9101 Assignment 4
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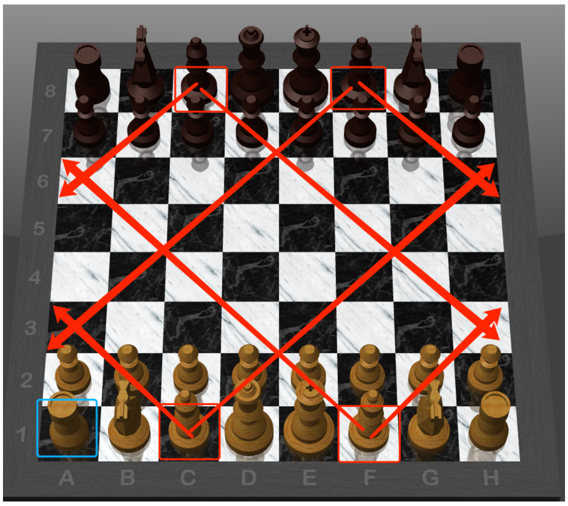
Q4.

4. Use max flow algorithm to solve the following problem. You are given a usual $n \times n$ chess board with k white bishops on the board at the given cells (a_i, b_i), $(1 \leq a_i, b_i \leq n$, $1 \leq i \leq k)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column or are under the attack of any of the k bishops (recall that bishops go diagonally).(25 pts)

Solution,



Create a flow network and it should be a bipartite graph:

1) All the rows as vertices on the left: $r_i - rows$
2) All the columns as vertices on the right: $c_i - columns$
3) Each square $S_{ij}(r_i, c_i)$ then can represent and edge from row r_i to column c_i
4) Setting a super source S and a super sink T,
   and the capacity (flow) of edge from S to r_i is 1, because there can only be one black rook in a row,
   meanwhile, the capacity (flow) of edge from c_i to T is also 1, because there can only be one black rook in a column.
5) Then, connecting between r_i and c_i to form edges.
   (The capacity of each edge is 1 also.)
   a) If the square $S_{ij}(r_i, c_i)$ is not within the attack of the range of white bishops, (The square outside the grid where the red line is located is as shown above.)
      i. then, r_i and c_i can be connected as an edge.
   b) Else if square $S_{ij}(r_i, c_i)$ is within the attack of the range of white bishops,
      i. Computer the maximum flow by the extension of the Preflow-Push algorithm. And therefore, we can get the largest number of the position

black rooks which can be placed on the board.

6) The complexity is $O(n^3)$