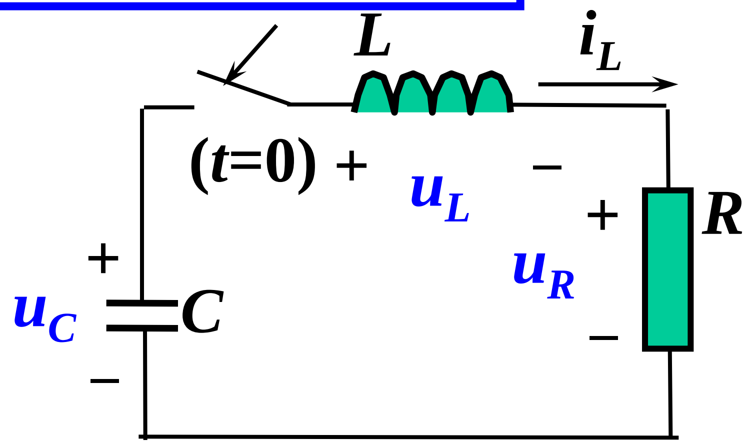
$\boxed{2\alpha}$ $\boxed{\omega_0^2}$

自由振荡角频率/自然角频率 ω_0

课外练习：

以u_R、u_L为变量列写微分方程。

零输入RLC串联



$$\frac{d^2 i_L}{dt^2}+2\alpha\frac{di_L}{dt}+\omega_0^2 i_L=0$$

$$\frac{d^2 u_C}{dt^2}+2\alpha\frac{du_C}{dt}+\omega_0^2 u_C=0$$

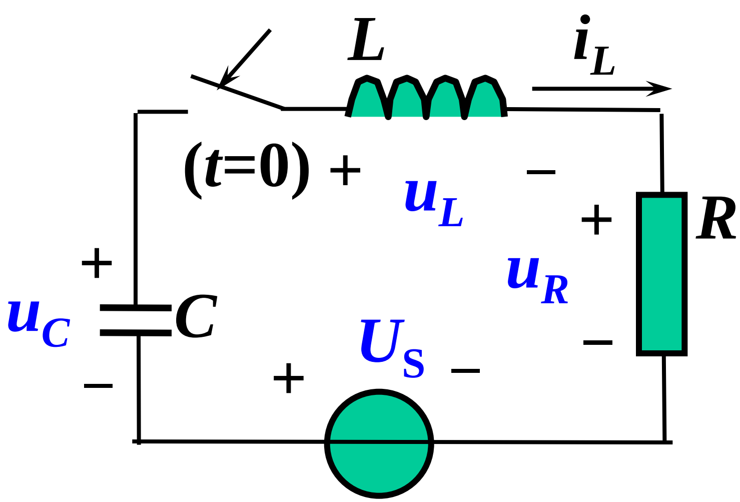
$$\frac{d^2 u_L}{dt^2}+2\alpha\frac{du_L}{dt}+\omega_0^2 u_L=0$$

$$\frac{d^2 u_R}{dt^2}+2\alpha\frac{du_R}{dt}+\omega_0^2 u_R=0$$

以不同的变量列写方程，得到的特征方程相同。

可先列写零输入电路方程，求得特征根。

有输入RLC串联



有独立源电路和零输入电路的特征方程相同。

$$\frac{d^2 i_L}{dt^2}+2\alpha\frac{di_L}{dt}+\omega_0^2 i_L=0$$

(2) 求自由分量

LC参数不变，随R增加，状态怎么变?

此处可以有弹幕

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 2\alpha \frac{\mathrm{d}u_C}{\mathrm{d}t} + \omega_0^2 u_C = 0$$

$$2\alpha = \frac{R}{L} \qquad \omega_0^2 = \frac{1}{LC}$$

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

$\alpha^2 > \omega_0^2$

$$p_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

过阻尼

$$u_C = Ae^{p_1 t} + Be^{p_2 t}$$

$\alpha^2 = \omega_0^2$

$$p_1 = p_2 = -\alpha$$

临界阻尼

$$u_C = Ae^{p_1 t} + Bte^{p_2 t}$$

$\alpha^2 < \omega_0^2$

$$p_{1,2} = -\alpha \pm \mathrm{j}\sqrt{\omega_0^2 - \alpha^2}$$

欠阻尼

$$u_C = Ke^{-\alpha t}\sin(\omega_\mathrm{d} t + \theta)$$

$\alpha = 0$

$$p_{1,2} = \pm \mathrm{j}\omega_0$$

无阻尼

$$u_C = K\sin(\omega_0 t + \theta)$$

有关RLC串联欠阻尼3个参数的讨论

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$

2α $\qquad$ ω_0^2

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 2\alpha\frac{\mathrm{d}u_C}{\mathrm{d}t} + \omega_0^2 u_C = 0$$

$$b^2 - 4ac < 0$$

$$p_{1,2} = \frac{-2\alpha \pm \mathrm{j}2\sqrt{\omega_0^2 - \alpha^2}}{2}$$

$$= -\alpha \pm \mathrm{j}\sqrt{\omega_0^2 - \alpha^2}$$

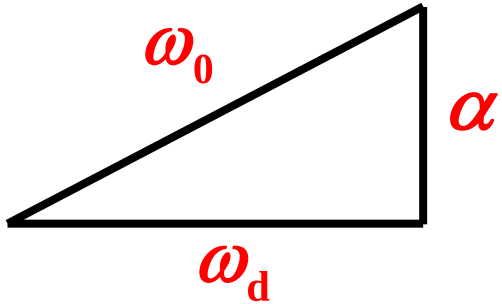
$$= -\alpha \pm \mathrm{j}\omega_\mathrm{d}$$

衰减系数 α

自由振荡角频率/自然角频率 ω_0

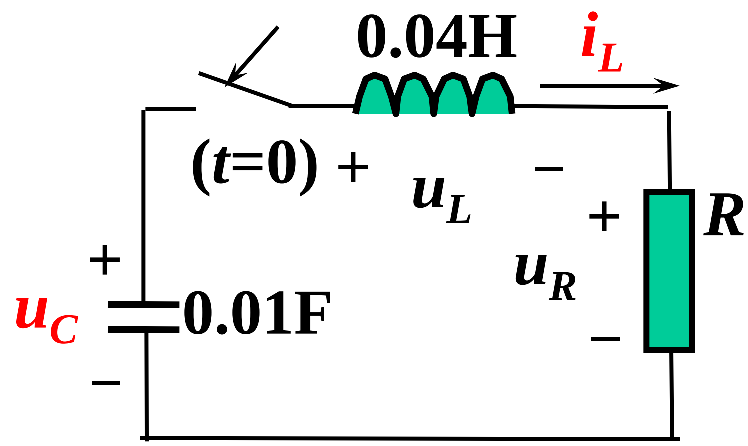
$$\omega_0^2 = \omega_\mathrm{d}^2 + \alpha^2$$

衰减振荡角频率 ω_d



数值例子 R分别为5Ω 、4Ω 、1Ω、 0Ω时求$u_C(t)$、 $i_L(t)$, $t \geq 0$

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{LC}u_C = 0$$



$u_C(0^-) = 3\text{V},\ i_L(0^-) = 0$

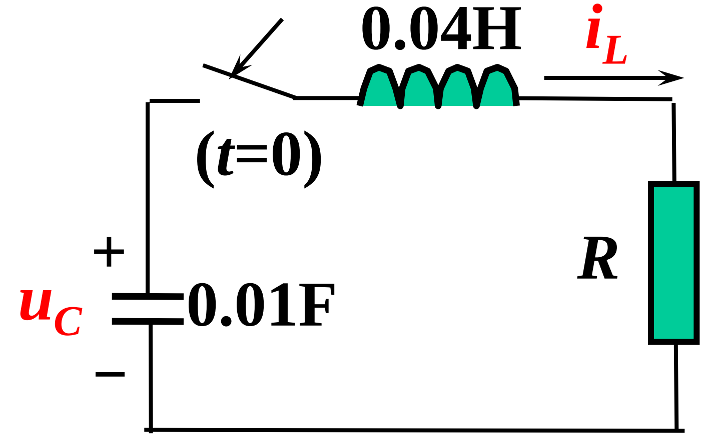
$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + 25R\frac{\mathrm{d}u_C}{\mathrm{d}t} + 2500u_C = 0$$

特征方程

$$p^2 + 25Rp + 2500 = 0$$

$$b^2 - 4ac = 625R^2 - 10000$$

$$p^2 + 25Rp + 2500 = 0$$
$$b^2 - 4ac = 625R^2 - 10000$$



$R=5\Omega$

$$b^2 - 4ac = 5625 > 0$$
$$p_1 = -25 \qquad p_2 = -100$$
$$u_C(t) = A_1 e^{-25t} + A_2\, e^{-100t}$$

$R=4\Omega$

$$b^2 - 4ac = 0$$
$$p_1 = p_2 = -50$$
$$u_C(t) = A_1 e^{-50t} + A_2 t\, e^{-50t}$$

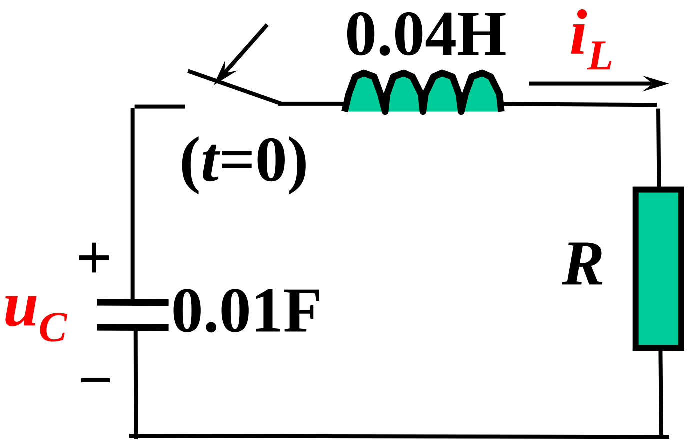
$R=1\Omega$

$$b^2 - 4ac = -9375 < 0$$
$$p_{1,2} = -12.5 \pm \mathrm{j}48.4$$
$$u_C(t) = K e^{-12.5t} \sin(48.4t + \theta)$$

$R=0\Omega$

$$p_{1,2} = \pm \mathrm{j}50$$
$$u_C(t) = K \sin(50t + \theta)$$

$$\left.\frac{du_C}{dt}\right|_{t=0^+} = \_\_\_\_ \text{V/s}$$



$u_C(0^-) = 3\text{V}$
$i_L(0^-) = 0$

A 0

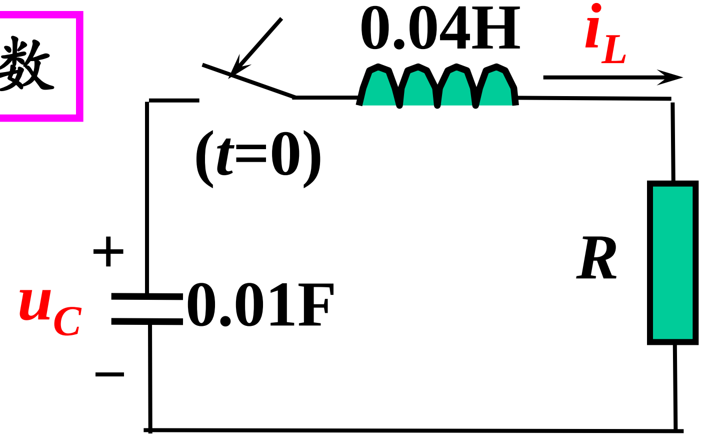
B 3

C 300

D −300

提交

(3) 用初值确定待定系数



$u_C(0^-) = 3\text{V}$

$i_L(0^-) = 0$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0^+} = 0$$

$R = 5\,\Omega$

$$\begin{cases} u_C(t) = A_1 e^{-25t} + A_2 e^{-100t} \\ A_1 + A_2 = 3 \\ -25A_1 - 100A_2 = 0 \end{cases} \longrightarrow A_1 = 4 \quad A_2 = -1$$

$$\boxed{u_C(t) = 4e^{-25t} - e^{-100t}\ \text{V} \quad t > 0^+}$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t} = -i_L$$

$$\boxed{i_L(t) = e^{-25t} - e^{-100t}\,\text{A} \quad t > 0^+}$$

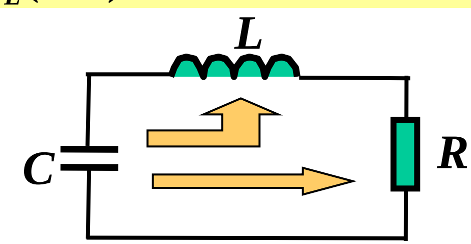
波形图和能量转换关系

$R=5\,\Omega$

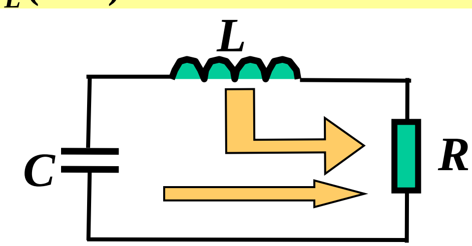
$$u_C(t)=4\mathrm{e}^{-25t}-\mathrm{e}^{-100t}\,\mathrm{V}\quad t>0^+$$

$$i_L(t)=\mathrm{e}^{-25t}-\mathrm{e}^{-100t}\,\mathrm{A}\quad t>0^+$$

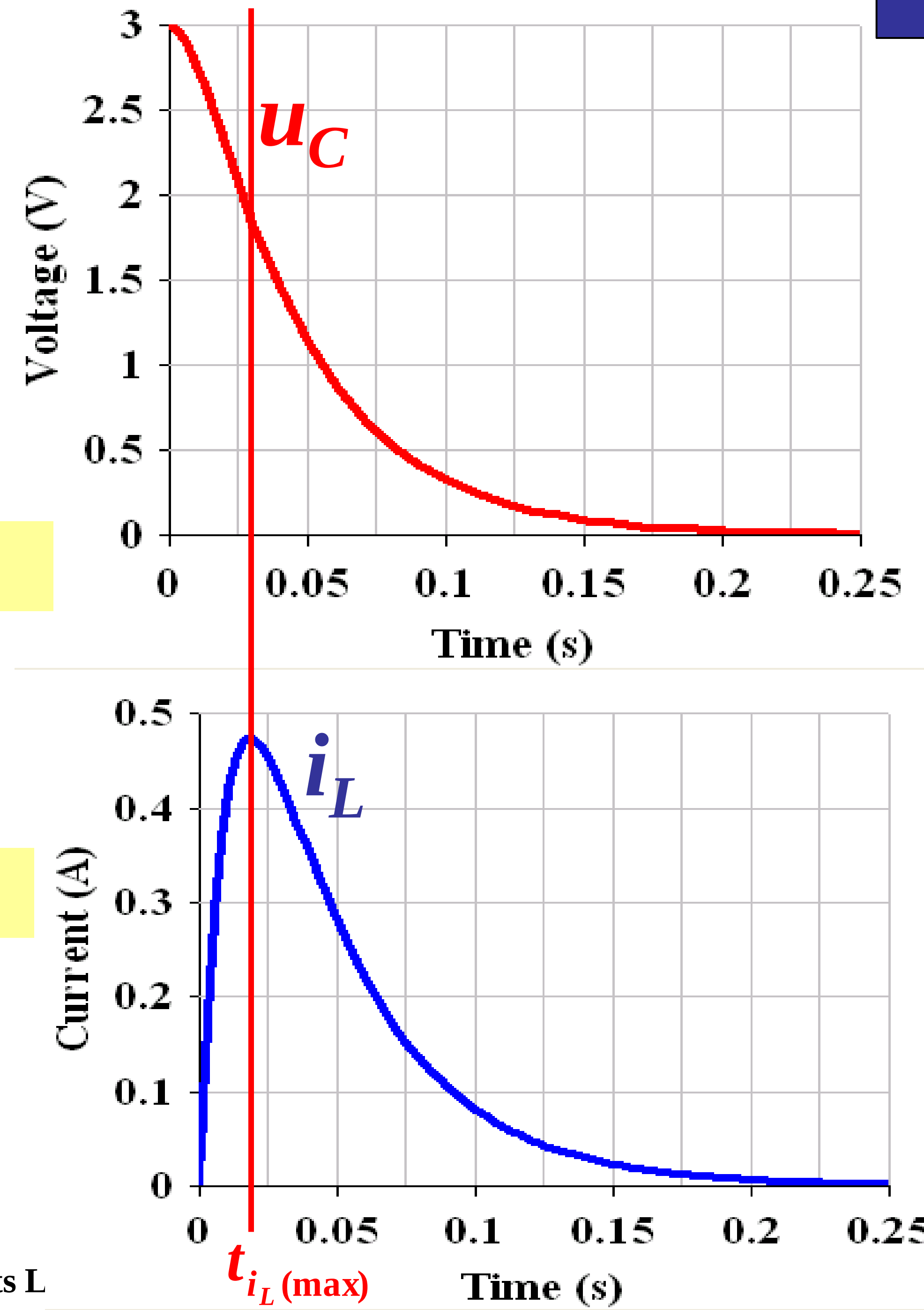
$0<t<t_{i_L(\max)}$ u_C 减小，i_L增大。



$t>t_{i_L(\max)}$ u_C 减小，i_L减小。



非振荡放电 过阻尼



单选题 1分

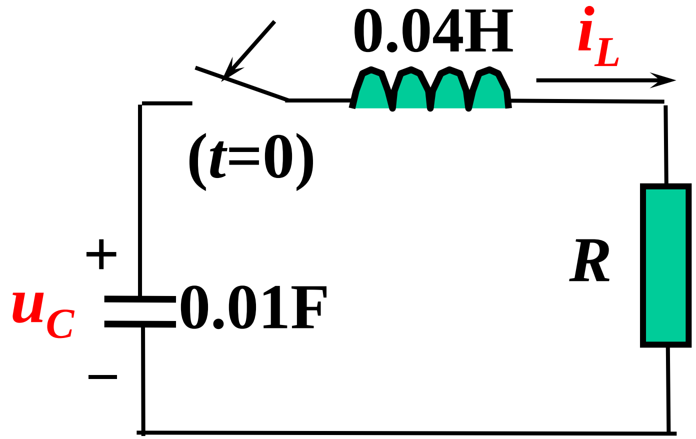
设置

过阻尼二阶系统，可以有一个储能元件给另一个储能元件的充电过程吗？

A 可以

B 不可以

提交



$$u_C(0)=3\text{V}$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0$$

$R=4\,\Omega$

$$\begin{cases} u_C(t)=A_1\mathrm{e}^{-50t}+A_2t\,\mathrm{e}^{-50t} \\ A_1=3 \\ -50A_1+A_2=0 \end{cases}$$

$$\longrightarrow A_1=3 \quad A_2=150$$

$$\boxed{u_C(t)=3\mathrm{e}^{-50t}(1+50t)\text{V} \quad t>0^+}$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-i_L$$

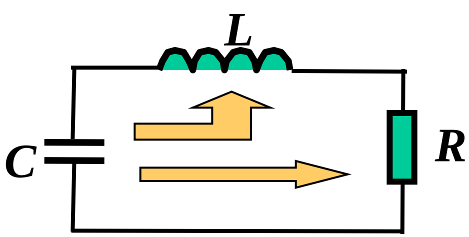
$$\boxed{i_L(t)=75t\mathrm{e}^{-50t}\text{A} \quad t>0^+}$$

$R = 4\ \Omega$

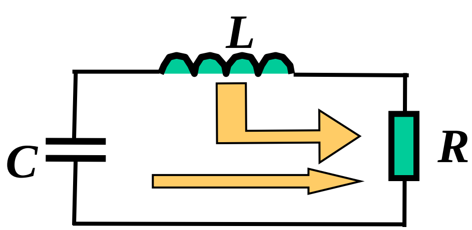
$$u_C(t) = 3\mathrm{e}^{-50t}(1+50t)\mathrm{V} \quad t > 0^+$$

$$i_L(t) = 75t\mathrm{e}^{-50t}\mathrm{A} \quad t > 0^+$$

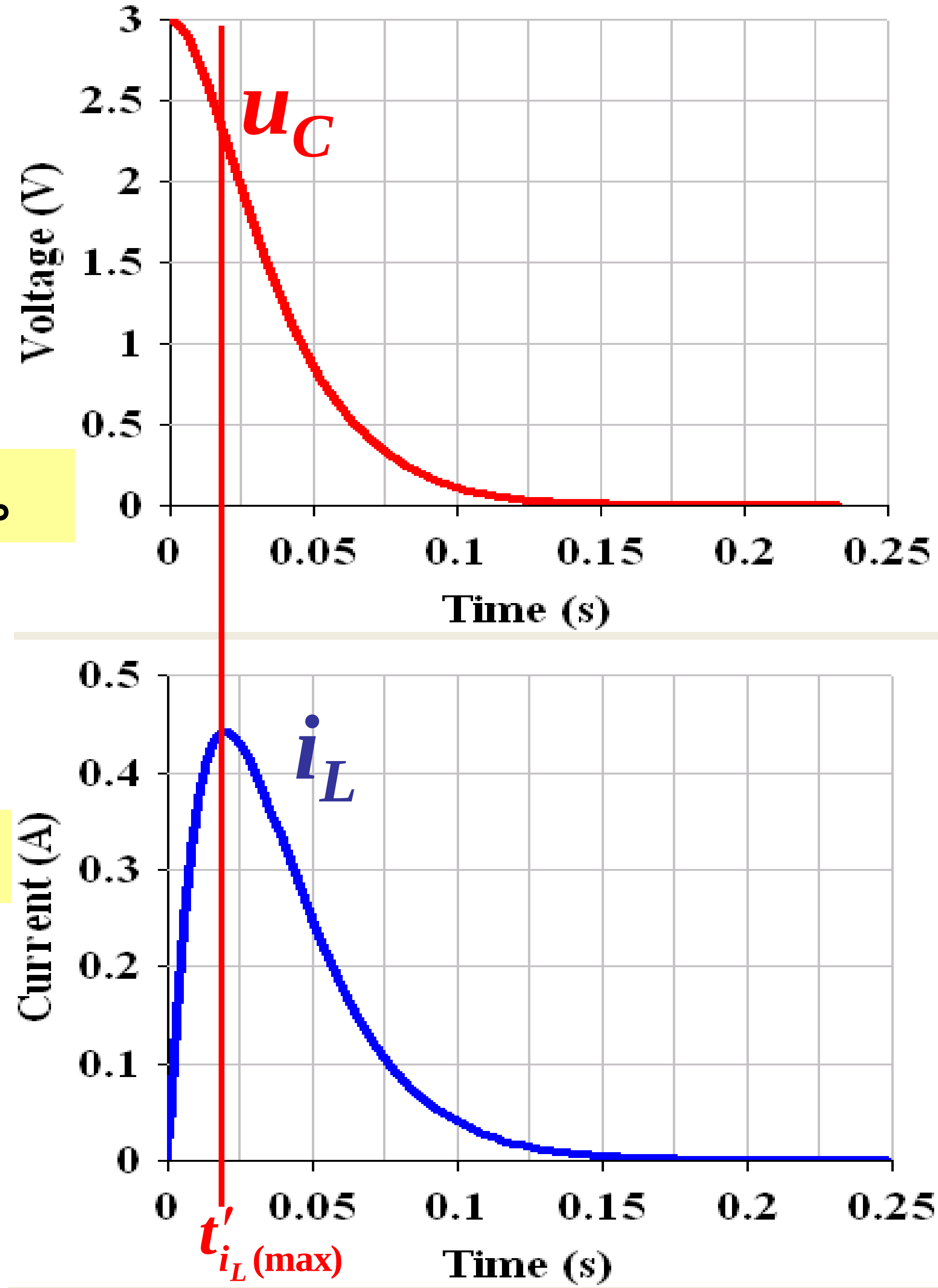
$0 < t < t'_{i_L(\max)}$ u_C 减小，i_L增大。

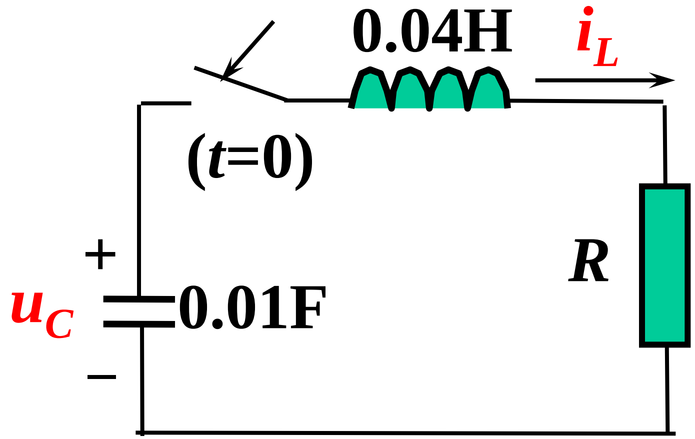


$t > t'_{i_L(\max)}$ u_C 减小，i_L增小。



非振荡放电 临界阻尼





$$u_C(0)=3\text{V}$$

$$\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0$$

$R=1\,\Omega$

$$\begin{cases} u_C(t)=K\mathrm{e}^{-12.5t}\sin(48.4t+\theta\,) \\ K\sin\theta=3 \\ -12.5K\sin\theta+48.4K\cos\theta=0 \end{cases}$$

$$K=3.1 \quad \theta=75.5^{\circ}$$

$$\boxed{u_C(t)=3.10\mathrm{e}^{-12.5t}\sin(48.4t+75.5^{\circ})\text{V} \quad t>0^+}$$

$$C\frac{\mathrm{d}u_C}{\mathrm{d}t}=-i_L$$

$$\boxed{i_L(t)=1.55\mathrm{e}^{-12.5t}\sin(48.4t)\text{A} \quad t>0^+}$$

$R = 1\,\Omega$

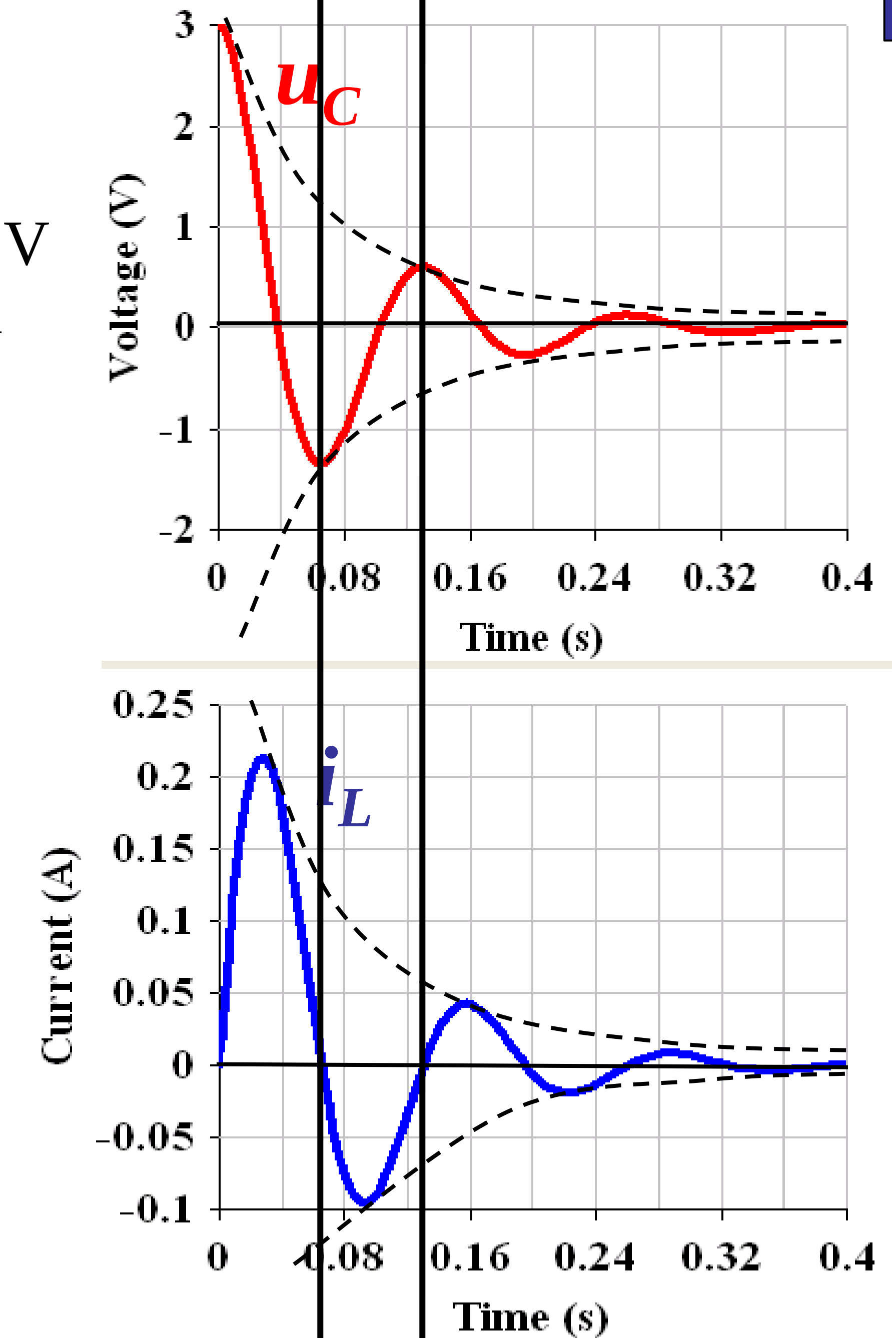
$$u_C(t) = 3.10e^{-12.5t}\sin(48.4t + 75.5^\circ)\mathrm{V}$$

$$t > 0^+$$

$$i_L(t) = 1.55e^{-12.5t}\sin 48.4t\mathrm{A}$$

$$t > 0^+$$

衰减振荡　　欠阻尼



单选题 1分

设置

RLC串联欠阻尼电路中，*R*越大，能量衰减得越

“红包”

A 快

B 慢

$$p^2 + \frac{R}{L}p + \frac{1}{LC} = 0$$

提交

设置

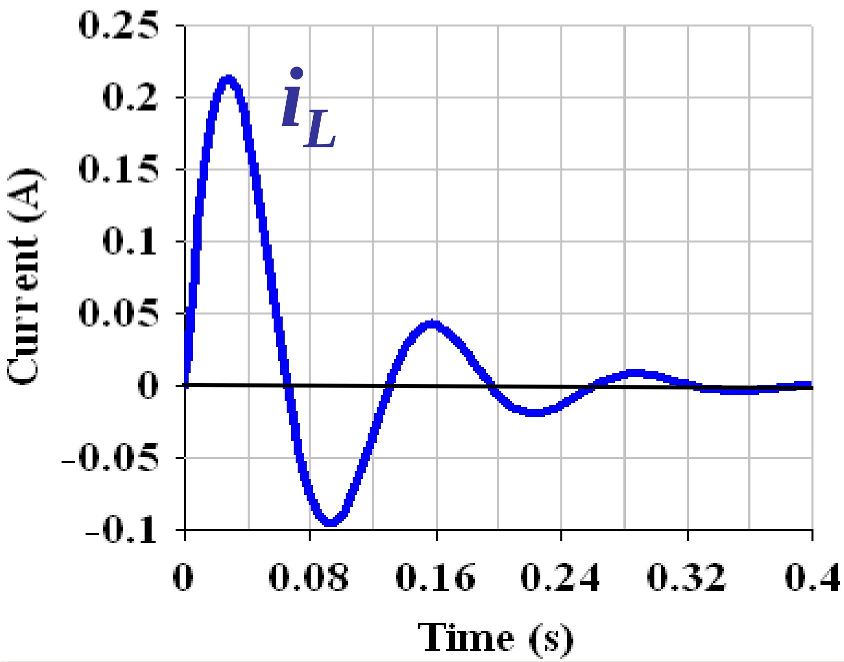
大致多久后趋于稳态?

$$i_L(t) = 1.55e^{-12.5t}\sin 48.4t\text{A} \quad t \geq 0$$

A 0.1s

B 0.3s

C 3s



提交

单选题 1分

设置

衰减振荡周期为____s

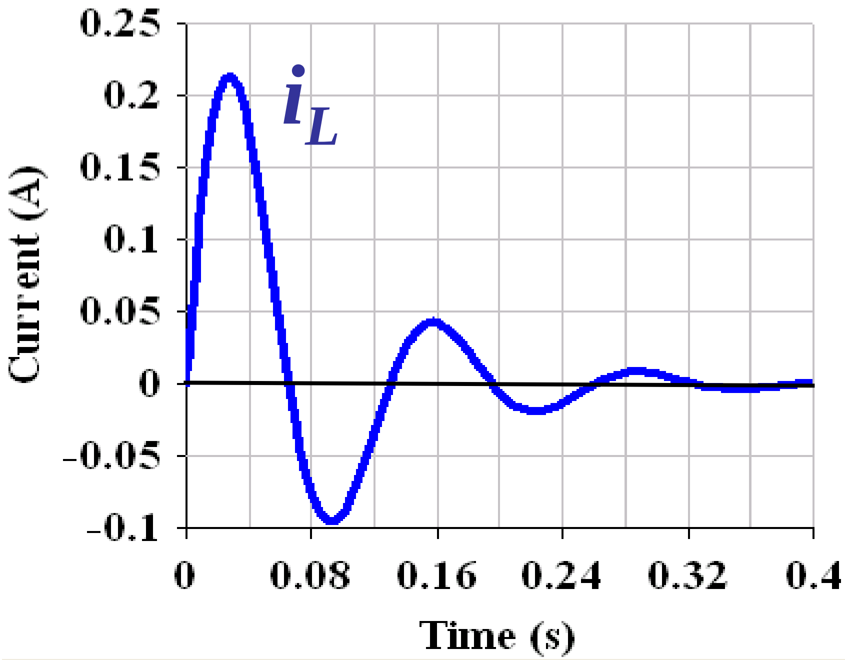
$$i_L(t) = 1.55e^{-12.5t}\sin 48.4t\text{A} \quad t \geq 0$$

A 0.1s

B 0.13s

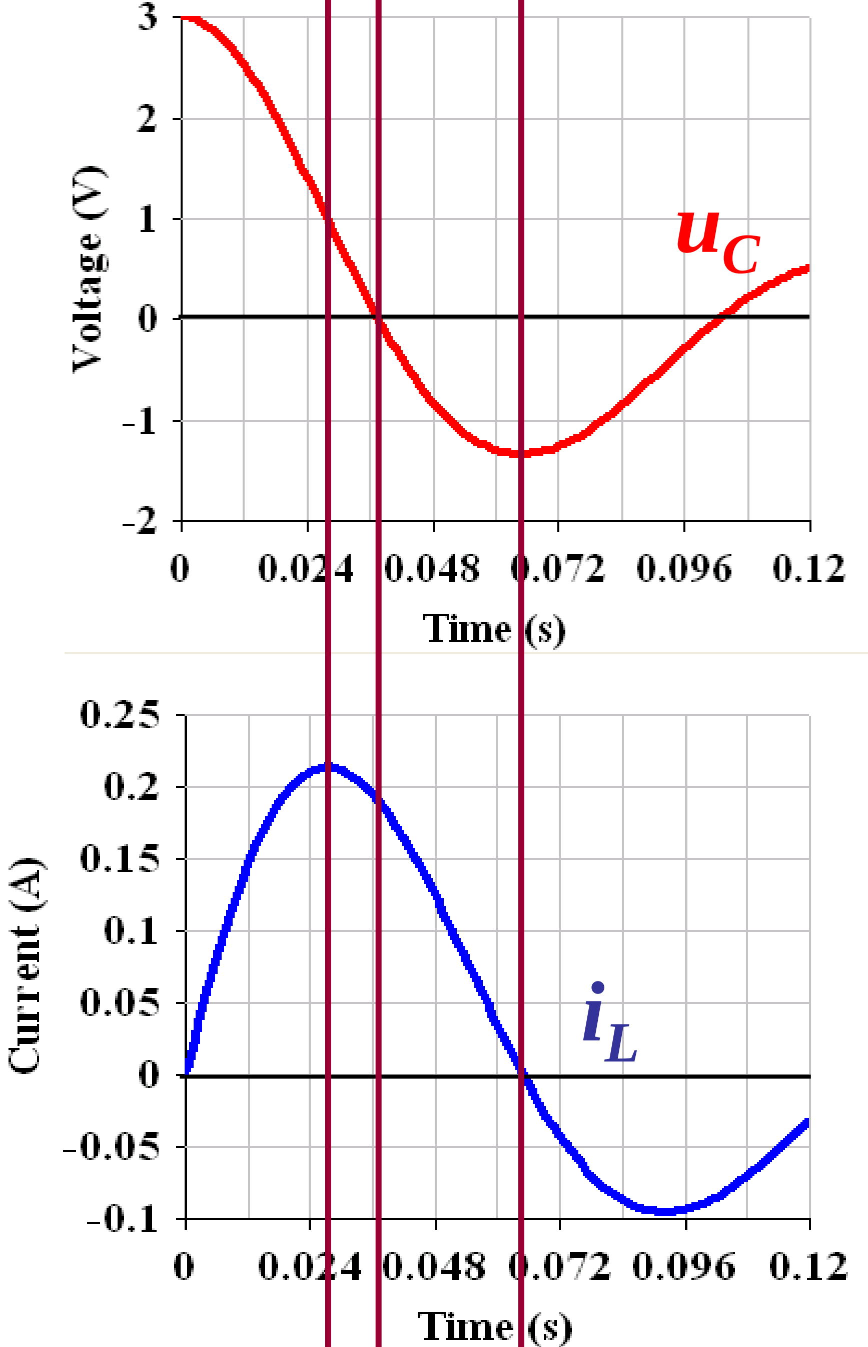
C 0.5s

D 1s



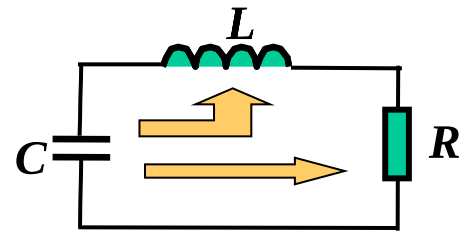
提交

讨论半个周期中能量的关系



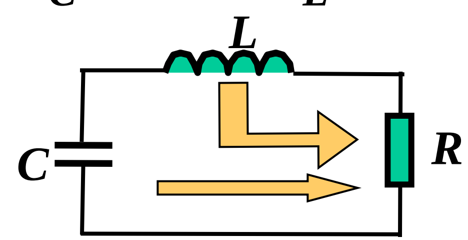
$0 \le 48.4t \le 75.5^{\circ}$

u_C减小，i_L增大



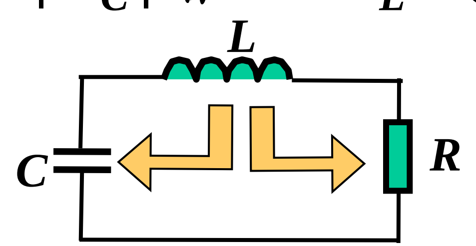
$75.5^{\circ} \le 48.4t \le 104.5^{\circ}$

u_C减小，i_L 减小



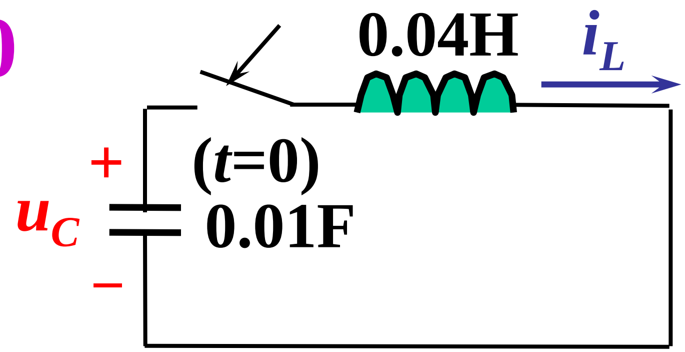
$104.5^{\circ} \le 48.4t \le 180^{\circ}$

$|u_C|$增大，i_L 减小



周而复始，电阻不断消耗能量，u_C i_L衰减到零

$R = 0$



$$LC\frac{\mathrm{d}^2 u_C}{\mathrm{d}t} + u_C = 0$$

$$p^2 + 2500 = 0 \quad p = \pm \mathrm{j}50$$

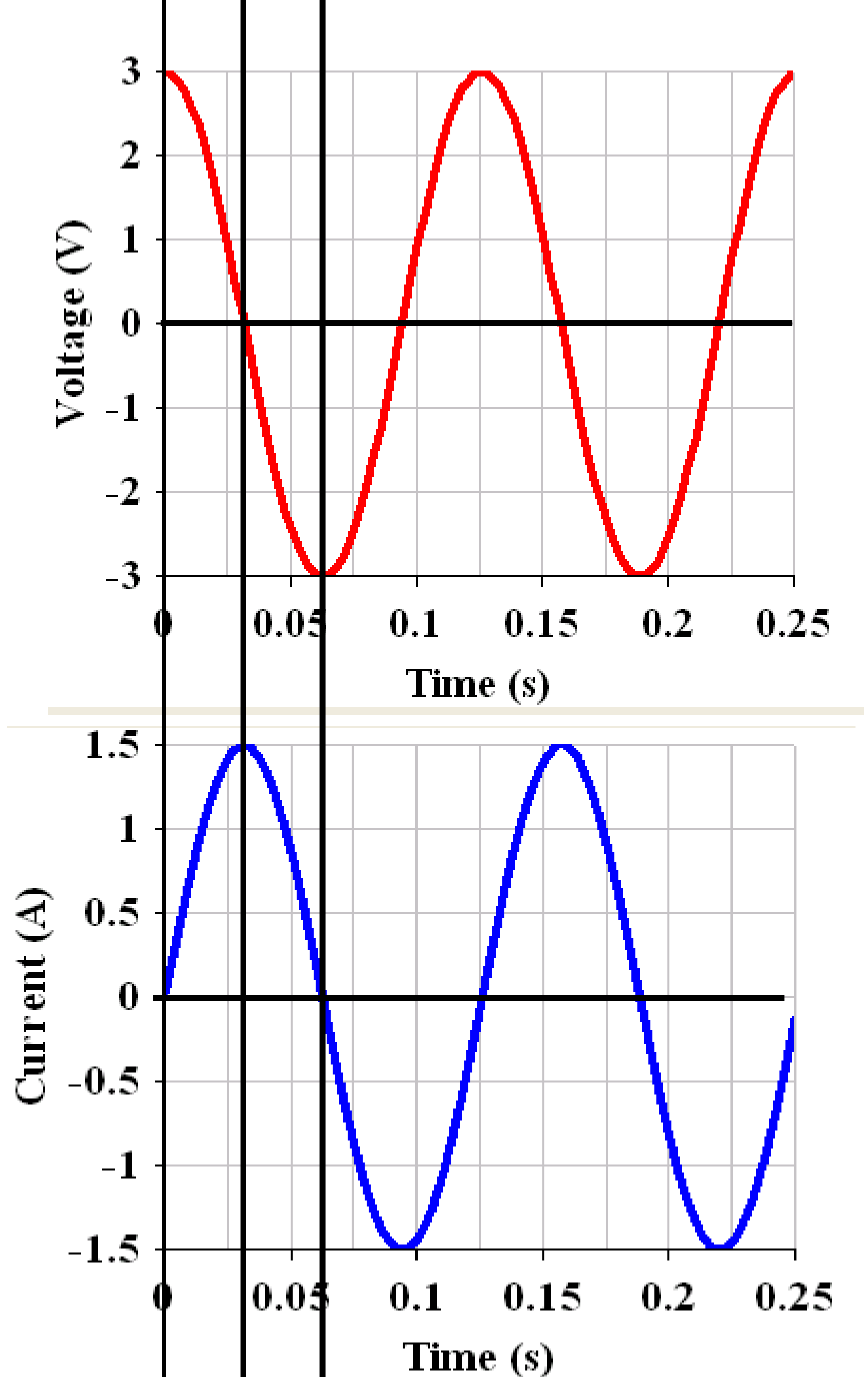
$$u_C(t) = K\sin(50t + \theta)$$

$$u_C(0) = 3 \qquad \left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+} = 0$$

$$K = 3 \quad \theta = 90^{\circ}$$

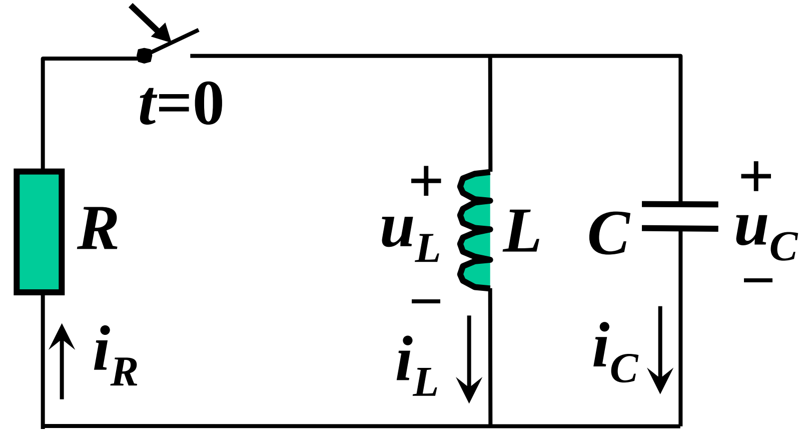
$$u_C(t) = 3\cos 50t \text{ V} \quad t > 0^+$$

$$i_L(t) = 1.5\sin 50t \text{ A} \quad t > 0^+$$



等幅振荡　　无阻尼

2 RLC并联二阶电路



零输入RLC并联

$$\begin{cases} i_R = i_L + C\dfrac{\mathrm{d}u_C}{\mathrm{d}t} \\ u_C = L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} \\ i_R = -\dfrac{u_C}{R} \end{cases}$$

$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + \frac{1}{RC}\frac{\mathrm{d}i_L}{\mathrm{d}t} + \frac{1}{LC}i_L = 0$$

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\Downarrow$$

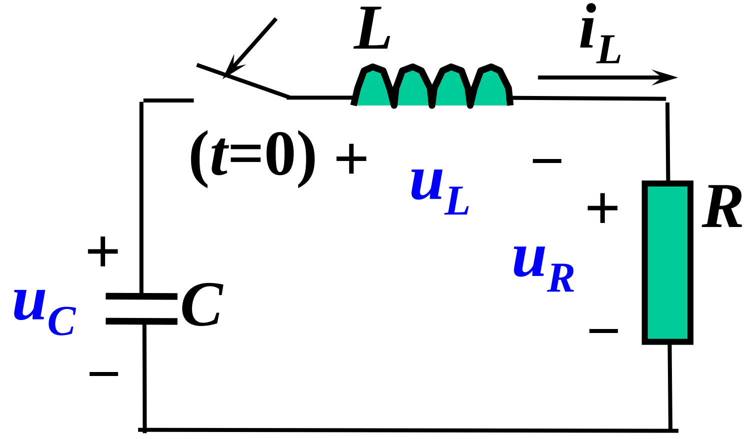
$$\frac{\mathrm{d}^2 i_L}{\mathrm{d}t^2} + 2\alpha\frac{\mathrm{d}i_L}{\mathrm{d}t} + \omega_0^2 i_L = 0$$

$$\Downarrow$$

RLC串联

$$2\alpha = \frac{R}{L}$$

$$\omega_0^2 = \frac{1}{LC}$$



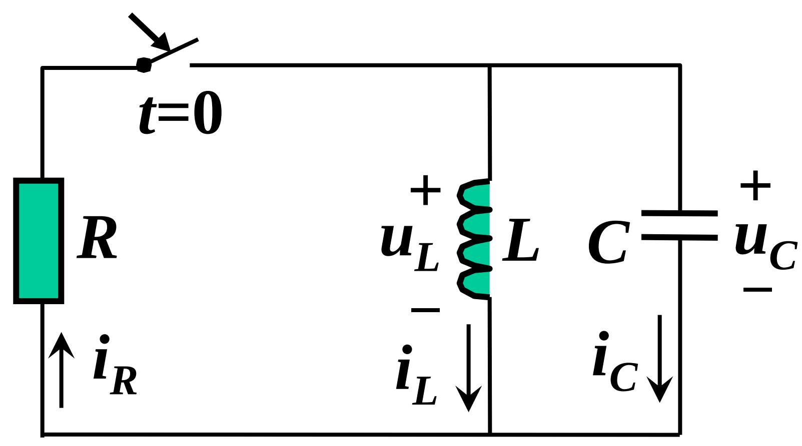
对偶的力量！

$$p^2 + 2\alpha p + \omega_0^2 = 0$$

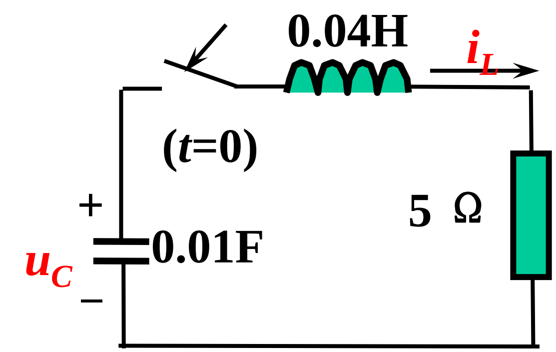
RLC并联

$$2\alpha = \frac{1}{RC}$$

$$\omega_0^2 = \frac{1}{LC}$$



3 二阶电路的直觉解法



不求待定系数**定性**画支路量的变化曲线

(1) 过阻尼或临界阻尼（无振荡衰减）

以过阻尼为例

$$p^2+\frac{R}{L}p+\frac{1}{LC}=0$$

$u_C(0^-) = 3\text{V}$

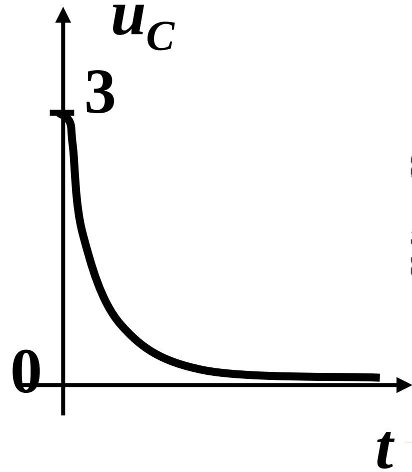
$i_L(0^-) = 0$

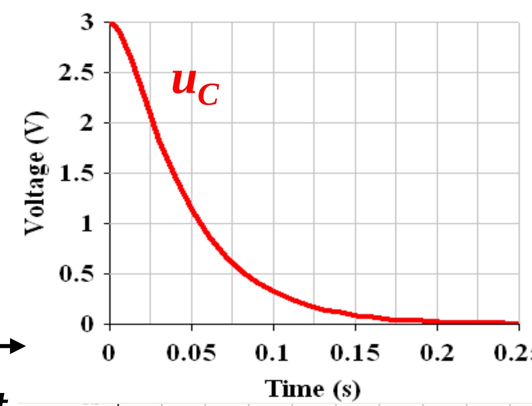
$$p_1=-25 \quad p_2=-100$$

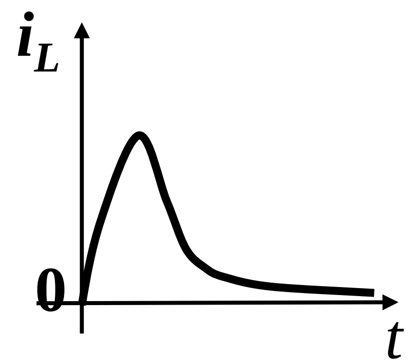
$$\begin{cases} u_C(0^+)=3\text{V} \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0 \end{cases}$$

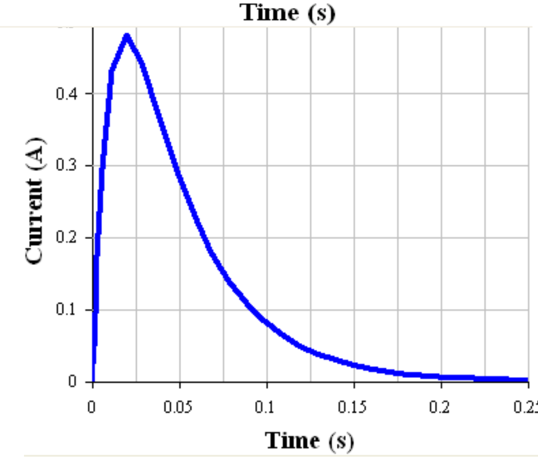
- 初值
- 导数初值
- 终值

$$\begin{cases} i_L\big|_{0^+}=0 \\ \left.\dfrac{\mathrm{d}i_L}{\mathrm{d}t}\right|_{0^+}=\dfrac{1}{L}u_L\Big|_{0^+}=\dfrac{3}{L} \end{cases}$$





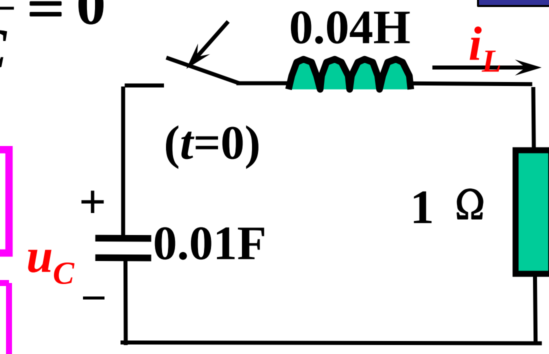




过（临界）阻尼，无振荡放电

(2) 欠阻尼（衰减振荡）

$$p^2+\frac{R}{L}p+\frac{1}{LC}=0$$



$$p_{1,2}=-12.5\pm j48.4$$

衰减振荡角频率 ω_d

衰减系数 α

$u_C(0^-)=3\text{V}$
$i_L(0^-)=0$

$$\begin{cases} u_C(0^+)=3\text{V} \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+}=0 \end{cases}$$

- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕

回忆一阶电路中的时间常数τ：3～5 τ 后过渡过程结束

$$3\times\frac{1}{\alpha}=\frac{3}{12.5}=0.24\,\text{s}$$

$$5\times\frac{1}{\alpha}=\frac{5}{12.5}=0.4\,\text{s}$$

后过渡过程结束

振荡周期为 $T=\dfrac{2\pi}{\omega_d}=\dfrac{2\pi}{48.4}=0.13\,\text{s}$

单选题 1分 设置

振荡几个周期后
可认为过渡过程结束？

$$p_{1,2} = -12.5 \pm \mathrm{j}48.4$$

A 1

B 2–3

C 5–10

$$3 \times \frac{1}{\alpha} = \frac{3}{12.5} = 0.24\ \mathrm{s}$$

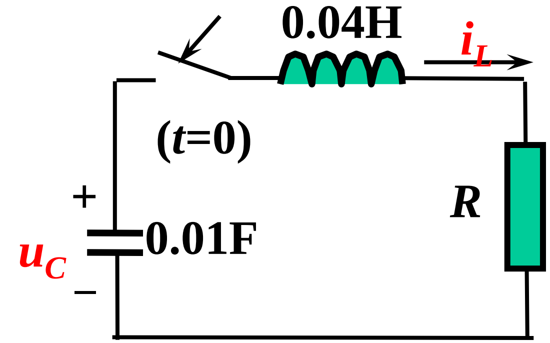
$$5 \times \frac{1}{\alpha} = \frac{5}{12.5} = 0.4\ \mathrm{s}$$

$$T = \frac{2\pi}{48.4} = 0.13\ \mathrm{s}$$

提交

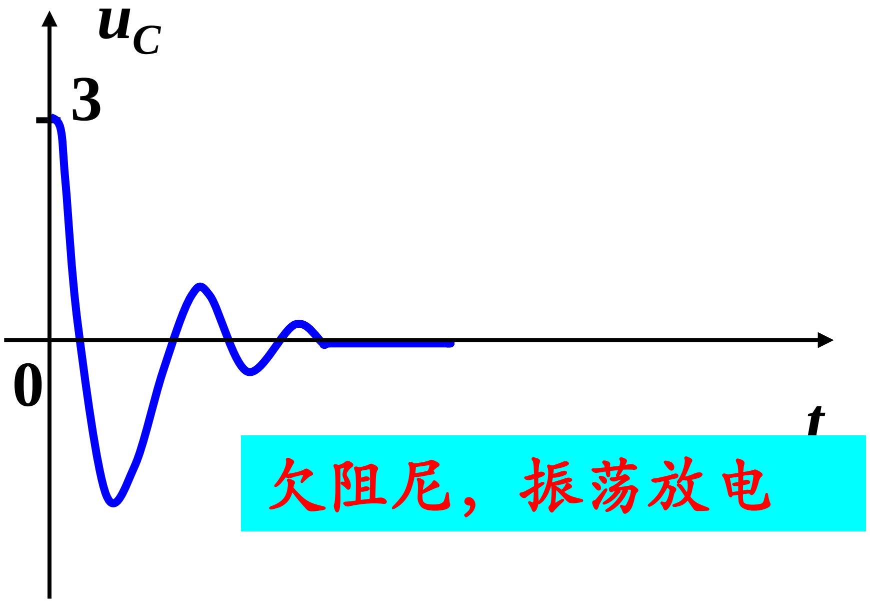
$$\lambda_{1,2} = -12.5 \pm \mathrm{j}48.4$$

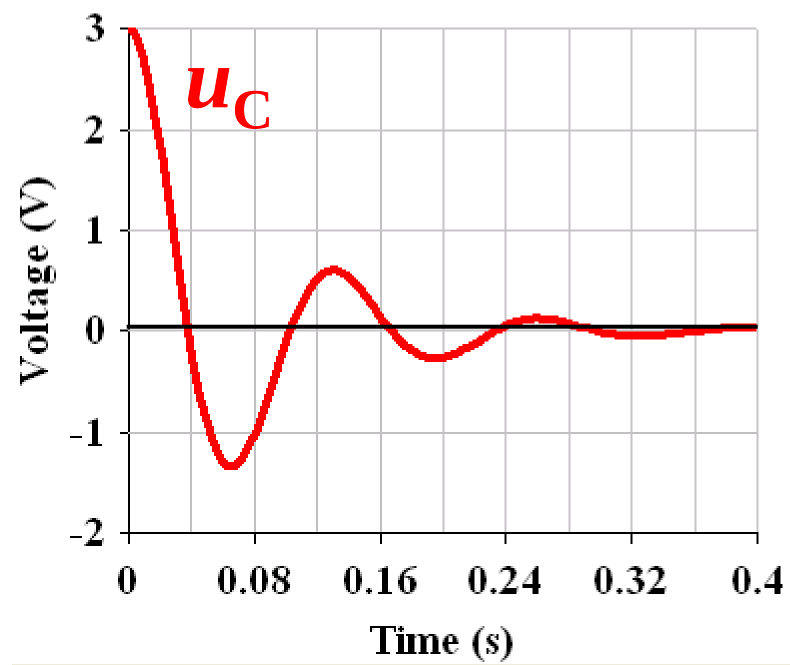
$$\begin{cases} u_C(0^+) = 3\mathrm{V} \\ \left.\dfrac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{t=0^+} = 0 \end{cases}$$



- 初值
- 导数初值
- 终值
- 经过多少周期振荡衰减完毕

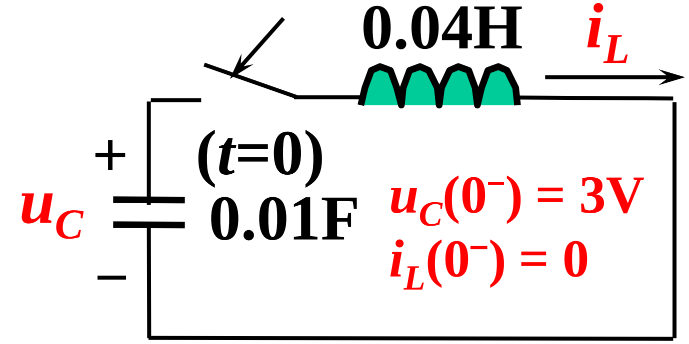
衰减过程中有
0.24/0.13≈2次振荡
或0.4/0.13≈3次振荡





(3) 无阻尼

- 初值
- 导数初值
- 最大值



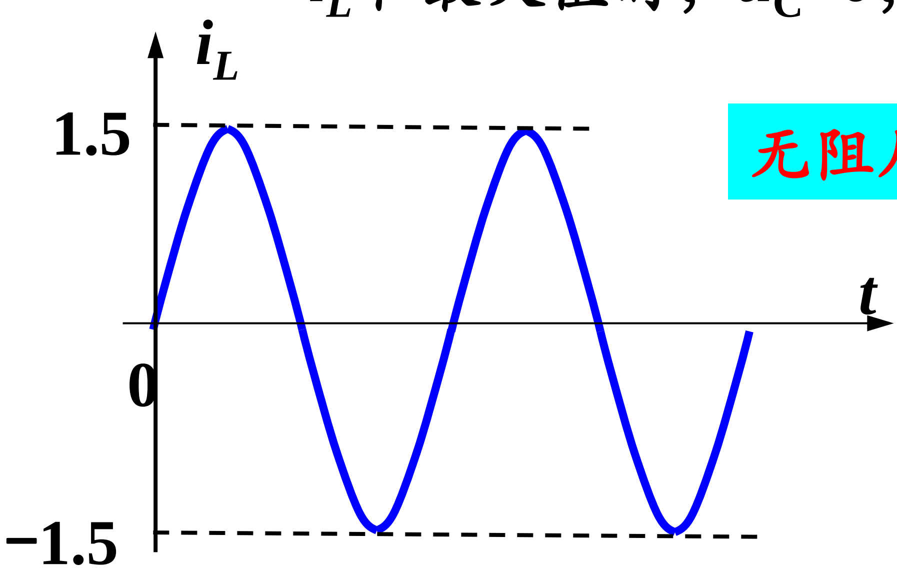
$$p_{1,2} = \pm \mathrm{j}50$$

$$\begin{cases} i_L\big|_{0^+} = 0 \\ \dfrac{\mathrm{d}i_L}{\mathrm{d}t}\Big|_{0^+} = \dfrac{1}{L}u_L = \dfrac{3}{L} \end{cases}$$

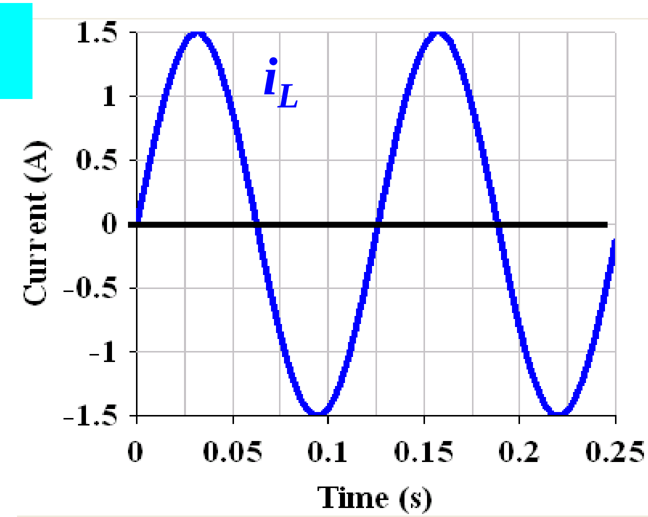
因为无阻尼，所以无能量损失

$$\frac{1}{2}Cu_C^2(0) + \frac{1}{2}Li_L^2(0) = \frac{1}{2}Cu_C^2(t) + \frac{1}{2}Li_L^2(t)$$

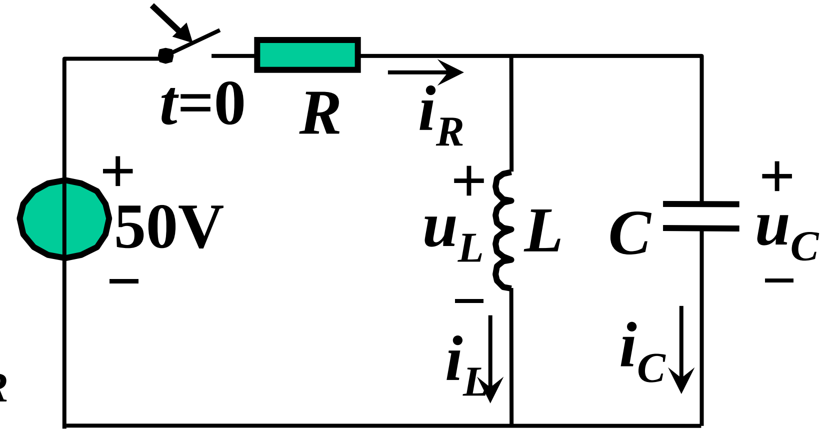
i_L取最大值时，u_C=0，因此 $i_{L,\max} = \sqrt{\dfrac{C}{L}}u_C(0) = 1.5\text{A}$



无阻尼振荡



例 已知 $i_L(0)=2\text{A}$ $u_C(0)=0$
$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。
求：$i_R(t)$。



法1： 列u_C的微分方程先求u_C再求i_R

法2： 列i_R的微分方程求解

法3： 通过求解一系列电阻电路求i_R

Step 1 由零输入电路得响应形式

零输入RLC并联

$$p^2+2\alpha p+\omega_0^2=0 \qquad 2\alpha=\frac{1}{RC}=200 \qquad \omega_0^2=\frac{1}{LC}=20000$$

$$p_{1,2}=-100\pm \text{j}100$$

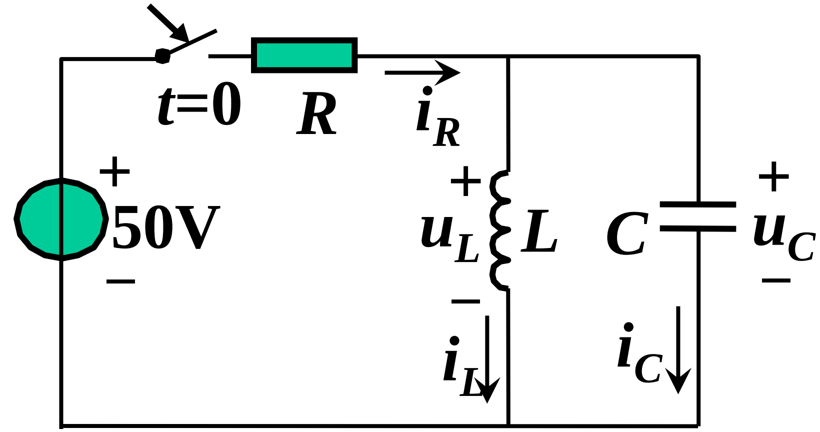
响应形式

$$\boxed{i_R=i_R(\infty)+K\text{e}^{-100t}\sin(100t+\theta)}$$

已知 $i_L(0)=2\text{A}$　$u_C(0)=0$

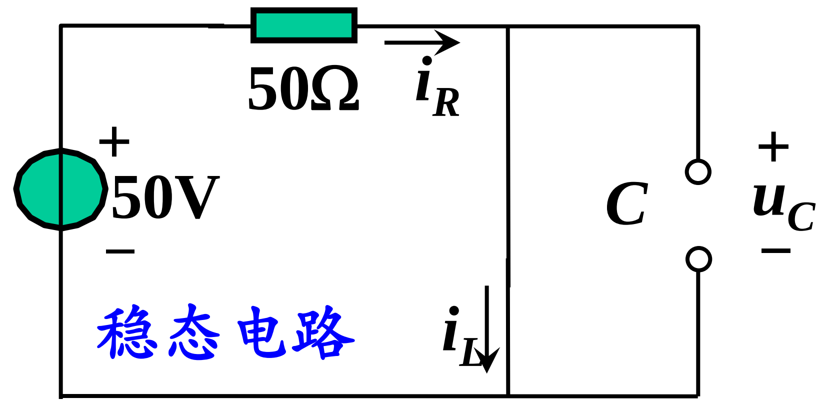
$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。

求：$i_R(t)$ 。



Step2　求稳态解

$$i_R(\infty)=1\text{A}$$

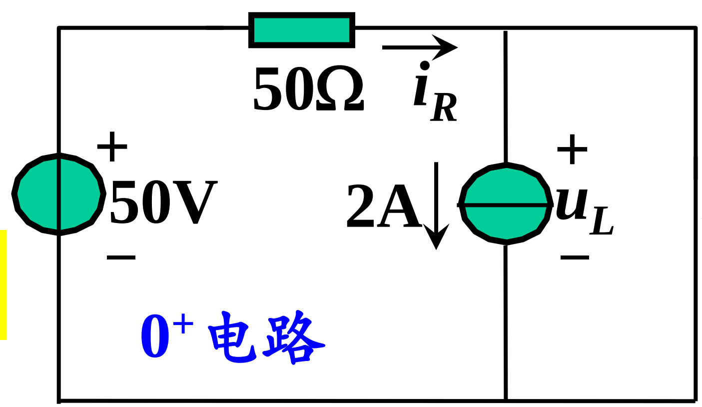


通解

$$i_R=1+Ke^{-100t}\sin\left(100t+\theta\right)$$

Step3　求初值　$i_L(0)=2\text{A}$　$u_C(0)=0$

$$i_R(0^+)=\frac{50-u_C(0^+)}{50}=1\ \text{A}$$

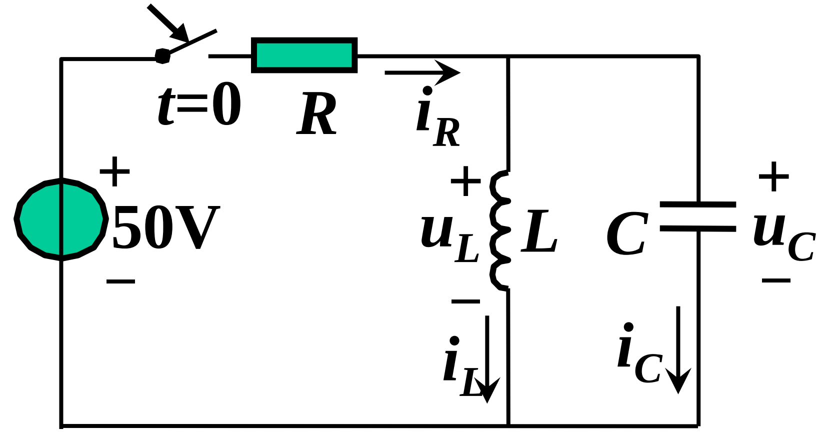


怎么求？

$$\left.\frac{\mathrm{d}i_R}{\mathrm{d}t}\right|_{t=0^+}$$

此处可以有弹幕

思路：用电源、u_C和i_L来表示i_R



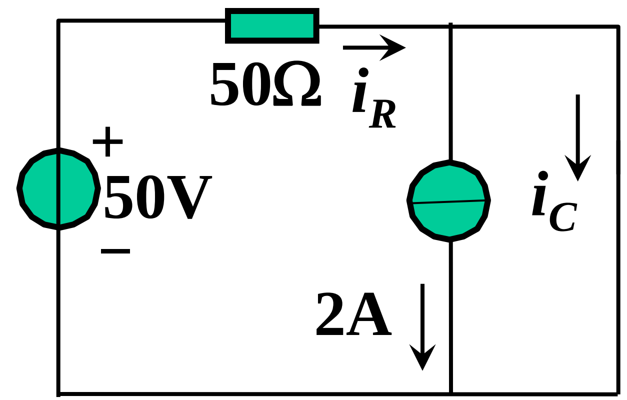
$$i_R = \frac{50 - u_C}{R}$$

$$\left.\frac{\mathrm{d}i_R}{\mathrm{d}t}\right|_{0+} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{50-u_C}{R}\right)\Big|_{0+} = -\frac{1}{R}\left.\frac{\mathrm{d}u_C}{\mathrm{d}t}\right|_{0+}$$

$$= -\frac{1}{RC} i_C(0^+)$$

$$= -\frac{-1}{50\times 100\times 10^{-6}} = 200\ \text{A/s}$$

0^+电路

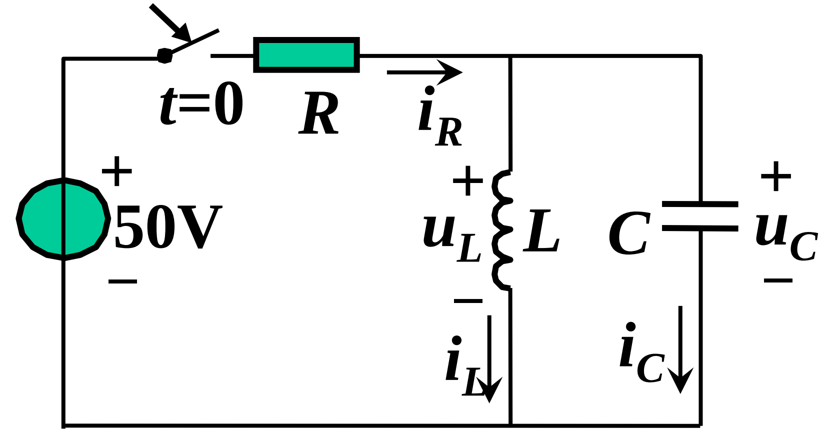


$$i_C(0^+) = -1\text{A}$$

已知： $i_L(0)=2\text{A}$　$u_C(0)=0$

$R=50\Omega$, $L=0.5\text{H}$, $C=100\mu\text{F}$。

求： $i_R(t)$ 。



Step4　求待定系数

通解　$$i_R = 1 + Ke^{-100t}\sin\left(100t+\theta\right)$$

$$\begin{cases} i_R(0^+) = 1\text{A} \\ \left.\dfrac{\mathrm{d}i_R}{\mathrm{d}t}\right|_{0^+} = 200 \ \text{A}/\text{s} \end{cases}$$

$$i_R(t) = 1 + 2e^{-100t}\sin 100t\ \text{A} \qquad t > 0^+$$

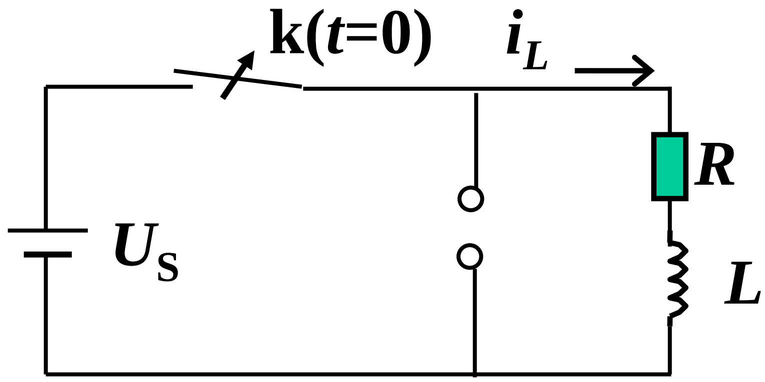
总结二阶电路的求解

- 求响应形式
  - RLC串联、RLC并联 → 直接得到特征方程
  - 状态方程 →（电阻电路）→ 求系数矩阵特征值（L12）
- 求稳态值 → 得通解表达式
  - 电阻电路
- 求初值
  - 电阻电路
- 求导数初值
  - 将支路量用独立源、u_C、i_L来表示
  - 输出方程（电阻电路）（L12）
- 用初值和导数初值确定通解待定系数

为什么一个动态电路中任意支路量都有相同的变化性质？（L12）

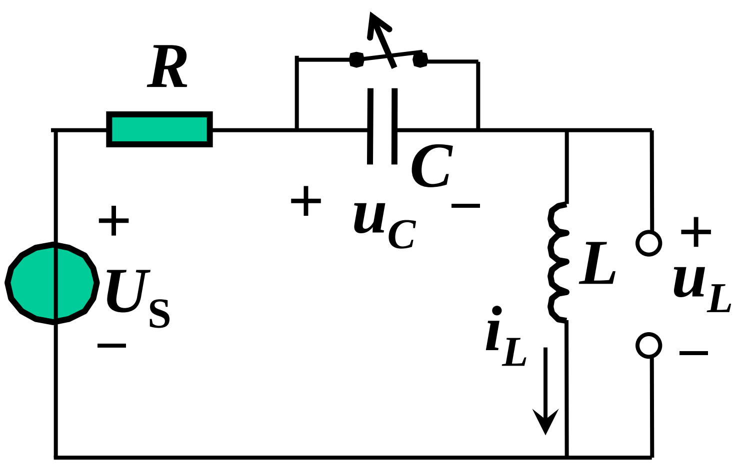
4 二阶电路的应用

(1) 汽车点火系统



一阶点火电路的问题:

开路开关和火花塞承受相同的无穷大电压

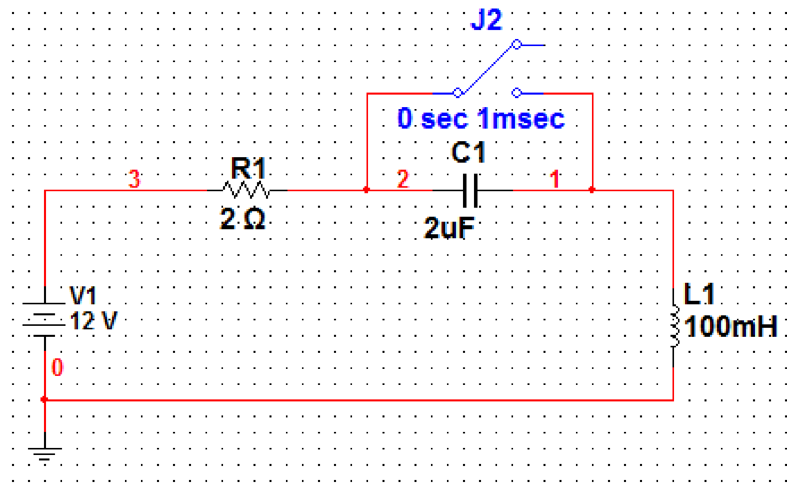


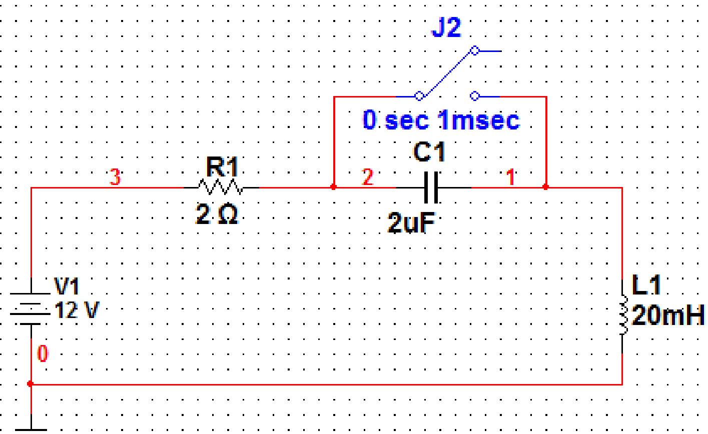
二阶点火电路的好处:

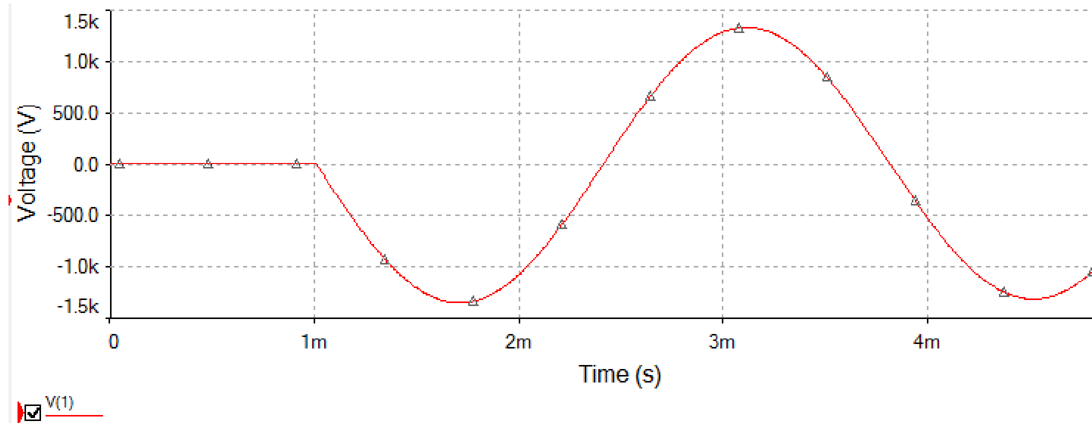
开路开关的电压被电容钳位

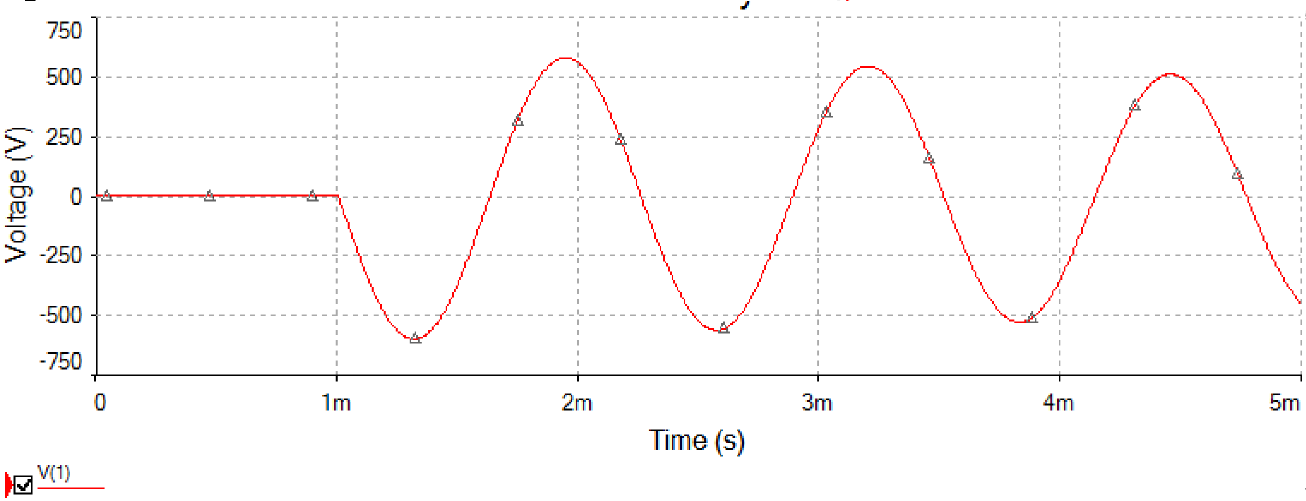
可通过电路参数控制开关电压

仿真


J2
0 sec 1msec
C1
R1
3
2
1
2 Ω
2uF
V1
12 V
L1
100mH
0


J2
0 sec 1msec
C1
R1
3
2
1
2 Ω
2uF
V1
12 V
L1
20mH
0

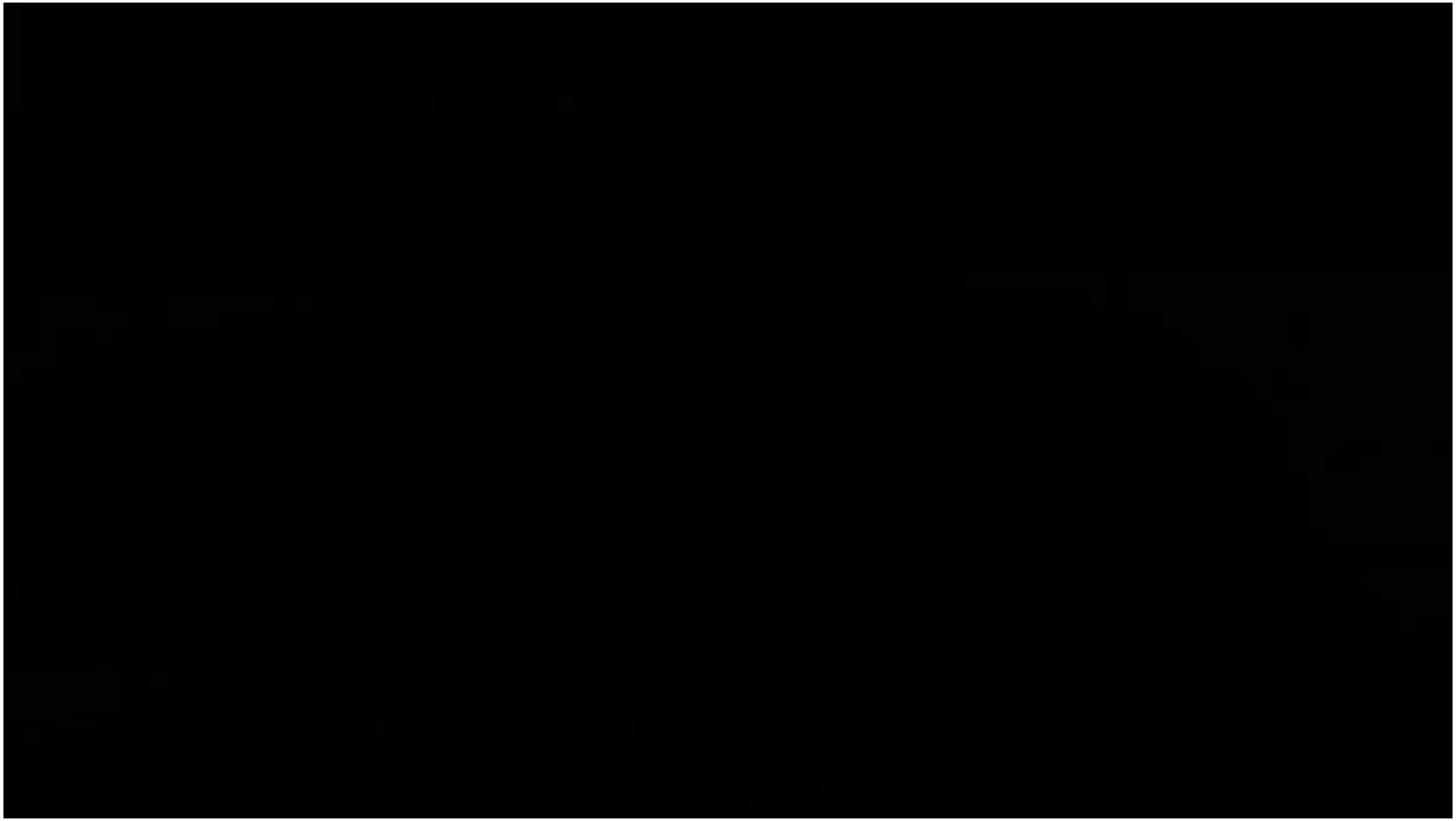

Voltage (V)
1.5k
1.0k
500.0
0.0
-500.0
-1.0k
-1.5k
0
1m
2m
3m
4m
Time (s)
V(1)


Voltage (V)
750
500
250
0
-250
-500
-750
0
1m
2m
3m
4m
5m
Time (s)
V(1)

（3）电磁轨道炮电源　　Dahlgren Surface Warfare Center

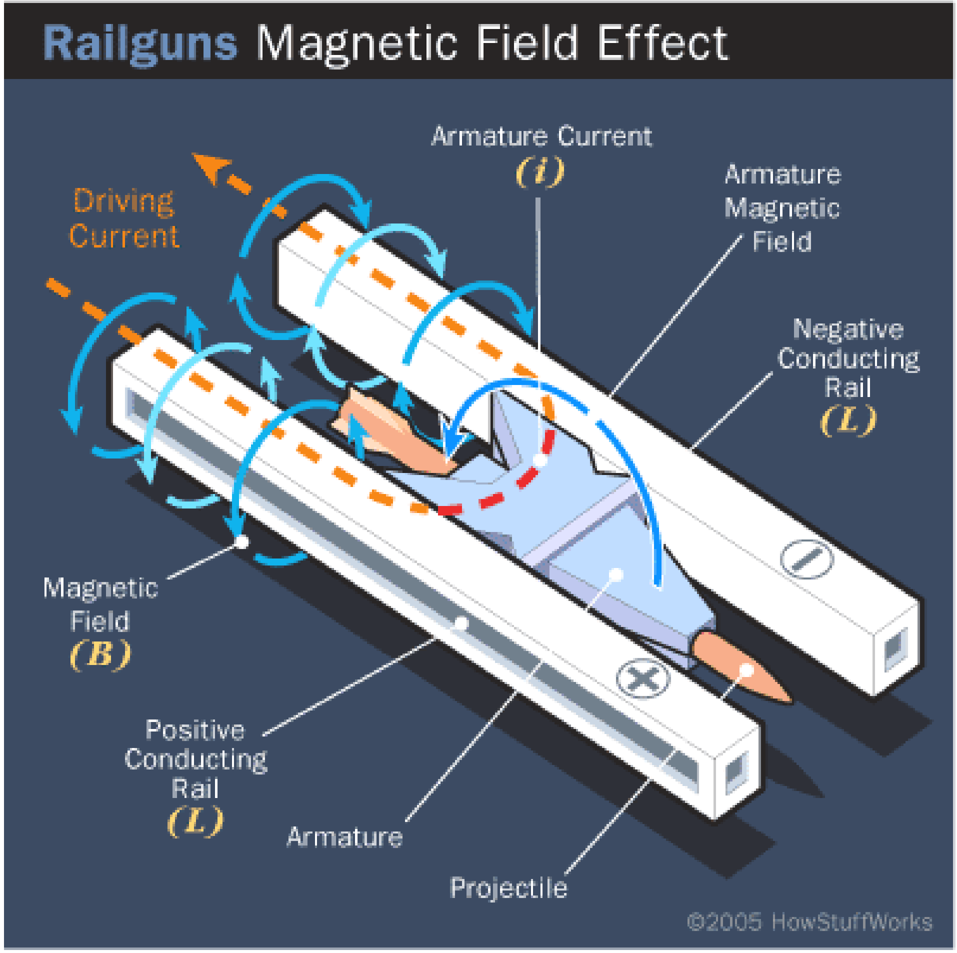
2012

2014

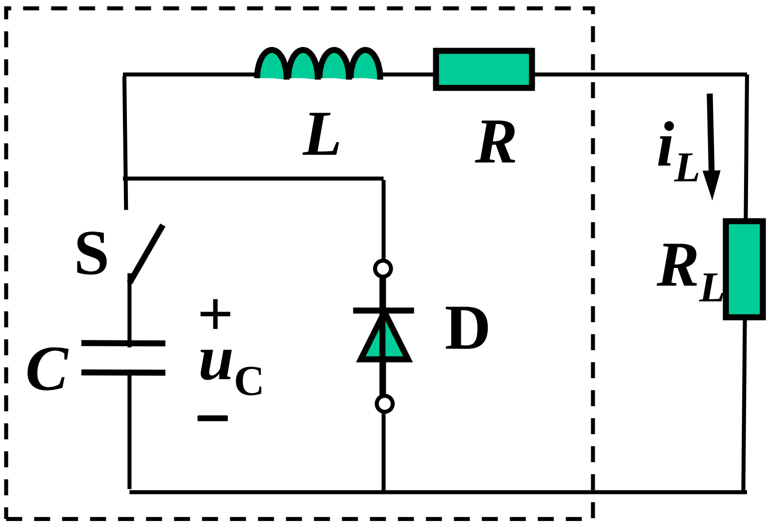


电磁轨道炮原理

关键是
可控的脉冲大电流



电磁轨道炮脉冲电源的基本电路（PFU）



4个PFU并联