9101 Assignment 4
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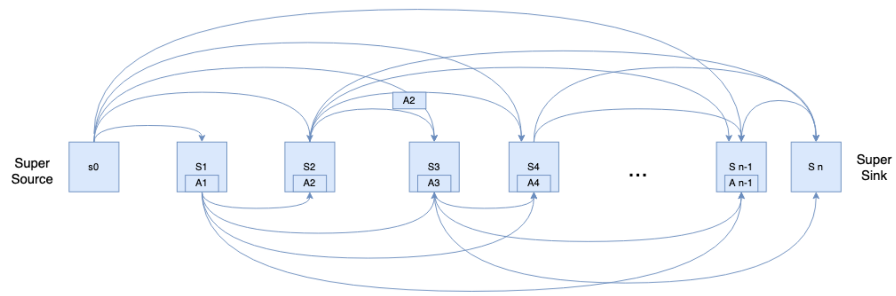
Q3.

3. A large group of children has assembled to play a modified version of hop-scotch. The squares appear in a line, numbered from 0 to n, and a child is successful if they start at square 0 and make a sequence of jumps to reach square n. However, each child can only jump at most $k < n$ squares at a time, i.e., from square i they can reach squares $i+1, i+2, \ldots, i+k$ but not $i+k+1$, and a child cannot clear the entire game in one jump. An additional rule of the game specifies an array $A[1, \ldots, n-1]$, where $A[i]$ is the maximum number of children who can jump on square i. Assuming the

children co-operate, what is the largest number of children who can successfully complete the game?(25 pts)

Hint: Connect every square i with squares $i+1, \ldots, i+k$ with a directed edge of infinite capacity. Now limit the capacity of each square appropriately and use max flow.

Solution,



Construct a flow network as a directed graph:

1) A total $n+1$ squares from $square\ 0$ to $square\ n$ are regarded as the vertices of the graph.
2) Connect every $square\ i$ with $square\ i+1, \ldots, i+k$ with a directed edge of infinite capacity.
3) Super source (S) is the $square\ 0$.
4) Super sink (T) is the $square\ n$.
5) Every vertex has the capacity $A[i]$
6) Divide each vertex $square\ i$ with capacity $A[i]$ into two vertices v_in and v_out, so that all edges (children) entering $square\ i$ enter v_in, and all leaving edges leave v_out.
7) Computer the maximum flow by the extension of the Preflow-Push algorithm. And therefore, we can get the maximum number of children that can reach T.
8) The complexity is $O((2n+2)^3)$