9101 Assignment 1
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Question 2

From the question, it can get that $y_i = c_i + E,$ where $c_i \geq 1$, E is a positive integer, y_i is positive integer.
Let $\text{y\_min} = \min\{y_i: 1 \leq i \leq n\}$.
Then, for all the c_i *and* y_i, $c_i = y_i - E \geq 1$,
And then, $c_i = \text{y\_min} - E \geq 1$,
and hence, $y_i \geq \text{y\_min} - E \geq 1$,
So, the range of E is $(0, \text{y\_min} - 1]$.

Then, the algorithm detail is,
Using binary search in the range of $[0, \text{y\_min} - 1]$, (the cost is $O(\log \text{y\_min})$)

Based on $y_i = c_i + E,$ each binary search for E, we can get a set of values of $\frac{x_i}{c_i}$, and do the sum operation $S' = \sum_{i=1}^{n} \frac{x_i}{c_i}$ which costs $O(n)$. If $S' < S$, binary search searches in the second half. If $S' > S$, binary search searches in the first half. Repeat the above operation until we find the value of E at the position $S' = S$.
(Notice that, $S' = \sum_{i=1}^{n} \frac{x_i}{c_i}$ is monotonic, because, $E \uparrow, c_i \downarrow, \frac{x_i}{c_i} \uparrow, S' \uparrow$)
Therefore, the total cost is,
$O(\log \text{y\_min}) * O(n) = O(n \log y\_min) = O(n \log \min\{y_i: 1 \leq i \leq n\})$.