9101 Assignment 3
Haojin Guo
z5216214

Q5.

5. Given a weighted **directed** graph $G(V, E)$, find a path in G (possibly self-intersecting) of length exactly K that has the maximum total weight. The path can visit a vertex multiple times and can traverse an edge also multiple times. It can also start and end at arbitrary vertices or even start and end at the same vertex. (20 pts)

Solution,

1) Subproblems.
   Let $dp[i][j][k]$ be the longest path from i vertex to the j vertex going through exactly K edges in total, where $i, j \in [0, V] and\ k \in [0, K]$.
   Also, let $edge_{weight}[i][j]$ be the weight of edge from vertex $i\ to\ j$,

2) Base case,
   For $m, n \in [0, V]$,
   $k = 0, dp[m][n][0] = 0$,
   $k = 1, dp[m][n][1] = edge_{weight}[m][n]$,
3) Recursion Step,
   For all the edges from $vertex - i\ to\ vertex - c$ (BFS algorithm), we have,
   $dp[i][j][k+1] = \max\{dp[i][c][k+1], dp[i][c][k] + edge_{weight}[c][i]\}$,

4) Final result,

And the result for the maximum total weight is $\max\{dp[i][j][K]\}$, where $i, j \in [0, V]$ and $dp[i][j][K]$ is a sequence.

5) Time complexity,
   The graph has V vertices,
   a) BFS algorithm costs $O(K \cdot V)$
   b) Total graph traversal costs $O(V \cdot (K \cdot V))$
   c) Get maximum total weight costs $O(V^2)$

   Therefore, total time cost is $O(K \cdot V^2)$