9101 Assignment 3
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Q4.

4. You are on vacation for N days at a resort that has three possible activities 1,2 and 3. For each day i, for each activity 1,2 or 3, you've determined how much enjoyment $e(i, j)$ $(1 \leq i \leq n;\ 1 \leq j \leq 3)$ you will get out of that activity if you do it on that particular day (the same activity might give you a different amounts of enjoyment at different days). However, you are not allowed to do the same activity two days in a row. Design an algorithm for determining the maximum total enjoyment possible over the entire stay of N days and the sequence of activities you should do at each day. (20 pts)

Solution,

1) Let $dp\ (i, j\ )$ represents the maximum total enjoyment we can get from day 1 to day n.
   For activities $1,2\ and\ 3$, their corresponding enjoyments in day i are $e(i, 1), e(i, 2)\ and\ e(i\ ,3)$.
2) Subproblem,
   We have the subproblem that in the i_th day, the maximum enjoyment is
   $$dp\ (i, j\ ) = \max\{(i, 1), e(i, 2), e(i\ ,3)\}$$
3) Base case,
   In the 1st day, recording the enjoyment directly.
   $dp\ (1,1\ ) = e(1,1)$,
   $dp\ (1,2\ ) = e(1,2)$,
   $dp\ (1,3\ ) = e(1,3)$,

4) Recursion,

   Assuming that we have solved all the subproblems f*or* all $j < i$,

   a) If we choose activity 1 at day i,

   $$dp(i, 1) = \max\{dp(i - 1,2) + e(i, 1), dp(i - 1,3) + e(i, 1)\}$$

   b) If we choose activity 2 at day i,

   $$dp(i, 2) = \max\{dp(i - 1,1) + e(i, 2), dp(i - 1,3) + e(i, 2)\}$$

   c) If we choose activity 3 at day i,

   $$dp(i, 3) = \max\{dp(i - 1,1) + e(i, 3),\quad dp(i - 1,2) + e(i, 3)\}$$

5) The final result for the maximum total enjoyment is
   $\max\{dp(n,1), dp(n,2), dp(n,3)\}$
6) The total complexity is O(n), where n means the vacation has n days.