

# Implementing Join

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## Join

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DBMSs are engines to *store*, *combine* and *filter* information.

Filtering is achieved via selection and projection.

The *join* operation ( $\bowtie$ ) is the primary means of *combining* information.

Because *join* is

- such an important operation in database applications/systems
- potentially very expensive to execute

many methods have been developed for its implementation.

(We use a running example to compare costs of the various join processing methods)

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## ... Join

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Types of join:

- simple equijoin (single-equality condition)  

```
select * from R,S where R.i = S.j
```
- partial-match join (conjunction of equality conditions)  

```
select * from R,S where R.a = S.b and R.c = S.d ...
```
- theta join (arbitrary expression as condition)  

```
select * from R,S where R.a < S.b and R.c <> S.d ...
```

Focus on simple equijoin, since common in practice ( $R.pk=S.fk$ )

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## Join Example

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Consider a university database with the schema:

```
create table Student(  
  id      integer primary key,  
  name    text, ...  
);  
create table Enrolled(  
  stude   integer references Student(id),  
  subj    text references Subject(code), ...  
);  
create table Subject(  
  code    text primary key,  
  title   text, ...  
);
```

And the following request on this database:

*List names of students in all subjects, arranged by subject.*

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## ... Join Example

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The result of this request would look like:

Subj	Name
-----	-----
COMP1011	Chen Hwee Ling
COMP1011	John Smith
COMP1011	Ravi Shastri
...	
COMP1021	David Jones
COMP1021	Stephen Mao
...	
COMP3311	Dean Jones
COMP3311	Mark Taylor
COMP3311	Sashin Tendulkar

### ... Join Example

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An SQL query to provide this information:

```
select E.subj, S.name
from   Student S, Enrolled E
where  S.id = E.stude
order by E.subj, S.name;
```

And its relational algebra equivalent:

$$\text{Sort}[\text{subj}] ( \text{Project}[\text{subj}, \text{name}] ( \text{Join}[\text{id}=\text{stude}](\text{Student}, \text{Enrolled}) ) )$$

The core of the query is the join  $\text{Join}[\text{id}=\text{stude}](\text{Student}, \text{Enrolled})$

To simplify writing of formulae,  $S = \text{Student}$ ,  $E = \text{Enrolled}$ .

### ... Join Example

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Some database statistics:

Sym	Meaning	Value
$r_S$	# student records	20,000
$r_E$	# enrollment records	80,000
$C_S$	student records/page	20
$C_E$	Enrolled records/page	40
$b_S$	# data pages in Student	1,000
$b_E$	# data pages in Enrolled	2,000

Also, in cost analyses below,  $N$  = number of memory buffers.

### ... Join Example

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out =  $\text{Student} \bowtie \text{Enrolled}$  relation statistics:

Sym	Meaning	Value
$r_{\text{Out}}$	# tuples in result	80,000
$C_{\text{Out}}$	result records/page	80
$b_{\text{Out}}$	# data pages in result	1,000

---

Notes:

- $r_{Out}$  ... one result tuple for each `Enrolled` tuple
  - $C_{Out}$  ... result tuples have only `subj` and `name`
- 

## Join via Cross-product

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Join can be defined as a cross-product followed by selection:

$$Join[Cond](R,S) = Select[Cond](R \times S)$$

For the example query, could implement

$$Join[id=stude](Student, Enrolled)$$

as

$$Select[id=stude](Student \times Enrolled)$$

Cross-product contains  $20,000 \times 80,000 = 1,600,000,000$  tuples.

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### ... Join via Cross-product

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For `Temp` =  $(Student \times Enrolled)$

I/O costs:

- size of `Temp` relation  $r = 16 \times 10^8$  records
- assuming  $C_{Temp}=16$ , then  $b_{Temp} = 10^8$
- `Temp` is written once, then scanned once
- total I/O =  $10^8 \cdot (T_w + T_r)$

Assuming  $T_w=T_r=0.01s$ , this will take around 500 hours!

---

### ... Join via Cross-product

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Because

- cross-products are infrequent in practice (except to describe join)
- cross-products are large (typically **much** larger than the final join result)

DBMSs do **not** implement join via cross-product.

DBMSs implement only join and provide cross-product as:

$$R \times S = Join[true](R,S)$$

or, in SQL

```
select * from R,S
```

---

## Nested-Loop Join

### Nested Loop Join

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The simplest join algorithm:

- iteratively generates the cross-product
- checks join condition on each tuple

Algorithm to compute  $Join[Cond](R,S)$ :

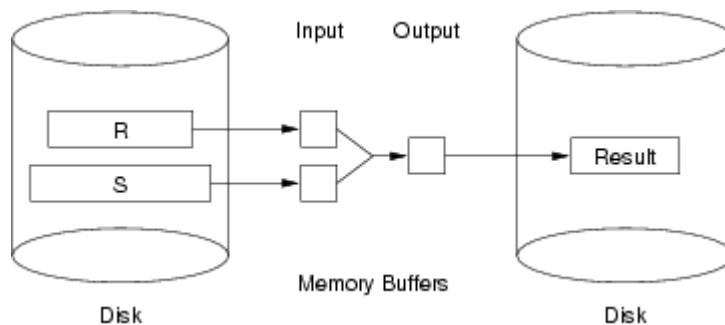
```
for each tuple r in R {
  for each tuple s in S {
    if ((r,s) satisfies join condition) {
      add (r,s) to result
    }
  }
}
```

$R$  is the *outer* relation;  $S$  is the *inner* relation.

### ... Nested Loop Join

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Requires (at least) three memory buffers (2 input, 1 output).



### ... Nested Loop Join

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Abstract algorithm for  $Join[Cond](R,S)$  (with 3 memory buffers):

```
for each page of relation R {
  read into buffer rBuf
  for each page of relation S {
    read into buffer sBuf
    for each record r in rBuf {
      for each record s in sBuf {
        if ((r,s) satisfies Cond) {
          add combined(r,s) to OutBuf
          write Outbuf when full
        }
      }
    }
  }
}
```

### ... Nested Loop Join

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Detailed algorithm for  $Join[Cond](R,S)$  (with 3 memory buffers):

```
// rf: file for R, sf: file for S, of: output file
outp = 0; clearBuf(oBuf);
for (rp = 0; rp < nPages(rf); rp++) {
  readPage(rf, rp, rBuf);
  for (sp = 0; sp < nPages(sf); sp++) {
    readPage(sf, sp, sBuf);
    for (i = 0; i < nTuples(rBuf); i++) {
      rTup = getTuple(rBuf, i);
      for (j = 0; j < nTuples(sBuf); j++) {
        sTup = getTuple(sBuf, j);
        if (satisfies(rTup,sTup,Cond)) {
          rsTup = combine(rTup,sTup);
          addTuple(oBuf, rsTup);
          if (isFull(oBuf)) {
            writePage(of, outp++, oBuf);
            clearBuf(oBuf);
          }
        }
      }
    }
  }
}
```

The three-memory-buffer nested loop join requires:

- read all  $b_R$  pages of  $R$  once
- for each of page of  $R$ , read  $b_S$  pages of  $S$

$$\text{Cost} = b_R + b_R b_S$$

If we use  $S$  as the outer relation in the join

$$\text{Cost} = b_S + b_S b_R$$

It is (slightly) better to use smaller relation as outer relation.

## Nested Loop Join on Example

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If `Student` is outer relation and `Enrolled` is inner:

$$\begin{aligned} \text{Cost} &= b_S + b_S b_E \\ &= 1,000 + 1,000 \times 2,000 = 2,001,000 \end{aligned}$$

If `Enrolled` is outer relation and `Student` is inner:

$$\begin{aligned} \text{Cost} &= b_E + b_E b_S \\ &= 2,000 + 2,000 \times 1,000 = 2,002,000 \end{aligned}$$

Cost of nested-loop join is too high (5 hours, if  $T_f=0.01$  sec)

## Implementing Join Better

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Aims of effective join computation:

- generate only relevant tuples from the cross-product
- generate these tuples with minimal disk I/O

Range of costs for  $Join(R,S)$

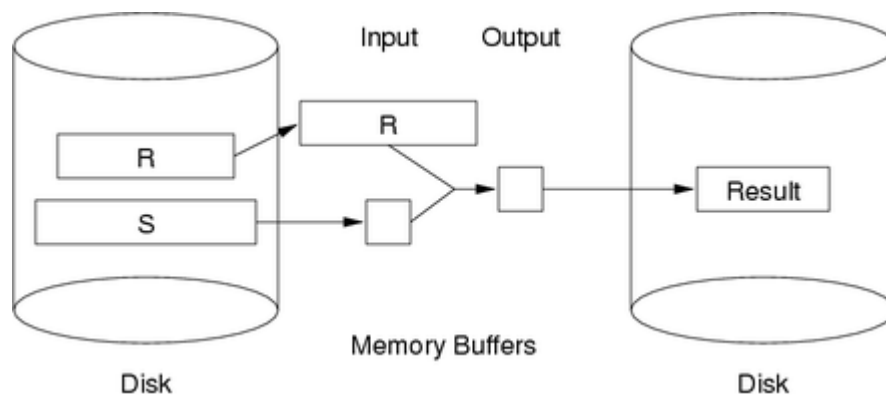
- worst case cost =  $b_R + b_R b_S$  (nested loop join)
- best case cost =  $b_R + b_S$  (read each page once)

## Block Nested Loop Join

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If at least  $b_R+2$  memory buffers available:

- read the entire  $R$  relation into memory
- for each  $S$  page, check join condition on all  $(r, s)$  pairs



### ... Block Nested Loop Join

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Algorithm for nested loop join with  $b_R+2$  memory buffers:

```

read all of R's pages into memory buffers
for each page of relation S {
  read page into S's input buffer
  for each tuple s in S's buffer {
    for each tuple r in R's memory buffers {
      if ((r,s) satisfies JoinCond) {
        add (r,s) to output buffer
      }
    }
    write output buffer when full
  }
}

```

Note that  $R$  effectively becomes the inner relation in this scheme.

### ... Block Nested Loop Join

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This method requires:

- read  $b_R$  pages of relation  $R$  into buffers
- while  $R$  is buffered, read  $b_S$  pages of  $S$

Cost =  $b_R + b_S$

Notes:

- minimal I/O cost, but considers all  $(r,s)$  pairs
- thus, requires  $r_R \cdot r_S$  checks of the join condition

### ... Block Nested Loop Join

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Further performance improvements:

- must reduce number of  $R$  tuples matched against each  $S$  tuple
- use access method to find small set of  $R$  tuples matching  $S$  tuple

Example:

- each  $S$  joins with  $k \ll r_R$  tuples of  $R$
- $R$  tuples are stored in sorted array of memory buffers
- for each  $S$  tuple, use binary search to find matching buffer
- scan around that buffer to find all matching  $(R,S)$  pairs
- requires approx  $C_R \cdot r_S$  checks of join condition

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## Block Nested Loop Join on Example

If  $\geq 1002$  memory buffers are available:

- read `Student` relation into memory
- scan `Enrolled` relation, computing join

$$\begin{aligned}\text{Cost} &= b_S + b_E \\ &= 1,000 + 2,000 = 3,000\end{aligned}$$

This is considerably better than  $10^6$  (30 secs vs 5 hours).

But what if we have only  $N$  memory buffers, where  $N < b_R$ ,  $N < b_S$ ?

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### ... Block Nested Loop Join on Example

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In general case, read *outer* relation in runs of  $N-2$  pages

```
for each run of N-2 pages from R {
  read N-2 of R's pages into memory buffers
  for each page of relation S {
    read page into S's input buffer
    for each tuple s in S's buffer do
      for each tuple r in R's memory buffers {
        if ((r,s) satisfies JoinCond)) {
          add (r,s) to output buffer
        }
      }
    }
  }
}
```

---

### ... Block Nested Loop Join on Example

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Block nested loop join requires

- read  $\lceil b_R/N-2 \rceil$  runs from  $R$
- for each run, scan  $b_S$  pages of  $S$

$$\text{Cost} = b_R + b_S \cdot \lceil b_R/N-2 \rceil$$

Notes:

- the final run will typically be "short" (i.e.  $< N-2$  pages)
- unless index/hash is used, we still do  $r_R.r_S$  tuple comparisons

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### ... Block Nested Loop Join on Example

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Costs for various buffer pool sizes:

$N$	Inner	Outer	#runs	Cost
22	Student	Enrolled	50	101,000
52	Student	Enrolled	20	41,000
102	Student	Enrolled	10	21,000
1002	Student	Enrolled	1	3,000
22	Enrolled	Student	100	102,000

52	Enrolled	Student	40	42,000
102	Enrolled	Student	20	22,000
1002	Enrolled	Student	2	4,000

## Block Nested Loop Join in Practice

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Why block nested loop join is very useful in practice ...

Many queries have the form

```
select * from R,S where r.i=s.j and r.x=k
```

This would typically be evaluated as

$$Join [i=j] ((Sel[r.x=k](R)), S)$$

If  $|Sel[r.x=k](R)|$  is small  $\Rightarrow$  may fit in memory (in small #buffers)

## Join Conditions and Methods

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Nested loop join makes no assumptions about join conditions.

```
for each pair of tuples (r,s) {
  check join condition on (r,s)
  if satisfied, add to results
}
```

To improve join:

- reduce the number of tuple pairs considered
- but not easy to do for arbitrary join condition

As noted above, simple equijoin is a common join condition.

Thus, a range of other join algorithms has been developed specifically for equality join conditions.

## Index Nested Loop Join

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Most joins considered so far have a common problem:

- repeated scans of *entire* inner relation  $S$  are required

If there is an index on  $S$ , we can avoid such repeated scanning.

Consider  $Join[R.i=S.j](R,S)$ :

```
for each tuple r in relation R {
  use index to select tuples
    from S where s.j = r.i
  for each selected tuple s from S {
    add (r,s) to result
  }
}
```

(For ordered indexes (e.g. Btree), this also assists join conditions like  $R.i < S.j$ )

## ... Index Nested Loop Join

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This method requires:



- one scan of  $R$  relation ( $b_R$ )
  - only one buffer needed, since we use  $R$  tuple-at-a-time
- for each *tuple* in  $R$  ( $r_R$ ), one index lookup on  $S$ 
  - cost depends on type of index and number of results
  - best case is when each  $R.i$  matches few  $S$  tuples

Cost =  $b_R + r_R \cdot Sel_S$  ( $Sel_S$  is the cost of performing a select on  $S$ ).

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### ... Index Nested Loop Join

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For index lookup:

- cost of locating first matching tuple
  - for B+ trees, typically 2–4 page reads
  - for hashing, typically 1–2 page reads
- cost of finding other matching tuples
  - if clustered, typically 1–2 page reads
  - if unclustered, up to  $b_q$  page reads

Note: building an index "on the fly" to perform a join can be very cost-effective.

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### Index Nested Loop Join on Example

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Case 1:  $Join[id=stude](Student, Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has a clustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- most of the time, the four matching records are in a single page

$$\begin{aligned} \text{Cost} &= b_S + r_S \cdot btree_E \\ &= 1,000 + 20,000 \times (3+1.01) = 80,000 \end{aligned}$$


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### ... Index Nested Loop Join on Example

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Case 2:  $Join[id=stude](Student, Enrolled)$

- `Student` is outer and `Enrolled` is inner
- `Enrolled` has an unclustered B+ tree index on `stude` field
- B+ tree has depth 3 (root + internal + leaf)
- assume worst case; matching records are all on different pages

$$\begin{aligned} \text{Cost} &= b_S + r_S \cdot btree_E \\ &= 1,000 + 20,000 \times (3+4) = 150,000 \end{aligned}$$


---

### ... Index Nested Loop Join on Example

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Case 3:  $Join[id=stude](Student, Enrolled)$

- Enrolled is outer and Student is inner
- Student is hashed on id field (e.g. linear hashing)
- there may be (short) overflow chains (e.g. 1.1 page reads/bucket)

$$\begin{aligned}\text{Cost} &= b_E + r_E \text{hash}_S \\ &= 2,000 + 80,000 \times 1.1 = 90,000\end{aligned}$$

---

## Optimised Index Nested Loop Join

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Consider the following scenario for  $\text{Join}[R.i=S.j](R,S)$ :

- $R.i$  is not a primary key (so many tuples have same  $R.i$  value)
- $R$  is sorted on  $R.i$  (or could be efficiently sorted on  $R.i$ )
- each  $R.i$  value does not match very many tuples

Could save repeated index scans with the same  $R.i$  value

- cache results of index scan for  $R.i=k$  in buffer
- if next  $R$  tuple also has  $R.i=k$ , re-use scan results

---

### ... Optimised Index Nested Loop Join

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Abstract algorithm for optimised index nested loop join:

```
for each tuple r in relation R {
  if (prev == r.i)
    use selected tuples in buffer(s)
  else {
    use index to select tuples
      from S where s.j = r.i
    store selected tuples in buffer(s)
  }
  for each selected tuple s from S
    add (r,s) to result
  prev = r.i
}
```

Cost savings depend on repetition factor, #buffers, size of index scans

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## Sort-Merge Join

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### Sort-Merge Join

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Basic approach:

- sort both relations on join attribute (reminder:  $\text{Join}[R.i=S.j](R,S)$ )
- scan together using merge to form result  $(r,s)$  tuples

Advantages:

- no need to deal with "entire"  $S$  relation for each  $r$  tuple
- deal with runs of matching  $R$  and  $S$  tuples

Disadvantages:

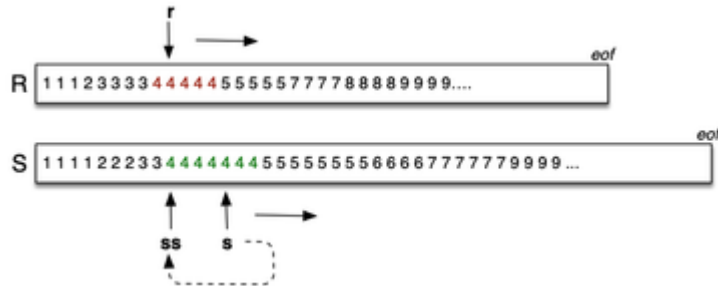
- cost of sorting both relations (relations may be sorted on join key?)
- some rescanning required when long runs of  $S$  tuples

## ... Sort-Merge Join

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Method requires several cursors to scan sorted relations:

- $r$  = current record in  $R$  relation
- $s$  = start of current run in  $S$  relation
- $ss$  = current record in current run in  $S$  relation



## ... Sort-Merge Join

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Abstract algorithm for merge phase of  $Join[R.i=S.j](R,S)$ :

```

r = first tuple in R
s = first tuple in S
while (r != eof and s != eof) {
    // align cursors to start of next common run
    while (r != eof and r.i < s.j) { r = next tuple in R }
    while (s != eof and r.i > s.j) { s = next tuple in S }
    // scan common run, generating result tuples
    while (r != eof and r.i == s.j) {
        ss = s // set to start of run
        while (ss != eof and ss.j == r.i) {
            add (r,s) to result
            ss = next tuple in S
        }
        r = next tuple in R
    }
    s = ss // start search for next run
}

```

## Sidetrack: Iterators

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Sort-merge join implementation is simplified by use of iterators.

- iterators give the appearance of tuple-at-a-time
- even when the underlying data is page-by-page
- and even in the presence of auxiliary index structures

Typical usage of iterator:

```

Iterator iter; Tuple tup;
iter = startScan("Rel", "i=5");
while ((tup = nextTuple(iter)) != NULL) {
    process(tuple);
}
endScan(iter);

```

## ... Sidetrack: Iterators

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```

typedef struct {
    File    inf; // input file
    Buffer buf; // buffer holding current page
    int     curp; // current page during scan
    int     curr; // index of current record in page
} Iterator;

// simple linear scan; no condition
Iterator *startScan(char *relName) {
    Iterator *iter = malloc(sizeof(Iterator));
    iter->inf = openFile(fileName(relName), READ);
    iter->curp = 0;
    iter->curr = -1;
    readPage(iter->inf, iter->curp, iter->buf);
}

```

---

... Sidetrack: Iterators

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```

Tuple nextTuple(Iterator *iter) {
    // check if reached end of current page
    if (iter->curr == nTuples(iter->buf)-1) {
        // check if reached end of data file
        if (iter->curp == nPages(iter->inf)-1)
            return NULL;
        iter->curp++;
        iter->buf = readPage(iter->inf, iter->curp);
        iter->curr = -1;
    }
    iter->curr++;
    return getTuple(iter->buf, iter->curr);
}
// curp and curr hold indexes of most recently read page/record

```

---

... Sidetrack: Iterators

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```

TupleID scanCurrent(Iterator *iter) {
    // form TupleID for current record
    return iter->curp + iter->curr;
}

void setScan(Iterator *iter, int page, int rec) {
    assert(page >= 0 && page < nPages(iter->inf));
    if (iter->curp != page) {
        iter->curp = page;
        readPage(iter->inf, iter->curp, iter->buf);
    }
    assert(rec >= 0 && rec < nTuples(iter->buf));
    iter->curr = rec;
}

void endScan(Iterator *iter) {
    closeFile(iter->buf);
    free(iter);
}

```

---

## Sort-Merge Join

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Concrete algorithm using iterators:

```

Iterator *ri, *si; Tuple rup, stup;

ri = startScan("SortedR");
si = startScan("SortedS");
while ((rtup = nextTuple(ri)) != NULL

```

```

    && (stup = nextTuple(si)) != NULL) {
// align cursors to start of next common run
while (rtup != NULL && rtup.i < stup.j)
    rtup = nextTuple(ri);
if (rtup == NULL) break;
while (stup != NULL && rtup.i > stup.j)
    stup = nextTuple(si);
if (stup == NULL) break;
    // must have (r.i == s.j) here
...

```

---

### ... Sort-Merge Join

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```

...
// remember start of current run in S
TupleID startRun = scanCurrent(si);
// scan common run, generating result tuples
while (rtup != NULL && rtup.i == stup.j) {
    while (stup != NULL and stup.j == rtup.i) {
        addTuple(outbuf, combine(rtup, stup));
        if (isFull(outbuf)) {
            writePage(outf, outp++, outbuf);
            clearBuf(outbuf);
        }
        stup = nextTuple(si);
    }
    rtup = nextTuple(ri);
    setScan(si, startRun);
}
}

```

---

### ... Sort-Merge Join

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Buffer requirements:

- for sort phase:
    - as many as possible (remembering that cost is  $O(\log \#Bufs)$ )
    - if insufficient buffers, sorting cost can dominate
  - for merge phase:
    - one output buffer for result
    - one input buffer for relation  $R$
    - (preferably) enough buffers for longest run in  $S$
- 

### ... Sort-Merge Join

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Cost of sort-merge join.

Step 1: sort each relation (if not already sorted):

- Cost =  $2 \cdot b_R (1 + \log_{N-1}(b_R / N)) + 2 \cdot b_S (1 + \log_{N-1}(b_S / N))$   
 (where  $N$  = number of memory buffers)

Step 2: merge sorted relations:

- if every run of values in  $S$  fits completely in buffers,  
 merge requires single scan, Cost =  $b_R + b_S$
  - if some runs in of values in  $S$  are larger than buffers,  
 need to re-scan run for each corresponding value from  $R$
- 

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## Sort-Merge Join on Example

Case 1:  $Join[id=stude](Student, Enrolled)$

- *Student* and *Enrolled* already sorted on *id#*
- memory buffers  $N=4$ ; all runs are of length  $< 2$

Cost =  $b_S + b_E = 3,000$  (i.e. minimal cost)

---

### ... Sort-Merge Join on Example

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Case 2:  $Join[id=stude](Student, Enrolled)$

- relations are not sorted on *id#*
- memory buffers  $N=32$ ; all runs are of length  $< 30$

$$\begin{aligned}\text{Cost} &= \text{sort}(S) + \text{sort}(E) + b_S + b_E \\ &= b_S \lceil \log_{30} b_S \rceil + b_E \lceil \log_{30} b_E \rceil + b_S + b_E \\ &= 1,000 \times 3 + 2,000 \times 3 + 1,000 + 2,000 \\ &= 12,000\end{aligned}$$

---

### ... Sort-Merge Join on Example

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Case 3:  $Join[id=stude](Student, Enrolled)$

- *Student* and *Enrolled* already sorted on *id#*
- memory buffers  $N=3$  (*S* input, *E* input, output)
- one-quarter of the "runs" in *E* span two pages
- there are no "runs" in *S*, since *id#* is a primary key

Cost depends on which relation is outer and which is inner.

---

### ... Sort-Merge Join on Example

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Case 3 (continued) ...

If *E* is outer relation:

- Cost =  $b_E + b_S = 3,000$

If *S* is outer relation:

- one-quarter of *E* runs require two page reads
  - each *E* run is processed once for matching *S.id* value
  - Cost =  $b_S + b_E + r_S/4 = 8,000$
- 

## Sidetrack 2: More on Iterators

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Above description of iterators:

- involved simple scan of a single table
- with no condition to select tuples

In the general case, an iterator involves:

- one (selection) or two (join) tables
- with a condition to determine relevant tuples

A typical SQL query involves many iterators

- one for each relational operator in query plan
- connected in a demand-driven network of query nodes

## ... Sidetrack 2: More on Iterators

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Requires a more general definition of execution state:

```
typedef struct {
    Oper    op;      // operation (sel,sort,join,...)
    Reln    r1;      // first relation
    Reln    r2;      // second relation (if any)
    Buffer   *bufs;   // buffers used by operation
    int     curp1;   // index of current page for r1
    int     curr1;   // index of current record in page
    int     curp2;   // index of current page for r2
    int     curr2;   // index of current record in page
    Cond    cond;    // condition for choosing tuple(s)
} Iterator;
```

For PostgreSQL details, see [include/nodes/execnodes.h](#)

# Hash Join

## Hash Join

56/87

Basic idea:

- use hashing as a technique to partition relations
- to avoid having to consider all pairs of tuples

Requires sufficient memory buffers

- to hold substantial portions of partitions
- (preferably) to hold largest partition of outer relation

Other issues:

- works only for equijoin  $R.i = S.j$  (but this is a common case)
- susceptible to data skew (or poor hash function)

Variations: *simple*, *grace*, *hybrid*.

## Simple Hash Join

57/87

Basic approach:

- hash part of the outer relation  $R$  into memory buffers (build)
- scan the inner relation  $S$ , using hash to search (probe)

- if  $R.i=S.j$ , then  $h(R.i)=h(S.j)$  (hash to same buffer)
- only need to check one memory buffer for each  $S$  tuple

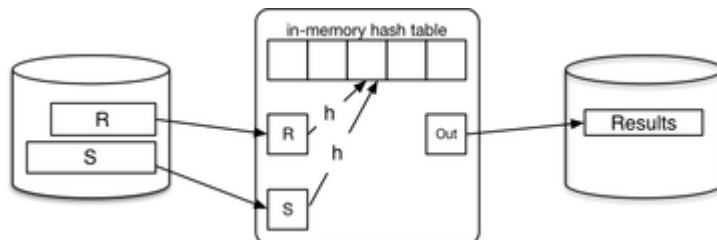
Makes the assumption: whole of  $S$  hashes into memory

- requires  $R$  to be smaller than memory buffers
- requires a uniform hash function (no overflows)

### ... Simple Hash Join

58/87

Data flow:



### ... Simple Hash Join

59/87

Algorithm for ideal simple hash join  $Join[R.i=S.j](R,S)$ :

```

for each tuple r in relation R
  { insert r into buffer[h(R.i)] }
for each tuple s in relation S {
  for each tuple r in buffer[h(S.j)] {
    if ((r,s) satisfies join condition) {
      add (r,s) to result
    }
  }
}

```

Cost =  $b_R + b_S$  (minimum possible cost)

### ... Simple Hash Join

60/87

Consider that we have  $N$  buffers available (2 input, 1 output,  $N-3$  hash)

If  $b_R \leq N-3$  buffers, no need to hash (use nested loop).

In practice, size of hash table  $b_{hR} > b_R$  (e.g. data skew)

$\Rightarrow$  hash table for  $R$  is even less likely to fit in memory

Can be handled by a variation on above algorithm:

- scan  $R$ , making hash table with  $N-3$  buffers
- once hash table built, scan  $S$  (standard probe phase)
- if more  $R$  tuples, build new table and repeat

### ... Simple Hash Join

61/87

Algorithm for realistic simple hash join  $Join[R.i=S.j](R,S)$ :

```

for each tuple r in relation R {
  if (buffer[h(R.i)] is full) {
    for each tuple s in relation S {
      for each tuple rr in buffer[h(S.j)] {

```



```

        if ((rr,s) satisfies join condition) {
            add (rr,s) to result
        }
    }
}
clear all hash table buffers
}
insert r into buffer[h(R.i)]
}

```

Note: requires multiple passes over the  $S$  relation.

---

### ... Simple Hash Join

62/87

Cost depends on  $N$  and on properties of data/hash.

Worst case:

- $h(i)=k$  so read only  $C_R$  tuples before hash table "full"
- each hash table for  $R$  occupies one buffer with  $C_R$  tuples
- degenerates to nested-loop-with-3-buffers case  $\Rightarrow b_R + b_R b_S$

Best case:

- perfect uniform distribution of hash values
- each hash table of  $R$  holds  $(N-3)C_R$  tuples from  $N-3$  pages
- number of hash tables built =  $n_{hR} = \lceil b_R / (N-3) \rceil$
- read all of  $S$  for each hash table  $\Rightarrow b_R + n_{hR} \cdot b_S$

---

## Grace Hash Join

63/87

Basic approach:

- partition both relations on join attribute using hashing
- scan through corresponding pairs of partitions to form results

Similar approach to sort-merge join, except:

- sort-merge: partitioning achieved by sorting (runs)
- hash: partitioning achieved by hashing

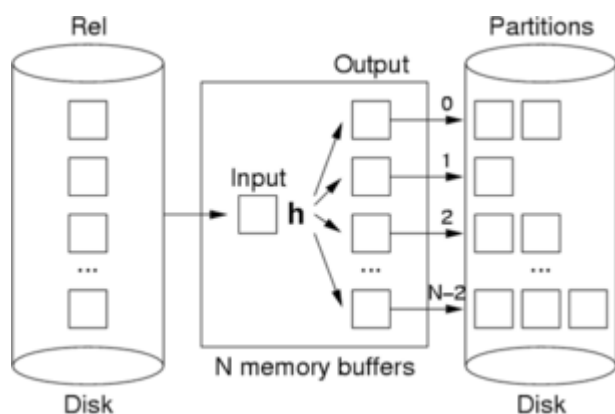
Requires enough buffer space to hold largest partition of inner relation.

---

### ... Grace Hash Join

64/87

Partition phase:



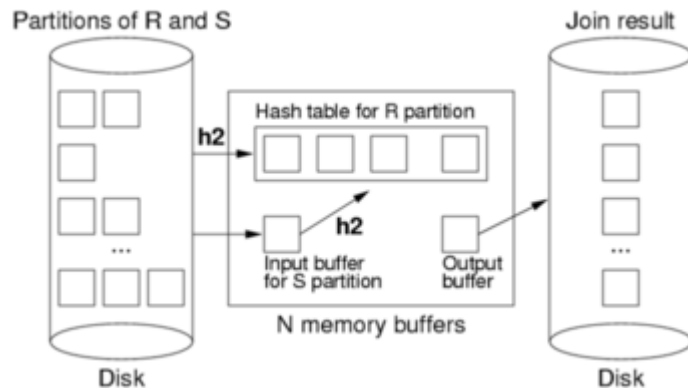
This is applied to each relation  $R$  and  $S$ .

---

### ... Grace Hash Join

65/87

Probe/join phase:



The second hash function ( $h2$ ) simply speeds up the matching process. Without it, would need to scan entire  $R$  partition for each record in  $S$  partition.

---

### ... Grace Hash Join

66/87

Abstract algorithm for  $Join[R.i=S.j](R,S)$ :

```
// assume h(val) generates [0..N-2]
// assume h2(val) generates [0..N-3]

// Partition phase (each relation -> N-1 partitions)
// 1 input buffer, N-1 output buffers

for each tuple r in relation R
  add r to partition h(r.i) in output file R'
for each tuple s in relation S
  add s to partition h(s.j) in output file S'
...
```

---

### ... Grace Hash Join

67/87

Abstract algorithm for  $Join[R.i=S.j](R,S)$  (cont.)

```
// Probe/join phase
// 1 input buffer for S, 1 output buffer
// N-2 buffers to build hash table for R partition

for each partition p = 0 .. N-2 {
  // Build in-memory hash table for partition p of R'
  for each tuple r in partition p of R'
    insert r into buffer h2(r.i)

  // Scan partition p of S', probing for matching tuples
  for each tuple s in partition p of S' {
    b = h2(s.j)
    for all matching tuples r in buffer b
      add (r,s) to result
  }
}
```

---

### ... Grace Hash Join

68/87

Concrete algorithm for partitioning:

```

Buffer iBuf, oBuf[N-1];
File inf, outf[N-1]; char rel[100];
int i, r, h, ip, op[N-1]; Tuple tup;
for (i = 0; i < N-1; i++) {
    clearBuf(oBuf[i]); op[i] = 0;
    rel = sprintf("%s%d", "Rel", i);
    outf[i] = openFile(fileName(rel), WRITE));
}
inf = openFile(fileName("Rel"), READ);
for (ip = 0; ip < nPages(inf); ip++) {
    iBuf = readPage(inf, ip);
    for (r = 0; r < nTuples(iBuf); r++) {
        tup = getTuple(iBuf, r);
        h = hash(tup.i, N-1);
        addTuple(oBuf[h], tup);
        if (isFull(oBuf[h])) {
            writePage(outf[h], op[h]++, oBuf[h]);
            clearBuf(oBuf[h]);
        }
    }
}

```

---

### ... Grace Hash Join

69/87

Cost of grace hash join:

- #pages in all partition files of  $Rel \approx b_{Rel}$  (maybe slightly more)
- partition relation  $R$  ... Cost =  $b_R.T_r + b_R.T_w = 2b_R$
- partition relation  $S$  ... Cost =  $b_S.T_r + b_S.T_w = 2b_S$
- probe/join requires one scan of each (partitioned) relation  
Cost =  $b_R + b_S$
- all hashing and comparison occurs in memory  $\Rightarrow \approx 0$  cost

Total Cost =  $3(b_R + b_S)$

---

### ... Grace Hash Join

70/87

The above cost analysis assumes:

- every partition of  $R$  fits in memory buffers at once

We achieve this situation if:

- data has uniform distribution
  - hash function gives uniform distribution  
(all partitions are similar size)
  - we have  $N-1 \geq \lceil \sqrt{b} \rceil$  memory buffers  
(giving  $N-1$  partitions, each with  $\approx b_R/(N-1)$  pages)
- 

### ... Grace Hash Join

71/87

Possibilities for dealing with "over-long" partitions of  $R$

- handle each over-long partition via scanning
  - requires over-long partitions to be scanned multiple times
  - essentially, such partitions are treated via nested loop join
- apply hash join recursively to over-long partitions
  - increases i/o by needing to partition parts of file multiple times
- use a different hash function with better distribution properties
  - but difficult to find such hash functions "on the fly"

- use the relation with the best partitioning as the "outer" relation

## Grace Hash Join on Example

72/87

For the example  $Join[id=stude](Student, Enrolled)$ :

- assume that we have a good hash function and  $N = \sqrt{1000} = 32$

$$\begin{aligned} \text{Cost} &= 3 (b_S + b_E) \\ &= 3 (1,000 + 2,000) = 9,000 \end{aligned}$$

## Hybrid Hash Join

73/87

An optimisation if we have  $\sqrt{b_R} < N < b_R + 2$

- create  $k$  partitions using  $N$  buffers where  $k \ll N$
- with grace join, would use  $k$  output buffers (one per partition)
- what to do with  $N - k$  remaining buffers? (ignore input buffer)
- use them to hold  $m$  partitions of  $R$  in memory (no disk writes)
- other partitions are handled as before (using  $k - m$  output buffers)

When we come to scan and partition  $S$  relation

- any tuple with hash in range  $0..m-1$  can be resolved
- other tuples are written to one of  $k - m$  partition files for  $S$

Final phase is same as grace join, but with only  $k - m$  partitions.

### ... Hybrid Hash Join

74/87

Some observations:

- for  $k$  partitions, each partition has expected size  $\text{ceil}(b_R/k)$
- holding  $m$  partitions in memory needs  $m \times \text{ceil}(b_R/k)$  buffers
- since we have  $k - m$  output buffers, we must have  $mb_R/k + (k - m) \leq N$
- for every partition/block held in memory, we save on disk i/o
- saving is  $m/k \times 2(b_R + b_S)$

Other notes:

- if  $N = b_R + 2$ , using block nested loop join is simpler
- cost depends on  $N$  (but less than grace hash join)

### ... Hybrid Hash Join

75/87

Need to choose appropriate  $m$  and  $k$  to minimise cost

- base cost:  $3 \times (b_R + b_S)$  (grace join)
- i/o saving:  $m/k \times 2(b_R + b_S)$
- constraint:  $mb_R/k + (k - m) \leq N$

Approach to maximise saving:

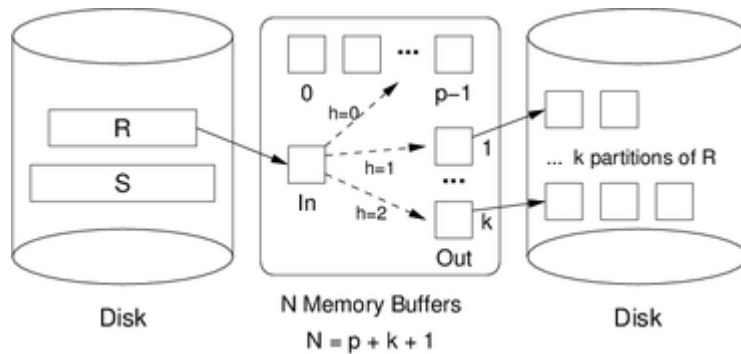
- have one large in-memory partition ( $m = 1$ )

- use as many as possible of  $N$  buffers for partition
- use as few output buffers as possible (minimise  $k$ )

### ... Hybrid Hash Join

76/87

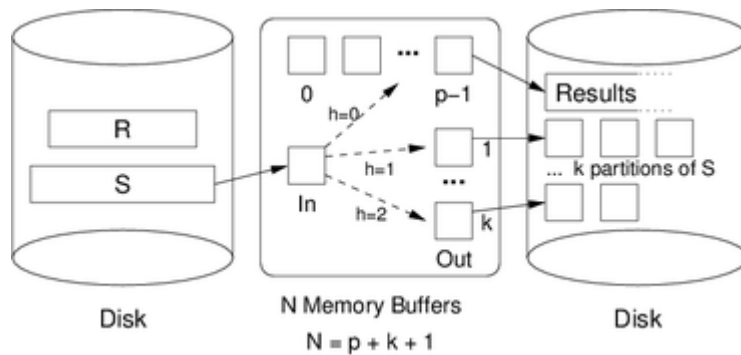
Data flow for hybrid hash join (partitioning  $R$ ):



### ... Hybrid Hash Join

77/87

Data flow for hybrid hash join (partitioning  $S$ ):



After this, proceed as for grace hash join.

### ... Hybrid Hash Join

78/87

Cost of hybrid hash join:

- assume: large  $N$  total buffers,  $m$  partitions in memory,  $k$  partitions on disk
- read both tables:  $b_R + b_S$
- total partitions for each table:  $m+k$
- assuming uniform hashing, #pages in each  $R$  partition  $P_R = \text{ceil}(b_R/(m+k))$
- assuming uniform hashing, #pages in each  $S$  partition  $P_S = \text{ceil}(b_S/(m+k))$
- in Pass 1,  $k \cdot P_R + k \cdot P_S$  pages written to disk partitions
- all joining of  $m$  in-memory partitions is handled in memory
- in Pass2,  $k \cdot P_R + k \cdot P_S$  pages read back from disk partitions

$$\begin{aligned}
 \text{Cost} &= b_R + b_S + k \cdot P_R + k \cdot P_S + k \cdot P_R + k \cdot P_S \\
 &= b_R + b_S + 2 \cdot k \cdot (P_R + P_S) \\
 &= b_R + b_S + 2 \cdot k \cdot (\text{ceil}(b_R/(m+k)) + \text{ceil}(b_S/(m+k)))
 \end{aligned}$$

How to determine  $k$ :

- set  $m=1$  and so size of partition  $\approx N \Rightarrow k \approx b_R/N$
- need to ensure that  $\lceil b_R/k \rceil + k \leq N$  (allowing for input buffer)

- choose  $k$  close to  $b_R/N$  but satisfying constraint

---

## Hybrid Hash Join on Example

79/87

Case 1:  $N = 100$  buffers,  $b_R = 1000$

- $k = 10 \Rightarrow 1000/10 + 10 = 110$  buffers; not less than 100
- $k = 12 \Rightarrow 1000/12 + 12 = 96$  buffers
- $\text{Cost} = (3 - 2/12) \cdot (1000 + 2000) = 8500$

Case 2:  $N = 200$  buffers,  $b_R = 1000$

- $k = 5 \Rightarrow 1000/5 + 5 = 205$  buffers; not less than 200
- $k = 6 \Rightarrow 1000/6 + 6 = 173$  buffers
- $\text{Cost} = (3 - 2/6) \cdot (1000 + 2000) = 8000$

Case 3:  $N = 502$  buffers,  $b_R = 1000$

- $k = 2 \Rightarrow 1000/2 + 2 = 502$  buffers
- $\text{Cost} = (3 - 2/2) \cdot (1000 + 2000) = 6000$

---

## Pointer-based Join

80/87

Conventional join algorithms set up  $R \leftrightarrow S$  connections via attribute values.

Join could be performed faster if direct connections already existed.

- in OODBMSs, they generally already exist in the form of object references (oids)
- in RDBMSs, they could be introduced via extra rid attributes

Such a modification to conventional RDBMS structure would be worthwhile:

- if we know in advance what kind of joins will be required
- adding the extra rid attributes into tuples is feasible

---

### ... Pointer-based Join

81/87

The basic idea for pointer-based join is:

```
for each tuple  $r$  in relation  $R$  {
  for each rid associated with  $r$  {
    fetch tuple  $s$  from  $S$  via rid
    add  $(r,s)$  to result relation
  }
}
```

Often, each  $R$  tuple is associated with only one rid, so the inner loop is not needed.

---

### ... Pointer-based Join

82/87

The advantage over value-based joins:

- rather than find  $S$  tuples via value-based lookup (e.g. hashing, index)
- we find  $S$  tuples by direct fetch with rid (much faster per tuple)
- requires no assumption about sorted-ness of relations
- does not require large numbers of buffers

The (potential) disadvantages:

- every *fetch* goes to a different page of *S*  
(this essentially returns us to the worst-case scenario for nested-loop join)
- the join only works in "one direction" (from *R* to *S*)
- requires additional data for each different join type
- requires tuples to be larger  $\Rightarrow b_R$  is larger

---

## General Join Conditions

83/87

Above examples all used simple equijoin e.g.  $\text{Join}[i=j](R,S)$ .

For theta-join e.g.  $\text{Join}[i<j](R,S)$ :

- index nested loop join: need B+ tree index on inner relation
- sort-merge join can be adapted, but is not very effective
- hash join is inapplicable
- other methods are essentially unchanged

---

## ... General Join Conditions

84/87

For multi-equality (pmr) join e.g.  $\text{Join}[i=j \wedge k=l](R,S)$

- index nested loop join:
  - build index on all join fields of inner relation
  - e.g. if *S* is inner, build index on (*S.j,S.l*)
- sort-merge join:
  - sort both relations on combined join fields
  - e.g. sort *R* on (*R.i,R.k*), sort *S* on (*S.j,S.l*)
- hash-join:
  - use multi-attribute hashing on combined join fields
- other methods are essentially unchanged

---

## Join Summary

85/87

No single join algorithm is superior in some overall sense.

Which algorithm is best for a given query depends on:

- sizes of relations being joined, size of buffer pool
- any indexing on relations, whether relations are sorted
- which attributes and operations are used in the query
- number of tuples in *S* matching each tuple in *R*

Choosing the "best" join algorithm is critical because the cost difference between best and worst case can be very large.

E.g.  $\text{Join}[id=stude](\text{Student}, \text{Enrolled})$ : 3,000 ... 2,000,000

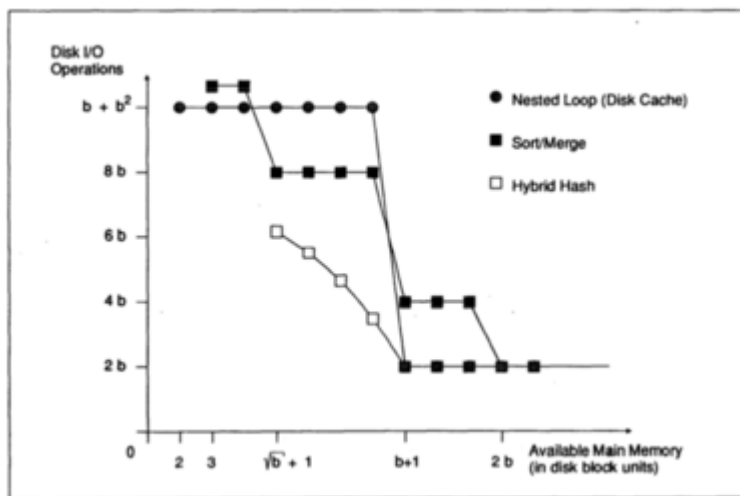
In some cases, it may be worth modifying access methods "on the fly" (e.g. add index) to enable an efficient join algorithm.

---

## ... Join Summary

86/87

Comparison of join costs (from Zeller/Gray VLDB90, assumes  $b_R = b_S = b$ )



## Join in PostgreSQL

87/87

Join implementations are under: [src/backend/executor](#)

PostgreSQL supports three kinds of join:

- nested loop join ([nodeNestloop.c](#))
- sort-merge join ([nodeMergejoin.c](#))
- hash join ([nodeHashjoin.c](#)) (hybrid hash join)

Query optimiser chooses appropriate join, by considering

- physical characteristics of tables being joined
- estimated selectivity (likely number of result tuples)

Produced: 30 Apr 2020