

# Week 08 Lectures

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## Implementing Join

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### Join

2/91

DBMSs are engines to *store*, *combine* and *filter* information.

*Join* ( $\Join$ ) is the primary means of *combining* information.

*Join* is important and potentially expensive

Most common join condition: equijoin, e.g.  $(R.pk = S.fk)$

Join varieties (natural, inner, outer, semi, anti) all behave similarly.

We consider three strategies for implementing join

- *nested loop* ... simple, widely applicable, inefficient without buffering
- *sort-merge* ... works best if tables are sorted on join attributes
- *hash-based* ... requires good hash function and sufficient buffering

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### Join Example

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Consider a university database with the schema:

```
create table Student(  
    id      integer primary key,  
    name    text, ...  
);  
create table Enrolled(  
    stude   integer references Student(id),  
    subj    text references Subject(code), ...  
);  
create table Subject(  
    code    text primary key,  
    title   text, ...  
);
```

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### ... Join Example

4/91

List names of students in all subjects, arranged by subject.

SQL query to provide this information:

```
select E.subj, S.name  
from   Student S, Enrolled E  
where  S.id = E.stude  
order  by E.subj, S.name;
```

And its relational algebra equivalent:

$$\text{Sort}[\text{subj}] ( \text{Project}[\text{subj}, \text{name}] ( \text{Join}[\text{id}=\text{stude}](\text{Student}, \text{Enrolled}) ) )$$

To simplify formulae, we denote *Student* by *S* and *Enrolled* by *E*

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Some database statistics:

Sym	Meaning	Value
$r_S$	# student records	20,000
$r_E$	# enrollment records	80,000
$c_S$	Student records/page	20
$c_E$	Enrolled records/page	40
$b_S$	# data pages in Student	1,000
$b_E$	# data pages in Enrolled	2,000

Also, in cost analyses below,  $N$  = number of memory buffers.

Out = *Student* ⋈ *Enrolled* relation statistics:

Sym	Meaning	Value
$r_{Out}$	# tuples in result	80,000
$c_{Out}$	result records/page	80
$b_{Out}$	# data pages in result	1,000

Notes:

- $r_{Out}$  ... one result tuple for each Enrolled tuple
- $c_{Out}$  ... result tuples have only subj and name
- in analyses, ignore cost of writing result ... same in all methods

## Nested Loop Join

Basic strategy ( $R.a \bowtie S.b$ ):

```

Result = {}
for each page i in R {
  pageR = getPage(R,i)
  for each page j in S {
    pageS = getPage(S,j)
    for each pair of tuples  $t_R, t_S$ 
      from pageR, pageS {
        if ( $t_R.a == t_S.b$ )
          Result = Result  $\cup$  ( $t_R:t_S$ )
      }
    }
  }

```

Needs input buffers for R and S, output buffer for "joined" tuples

Terminology: R is outer relation, S is inner relation

Cost =  $b_R \cdot b_S$  ... ouch!

Method (for  $N$  memory buffers):

- read  $N-2$ -page chunk of  $R$  into memory buffers
- for each  $S$  page  
check join condition on all  $(t_R, t_S)$  pairs in buffers
- repeat for all  $N-2$ -page chunks of  $R$

### ... Block Nested Loop Join

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Best-case scenario:  $b_R \leq N-2$

- read  $b_R$  pages of relation  $R$  into buffers
- while whole  $R$  is buffered, read  $b_S$  pages of  $S$

$$\text{Cost} = b_R + b_S$$

Typical-case scenario:  $b_R > N-2$

- read  $\text{ceil}(b_R/(N-2))$  chunks of pages from  $R$
- for each chunk, read  $b_S$  pages of  $S$

$$\text{Cost} = b_R + b_S \cdot \text{ceil}(b_R/(N-2))$$

Note: always requires  $r_R, r_S$  checks of the join condition

## Exercise 1: Nested Loop Join Cost

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Compute the cost (# pages fetched) of  $(S \bowtie E)$

Sym	Meaning	Value
$r_S$	# student records	20,000
$r_E$	# enrollment records	80,000
$c_S$	Student records/page	20
$c_E$	Enrolled records/page	40
$b_S$	# data pages in Student	1,000
$b_E$	# data pages in Enrolled	2,000

for  $N = 22, 202, 2002$  and different inner/outer combinations

If the query in the above example was:

```
select j.code, j.title, s.name
from   Student s
       join Enrolled e on (s.id=e.student)
       join Subject j on (e.subj=j.code)
```

how would this change the previous analysis?

What join combinations are there?

Assume 2000 subjects, with  $c_J = 10$

How large would the intermediate tuples be? What assumptions?

Compute the cost (# pages fetched, # pages written) for  $N = 202$

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### ... Block Nested Loop Join

12/91

Why block nested loop join is actually useful in practice ...

Many queries have the form

```
select * from R,S where r.i=s.j and r.x=K
```

This would typically be evaluated as

```
Tmp = Sel[x=K](R)
Res = Join[i=j](Tmp, S)
```

If Tmp is small  $\Rightarrow$  may fit in memory (in small #buffers)

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### Index Nested Loop Join

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A problem with nested-loop join:

- needs repeated scans of *entire* inner relation  $S$

If there is an index on  $S$ , we can avoid such repeated scanning.

Consider  $Join[i=j](R,S)$ :

```
for each tuple r in relation R {
  use index to select tuples
  from S where s.j = r.i
  for each selected tuple s from S {
    add (r,s) to result
  }
}
```

---

### ... Index Nested Loop Join

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This method requires:

- one scan of  $R$  relation ( $b_R$ )
  - only one buffer needed, since we use  $R$  tuple-at-a-time
- for each *tuple* in  $R$  ( $r_R$ ), one index lookup on  $S$ 
  - cost depends on type of index and number of results
  - best case is when each  $R.i$  matches few  $S$  tuples

Cost =  $b_R + r_R \cdot Sel_S$  ( $Sel_S$  is the cost of performing a select on  $S$ ).

Typical  $Sel_S = 1-2$  (hashing) ..  $b_q$  (unclustered index)

Trade-off:  $r_R \cdot Sel_S$  vs  $b_R \cdot b_S$ , where  $b_R \ll r_R$  and  $Sel_S \ll b_S$

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## Exercise 2: Index Nested Loop Join Cost

15/91

Consider executing  $Join_{[i=j]}(S,T)$  with the following parameters:

- $r_S = 1000$ ,  $b_S = 50$ ,  $r_T = 3000$ ,  $b_T = 600$
- $S.i$  is primary key, and  $T$  has index on  $T.j$
- $T$  is sorted on  $T.j$ , each  $S$  tuple joins with 2  $T$  tuples
- DBMS has  $N = 12$  buffers available for the join

Calculate the costs for evaluating the above join

- using block nested loop join
- using index nested loop join

$Cost_r = \# \text{ pages read}$  and  $Cost_j = \# \text{ join-condition checks}$

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## Sort-Merge Join

16/91

Basic approach:

- sort both relations on join attribute (reminder:  $Join_{[i=j]}(R,S)$ )
- scan together using *merge* to form result  $(r,s)$  tuples

Advantages:

- no need to deal with "entire"  $S$  relation for each  $r$  tuple
- deal with runs of matching  $R$  and  $S$  tuples

Disadvantages:

- cost of sorting both relations (already sorted on join key?)
- some rescanning required when long runs of  $S$  tuples

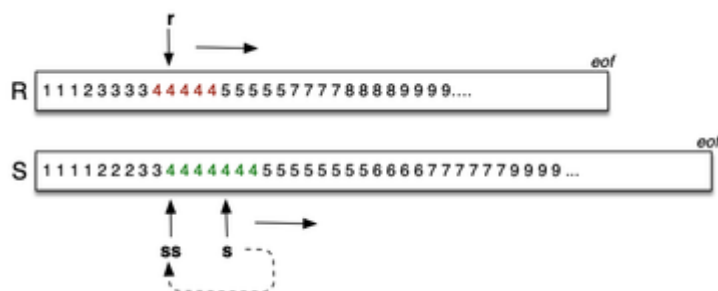
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### ... Sort-Merge Join

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Method requires several cursors to scan sorted relations:

- $r$  = current record in  $R$  relation
- $s$  = start of current run in  $S$  relation
- $ss$  = current record in current run in  $S$  relation



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### ... Sort-Merge Join

18/91

Algorithm using query iterators/scanners:

```
Query ri, si; Tuple r,s;
```

```
ri = startScan("SortedR");  
si = startScan("SortedS");  
while ((r = nextTuple(ri)) != NULL  
      && (s = nextTuple(si)) != NULL) {
```

```

// align cursors to start of next common run
while (r != NULL && r.i < s.j)
    r = nextTuple(ri);
if (r == NULL) break;
while (s != NULL && r.i > s.j)
    s = nextTuple(si);
if (s == NULL) break;
// must have (r.i == s.j) here
...

```

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### ... Sort-Merge Join

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```

...
// remember start of current run in S
TupleID startRun = scanCurrent(si)
// scan common run, generating result tuples
while (r != NULL && r.i == s.j) {
    while (s != NULL and s.j == r.i) {
        addTuple(outbuf, combine(r,s));
        if (isFull(outbuf)) {
            writePage(outf, outp++, outbuf);
            clearBuf(outbuf);
        }
        s = nextTuple(si);
    }
    r = nextTuple(ri);
    setScan(si, startRun);
}
}

```

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### ... Sort-Merge Join

20/91

Buffer requirements:

- for sort phase:
    - as many as possible (remembering that cost is  $O(\log N)$ )
    - if insufficient buffers, sorting cost can dominate
  - for merge phase:
    - one output buffer for result
    - one input buffer for relation  $R$
    - (preferably) enough buffers for longest run in  $S$
- 

### ... Sort-Merge Join

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Cost of sort-merge join.

Step 1: sort each relation (if not already sorted):

- Cost =  $2 \cdot b_R (1 + \log_{N-1}(b_R / N)) + 2 \cdot b_S (1 + \log_{N-1}(b_S / N))$   
 (where  $N$  = number of memory buffers)

Step 2: merge sorted relations:

- if every run of values in  $S$  fits completely in buffers,  
 merge requires single scan, Cost =  $b_R + b_S$
  - if some runs in of values in  $S$  are larger than buffers,  
 need to re-scan run for each corresponding value from  $R$
- 

## Sort-Merge Join on Example

22/91

Case 1:  $Join[id=stude](Student, Enrolled)$

- relations are not sorted on  $id\#$
- memory buffers  $N=32$ ; all runs are of length  $< 30$

$$\begin{aligned} \text{Cost} &= \text{sort}(S) + \text{sort}(E) + b_S + b_E \\ &= 2b_S(1+\log_{31}(b_S/32)) + 2b_E(1+\log_{31}(b_E/32)) + b_S + b_E \\ &= 2 \times 1000 \times (1+2) + 2 \times 2000 \times (1+2) + 1000 + 2000 \\ &= 6000 + 12000 + 1000 + 2000 \\ &= 21,000 \end{aligned}$$

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### ... Sort-Merge Join on Example

23/91

Case 2:  $Join[id=stude](Student, Enrolled)$

- *Student* and *Enrolled* already sorted on  $id\#$
- memory buffers  $N=4$  ( $S$  input,  $2 \times E$  input, output)
- 5% of the "runs" in  $E$  span two pages
- there are no "runs" in  $S$ , since  $id\#$  is a primary key

For the above, no re-scans of  $E$  runs are ever needed

$$\text{Cost} = 2,000 + 1,000 = 3,000 \quad (\text{regardless of which relation is outer})$$

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### Exercise 3: Sort-merge Join Cost

24/91

Consider executing  $Join[i=j](S, T)$  with the following parameters:

- $r_S = 1000$ ,  $b_S = 50$ ,  $r_T = 3000$ ,  $b_T = 150$
- $S.i$  is primary key, and  $T$  has index on  $T.j$
- $T$  is sorted on  $T.j$ , each  $S$  tuple joins with 2  $T$  tuples
- DBMS has  $N = 42$  buffers available for the join

Calculate the cost for evaluating the above join

- using sort-merge join
- compute #pages read/written
- compute #join-condition checks performed

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### Hash Join

25/91

Basic idea:

- use hashing as a technique to partition relations
- to avoid having to consider all pairs of tuples

Requires sufficient memory buffers

- to hold substantial portions of partitions
- (preferably) to hold largest partition of outer relation

Other issues:

- works only for equijoin  $R.i=S.j$  (but this is a common case)
- susceptible to data skew (or poor hash function)

Variations: *simple*, *grace*, *hybrid*.

## Simple Hash Join

26/91

Basic approach:

- hash part of outer relation  $R$  into memory buffers (build)
- scan inner relation  $S$ , using hash to search (probe)
  - if  $R.i=S.j$ , then  $h(R.i)=h(S.j)$  (hash to same buffer)
  - only need to check one memory buffer for each  $S$  tuple
- repeat until whole of  $R$  has been processed

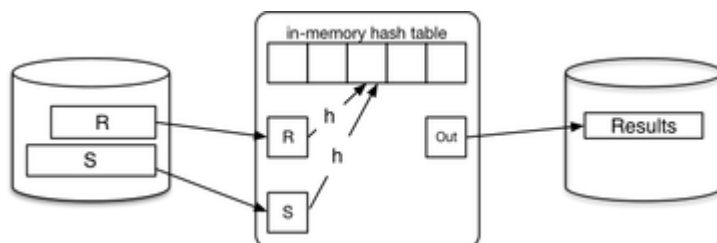
No overflows allowed in in-memory hash table

- works best with uniform hash function
- can be adversely affected by data/hash skew

### ... Simple Hash Join

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Data flow:



### ... Simple Hash Join

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Algorithm for simple hash join  $Join[R.i=S.j](R,S)$ :

```

for each tuple r in relation R {
  if (buffer[h(R.i)] is full) {
    for each tuple s in relation S {
      for each tuple rr in buffer[h(S.j)] {
        if ((rr,s) satisfies join condition) {
          add (rr,s) to result
        }
      }
    }
    clear all hash table buffers
  }
  insert r into buffer[h(R.i)]
}

```

Best case:  $\# \text{ join tests} \leq r_S \cdot c_R$  (cf. nested-loop  $r_S \cdot r_R$ )

### ... Simple Hash Join

29/91

Cost for simple hash join ...

Best case: all tuples of  $R$  fit in the hash table



- Cost =  $b_R + b_S$
- Same page reads as block nested loop, but less join tests

Good case: refill hash table  $m$  times (where  $m \geq \text{ceil}(b_R / (N-3))$ )

- Cost =  $b_R + m \cdot b_S$
- More page reads than block nested loop, but less join tests

Worst case: everything hashes to same page

- Cost =  $b_R + b_R \cdot b_S$

## Exercise 4: Simple Hash Join Cost

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Consider executing  $\text{Join}[i=j](R,S)$  with the following parameters:

- $r_R = 1000$ ,  $b_R = 50$ ,  $r_S = 3000$ ,  $b_S = 150$ ,  $c_{Res} = 30$
- $R.i$  is primary key, each  $R$  tuple joins with 2  $S$  tuples
- DBMS has  $N = 42$  buffers available for the join
- data + hash have uniform distribution

Calculate the cost for evaluating the above join

- using simple hash join
- compute #pages read/written
- compute #join-condition checks performed
- assume that hash table has  $L=0.75$  for each partition

## Grace Hash Join

31/91

Basic approach (for  $R \bowtie S$ ):

- partition both relations on join attribute using hashing ( $h1$ )
- load each partition of  $R$  into  $N$ -buffer hash table ( $h2$ )
- scan through corresponding partition of  $S$  to form results
- repeat until all partitions exhausted

For best-case cost ( $O(b_R + b_S)$ ):

- need  $\geq \sqrt{b_R}$  buffers to hold largest partition of outer relation

If  $< \sqrt{b_R}$  buffers or poor hash distribution

- need to scan some partitions of  $S$  multiple times

### ... Grace Hash Join

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Partition phase (applied to both  $R$  and  $S$ ):

### ... Grace Hash Join

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Probe/join phase:

The second hash function (h2) simply speeds up the matching process.  
Without it, would need to scan entire  $R$  partition for each record in  $S$  partition.

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### ... Grace Hash Join

34/91

Cost of grace hash join:

- #pages in all partition files of  $Rel \approx b_{Rel}$  (maybe slightly more)
- partition relation  $R$  ... Cost =  $b_R.T_r + b_R.T_w = 2b_R$
- partition relation  $S$  ... Cost =  $b_S.T_r + b_S.T_w = 2b_S$
- probe/join requires one scan of each (partitioned) relation  
Cost =  $b_R + b_S$
- all hashing and comparison occurs in memory  $\Rightarrow \approx 0$  cost

$$\text{Total Cost} = 2b_R + 2b_S + b_R + b_S = 3(b_R + b_S)$$

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### Exercise 5: Grace Hash Join Cost

35/91

Consider executing  $\text{Join}[i=j](R,S)$  with the following parameters:

- $r_R = 1000$ ,  $b_R = 50$ ,  $r_S = 3000$ ,  $b_S = 150$ ,  $c_{Res} = 30$
- $R.i$  is primary key, each  $R$  tuple joins with 2  $S$  tuples
- DBMS has  $N = 43$  buffers available for the join
- data + hash have reasonably uniform distribution

Calculate the cost for evaluating the above join

- using Grace hash join
  - compute #pages read/written
  - compute #join-condition checks performed
  - assume that no  $R$  partition is larger than 40 pages
- 

### Exercise 6: Grace Hash Join Cost

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Consider executing  $\text{Join}[i=j](R,S)$  with the following parameters:

- $r_R = 1000$ ,  $b_R = 50$ ,  $r_S = 3000$ ,  $b_S = 150$ ,  $c_{Res} = 30$
- $R.i$  is primary key, each  $R$  tuple joins with 2  $S$  tuples
- DBMS has  $N = 42$  buffers available for the join
- data + hash have reasonably uniform distribution

Calculate the cost for evaluating the above join

- using Grace hash join
  - compute #pages read/written
  - compute #join-condition checks performed
  - assume that one  $R$  partition has 50 pages, others  $< 40$  pages
  - assume that the corresponding  $S$  partition has 30 pages
- 

### Hybrid Hash Join

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A variant of grace join if we have  $\sqrt{b_R} < N < b_R + 2$

- create  $k \ll N$  partitions,  $m$  in memory,  $k-m$  on disk

- buffers: 1 input,  $k-m$  output,  $p = N-(k-m)-1$  for in-memory partitions

When we come to scan and partition  $S$  relation

- any tuple with hash in range  $0..m-1$  can be resolved
- other tuples are written to one of  $k$  partition files for  $S$

Final phase is same as grace join, but with only  $k$  partitions.

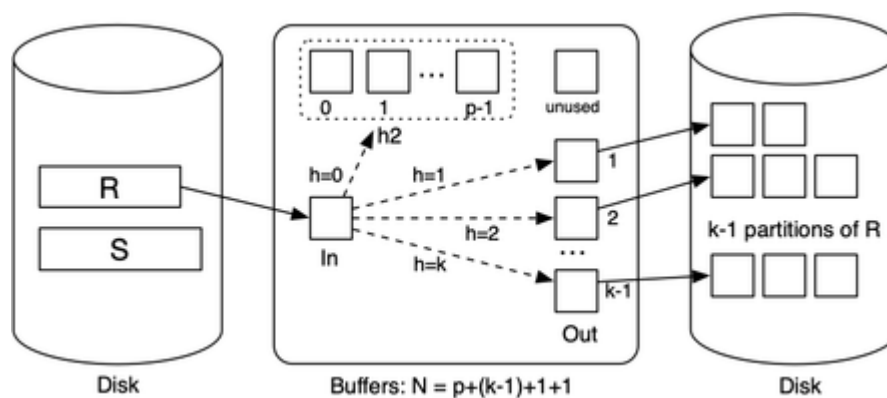
Comparison:

- grace hash join creates  $N-1$  partitions on disk
- hybrid hash join creates  $m$  (memory) +  $k$  (disk) partitions

### ... Hybrid Hash Join

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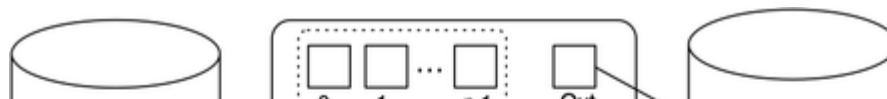
First phase of hybrid hash join with  $m=1$  (partitioning  $R$ ):



### ... Hybrid Hash Join

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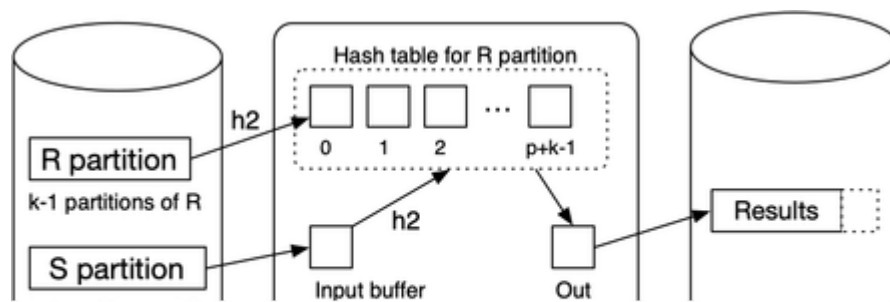
Next phase of hybrid hash join with  $m=1$  (partitioning  $S$ ):



### ... Hybrid Hash Join

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Final phase of hybrid hash join with  $m=1$  (finishing join):



## ... Hybrid Hash Join

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Some observations:

- with  $k$  partitions, each partition has expected size  $b_R/k$
- holding  $m$  partitions in memory needs  $\lceil mb_R/k \rceil$  buffers
- trade-off between in-memory partition space and #partitions

Best-cost scenario:

- $m = 1, \quad k \approx \lceil b_R/N \rceil$  (satisfying above constraint)

Other notes:

- if  $N = b_R + 2$ , using block nested loop join is simpler
- cost depends on  $N$  (but less than grace hash join)

## Exercise 7: Hybrid Hash Join Cost

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Consider executing  $\text{Join}[i=j](R, S)$  with the following parameters:

- $r_R = 1000, \quad b_R = 50, \quad r_S = 3000, \quad b_S = 150, \quad c_{Res} = 30$
- $R.i$  is primary key, each  $R$  tuple joins with 2  $S$  tuples
- DBMS has  $N = 42$  buffers available for the join
- data + hash have reasonably uniform distribution

Calculate the cost for evaluating the above join

- using hybrid hash join with  $m=1, p=40$
- compute #pages read/written
- compute #join-condition checks performed
- assume that no  $R$  partition is larger than 40 pages

## Join Summary

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No single join algorithm is superior in some overall sense.

Which algorithm is best for a given query depends on:

- sizes of relations being joined, size of buffer pool
- any indexing on relations, whether relations are sorted
- which attributes and operations are used in the query
- number of tuples in  $S$  matching each tuple in  $R$

- distribution of data values (uniform, skew, ...)

Choosing the "best" join algorithm is critical because the cost difference between best and worst case can be very large.

E.g. `Join[id=stude](Student,Enrolled)`: 3,000 ... 2,000,000

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## Join in PostgreSQL

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Join implementations are under: `src/backend/executor`

PostgreSQL supports three kinds of join:

- nested loop join (`nodeNestloop.c`)
- sort-merge join (`nodeMergejoin.c`)
- hash join (`nodeHashjoin.c`) (hybrid hash join)

Query optimiser chooses appropriate join, by considering

- physical characteristics of tables being joined
  - estimated selectivity (likely number of result tuples)
- 

## Exercise 8: Outer Join?

45/91

Above discussion was all in terms of theta inner-join.

How would the algorithms above adapt to outer join?

Consider the following ...

```
select *
from   R left outer join S on (R.i = S.j)

select *
from   R right outer join S on (R.i = S.j)

select *
from   R full outer join S on (R.i = S.j)
```

---

## Query Evaluation

### Query Evaluation

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#### ... Query Evaluation

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A *query* in SQL:

- states *what* kind of answers are required (declarative)
- does not say *how* they should be computed (procedural)

A *query evaluator/processor* :

- takes declarative description of query (in SQL)

- parses query to internal representation (relational algebra)
- determines plan for answering query (expressed as DBMS ops)
- executes method via DBMS engine (to produce result tuples)

Some DBMSs can save query plans for later re-use.

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## ... Query Evaluation

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Internals of the query evaluation "black-box":

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## ... Query Evaluation

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DBMSs provide several "flavours" of each RA operation.

For example:

- several "versions" of selection ( $\sigma$ ) are available
- each version is effective for a particular kind of selection, e.g

```
select * from R where id = 100  -- hashing
select * from S                -- Btree index
where age > 18 and age < 35
select * from T                -- MALH file
where a = 1 and b = 'a' and c = 1.4
```

Similarly,  $\pi$  and  $\bowtie$  have versions to match specific query types.

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## ... Query Evaluation

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We call these specialised version of RA operations *RelOps*.

One major task of the query processor:

- given a RA expression to be evaluated
- find a combination of RelOps to do this efficiently

Requires the query translator/optimiser to consider

- information about relations (e.g. sizes, primary keys, ...)
- information about operations (e.g. selection reduces size)

RelOps are realised at execution time

- as a collection of inter-communicating *nodes*
- communicating either via pipelines or temporary relations

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## Terminology Variations

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Relational algebra expression of SQL query

- intermediate query representation
- logical query plan

Execution plan as collection of RelOps

- query evaluation plan

- query execution plan
- physical query plan

Representation of RA operators and expressions

- $\sigma = \text{Select} = \text{Sel}, \quad \pi = \text{Project} = \text{Proj}$
- $R \bowtie S = R \text{ Join } S = \text{Join}(R, S), \quad \wedge = \&, \quad \vee = |$

## Query Translation

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Query translation: SQL statement text  $\rightarrow$  RA expression

## Query Translation

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Translation step: SQL text  $\rightarrow$  RA expression

Example:

SQL: `select name from Students where id=7654321;`  
 -- is translated to  
 RA: `Proj[name](Sel[id=7654321]Students)`

Processes: lexer/parser, mapping rules, rewriting rules.

Mapping from SQL to RA may include some optimisations, e.g.

```
select * from Students where id = 54321 and age > 50;
-- is translated to
Sel[age>50](Sel[id=54321]Students)
-- rather than ... because of index on id
Sel[id=54321&age>50](Students)
```

## Parsing SQL

55/91

Parsing task is similar to that for programming languages.

Language elements:

- keywords: `create`, `select`, `from`, `where`, ...
- identifiers: `Students`, `name`, `id`, `CourseCode`, ...
- operators: `+`, `-`, `=`, `<`, `>`, `AND`, `OR`, `NOT`, `IN`, ...
- constants: `'abc'`, `123`, `3.1`, `'01-jan-1970'`, ...

PostgreSQL parser ...

- implemented via `lex/yacc` ([src/backend/parser](#))
- maps all identifiers to lower-case (`A-Z`  $\rightarrow$  `a-z`)
- needs to handle user-extendable operator set
- makes extensive use of catalog ([src/backend/catalog](#))

## Expression Rewriting Rules

56/91

Since RA is a well-defined formal system

- there exist many algebraic laws on RA expressions
- which can be used as a basis for expression rewriting

- in order to produce *equivalent* (more-efficient?) expressions

Expression transformation based on such rules can be used

- to simplify/improve SQL  $\rightarrow$  RA mapping results
- to generate new plan variations to check in query optimisation

## Relational Algebra Laws

57/91

Commutative and Associative Laws:

- $R \bowtie S \leftrightarrow S \bowtie R$ ,  $(R \bowtie S) \bowtie T \leftrightarrow R \bowtie (S \bowtie T)$  (natural join)
- $R \cup S \leftrightarrow S \cup R$ ,  $(R \cup S) \cup T \leftrightarrow R \cup (S \cup T)$
- $R \bowtie_{Cond} S \leftrightarrow S \bowtie_{Cond} R$  (theta join)
- $\sigma_c(\sigma_d(R)) \leftrightarrow \sigma_d(\sigma_c(R))$

Selection splitting (where  $c$  and  $d$  are conditions):

- $\sigma_{c \wedge d}(R) \leftrightarrow \sigma_c(\sigma_d(R))$
- $\sigma_{c \vee d}(R) \leftrightarrow \sigma_c(R) \cup \sigma_d(R)$

## ... Relational Algebra Laws

58/91

Selection pushing (  $\sigma_c(R \cup S)$  and  $\sigma_c(R \cap S)$  ):

- $\sigma_c(R \cup S) \leftrightarrow \sigma_c R \cup \sigma_c S$ ,  $\sigma_c(R \cap S) \leftrightarrow \sigma_c R \cap \sigma_c S$

Selection pushing with join ...

- $\sigma_c(R \bowtie S) \leftrightarrow \sigma_c(R) \bowtie S$  (if  $c$  refers only to attributes from  $R$  )
- $\sigma_c(R \bowtie S) \leftrightarrow R \bowtie \sigma_c(S)$  (if  $c$  refers only to attributes from  $S$  )

If *condition* contains attributes from both  $R$  and  $S$ :

- $\sigma_{c' \wedge c''}(R \bowtie S) \leftrightarrow \sigma_{c'}(R) \bowtie \sigma_{c''}(S)$
- $c'$  contains only  $R$  attributes,  $c''$  contains only  $S$  attributes

## ... Relational Algebra Laws

59/91

Rewrite rules for projection ...

All but last projection can be ignored:

- $\pi_{L1}(\pi_{L2}(\dots \pi_{Ln}(R))) \rightarrow \pi_{L1}(R)$

Projections can be pushed into joins:

- $\pi_L(R \bowtie_c S) \leftrightarrow \pi_L(\pi_M(R) \bowtie_c \pi_N(S))$

where

- $M$  and  $N$  must contain all attributes needed for  $c$
- $M$  and  $N$  must contain all attributes used in  $L$  ( $L \subset M \cup N$ )

## Query Rewriting

60/91



Subqueries  $\Rightarrow$  convert to a join

Example: (on schema Courses(id,code,...), Enrolments(cid,sid,...), Students(id,name,...))

```
select c.code, count(*)
from   Courses c
where  c.id in (select cid from Enrolments)
group by c.code
```

becomes

```
select c.code, count(*)
from   Courses c join Enrolments e on c.id = e.cid
group by c.code
```

---

### ... Query Rewriting

61/91

But not all subqueries can be converted to join, e.g.

```
select e.sid as student_id, e.cid as course_id
from   Enrolments e
where  e.sid = (select max(id) from Students)
```

has to be evaluated as

$Val = \max[id]Students$

$Res = \pi_{(sid,cid)}(\sigma_{sid=Val}Enrolments)$

---

### ... Query Rewriting

62/91

In PostgreSQL, views are implemented via rewrite rules

- a reference to view in SQL expands to its definition in RA

Example:

```
create view COMP9315studes as
select stu,mark from Enrolments where course='COMP9315';
-- students who passed
select stu from COMP9315studes where mark >= 50;
```

is represented in RA by

```
COMP9315studes
  = Proj[stu,mark](Sel[course=COMP9315](Enrolments))
-- with query ...
Proj[stu](Sel[mark>=50](COMP9315studes))
-- becomes ...
Proj[stu](Sel[mark>=50](
  Proj[stu,mark](Sel[course=COMP9315](Enrolments))))
-- which could be rewritten as ...
Proj[stu](Sel[mark>=50 & course=COMP9315]Enrolments)
```

---

## Exercise 9: SQL $\rightarrow$ RelAlg

63/91

Convert the following queries into (efficient?) RA expressions

```
select * from R where a > 5;
```

```
select * from R where id = 1234 and a > 5;
```

```
select R.a from R, S where R.i = S.j;

select R.a from R join S on R.i = S.j;

select * from R, S where R.i = S.j and R.a = 6

select R.a from R, S, T where R.i = S.j and S.k = T.y;
```

Assume R.id is a primary key and R is hashed on id

Assume that there is a B-tree index on R.b

---

## Query Optimisation

---

### Query Optimisation

65/91

Query optimiser: RA expression  $\rightarrow$  efficient evaluation plan

---

#### ... Query Optimisation

66/91

*Query optimisation* is a critical step in query evaluation.

The query optimiser

- takes relational algebra expression from SQL compiler
- produces sequence of RelOps to evaluate the expression
- *query execution plan* should provide efficient evaluation

"Optimisation" is a misnomer since query optimisers

- aim to find a good plan ... but maybe not optimal

Observed Query Time = Planning time + Evaluation time

---

#### ... Query Optimisation

67/91

Why do we not generate optimal query execution plans?

Finding an optimal query plan ...

- requires exhaustive search of a *space of possible plans*
- for each possible plan, need to estimate cost (not cheap)

Even for relatively small query, search space is *very large*.

Compromise:

- do limited search of query plan space (guided by heuristics)
  - *quickly* choose a *reasonably efficient* execution plan
- 

## Approaches to Optimisation

68/91

Three main classes of techniques developed:

- algebraic (equivalences, rewriting, heuristics)

- physical (execution costs, search-based)
- semantic (application properties, heuristics)

All driven by aim of minimising (or at least reducing) "cost".

Real query optimisers use a combination of algebraic+physical.

Semantic QO is good idea, but expensive/difficult to implement.

---

## ... Approaches to Optimisation

69/91

Example of optimisation transformations:

For join, may also consider sort/merge join and hash join.

---

## Cost-based Query Optimiser

70/91

Approximate algorithm for cost-based optimisation:

```

translate SQL query to RAexp
for enough transformations RA' of RAexp {
  while (more choices for RelOps) {
    Plan = {}; i = 0; cost = 0
    for each node e of RA' (recursively) {
      ROp = select RelOp method for e
      Plan = Plan U ROp
      cost += Cost(ROp) // using child info
    }
    if (cost < MinCost)
      { MinCost = cost; BestPlan = Plan }
  }
}

```

Heuristics: push selections down, consider only left-deep join trees.

---

## Exercise 10: Alternative Join Plans

71/91

Consider the schema

```

Students(id,name,...)   Enrol(student,course,mark)
Staff(id,name,...)      Courses(id,code,term,lic,...)

```

the following query on this schema

```

select c.code, s.id, s.name
from   Students s, Enrol e, Courses c, Staff f
where  s.id=e.student and e.course=c.id
      and c.lic=f.id and c.term='19T2'
      and f.name='John Shepherd'

```

Show some possible evaluation orders for this query.

---

## Cost Models and Analysis

72/91

The cost of evaluating a query is determined by:

- size of relations (database relations and temporary relations)
- access mechanisms (indexing, hashing, sorting, join algorithms)

- size/number of main memory buffers (and replacement strategy)

Analysis of costs involves *estimating*:

- size of intermediate results
- number of secondary storage accesses

## Choosing Access Methods (RelOps)

73/91

Performed for each node in RA expression tree ...

Inputs:

- a single RA operation ( $\sigma$ ,  $\pi$ ,  $\bowtie$ )
- information about file organisation, data distribution, ...
- list of operations available in the database engine

Output:

- specific DBMS operation to implement this RA operation

## ... Choosing Access Methods (RelOps)

74/91

Example:

- RA operation:  $Sel_{[name='John' \wedge age>21]}(Student)$
- Student relation has B-tree index on name
- database engine (obviously) has B-tree search method

giving

```
tmp[i] := BtreeSearch[name='John'](Student)
tmp[i+1] := LinearSearch[age>21](tmp[i])
```

Where possible, use pipelining to avoid storing  $tmp[i]$  on disk.

## ... Choosing Access Methods (RelOps)

75/91

Rules for choosing  $\sigma$  access methods:

- $\sigma_{A=c}(R)$  and R has index on A  $\Rightarrow$   $indexSearch[A=c](R)$
- $\sigma_{A=c}(R)$  and R is hashed on A  $\Rightarrow$   $hashSearch[A=c](R)$
- $\sigma_{A=c}(R)$  and R is sorted on A  $\Rightarrow$   $binarySearch[A=c](R)$
- $\sigma_A \geq c(R)$  and R has clustered index on A  
 $\Rightarrow$   $indexSearch[A=c](R)$  then scan
- $\sigma_A \geq c(R)$  and R is hashed on A  
 $\Rightarrow$   $linearSearch[A \geq c](R)$

## ... Choosing Access Methods (RelOps)

76/91

Rules for choosing  $\bowtie$  access methods:

- $R \bowtie S$  and R fits in memory buffers  $\Rightarrow$   $bnlJoin(R, S)$
- $R \bowtie S$  and S fits in memory buffers  $\Rightarrow$   $bnlJoin(S, R)$
- $R \bowtie S$  and R, S sorted on join attr  $\Rightarrow$   $smJoin(R, S)$

- $R \bowtie S$  and  $R$  has index on join attr  $\Rightarrow$  `inlJoin(S,R)`
- $R \bowtie S$  and no indexes, no sorting  $\Rightarrow$  `hashJoin(R,S)`

(bnl = block nested loop; inl = index nested loop; sm = sort merge)

## Cost Estimation

77/91

Without executing a plan, cannot always know its precise cost.

Thus, query optimisers *estimate* costs via:

- cost of performing operation (dealt with in earlier lectures)
- size of result (which affects cost of performing next operation)

Result size estimated by statistical measures on relations, e.g.

$r_S$             cardinality of relation  $S$   
 $R_S$             avg size of tuple in relation  $S$   
 $V(A,S)$        # distinct values of attribute  $A$   
 $\min(A,S)$     min value of attribute  $A$   
 $\max(A,S)$     max value of attribute  $A$

## Estimating Projection Result Size

78/91

Straightforward, since we know:

- number of tuples in output

$$r_{out} = |\pi_{a,b,\dots}(T)| = |T| = r_T \quad (\text{in SQL, because of bag semantics})$$

- size of tuples in output

$$R_{out} = \text{sizeof}(a) + \text{sizeof}(b) + \dots + \text{tuple-overhead}$$

Assume page size  $B$ ,  $b_{out} = \lceil r_T / c_{out} \rceil$ , where  $c_{out} = \lfloor B / R_{out} \rfloor$

If using `select distinct ...`

- $|\pi_{a,b,\dots}(T)|$  depends on proportion of duplicates produced

## Estimating Selection Result Size

79/91

Selectivity = fraction of tuples expected to satisfy a condition.

Common assumption: attribute values uniformly distributed.

**Example:** Consider the query

```
select * from Parts where colour='Red'
```

If  $V(\text{colour}, \text{Parts})=4$ ,  $r=1000 \Rightarrow |\sigma_{\text{colour}=\text{red}}(\text{Parts})|=250$

In general,  $|\sigma_{A=c}(R)| \approx r_R / V(A,R)$

Heuristic used by PostgreSQL:  $|\sigma_{A=c}(R)| \approx r/10$

---

### ... Estimating Selection Result Size

80/91

Estimating size of result for e.g.

```
select * from Enrolment where year > 2015;
```

Could estimate by using:

- uniform distribution assumption,  $r$ , min/max years

Assume:  $\min(\text{year})=2010$ ,  $\max(\text{year})=2019$ ,  $|Enrolment|=10^5$

- $10^5$  from 2010–2019 means approx 10000 enrolments/year
- this suggests 40000 enrolments since 2016

Heuristic used by some systems:  $|\sigma_{A>c}(R)| \approx r/3$

---

### ... Estimating Selection Result Size

81/91

Estimating size of result for e.g.

```
select * from Enrolment where course <> 'COMP9315';
```

Could estimate by using:

- uniform distribution assumption,  $r$ , domain size

e.g.  $|V(\text{course}, Enrolment)| = 2000$ ,  $|\sigma_{A<>c}(E)| = r * 1999/2000$

Heuristic used by some systems:  $|\sigma_{A<>c}(R)| \approx r$

---

## Exercise 11: Selection Size Estimation

82/91

Assuming that

- all attributes have uniform distribution of data values
- attributes are independent of each other

Give formulae for the number of expected results for

1. `select * from R where not A=k`
2. `select * from R where A=k and B=j`
3. `select * from R where A in (k,l,m,n)`

where  $j, k, l, m, n$  are constants.

Assume:  $V(A,R) = 10$  and  $V(B,R)=100$  and  $r=1000$

---

### ... Estimating Selection Result Size

83/91

How to handle non-uniform attribute value distributions?

- collect statistics about the values stored in the attribute/relation
- store these as e.g. a histogram in the meta-data for the relation

So, for part colour example, might have distribution like:

White: 35% Red: 30% Blue: 25% Silver: 10%

Use histogram as basis for determining # selected tuples.

Disadvantage: cost of storing/maintaining histograms.

### ... Estimating Selection Result Size

84/91

Summary: analysis relies on operation and data distribution:

E.g. `select * from R where a = k;`

Case 1:  $uniq(R.a) \Rightarrow 0 \text{ or } 1 \text{ result}$

Case 2:  $r_R \text{ tuples \&\& } size(dom(R.a)) = n \Rightarrow r_R / n \text{ results}$

E.g. `select * from R where a < k;`

Case 1:  $k \leq min(R.a) \Rightarrow 0 \text{ results}$

Case 2:  $k > max(R.a) \Rightarrow = r_R \text{ results}$

Case 3:  $size(dom(R.a)) = n \Rightarrow ? min(R.a) \dots k \dots max(R.a) ?$

### Estimating Join Result Size

85/91

Analysis relies on semantic knowledge about data/relations.

Consider equijoin on common attr:  $R \bowtie_a S$

Case 1:  $values(R.a) \cap values(S.a) = \{\}$   $\Rightarrow size(R \bowtie_a S) = 0$

Case 2:  $uniq(R.a) \text{ and } uniq(S.a) \Rightarrow size(R \bowtie_a S) \leq min(|R|, |S|)$

Case 3:  $pkey(R.a) \text{ and } fkey(S.a) \Rightarrow size(R \bowtie_a S) \leq |S|$

### Exercise 12: Join Size Estimation

86/91

How many tuples are in the output from:

1. `select * from R, S where R.s = S.id`  
where `S.id` is a primary key and `R.s` is a foreign key referencing `S.id`
2. `select * from R, S where R.s <> S.id`  
where `S.id` is a primary key and `R.s` is a foreign key referencing `S.id`
3. `select * from R, S where R.x = S.y`  
where `R.x` and `S.y` have no connection except that  $dom(R.x) = dom(S.y)$

Under what conditions will the first query have maximum size?

### Cost Estimation: Postscript

87/91

Inaccurate cost estimation can lead to poor evaluation plans.

Above methods can (sometimes) give inaccurate estimates.

To get more accurate cost estimates:

- more time ... complex computation of selectivity
- more space ... storage for histograms of data values

Either way, optimisation process costs more (more than query?)

Trade-off between optimiser performance and query performance.

---

## PostgreSQL Query Optimiser

---

### PostgreSQL Query Optimization

89/91

Input: tree of **Query** nodes returned by parser

Output: tree of **Plan** nodes used by query *executor*

- wrapped in a **PlannedStmt** node containing state info

Intermediate data structures are trees of **Path** nodes

- a path tree represents one evaluation order for a query

All **Node** types are defined in **include/nodes/\*.h**

---

### ... PostgreSQL Query Optimization

90/91

Query optimisation proceeds in two stages (after parsing)...

*Rewriting:*

- uses PostgreSQL's *rule* system
- query tree is expanded to include e.g. view definitions

*Planning and optimisation:*

- using cost-based analysis of generated paths
- via one of *two* different path generators
- chooses least-cost path from all those considered

Then produces a **Plan** tree from the selected path.

---

### ... PostgreSQL Query Optimization

91/91

---