Mini Project Four

ELEC 301

University of British Columbia

Submitted By:

Hanlin Yu

36958502

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Professor:

Nicolas Jaeger

Content

1.Introduction

This report includes implementing a second-order Butterworth filter, a phase-shift oscillator, and a feedback circuit. The focus is on understanding circuit behavior, validating theoretical models, and analyzing the impact of feedback on gain, stability, and frequency response through simulations and experiments.

2. Part A – Experiment

1.

$H(s) = A_M * \frac{\frac{1}{RC}^2}{s^2+s*\frac{3-A_M}{R*C}+\frac{1}{RC}^2}$, $A_M = 1+\frac{R_2}{R_1}$, $\boxed{\boldsymbol{R = 10k\Omega}}$.

Since 3dB frequency is 10kHz. $\omega_C = (10kHz * 2 * \pi)\frac{1}{s} = \frac{1}{R*C} = \frac{1}{10k\Omega*C}$.

Solve to get $\boxed{\boldsymbol{C = 1.59 * 10^{-9}F}} \approx 1.6nF$.

Since it is a second-order Butterworth filter with 3dB frequency at 10kHz.

The denominator of $H(s)$ will be $\left(s - \frac{1}{R*C*\sqrt{2}} + \frac{1}{R*C*\sqrt{2}} * j\right) * \left(s - \frac{1}{R*C*\sqrt{2}} - \frac{1}{R*C*\sqrt{2}} * j\right)$.

Which can be simplified as $s^2 + s * \frac{\sqrt{2}}{R*C} + \frac{1}{RC}^2$.

So $3 - A_M = \sqrt{2}$, $A_m = (3-\sqrt{2})\frac{V}{V} = \boxed{\boldsymbol{1.5858\frac{V}{V}}}$.

$\begin{cases} R_1 + R_2 = 10k\Omega \\ 1+\frac{R_2}{R_1} = 3-\sqrt{2} \end{cases}$, $Get \begin{cases} \boxed{\begin{matrix}\boldsymbol{R_1 = 6306.01937\Omega} \\ \boldsymbol{R_2 = 3693.98\Omega}\end{matrix}} \end{cases}$.

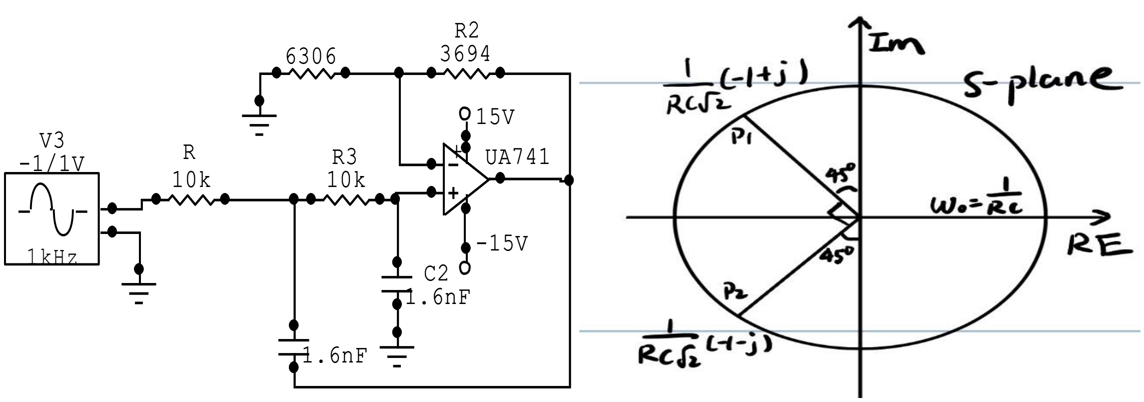


Figure 1Figure 1 Filter circuit diagram and s-plane plot

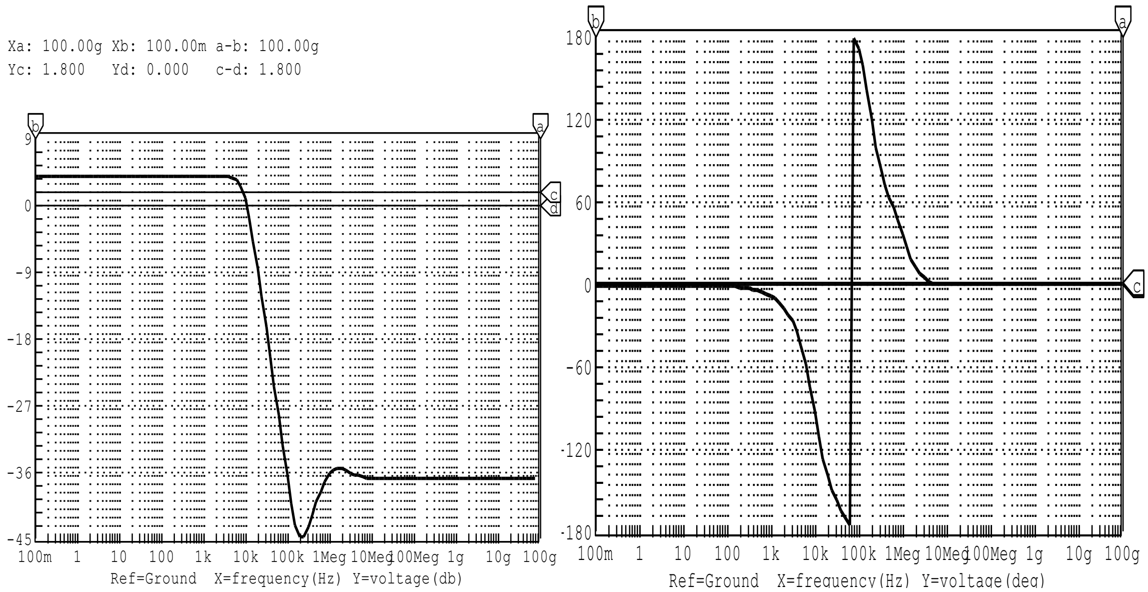


Figure 2 Magnitude and phase bode plots

2.

Expected A_m to start oscillate is $\boxed{\boldsymbol{A_m = 3\frac{V}{V}}}$, because when short the input, s^2 is $(j * \omega_c)^2 = (\frac{-1}{R*C})^2$. If $A_m = 3\frac{V}{V}$, the denominator of $H(s)$ becomes $s^2 + s * \frac{3-A_M}{R*C} + \frac{1}{RC}^2 = 0$. So it starts to oscillate.$R_2 = 2 * R_1 = 6.667k\Omega$. Without input source, $s = j * \omega_c$.

Simulate:Then, adjust values of $R_1$and R_2,run the simulation on Circuitmaker and find out when $\boxed{\boldsymbol{R_2 = 6.667k\Omega, R_1 = 3.333k\Omega}}$ the output of the circuit starts to oscillate, which is the same as expected value. $\boldsymbol{A_m = 1 + \frac{R_2}{R_1} = 3\frac{V}{V}}$

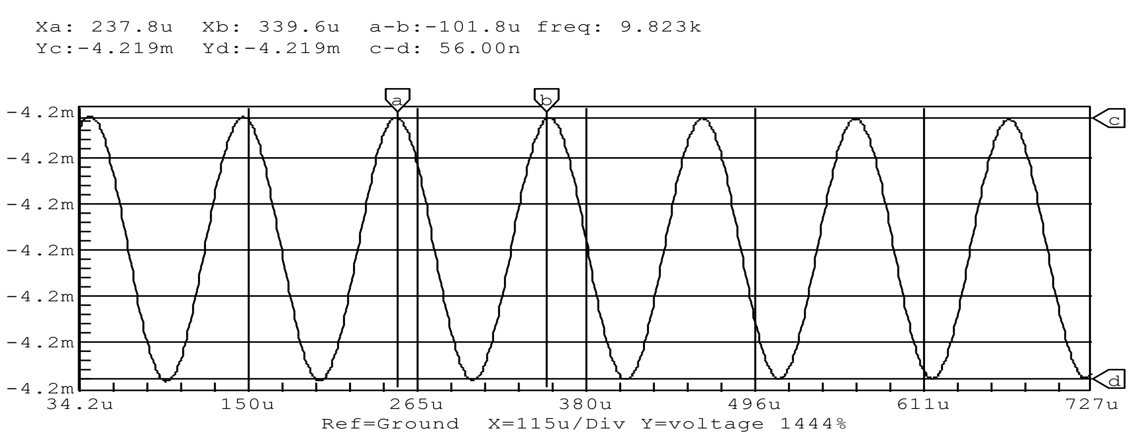


Figure 3 Output of circuit

Shown in the graph is that the output frequency is about $\boxed{\mathbf{9.823}\boldsymbol{kHz}}$, which is close to 10kHz.

$\boldsymbol{A_{m(oscillate)}} = \mathbf{3}\frac{\boldsymbol{V}}{\boldsymbol{V}}$, $.A_{m(Butterworth)} = (3-\sqrt{2})\frac{V}{V}$. $H(s) = A_M * \frac{\frac{1}{RC}^2}{s^2+s*\frac{3-A_M}{R*C}+\frac{1}{RC}^2}$

Draw the root locus for both the oscillating case and the Butterworth case using Matlab.

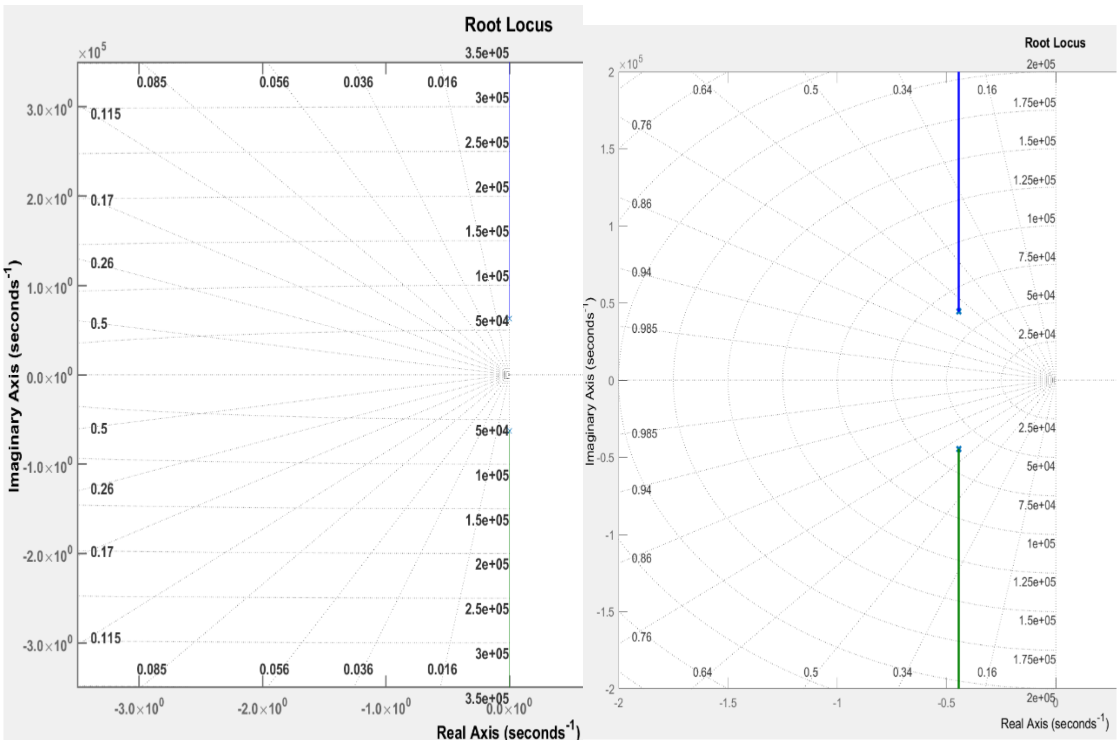


Figure 4 root locus graphs of oscillating case and Butterworth case

For the first one the transfer function is $H(s) = 3 * \frac{\frac{1}{RC}^2}{s^2+\frac{1}{RC}^2}$, and the second one is $H(s) = (3-\sqrt{2}) * \frac{\frac{1}{RC}^2}{s^2+s*\frac{\sqrt{2}}{R*C}+\frac{1}{RC}^2}$. For the first case, poles are imaginary figures and poles are complex number at the left of imaginary axis for the second case. When increasing $A_m > 3\frac{V}{V}$, the root locus will be at the right of imaginary axis(not stable). Overall, increasing A_m will make root locus shift to the right while decreasing A_m will make the root locus shift to the left. The radius of the circle remains the same with $r = \omega_0 = \frac{1}{RC}$.

3. Part B - A Phase Shift Oscillator

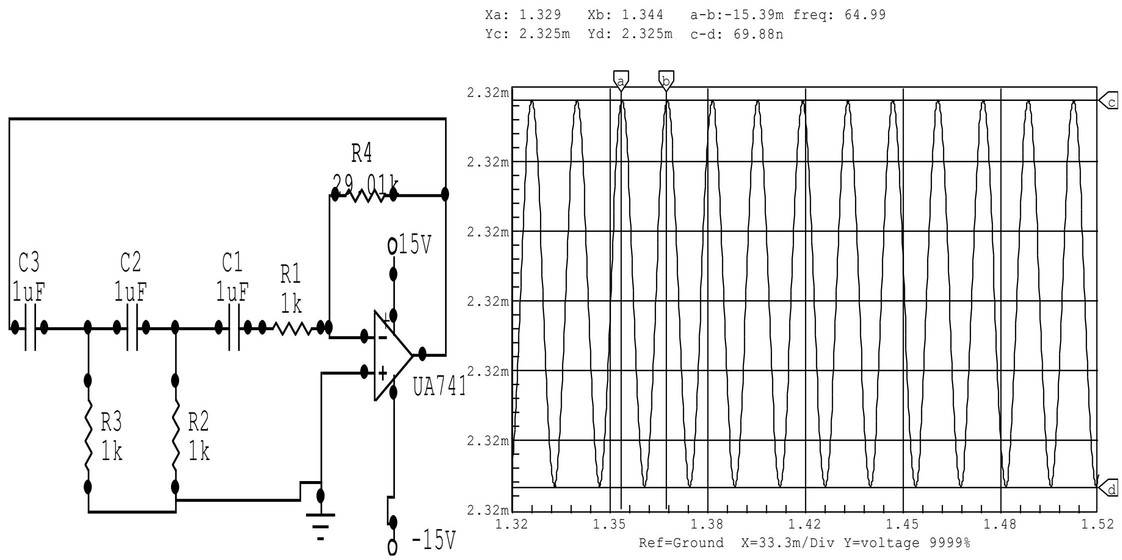


Figure 5 Phase shift oscillator circuit diagram and 1R,1C bode plot

Wire up the circuit on Circuitmaker and increase R_4 slightly until finding a stabilized oscillatior. Run the simulation and use the "transient and fourier analysis" tool to measure. When $\boxed{\boldsymbol{R_4 = 29.01k\Omega = 29k\Omega + 0.01k\Omega}}$, the circuit is a stable oscillator(When R_4 is smaller than $29.01k\Omega$, the output will gradually decay and when R_4 is larger than $29.01k\Omega$, the output increase to infinity). The transient plot shows that the frequency is $\boxed{\boldsymbol{f_R = 64.99Hz}}$.

Increase R and C by factors of 2, ($R = 2k\Omega, C = 2\mu F$)and plot the graph. Measure $\boxed{\boldsymbol{f_{2R} = 16.20Hz}}$.

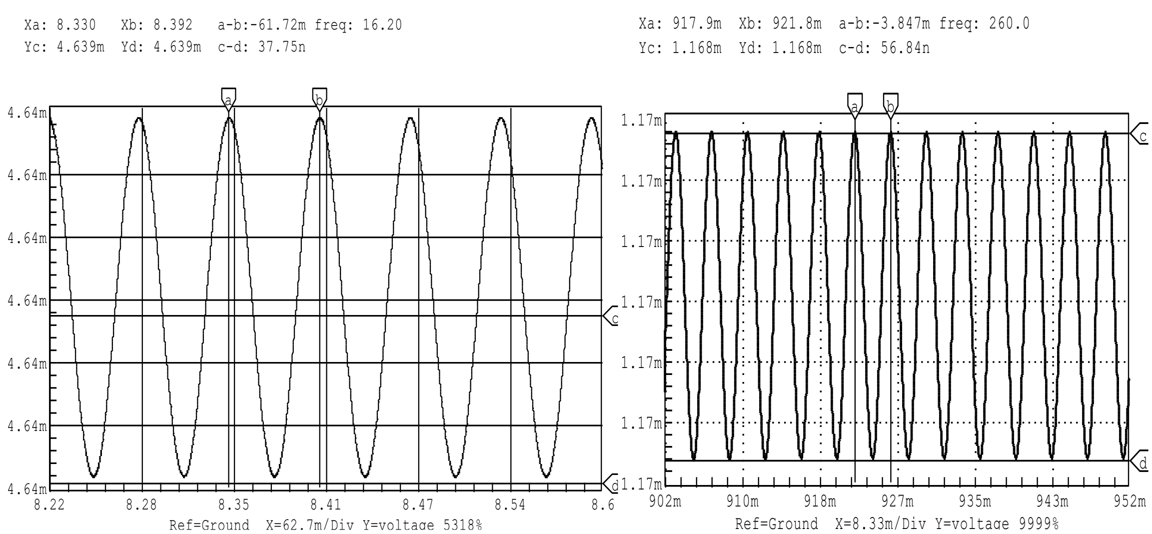


Figure 6 plotted graphs of oscillating frequency of 2R 2C and 1/2R 1/2C respectively

Then, decrease resistors and capacitors by factors of 2, ($R = 0.5k\Omega, C = 0.5\mu F$), and plot the frequency. $\boxed{\boldsymbol{f_{\frac{1}{2}R} = 260Hz}}$.

Calculated:

The feedback circuit is identical to the one in the course note. Use equations given in the course(I derive the same formula) to calculate oscillating frequency(derived by KVL):

$$f = \frac{1}{2 * \pi * \sqrt{6} * R * C}$$

	Measured oscillating frequency	Calculated oscillating frequency	Error
$R = 1k\Omega$ $C = 1\mu F$	$\boxed{\boldsymbol{f_R = 64.99Hz}}$	$\boxed{\boldsymbol{f_R = 64.97Hz}}$	0.03%
$R = 2k\Omega$ $C = 2\mu F$	$\boxed{\boldsymbol{f_{2R} = 16.20Hz}}$	$\boxed{\boldsymbol{f_{2R} = 16.24Hz}}$	0.24%
$R = 0.5k\Omega$ $C = 0.5\mu F$	$\boxed{\boldsymbol{f_{\frac{1}{2}R} = 260Hz}}$	$\boxed{\boldsymbol{f_{\frac{1}{2}R} = 259.90Hz}}$	0.038%

Table 1 Measured and calculated oscillating frequency

The table shows that the error is almost 0 and it may be from human error or due to resolution (resolution of R_4). The result proves that the equations from course notes work well, which makes sense because the input resistance of the op-amp is so large and the output resistance is so small that they can be neglected.

Report: The three RC stages connected in cascade create a feedback network to provide a total 180° phase shift. Shown in the course note handout is the attenuation of RC network being $\frac{1}{29}$, so the op-amp gain should be 29. Hence, the resistor is 29R. Also, inverting op-amp provides 180° phase shift, resulting in 0 total phase shift. Overall, $A = -29, \beta = \frac{1}{29}$. $A * \beta = -1$. Increase the 29R increase the gain of the amplifier, so increase the loop gain $|A\beta|$ to be ≥1 to oscillate the circuit. (Barkhausen criterion). The slight increase in 29R helps to overcome the imperfection of resistors and capacitors, which affect the attenuation of the circuit.

4. Part C - A Feedback Circuit

Wire up and test the circuit, sweeping the R_{B2} from $15k\Omega$ to $25k\Omega$ with step value of $1k\Omega$ to obtain the largest open loop gain at 1kHz.

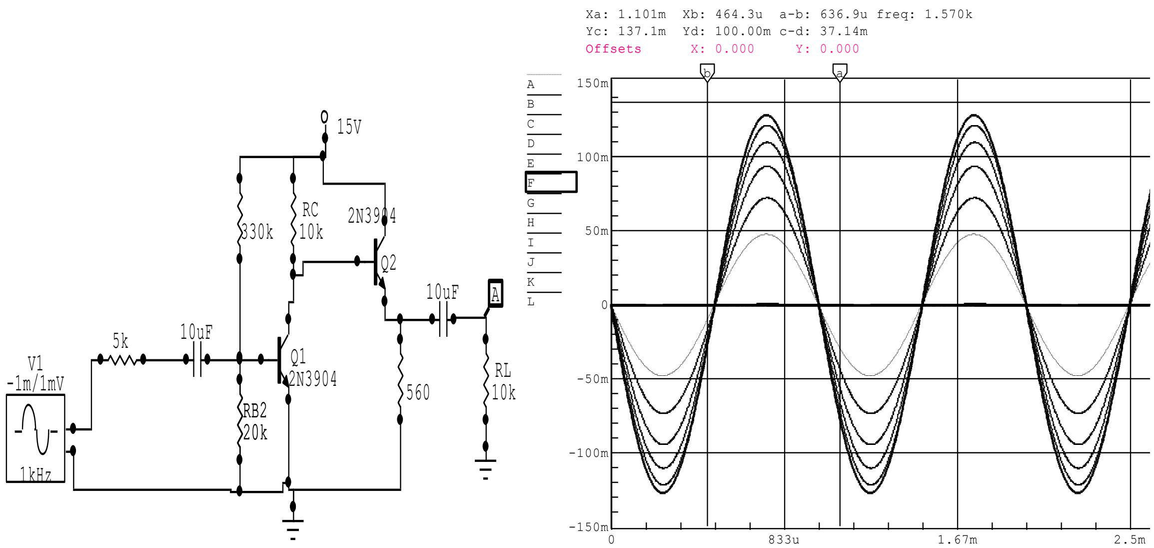


Figure 7 Circuit diagram and transient analysis

Shown in the graph is that when $\boxed{\boldsymbol{R_{B2} = 20k\Omega}}$, the open loop gain is maximum $|A| = \frac{255mV}{2mV} = \boxed{\mathbf{127.5}\frac{\boldsymbol{V}}{\boldsymbol{V}}}$, with 180° phase shift.

1.

To measure DC operating point, open capacitors.

	V_C	V_B	V_E	I_C	I_B	I_E
Q_2	**15V**	**1.9V**	**1.235V**	**2.19mA**	**15.39$\boldsymbol{\mu A}$**	**2.205mA**
Q_1	**1.9V**	**0.654V**	**0V**	**1.295mA**	**10.77$\boldsymbol{\mu A}$**	**1.305mA**

Table 2 DC operating point

Then, use formulas: $g_m = \frac{I_C}{V_T}, , h_{FE}(\beta) = \frac{I_C}{I_B}, r_\pi = \frac{h_{FE}}{g_m}$ to calculate the parameters.

	g_m	h_{FE}	r_π
Q_2	$\mathbf{87.6m\mho}$	**142.3**	**1.624 $\boldsymbol{k\Omega}$**
Q_1	**51.8$\boldsymbol{m\mho}$**	**120.24**	**2.231$\boldsymbol{k\Omega}$**

Table 3 Parameters' values

2.

Measure the open loop frequency response from 10mHz up to 100MegHz to find 3dB frequencies.

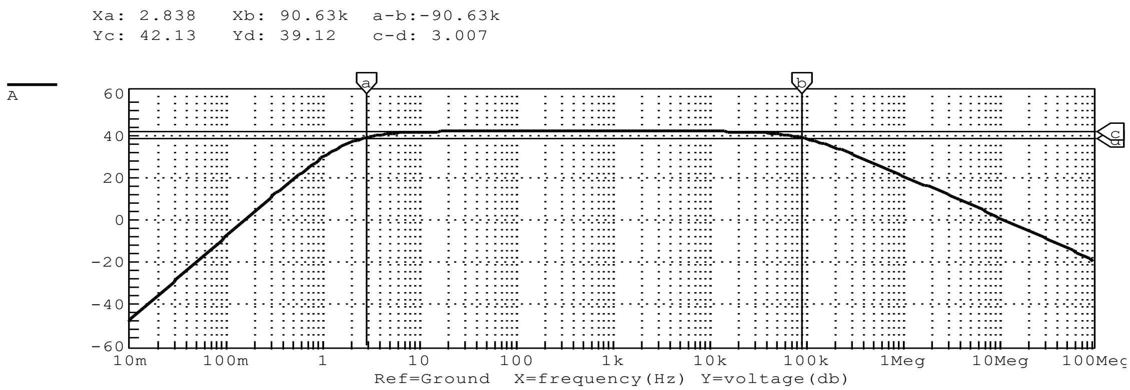


Figure 8 Bode plot of open-loop circuit

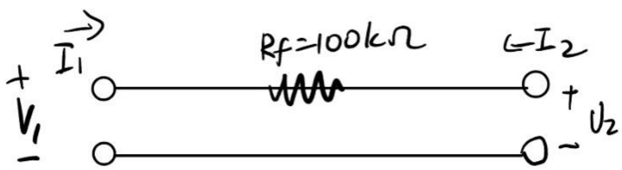
Shown in the amplitude bode plot is $\boxed{\boldsymbol{\omega_{L3dB} = 2.838Hz * 2 * \pi = 17.83\frac{1}{s}}}$, $\boxed{\boldsymbol{\omega_{H3dB} = 90.63kHz * 2 * \pi = 5.69 * 10^5\frac{1}{s}}}$. $A_m = 42.13dB = 127.79\frac{V}{V}$. Since CE amplifier is an inverting amplifier, $\boxed{\boldsymbol{A_m = -127.79\frac{V}{V}}}$.

Then use a test source to measure the input and output resistance, (short source and load resistance respectively)

$\boxed{\boldsymbol{R_{in} = \frac{V_{test}}{I_{test}} = \frac{706.2\mu V}{273.4nA} = 2.583k\Omega}}$. $\boxed{\boldsymbol{R_{out} = \frac{V_{test}}{I_{test}} = \frac{706.2\mu V}{11.27\mu A} = 62.661\Omega}}$.

Calculated:

Since the feedback network samples the output voltage and converts it into a current, $\beta = \frac{I_1}{V_2} = y_{12}$. Use shunt-shunt topology (**y-parameters**).



$$\begin{cases} I_1 = y_{11}V_1 + y_{12}V_2 \\ I_2 = y_{21}V_1 + y_{22}V_2 \end{cases}$$

Get: $y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{100k\Omega} = 1 * 10^{-5}\mho$, $y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = -\frac{1}{R_f} = \boxed{\boldsymbol{-1 * 10^{-5}\mho = \beta}}$, $y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{1}{R_f} = 1 * 10^{-5}\mho$.

Previously, measured $A_m = \frac{V_{out}}{V_{in}} = -127.79\frac{V}{V}$. So for $A' = \frac{V_{out}}{i_{in}} = A_m * R_s = \boxed{\boldsymbol{-638.95}}\boldsymbol{\frac{kV}{A}}$.(Norton circuit is applied to input voltage source and source resistance)

So the feedback loop gain is: $A_f = \frac{A'}{1 + A' * \beta} = \frac{-638.95*10^3}{1 + -638.95*10^3*(-1*10^{-5})} = -86.467\frac{kV}{A}$.

Then, the voltage gain is $\boxed{\boldsymbol{A_{mf} = \frac{A_f}{R_s} = -17.29\frac{V}{V}}}$.

With feedback circuit, the mid-band will extend:

$\omega_{Hf} = \omega_{H3dB} * (1 + A_M\beta) = 5.69 * 10^5 \frac{1}{s} * \left(1 + \left(638.95 \frac{kV}{A}\right) * (-1 * 10^{-5}℧)\right) =$

$\boxed{4.2 * 10^6 \frac{1}{s}} = 669.2kHz.$

$\omega_{Lf} = \frac{\omega_{LdB}}{1+A_M\beta} = \boxed{2.4124 \frac{1}{s}} = 0.384Hz.$

Previously, without feedback, we measured input and output resistance without source and load resistors. To calculate input and output resistance with feedback, we need to first embed source and load resistors and then decrease them both by the factor $\frac{1}{1+A\beta}$ (because of shunt-shunt topology). Then de-embed the source and load resistors. $R_s = 5k\Omega$.

$R_{in} = 2.583k\Omega$, $R_{out} = 62.661\Omega$ (without feedback and source, load resistors)

$\boldsymbol{R_{in_{with\ source\ resistor}} = R_s||R_{in} = 1.703k\Omega}$, $\boldsymbol{R_{out_{with\ source\ resistor}} = R_L||R_{out} = 62.27\Omega}$,

$R_{if} = \frac{R_{in_{with\ source\ resistor}}}{1+A'\beta} = \frac{1.703k\Omega}{1+(-638.95k\Omega)*(-1*10^{-5}℧)} = 230.462\Omega$. **Then we de-embedded** the source resistor to get the final calculated input resistance with feedback and without source resistor.

$R_{if} = R_s||R_{input}$

Get: $\boxed{\boldsymbol{R_{input} = 241.598\Omega}}$

Similar approach to get $\boxed{\boldsymbol{R_{output} = 8.434\Omega}}$

Simulated:

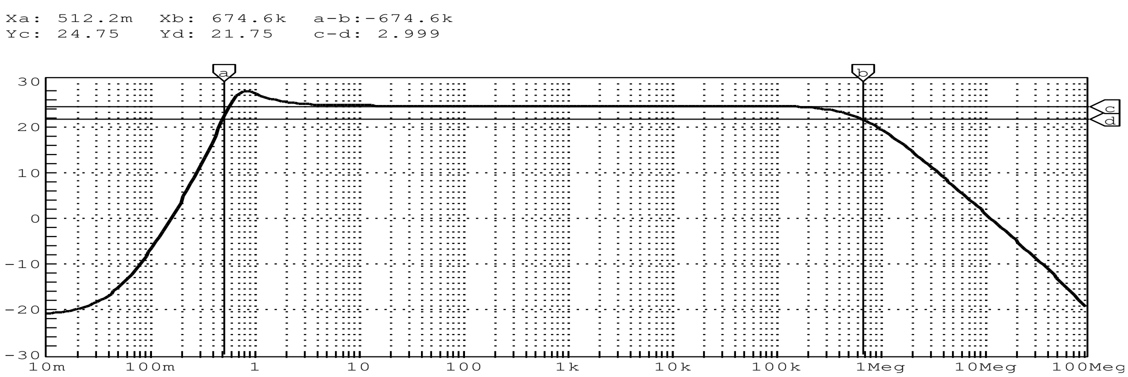


Figure 9 Magnitude bode plot of closed-loop feedback circuit

Plot the magnitude bode plot with feedback circuit to determine midband gain A_{mf} and ω_{Lf} and ω_{Hf}

As shown on the graph: $\omega_{Lf} = 0.512mHz = \boxed{3.21699 \frac{1}{s}}$.

$\omega_{Hf} = 674.6kHz = \boxed{4.238 * 10^6 \frac{1}{s}}$.

$|A_{mf}| = 24.75dB = 17.278$, since CE amplifier is an inverting amplifier.

$\boxed{\boldsymbol{A_{mf} = -17.278 \frac{V}{V}}}$.

Use the same approach as before to measure the input and output resistance:

$\boxed{\boldsymbol{R_{input}} = \frac{V_{test}}{I_{test}} = \frac{706.2\mu V}{2.927\mu A} = \mathbf{241.27\Omega}}$. $\boxed{\boldsymbol{R_{output}} = \frac{V_{test}}{I_{test}} = \frac{706.2\mu V}{82.36\mu A} = \mathbf{8.57455\Omega}}$.

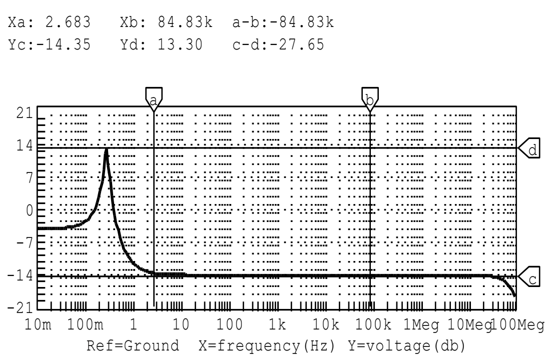
	R_{input}	R_{output}	ω_{H3dB-f}	ω_{L3dB-f}	A_{mf}
Calculated	241.598Ω	8.434Ω	$4.2 * 10^6 \frac{1}{s}$	$2.4124 \frac{1}{s}$	$-17.29 \frac{V}{V}$
Simulated	241.27Ω	8.57455Ω	$4.238 * 10^6 \frac{1}{s}$	$3.21699 \frac{1}{s}$	$-17.278 \frac{V}{V}$
Error	0.1359%	1.639%	0.8966%	25.01%	0.00695%

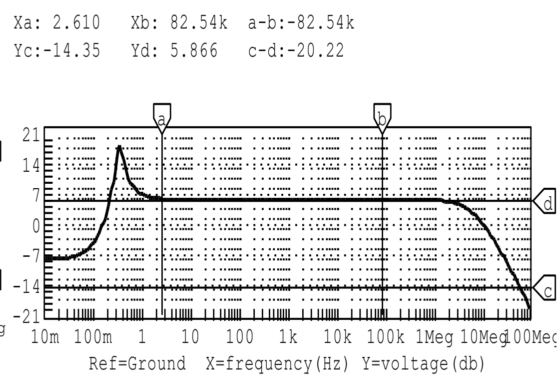
Table 4 Frequency response and input and output impedance

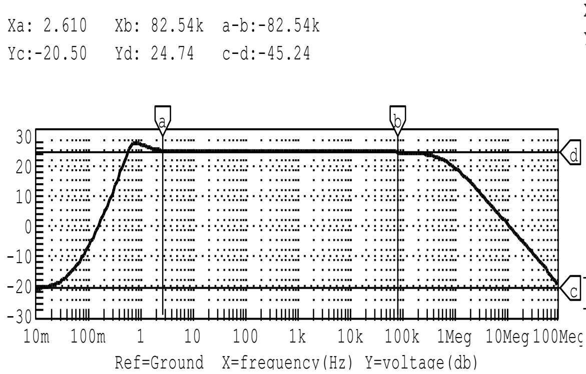
The table shows that the error of ω_{L3dB-f} is large. The bode plot shows that there is an overshoot at low frequency, which makes the simulated ω_{L3dB-f} not close to the calculated value. The difference is because the feedback will bring two poles closer and when they are too close, they will go to imaginary axis. When they start to go to imaginary axis, the real part of poles remains the same so the real part of ω_{L3dB} remains the same. But the calculated ω_{L3dB-f} keeps changing, which causes a difference. However, the other value on the table is close.

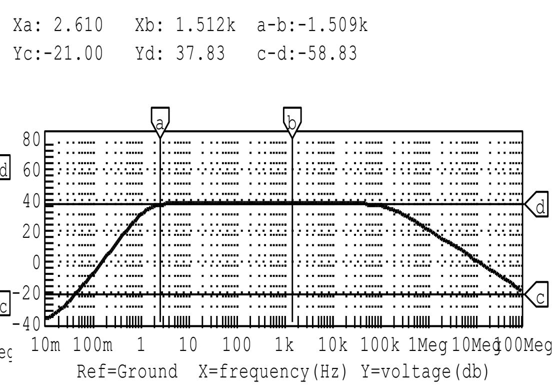
3.

Measure the closed-loop frequency response over the same range of frequencies, for $R_f = 1k\Omega, 10k\Omega, 100k\Omega, 1M\Omega, 10M\Omega$.









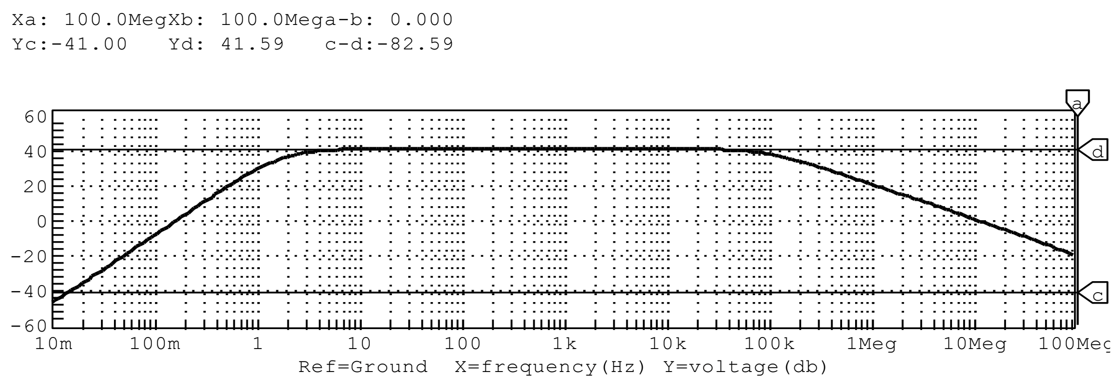


Figure 10The amplitude response of 1k,10k,100k,1M,10M Rf, respectively

The plots show the $A_{m(dB)}$ for different R_f. $A_{mf} = 10^{\frac{A_{m(dB)}}{20}} \frac{V}{V}$. Use same approach above to calculate $A_f = (A_{mf}) * R_s \frac{V}{A} (R_s\ is\ 5k\Omega)$. Also, calculated before, the open loop gain $A' = -638.95 \frac{kV}{A}$. Use formula :$\boldsymbol{A_f = \frac{A'}{1 + A' * \beta}}$ to get $\beta_{simulated}$.

Also, as mentioned before, $\beta_{calculated} = -\frac{1}{R_f}$.

R_f	$A_{m(dB)}$	$A_{mf}\ \frac{V}{V}$	$A_f\ \frac{V}{A}$	$\beta_{simulated}$ ℧	$\beta_{calculated}$ ℧	Error
$1k\Omega$	-14.35	-0.1916	-958.23	$-1.042 * 10^{-3}$	$-1 * 10^{-3}$	4%
$10k\Omega$	5.866	-1.9647	-9823.58	$-1.0023 * 10^{-4}$	$-1 * 10^{-4}$	0.229%
$100k\Omega$	24.74	-17.2584	-86291.89	$-1.0024 * 10^{-5}$	$-1 * 10^{-5}$	0.239%
$1M\Omega$	37.83	-77.8923	-389466.4	$-1.0025 * 10^{-6}$	$-1 * 10^{-6}$	0.249%
$10M\Omega$	41.59	-120.088	-600440.5	$-1.0038 * 10^{-7}$	$-1 * 10^{-7}$	0.379%

Table 5 Measured and calculated feedback factor

The error is small in all cases. And when β is smaller the overshoot is smaller(Bigger β will bring two poles closer to bring overshoot).

4.

Recall that $R_{in_{with\ source\ resistor}} = 1.703k\Omega$ $R_{out_{with\ load\ resistor}} = 62.27\Omega$. $\beta_{calculated\ in\ part\ 3} = -\frac{1}{R_f}$, $A_{in\ part\ 2} = -638.95 \frac{kV}{A}$.

Determine both the input and the output resistance of your amplifier, both at 1kHz, for $R_f = 10k\Omega, 100k\Omega, 1M\Omega$. Use the same approach, test source. So the amount of feedback is $(1 + A\beta_i) = \frac{R_{in_{with\ source\ resistor}}}{R_{input}||R_s}$, $(1 + A\beta_o) = \frac{R_{out_{with\ load\ resistor}}}{R_{output}||R_s}$. And the **predicted amount of feedback is $\boldsymbol{1 + A_{in\ part\ 2} * \beta_{calculated\ in\ part\ 3}}$.**

R_f	R_{input}	R_{output}	$1 + A\beta_i$	$1 + A\beta_o$	Predicted feedback	Error 1	Error 2
$10k\Omega$	26.489Ω	1.12992Ω	64.6314	55.456	64.895	0.408%	17.02%
$100k\Omega$	241.27Ω	8.57455Ω	7.399	7.3079	7.3895	0.128%	1.118%
$1M\Omega$	1308.5Ω	38.2Ω	1.642	1.64	1.63895	0.186%	0.064%

Table 6 Measured and calculated the amount of feedback

The error 1 and error 2 compare $1 + A\beta_i$ and $1 + A\beta_o$ with predicted feedback respectively. The error is small (except for when $R_f = 10k\Omega$, Error 2 is large). As R_f gets larger, β is smaller, A_f is close to A', the error gets smaller and the calculated amount of feedback is almost consistent with predicted feedback(get closer to 1).

5.

In the course note we know that $\frac{dA_f}{A_f} * 100 = \frac{1}{1+A*\beta}\frac{dA}{A} * 100$ and $1 + A\beta$ is the "de-sensitivity factor". So $1 + A\beta = \frac{dA}{A} * \frac{A_f}{dA_f} = \boxed{\frac{\Delta A}{A} * \frac{A_f}{\Delta A_f}}$. Change R_C from $9.9k\Omega, 10k\Omega, to$

$10.1k\Omega.A_f$ is when $R_f = 100k\Omega$, and A is open loop gain, when $R_f = \infty k\Omega$.

Similar as in the previous question: $R_s = 5000\Omega, (A_f \frac{V}{V}) * R_s = A_f \frac{kV}{A}$. Run circuit simulation and measure the value below.

R_C	A_f	$A_f \frac{kV}{A}$	$\Delta A_f \frac{kV}{A}$	A	ΔA	$1 + A\beta$
$9.9k\Omega$	-17.24$\frac{V}{V}$	86.20	\	-127.0$\frac{V}{V}$	\	\
$10.0k\Omega$	-17.25$\frac{V}{V}$	86.25	0.5	-127.6 $\frac{V}{V}$	0.6$\frac{V}{V}$	9.463
$10.1k\Omega$	-17.26$\frac{V}{V}$	86.3	0.5	-128.2$\frac{V}{V}$	0.6$\frac{V}{V}$	8.078

Table 7 De-sensitivity factor

Average the two $1 + A\beta$ to get $\frac{9.463+8.078}{2} = \boxed{\mathbf{8.77}}$. Expected value using the measured values of A and β from parts 2 and 3 above gets (1+Aβ)=**7.3895** ($R_f = 100k\Omega$). The error is 15.74%. The error is acceptable and may due to simulation error.

Report:

The graphs in "3" show that when R_f gets smaller, β gets larger, which will bring two poles closer causing the overshoot. At the overshoot, the gain is larger than the midband gain. When the two poles are close, they may boost the resonance causing larger gain.

Conclusion:

The project demonstrated the principles of circuit design and analysis. The Butterworth filter and phase-shift oscillator results closely matched theoretical predictions, showcasing the accuracy of derived models. The feedback circuit analysis highlighted the effects of feedback on gain and bandwidth. Overall, the project reinforced critical concepts in circuit behavior and feedback systems

References:

EECE 356 Course notes.

A. Sedra and K. Smith, "Microelectronic Circuits," 5th, 6th, or 7th Ed., Oxford University Press, New York.