9101 Assignment 1
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Question1
You are given an array A of n distinct positive integers. (a) Design an algorithm which decides in time O($n^2 log\ n$) (in the worst case) if there exist four distinct integers m, s, k, p in A such that $m^2 + s = k + p^2$ (b) Solve the same problem but with an algorithm which runs in the expected time of O($n^2$).

Solution:
a)

Step (i): Create an array B, and then calculate the square value of each element $i$ in the array A, and then add the value $i^2$ to the new array B.
(The time complexity in this step is $O(n)$).

Step (ii): Create an array B, by iterating through the array A and B, we can get the value of $i + j^2$ $(i \neq j)$, and then put the set $(i + j^2, i, j)$ value into the array C.
(The time complexity in this step is $O(n^2)$).

Step(iii): Sort the array C, using Merge Sort, which costs $O(n^2 logn^2) = O(2n^2 logn) = O(n^2 logn)$

Step(iv): Using binary search to check the array C.

If exists three first elements of in three different tuples, and then there exist four distinct integers $m, s, k, p$ such that $m^2 + s = k + p^2$.

If exists two first elements of in two different tuples, and meanwhile, $i_1 \neq j_1 \neq i_2 \neq j_2$, then there exist four distinct integers $m, s, k, p$ such that $m^2 + s = k + p^2$.

Otherwise, there are no target integers.

(The cost in this step is $O(n^2 logn)$ )

Therefore, the total cost = $O(n^2 logn)$ + $O(n^2 logn)$ + $O(n^2)$ + $O(n)$ = $O(n^2 logn)$

b)

In order to shorten the expected time complexity, it is need to create a hash table to

make sure the search time complexity is $O(1)$.

Step (i): Same to the step(i) in (a) above, which cost $O(n)$.

Step (ii): Create an array B, b iterating through the array A and B, we can get the value of $i + j^2$ $(i \neq j)$, then put the set $(i + j^2, i, j)$ value into the array C, and construct a hash table with the key is $(i + j^2, i, j)$ and the corresponding result is 0 for the first time. And then each time a new $(i + j^2, i, j)$ $(where\ i \neq j)$ is obtained, it is required to search in the hash table, if it exists in the map, it means that the array A has the target integers, otherwise it does not exist. (The time complexity in this step is $O(n^2)$)

Therefore, the total cost = $O(n^2)$ + $O(n)$ = $O(n^2)$