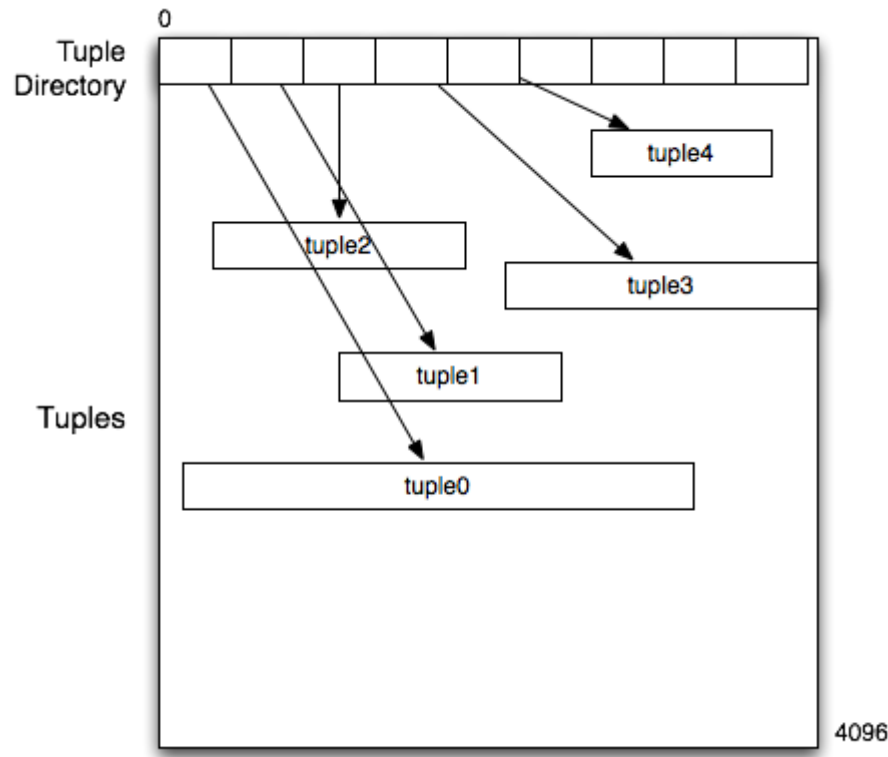# Exercises 03

Pages and Tuples

1. Assume that a data file is composed of 4KB pages, where each page is structured as follows:

The start of the page contains a tuple directory which is a sequence of three-byte values, where the first 12 bits contain the offset into the page of the tuple and the second 12 bits contain the tuple length.

Write a C function that takes three parameters: an open file descriptor, a page number and a record number and reads the data for the corresponding record. Do *not* read the whole page; read just enough of the data to solve the problem. Dynamically allocate a memory buffer large enough to hold the tuple data when read in. The function should return a pointer to the start of the tuple memory buffer.

The function should do appropriate error-checking and return `NULL` in the case that any operation cannot be completed. Use the following function template:

```
char *getTuple(int inFile, int pageNumber, int recNumber) { ... }
```

Hint: when a directory value is read into memory, the high-order bits contain the offset and the low-order bits contain the length.

Use only the low-level i/o operators (system calls) such as `open()`, `read()`, `write()`, `lseek()`, etc.

Answer:

We assume that all of the relevant `.h` files have been included.

```c
#define PAGE_SIZE 4096

char *getTuple(int inFile, int pageNumber, int recNumber)
{
        // position file at start of page

        off_t pageAddr = pageNumber * PAGE_SIZE;
        if (lseek(inFile, pageAddr, SEEK_SET) < 0)
                return NULL;

        // re-position the file to the start of the tuple directory entry

        off_t dirOffset = recNumber * 3; // 3 bytes per directory entry
        if (lseek(inFile, dirOffset, SEEK_CUR) < 0)
                return NULL;

        // read 3-byte directory entry for this tuple

        unsigned int dirEntry;
        if (read(inFile, &dirEntry, 3) != 3)
                return NULL;

        // extract tuple offset and length from directory entry

        unsigned int tupOffset, tupLength;
        unsigned int lengthMask = 0x00000fff; // low-order 12 bits
        unsigned int offsetMask = 0x00fff000; // high-order 12 bits

        tupOffset = (dirEntry & offsetMask) >> 12;
        tupLength = dirEntry & lengthMask;

        // allocate memory buffer to hold tuple data

        char *tupBuf;
        if ((tupBuf = malloc(tupLength)) == NULL)
                return NULL;
```

```
        // position file at tuple location

        off_t tupAddr = pageAddr + tupOffset;
        if (lseek(inFile, tupAddr, SEEK_SET) < 0)
                return NULL;

        // read tuple data into buffer

        if (read(inFile, tupBuf, tupLength) != tupLength)
                return NULL;

        return tupBuf;
}
```

2. Consider a data file containing tuples with a page structure similar to that in the previous question. Pages are 4KB in size, and each page contains a tuple directory with 100 entries in it, where each entry is 3-bytes long. Assuming that the (minimum,average,maximum) tuple lengths are (32,64,256) bytes and that the file has 100 pages, determine the following:

   a. The minimum number of tuples that the file can hold

   b. The maximum number of tuples that the file can hold

   Answer:

   a. The minimum number of tuples is zero (trick question)

   b. Maximum occurs when all tuples are minimum size (or close to it). Each page uses 300 bytes for the tuple directory, leaving 4096-300=3796 bytes of space for tuples. In theory, this amount of space could hold floor(3796/32) = 118 tuples; however, the page directory only has space for 100 tuples, so 100 tuples is the maximum number of tuples per page. Since we have 100 pages, the file can hold 100*100=10000 tuples.

3. Consider a variation on the above scenario. Rather than pages having a fixed size tuple directory, the tuple directory can grow and shrink depending on the number of tuples in the page. For this to work, the tuple directory starts at the bottom of the page (address 0) and grows up, while tuples are added from the top of the page (address 4095) and grow down. If all other factors are the same (total 100 pages, (min,avg,max) tuple lengths (32,64,128)), what is the maximum number of tuples that the file can hold? You may assume that tuples can begin at any address (i.e. they do not have to start at a 4-byte address).

   Answer:

The maximum number of tuples still occurs when all tuples are minimum size. However, in this case we need to balance the tuple space against the directory space. For example, if we have 100 tuples, then the top 3200 bytes of the page are occupied by tuple data, leaving 896 (4096-3200) bytes for the slot directory. We can clearly add more tuples, since we have space for them and space to hold their directory entries. Eventually, though, there will be enough tuples that there is no more room to add directory entries for them and the page is full. Since each tuple requires space for its data (32 bytes) plus 3 bytes for a directory entry, we can compute the maximum tuples that will fit in a page by finding the maximum N such that $(3*N + 32*N) < 4096$. In this case, N=117 and so this the file can hold at most 11700 tuples.

This scenario is not totally implausible since some common tables have fixed-size tuples (consider a "link" table with just two foreign keys). Of course, in such a case, we wouldn't need the tuple directory either, since we could simply compute the tuple offset based on its number in the page.

4. [Based on Garcia-Molina/Ullman/Widom 13.7.1]
Consider the following relational table:

```
create table Patient (
    id        integer primary key,
    ssn       char(20),
    name      varchar(30),
    addr      varchar(60),
    history   varchar(200),
    born      date
);
```

Consider how records of this type might be represented when stored in disk blocks. If a pointer within a record requires 4 bytes and the record length is also stored as a 4-byte quantity, how many bytes would be needed for each `Patient` record, excluding of the space required to store the variable-length fields?

For variable-length records (`varchar`), assume that we don't need a terminating char (e.g. `'\0'`) but we do need to allocate a multiple of 4 bytes to the field to ensure alignment of possible following integer fields. (These additional bytes would not be necessary if the fixed-length fields were all stored at the front of the record.)

How many bytes would be required for the following instances of `Patient` records:

```
insert into Patient values (
    12345678, '222-444-555-7777',
    'John Smith', (10)
    '50 Smith  St, Smithton, 2345', (28)
    'Previously diagnosed with URTI, also weak ankles', (48)
    '1966-12-2'
);
insert into Patient values (
    87654321, '123-456-654-4321',
```
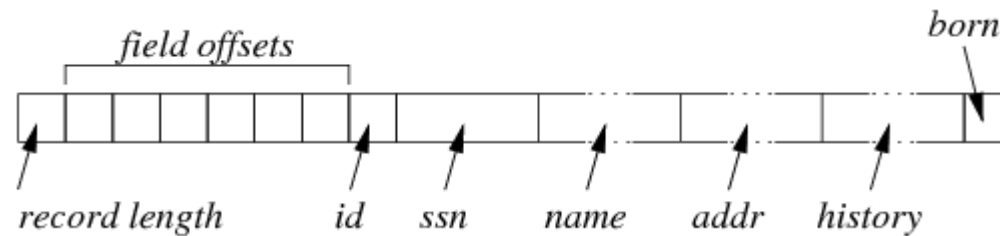
```
        'Jane Brown', (10)
        '15 Brown  St, Brownsville, 2427', (31)
        'Prior history of urinary tract infections', (41)
        '1966-12-2'
);
```

(Note that the (string lengths) given after each string are not part of the `insert` statement).

Answer:

Assume that the record has the following structure:

The fixed storage cost includes:

- the record length (4 bytes)
- offsets for each of the fields (4 bytes times 6)
- fixed-length fields `id` (4 bytes), `ssn` (20 bytes), `born` (4 bytes for `date`)

This gives a total fixed storage cost of 4+24+4+20+4 = 56 bytes

For the John Smith record, add additional bytes for

- `name` (12 bytes ... 10 bytes rounded up for alignment)
- `addr` (28 bytes ... no rounding up needed)
- `name` (48 bytes ... no rounding up needed)

giving a total of 56+12+28+48 = 144 bytes

For the Jane Brown record, add additional bytes for

- `name` (12 bytes ... 10 bytes, rounded up for alignment)
- `addr` (32 bytes ... 31 bytes, rounded up)
- `name` (44 bytes ... 41 bytes, rounded up)

giving a total of 56+12+32+44 = 144 bytes

It is a coincidence that both records come out with the same length.

5. PostgreSQL tuples have an array of flags to indicate where NULL values occur in tuples. Two of the critical functions for manipulating tuples in PostgreSQL (`heap_form_tuple()` and `heap_modify_tuple()`) accept a parameter which is an an array of flags for NULLs, as well as accepting an array of `Datum` values. This array contains only the non-NULL attribute values.

For example, a tuple like *R(42,null,'abc',-5)* would be represented by the two arrays: flags=(0,1,0,0) and values=(42,'abc',-5).

Why doesn't PostgreSQL simply include the NULL values in the array of `Datum` values?

Answer:

Every possible collection of bits/bytes represents a valid `Datum` value (e.g. you can't simply user zero to represent NULL, because zero is a perfectly useful integer value). Since there is no way to represent NULL as a `Datum`, we clearly can't include NULL values in the `Datum` array. This means that we need a separate representation for NULLs; it makes sense to simply use a bit-string, with one bit for each attribute, where a value of 1 means "this attribute is NULL", and a value of 0 means "this attribute has a value; look for it in the `Datum` array".

6. Consider a `Students` relation defined as:

```
CREATE TABLE Students (
    id#     integer,      name  varchar(30),
    gender varchar(1),    birth date,
    degree varchar(10),   score real
);
```

Assume that:

- There are 20,000 student records
- The relation is stored in one file
- The file is composed of 1024-byte blocks
- Average block-fetch time (T_r) is 10ms
- A `date` is represented by an 8-byte `long` value
- All numeric values must be stored on 4-byte address boundaries
- Average length of `name` value is 15 chars
- Average length of `degree` value is 5 chars

Consider two possible storage structures:

   i. fixed-length records with a presence bit-vector
  ii. variable-length records with a fixed-size directory containing one-byte offset values

For each of these structures:

   a. Show the internal record structure and compute the (average) size of a record

b. Compute how many blocks are needed to store the whole relation

c. Compute how long it takes to answer a query on `id#` if the file is sorted on this field (worst case value)

Answer:

i. fixed-length records with a presence bit-vector ...

a. Show the internal record structure and compute the (average) size of a record ...

Each record has something like the following structure, where fields are arranged to ensure that no numeric field begins on a non-aligned address boundary.

Offset		Field
0		id#
4		birth
12		score
20	21	name
	sex	
51		degree
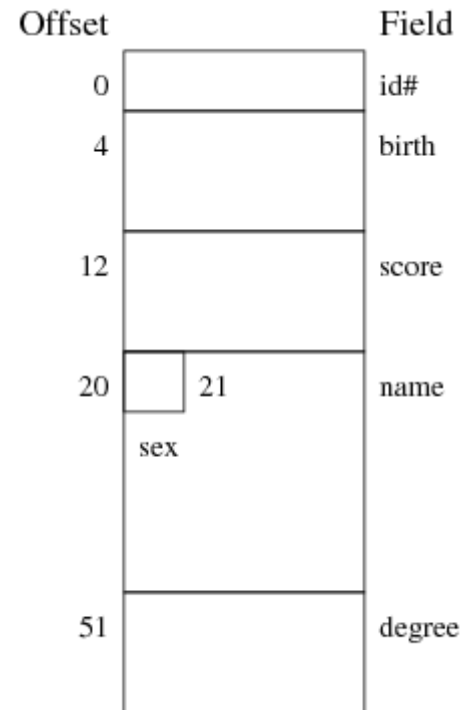
While character fields don't need to be aligned on 4-byte addresses, they do need to be as large as the maximum number of characters that might be stored in them (e.g. `varchar(10)` always occupies 10-bytes, regardless of the actual size of the string).

The size of each record is thus:

- 4 bytes for the `id#` field
- 8 bytes for the `birth` field
- 8 bytes for the `score` field
- 1 bytes for the `gender` field

- 30 bytes for the `name` field
- 10 bytes for the `degree` field

giving a total of 4 + 8 + 8 + 1 + 30 + 10 = 61 bytes.

This will need to be padded to 64 bytes to ensure that the next record in the page also begins on a 4-byte address.

Solution: *R = 64*

b. Compute how many blocks are needed to store the whole relation

If each record is 52 bytes long, and there are 1024 bytes in a block, then we could potentially store N_r = *floor(1024/64) = 16* records in a block. However, we also need to store the presence vector to indicate which record slots are actually filled. This requires at last N_r bits, thus reducing the effective number of records per block to 15. The block contains 15*64-byte records along with a 15-bit (= 2-byte) presence vector. This "wastes" 62 bytes in each block, which is unfortunate but unavoidable. Thus, N_r = *15*

If there are 15 records in each block, then we need *b = ceil(20,000/15) = 1334* blocks to store all of the records.

Solution: *b = 1334*

c. Compute how long it takes to answer a query on `id#` if the file is sorted on this field (worst case value)

Performing a binary search requires us to examine at most *ceil($log_2 b$) = ceil($log_2 1334$) = 11* blocks. Since the cost of reading each block is T_r=*10ms*, then the total i/o cost is *110 ms*

Solution: $T_{BinarySearch}$ = *110ms*

ii. Variable-length records with a fixed-size directory ...

a. Show the internal record structure and compute the (average) size of a record

Each record has something like the following structure, where fields are arranged to ensure that no numeric field begins on a non-aligned address boundary.
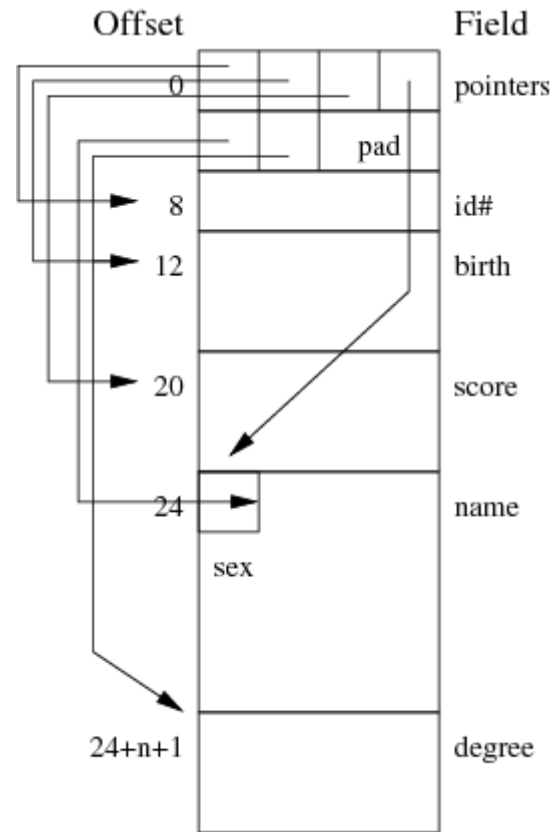
In this case, one byte of storage is required for each field to hold the offset of the field. Since there are 6 fields, this will require 6 bytes, which then needs to be padded to 8 bytes to ensure that the first numeric field starts on a 4-byte address boundary.

The offset block will be followed by four fixed-size fields:

- 4 bytes for the `id#` field
- 8 bytes for the `birth` field
- 8 bytes for the `score` field
- 1 bytes for the `gender` field

These will be followed by the variable-length fields:

- `name`, with an average of 15 characters (15 bytes)
- `degree`, with an average of 5 characters (5 bytes)

On average, this gives a total record size of 8 + 4 + 8 + 8 + 1 + 15 + 5 = 49 bytes. This will need to be padded to a multiple of 4 bytes, and so we would expect an effective average record size of 52 bytes.

Solution: *R = 52*

b. Compute how many blocks are needed to store the whole relation

If each record is 64 bytes long, and there are 1024 bytes in a block, then we could potentially store N_r = *floor(1024/52) = 19* records in a block. However, we also need to store a directory to indicate where each record is located. This requires at least N_r bytes. If the block contains 19*52-byte records, then the amount of space available for dictionary is 1024-19*52 = 36 bytes, so there is room for both the dictionary and all N_r records. presence vector. This "wastes" 17 bytes in each block (on average), which is unfortunate but unavoidable. Thus, N_r = *19*

If there are 19 records in each block, then we need *b = ceil(20,000/19) = 1053* blocks to store all of the records.

Solution: *b = 1053*

c. Compute how long it takes to answer a query on `id#` if the file is sorted on this field (worst case value)

Performing a binary search requires us to examine at most *ceil(log_2b) = ceil($log_2$1053) = 11* blocks. Since the cost of reading each block is T_r=*10ms*, then the total i/o cost is *110 ms*

Solution: $T_{BinarySearch}$ *= 110ms*